Logic

Connectives

Define negation: ($\neg$, $\sim$)

If p denotes a statement, then the negation of p is denoted by $\sim p$

Truth table

| p | $\sim p$ |
|---|---|
| T | F |
| F | T |

Define Disjunction: ($\vee$)

The disjunction of two statements p and q is denoted by $p \vee q$ and it is read as p or q. The statement $p \vee q$ has the truth value F, if both p and q have the truth value F, otherwise, it has the truth value T.

Truth table

| p | q | $p \vee q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

(eg) p: There is Hyundai i10

q: There is i20

Define conjunction: ($\wedge$)

The 2 statements p and q are connected

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Truth table

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Define conditional statement:

If P and Q are any two statements, then the statement $P \rightarrow Q$ which is read as if P then Q is called as conditional statement. The statement $P \rightarrow Q$ has the truth value F if p has the truth value T and Q has the truth value F, otherwise it has the truth value T.

(eg) If it is raining today, then today is a holiday.

The given statement is conditional where

p: It is raining today.

q: Today is a holiday.

Truth table:

| $p$ | $q$ | $p \rightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Define biconditional statements:

If p and q are any 2 statements, then the statement $p \Leftrightarrow$ is read as 'p if and only if q' is called biconditional statement. The statement p has the truth value T only if both p and q has the same truth value, otherwise it has the truth value F.

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Truth table:

| $p$ | $q$ | $p \Leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

Look for 'if and only if' in a given sentence.

Define tautology:

A logical expression is said to be a tautology if it is true under all possible assignments.

Define contradiction:

A logical expression is said to be a contradiction if it is false under all possible assignments.

Contingent: not tautology nor contradiction.

Define logically equivalent: biconditional is tautology

Two statements, A and B are said to be equivalent if and only if $A \Leftrightarrow B$ is a tautology and it is denoted by $A \Leftrightarrow B$ and it is read as A is equivalent to B.

(eg) $p \Leftrightarrow q \Leftrightarrow (p \to q) \wedge (q \to p)$

| $p$ | $q$ | $p \to q$ | $q \to p$ | $p \leftrightarrow q$ | $(p \to q) \wedge (q \to p)$ | |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| T | F | F | T | F | F | T |
| F | T | T | F | F | F | T |
| F | F | | | T | T | T |

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Note: If $A \to B$ is tautology, then we ...
it is read as A is logically implied to B.

Show that $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$

| P | Q | R | $Q \to R$ | $P \to (Q \to R)$ A | $P \to Q$ | $P \to R$ | $((P \to Q) \to (P \to R))$ B | $A \to B$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T | T |
| T | T | F | F | F | T | F | F | T |
| T | F | T | T | T | F | T | T | T |
| T | F | F | T | T | F | F | T | T |
| F | T | T | T | T | T | T | T | T |
| F | T | F | F | T | T | T | T | T |
| F | F | T | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T | T |

Equivalence Formula:

1) $P \vee P \Leftrightarrow P$ Idempotent laws
$P \wedge P \Leftrightarrow P$

2) $P \vee F \Leftrightarrow P$ Identity laws
$P \wedge T \Leftrightarrow P$

3) $P \vee T \Leftrightarrow T$ Domination laws
$P \wedge F \Leftrightarrow F$

4) $P \vee \sim P \Leftrightarrow T$ Tautology
$P \wedge \sim P \Leftrightarrow F$ Contradiction

5) $\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$ De Morgan's laws
$\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$

6) $P \vee (P \wedge Q) \Leftrightarrow P$ Absorption laws
$P \wedge (P \vee Q) \Leftrightarrow P$

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$P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$ Associative laws

$P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ Distributive laws

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

$\sim(\sim P) \Leftrightarrow P$

Show that $((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R))$ is a tautology.

$((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R))$

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Given:

$(\sim P \wedge (\sim Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))$

$\Leftrightarrow (\sim P \wedge (\sim Q \wedge R)) \vee (R \wedge (P \vee Q))$

(By Distributive law)

$\cancel{\Leftrightarrow (\sim P \wedge \sim (Q \vee}$

$\Leftrightarrow ((\sim P \wedge \sim Q) \wedge R) \vee (R \wedge (P \vee Q))$

(By associative law)

$\Leftrightarrow (\sim (P \vee Q) \wedge R) \vee (R \wedge (P \vee Q))$

$\Leftrightarrow$ (By De Morgan's law)

$\Leftrightarrow R \wedge (\sim (P \vee Q) \vee (P \vee Q))$

(By Distributive law)

$\Leftrightarrow R \wedge T$

(Tautology: $P \vee \sim P = T$)

$\Leftrightarrow R$ (By Identity law)

7) Show that $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a tautology.

Given

$Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\Leftrightarrow Q \vee (P \wedge \sim Q) \vee (\sim ((P \vee Q) \wedge (P \vee Q)))$

(By De Morgan's law)

$\Leftrightarrow ((Q \vee P) \wedge (Q \vee \sim Q)) \vee (\sim ((P \vee Q) \wedge (P \vee Q)))$

(By Distributive law)

$\Leftrightarrow ((Q \vee P) \wedge T) \vee (\sim (P \vee Q))$

(By Tautology) (By Idempotent law)

$\Leftrightarrow (P \vee Q) \vee (\sim (P \vee Q))$

(By Identity law)

$\Leftrightarrow T$ (By Tautology)

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$\sim(P \rightleftarrows Q)$

$\Leftrightarrow \sim((\sim P) \vee Q)$

$\Leftrightarrow \sim((P \rightarrow Q) \wedge (Q \rightarrow P))$

(By definition of biconditional)

$\Leftrightarrow \sim((\sim P \vee Q) \wedge (\sim Q \vee P))$

$\Leftrightarrow$

| $P$ | $Q$ | $P \rightleftarrows Q$ | $\sim(P \rightleftarrows Q)$ | $(P \vee Q)$ | $(P \wedge Q)$ | $\sim(P \wedge Q)$ | $(P \vee Q) \wedge \sim(P \wedge Q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | F | F |
| T | F | F | T | T | F | T | T |
| F | T | F | T | T | F | T | T |
| F | F | T | F | F | F | T | F |

b) $(P \wedge Q) \rightarrow R \Leftrightarrow (P \rightarrow R) \vee (Q \rightarrow R)$

| $P$ | $Q$ | $R$ | $P \wedge Q$ | $(P \wedge Q) \rightarrow R$ (A) | $(P \rightarrow R)$ | $(Q \rightarrow R)$ | $(P \rightarrow R) \vee (Q \rightarrow R)$ (B) |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | F |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | F | T | T |
| F | T | T | F | T | T | T | T |
| F | T | F | F | T | T | F | T |
| F | F | T | F | T | T | T | T |
| F | F | F | F | T | T | T | T |

$A \Leftrightarrow B$

T

T

T

T

T

T

T

T

7 Show that

a) $((P \vee Q) \wedge (\sim P \vee \sim Q)) \rightarrow (Q \vee R)$ is tautology.

