# More Divide and Conquer

# Lecture 15

Today

- Merge Sort:  a Divide and Conquer Sort

# Different Sorts of Sorts

So far, we have seen two implementations of sorting:

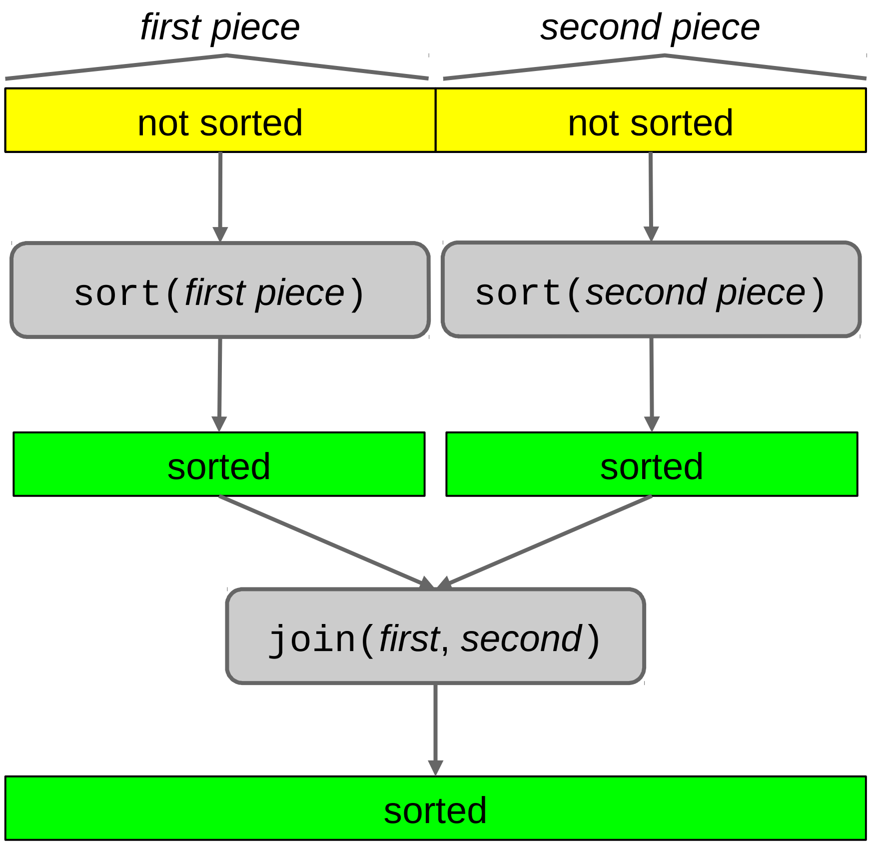
- Selection Sort - find the min, swap it with position 0; find the second min, swap it with position 1; . . . ; working incrementally - $O(N^2)$
- Insertion Sort - incrementally insert an element to a growing list of sorted elements - also $O(N^2)$

To get better performance, we need a non-incremental algorithm

# Sorting by Recursion (Review)

Use Divide and Conquer to sort recursively.

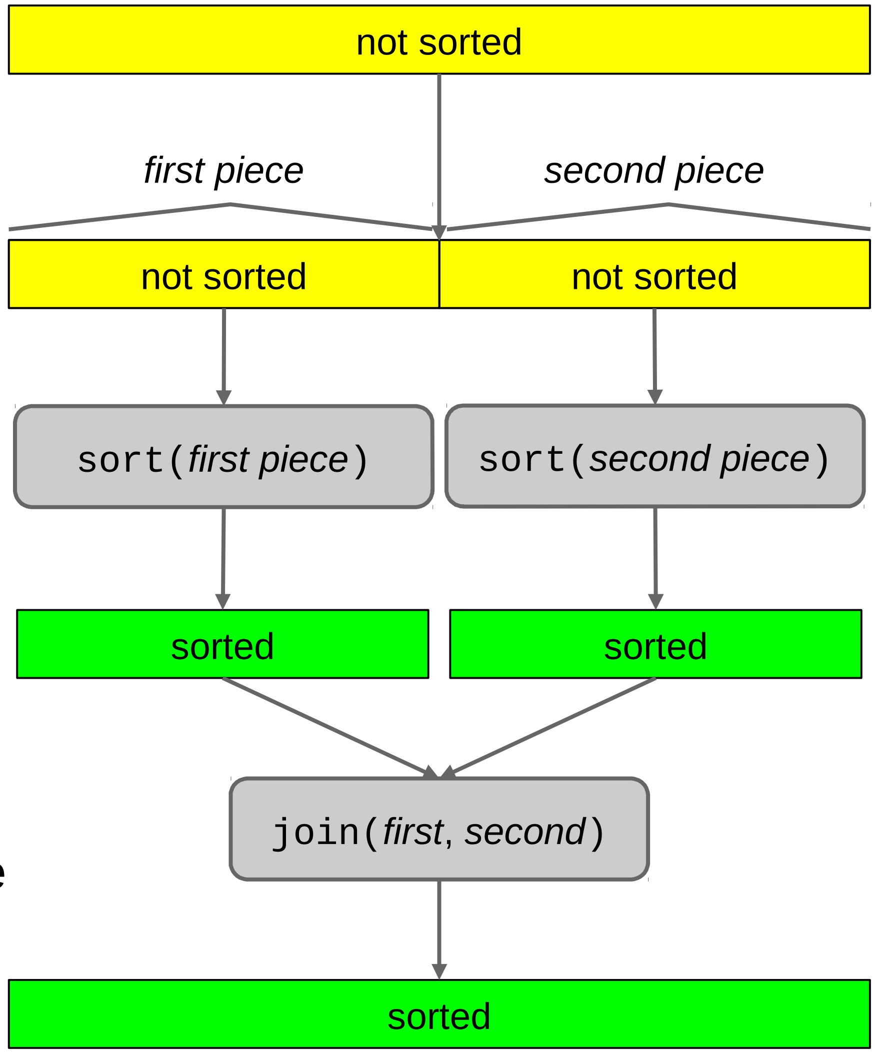
1. Split the array into two roughly equal pieces.
2. Recursively sort each half.
   - This works because each piece is *smaller*.
3. Join the two pieces together to make one sorted array.



