MA2040: Probability, Statistics and Stochastic Processes
Problem Set-VII

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1. Each of n packages is loaded independently onto either a red truck (with probability p) or onto a green truck (with a probability $1 - p$). Let R be the total number of items selected for the red truck and let G be the total number of items selected for the green truck.

   - Determine the PMF, expected value and variance of the random variable R.
   - Evaluate the probability that the first item to be loaded ends up being the only one on its truck.
   - Evaluate the probability that at least one truck ends up with a total of exactly one package.
   - Evaluate the expected value and the variance of the difference R G.
   - Assume $n \geq 2$. Given that both of the first two packages to be loaded go onto the red truck, find the conditional expectation, variance and PMF of the random variable R.

2. Hilary fails quizzes with probability 1/4, independent of other quizzes.

   - What is the probability that Hilary fails exactly two of the next six quizzes?
   - What is the expected number of quizzes that Hilary will pass before she has failed three times?
   - What is the probability that the second and third time Hilary fails a quiz will occur when she takes her eighth and ninth quizzes, respectively?
   - What is the probability that Hilary fails two quizzes in a row before she passes two quizzes in a row?

3. Consider a Bernoulli process with probability of success in each trial equal to p.

   - Relate the number of failures before the rth success (sometimes called a negative binomial random variable) to a Pascal random variable and derive its PMF.
   - Find the expected value and variance of the number of failures before the rth success.
   - Obtain an expression for the probability that the ith failure occurs before the rth success.

4. A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

   - If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.
   - Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.
   - Find the probability that no trains arrive in the first two days, but 4 trains arrive on the 4th day.
   - Find the probability that it takes 2 days for the 5th train to arrive at the bridge.

5. A store opens at $t = 0$ and potential customers arrive in a Poisson manner at an average arrival rate of λ potential customers per hour. As long as the store is open, and independently of all other events, each particular potential customer becomes an actual customer with probability p. The store closes as soon as ten customers have arrived.

- What is the probability that exactly three of the first five potential customers become actual customers?
- What is the probability that the fifth potential customer to arrive becomes the third actual customer?
- What is the PDF and expected value for L, the duration of the interval from store opening to store closing?
- Given only that exactly three of the first five potential customers became actual customers, what is the conditional expected value of the total time the store is open?