CLOE-19 optimisation technique

ASSIGNMENT-2

SARISHMA BHANDARI 10317086

Q:1

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply
f_1	5	6	4	3	7	5	4	7000
f_2	9	4	3	4	3	2	1	4000
f_3	8	4	2	5	4	8	3	10,000
Demand	1500	2000	4500	4000	2500	3500	3000	21,000 / 21,000

⇓ Balanced TP.

Soln :-

LEAST - COST METHOD :-

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply
f_1	1500 5	6	4	4000 3	7	1500 5	4	7000 → 0
f_2	9	2000 4	3	4	3	2	3000 1	4000 → 0
f_3	8	2000 4	4000 2	5	2500 4	1000 8	3	10000 → 5000 → 3000 → 1000
Demand	1500 → 0	2000 → 0	4500 → 0	4000 → 0	2500 → 0	3500 → 0	3000 → 0	

Consolidating the table we get:-

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply
f_1	5 × 1500	6	4	3 × 4000	7	5 × 1500	4	7000
f_2	9	4	3	4	3	2 × 1000	1 × 3000	4000
f_3	8	4 × 2000	2 × 4500	5	4 × 2500	3 × 1000	3	10,000
Demand	1500	2000	4500	4000	2500	3500	3000	

Minimize $Z = 5\times1500 + 3\times4000 + 5\times1500 + 2\times1000 + 1\times3000 + 2\times4500 + 4\times2500 + 8\times1000$

$\text{miz}(Z) = 67,000$

Total cost of transportation :- Z 67,000.

* VOGEL'S APPROXIMATION METHOD :-

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
f_1	5	6	4	3	7	5	4	7000	1
f_2	9	4	3	4	3	2 (3500)	1	5000 4000	1
f_3	8	4	2	5	4	8	3	10000	1
Demand	1500	2000	4500	4000	2500	3500 0	3000		
Penalty	3	0	1	1	1	3	2		

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	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
f_1	1500 5	6	4	3	7	-	4	~~5500~~ ~~2000~~	1
f_2	9	4	3	4	3		1	500	2
f_3	8	4	2	5	4		3	10000	1
Demand	~~1500~~ 0	~~2000~~	4500	4000	2500		3000		
Penalty	3	0	1	1	1		2		

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
f_1		6	4	3	7		4	5500	1
f_2		4	3	4	3		1500 1	0 ~~500~~	2
f_3		4	2	5	4		3	10000	1
Demand		2000	4500	4000	2500		2500 ~~3000~~		
Penalty		0	1	1	1		2		

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	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
f_1		6	4	3	7	-	4	5500	1
f_2									
f_3		4	2	5	4 (2500)		3	7500	1
Demand		2000	4500	4000	0		2500		
Penalty		2	2	2	3		1		

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
f_1		6	4	3			4	5500	1
f_2									
f_3		4	2 (4500)	5			3	3000	1
Demand		2000	0 4500	4000			2500		
Penalty		2	2	2			1		

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	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
F_1		6		3 (4000)		-	4	1500	1
F_2									
F_3		4		5			3	3000	1
Demand		2000		4000 0			2500		
Penalty		2		2			1		

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Penalty
F_1		6					4 (1500)	0	2
F_2									
F_3		4 (2000)					3 (1000)		1
Demand		2000					2500 3000		
Penalty		2					1		

Since only one column is left, Apply the least cost methode

Finally, we got all zeroes.

