

B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2011

First Semester

MATRICES, CALCULUS AND LAPLACE TRANSFORMS

(Complementary Course to B.C.A.)

Time : Three Hours

Maximum Weight : 25

Part A (Objective Type Questions.

*Answer all questions.**Each Bunch of four questions has weight 1.*

I. 1 Define a skew-symmetric matrix with an example.

2 If $A = \begin{bmatrix} 3 & 6 \\ 7 & 2 \end{bmatrix}$, find A^{-1} .

3 Show that is idempotent.

4 Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$ is idempotent.II. 5 Evaluate: $\lim_{x \rightarrow -5} \frac{x^2 + 3x - 10}{x + 5}$.6 Find the equation of the tangent line at (6, 4) on the graph of the function $f(x) = \frac{8}{\sqrt{x-2}}$.7 If $y = \frac{x^4}{2} - \frac{3}{2}x^2 - x$, find $\frac{d^2y}{dx^2}$.

8 Explain local and global extrema.

Turn over

III. 9 Form the partial differential equation by eliminating the arbitrary constants from

$$Z = f(x^2 - y^2).$$

10 Define the order and degree of a partial differential equation.

11 Write the Lagrange's linear equation of the first order.

12 Explain the term particular Integral for a partial differential equation.

IV. 13 If $L\{F(t)\} = f(s)$, how to find $L\left\{\frac{F(t)}{t}\right\}$.

14 Find $L^{-1}\left\{\frac{1}{s^2 - 1}\right\}$.

15 State the Convolution theorem of Laplace transforms.

16 Find $L\{e^{-3t} \cos 5t\}$.

(4 × 1 = 4)

Part B (Short Answer Questions)

Answer any five questions.

Each question has weight 1.

17. Explain the normal form of a matrix.

18. Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ by reducing it to the Canonical form.

19. Prove that $\frac{d}{dx}(\sin x) = \cos x$.

20. State the Mean Value theorem.

21. Solve $\frac{\partial^2 z}{\partial x^2} = xy$.

22. Find the differential equation of all spheres whose centres lie on the z-axis.

23. Show that $L\{t \sin at\} = \frac{2as}{(s^2 + a^2)^2}$.

24. Find $L^{-1}\left\{\frac{s+3}{s^2 - 4s + 13}\right\}$.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any four questions.

Each question has weight 2.

25. If A is a non-singular square matrix then prove the following :—

(i) A^T is invertible.

(ii) $(A^T)^{-1} = (A^{-1})^T$.

26. Find the rank of A where $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ by reducing it to the normal form.

27. State the Rolle's theorem. Give its geometrical meaning.

28. A dynamic blast blows a heavy rock straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after t seconds.

(a) How high does the rock go ?

(b) What is the velocity and speed of the rock when it is 256 ft above the ground on the way up ? One the way down ?

29. Use Lagrange method to solve $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.

30. Use Convolution theorem to evaluate $L^{-1} \left(\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right)$.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any two questions.

Each question has weight 4.

31. Test for consistency and solve the system of linear equations

$$x + 2y + z = 2$$

$$3x + y - 2z = 1$$

$$4x - 3y - z = 3$$

$$2x + 4y + 2z = 4$$

Turn over

32. (i) If $x^5 y^3 = (x + y)^8$, show that $y = x \frac{dy}{dx}$.

(ii) Evaluate $\sqrt[3]{25}$ approximately by use of differentials.

33. (i) If $F(t)$ is a periodic function with period T , Prove that $L\{F(t)\} = \frac{\int_0^T e^{-st} F(t) dt}{1 - e^{-sT}}$.

(ii) Find $L\{F(t)\}$ if $F(t) = \begin{cases} 1, & 0 < t \leq 1 \\ t, & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$.

(2 × 4 = 8)