TASK2

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1 Second task

For building circuit, RY and CX gates are used as shown in figure:1.

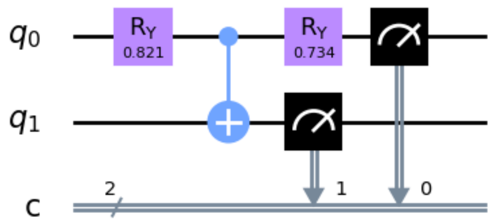

Figure 1: *Circuit*

If x,y are parameters(chosen to be random initially) of the first and second RY gates in figure:1 respectively and starting with $|00\rangle$, output state can be written as shown in (1).Which is a general state for two qubit system(apart from complex phase factors).

$$|\psi\rangle = cos\frac{x}{2}cos\frac{y}{2}|00\rangle - sin\frac{x}{2}sin\frac{y}{2}|01\rangle + cos\frac{x}{2}sin\frac{y}{2}|10\rangle + sin\frac{x}{2}cos\frac{y}{2}|11\rangle \tag{1}$$

Cost function is as shown in equation (2) and appropriate derivatives are formed to perform Gradient descent technique.

$$C = \left(\frac{1}{2} - (cos\frac{x}{2}sin\frac{y}{2})^2\right)^2 + \left(\frac{1}{2} - (sin\frac{x}{2}sin\frac{y}{2})^2\right)^2 \tag{2}$$

Measurements are taken after giving each gate 5 % depolarizing probability of error. Figure 2,3,4,5 show measurement result for shots = 1,10,100,1000 respectively.

2 Bonus task

Two ways are possible to produce the required state($|\psi 1\rangle = \frac{|01\rangle+|10\rangle}{\sqrt{2}}$).First way is to build the following cost function obtained by setting coefficients of $|01\rangle$ and$|10\rangle$ in (1) to $\frac{1}{\sqrt{2}}$ and proceed according to normal gradient descent method

$$C1 = \left(\frac{1}{\sqrt{2}} + sin\frac{x}{2}sin\frac{y}{2}\right)^2 + \left(\frac{1}{\sqrt{2}} - cos\frac{x}{2}sin\frac{y}{2}\right)^2 \tag{3}$$

OTHER way is, it can be understood that we can get only one among $|\psi 1\rangle = \frac{|01\rangle+|10\rangle}{\sqrt{2}} and\, |\psi 2\rangle = \frac{|01\rangle-|10\rangle}{\sqrt{2}}$ from Second task because RY and CX matrices have only real entries, Since $|\psi 1\rangle$, $|\psi 2\rangle$ are orthogonal they can be distinguished.