$((P \vee Q) \wedge (\sim P \vee \sim Q)) \rightarrow (Q \vee R)$

$\Leftrightarrow \sim((P \vee Q) \wedge (\sim P \vee \sim Q)) \vee (Q \vee R)$

(By definition of biconditional)

$\Leftrightarrow \sim((P \vee Q) \wedge \sim(P \wedge Q)) \vee (Q \vee R)$

(By De Morgan's law)

$\Leftrightarrow \sim(F) \vee (Q \vee R)$

(by contradiction)

$\Leftrightarrow T \vee (Q \vee R)$

$\Leftrightarrow T$ (By Domination law)

b) $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\Leftrightarrow Q \vee (P \wedge \sim Q) \vee \sim(P \vee Q)$

(By De Morgan's law)

$\Leftrightarrow (Q \vee P) \wedge (Q \vee \sim Q) \vee \sim(P \vee Q)$

(By distributive law)

$\Leftrightarrow (Q \vee P) \wedge T \vee \sim(P \vee Q)$

(By Tautology)

$\Leftrightarrow (Q \vee P) \vee \sim(P \vee Q)$

(By Identity law)

$\Leftrightarrow (Q \vee P) \vee \sim(Q \vee P)$

(By commutative law)

$\Leftrightarrow T$ (By tautology)

Normal Forms:

Sum → Disjunction

Product → Conjunction

Normal forms: If P, Q are given:

Elementary sum $P \vee \sim P$, $\sim P \vee \sim Q$, $\sim P \vee Q$, $P \vee \sim Q$

| P | Q |
|---|---|
| P | Q |
| $\sim P$ | $\sim P$ |

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Elementary product - $P \wedge Q$, $\neg P \wedge \sim Q$, ...
$P \wedge \sim Q$

PDNF - Principle Disjunctive Normal Form
PCNF - Principle Conjunctive Normal Form

Define elementary product:
A product (conjunction) of the variables and their negations in a formula is called elementary product.
A sum (disjunction) of the variables and their negations in a formula is called elementary sum.

Define disjunctive normal form:
A formula which is equivalent to a given formula, which consists of sum of elementary products is called disjunctive normal form of the given formula.

Define Principle Disjunctive Normal Form (PDNF):
A formula which is equivalent to a given formula and which consists of sum of its minterms or sum of elementary product terms is called principle Disjunctive Normal Form or sum of product ~~of~~ canonical form.

Define Principle Conjunctive Normal Form (PCNF)
A formula which is equivalent to a formula and which consists of product elementary

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1) Obtain the PDNF of $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

Solution:

Given

$(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

$\Leftrightarrow ((P \wedge Q) \wedge T) \vee ((\sim P \wedge R) \wedge T) \vee ((Q \wedge R) \wedge T)$

$\Leftrightarrow ((P \wedge Q) \wedge (R \vee \sim R)) \vee ((\sim P \wedge R) \wedge (Q \vee \sim Q))$
$\vee ((Q \wedge R \wedge (P \vee \sim P))$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge R \wedge Q) \vee$
$(\sim P \wedge R \wedge \sim Q) \vee (Q \wedge R \wedge P) \vee (Q \wedge R \wedge \sim P)$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge R \wedge Q) \vee$
$(\sim P \wedge R \wedge \sim Q)$

This is the required PDNF for the given statement.

$P \rightleftarrows Q \Leftrightarrow \sim P \vee (\sim P \vee Q)$

2) Obtain the PDNF: $(\sim P \vee \sim Q) \rightarrow (P \rightleftarrows \sim Q)$

3) Obtain the PCNF: $(\sim P \rightarrow R) \wedge (Q \rightleftarrows P)$

2) Given

$(\sim P \vee \sim Q) \rightarrow (P \rightleftarrows \sim Q)$

$\Leftrightarrow (\sim P \vee \sim Q) \rightarrow (\sim P \vee (\sim P \vee \sim Q))$

$\Leftrightarrow (\sim P \vee \sim Q) \rightarrow \sim (P \wedge (P \wedge Q))$

$\Leftrightarrow \sim (\sim P \vee \sim Q) \vee \sim (P \wedge (P \wedge Q))$

$\Leftrightarrow (P \wedge Q) \vee \sim (P \wedge (P \wedge Q))$

$\Leftrightarrow \sim ((P \wedge Q \vee P) \wedge (P \wedge Q \vee (P \wedge Q)))$

$\Leftrightarrow \sim ((P) \wedge (P \wedge Q))$

X $\Leftrightarrow$ ~~$\sim P \vee (\sim P \vee \sim Q)$~~

$\Leftrightarrow \sim ((P \wedge (Q \vee \sim Q))$

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$\Leftrightarrow P \vee R \wedge (Q \to P) \wedge (P \to Q)$

$\Leftrightarrow (P \vee R) \wedge (\sim Q \vee P) \wedge (\sim P \vee Q)$

which is CNF

$\cancel{\Leftrightarrow (P \vee R) \wedge F}$

$\Leftrightarrow (P \vee R) \vee F \wedge (\sim Q \vee P \vee F) \wedge (\sim P \vee Q \vee F)$

$\Leftrightarrow (P \vee R) \vee (Q \wedge \sim Q) \wedge ((\sim Q \vee P) \vee (R \wedge \sim R)) \wedge$
$((\sim P \vee Q) \vee (R \wedge \sim R))$

$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge (\sim Q \vee P \vee R) \wedge$
$(\sim Q \vee P \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge$
$(P \vee \sim Q \vee \sim R) \wedge (P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (\sim P \vee Q \vee R) \wedge$
$(P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee \sim R)$

Now, PCNF of ~~S~~ ~S is:

$\sim S \Leftrightarrow (\sim P \vee \sim Q \vee R) \wedge (P \vee Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee R)$

Taking negation of both sides

$S \Leftrightarrow \sim(\sim P \vee \sim Q \vee R) \vee \sim(P \vee Q \vee \sim R) \vee$
$\sim(\sim P \vee \sim Q \vee R)$

$\Leftrightarrow (P \wedge Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R)$

which is ~~PDNF~~ the required PDNF of given formula

Theory of inference:

Rule : 1 :

Rule P: A given premise.

