

Notes on Electromagnetism

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Lesson 1: Coulomb's Law

Coulomb's Law (Legge di Coulomb) describes the electrostatic force (forza elettrostatica) between two static charged particles (particelle cariche). The force \vec{F}_{12} that a charge (carica) q_1 exerts on a second charge q_2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The force is directed along the line connecting the two charges.

The vector form of the equation is:

$$\vec{F}_{12} = k \frac{q_1 q_2}{R^2} \hat{R}_{12} \quad (1)$$

Here, k is Coulomb's constant (costante di Coulomb), R is the distance between the charges, and \hat{R}_{12} is the unit vector (versore) pointing from q_1 to q_2 . The law is reciprocal, meaning the force exerted by q_2 on q_1 is equal in magnitude and opposite in direction, so $\vec{F}_{12} = -\vec{F}_{21}$.

Coulomb's constant is expressed in terms of the vacuum permittivity (permittività del vuoto), ϵ_0 , as:

$$k = \frac{1}{4\pi\epsilon_0} \quad (2)$$

Following the 2019 redefinition of the SI base units (Unità di base del SI), the elementary charge (carica elementare) e is a defining constant. As a result, ϵ_0 is now an experimentally determined value with an associated uncertainty, not a defined quantity.

The electric force is significantly stronger than the gravitational force. For instance, the electric force (forza elettrica) F_E between a proton and an electron is many orders of magnitude greater than the gravitational force (forza gravitazionale) F_G between them:

$$F_G = G \frac{m_p m_e}{R^2} \approx 10^{-47} N \quad (3)$$

$$F_E = \frac{1}{4\pi\epsilon_0} \frac{e^2}{R^2} \approx 10^{-7} N \quad (4)$$

Lesson 2: Electric Flux and Gauss's Law

The concept of electric flux quantifies the flow of an electric field (campo elettrico) through a surface. For a uniform electric field \vec{E} and a flat surface area \vec{A} , the flux is the scalar product $\Phi_E = \vec{E} \cdot \vec{A}$. For non-uniform fields and curved surfaces, the flux is found by integrating the field over the surface:

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A} \quad (5)$$

This integral effectively sums the component of the electric field perpendicular to the surface at every point.

Gauss's Law (Legge di Gauss), a cornerstone of electrostatics (elettrostatica) and one of the four fundamental Maxwell's Equations (Equazioni di Maxwell), relates the electric flux through a closed surface (superficie chiusa) to the net electric charge enclosed within it. The law states that the net electric flux is directly proportional to the enclosed charge.

In its integral form, Gauss's Law is expressed as:

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \quad (6)$$

The integral is performed over a conceptual closed surface known as a Gaussian surface (superficie Gaussiana), and Q_{enc} is the total charge it encloses. By applying the divergence theorem (teorema della divergenza), which connects the surface integral of a vector field to the volume integral of its divergence, we can derive the differential form of Gauss's Law:

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ (7) Here, ρ represents the volume charge density. This form locally relates the divergence of the electric field to the charge density at that point.

A key application of Gauss's Law is to calculate electric fields in situations with high symmetry. By choosing a Gaussian surface that mirrors the symmetry of the charge distribution (distribuzione di

carica), the flux integral becomes simple to solve. For example, to find the field from an infinite line of charge with a uniform linear charge density (densità di carica lineare) λ , we use a cylindrical Gaussian surface. The symmetry dictates that the electric field must point radially outward. The flux through the top and bottom caps of the cylinder is zero, and the flux through the side wall is $E \cdot (2\pi rL)$. The enclosed charge is λL . Applying Gauss's Law, $E(2\pi rL) = \lambda L/\epsilon_0$, yields the field:

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (8)$$

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Lesson 3: Applications of Gauss's Law

Gauss's Law is a powerful tool for calculating the electric field for charge distributions with a high degree of symmetry, such as spherical, cylindrical, or planar symmetry. The symmetry of the charge density ρ is inherited by the electric field \vec{E} , which simplifies the calculation.

