

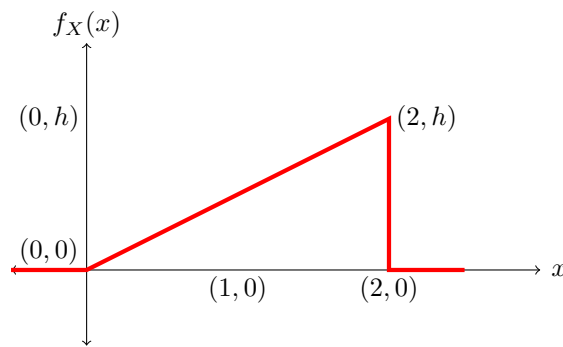
MA2040: Probability, Statistics and Stochastic Processes

Problem Set-III

Sivaram Ambikasaran

March 6, 2019

1. If X_1, X_2, \dots, X_n are independent random variables having the same probability density function $f_X(x)$, what is the probability density function for the random variable $Y = \min\{X_1, X_2, \dots, X_n\}$?
2. A random variable X has a probability density function as shown below.



- (a) Determine h
 - (b) Determine the cumulative distribution function
 - (c) Compute the mean
 - (d) Compute the variance
 - (e) Determine the probability that $X \in (1, 2)$.
3. The median m of a probability density function is defined as the value of m such that

$$\int_{-\infty}^m f(x)dx = \int_m^{\infty} f(x)dx = 1/2$$

Essentially, the median splits the distribution into two equal halves. Prove that the median is the best predictor if one wants to minimize the expected value of the absolute error, i.e., $\mathbb{E}(|X - c|)$ is minimized when c is the median of the underlying distribution.

4. Let X be a random variable, whose pdf is given by

$$f_X(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ xe^{-x^2/2} & \text{if } x > 0 \end{cases}$$

Find the pdf for the random variable $Y = X^2$.

5. Let X be a uniform random variable on the interval $[0, 1]$. Consider the random variable $Y = g(X)$, where

$$g(x) = \begin{cases} 1 & \text{if } x \leq 1/3 \\ 2 & \text{else} \end{cases}$$

Find the probability mass function of Y and compute its expected value.

6. Show the expected value of a random variable X can also be obtained as

$$\mathbb{E}(X) = \int_0^\infty \mathbb{P}(X > x) dx - \int_0^\infty \mathbb{P}(X < -x) dx$$

7. A defective coin minting machine produces coins whose probability of heads is a random variable Y with PDF

$$f_Y(y) = \begin{cases} y \exp(y) & \text{if } y \in [0, 1] \\ 0 & \text{otherwise} \end{cases}$$

A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent.

- Find the probability that a coin toss results in head.
 - Given that a coin toss resulted in heads, find the conditional PDF of Y .
 - Given that the first coin toss resulted in heads, find the conditional probability of heads on the next toss.
8. Let the random variables X and Y have a joint PDF, which is uniform over the triangles with vertices $(0, 0)$, $(0, 1)$ and $(1, 0)$.
- Find the joint PDF of X and Y .
 - Find the marginal PDFs.
 - Find the conditional PDFs.
9. Chennai's temperature is modeled as a normal random variable with a mean temperature of 34°C and a standard deviation of 5°C . What is the probability that the temperature at a randomly chosen time will exceed 45°C ?
10. A surface is ruled with parallel lines, which are at a distance d from each other. Suppose that we throw a needle of length l on the surface at random. What is the probability that the needle will intersect one of the lines? (NOTE: You will need to treat the case $d < l$ and $d > l$ separately.)
11. Consider two continuous random variables Y and Z and a random variable X that is equal to Y with a probability p and equals Z with a probability $1 - p$. Obtain the pdf of X in terms of the pdf's of Y and Z .