Analisi 1

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Differential Equations

"Would you tell me, please, which way I ought to go from here?"
"That depends a good deal on where you want to get to," said the Cat.

ODEs and PDEs

Ordinary Differential Equations (ODEs) are a differential equation which has a single variable. ODEs have a general form:

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, ..., \frac{d^ny}{dx^n}\right) = 0 \tag{1}$$

where
- x independent
- y dependent

Partial Differential Equations (PDEs) are a Differential equation which has multiple independent variables. Instead of using the standard d, they use partial derivatives (∂) to show the change with respect for multiple variables. The general form is:

$$F\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x^2}, ...\right) = 0 \tag{2}$$

where
- x,y independent
- u(x,y) dependent

Fundamentally image a ODE as a means to track a single car, while PDE track all the traffic in the city.

Types of ODEs

ODEs are usually classified by 2 primary things. their order, aka the degree of their derivative, and by whether they are linear or non-linear.

First order ODEs are pretty self explanatory, they involve only the 1st derivative. Here is a basic first order ODE:

$$\frac{dy}{dx} + y = x \tag{3}$$

Second order ODEs involve UP to the 2nd derivative. Here is a example:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0 \tag{4}$$

Higher order ODEs involve everything 3rd derivative or higher. It is unlikely to ever appear in a 1st year analisis exam, but you never know.

Liniar and Non-liniear ODEs. An ODE is linear if the dependent variable and the derivatives are in a linear form. Basically: they are not multiplied together. Anything else is considered non-liniear. A linear ODE can be written in the form:

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + ... + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x) \tag{5}$$

- a(x) is a function of x

Here are some basic examples. We will go in much more detail when solving ODEs.

$\frac{dy}{dx} + 3y = x$, and $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = sinx$

(NON-LINIAR) To be added

The weird classifications of ODEs

A Homogeneous ODE, is a differential equation where L is a linear differential operator.

$$L[y] = 0$$

A Non-Homogeneous ODE, is a differential equation where f(x) $\neq 0$

$$L[y] = f(x)$$

A Autonomous ODE, is a differential equation where the independent variable (usually x or t) does not appear in the equation.

$$\frac{d^ny}{dx^n} = F\left(y, y', ..., y^{(n-1)}\right)$$

A Non-Autonomous ODE, is a differential equation if the independent variable appears eq (1).

*You can be multiple classifications at once, use your head.

Dealing and solving ODEs

A separable ODE can be written as

$$\frac{dy}{dx} = f(x)g(y)$$

$$\frac{dy}{g(y)} = f(x)dx$$

$$\int \frac{dy}{g(y)} = \int f(x)dx$$

Integrating Method

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$\mu(x) = e^{\int P(x)dx}$$

$$\frac{d}{dx}(\mu(x)y) = \mu(x)Q(x)$$

$$y = \frac{1}{\mu(x)}\int \mu(x)Q(x)dx + C$$

Quick note on integrating factor.

A integrating factor is the method used to solve derivative equation above. The μ(x) also called the integrating factor, works for any, first-order linear differential equations. A function is derived by multiplying the equation with μ(x), which makes the left-hand side a derivative of μ(x)y

Second Order Equations Homogeneous Equations with Constant Coefficients

$$Not$$

Nonhomogeneous Equations: Method of Undetermined Coefficients

$$Finished$$

Special theorems and Problems

Cauchy's Problem