Announcements

- Assignment 3 due 15:20:00 on Fri. Feb. 1

- Assignment 4 due 15:20:00 on Wed. Feb. 6
  - Will be posted tonight

- Please hand in assignments by uploading a pdf to CourSys

Binary Search

CMPT 125
Jan. 30

Lecture 12

Today

- Binary Search

Searching Overview (Review)

- It is often useful to find out whether or not an array contains a particular item
  - A search can either return true or false
  - OR . . . the position of the item in the array (-1 for fail)
- Searching is one of those activities that can be done much more efficiently if the set is sorted ahead of time
  - Best for unordered array is a *linear search* - $O(N)$

What if the array was ordered?

Think of searching a dictionary for a word?

- Strategy: *Not* one word at a time in sequential order starting from aardvark, etc.

- Strategy: Jump to where you estimate the word to be based on what you know about the alphabet.

  Refine your jumps + hone in on the correct page quickly.

This is the main idea behind *binary search*.

Divide and Conquer

Generic Strategy (*Paradigm*):

1. **Divide:** Cut the array into 2 or more roughly equally sized pieces

1. **Conquer:** Use what you know about the pieces to solve the original problem

Binary Search

Strategy: Divide and Conquer

1. Examine the *middle* element of the array of candidates. This divides the array into two [roughly] equal halves.
2. Compare the middle element with the target.
   - If middle < target then throw out the first half.
   - But if middle > target then throw out second half.
3. Repeat 1-3 until middle == target (found!) or no candidates remain (fail!).

E.g., target = `42`:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-8	-7	-5	-2	0	4	6	7	17	20	28	29	42	49	64

return true
or index=12

Binary Search

Requirements (Pre-Conditions):

- Candidate array must be sorted

How to keep track of the list of candidates?

- Use integers `first` and `last` for remaining candidates `arr[first..last]`
- Initially, `first=0`; `last=len-1`
- Middle element is at index `(first+last)/2`

E.g., target = `42`:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
-8	-7	-5	-2	0	4	6	7	17	20	28	29	42	49	64

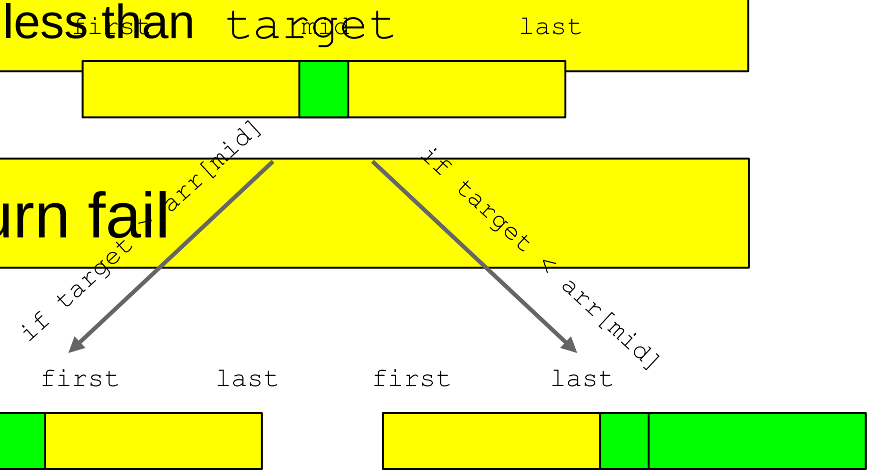
first =	0	8	12	12
last =	14	14	14	12
mid =	7	11	13	12

Binary Search Code

```
int BinarySearch(int arr[], int len, int target) {
```

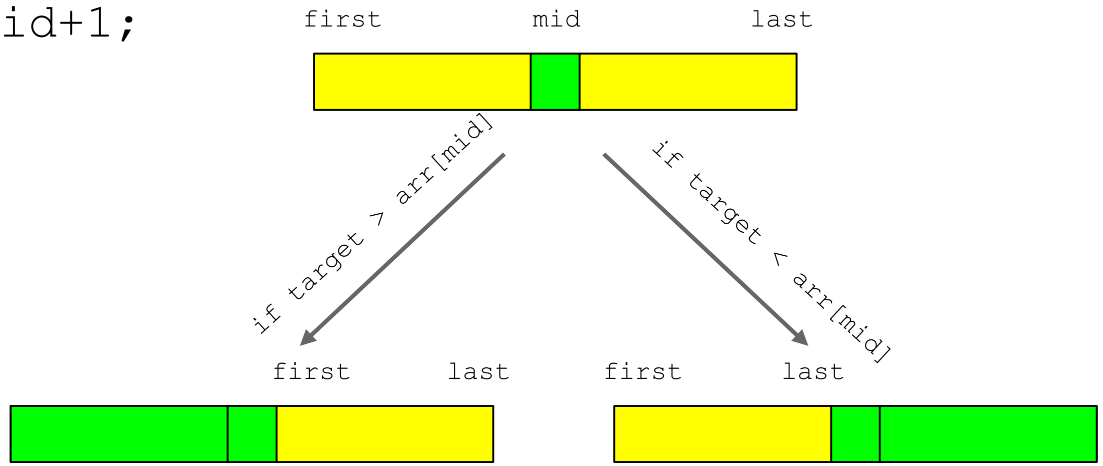
- Search candidate array `arr[first..last]` while not empty
  - Compare with the middle element
  - Algorithm:
    - found if equal to `target`, so return position
    - throw out second half if greater than `target` OR
    - throw out first half if less than `target`
- No candidates, so return fail

```
}
```



