Patient	Fever	Cough	Diagnosis
1	Yes	No	Cold
2	No	Yes	Cold
3	Yes	Yes	Pneumonia
4	Yes	Yes	Pneumonia
5	No	No	Healthy
6	Yes	No	Cold
7	No	Yes	Cold
8	Yes	Yes	Pneumonia
9	No	No	Healthy
10	Yes	Yes	Pneumonia

- P(Cold)= 4/10 =0.4
- P(Pneumonia) = 4/10 =0.4
- P(Healthy) =2/10= 0.2

Likelihoods:

For Cold:

- P(Fever=Yes|Cold)=4/4=1.0
- P(Cough=Yes|Cold)=2/4=0.5

For Pneumonia:

- P(Fever=Yes|Pneumonia)=4/4=1.0
- P(Cough=Yes|Pneumonia)=4/4=1.0

For Healthy:

- P(Fever=Yes|Healthy)=0
- P(Cough=Yes|Healthy)=0

Apply Bayes' Theorem,

Let E denote the observed symptoms (Fever = Yes, Cough = Yes).

Calculate P(E) the total probability of observing Fever and Cough,

$P(E) = P(E|Cold) \cdot P(Cold) + P(E|Pneumonia) \cdot P(Pneumonia) + P(E|Healthy) \cdot P(Healthy)$

$P(E) = (1.0 * 0.5) * 0.4 + (1.0 * 1.0) * 0.4 + (0 * 0) * 0.2$

$P(E) = 0.2+0.4+0$

$P(E) = 0.6$

Calculate P(Cold | E) the probability of having Cold given Fever and Cough,

$P(Cold \mid E) = P(E \mid Cold) \cdot P(Cold) / P(E)$

$P(Cold \mid E) = (1.0 * 0.4) / 0.6$

$P(Cold \mid E) = 0.4 / 0.6$

$P(Cold \mid E) = 2 / 3$

$P(Cold \mid E) \approx 0.67$

Calculate P(Pneumonia | E) the probability of having Pneumonia given Fever and Cough,

$P(Pneumonia \mid E) = P(E \mid Pneumonia) \cdot P(Pneumonia) / P(E)$

$P(Pneumonia \mid E) = (1.0 * 0.4) / 0.6$

$P(Pneumonia \mid E) = 0.4 / 0.6$

$P(Pneumonia \mid E) = 2 / 3$

$P(Pneumonia \mid E) \approx 0.67$

According to Bayes' Theorem and the given data, if a patient presents with both fever and cough, the probabilities of diagnoses are near about,

$P(Cold \mid Fever = Yes, Cough = Yes) \approx 0.67$

$P(Pneumonia \mid Fever=Yes, Cough=Yes) \approx 0.67$

For probability matrix :

A = [[0.67 0.33 0.00]

[0.33 0.67 0.00]

[0.00 0.00 0.00]]

Eigenvalues :

det(A−λ*I) = 0, where I is the identity matrix.

Eigenvalues of A,

$\lambda 1 \approx 1.00$,

$\lambda 2 \approx 0.34$,

$\lambda 3 \approx 0.00$

Eigenvectors :

AV = λV

Corresponding eigenvectors,

- For $\lambda 1 \approx 1.00$:

  V1 ≈ [[0.71 0.71 0.00]]

- For $\lambda 2 \approx 0.34$:

  V2 ≈ [[- 0.71 0.71 0.00]]

- For $\lambda 3 \approx 0.00$:

  V3 ≈ [[0.00 0.00 1.00]]

Determinant of the Matrix

The determinant det(A) of matrix A,

det(A) = 0.67 * (0.67 * 1.00 − 0.00 * 0.33) − 0.33 * (0.33 * 1.00 − 0.00 * 0.33)

det(A) = 0.67 * 0.67 − 0.33 * 0.33

det(A) = 0.4489 − 0.1089

det(A) = 0.34