MA2040: Probability, Statistics and Stochastic Processes
Problem Set-VIII

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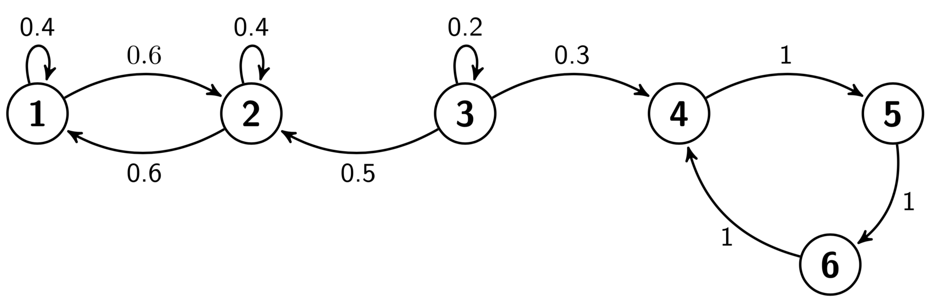
April 10, 2019

1. A mouse moves along a tiled corridor with $2m$ tiles, where $m > 1$. From each tile $i \neq 1, 2m$, it moves to either tile $i-1$ or $i+1$ with equal probability. From tile 1 or tile $2m$, it moves to tile 2 or $2m-1$, respectively, with probability 1. Each time the mouse moves to a tile $i \neq m$ or $i > m$, an electronic device outputs a signal L or R respectively. Can the generated sequence of signals L and R be described as a Markov chain with states L and R?

2. A spider and a fly move along a straight line in unit increments. The spider always moves towards the fly by one unit. The fly moves towards the spider by one unit with probability 0.3, moves away from the spider by one unit with probability 0.3, and stays in place with probability 0.4. The initial distance between the spider and the fly is an integer. When the spider and the fly land in the same position, the spider captures the fly.
   - Construct a Markov chain that describes the relative location of the spider and fly.
   - Identify the transient and recurrent states.

3. Consider a Markov chain with states in $\{1, 2, \ldots, 9\}$ and the following transition probabilities: $p_{1,2} = p_{1,7} = 1/2$ and $p_{i,i+1} = 1$ for $i \neq 1, 6, 9$ and $p_{6,1} = p_{9,1} = 1$. Is the recurrent class of the chain periodic or not?

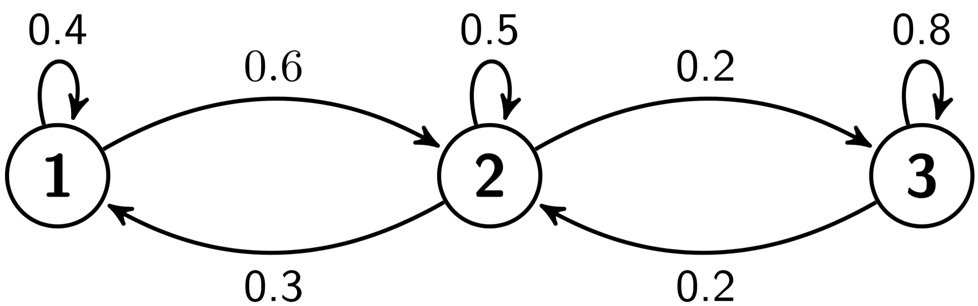
4. The Markov chain shown below is in state 3 immediately before the first trial.



   - Indicate which states, if any, are recurrent, transient, and periodic.
   - Find the probability that the process is in state 3 after n trials.
   - Find the expected number of trials up to and including the trial on which the process leaves state 3.
   - Find the probability that the process never enters state 1.
   - Find the probability that the process is in state 4 after 10 trials.

- Given that the process is in state 4 after 10 trials, find the probability that the process was in state 4 after the first trial.

5. Consider the Markov chain below. Let us refer to a transition that results in a state with a higher index as a birth (and a lower index as death). Calculate the following quantitites, assuming that when we start observing the chain, it is already in steady-state.



- For each state i, compute the probability that the current state is i.
- The probability that the first transition we observe is a birth.
- The probability that the first change of state we observe is a birth.
- The conditional probability that the process was in state 2 before the first transition that we observe, given that this transition was a birth.
- The conditional probability that the process was in state 2 before the first change of state that we observe, given that this change of state was a birth.
- The conditional probability that the first observed transition is a birth given that it resulted in a change of state.
- The conditional probability that the first observed transition leads to state 2, given that it resulted in a change of state.