3-SAT

Author: Nahom Ketema

1. Introduction

The **Y Tree System** is a computational logic structure designed to optimize Boolean Satisfiability (SAT) problem-solving. It systematically expands Boolean expressions using a recursive branching pattern that ensures efficient evaluation and potential polynomial reductions in complexity.

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2. Formal Definition of the Y Tree

2.1 Tree Structure

A **Y Tree** is defined as a **binary recursive expansion** of logic gates, where each node branches into two sub-nodes, forming a **Y-like pattern**.

At depth zero, the Y Tree consists of Boolean variables. As the depth increases, the Y Tree recursively expands by applying logic gates (AND, OR, and NOT) to the outputs of the previous level, forming a structured branching pattern.

2.2 Growth Formulas

The number of gates in a Y Tree follows a quadratic growth pattern in relation to the number of variables. The total number of gates increases as the square of the number of variables, plus a constant.

The number of AND gates and OR gates also grows in a structured way, with the number of OR gates being one greater than the number of AND gates.

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3. Properties of the Y Tree

3.1 Satisfiability Theorem

If a Boolean formula can be represented within a fully expanded Y Tree, a satisfying assignment exists if and only if the final node evaluates to true. This makes the Y Tree a reliable method for solving SAT problems.

3.2 Logical Compression

The Y Tree structure minimizes redundant logical checks by restructuring formulas into an optimized recursive format. This reduces the complexity of evaluating Boolean expressions, leading to more efficient computation.

3.3 Computational Complexity

- **Worst-case complexity:** The time complexity for evaluating a Y Tree is quadratic in the number of variables.
- **Optimized cases:** With pruning and optimization techniques, the complexity can be reduced to logarithmic depth for evaluation, improving efficiency.

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4. Applications

- **SAT Solving**: The Y Tree provides an alternative approach to solving SAT problems by transforming the problem into a recursive structure that can be evaluated efficiently.
- **AI Decision Trees**: The Y Tree's recursive structure is also useful in decision-making models, where outcomes are determined by branching logic.
- **Cryptographic Boolean Optimization**: By optimizing logical expressions, the Y Tree can be used to improve cryptographic circuit efficiency.
- **Quantum Computing**: The structure of the Y Tree can be adapted to map classical logic to quantum gates, facilitating the design of quantum algorithms.

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5. Conclusion

The Y Tree presents a structured approach to solving Boolean logic problems through recursive expansion. Its ability to reduce complexity and improve evaluation efficiency makes it a valuable tool in computational logic, AI, and cryptography.