Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : S1352

M.C.A. DEGREE EXAMINATION, FEBRUARY/MARCH 2016.

First Semester

DMC 7101 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the negation of the statement "2 is even and –3 is negative"?

2. Define *Inconsistency of Premises.*

3. State the principle of Inclusion and Exclusion.

4. How many words of three distinct letters can be formed from the letters of the word MAST?

5. Define Cyclic group.

6. Prove that the inverse of an element 'a' in a group G is unique.

7. Draw the Hasse diagram of the set of all divisors of 16.

8. What is meant by minterm and maxterm in a poset?

9. Define an ambiguous grammar.

10. Write regular expression for the language L = "All strings ending in 01" over the alphabet set {0,1}.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove by indirect method that $(\neg Q), P \to Q, P \vee R \Rightarrow R$. (8)

(ii) Demonstrate that S is a valid inference from the premises $P \to \neg Q, Q \vee R, \neg S \to P$ and $\neg R$. (8)

Or

(b) (i) Obtain a DNF of $(Q \vee (P \wedge R)) \wedge \neg((P \vee R) \wedge Q)$. (8)

(ii) Obtain a CNF of $P \to [(P \to Q) \wedge \neg(\neg Q \vee \neg P)]$. (8)

12. (a) (i) Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 5 or 7. Also indicate how many are divisible by 3 or 7 but not by 5. (8)

(ii) Find the positive integer solutions of $x + y + z = 9$. (8)

Or

(b) (i) Prove by mathematical induction that, $2n + 1 \leq 2^n$ for $n \geq 3$. (8)

(ii) Find the generating function of $n.a^n$ (a is a constant). (8)

13. (a) (i) Prove that the group of multiples of four forms a subgroup of additive group of integers. (8)

(ii) Define Ring with Example. (8)

Or

(b) State and prove Lagrange's theorem. (16)

14. (a) (i) If R is a relation defined on the set Z by a R b if a – b is a non – negative even integer. Determine if R is a partial order relation. (8)

(ii) Find the values of Boolen function $f(x, y, z) = xy + z'$. (8)

Or

(b) Let A be the set of factors of a particular positive integer m and let $\leq$ be the relation 'divides' ie, $\leq = \{(x, y) / x \in A \wedge y \in A \wedge (x \text{ divides } y)\}$. Draw Hasse diagrams for

(i) $m = 30$,

(ii) $m = 210$,

(iii) $m = 12$ and

(iv) $m = 45$. (16)

15. (a) (i) Construct the rightmost and leftmost derivation tree for $+*-xyxy$ using the grammar $E = +EE/*EE/-EE/x/y$. (10)

(ii) Describe the operations involved in Regular expression and the applications of Regular expressions. (6)

Or

(b) Define Finite Automata. Describe the ways to represent it. Construct FA accepting the following languages, and show the sequence of transitions for a sample string each : (16)

(i) L = (0 + 1) * (00 + 11)

(ii) L = {The set of strings of a's and b's starting with 'a' and ending with 'ab'.

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Question Paper Code : 80352

M.C.A. DEGREE EXAMINATION, AUGUST 2015.

First Semester

DMC 7101 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the negation of the given statement in Symbolic form: "If the processor is fast, then the printer is slow".

2. Verify whether $(P \vee Q) \rightarrow P$ is a tautology.

3. State Pigeonhole principle.

4. What is the minimum number of students required in discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades.

5. Define group homomorphism.

6. Show that the inverse of an element of a group $(G, *)$ are always unique.

7. Define Poset with an example.

8. State and prove De'Morgan laws in Boolean algebra.

9. Give the regular expression to denote the floating point constant in C programming language.

10. Distinguish Deterministic Finite Automata and Non-deterministic Finite Automata.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$. (8)

(ii) Show that r is valid conclusion from the premises : $(p \wedge \neg q) \to r$, $s \to p, q \to \neg u, u \wedge s$. (8)

Or

(b) Define PDNF and PCNF. By not using truth table, find the PDNF and PCNF of $p \to ((p \to q) \wedge \neg(\neg q \vee \neg p))$. (16)

12. (a) (i) Find the generating function of Fibonacci sequence $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$ with $F(0) = F(1) = 1$. (8)

(ii) Using generating function solve $y_{n+2} - 2y_{n+1} + y_n = 2^n$, $n \geq 0$ with $y_0 = 2$ and $y_1 = 1$. (8)

Or

(b) (i) Define the order of a recurrence relation. Solve the recurrence relation $U_n = 3U_{n-1} + 2$ where $n \geq 1$, $u_0 = 1$. (8)