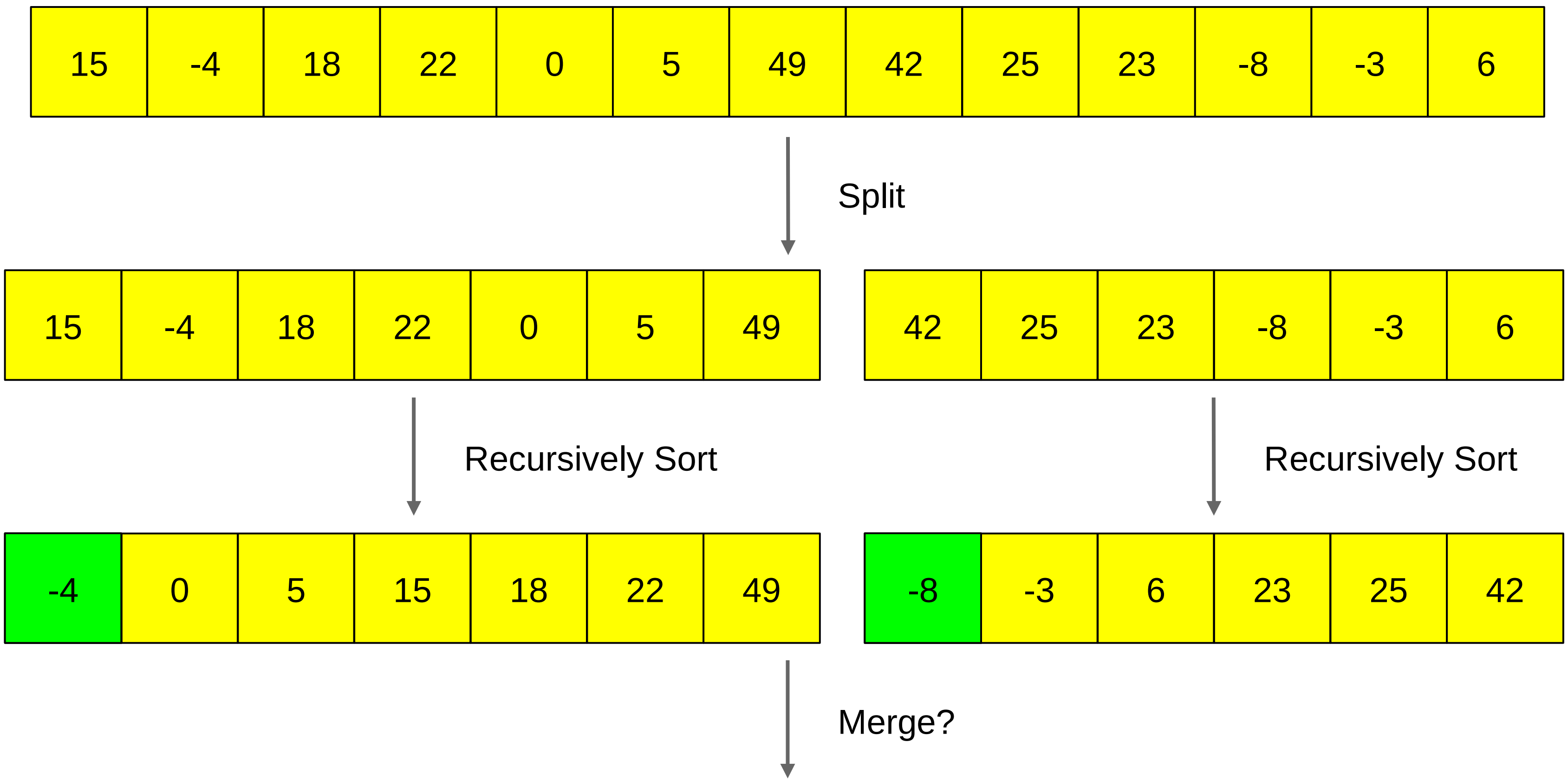

Two famous sorts behave this way: *mergesort* and *quicksort*.

