MA2040: Probability, Statistics and Stochastic Processes
Problem Set-IV

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1. Let X and Y have joint pdf $f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$
   - Are X and Y independent?
   - Find $\mathbb{E}(Y \mid X > 2)$
   - Find $\mathbb{P}(X > Y)$
2. Let $X \sim \text{Uniform}(1,2)$ and given $X = x$, Y follows an exponential distribution with $Y \sim \text{EXP}(\lambda = x)$. Find the covariance of X and Y.
3. Let X and Y be independent standard normal random variables. Find covariance of Z and W, where $Z = 1 + X + XY^2$ and $W = 1 + X$.
4. A fair die is rolled n times. Let X be the number of 1's and Y be the number of 2's. Find the correlation coefficient $\rho(X,Y)$.
5. Let 32 people sit around a round table. Each person tosses a fair coin. Anyone whose outcome is different from both his neighbors is taken out from the group. If X is the total number of such persons, find variance of X.
6. The moment generating function of a random variable X is given by
$$M_X(s) = \frac{2}{2-s}, \quad \forall s \in (-\infty, 2)$$
   Find the distribution of X.
7. Let $X \sim \text{Binomial}(n,p)$ and $Y \sim \text{Binomial}(m,p)$ be independent random variables. Show that $X+Y \sim \text{Binomial}(m+n,p)$.
8. Show that the function e^{-s} is a moment generating function.
9. Show that if $M(s)$ is a moment generating function, so is $M(cs)$.
10. Show that if $M(s)$ is a moment generating function, $cM(s)$ cannot be a moment generating function for $c \neq 1$.
11. Show that if $M(s)$ is a moment generating function, then $e^{-s}M(s)$ is also a moment generating function.
12. The moment generating function of a random variable X is
$$M_X(s) = \frac{e^{-2s}}{6} + \frac{e^{-s}}{3} + \frac{e^{s}}{4} + \frac{e^{2s}}{4}$$
   Show that $P(|X| \leq 1) = \dfrac{7}{12}$.