(ii) Suppose that a person deposits Rs. 10,000 in a savings account at a bank yielding 11% per year with interest compounded annually. How much will be in the account after 30 years? (Use recurrence relation). (8)

13. (a) (i) Define a group. Prove that the addition modulo 5 is a group. (8)

(ii) Define field with an example. (8)

Or

(b) State and prove Lagrange's theorem. (16)

14. (a) (i) If $R = ((a,b)/a,b \in Z, a = 4b)$. Find whether R is partial order relation or not. (8)

(ii) Prove that a subgroup of a cyclic group is cyclic. (8)

Or

(b) Define partial ordered set. Describe the characteristics of partially ordered set and the procedure for drawing Ruse diagram for the divisors of 32 with the partial order of divisibility. (16)

15. (a) (i) Construct a finite Automata accepting all strings in $\{0,1\}^*$ having even number of 0's, and also give its transition table and transition diagram. (8)

(ii) Show that the grammar G with production rules $S \to aS / aSbS / \in$ is ambiguous and find the unambiguous grammar. (8)

Or

(b) (i) Illustrate with an example the derivation methods of CFG. (8)

(ii) Construct a finite automata with reduced states equivalent to the regular expression $(ab + (b + aa))b^* a$. (8)

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Question Paper Code : 22354

M.C.A. DEGREE EXAMINATION, FEBRUARY/MARCH 2015.

First Semester

DMC 7101 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Write in symbolic form the statement "The crop will be destroyed if there is a flood".

2. Define Tautology.

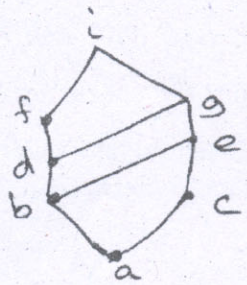
3. How many permutations of the letters A, B, C, D, E, F, G contain the string CFGA?

4. State the principle of inclusion-exclusion as applied to two finite subsets. Extend it for three finite subsets.

5. Show that "*A* cyclic group must be Abelian".

6. The ring of rational numbers $(Q, +, .)$ is a field-Justify.

7. For Hasse diagram of poset given below, find the Greatest Lower Bound and least upper bound of the following subsets : $\{b, d, e\}, \{c, e, f\}$.



8. Prove the Boolean idnetity, $a * (a' \oplus b) = a * b$.

9. Create regular expression for the language of strings of 0's and 1's such that all strings are of even length.

10. Write CFG for defining palindrome over $\Sigma = \{a, b\}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove the implication by using truth table : $(p \to (q \to s)) \wedge (\neg r \vee p) \wedge q \Rightarrow r \to s$. (8)

(ii) Without constructing the truth table, find the principal disjunctive normal form of the statement $P \wedge \neg(q \wedge r) \vee (p \to q)$. (8)

Or

(b) (i) Prove that the premises $a \to (b \to c)$, $d \to (b \wedge \neg c)$ and $(a \wedge d)$ are inconsistent. (8)

(ii) Show by indirect method of proof, that $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x P(x)) \vee (\exists x\, q(x))$. (8)

12. (a) (i) How many positive integers ≤ 10,00,000 have the sum of their digits equal to 19? (8)

(ii) Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n;\ n \geq 0,\ a_0 = 3$ by characteristic equation. (8)

Or

(b) (i) Solve the recurrence relation $a_{n+1} - 8a_n + 16a_{n-1} = 4^n;\ n \leq 1; a_0 = 1$, $a_1 = 8$ by using generating function. (8)

(ii) Use mathematical induction to prove that $H_2 n \geq 1 + \frac{n}{2}$, where $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{j}$. (8)

13. (a) (i) If M_2 is the set of 2×2 non-singular matrices over R, viz, $M_2 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Big/ a,b,c,d \in R \text{ and } ad - bc \neq 0 \right\}$ prove that M_2 is a group under the operating of usual matrix multiplication. (8)

(ii) If H is a subgroup of G such that $x^2 \in H$ for every $x \in G$, prove that H is a normal subgroup of G. (8)

Or

(b) (i) State and prove Lagrange's theorem. (8)

(ii) If * is defined on $S = N \times N$ by $(a,b) * (a',b') = (a + a', b + b')$ and if the mapping $f : (S,*) \to (z,+)$ is defined by $f(a,b) = a - b$, show that f is a homomorphism. (8)

14. (a) (i) If R is a partially ordered relation on a set x and $A \leq x$, show that $R \cap (A \times A)$ is a partial ordering relation on A. (8)

(ii) Let $A = \{1, 2, 3, 4\}$ and $P(A)$ be its power set. Let $\subseteq$ be the inclusion relation on $P(A)$. Draw the Hasse diagram. (8)