Rule T: A ~~fosr~~ sufficient formula can be introduced

logically implied by symbol $\Rightarrow$

Rule CP: If we can derive S from R and a set of premises, then we can write $R \to S$ as the resulting solution

Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

Solution:

| Step: | Derivation | Rule |
|---|---|---|
| 1 | $P \vee Q$ | P |
| 2 | $\sim P \to Q$ | T, $(\sim A \to B \Leftrightarrow A \vee B)$ |
| 3 | $Q \to S$ | P |
| 4 | $\sim P \to S$ | T, $((A \to B) \wedge (B \to C) \Leftrightarrow (A \to C))$ |
| 5 | $P \to R$ | P |
| 6 | $\sim R \to \sim P$ | T, $(A \to B \Leftrightarrow \sim B \to \sim A)$ |
| 7 | $\sim R \to S$ | T, $[(\sim R \to \sim P) \wedge (\sim P \to S) \Rightarrow (\sim R \to S)]$ |
| 8 | $R \vee S$ | T |

$\therefore$ $S \vee R$ is logically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

(or)

$(P \vee Q) \wedge (P \to R) \wedge (Q \to S) \Rightarrow S \vee R$

Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$ and $Q$

Solution:

| Step: | Derivation | Rule |
|---|---|---|
| 1 | $\sim R \vee P$ | P |
| 2 | $R \to P$ | T, $(\sim A \vee B \Leftrightarrow A \to B)$ |
| 3 | $P \to (Q \to S)$ | P |
| 4 | $R \to (Q \to S)$ | T (2 & 3) |
| 5 | $R$ | P (assumed premise) |
| 6 | $Q \to S$ | T (4 & 5) |

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$\therefore$ $R \to S$ is logically implied by

1. Prove by indirect method $\sim Q, P \to Q, P \vee R \Rightarrow R$
2. Show that $P \to S$ follows logically from $\sim P \vee Q$, $\sim Q \vee R$ and $R \to S$

1. Solution: We show that $\sim Q, P \to Q, P \vee R \Rightarrow F$

| Step | Derivation | Rule |
|---|---|---|
| 1 | $P \vee R$ | P |
| 2 | $\sim R$ | CP (assumed premise) |
| 3 | $\sim R \to P$ | T, (from 1) |
| 4 | $P$ | T, (from 1 & 2) |
| 5 | $P \to Q$ | P |
| 6 | $Q$ | T (from 4 and 5) |
| 7 | $\sim Q$ | P |
| 8 | $Q \wedge \sim Q$ | T (from 6 & 7) $(Q \wedge \sim Q \Leftrightarrow F)$ |

which gives the contradiction

Hence, $\sim Q \wedge (P \to Q) \wedge (P \vee R) \Rightarrow R$

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UNIT -

Combinatorics

Define the Pigeon Hole principle:

If n pigeons are assigned to m ~~holes~~ pigeon holes, one of the pigeon hole must contain 2 or more pigeons where $m < n$.

Define extended Pigeon Hole principle:

If n pigeons are assigned to m pigeon holes, then one of the pigeon hole must contain atleast $\left\lfloor \frac{(n-1)}{m} \right\rfloor + 1$ pigeons

Example:

2M Show that if 7 colours are used to paint 50 bicycles, atleast 8 bicycles will be the same colour.

Solution:

Here $n = 50$

$m = 7$

By extended Pigeon hole principle

$$\frac{49}{7} + 1$$

$$= 7 + 1 = 8$$

$\therefore$ Atleast 8 bicycles will be the same colour

2M Among 100 people, show that atleast 9 of them were born in the same month.

Solution:

Here $n = 100$

$m = 12$

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By extended Pigeon hole principle:

$$\frac{(100-1)}{12}+1$$

$$\left(\frac{99}{12}\right)+1$$

$$=(8.25)+1$$

$$=8+1$$

$$=9$$

8M Set Inclusion and Exclusion

2M Formula: $2^2-1$ terms

1) $|A \cup B| = |A| + |B| - |A \cap B|$

2) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| + |A \cap B \cap C|$

3) $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap B \cap D| + |B \cap C \cap D| + |A \cap C \cap D| - |A \cap B \cap C \cap D|$

1) How many positive integers not exceeding

apply inclusion

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$$|A \cap B| = \left\lfloor \frac{1000}{7 \times 11} \right\rfloor = 12$$

w.r.t, $|A \cup B| = |A| + |B| - |A \cap B|$

$= 142 + 90 - 12 = 232 - 12$

$= 220$

2) Find the number of integers between 1 and 250 that are (complement of divisible) not divisible by any of the integers 2, 3, 5 and 7.

