0/1 knapsack problem using Dynamic Programming

$M = 8$ $\quad P = \{1, 2, 5, 6\}$

$n = 4$ $\quad W = \{2, 3, 4, 5\}$

This we can solve with table and set method

⇒ Here we can take solid object, we are not suppose to take fraction.

- $x_i = 0/1$

$$\max \sum p_i x_i$$

- It demands maximum profit so it is optimization problem.

- It takes sequence of decision yes all items are require to check.

- DP allow try ~~of~~ for all possible decisions or solution and pick up the best.

$2^4 = 16$ ⇒ no. of solutions.

- It is for n is 2^n ⇒ ~~$O(2^n)$~~ $O(2^n)$

Tabulation Method.

P_i	W_i
1	2
2	3
5	4
6	5

	0	1	2	3	4	5	6	7	8 ← weights
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1
2	0	0	1	2	3	3	3	3	3
3	0	0	1	2	5	5	6	7	7
4	0	0	1	2	5	6	6	7	8

ep ①

row → column   ↑ column   Column ↑   ↑ object we   ↑ Prof

$$V[i, w] = \max\{ V[i-1, w], V[i-1, w - w[i]] + P[i] \}$$

$$V[4,1] = \max\{ V[3,1], V[3, 1-5] + 6 \}$$

$$= \max\{ 0, V[3,-4] + 6 \}$$

→ value undefined upto 5th weight
Value is negative
so fill weight
it is

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1
2	0	0	1	2	3	3	3	3	3
3	0	0	1	2	5	5	6	7	7
4	0	0	1	2	5	6			
4	0								

$$V[i, w] = \max\{V[i-1, w],\ V[i-1, w - w[i]] + P[i]\}$$

$$V[4,5] = \max\{\underset{\text{column}}{}V[3,5],\ V[3, \overset{\text{row}}{5-5}] + 6\}$$

$$= \max\{5,\ V[3,0] + 6\}$$

$$= \max\{5,\ 0+6\}$$

$$= \max\{5, 6\}$$

$$\boxed{V[4,5] = 6}$$

$$V[4,6] = \max\{V[3,6],\ V[3, \overset{\text{row}}{\cancel{6}-5}] + 6\}$$

$$6-5 \qquad V[3,1]+6$$

$$= \max\{5,\ 0+6\}$$

$$= \max\{6\}$$

$$\boxed{V[4,6] = 6}$$

$$V[4,7] = \max\{V[3,7],\ V[3, \cancel{7}-5] + 6\}$$

$$= \max\{7,\ \overset{1}{\cancel{\ }}+6\}$$

$$= \max\{7\}$$

$$\boxed{V[4,7] = 7}$$

↑ column   column ↑ row

$$V[4,8] = \max\{V[3,8],\ V[3,\ 8-5]+6\}$$

$$= \max\{7,\ 2+6\}$$

$$= \max\{8\}$$

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1
2	0	0	1	2	3	3	3	3	3
3	0	0	1	2	5	5	6	7	7
4	0	0	1	2	5	6	6	7	8

x_1 x_2 x_3 x_4 ← sequence of decisions.

maximum profit = 8 including 4th object.

x_1 x_2 x_3 x_4

8      $8 - 6 = 2$
↑ profit of 4th object.

x_1 x_2 x_3 x_4

$\{0 \quad 1 \quad 0 \quad 1\}$ $2 - 2 = 0$

solve the problem using set's method.

$M = 8$ $\quad P = \{1, 2, 5, 6\}$

$S = (P, W)$ $\quad n = 4$ $\quad W = \{2, 3, 4, 5\}$

$S^0 = \{(0,0)\}$ No profit & No weight.

$S^0_1 = \{(1,2)\}$

$S^1 = \{(0,0), (1,2)\}$

$S^1_1 = \{(2,3), (3,5)\}$ Merge

$S^2 = \{(0,0), (1,2), (2,3), (3,5)\}$

$S^2_1 = \{(5,4), (6,6), (7,7), \cancel{(8,9)}\}$ Merge

don't include

descarde one with lesser profit

P W

$S^3 = \{(0,0)(1,2)(2,3)\cancel{(3,5)}(5,4), (6,6), (7,7)\}$

Dominance Rule

$S^3_1 = \{(6,5), (7,7)(8,8)\cancel{(11,9)}\cancel{(12,11)}, \cancel{(13,12)}\}$

$S^4 = \{(0,0)(1,2)(2,3)(5,4)\cancel{(6,6)}(6,5)(7,7), (8,8)\}$

It is the easy method as compare to ~~this~~ tabular method.

$(8,8) \in S^4$	$(2,2) \notin S^2$
but $(8,8) \notin S^3$ $\therefore x_4 = 1$	but $(2,3) \notin S^1$ $\therefore x_2 = 1$
$(8-6, 8-5) = (2,3)$	$(2-2, 3-3) = (0,0)$
$(2,3) \notin S^3$	$(0,0) \in S^1$ & $(0,0) \in S^0$
& $(2,3) \notin S^2$ $\therefore x_3 = 0$	$x_1 = 0$