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Active Filtering and Amplifying Circuits

Laboratory 5 Report

11 March 2016

Electrical Engineering 101 Lab

University of California, Santa Cruz

Introduction

In previous labs, we discussed the mechanism of constructing and analyzing passive RLC filters and how to construct and analyze operational amplifiers. In this lab, we will combine our previous knowledge of these two constructs to build a bass and treble boosting circuit to play music out of with a speaker. We will design a circuit that incorporates our knowledge of filters to hit low lows and high highs. Since the human ear can hear a wide spectrum of frequencies (around 20 Hz to 20 KHz), and designed a treble and bass booster with an active low-pass filter with a cutoff frequency low enough so that only the lower (bass) frequencies, 400 Hz maximum, can be amplified, and a high-pass filter that will amplify the frequencies above about 15 Khz (treble) range. We will go into further detail on how we achieved the correct cutoff frequencies in Part I of the experiment. We then linearly added the two sources using a summing circuit.

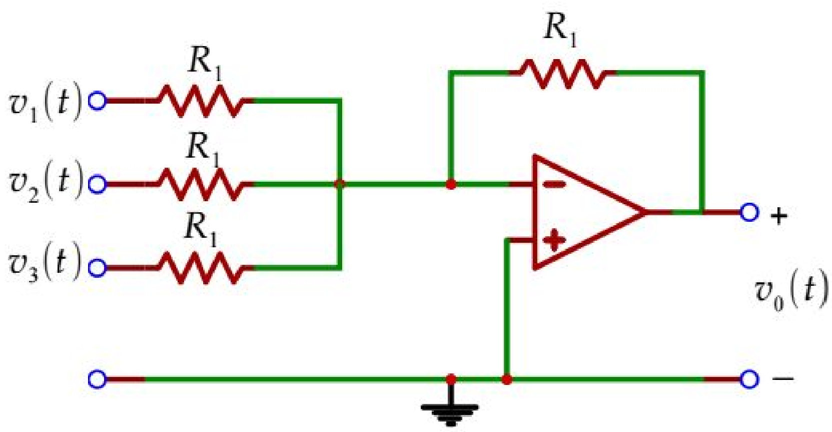


Figure 1: *An active summing circuit*

Figure 1 shows a summing circuit with three inputs. The op-amp is in a negative-feedback, so the summing point constraint applies.

$$\text{Let } i_1, i_2, i_3 \text{ be the currents across the } V_n \text{ and } R_1 \text{ branches respectively.}$$

$$\text{Let } i_f \text{ be the current through the feedback resistor.}$$

$$\text{We know (by Ohm's Law)}: \; i_1 = \frac{V_1(t)-0}{R_1},\; i_2 = \frac{V_2(t)-0}{R_1},\; i_3 = \frac{V_3(t)-0}{R_1}$$

$$\text{The sum of these currents } = i_{in} = i_1 + i_2 + i_3 = \frac{V_1(t)}{R_1} + \frac{V_2(t)}{R_1} + \frac{V_3(t)}{R_1} = i_f = \frac{0-V_o(t)}{R_1}$$

$$\text{Therefore}: V_o(t) = -[V_1(t) + V_2(t) + V_3(t)]$$

If we place variable resistors in place of the equivalent input resistors, R1, Vo(t) will equal a linear combination of the input voltages and the inverse of the variable resistances.

$$V_o(t) = -[\tfrac{1}{R_1}V_1(t) + \tfrac{1}{R_2}V_2(t) + \ldots + \tfrac{1}{R_n}V_n(t)]$$

Since the output is negative, we will compensate for this in Part II by inverting the signal as it comes into the summer.

We will use this approach to allow the user to control the proportionality of bass/treble boosting to the output.

Experiments

Part I: Active Filters

Active filters differ from passive filters mainly due to the use of the amplifier in the active circuit. Active filters also provide the advantages of fewer components, they have a transfer function that is insensitive to component tolerances, they place modest demands on the op-amp's gain–bandwidth product, output impedance, slew rate, and other specifications, can be easily adjusted, and they require a small spread of component values.