Solution:

A, B, C, D are the set of integers between 1 to 250 that are divisible by 2, 3, 5 and 7 respectively.

$$|A| = \left\lfloor \frac{250}{2} \right\rfloor = 125$$

$$|B| = \left\lfloor \frac{250}{3} \right\rfloor = 83$$

$$|C| = \left\lfloor \frac{250}{5} \right\rfloor = 50$$

$$|D| = \left\lfloor \frac{250}{7} \right\rfloor = 35$$

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$$|A \cap B| = \left\lfloor \frac{1000}{7 \times 11} \right\rfloor = 12$$

w.r.t, $|A \cup B| = |A| + |B| - |A \cap B|$

$= 142 + 90 - 12 = 232 - 12$

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Solution:

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$$|A| = \left\lfloor \frac{250}{2} \right\rfloor = 125$$

$$|B| = \left\lfloor \frac{250}{3} \right\rfloor = 83$$

$$|C| = \left\lfloor \frac{250}{5} \right\rfloor = 50$$

$$|D| = \left\lfloor \frac{250}{7} \right\rfloor = 35$$

$$|A \cap B| = \left\lfloor \frac{250}{2 \times 3} \right\rfloor = 41$$

$$|A \cap C| = \left\lfloor \frac{250}{2 \times 5} \right\rfloor = 25$$

$$|A \cap D| = \left\lfloor \frac{250}{2 \times 7} \right\rfloor = 17$$

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$$|B \cap C| = \left\lfloor \frac{250}{3 \times 5} \right\rfloor = 16$$

$$|B \cap D| = \left\lfloor \frac{250}{3 \times 7} \right\rfloor = 11$$

$$|C \cap D| = \left\lfloor \frac{250}{5 \times 7} \right\rfloor = 7$$

$$|A \cap B \cap C| = \left\lfloor \frac{250}{2 \times 3 \times 5} \right\rfloor = 5$$

$$|A \cap C \cap D| = \left\lfloor \frac{250}{2 \times 5 \times 7} \right\rfloor = 3$$

$$|B \cap C \cap D| = \left\lfloor \frac{250}{3 \times 5 \times 7} \right\rfloor = 2$$

$$|A \cap B \cap C \cap D| = \left\lfloor \frac{250}{2 \times 3 \times 5 \times 7} \right\rfloor = 1$$

$$\therefore |A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap C \cap D| + |A \cap B \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$$

Substituting all the values:

$$|A \cup B \cup C \cup D| = 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7 + 5 + 3 + 2 - 1$$

$$= 193$$

Hence, the number of integers between 1 and 250 that are not divisible by any of the integers 2, 3, 5 and 7 = 250 − 193

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$$|B \cap C| = \left\lfloor \frac{250}{3 \times 5} \right\rfloor = 16$$

$$|B \cap D| = \left\lfloor \frac{250}{3 \times 7} \right\rfloor = 11$$

$$|C \cap D| = \left\lfloor \frac{250}{5 \times 7} \right\rfloor = 7$$

$$|A \cap B \cap C| = \left\lfloor \frac{250}{2 \times 3 \times 5} \right\rfloor = 5$$

$$|A \cap C \cap D| = \left\lfloor \frac{250}{2 \times 5 \times 7} \right\rfloor = 3$$

$$|B \cap C \cap D| = \left\lfloor \frac{250}{3 \times 5 \times 7} \right\rfloor = 2$$

$$|A \cap B \cap C \cap D| = \left\lfloor \frac{250}{2 \times 3 \times 5 \times 7} \right\rfloor = 1$$

$$\therefore |A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap C \cap D| + |A \cap B \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$$

Substituting all the values:

$$|A \cup B \cup C \cup D| = 125 + 83 + 50 + 35 - 41 - 25 - 17 - 16 - 11 - 7 + 5 + 3 + 2 - 1$$

$$= 193$$

Hence, the number of integers between 1 and 250 that are not divisible by any of the

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3) Find the number of integers between 1 and 500 that are not divisible by 2, 3, 5 and 6.

Solution:

A, B, C, D are the sets of integers between 1 and 500 that are divisible by 2, 3, 5 and 6 respectively

$$|A| = \left\lfloor \frac{500}{2} \right\rfloor = 250$$

$$|B| = \left\lfloor \frac{500}{3} \right\rfloor = 166$$

$$|C| = \left\lfloor \frac{500}{5} \right\rfloor = 100$$

$$|D| = \left\lfloor \frac{500}{6} \right\rfloor = 83$$

$$|A \cap B| = \left\lfloor \frac{500}{2 \times 3} \right\rfloor = 83$$

$$|A \cap C| = \left\lfloor \frac{500}{2 \times 5} \right\rfloor = 50$$

$$|A \cap D| = \left\lfloor \frac{500}{2 \times 6} \right\rfloor = 41$$

5 6
18 18
7 8
12 & 144

$$|B \cap C| = \left\lfloor \frac{500}{3 \times 5} \right\rfloor = 33$$

$$|B \cap D| = \left\lfloor \frac{500}{3 \times 6} \right\rfloor = 27$$

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$$|C \cap D| = \left\lfloor \frac{500}{5 \times 6} \right\rfloor = 16$$

$$|A \cap B \cap C| = \left\lfloor \frac{500}{2 \times 3 \times 5} \right\rfloor = 16$$

$$|A \cap C \cap D| = \left\lfloor \frac{500}{2 \times 5 \times 6} \right\rfloor = 8$$

$$|B \cap C \cap D| = \left\lfloor \frac{500}{3 \times 5 \times 6} \right\rfloor = 5$$

$$|A \cap B \cap C \cap D| = \left\lfloor \frac{500}{2 \times 3 \times 5 \times 6} \right\rfloor = 2$$

$$\therefore |A \cup B \cup C \cup D| = |A| + |B| + |C| + |D| - |A \cap B| - |A \cap C| - |A \cap D| - |B \cap C| - |B \cap D| - |C \cap D| + |A \cap B \cap C| + |A \cap C \cap D| + |A \cap B \cap D| + |B \cap C \cap D| - |A \cap B \cap C \cap D|$$

Substituting all the values

$$|A \cup B \cup C \cup D| = 250 + 166 + 100 + 83 - 83 - 50 - 41 - 33 - 27 - 16 + 16 + 8 + 5 - 2$$

$$=$$

Hence the number of integers between 1 and 500 that are not divisible

10-11-2016

8M There are 2500 students in a school, of these 1700 taken a course in C, 1000 have taken a course in Pascal and 550 have taken a course in Networking. Further, 750 have taken courses in both C and Pascal, 400 have taken courses in both C and networking, 275 have taken course in both Pascal and networking. If 200 of these students have taken course in C, Pascal and networking.

i) How many of these 2500 students have taken a course in any of these 3 course C, Pascal and Networking.

ii) How many of these 2500 students have not taken a course in any of these three courses

Solution:

Let E be the set of students in the school.

Let A be the set of students who have taken a course C,

Similarly B and C have taken a course Pascal and Networking respectively.