# Merge Sort

1. Split the array into two roughly equal pieces.
   - split by index: `[first..mid]` and `[mid+1..last]`
2. Recursively sort each half.
   - two recursive calls to `sort()`
   - assume smaller cases are sorted correctly
3. Join the two pieces together to make one sorted array.
   - Q. How can you quickly combine two sorted pieces into one?
   - *Merge* the two arrays



# Example




Merge strategy is similar to Selection Sort: repeatedly find the min and place it.

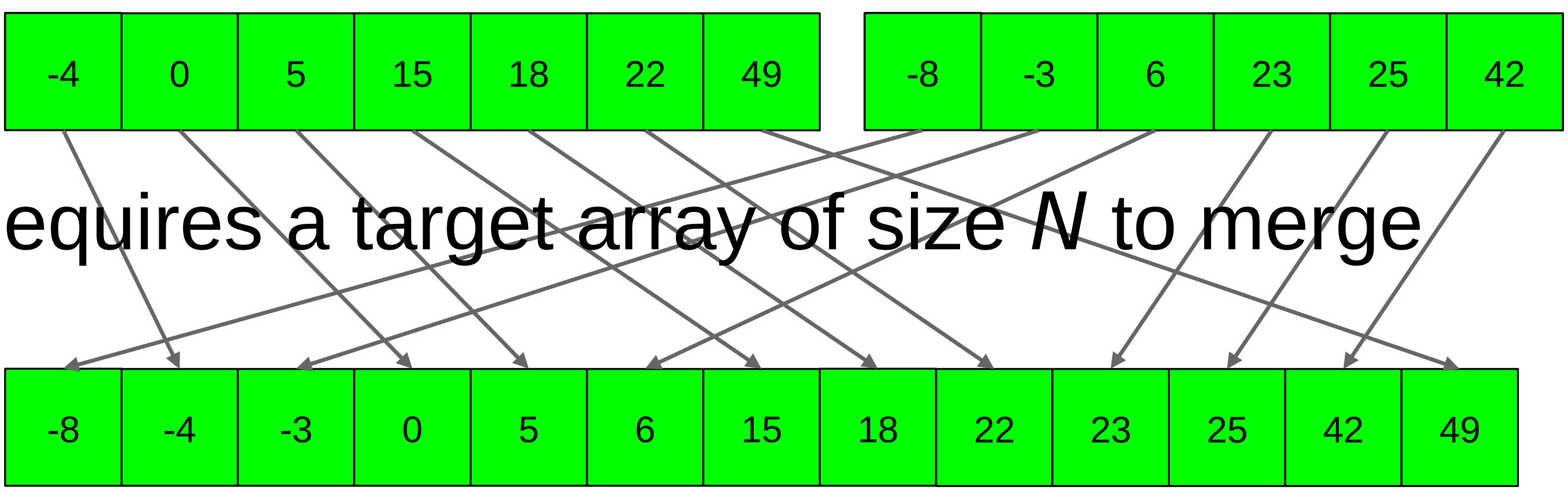
Q. How much time is required to find the min?

- it must be one of the heads of the two sorted subarrays. $\Rightarrow O(1)$

# Merge Example

Strategy:

1. Find the min.  Where is it?
    - It must be one of the heads of the two sorted subarrays
    - Compare and take the smaller.
2. Place the min into the next sequential position.




Requires a target array of size *N* to merge

# MergeSort Code

```
//  Post:  arr[first..last] are sorted
void mergeSort(int arr[], int first, int last) {
    //  Base case
```

- Base case
  - return if fewer than 2 elements

```
    //  Split array
```

- Split array into two roughly equal pieces
  - compute `mid` element

```
    //  Recursively sort
```

- Recursively sort each piece

- Join the two sorted pieces together by merging
  - place the smallest min of each sorted piece

```
}
```

# MergeSort Code

```
//  Post:  arr[first..last] are sorted
void mergeSort(int arr[], int first, int last) {
   //  Base case
   if (last <= first) return;

   //  Split array
   int mid = (first+last) / 2;

   //  Recursively sort
   mergeSort(arr, first, mid);
   mergeSort(arr, mid+1, last);

   //  Join
   merge(arr, first, mid, last);
}
```

