Notes on Electromagnetism

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September 17, 2025

Class Information

Recommended Textbooks

- The Feynman Lectures on Physics
- Picasso - Lezioni di fisica

1 Coulomb's Law

Coulomb's law gives the electrostatic force between two charged particles. The force $\vec{F}_{12}$ exerted by a charge q_1 on a charge q_2 is:

$$\vec{F}_{12} = k\frac{q_1 q_2}{R^2}\hat{\vec{R}}_{12} \tag{1}$$

where k is Coulomb's constant, R is the distance between the charges, and $\hat{\vec{R}}_{12}$ is the unit vector pointing from q_1 to q_2. The force is reciprocal:

$$\vec{F}_{12} = -\vec{F}_{21} \tag{2}$$

Coulomb's constant k is:

$$k = \frac{1}{4\pi\epsilon_0} \tag{3}$$

where ϵ_0 is the permittivity of free space.

2 2019 Redefinition of SI Base Units

Since 2019, the SI base units are defined by setting the numerical values of seven defining constants, including the elementary charge, e. Consequently, the vacuum permittivity ϵ_0 is now a measured value with an associated uncertainty, rather than a defined constant.

For example, a comparison of the gravitational force F_G and the electric force F_E:

$$F_G = G\frac{m_p m_e}{R^2} \approx 10^{-47}N \tag{4}$$

$$F_E = \frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} \approx 10^{-7}N \tag{5}$$

3 The Electric Field

The electric field (*campo elettrico*) is a vector field representing the force per unit charge. It extends the concept of Coulomb's force (*forza di Coulomb*) to describe the influence of charges in space.

The electric field $\vec{E}$ at a point is defined as the force $\vec{F}$ on a test charge (*carica di prova*) q_0 at that point, divided by the charge. The limit as $q_0 \to 0$ is taken to not disturb the source charge:

$$\vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0} \tag{6}$$

The unit is Newtons per Coulomb (N/C), or Volts per meter (V/m).

For a single point charge (*carica puntiforme*) Q at the origin, the electric field is:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \tag{7}$$

where $\hat{r}$ is the radial unit vector (*versore*). For a charge Q_1 at $\vec{r}_1$, the field at $\vec{r}$ is:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{|\vec{r}-\vec{r}_1|^3}(\vec{r}-\vec{r}_1) \tag{8}$$

The superposition principle (*principio di sovrapposizione*) states that the total electric field from multiple charges is their vector sum.

Field lines (*linee di campo*) are used to visualize the electric field. They are tangent to the field vector at every point, originate from positive charges, and terminate on negative charges or at infinity. Their density indicates the field's strength.

For continuous charge distributions (*distribuzioni continue di carica*) (linear λ, surface σ, or volume ρ), the field is the integral of the contributions from each infinitesimal charge element. For a volume distribution:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^3}(\vec{r}-\vec{r}')dV' \tag{9}$$

For an infinite straight wire (*filo rettilineo infinito*) with uniform charge density λ, the field is $E = \frac{\lambda}{2\pi\epsilon_0 z}$ at distance z. For a charged ring (*anello carico*) of radius R and charge Q, the axial field is $E_z = \frac{1}{4\pi\epsilon_0}\frac{Qz}{(z^2+R^2)^{3/2}}$. For a uniformly charged disk (*disco carico*) of radius R and density σ, the axial field is $E_z = \frac{\sigma}{2\epsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$. For an infinite plane (*piano infinito*), the field is uniform (*campo uniforme*): $E = \frac{\sigma}{2\epsilon_0}$.

Gauss's Law (*teorema di Gaus*) relates the electric flux (*flusso*), Φ_E, through a closed surface (*superficie chiusa*) to the enclosed charge Q_{int}:

$$\Phi_E = \oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0} \tag{10}$$

where Q_{int} is the total internal charge (*cariche interne*).