ASSIGNMENT - 4

ML & MAP

→ Consider BPSK modulation

→ x → Bits which are been send

y → Received Bits

→ By looking at y, Receiver has to determine which x has been send

ie $\cancel{P(y/x) \propto P(x/y) \cdot P(y)}$ [Bayes's Rule]

Posterior   Likelihood   Prior

ie

$$\underset{\text{Posterior}}{P_{X/Y}(x/y)} \propto \underset{\text{Likelihood}}{P_{Y/X}(y/x)} \cdot \underset{\text{Prior}}{P_X(x)}$$

→ In this experiment we ~~the~~ take ① $y = x + n$
② $y = hx + n$

→ When Priors are equal (ie uniformly dxn) the Maximum likelihood = MAP

→ ~~We~~ For a receiver we need :

$$\arg\max_{x} \left[P(y/x) \cdot P(x) \right]$$

$\Rightarrow$ If $P(x/y=0) > P(x/y=1)$ Then $x = 0$

$P(y/x=0)\,P(x=0) > P(y/x=1)\,P(x=1)$ Then 0 is sent.

~~Then y~~

ML Detection

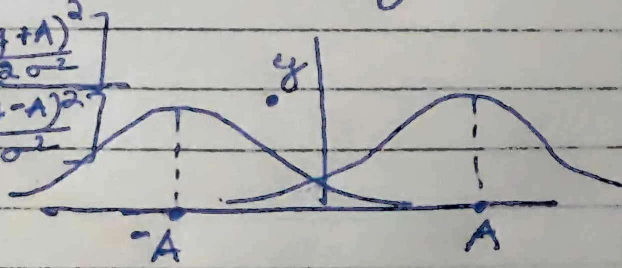
$P(x=0) = P(x=1)$

$$\Rightarrow \frac{P(y/x=0)}{P(y/x=1)} \geq 1 \Rightarrow 0 \text{ is sent} \Rightarrow H_0$$

$$\frac{P(y/x=0)}{P(y/x=1)} < 1 \Rightarrow 1 \text{ is sent} \Rightarrow H_1$$

If we assume Gaussian dxn for the conditional density

$$\frac{P(y/x=0)}{P(y/x=1)} = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(y+A)^2}{2\sigma^2}\right]}{\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(y-A)^2}{2\sigma^2}\right]}$$



Comparing terms inside the exponent

$$\left.\begin{aligned} \text{For } H_0 &: (y+A)^2 \leq (y-A)^2 \\ H_1 &: (y+A)^2 > (y-A)^2 \end{aligned}\right\} \text{Min distance decoding}$$

MAP :

$$\frac{P(y/x=0)}{P(y/x=1)} \geq \frac{P(x=1)}{P(x=0)} \Rightarrow H_0$$

$$\frac{\exp\left(-\frac{(y+A)^2}{2\sigma^2}\right)}{\exp\left(-\frac{(y-A)^2}{2\sigma^2}\right)} \geq \frac{P(x=1)}{P(x=0)}$$

$$\exp\left[-\frac{[(y+A)^2-(y-A)^2]}{2\sigma^2}\right] \geq \frac{P[x=1]}{P[x=0]}$$

Taking log of likelihood ratio :

$$\left\{ [(y+A)^2-(y-A)^2] \leq -2\sigma^2 \cancel{\frac{P[x=1]}{P[x=0]}} \ln \left[\frac{P[x=1]}{P[x=0]}\right] \right\}$$

~~H_0~~ $y = -A + N$ for H_0

& $y = A + N$ for H_1

$\sigma^2 \Rightarrow Var(y) = Var(N)$

$\Rightarrow$ We are taking $A = 1$ in our experiment

Case 1:

$P(x=0) = 0.1$
$P(x=1) = 0.9$

$\Rightarrow \left[(y+1)^2 - (y-1)^2\right] \leq -2\sigma^2 \ln(9)$ $\sigma^2 \rightarrow$ Var of noise

$[4y] \leq -2\sigma^2 \ln(9)$

Case 2:

$P(x=0) = 0.2$
$P(x=1) = 0.8$

$[4y] \leq -2\sigma^2 \ln(4)$

Case 3:

$P(x=0) = 0.3$
$P(x=1) = 0.7$

$[4y] \leq -2\sigma^2 \ln(7/3)$