Binary Search Code

```
int BinarySearch(int arr[], int len, int target) {
    int first = 0;
    int last = len-1;
    while(first <= last) {
        //   Q.  What's a good assertion this time?
        int mid = (first+last) / 2;
        if (target == arr[mid]) return mid;
        if (target < arr[mid]) last = mid-1;
        else first = mid+1;
    }
    return -1;
}
```



Binary Search - Loop Free Version

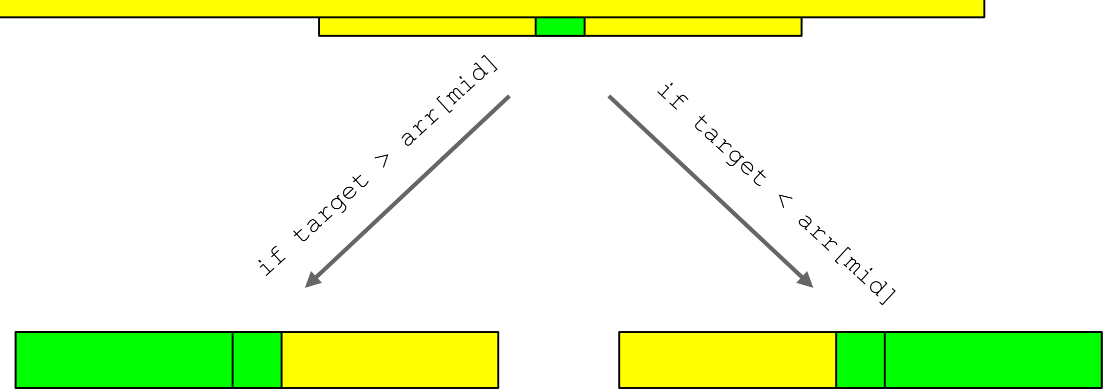
```
int BinarySearch(int arr[], int len, int target) {
```

- Search candidate array `arr[0..len-1]`
- Algorithm:
  - return fail if empty

```
    int mid = len/2;
```

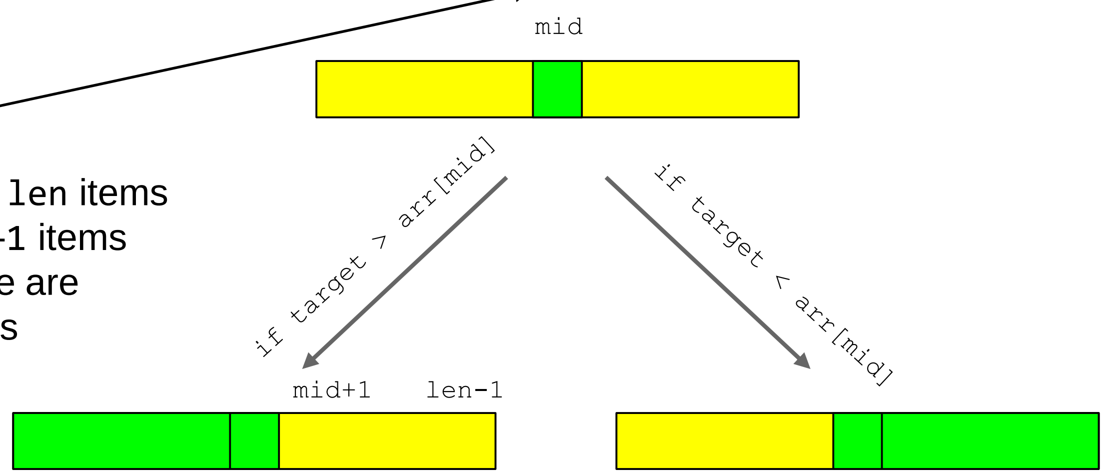
- Compare with the middle element + re-search
- Algorithm:
  - found if equal to `target`, so return true
  - throw out second half if greater than `target` OR
  - throw out first half if less than `target`

```
                                                                  arget);
                                                              ;
}
```



Binary Search - Loop Free Version

```
int BinarySearch(int arr[], int len, int target) {
    if (len <= 0) {
        return 0;
    }
    int mid = len/2;
    if (target == arr[mid]) return 1;
    if (target < arr[mid]) return BinarySearch(arr,mid,target);
    else return BinarySearch(arr+mid+1,len-mid-1,target);
}
```



If we go from index `0` to `len-1`, there are `len` items
If we go from index `a` to `b`, there are `b-a+1` items
If we go from index `mid+1` to `len-1`, there are
`(len-1) - (mid+1) + 1` items

Analysis of Binary Search

What's the worst case on an array of length N?

- After one iteration, the possible candidates are [roughly] cut in half.

After k iterations, how many candidates remain?
- Roughly $N / 2^k$

When do you run out of candidates?

- when $2^k \geqslant N$
- i.e., after $k \geqslant \log_2 N$ iterations

Thus binary search runs in $O(\log N)$.

Linear Search vs Binary Search

Even though the inner loop of binary search is more complex than linear search, we expect $O(\log N)$ to outperform $O(N)$ as N gets large.

N	Linear Search $(3 + 4N)$	Binary Search $(4 + 12\log_2(N+1))$
1	7	16
3	15	28
7	31	40
15	63	52
100	403	88
1000	4003	124
10^6	4000003	244
10^9	4×10^9	364

Linear Search vs Binary Search

- Binary search has a fast running time.
- Disadvantages?
  - Harder to code
  - Requires the array be sorted
- Keeping the array sorted can be expensive!
  - Significantly more searching than update? Keep list sorted (slow) and use (fast) binary search
  - Significantly more update than search? Keep array unsorted (fast) and use (slow) linear search