Now, Consolidate the final table.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	Supply	Pen
f_1	5 x 1500	6	4	3×1000	7	5	4×1500	7000	
f_2	9	4	3	4	3	2×3000	1×1500	4000	
f_3	8	4×2000	2×4500	5	4×2400	8	3×1000	10000	
Demand	1500	2500	4500	4000	2500	3500	3000		

$$\min z = 5\times1500 + 3\times4000 + 4\times1500 + 2\times3500 + 1\times1500 + 4\times2000 + 2\times4500 + 4\times2500 + 3\times1000$$

$$\min z = 63,000.$$

∴ minimum total ~~trans~~ portation cost = ₹ 63,000

Question 2] Linear programming problem:

minimize $Z = 10x_1 + 8x_2$

Subject to constraints

$$2x_1 + 4x_2 \leq 25$$

$$4x_1 + 6x_2 \leq 27$$

where,

x_1 and $x_2 \geq 0$

Solution:

Let's Convert this into standard form:

minimize $Z = 10x_1 + 8x_2 + 0s_1 + 0s_2$

Subject to constraints:

$$2x_1 + 4x_2 + 1s_1 + 0s_2 = 25$$

$$4x_1 + 6x_2 + 0s_1 + 1s_2 = 27$$

where, x_1, x_2, s_1 and $s_2 \geq 0$.

ITERATION - 1

C_{Bi}	C_j	10	8	0	0		
	Basic Variable	x_1	x_2	s_1	s_2	Sol.	Ratio
0	s_1	2	4	1	0	25	12.5
0	s_2	4	6	0	1	27	6.75 K.R
$-Z_j$		0	0	0	0		
$C_j - Z_j$		10 ✓	8 ✗	0 ✓	0 ✓		

KC

∴ key element = 4

x_1 = entering variable

s_2 = leaving variable

ITERATION 2

C_{Bi}	C_j	10	8	0	0	
	Basic Var.	x_1	x_2	s_1	s_2	Sol
0	s_1	0	1	1	−0.5	11.5
10	x_1	1	1.5	0	0.25	6.75
$-Z_j$		10	15	0	2.5	
$C_j - Z_j$		0 ✓	−7 ✓	0 ✓	−2.5 ✓	

∴ $C_j - Z_j \leq 0$ & optimality reached.

∴ $x_1 = 6.75$, $x_2 = 0$ & solution

max. $Z = 10 \times 6.75 + 8 \times 0 = 67.5$

∴ Optimized Value = 67.5

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Question 3] Linear programming problem.

maximize $Z = 8x_1 + 6x_2$

subject to

$$3x_1 + 4x_2 \leq 85$$

$$3x_1 + 6x_2 \leq 95$$

where $x_1, x_2 \geq 0$.

Solution:

Converting into standard form.

maximize $Z = 8x_1 + 6x_2 + 0S_1 + 0S_2$

subject to

$$3x_1 + 4x_2 + 1S_1 + 0S_2 = 85$$

$$3x_1 + 6x_2 + 0S_1 + 1S_2 = 95$$

where, x_1, x_2, S_1 and $S_2 \geq 0$

ITERATION 1

C_{Bi}	C_j	8	6	0	0		
	Basic variable	x_1	x_2	S_1	S_2	Sol.	Ratio
0	S_1	8	4	1	0	85	10.625 KR
0	S_2	3	6	0	1	95	31.67
Z_j		0	0	0	0		
$C_j - Z_j$		8 X KC	6 X	0 ✓	0 ✓		

∴ Key element = 8

$x_1 \rightarrow$ Entering variable, S_1 = leaving variable

ITERATION - 2

C_{Bj}	C_j	8	6	0	0		
	Basic variables	x_1	x_2	s_1	s_2	Sol.	Ratio
8	x_1	1	0.5	0.125	0	10.125	2.25
0	s_2	0	4.5	−0.33	1	63.125	14.03
Z_j		8	4	1	0		
$C_j - Z_j$		0	2	−1	0		
		✓	★ kc	✓	✓		

∴ key element = 4.5

x_1 = Entering variable

s_2 = leaving variable

ITERATION 3

C_{Bi}	C_j	8	6	0	0	
	Basic variables	x_1	x_2	s_1	s_2	Sol
8	x_1	1	0	0.117	−0.11	3.61
6	x_2	0	1	−0.083	0.22	14.03
Z_1	.	8	6	+0.833	0.44	
$C_j - Z_j$		0	0	−0.833	−0.44	
		✓	✓	✓	✓	

∴ $C_j - Z_j \leq 0$ & optimally reached

Solution is $x_1 = 3.61$, $x_2 = 14.03$

max $Z = 8 \times 3.61 + 6 \times 14.03 = 113.056$

∴ optimized value = 113.056

Question 4

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Assignment problem :-

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10
F	0	0	0	0	0	0

Soln

New no. of rows = no. of Columns

∴ Balanced assignment problem

Row optimization.