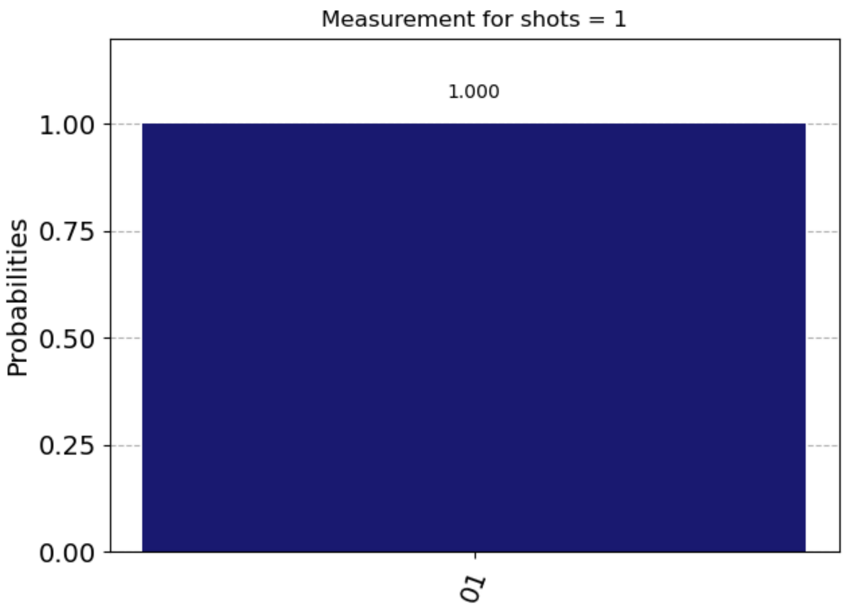

Figure 2: $\#of shots = 1$

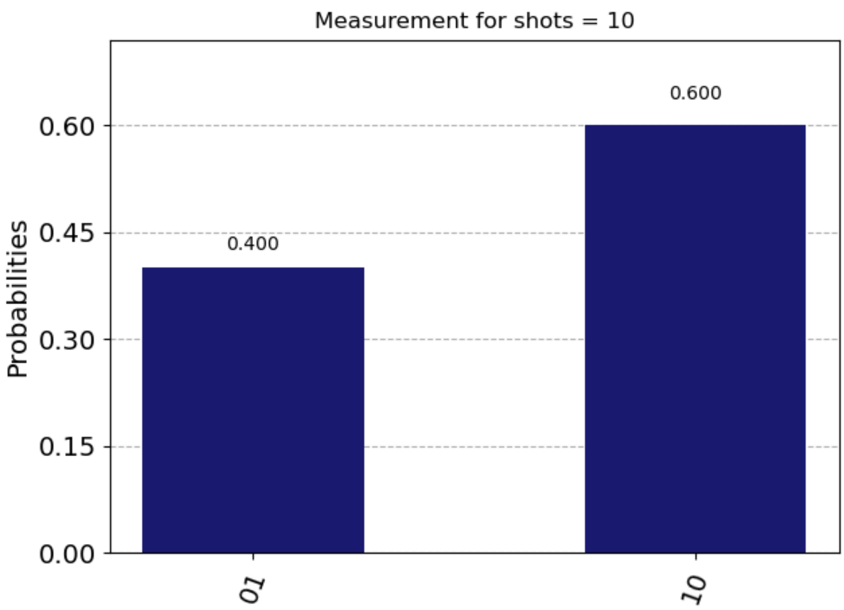

Figure 3: $\#of shots = 10$

If $RY(\frac{\pi}{2})$ is acted at output of circuit in Figure:1 on two qubits (before measurement) with parameters of initial RY gates as $x(=\alpha)$,$y(=\beta)$ obtained from second task ,we get $|\phi1\rangle = \frac{|00\rangle - |11\rangle}{2}$ for $|\psi1\rangle$ and $|\phi2\rangle = \frac{|01\rangle - |10\rangle}{2}$ for $|\psi2\rangle$. So after $SOME^{\dagger}$ shots of measurement we can easily distinguish both the states and if we get $|\phi1\rangle$ we arrived at correct parameters , else we can change α to $-\alpha$ to get $|\psi1\rangle$.

†SOME shots of measurement are required because we have noise, otherwise measurement of one qubit after applying $RY(\frac{\pi}{2})$ on both qubits, yields only $|00\rangle$ or $|11\rangle$ for $|\psi1\rangle$ and $|01\rangle$ or $|10\rangle$ for $|\psi2\rangle$ which can easily be distinguished by looking at output registers.

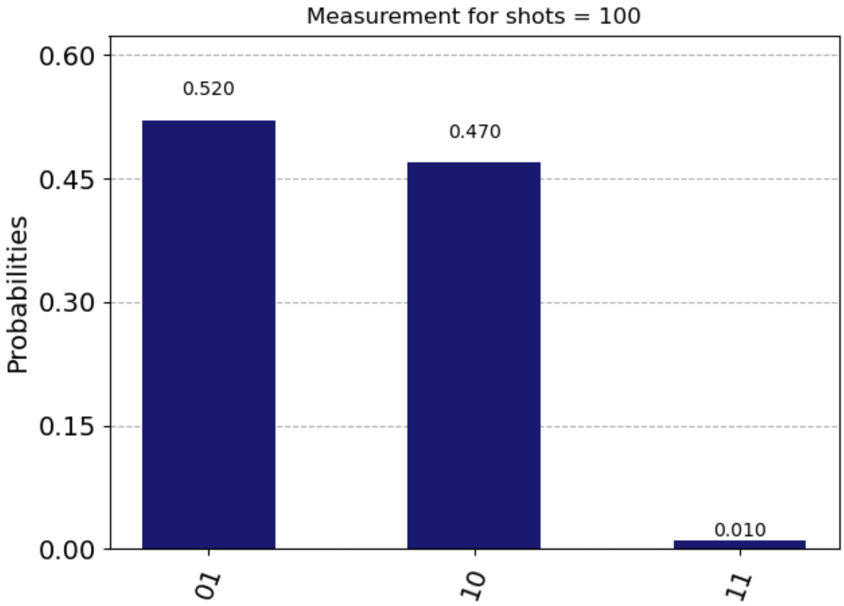

Figure 4: $\#of shots = 100$

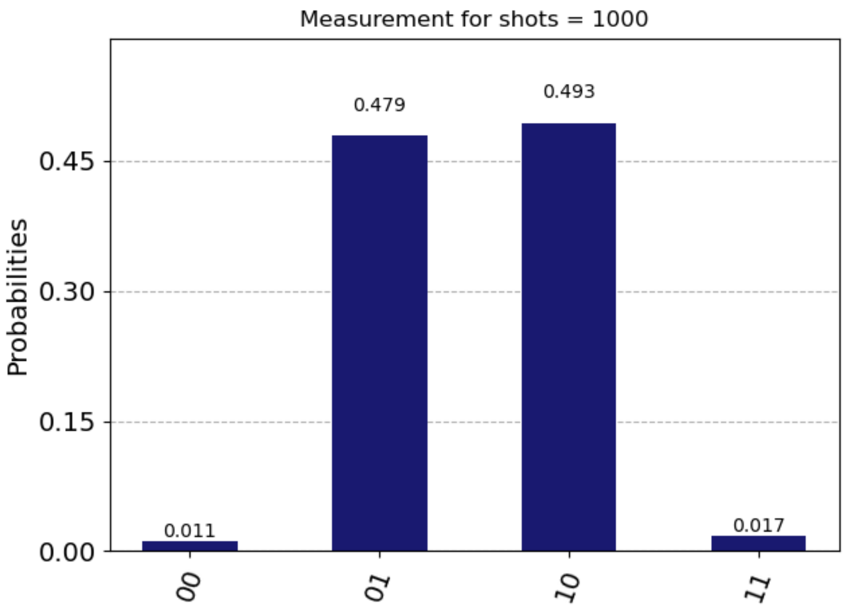

Figure 5: $\#of shots = 1000$