Let $A \cap B$ be the set of students who have taken course in both C and Pascal. Similarly we have $A \cap C$ and $B \cap C$.

Then, $|E| = 2500$

[Note: Modulus symbol means number of elements in the set]

$|E| = 2500$

$|A| = 1700$

$|B| = 1000$

$|C| = 550$

$|A \cap B| = 750$

$|A \cap C| = 400$

$= 1 - 200$

By Inclusion principle:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$= 2025$$

(ie) 2025 of 2500 students have taken course in any of these three courses

The number of students who have not taken ~~in~~ any of these 3 courses $= |E| - |A \cup B \cup C|$

$$= 2500 - 2025$$

$$= 475$$

Form the recurrence relation

Find the the recurrence relation $y_n = A(3)^n + B(-4)^n$, for $n \geq 0$

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$y_n(48+36) - 1(16y_{n+1} + 4y_{n+2}) + 1(9.y_{n+1} - 3y_{n+2}) = 0$

$\Rightarrow 84y_n - 7y_{n+1} - 7y_{n+2} = 0$

$(\div) \Rightarrow 7y_{n+2} + 7y_{n+1} - 84y_n = 0$

$(-1) \Rightarrow y_{n+2} + y_{n+1} - 12y_n = 0$

2) $S_n = (A + Bn)5^n$

$y_n = 1.(A5^n + Bn.5^n)$

$y_{n+1} = A5^{n+1} + B(n+1)5^{n+1}$

$= A5^{n+1} + Bn5^{n+1} + B5^{n+1}$

$= 5.A5^n + (n+1).5.B5^n$

$y_{n+2} = A5^{n+2} + B(n+2)5^{n+2}$

$= A5^{n+2} + Bn5^{n+2} + 2B5^{n+2}$

$= 5^2.A5^n + (n+2)5^2.B.5^n$

By taking coefficients in the above 3 equations

$$\begin{vmatrix} y_n & 1 & n \\ y_{n+1} & 5 & 5(n+1) \\ y_{n+2} & 25 & 25(n+2) \end{vmatrix} = 0$$

$y_n(125(n+2) - 125(n+1) - y_{n+1}(25(n+2)y_n - n(y_{n+2})) + \cancel{5(n+1)} \; y_{n+2}(5(n+1) - 5n)$

$y_n(125n + 250 - 125n - 125) - y_{n+1}(25ny_n + 50y_n - ny_{n+2}) + y_{n+2}(5n+1-5n)$

$= 125y_n - 25ny_n y_{n+1} - 50y_n y_{n+1} + ny_{n+1}y_{n+2} + y_{n+2}$

Ans $y_n - 10y_{n+1} + 25y_{n+2} = 0$

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H.W 3 Find the recurrence relation of $y_n = A(-3)^n + B(4)^n$ for $n \geq 0$

Solution of recurrence relation:

1) Solve the recurrence relation for Fibonacci numbers.

(or) Solve the recurrence relation } same

$F(n) = F(n-1) + F(n-2)$, $n \geq 2$

with $F(0) = 0$ and $F(1) = 1$

Solution:

Given recurrence relation of Fibonacci numbers.

$F(n) = F(n-1) + F(n-2)$, $n \geq 2 \longrightarrow (1)$

$\Rightarrow$ with $F(0) = 0$ and $F(1) = 1$

$(1) \Rightarrow F(n) - F(n-1) - F(n-2) = 0 \longrightarrow (2)$

The characteristic equation of (2) is

$a^2 - a - 1 = 0 \longrightarrow (2')$

$a = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | $a = 1, b = -1$, $c = -1$

$= \dfrac{+1 \pm \sqrt{1+4}}{2}$

$= \dfrac{1 \pm \sqrt{5}}{2}$

$\therefore$ The roots are $\dfrac{1+\sqrt{5}}{2}, \dfrac{1-\sqrt{5}}{2}$

(Different roots so $C_1(\cdot) + C_2(\cdot)$)

$\therefore$ The solution is: $F(n) = C_1\left(\dfrac{1+\sqrt{5}}{2}\right)^n + C_2\left(\dfrac{1-\sqrt{5}}{2}\right)^n$

$a = -R, R$

If roots are repeated then $\boxed{y_n = (A + Bn)(-R)^n}$

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∴ The solution is

$F(n) =$

i) Put $F(0) = 0$

$$0 = C_1 \cdot 1 + C_2 \cdot 1 \rightarrow (5)$$

ii) Put $F(1) = 1$

$$1 = C_1\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right) \rightarrow (6)$$

$$(5) \Rightarrow C_2 = -C_1$$

$$(6) \Rightarrow 1 = C_1\left(\frac{1+\sqrt{5}}{2}\right) - C_1\left(\frac{1-\sqrt{5}}{2}\right)$$

$$1 = C_1\left[\frac{1}{2} + \frac{\sqrt{5}}{2} - \frac{1}{2} + \frac{\sqrt{5}}{2}\right]$$

$$1 = C_1\left(\frac{2\sqrt{5}}{2}\right)$$

$$\therefore C_1 = \frac{1}{\sqrt{5}}$$

$$\therefore C_2 = \frac{-1}{\sqrt{5}}$$

$$(4) \Rightarrow F(n) = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

2M Write the characteristic equation of

$$S(n+3) + 4S(n+2) - 7S(n+1) + 5S(n) = 0$$

for $n \geq 0$

The required characteristic equation ~~of~~ is

$$a^3 + 4a^2 - 7a + 5 = 0$$

Case

$$S(n-3) - 7S(n) = 0$$

$$a^3 + 0 \cdot a^2 + 0 \cdot a - 7 = 0$$

(7) Solving recurrence relation using generating function.

