# Analisi 1

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# Pre-Derivatives

## Theorems, functions and axioms

"Calvin: You know, I don't think math is a science, I think it's a religion.

Hobbes: A religion?

Calvin: Yeah. All these equations are like miracles. You take two numbers and when you add them, they magically become one NEW number! No one can say how it happens. You either believe it or you don't. [Pointing at his math book] This whole book is full of things that have to be accepted on faith! It's a religion!"

## Complex numbers

A complex number is defined, by $x, y \in \mathbb{R}$ and i as the imaginary unit.

$$z = x + iy \tag{1}$$

Imagine $\mathbb{R}$ covering the whole x axis, and $\mathbb{C}$ covering the whole y axis, that's the complex plane.

### Operations

Addition

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2) \tag{2}$$

Subtraction

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2) \tag{3}$$

Multiplication

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2) = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1) \tag{4}$$

Complex Conjugate

$$\bar{z} = x - iy \tag{5}$$

Modulus

$$|z| = \sqrt{x^2 + y^2} \tag{6}$$

Inverse

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{x - iy}{x^2 + y^2} \tag{7}$$

## Sums and Sequences

## Series

# Post-Derivatives

## Derivatives

"It's dangerous to go alone! Take this."
— The Legend of Zelda

## Integrals

"I calculate the odds of this succeeding... never tell me the odds!"
— Han

### Reiman Integral

#### Fundamental theorem of calculus

#### Methods for integration

Substitution. if $u = f(x)$, then $du = g'(x)dx$

$$\int f(g(x))g'(x)dx = \int f(u)du \tag{8}$$

Integration by parts

$$\int udv = uv - \int vdu \tag{9}$$

Lebiz rule for differentiation. If f(t) is continuous and g(x) is differentiable

$$f(x) = \int_{a}^{g(x)} f(t)dt$$

If this is true, then:

$$F(x)' = f\left(g(x)\right) \cdot g'(x) \tag{10}$$

If both the limits are dependent on x:

$$I(x) = \int_{g_1(x)}^{g_2(x)} f(t)dt$$

then:

$$I'(x) = f\left(g_2(x)\right) g_2'(x) - f\left(g_1(x)\right) g_1'(x) \tag{11}$$

Limit under a derviative

# Differential Equations

"Would you tell me, please, which way I ought to go from here?"

"That depends a good deal on where you want to get to," said the Cat.

### ODEs and PDEs

Ordinary Differential Equations (ODEs) are a differential equation which has a single variable. ODEs have a general form:

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, ..., \frac{d^ny}{dx^n}\right) = 0 \tag{12}$$

where
- x independent
- y dependent

Partial Differential Equations (PDEs) are a Differential equation which has multiple independent variables. Instead of using the standard d, they use partial derivatives ($\partial$) to show the change with respect for multiple variables. The general form is:

$$F\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x^2}, ....\right) = 0 \tag{13}$$

where
- x,y independent
- u(x,y) dependent

Fundamentally image a ODE as a means to track a single car, while PDE track all the traffic in the city.

**Types of ODEs**

ODEs are usually classified by 2 primary things. their order, aka the degree of their derivative, and by whether they are linear or non-linear.

**First order ODEs** are pretty self explanatory, they involve only the 1st derivative. Here is a basic first order ODE:

$$\frac{dy}{dx} + y = x \tag{14}$$

**Second order ODEs** involve UP to the 2nd derivative. Here is a example:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0 \tag{15}$$

**Higher order ODEs** involve everything 3rd derivative or higher. It is unlikely to ever appear in a 1st year analisis exam, but you never know.

**Liniar and Non-liniear ODEs.** An ODE is linear if the dependent variable and the derivatives are in a linear form. Basically: they are not multiplied together. Anything else is considered non-liniear. A linear ODE can be written in the form:

$$a_n(x)\frac{d^ny}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + ... + a_1(x)\frac{dy}{dx} + a_0(x)y = f(x) \tag{16}$$

- a(x) is a function of x

Here are some basic examples. We will go in much more detail when solving ODEs.

$\frac{dy}{dx} + 3y = x$, and $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = sinx$

(NON-LINIAR) To be added

**The weird classifications of ODEs**

**A Homogeneous ODE** , is a differential equation where L is a linear differential operator.

$$L[y] = 0$$

**A Non-Homogeneous ODE** , is a differential equation where f(x) $\neq 0$

$$L[y] = f(x)$$

**A Autonomous ODE** , is a differential equation where the independent variable (usually x or t) does not appear in the equation.

$$\frac{d^n y}{dx^n} = F\left(y, y', ..., y^{(n-1)}\right)$$

**A Non-Autonomous ODE** , is a differential equation if the independent variable appears eq (1).

*You can be multiple classifications at once, use your head.

## Dealing with ODEs

**Separable ODE** A separable ODE can be written as

$$\frac{dy}{dx} = f(x)g(y)$$
$$\frac{dy}{g(y)} = f(x)dx$$
$$\int \frac{dy}{g(y)} = \int f(x)dx$$

**Integrating Method**

$$\frac{dy}{dx} + P(x)y = Q(x)$$
$$\mu(x) = e^{\int P(x)dx}$$
$$\frac{d}{dx}(\mu(x)y) = \mu(x)Q(x)$$
$$y = \frac{1}{\mu(x)} \int \mu(x)Q(x)dx + C$$

Quick note on integrating factor. A integrating factor is the method used to solve derivative equation above. The μ(x) also called the integrating factor, works for any, first-order linear differential equations. A function is derived by multiplying the equation with μ(x), which makes the left-hand side a derivative of μ(x)y.

### Solving a Initial Value Problem (IVP)

Generally, IVPs are a DE and a <u>initial condition</u> or condition's which when used in unison they can be used to solve a function, that will also fit the DE. The steps are pretty straight forward.

1. Solve the DE

$$y(x) = \int f(x)dx + C$$

2. Use the initial condition, lets say that $y(x_0) = y_0$

$$y_0 = \int f(x_0)dx + C$$

Where C is:

$$C = y_0 - V$$

*V is the value of the integral at $x_0$, therefore if we replace C, the final answer is

$$y(x) = \int f(x)dx + (y_0 - V)$$

**Second Order Equations**

Homogeneous Equations with Constant Coefficients

$$N0t$$

Non-homogeneous Equations: Method of Undetermined Coefficients

$$Finished$$

## Special theorems and Problems

### Cauchy's Problem