	1	2	3	4	5	6
A	4	2	7	14	10	6
B	0	8	15	5	6	2
C	8	7	0	5	2	6
D	0	8	4	7	7	6
E	1	5	4	0	6	3
F	0	0	0	0	0	0

Column optimisation :-

	1	2	3	4	5	6
A	4	2	7	14	10	6
B	0	8	15	5	6	2
C	8	7	0	5	2	6
D	0	8	4	7	7	6
E	1	5	4	0	6	3
F	0	0	0	0	0	0

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⇒ Row Scanning, Column Scanning:

	1	2	3	4	5	6
A	4	12	7	14	10	[0]
B	[0]	8	15	5	6	2
C	8	7	[0]	5	2	6
D	0	8	4	7	7	6
E	1	5	4	[0]	6	3
F	0	[0]	0	0	0	0

All Zeros are deleted

No. of boxes = 5 ; no of rows = 6

$5 \neq 6$

Next iteration

	1	2	3	4	5	6
A	4	0	7	14	18	0
B	0	6	15	5	4	2
C	8	5	0	5	0	6
D	0	6	4	7	5	6
E	1	3	4	0	4	3
F	2	0	2	2	0	2

⇒ Row Scanning, Column Scanning

	1	2	3	4	5	6
A	4	0	7	14	8	[0]
B	[0]	6	15	5	4	2
C	8	5	[0]	5	0	6
D	[0]	6	4	7	5	6
E	1	3	4	[0]	4	3
F	2	0	2	2	[0]	2

no of boxes = 5

no of rows = 6

$5 \neq 6$

⇒ Next iteration:

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	1	2	3	4	5	6
A	6	0	7	13	8	0
B	0	4	13	5	2	0
C	10	5	0	7	0	6
D	0	4	2	7	3	4
E	1	1	2	0	2	1
F	4	0	2	4	0	2

⇒ row Scanning, Column Scanning, row Scanning
Column Scanning

	1	2	3	4	5	6
A	6	[0]	7	13	8	0
B	0	4	13	5	2	[0]
C	10	5	[0]	7	0	6
D	[0]	4	2	7	3	4
E	1	1	2	[0]	2	1
F	4	0	2	4	[0]	2

All Zeroes are deleted.

No. of boxes = 6

No. of rows = 6

∴ optimality is reached.

Consolidate the table:-

Cities Surplus	Cities Deficit	Distance
A	1	10
B	2	12
C	3	3
D	4	1
E	5	7
F	1	0
	total =	38 km

∴ minimum total distance = 38 km.

Question - 5

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	1	2	3	4	5	6
1	9	22	58	11	19	27
2	43	78	72	50	63	48
3	41	28	91	37	45	33
4	74	42	27	49	39	32
5	36	11	57	22	25	18
6	3	56	53	31	17	28

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No. of rows = no. of columns

6 = 6

∴ Balanced assignment problem

Row minimization:-

	1	2	3	4	5	6
1	0	13	49	2	10	18
2	0	35	29	7	20	5
3	13	0	63	9	17	5
4	47	15	0	22	12	5
5	25	0	46	11	14	7
6	6	53	50	28	14	25

Column minimization

	1	2	3	4	5	6
1	0	13	49	0	0	13
2	0	35	29	5	10	0
3	13	0	63	7	7	0
4	47	15	0	20	2	0
5	25	0	46	9	4	2
6	0	53	50	26	4	20