Let generating function is of the form

$$G(S(n) z) = \sum_{n=0}^{\infty} S(n) z^n = S(0).1 + S(1).z + S(2) z^2$$

$$(1+z)^{-1} = 1 - z + z^2 - z^3 + \cdots$$
$$(1-z)^{-1} = 1 + z + z^2 + z^3 + \cdots$$
$$(1-z)^{-2} = 1 + 2z + 3z^2 + 4z^3 + \cdots$$

Solve the recurrence relation using generating function

$$S(n+1) - 2S(n) = 4^n$$

with $S(0) = 1$, for $n \geq 0$

Solution:

Given recurrence relation

$S(n+1) - 2S(n) = 4^n$, $n \geq 0 \rightarrow (1)$

Let $G(S, z) = \sum_{n=0} S(n).z^n$ be the required generating function for the given recurrence relation

or $G(z)$

~~Let~~ $\cancel{G(S, z)} =$

Multiply equation (1) by $z^n$ and take summation from ~~$n=1$~~ $n=0$ to $\infty$

$$S(n) - 4^n = 0$$

$$\Rightarrow \frac{1}{z}\left[S(1).z + S(2).z^2 + S(3).z^3 + \cdots\right] -$$

$$2G(S,z) - (1-4z)^{-1} = 0$$

$$\left(\because (1-x)^{-1} = 1 + x + x^2 + \cdots\right) \quad (x = 4z$$

$$= \frac{1}{z}\left[G(S,z) - S(0)\right] - 2G(S,z) - \frac{1}{1-4z} = 0$$

$$\Rightarrow \frac{1}{z}\left[G(S,z) - 1\right] - 2G(S,z) = \frac{1}{1-4z}$$

$$\Rightarrow \frac{1}{z}G(S,z) - \frac{1}{z} - 2G(S,z) = \frac{1}{1-4z}$$

$$\Rightarrow \left(\frac{1}{z} - 2\right)G(S,z) = \frac{1}{1-4z} + \frac{1}{z}$$

$$\Rightarrow \left(\frac{1-2z}{z}\right)G(S,z) = \frac{1}{1-4z} + \frac{1}{z}$$

$$\Rightarrow G(S,z) = \frac{z}{1-2z}\left[\frac{1}{1-4z} + \frac{1}{z}\right]$$

$$= \frac{z}{(1-2z)(1-4z)} + \frac{1}{1-2z}$$

Denominator is made it be single factor

$$\therefore G(S,z) = \frac{z + 1 - 4z}{(1-2z)(1-4z)} = \frac{1-3z}{(1-2z)(1-4z)}$$

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4 2 — right hand side value 0

$$\boxed{B = \frac{1}{2}}$$

Put $z = \frac{1}{2}$

$$1 - \frac{3}{2} = -A + 0$$

$$\frac{-1}{2} = -A$$

$$\boxed{A = \frac{1}{2}}$$

$$\therefore \quad G(s,z) = \frac{1/2}{(1-2z)} + \frac{1/2}{(1-4z)}$$

$$= \frac{1}{2}(1-2z)^{-1} + \frac{1}{2}(1-4z)^{-1}$$

$$= \frac{1}{2}\left[1 + (2z) + (2z)^2 + \dots + (2z)^n + \dots\right]$$

$$+ \frac{1}{2}\left[1 + (4z) + (4z)^2 + \dots + (4z)^n + \dots\right]$$

Taking coefficient of $z^n$:

$$S(n) = \frac{1}{2}(2^n) + \frac{1}{2}(4)^n$$

H.W

⇒ Solve the recurrence relation using generating function of $S(n) - 8S(n-1) - 9S(n-2) = 0$ for $n \geq 2$

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# Algebraic Structure

Define Groups:

A group $\langle G, * \rangle$ is an algebraic system in which binary operation $*$ on $G$ satisfies the following properties,

i) $\forall\, x, y, z \in G$
$x * (y * z) = (x * y) * z$ (Associative)

ii) there exists an element $e \in G$ such that for any $x \in G$
$e * x = x * e = x$ (Identity)

iii) For every $x \in G$ there exists an element $x^{-1} \in G$ such that $x * x^{-1} = x^{-1} * x = e$
(Inverse)

Ex: A nonempty set of integers with the operator addition is a group (i.e) $\langle I, \oplus \rangle$

Theorem

In any group $\langle G, * \rangle$, $(a * b)^{-1} = b^{-1} * a^{-1}$ for any $a, b \in G$

Solution.

Let $\langle G, * \rangle$ be a given group, we have to show that $(a * b)^{-1} = b^{-1} * a^{-1}$

Solution:

Now

$(a * b) * (b^{-1} * a^{-1})$
$= a * (b * b^{-1}) * a^{-1}$
$= a * e * a^{-1}$
$= a * a^{-1}$
$= e$

& $(b^{-1} * a^{-1}) * (a * b) = b^{-1} * (a^{-1} * a) * b$
$= b^{-1} * e * b$
$= b^{-1} * b$
$= e$

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$\langle G, * \rangle$ is abelian for all $a, b \in G$ such that $\Rightarrow a*b = b*a$

Hence, $(a*b)^{-1} = b^{-1} * a^{-1}$

Show that in a group $\langle G, * \rangle$, if for any $a, b \in G$, $(a*b)^2 = a^2 * b^2$, then $\langle G, * \rangle$ must be abelian.

Let $\langle G, * \rangle$ be a given group for any $a, b \in G$

$(a*b)^2 = a^2 * b^2$

$\Rightarrow (a*b)^2 = (a*a)*(b*b)$

$(a*b)*(a*b) = a*(a*b)*b$

$\cancel{a}*(b*a)*\cancel{b} = \cancel{a}*(a*b)*\cancel{b}$

$\Rightarrow b*a = a*b$

Hence, by definition we have $\langle G, * \rangle$ must be an abelian.

Define cyclic group:

A group $\langle G, * \rangle$ is said to be cyclic, if there exists an element $a \in G$ such that for every $x \in G$, $x = a^n$ for some integer $n$

Let $G = \{1, -1, i, -i\}$

$(i)^1 = \cancel{i (i)^3 = i^4} = i$

$(i)^2 = -1$

$(i)^3 = i^2 \cdot i = -1 \cdot i = -i$

$(i)^4 = 1$

Here, $i$ is the generator of the cyclic group $G$

Every cyclic group is abelian

Solution:

Let $\langle G, * \rangle$ be a cyclic group with

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Now, $x * y = \cancel{y * x}\ a^m * a^n$
$= a^{m+n}$
$= a^{n+m}$
$= a^n * a^m$
$= y * x$

Hence, $\langle G, * \rangle$ is abelian

Define Group Homomorphism:
Let $\langle G, * \rangle$ and $\langle G', \Delta \rangle$ be any two groups, a mapping
$f: G \to G'$ is called a group homomorphism if any $a, b \in G$ then
$f(a * b) = f(a) \Delta f(b)$

Define kernal of a homomorphism:
Let $\langle G, * \rangle$ and $\langle G', \Delta \rangle$ be any two groups, a mapping $f$ is from $G \to G'$ is a group homomorphism, then the kernal of $f$ defined by
$Ker(f) = \{x \in G \mid f(x) = e',$
$e'$ is the identity of $G'\}$

Show that any subgroup of a finite group divides the order of finite group.