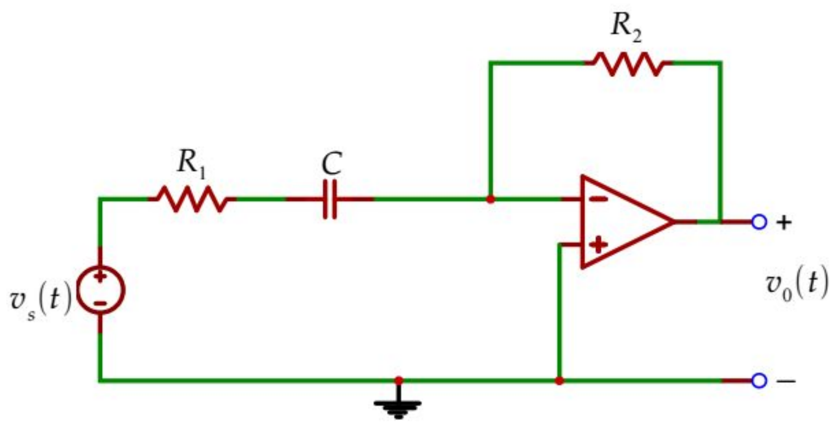


Figure 2: *An active high-pass filter*

Similarly to a passive configuration, the active high-pass filter circuit provided a form of amplifying the filtering process. In order to determine the gain, magnitude, and angle between the input and output of the circuit, we will first derive a transfer function as a function of frequency. Since the op-amp is in a negative-feedback loop, and the summing point constraint applies, we will derive the following equation.

$$i_{in} = \frac{V_{in}(t)}{R_1 + \frac{1}{2\pi jfC}} = i_f = \frac{-V_{out}(t)}{R_2}$$

$$\rightarrow H(f) = -\frac{R_1 R_2}{{R_1}^2 + \frac{1}{4\pi^2 C^2 f^2}} - j\frac{R_2}{2\pi C f {R_1}^2 + \frac{1}{4\pi^2 C^2 f^2}}$$

Plugging in our values for the components, where: R1 = 1KΩ (.99 KΩ actual), C = 1 μF, R2 = 1.5 KΩ (1.48 KΩ actual), we get:

$$H(f) = -\frac{1500000}{\frac{250000000000}{\pi^2 f^2} + 1000000} - j\frac{1500}{\frac{250000000000}{\pi^2 f^2} + \frac{\pi f(1000000)}{500000}}$$

To obtain the magnitude of the function we will use Pythagorean's theorem to derive the magnitude of the phasors, since we now have the real and imaginary vectors (above).

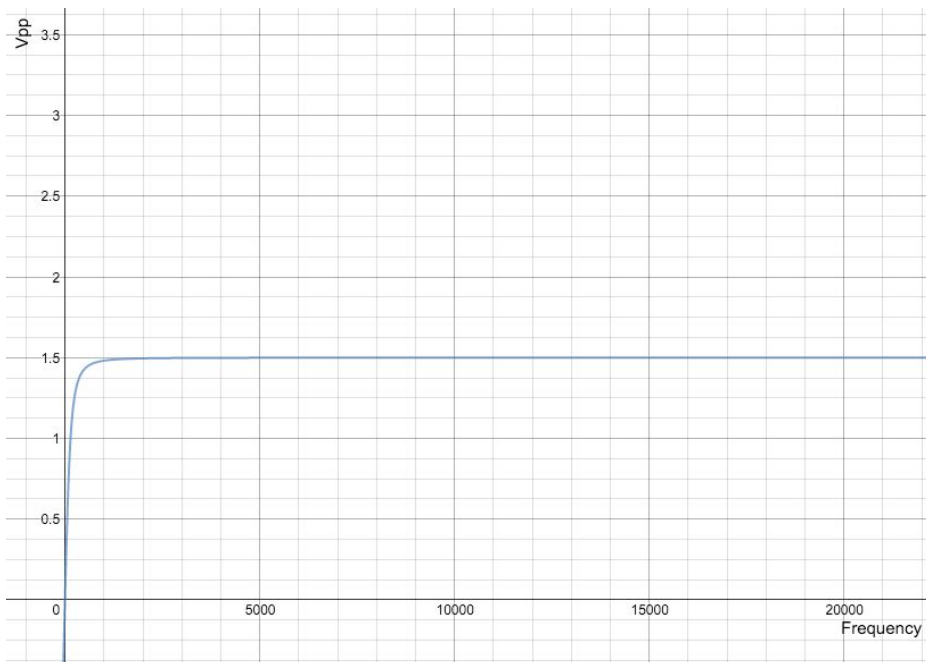
$$|H(f)| = \frac{R_2}{\sqrt{{R_1}^2 + \frac{1}{4\pi^2 C^2 f^2}}} = \frac{1500}{\sqrt{(1000)^2 + \frac{1}{4\pi^2 (1E-6)^2 f^2}}}$$

$$Half - Power\ Frequency\ = f\ @\ |H(f)| = \frac{1}{\sqrt{2}} \rightarrow f \approx 75\ Hz$$

Our phase response between the real and imaginary vectors becomes:

$$\Phi(f) = -\tan^{-1}\left(\frac{\frac{-250000000000}{\pi^2 f^2} - 1000000}{1000\left(\frac{-250000000000}{\pi^2 f^2}\right) + 2\pi f}\right)$$

We plotted the theoretical values for magnitude and phase response, to compare with our experimental results later on.



Graph 1: *Theoretical magnitude versus frequency for a high pass filter*

Since the cutoff frequency of a high-pass filter is:

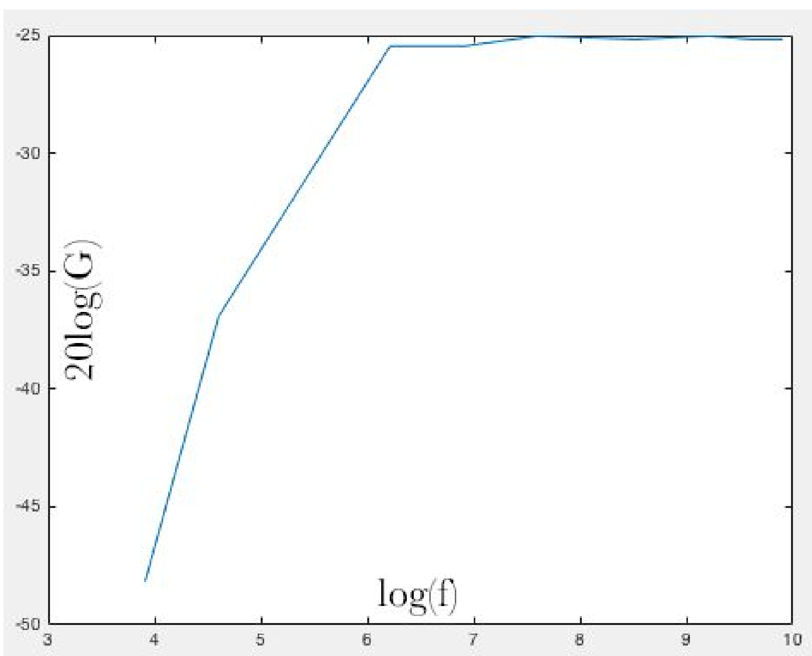
$$f_c = \frac{1}{2\pi R_1 C} = \frac{1}{2\pi(1000)(1E-6)} = 159.15\ Hz$$

This theoretical cutoff frequency tells us where the frequency in the circuit begins to produce higher gain. The value is consistent with our experimental gain. Our gain begins to increase dramatically passed the cut off frequency, as shown in Table 1, as expected.

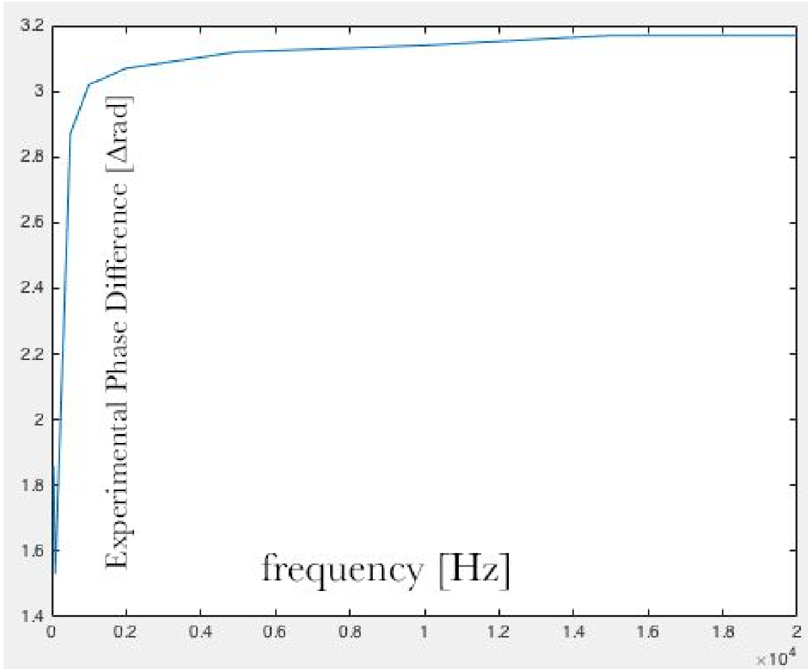
$f \quad [Hz]$	$\Delta t \ \ (s)$	$\Delta\Phi = 2\pi f \Delta t$	$Vpp_{out} \quad [V]$	$G \ = \ \frac{V_{out}}{V_{in}}$
50	**5.92 m**	**1.86**	**.9**	**0.0900**
100	**2.44 m**	**1.53**	**1.58**	**0.1580**
500	**912 *μ***	**2.87**	**2.80**	**0.2800**
1K	**480 *μ***	**3.02**	**2.80**	**0.2800**
2K	**244 *μ***	**3.07**	**2.86**	**0.2860**
5K	**99.2 *μ***	**3.12**	**2.84**	**0.2840**
10K	**50 *μ***	**3.14**	**2.86**	**0.2860**
15K	**33.6 *μ***	**3.17**	**2.84**	**0.2840**
20K	**25.2 *μ***	**3.17**	**2.84**	**0.2840**

***Table 1**: experimental frequency, time, angle, voltage out, and gain of the active high-pass filter*

We then graphed the output gain versus frequency to get an experimental plot of the filter. The function plotted in Graph 2 from the data in Table 1 aligns with our theoretical plot in Graph 1



***Graph 2:** A graph showing the gain 20log(G) v.s. log(f). This is a bode plot for our high pass filter.*



Graph 3: *A graph showing the experimental phase difference versus frequency for the high-pass filter.*

The experimental phase difference graph (Graph 3), shows that the angle between the input and output. Notice how the phase difference increases as frequency increases, showing a destabilizing effect between the input and the output as frequency increases.

1.2 Active Low-Pass Filter

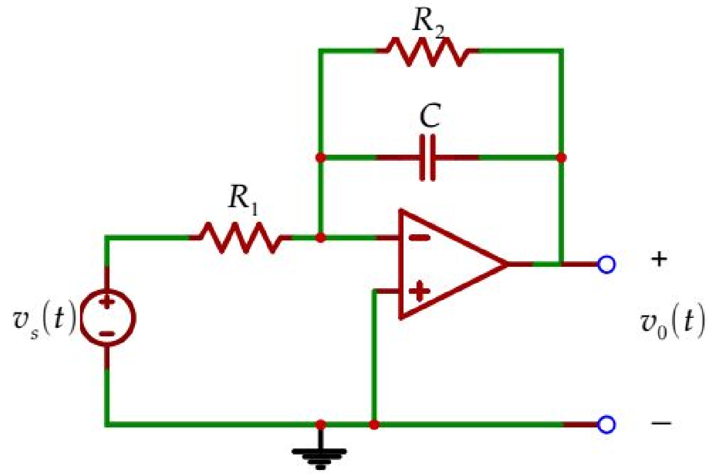


Figure 3: *An active low-pass filter*

We then constructed the low-pass filter in Figure 3. In order to determine the gain, magnitude, and angle between the input and output of the low-pass filter in the circuit depicted in Figure 3, we will first derive a transfer function as a function of frequency. Since the op-amp is in a negative-feedback loop, and the summing point constraint applies, we will derive the following equation.

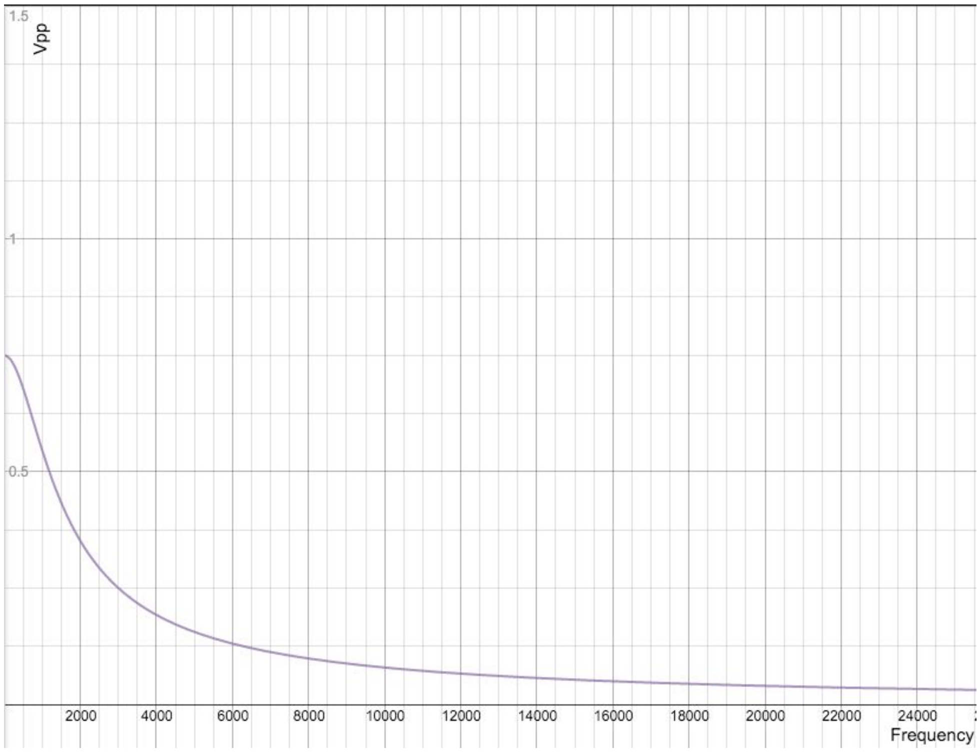
$$i_{in} = \frac{V_{in}(t)}{R_1} = i_f = -[\frac{1}{R_2} + j2\pi f C]V_{out}(t) = \frac{-1-j2\pi fC}{R_2}V_{out}(t)$$
$$H(f) = \frac{V_{out}(t)}{V_{in}(t)} = \frac{R_2}{-R_1 - j2\pi fCR_1R_2} = \frac{-R_1R_2}{R_1^2+4\pi^2f^2C^2R_1^2R_2^2} + \frac{2\pi fCR_1R_2^2}{R_1^2+4\pi^2f^2C^2R_1^2R_2^2}j$$

Plugging in our values for the components, where: R1 = 2KΩ (1.98 KΩ actual), C = .1 µF, R2 = 1.5 KΩ (1.48 KΩ actual), we get:

$$H(f) = -\frac{3000000}{3.55306f^2+4000000} + j\frac{2827.43f}{3.55306f^2+4000000}$$

$$|H(f)| = \sqrt{\frac{9000000000000}{(3.55306f^2+4000000)^2} + \frac{8000000f^2}{(3.55306f^4+4000000)^2}}$$

$$Half-Power\ Frequency\ = f\ @\ |H(f)| = \frac{1}{\sqrt{2}} \rightarrow f \approx 2000\ Hz$$



Graph 4: *Theoretical plot for the magnitude of the transfer function*

$$\Phi(f) =- arctan(\frac{0.00094f(3.55306f^2+4000000)}{3.55306f^4+4000000})$$

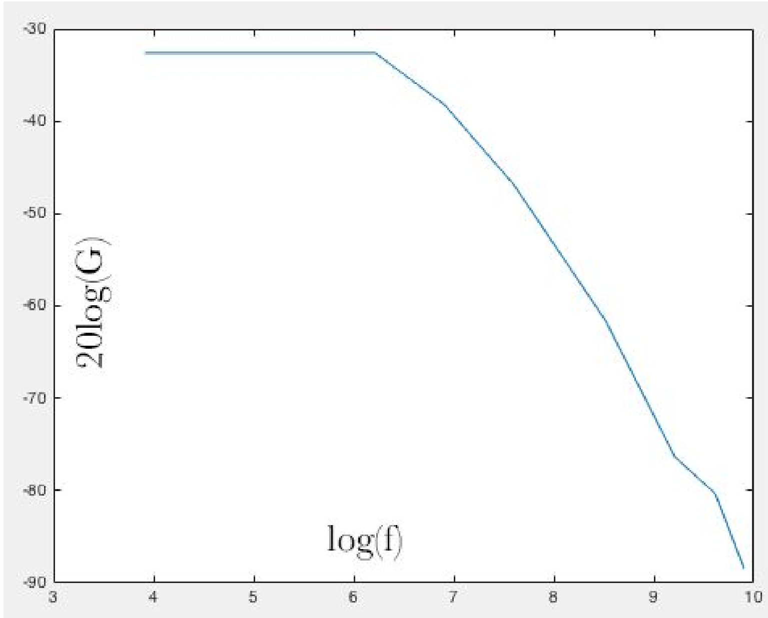
$$f_c = \frac{1}{2\pi R_1 C} = \frac{1}{2\pi(1500)(.1E-6)} = 1061.03\ Hz$$

This theoretical cutoff frequency tells us where the frequency in the circuit stops to produce higher gain for low-pass filter. The value is consistent with our experimental gain. Our gain begins to decrease dramatically passed the cut off frequency, as shown in Table 2, as expected.

$f \quad [Hz]$	$\Delta t \ (s)$	$\Delta\Phi = 2\pi f \Delta t$	$Vpp_{out} \quad [V]$	$G = \frac{V_{out}}{V_{in}}$
50	**9.6 m**	**3.0159**	**1.96**	**.1960**
100	**5.12 m**	**3.2170**	**1.96**	**.1960**
300	**1.78 m**	**3.3552**	**1.96**	**.1960**
500	**1.13 m**	**3.5500**	**1.96**	**.1960**
1K	**620 *μ***	**3.8956**	**1.48**	**.1480**
2K	**336 *μ***	**4.2223**	**.96**	**.0960**
5K	**144 *μ***	**4.5239**	**.46**	**.0460**
10K	**73.6 *μ***	**4.6244**	**.22**	**.0220**
15K	**51.2 *μ***	**4.8255**	**.18**	**.0180**
20K	**38.8 *μ***	**4.8758**	**.12**	**.0120**

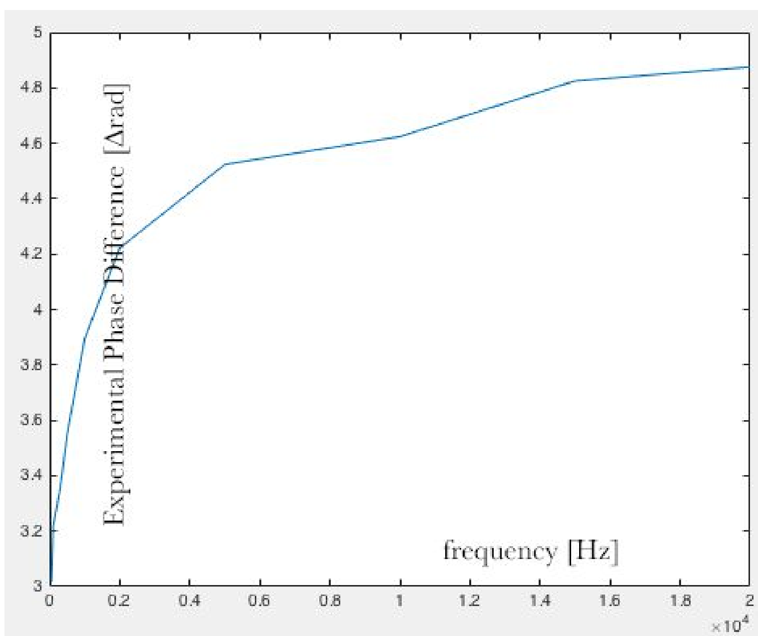
Table 2: *experimental frequency, time, angle, voltage out, and gain of the active low-pass filter*

The plot in Graph 5 shows the experimental frequency versus experimental gain. The behavior depicted in the graph is consistent with the bode plot expected from a low-pass filter, where the gain is higher before the cutoff frequency (~1.06 KHz), and it begins to attenuate after this cutoff frequency.



Graph 5: *A bode plot with experimental data, collected into Table 2 for a low-pass filter.*

The experimental phase difference graph (Graph 6), shows that the angle between the input and output. Notice how the phase difference increases as frequency increases, showing a destabilizing effect between the input and the output as frequency increases.



Graph 6: *A graph showing the experimental phase difference versus frequency for the low-pass filter.*

Part II: Bass & Treble Boosting

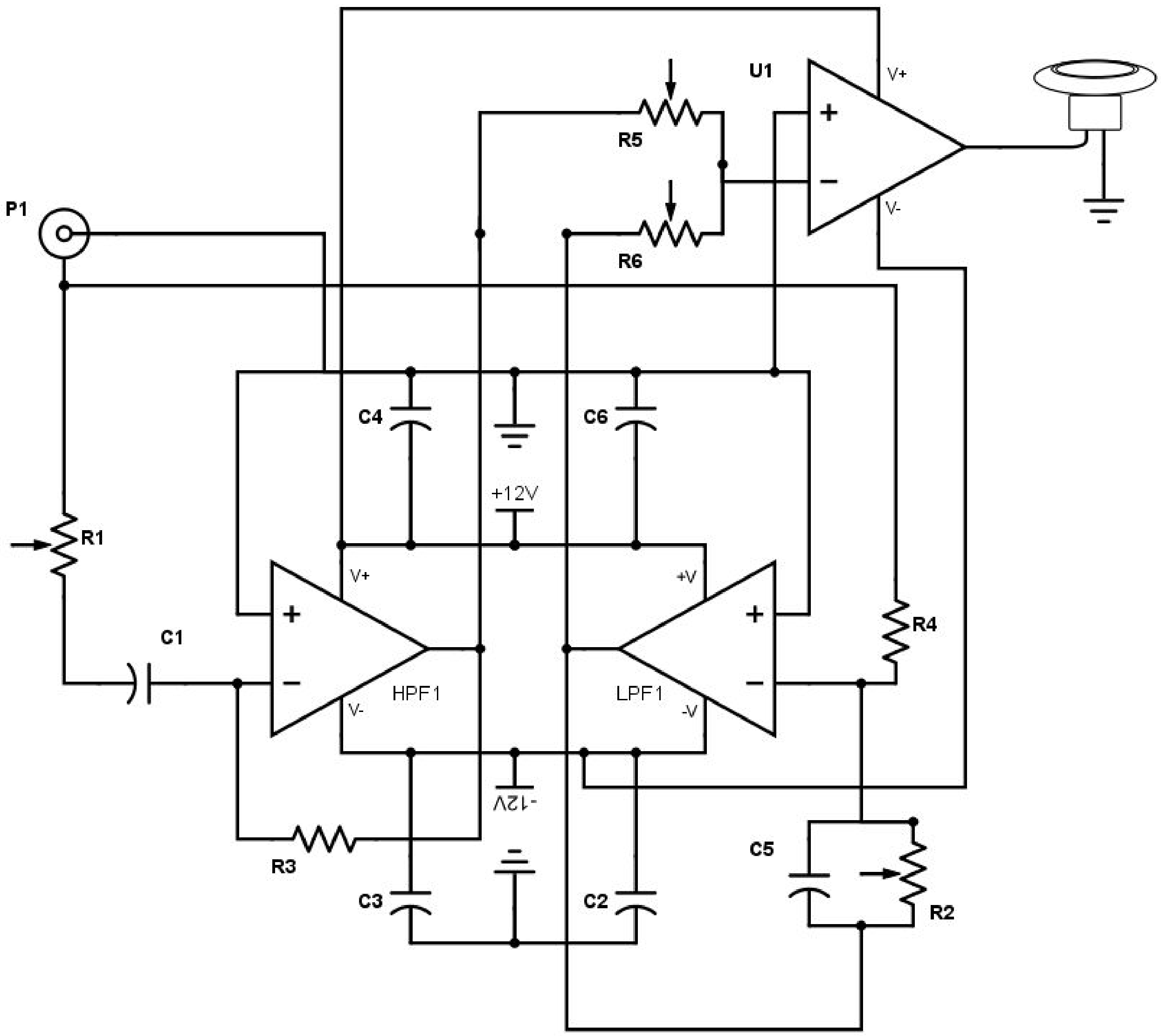


Figure 3: *Our bass/treble boosting circuit with a proportionality controller on a summing circuit to allow difference in bass/treble output.*

To conclude the lab, we constructed a applicable circuit to boost bass and treble in audio from an input source (P1), to both the high-pass and low-pass filters described above. We then added the two circuits together using a summing circuit (described in the introduction).
We used the following values for our components:
Low Pass Filter (LPF1): Cutoff Frequency = 250 Hz
R2 = 636 Ohms (actual - variable resistor)
C5 = 1 microfarads
R4 (feedback resistor) = 1 K Ohms (.99 K Ohm actual)
High Pass Filter (HPF1): Cutoff Frequency = 5000 Hz
R1 = 318 Ohms (actual - variable resistor)

C1 = 1 microfarads
R3 (feedback) = 1 K Ohm (.99 K Ohm actual)
Other:
C2 = C4 = C6 = C3 = 10 nanofarads (noise cancelling stabilizing capacitors)
R5 = variable proportionality resistor for treble boosting
R6 = variable proportionality resistor for bass boosting
All Op-Amps = LM741 Dual Powered with +/- 12 Volts
P1 = input jack for audio source
Speaker = 8 Ohm 4-inch diameter speaker

We used a 5000 Hz cutoff frequency for the treble booster since treble is associated with higher frequencies. The treble range is a frequency spectrum between 4-20 KHz. With the desired cutoff frequency:

$$5000\ Hz\ =\ \frac{1}{2\pi R_1 C_1} \rightarrow R_1 = \frac{1}{2\pi(.1\ microfarad)(5000)} = 318\ \Omega$$

We used a 250 Hz cutoff frequency for the bass booster since bass is associated with lower frequencies. The bass range is a frequency spectrum between 20 - 250 Hz. With the desired cutoff frequency:

$$250\ Hz\ =\ \frac{1}{2\pi R_1 C_1} \rightarrow R_1 = \frac{1}{2\pi(1\ microfarad)(5000)} = 636\ \Omega$$

After creating the two filters, we tested them separately with a variety of genres of music ranging from early synthesizer electronic music (with high treble), and funk (high bass) to make sure the treble allowed most of the synthesizer music through, and not too much funk, and vica versa with the bass boosting low-pass filter.
In order to combine the two frequency filters, we joined the two outputs from the filters linearly using a summing circuit. We included variable resistors on the inputs to the summer in order to allow the user to adjust proportionally the right bass/treble boosting ratio for their tastes.
The signals from the filters are inverted with respect to their inputs, but the summer inverts the signals as well, making the output from the summer the correct phase. We found that a +/- 12 volt power supply for the amplifiers was sufficient enough to not produce clipping, with a decent volume. In order to get the mid-range signals, we also tied the outputs of P1 backwards to the input of the speaker, we found that the combination of just the treble and bass boosting circuits was sufficient for most tunes.