A classic example is the field of a uniformly charged sphere of radius R and total charge Q . Due to spherical symmetry (simmetria sferica), the electric field must be radial, $\vec{E} = E(r)\hat{r}$. By choosing a spherical Gaussian surface of radius r , the flux integral simplifies to $\oint_S \vec{E} \cdot d\vec{S} = E(r) \cdot (4\pi r^2)$. Outside the sphere ($r > R$), the enclosed charge is the total charge Q . Applying Gauss's law gives $E(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$, which is identical to the field of a point charge (carica puntiforme) Q at the origin. Inside the sphere ($r \leq R$), the enclosed charge is only the fraction of charge within the radius r , $Q_{\text{int}} = Q \frac{r^3}{R^3}$. Gauss's law then yields $E(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r$, showing that the field increases linearly from the center.

This method can be extended to other symmetric configurations. For a charged spherical shell, the electric field inside the inner cavity is zero. For a sphere with an off-center cavity, the superposition principle (principio di sovrapposizione) can be used by treating the cavity as a superposition of a sphere with negative charge density over a larger sphere with positive charge density. This reveals a uniform electric field inside the cavity. Gauss's law also efficiently re-derives the fields for an infinite line of charge and an infinite plane of charge.

The concept of divergence of a vector field (campo vettoriale) measures the field's tendency to originate from or converge to a point. For an electric field, divergence signifies the presence of a charge source or sink. The Divergence Theorem (Teorema della Divergenza) relates the flux of a field through a closed surface to the integral of its divergence over the enclosed volume. Combining this theorem with Gauss's Law leads to its differential form:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (9)$$

This is the first of Maxwell's Equations, stating that the divergence of the electric field at any point is proportional to the charge density at that same point.

Lesson 4: Superposition, Conductors, and Plasma Oscillations

The superposition principle (principio di sovrapposizione) is a fundamental concept for calculating electric fields from complex charge distributions. A powerful illustration is the case of two overlapping spheres of the same radius R , one with uniform positive charge density $+\rho_0$ and the other with uniform negative charge density $-\rho_0$. The total electric field is the vector sum of the fields from each sphere. Using the known result for the field inside a uniformly charged sphere, $\vec{E} = \frac{\rho_0}{3\epsilon_0} \vec{r}$, we can calculate the field in the overlapping region. If the centers of the spheres are separated by a vector $\vec{\delta}$, the field in the overlapping region is found to be uniform and constant:

$$\vec{E}_{\text{overlap}} = -\frac{\rho_0}{3\epsilon_0} \vec{\delta} \quad (10)$$

This model provides an excellent analogy for the behavior of conductors (conduttori) in an external electric field. In a conductor, conduction electrons are free to move. When an external field \vec{E}_{ext} is applied, the "sea" of electrons displaces slightly, creating a charge separation, or polarization (polarizzazione). This separation induces an internal electric field (campo elettrico interno) \vec{E}_{int} that opposes the external field. The displacement continues until the net electric field inside the conductor is zero, a

condition known as electrostatic equilibrium (equilibrio elettrostatico). At this point, $\vec{E}_{ext} + \vec{E}_{int} = 0$. The conductor effectively shields its interior from the external static electric field.

If the external field is suddenly removed, the displaced electron sea is no longer in equilibrium and experiences a restoring force from the fixed positive ions. This force acts like a spring, leading to oscillations. The equation of motion for the electron sea is that of a simple harmonic oscillator (oscillatore armonico semplice). This collective, high-frequency oscillation of the electrons is a quantum mechanical effect known as ...IDK this ???. The characteristic angular frequency of this oscillation is the plasma frequency, ω_p , a property of the material that depends on the density of conduction electrons.

$$\omega_p^2 = \frac{n_e e^2}{m_e \epsilon_0} \quad (11)$$

This phenomenon is crucial in understanding the optical properties of metals.