⇒ row, column, row Scanning 10311 7086

	1	2	3	4	5	6
1	0	13	49	[0]	0	13
2	0	35	29	5	10	[0]
3	13	0	63	7	7	0
4	47	15	[0]	20	2	0
5	25	[0]	46	9	4	2
6	[0]	53	50	26	4	20

All Zeros are deleted

No of boxes = 5

No. of rows = 6

$5 \neq 6$

⇒ Next iteration:

	1	2	3	4	5	6
1	4	17	49	0	0	17
2	0	35	25	1	6	0
3	13	0	59	3	3	0
4	51	19	0	20	2	4
5	25	0	42	5	0	2
6	6	53	41	22	0	20

⇒ row, column, row, column Screening.

	1	2	3	4	5	6
1	4	17	49	[0]	0	17
2	[0]	35	25	1	6	0
3	13	0	59	3	3	[0]
4	51	17	[0]	20	2	4
5	25	[0]	42	5	0	2
6	0	53	41	22	[0]	20

∴ No. of boxes = 6

No. of rows = 6

6 = 6 ∴ optimality is reached.

Considated table :-

Tol	Operation	Cost
1	1	11
2	2	43
3	3	33
4	4	27
5	5	11
6	6	17

total = 142

∴ minimal cost = ₹ 142.

Question-6: Linear programming problem

maximize $Z = 8x_1 + 6x_2 + 10x_3$

subject to

$8x_1 + 4x_2 + 2x_3 \leq 155$

$3x_1 + 6x_2 + 12x_3 \leq 135$

where

$x_1, x_2, x_3 \geq 0.$

Converting the problem into standard form:-

maximize $z = 8x_1 + 6x_2 + 10x_3 + 0s_1 + 0s_2$.

Subject to

$$8x_1 + 4x_2 + 2x_3 + 1s_1 + 0s_2 = 155$$

$$3x_1 + 6x_2 + 12x_3 + 0s_1 + 1s_2 = 135$$

where

x_1, x_2, x_3, s_1 and $s_2 \geq 0$.

ITERATION - 1.

Cbi	Cj	8	6	10	0	0	~~Sol~~	~~Ratio~~
	Basic Variables	x_1	x_2	x_3	s_1	s_2	Sol	Ratio
0	s_1	8	4	2	1	0	155	77.5
0	s_2	3	6	12	0	1	135	11.25 KR
Zi		0	0	0	0	0		
Cj-Zi		8	6	10	0	0		
		↑	↑	↑ KC	~	~		

∴ Key element = 12

Entering Variable = x_3

leaving Variable = s_2

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ITERATION-2

C_{Bi}	C_j	8	6	10	0	0		
	Basic Variable	x_1	x_2	x_3	s_1	s_2	Sol	Ratio
0	s_1	7.5	3	0	1	-0.167	132.5	17.67
10	x_3	0.25	0.5	1	0	0.083	11.25	45
Z_j		2.5	5	10	0	-0.833		
$C_j - Z_j$		5.5	1	0	0	-0.833		
		✗	✗	✓	✓	✓		

kc

∴ Key element = 7.5

x_1 = Entering Variable , s_1 = leaving Variable.

ITERATION-3

C_{Bi}	C_j	8	6	0	0	0	Sol
	Basic Variable	x_1	x_2	x_3	s_1	s_2	Sol
	x_1	1	0.4	0	0.133	-0.32	17.67
	x_3	0	0.4	1	-0.033	0.089	6.83
Z_j		8	7.2	10	0.733	0.711	
$C_j - Z_j$		0	-1.2	0	-0.733	-0.711	
		✓	✓	✓	✓	✓	

∴ $C_j - Z_j \leq 0$; optimality is reached

Sol^n ⇒ $x_1 = 17.67$, $x_2 = 0$, $x_3 = 6.83$

∴ max $Z = 8 \times 17.67 + 6 \times 0 + 10 \times 6.83$

max $Z = 209.67$

∴ optimized Value = 209.67.