Or

(b) (i) Show that in a complemented lattice $(L, \leq)$, $a \leq b \Leftrightarrow a' \vee b = 1 \Leftrightarrow a \wedge b' = 0 \Leftrightarrow b' \leq a'$. (8)

(ii) Express the following Boolean expressions E in terms of its minterm canonical form :

(1) $E = x(xy' + x'y + y'z)$

(2) $E = (x' + y)' + y'z$

(3) $E = y(x + y + z)'$

(4) $E = (x + y'z)(y + z')$. (8)

15. (a) (i) Given the regular expression $(a + b)^* b(a + b + \wedge)$

(1) Construct the $\wedge$-NFA

(2) Construct DFA from NFA

(3) Show the transitions of the DFA for a string "*abbb*". (8)

(ii) Construct derivation trees for the words (1) *ababbbba* (ii) *bbbcbba* using the grammars G_1 and G_2, respectively, where G_1 consists of the production $\{S \to AbS, A \to aS, S \to ba \text{ and } A \to b\}$ and G_2 consists of the productions $\{S \to bcS, S \to bbS, S \to cb, S \to a\}$. (8)

Or

(b) (i) Find a regular grammar that generates the language $L = (a^m b^n c^p / m, n, p \geq 1)$. (8)

(ii) Design a finite state automation that accepts the string of natural numbers (0 through 9) which are divisible by 3. (8)

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Question Paper Code : 46351

M.C.A. DEGREE EXAMINATION, AUGUST 2014.

First Semester

DMC 7101 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $p \to (q \to p)$ is a tautology.

2. "Not everybody is your friend or someone is not perfect" translate into logical expression using predicates and quantifiers.

3. How many 8-bit strings contain exactly three 0's?

4. Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n,\ n \geq 0$.

5. Prove that identity of a subgroup is same as that of the group.

6. Define a cyclic group.

7. Define a lattice and give a an example of a lattice.

8. State and prove absorption law in Boolean Algebra.

9. Define finite state automation.

10. Construct a Turing machine that computes the function $f(m, n) = m$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Without constructing the truth tables, find the principal conjunctive normal forms of $(p \wedge q) \vee (\neg p \wedge q \wedge r)$. (8)

(ii) Show that $(t \wedge s)$ can be derived from the premises $p \to q, q \to \neg r, r, p \vee (t \wedge s)$. (8)

Or

(b) (i) Without constructing the truth tables, find the principal disjunctive normal forms of $p \wedge \neg(q \wedge r) \vee (p \to q)$. (8)

(ii) Show that the conclusion $\forall x(P(x) \to \neg Q(x))$ follows from the premises $\exists x(P(x) \wedge Q(x)) \to \forall y(R(y) \to S(y))$ and $\exists y(R(y) \wedge \neg S(y))$. (8)

12. (a) (i) Solve the reccurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, $n \geq 0$, given that $a_0 = 1$ and $a_1 = 4$. (8)

(ii) Show that $1.1!+2.2!+3.3!+.... + n.n! = (n+1)!-1$ when ever n is a positive integer using mathematical induction. (8)

Or

(b) (i) Find the number of primes less than 100 using the principle of inclusion and exclusion. (8)

(ii) Use generating functions to solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$. (8)

13. (a) (i) If {G, *} is an abelian group, show that $(a * b)^n = a^n * b^n$, for all $a, b \in G$ where n is a positive integer. (8)

(ii) Show that every group of prime order is cyclic. (8)

Or

(b) (i) Show that the intersection of two normal subgroups of a group G is also a normal subgroup of G. (8)

(ii) Prove that the kernel of a homomorphism f from a group $(G, *)$ to another (G', Δ) is a Sub group of (G, *). (8)

14. (a) (i) If $\{L, \leq\}$ is a Lattice in which $\vee$ and $\wedge$ denote the operations of join and meet respectively, then for $a, b \in L$,

$$a \leq b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a.$$ (8)

(ii) If $P(S)$ is the power set of a set S and $\cup$ and $\cap$ are taken as the join and meet, prove that $\{P(S), \subseteq\}$ is a lattice. (8)

Or

(b) (i) Show that in a distributive $(L, \vee, \wedge\}$ lattice if an element $a \in L$ has a complement, then it is unique. (8)

(ii) In a Boolean algebra, prove that the following statements are equivalent :

(1) $a + b = b$

(2) $a.b = a$

(3) $a' + b = 1$

(4) $a.b' = 0$. (8)

15. (a) (i) Construct a deterministic finite-state automation that recognizes the set of all bit strings that contain exactly three 0's. (8)

(ii) Prove that a set is generared by a regular grammar if and only if it is a regular set. (8)

Or

(b) (i) Prove that if $G = \{V_N, V_r, S_0, P\}$ is a regular grammar which generates the regular Language $L(G)$, then there exists a FSA 'M' which accepts $L(G)$ and conversely. (8)

(ii) Construct a Turing machine that will accept the language L on $\{a, b\}$, given by $L = \{w : |w| \text{ is a multiple of } 3\}$. (8)

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Question Paper Code : 96351

M.C.A. DEGREE EXAMINATION, FEBRUARY/MARCH 2014.