# MergeSort Code

```
//  Pre:  arr[first..mid] and arr[mid+1..last] are sorted
//  Post:  arr[first..last] are sorted
void merge(int arr[], int first, int mid, int last) {
```

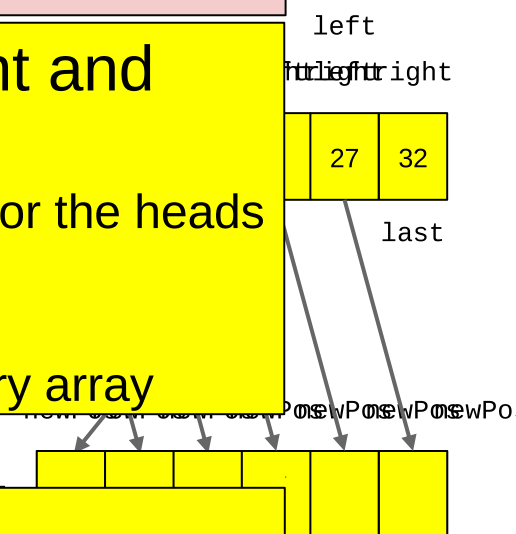
An array bounds error occurs when you run out of elements from the left piece or on the right piece.

- Repeat for N elements
  - Take the smallest unplaced element and place into position
    - Maintain indices `leftPos`, `rightPos` for the heads of each piece
    - Compare the heads
    - Place the min in sequence into a temporary array

```
}
```

- Copy temporary array → `arr[]`

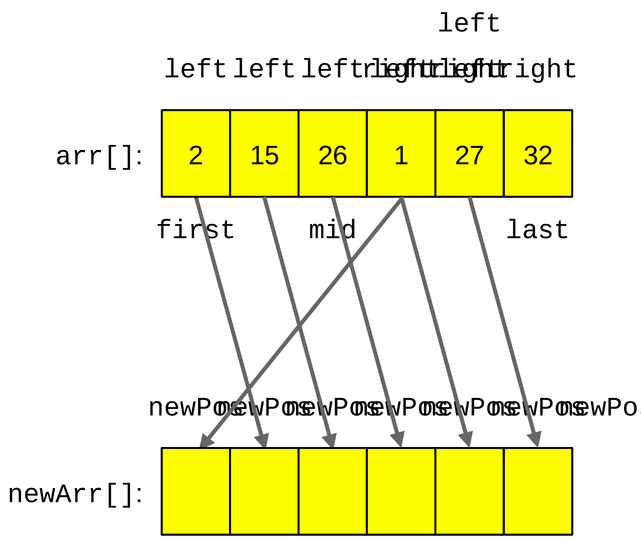
post-increment operator



# Merge Code

An array bounds error occurs when you run out of elements from the left piece or on the right piece.

```
//  Pre:  arr[first..mid] and arr[mid+1..last] are sorted
//  Post:  arr[first..last] are sorted
void merge(int arr[], int first, int mid, int last) {
     int len = last-first+1;  int newArr[len];
     int leftPos = first;  int rightPos = mid+1;
     for (int newPos = 0; newPos < len; newPos++) {
          if (arr[leftPos] < arr[rightPos]) {
               newArr[newPos] = arr[leftPos++];
          } else {
               newArr[newPos] = arr[rightPos++];
          }
     }
     arrCpy(arr + first, newArr, len);
}
```




post-increment operator

# A Bug!

The merge strategy:

- Take the smallest [remaining] element of each sorted piece and place into position
- Fails when one piece runs out of elements

Solutions:

- Append $+\infty$ to the end of each piece
    - good in theory, but has practical issues
- Copy remaining elements from unfinished piece
    - a while loop will be required

# Merge Code - Fixed

```
//  Pre:  arr[first..mid] and arr[mid+1..last] are sorted
//  Post:  arr[first..last] are sorted
void merge(int arr[], int first, int mid, int last) {
     int len = last-first+1;  int newArr[len];
     int leftPos = first;  int rightPos = mid+1;  int newPos = 0;
     while(leftPos <= mid && rightPos <= last) {
          if (arr[leftPos] < arr[rightPos]) {
               newArr[newPos++] = arr[leftPos++];
          } else {
               newArr[newPos++] = arr[rightPos++];
          }
     }

     //  Flush non empty piece
     arrCpy(newArr + newPos, arr + leftPos, mid - leftPos + 1);
     arrCpy(newArr + newPos, arr + rightPos, last - rightPos + 1);

     arrCpy(arr + first, newArr, len);
}
```

Q.  What's the running time for `merge()`?

# Running Time Analysis

- $O(N)$ work per row
- $O(\log N)$ rows

$\Rightarrow$ $O(N \log N)$ running time

Visualize with a *recursion tree*: