Data snooping and the multiple testing fallacy

In 1992 a Swedish study examined whether living near a power line causes adverse health effects. It reported a statistically highly significant increase in childhood leukemia.

A careful follow-up analysis failed to confirm this result.

Why did the study find a statistically significant result?

The study looked at 800 different health effects:

There were 800 statistical tests involved.

Multiple comparisons

A statistical test summarizes the evidence for an effect by reporting a p-value:
A smaller p-value means stronger evidence.

p-value $< 1\% \ \rightarrow$ test is 'highly significant'

Interpretation: If there is no effect, then there is only a 1% chance to get such a highly significant result.

So if we do 800 tests, then even if there is no effect at all we expect to see $800 \times 1\% = 8$ highly significant results just by chance!

This is called the **multiple testing fallacy** or **look-elsewhere effect**.

When analyzing large amounts of data it is easy to fall into this trap because there are so many potential relationships to explore, which leads to **data snooping** (=**data dredging**).

Reproducibility and Replicability

Data snooping and other problems have lead to a crisis with regard to **replicability** (getting similar conclusions with different samples, procedures and data analysis methods) and **reproducibility** (getting the same results when using the same data and methods of analysis.)

- 'How science goes wrong' in The Economist (10/13/2013)
- 'Why most published research findings are false' by J. Ioannidis (2005)

How can one account for multiple testing?

Bonferroni correction: If there are m tests, multiply the p-values by m.

The Bonferroni correction makes sure that P(any of the m tests rejects in error) $\leq 5\%$.

The Bonferroni correction is often very restrictive: It guards against having even one false positive among the m tests.

As a consequence the adjusted p-values may not be significant any more even if a noticeable effect is present.

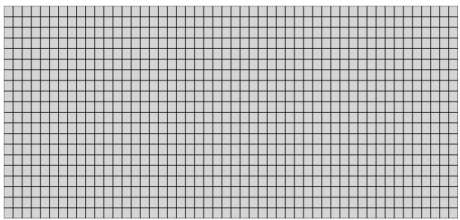
Accounting for multiple testing

Alternatively, we can try to control the **False Discovery Proportion (FDP)**:

$$\text{FDP} = \frac{\text{number of false discoveries}}{\text{total number of discoveries}}$$

where a 'discovery' occurs when a test rejects the null hypothesis.

As an example, we test 1,000 hypotheses.



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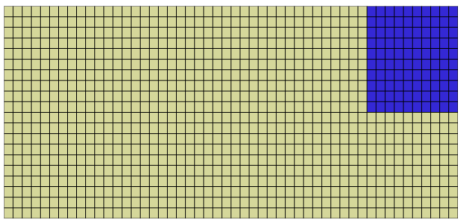

In 900 cases the null hypothesis is true ("Nothing is going on"), and in 100 cases an alternative hypothesis is true ("There is an effect: something is going on").

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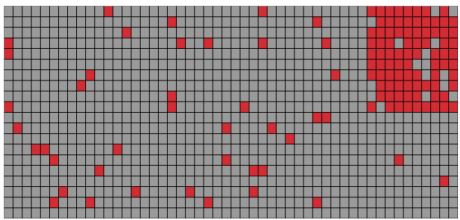

Doing 1,000 tests results in Discoveries and Non-discoveries.

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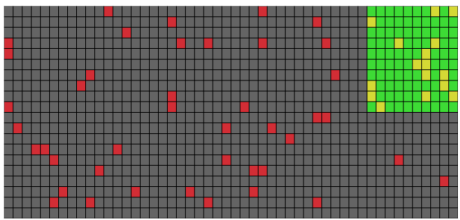

We made 80 true discoveries and 41 false discoveries. The false discovery proportion is 41/121=0.34.

Accounting for multiple testing with FDR

False discovery rate (FDR): Controls the expected proportion of discoveries that are false.

Benjamini-Hochberg procedure to control the FDR at level $\alpha = 5\%$ (say):

1. Sort the p-values: $p_{(1)} \leq \ldots \leq p_{(m)}$
2. Find the largest k such that $p_{(k)} \leq \frac{k}{m}\alpha$
3. Declare discoveries for all tests i from 1 to k.

Accounting for multiple testing with validation set

Using a validation set: Split the data into a *model-building set* and a *validation set* before the analysis.

You may use data snooping on the model-building set to find something interesting.

Then test this hypothesis on the validation set.

This approach requires strict discipline: You are not allowed to look at the validation set during the exploratory step!