4-Dec. 2014 Lagrange's theorem
State and prove Lagrange's theorem

Statement:
In a finite group, the order of any subgroup of $G$ divides the order of the group.

Proof:
Let $G$ be a finite group.
(i.e) $O(G) = n$ (finite)

Let $H$ be any subgroup of $G$

Now, $x * y = a^m * a^n$

$= a^{m+n}$

$= a^{n+m}$

$= a^n * a^m$

$= y * x$

Hence, $\langle G, * \rangle$ is abelian

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Proof:

Let $G$ be a finite group.

(i.e) $O(G) = n$ (finite)

Let $H$ be any subgroup of $G$

and Let $G = \{a_1 = e, a_2, a_3, \ldots, a_n\}$

Consider the ~~left~~ Left coset of H as follows

$$e * H = \{e * h / h \in H\}$$

$$a_2 * H = \{a_2 * h / h \in H\}$$

$$\ldots\ldots$$

$$a_n * H = \{a_n * h / h \in H\}$$

We know that any two left cosets are either identical or disjoint.

Also, we have, $O(e * H) = O(H)$

$$\Rightarrow O(a_i * H) = O(H), \forall i$$

Let there be k-disjoint cosets of H in G.
Nearly, G is the union of k-disjoint cosets of H in G.

(i.e) $O(G) = O(e * H) \cup (a_1 * H) \cup \ldots\ldots \cup O(a_k * H)$

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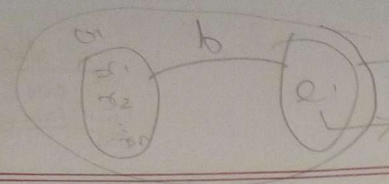



Let $\langle G, * \rangle$ and $\langle G', \Delta \rangle$ be any two groups then by definition, the homomorphism $f$ be defined by

$f: G \longrightarrow G'$ be defined by

$$f(a * b) = f(a) \Delta f(b) \text{ for all } a, b \in G$$

and let $e$ & $e'$ be the identity elements of $G$ and $G'$ respectively.

By definition, $\ker(f) = \{ x \mid f(x) = e', \forall x \in G \}$

Clearly $f(e) = e'$, where $e \in G$

To prove: $\ker(f)$ is a subgroup.

~~Let $a, b \in G \Rightarrow$~~

Let $a, b \in \ker(f) \Rightarrow f(a) = e'$ & $f(b) = e'$

$\Rightarrow f(a * b^{-1}) = f(a) \Delta f(b^{-1}) = e' \Delta e' = e'$

$(\because f(b^{-1}) = (f(b))^{-1} = e')$

$\Rightarrow a * b^{-1} \in \ker(f)$

$\Rightarrow \ker(f)$ is a subgroup

Define Ring:

A non-empty set $R$ is said to be a 'Ring' if it satisfies the following conditions

i) $\langle R, + \rangle$ is an abelian group

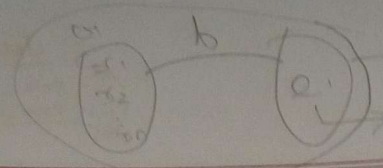
ii) $\langle R, . \rangle$ is a semigroup

iii) the binary operation '$\cdot$' distributive over the binary operation '$+$'

$a, b, c \in R$

$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

$a + (b \cdot c) = (a + b)(a + c)$

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ii) $\langle R, \cdot\rangle$ is a semigroup
iii) the binary operation '$\cdot$' distributive over the binary operation '$+$'

$a, b, c \in R$

$a \cdot (b+c) = (a \cdot b) + (a \cdot c)$

$a + (b \cdot c) = (a+b) \cdot (a+c)$

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(eg) The set of all real numbers R, under the binary operations '·' and '+' satisfying the property of abelian group under the operation addition and semigroup under the operation multiplication, then it is called Ring.
It can be written as $\langle R, +, \cdot \rangle$ is a Ring, where R is a set of real numbers.

Unit IV - Lattices

Define equivalence relation:
Let R be a binary relation on X. R is said to be equivalence relation on X, if
i) R is Reflexive (ie) $\langle x, x \rangle \in R$, $\forall x \in X$
ii) R is Symmetric (ie) $\langle x, y \rangle \in R$
$\Rightarrow \langle y, x \rangle \in R$, $\forall x, y \in X$
iii) R is transitive (ie) $\langle x, y \rangle \in R$ and $\langle y, z \rangle \in R$
$\Rightarrow \langle x, z \rangle \in R$, $\forall x, y, z \in X$

Define partial ordering relation:
Let R be a relation on X. R is said to be partial ordering relation on X if it satisfies that R is Reflexive
R is anti-symmetric
(ie) $\langle x, y \rangle \in R$ and $\langle y, x \rangle \in R$
$\Rightarrow x = y$, $\forall x, y \in R$
R is transitive

Define partially ordered set:
A set P together with a partial ordering relation R is called partially ordered set
[illegible] $\langle P, R \rangle$ [illegible] relation

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Let R be the set of real numbers, the relation $\leq$ less than or equal to ($\leq$) is a partial ordering relation on R.

