MA 2040 $\boxed{A}$ **Probability, Statistics and Stochastic Processes** **Max. Marks 20**

Duration 50 minutes **Quiz-II** **March 25, 2019**

Student Name ______________________ **Student Roll number** ______________

Instructor Name ______________________

1. Random variables X and Y have a joint probability density function that is uniform on the disc with centre as origin and radius 2.

   - (1 point) The marginal pdf of X, i.e., $f_X(x)$ is ________

     Answer: $\boxed{f_X(x) = \dfrac{\sqrt{4-x^2}}{2\pi}}, \quad x \in (-2, 2)$

   - (1 point) The conditional pdf of X given $Y = y$, i.e., $f_{X|Y}(x \mid y)$ is ________

     Answer: $\boxed{f_{X|Y}(x \mid y) = \dfrac{1}{2\sqrt{4-y^2}}, \quad x \in \left(-\sqrt{4-y^2}, \sqrt{4-y^2}\right)}$

   - (1 point) The $\mathrm{Cov}(X, Y)$ is ________

     Answer: $\boxed{0}$

   - (1 point) The value of $f_{XY}(0,0) - f_X(0)\, f_Y(0)$ is ________

     Answer: $\boxed{\dfrac{1}{4\pi} - \dfrac{1}{\pi^2} = -0.0217}$

2. $\{X_i\}_{i=1}^{n}$ are drawn from a PDF given by

$$f_X(x) = \frac{a}{2} \exp\left(-a\,|x|\right), \quad \forall x \in \mathbb{R}$$

   where $a > 0$.

   - (2 points) The moment generating function $M_X(s)$ for the random variable X is ________

     Answer: $\boxed{M_X(s) = \dfrac{a^2}{a^2 - s^2}, \quad s \in (-a, a)}$

   - (1 point) The domain of the variable s in the moment generating function $M_X(s)$ is ________

     Answer: $\boxed{s \in (-a, a)}$

   - (2 points) The maximum likelihood estimator for a is ________

     Answer: $\boxed{\hat{a} = \dfrac{n}{|X_1| + |X_2| + \cdots + |X_n|}}$

3. A manufacturer measures the diameter of n pipe fittings by drawing them at random. Assume that the diameter is normally distributed with standard deviation of 10^{-1} cm.

   - (2 points) The sample mean for $n = 20$ is 2cm; 95% confidence interval for mean is ________

     Answer: $\boxed{(1.956\text{cm}, 2.044\text{cm})}$

- (1 point) Value of n for a 99% confidence interval of length 10^{-2}cm is __________

  Answer: $\boxed{n = 2642}$

4. Let X be the annual household income in lakhs. It is given that the mean annual income of a household is 1 lakh.

   - (1 point) Upper bound on the $\mathbb{P}(X \geq 5)$ (**using Markov inequality**) is __________

     Solution: $\boxed{\mathbb{P}(X \geq 5) \leq 0.2}$

   - (2 points) If standard deviation is 0.4 lakhs, $\mathbb{P}(X \geq 5)$ (**using Chebyshev inequality**) is __________

     Solution: $\boxed{\mathbb{P}(X \geq 5) \leq 0.01}$

5. The processing times (in minutes) of different parts are independent random variables, uniformly distributed in $[1, 6]$ and 100 parts need to be processed. (**Use Central Limit Theorem**)

   - (2 points) The time taken to guarantee 98% probability for the above task is __________

     Answer: $\boxed{T = 379.589 \text{ minutes}}$

   - (1 point) The probability that the total process time is within 6 hours is __________

     Answer: $\boxed{\mathbb{P}(T \leq 6 \text{ hours}) = 0.755}$

6. Probability density function of a random variable X is $f_X(x) = \lambda \exp(-\lambda x)$ with $x \geq 0$. Let $Y = \sqrt{X}$.

   - (1 point) The probability density function for Y, i.e., $f_Y(y)$ is __________

     Answer: $\boxed{f_Y(y) = 2\lambda y \exp\left(-\lambda y^2\right), \quad y \geq 0}$

   - (1 point) $\mathbb{E}\left(Y^2\right)$ is __________

     Answer: $\boxed{\mathbb{E}\left(Y^2\right) = \mathbb{E}(X) = \frac{1}{\lambda}}$