

# MA2040: Probability, Statistics and Stochastic Processes

## Problem Set-IV

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March 14, 2019

1. Let  $X$  and  $Y$  have joint pdf  $f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$ 
  - Are  $X$  and  $Y$  independent?
  - Find  $\mathbb{E}(Y \mid X > 2)$
  - Find  $\mathbb{P}(X > Y)$
2. Let  $X \sim \text{Uniform}(1, 2)$  and given  $X = x$ ,  $Y$  follows an exponential distribution with  $Y \sim \text{EXP}(\lambda = x)$ . Find the covariance of  $X$  and  $Y$ .
3. Let  $X$  and  $Y$  be independent standard normal random variables. Find covariance of  $Z$  and  $W$ , where  $Z = 1 + X + XY^2$  and  $W = 1 + X$ .
4. A fair die is rolled  $n$  times. Let  $X$  be the number of 1's and  $Y$  be the number of 2's. Find the correlation coefficient  $\rho(X, Y)$ .
5. Let 32 people sit around a round table. Each person tosses a fair coin. Anyone whose outcome is different from both his neighbors is taken out from the group. If  $X$  is the total number of such persons, find variance of  $X$ .
6. The moment generating function of a random variable  $X$  is given by

$$M_X(s) = \frac{2}{2-s}, \quad \forall t \in (-\infty, 2)$$

Find the distribution of  $X$ .

7. Let  $X \sim \text{Binomial}(n, p)$  and  $Y \sim \text{Binomial}(m, p)$  be independent random variables. Show that  $X+Y \sim \text{Binomial}(m+n, p)$ .
8. Show that the function  $e^{-s}$  is a moment generating function.
9. Show that if  $M(t)$  is a moment generating function, so is  $M(ct)$ .
10. Show that if  $M(t)$  is a moment generating function,  $cM(t)$  cannot be a moment generating function for  $c \neq 1$ .
11. Show that if  $M(t)$  is a moment generating function, then  $e^{-t}M(t)$  is also a moment generating function.
12. The moment generating function of a random variable  $X$  is

$$M_X(t) = \frac{e^{-2t}}{6} + \frac{e^{-t}}{3} + \frac{e^t}{4} + \frac{e^{2t}}{4}$$

Show that  $P(|X| \leq 1) = \frac{7}{12}$ .