Therefore, $<R, \leq>$ is a partially ordered set $<R, \geq>$

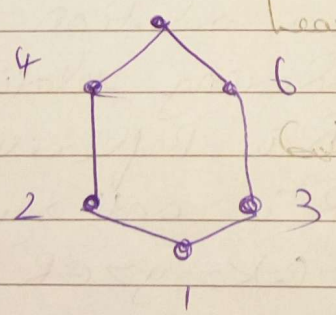
Define Hasse diagram:

A partial ordering relation '$\leq$' on a finite set P can be represented in a plane by means of a diagram is called Hasse diagram or a partially ordered set diagram

Eg 1 Draw a Hasse diagram of $<P, \leq>$, where $P = \{1, 2, 3, 4, 6, 12\}$ and a partial order relation $\leq$ be such that $x \leq y$ if $x/y$

The solution is given that $P = \{1, 2, 3, 4, 6, 12\}$ and the partial ordering relation is '/' (divides) ~~the~~

The required Hasse diagram,




Least Upper Bound $\{2, 3\} = 6$
$\{4, 6, 12\}$

Greatest Lower Bound $\{4, 6\} = 2$
$\{1, 2\}$

Hasse diagram of $<P, '/'>$ divides

2) Draw the Hasse diagram of power set of A with partial ordering relation is set inclusion ($\subseteq$)

Given $A = \{a, b, c\}$ $\quad \therefore P(A) = 2^3 = 8$

Let R be the set of real numbers, the relation $\leq$ less than or equal to ($\leq$) is a partial ordering relation on R.

Therefore, $\langle R, \leq \rangle$ is a partially ordered set $\langle R, \geq \rangle$

Define Hasse diagram:

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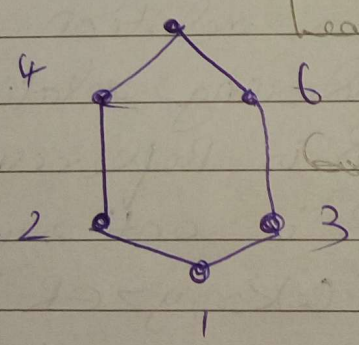
Eg 1 Draw a Hasse diagram of $\langle P, \leq \rangle$, where $P = \{1, 2, 3, 4, 6, 12\}$ and a partial order relation $\leq$ be such that $x \leq y$ iff $x | y$

The solution is given that $P = \{1, 2, 3, 4, 6, 12\}$ and the partial ordering relation is '|' (divides) ~~the~~

The required Hasse diagram.




Least Upper Bound $\{2, 3\} = 6$
$\{4, 6, 12\}$

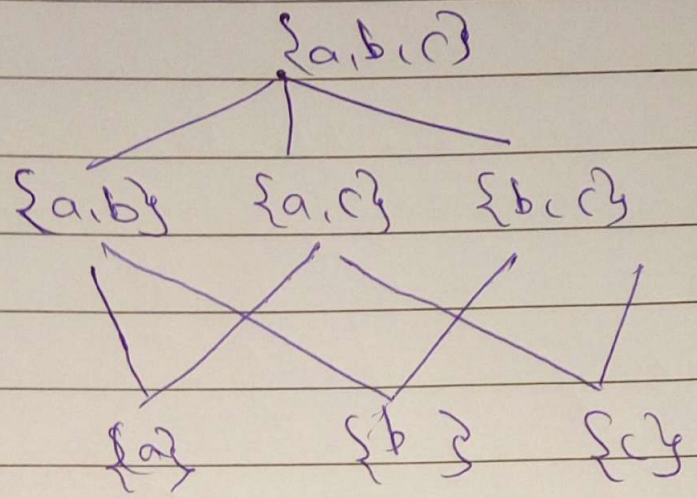
Greatest Lower Bound $\{4, 6\} = 2$
$\{1, 2\}$

Hasse diagram of $\langle P, \underset{\text{divides}}{|} \rangle$

2) Draw the Hasse diagram of power set of A with partial ordering relation is set inclusion ($\subseteq$)

Given $A = \{a, b, c\}$ $\quad \therefore P(A) = 2^3 = 8$

Given partial order relation is inclusion ($\subseteq$)




Hasse diagram of $\langle P(A), \subseteq \rangle$

Divisors of 24

Draw the Hasse diagram of $\langle D_{24}, | \rangle$, where | means divides.

$\langle P, | \rangle$, where $P = \{2, 3, 6, 12, 24, 36\}$ and | means divides

Define Lattice - 5.40 with example

Theorem - 3 - Every chain is lattice 5.10

Pg. ~~Pro~~ Theorem - 9 (Isomorphism property of lattice)

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7) $P \vee Q \Leftrightarrow Q \vee P$ Commutative laws
$P \wedge Q \Leftrightarrow Q \wedge P$

8) $P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$ Associative laws
$P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

9) $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ Distributive laws
$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

10) $\sim(\sim P) \Leftrightarrow P$

Show that $((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R))$ is a tautology.

$((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R))$

$\Leftrightarrow ((P \vee Q) \wedge (P \vee (Q \wedge R))) \vee ((\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R))$
$\Leftrightarrow ((P \vee Q) \wedge (P \vee (Q \wedge R))) \vee \sim ((P \vee Q) \wedge (P \vee R))$
$\Leftrightarrow ((P \vee Q) \wedge (P \vee (Q \wedge R))) \vee \sim ((P \vee (Q \wedge R)))$
$\times \Leftrightarrow ((P \wedge (P \vee Q)) \vee ((P \vee Q) \wedge (Q \wedge R))) \vee \sim ((P \vee (Q \wedge R)))$
$\times \Leftrightarrow ((\cancel{P \wedge (P \vee Q)}) \vee ((\cancel{\ }$
$\times \Leftrightarrow (P \vee ((P \vee Q) \wedge (Q \wedge R))) \vee \sim ((P \vee (Q \wedge R)))$
$\times \Leftrightarrow ((P \vee (P \vee Q)) \wedge (P \vee (Q \wedge R))) \vee \sim ((P \vee (Q \wedge R)))$
$\Leftrightarrow$
$\Leftrightarrow ((P \vee Q) \wedge ((P \vee Q) \wedge (P \vee R))) \vee \sim ((P \vee (Q \wedge R)))$
$\Leftrightarrow (((P \vee Q) \wedge (P \vee Q)) \wedge (P \vee R)) \vee \sim ((P \vee (Q \wedge R)))$
$\Leftrightarrow ((P \vee Q) \wedge (P \vee R)) \vee \sim ((P \vee (Q \wedge R)))$
$\Leftrightarrow (P \vee (Q \wedge R)) \vee \sim ((P \vee (Q \wedge R)))$
$\Leftrightarrow T$

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