First Semester

DMC 7101 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulations 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the predicate "x is the father of the mother of y".

2. Find the truth table for $P \to Q$

3. There are n stations on a Railway. Prove that a train can stop at three of these in $\frac{1}{6}(n-2)(n-3)(n-4)$ ways, no two of the stopping stations being consecutive.

4. Form the recurrence relation, given $f_n = 3.5^n, n \geq 0$.

5. Let $(G,*)$ be a group. Then show that for any $a \in G$, the inverse of a is unique.

6. Define normal subgroup.

7. Draw the Harse diagram of a set of all divisors of 24.

8. Write the dual of the following statements.

   (a) $(0.a)+(b.1)=b$.

   (b) $(a+0)+(1+a')=1$.

9. Obtain the grammar for Language, $L = \{a^n b^n / n \geq 0\}$.

10. Construct on NFA with 3 states that accepts the language $0^*1^*0^*0^*$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Construct the truth table for $(p \vee q) \vee (p \wedge s)$. (8)

(ii) Obtain the principal conjunctive normal form for $(p \wedge q) \vee (\sim p \wedge q \wedge r)$. (8)

Or

(b) (i) Is the conclusion $(\exists z)\, p(z)$ valid from the premises $(\forall x)[p(x) \to \phi(x)]$, $(\exists y)\, p(y)$. (8)

(ii) Show that $\sim (p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. (8)

12. (a) (i) Using generating function, solve the differences equation $y_{n+2} - 6y_{n+1} + 8y_n = 0$, $y_0 = 1, y_1 = 4$. (8)

(ii) Use mathematical induction to show that 3 divides $n^3 + 2n$ whenever n is a non-negative integer. (8)

Or

(b) (i) Find the general solution of $f(n) - 3f(n-1) - 4f(n-2) = 4^n$. (8)

(ii) In a certain village, there are 3 sports clubs, the rugby club, the soccer club and the cricket club. Every one who belongs to the cricket club also belongs to the soccer club or rugby club (or both). The following additional information is known.

42 people belong to the soccer club;

45 people belong to the rugby club;

7 people belong to the both the soccer and rugby clubs;

11 people belong to the both the soccer and cricket clubs;

28 people belong to both rugby and cricket clubs; twice as many people belong only to the soccer club as belong only to the rugby club. Find the number of people in the village who belong to

(1) All three clubs

(2) The cricket club

(3) Only the soccer club. (8)

13. (a) (i) List all the elements of the permutation group $(D_3, 0)$ and form the composition table. (8)

(ii) If $f : (G \to G_1)$ is a group homomorphism, prove that f is one-to-one if and only if $\ker f = \{e\}$. (8)

Or

(b) (i) State and prove Lagrange's theorem. (12)

(ii) Define ring. (4)

14. (a) (i) Let L be a set of positive integers. Show that the "divide" relation is a partial ordered relation. (6)

(ii) In a Lattice, show that

$(a * b) \oplus (b * c) \oplus (c * a) \leq (a \oplus b) * (b \oplus c) * (c \oplus a)$. (10)

Or

(b) (i) Show that De Morgan's law $(a \wedge b)' = a' \vee b'$ holds in a complemented distributive lattice. (6)

(ii) Describe the truth table for the given K–Map. (10)

CD \ AB	00	01	11	10
00	0	4	12 1	8
01	1	5 1	13	9 1
11	3 1	7	15 1	11
10	2	6	14	10

15. (a) (i) Show that the set $L = \{a^{i^2} / i \geq 1\}$ is not regular. (8)

(ii) What are the applications of finite state machines? Construct a DFA with reduced states equivalent to the regular expression $(ab + (b + aa))b * a$. (8)

Or

(b) (i) Prove the theorem : For every context-free grammar G_1, there is an equivalent grammar G_2 in chomsky normal form. Find a grammar in Chomsky Normal form equivalent to $S \to cCdD,\ C \to cC / c,\ D \to dD / d$. (8)

(ii) Construct a DFA equivalent to an NFA whose transition table is defined by (8)

state	a	b
(initial) q_0	$\{q_1, q_3\}$	$\{q_2, q_3\}$
q_1	q_1	q_3
q_2	q_3	q_2
(final) q_3	ϕ	ϕ