

ASSIGNMENT - I

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Q = 21

maximize $Z = 5x_1 - 2x_2 + 3x_3$.

Subject to constraints,

$$2x_1 + 2x_2 - x_3 \geq 2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_2 + 3x_3 \leq 5$$

where x_1, x_2 and $x_3 \geq 0$.

Sol :-

Converting the problem into standard form:

$$\text{maximize } Z = 5x_1 - 2x_2 + 3x_3 + 0s_1 + 0s_2 + 0s_3 - M R_1$$

Subject to constraints,

$$2x_1 + 2x_2 - x_3 - 1s_1 + 0s_2 + 0s_3 + 1R_1 = 2$$

$$3x_1 - 4x_2 + 0s_1 + 1s_2 + 0s_3 = 3$$

$$x_2 + 3x_3 + 0s_1 + 0s_2 + 1s_3 = 5$$

where $x_1, x_2, x_3, s_1, s_2, s_3$ and $R_1 \geq 0$

ITERATION-1

C_{Bi}	C_i	5	-2	3	0	0	0	-M	
Basic variables	x_1	x_2	x_3	S_1	S_2	S_3	R_1	S_{st}	Ratio
-M	<u>R_1</u>	(2)	2	-1	-1	0	0	1	2 1
0	S_2	3	-4	0	0	1	0	0	3 1
0	S_3	0	1	3	0	0	1	0	5 ∞
Z_i		$-2m$	$-2m$	M	M	0	0	-M	
$C_j - Z_i$		$5+2m$	$-2+2m$	$3-M$	$-M$	0	0	0	
		X	X	<u>$3-M$</u>	<u>$-M$</u>	<u>0</u>	<u>0</u>	<u>0</u>	
									RC

Key element : 2

, . . . entering variable: x_1

leaving variable: S_3

ITERATION-2

C_{Bi}	C_j	5	-2	3	0	0	0	-M	
Basic variables		x_1	x_2	x_3	S_1	S_2	S_3	R_1	Slacks Ratio
-M	<u>R_1</u>	1	1	-0.5	-0.5	0	0	0.5	1 -2
0	S_2	0	-7	1.5	1.5	1	0	-1.5	0 0 KR
0	S_3	0	1	3	0	0	1	0	5 1.67
Z_i		5	5	-2.5	-2.5	0	0	2.5	
$(C_j - Z_i)$		0	1 -7	5.5	2.5	0	0	-M -2.5	
				X					
				KC					

Key element : 2.5

\therefore entering Variable: x_3

leaving Variable: S_2

ITERATION - 3

C_{Bi}	C_j	5	-2	3	0	0	0	-M	
Basic variables		x_1	x_2	x_3	S_1	S_2	S_3	R_1	Slack Ratio
-M	x_1	1	-1.33	0	0	0.33	0	0	1 -0.75
0	x_3	0	-4.67	1	1	0.67	0	-1	0 -%
0	S_3	0	15	0	-3	-2	1	3	5 0.33
Z_i		5	-20.67	3	3	3.67	0	-3	
$(C_j - Z_i)$		0	18.67	0	-3	-3.67	0	-M+3	
		✓	x_2	✓	✓	✓	✓	✓	

Key element : 15

∴ entering variable: x_2

leaving variable: S_3

ITERATION - 4

C_{Bi}	C_j	5	-2	3	0	0	0	-M	
Basic variables		x_1	x_2	x_3	S_1	S_2	S_3	R_1	Ratio
-M	x_1	1	0	0	-0.267	0.186	0.089	0.267	1.444
0	x_3	0	0	1	0.067	0.044	0.311	1.567	23.33
0	x_2	0	1	0	-0.2	-0.13	0.067	0.333	0.33
Z_i		5	-2	3	-0.733	1.178	1.24	5.376	
$C_j - Z_i$		0	0	0	0.733	-1.178	-1.24	-M	
		0	0	0	X	✓	✓	-5.376	✓
									NC

Key element : 0.067

∴ entering variable: S_1

leaving variable: x_3 .

C_B	C_j	5	-2	3	0	0	0	
Basic Variable		x_1	x_2	x_3	S_1	S_2	C_3	Sol
	x_1	1	0	4	0	0.333	1.33	7.667
	S_1	0	0	15	1	0.17	4.17	23.33
	x_2	0	1	3	0	0	-1	5
Z_j		5	-2	14	0	1.667	4.667	
$C_j - Z_j$		0	0	-11	0	-1.667	-4.667	
		✓	✓	✓	✓	✓	✓	

$\therefore C_j - Z_j \leq 0$ optimality reached

$$\therefore x_1 = 7.667 \quad x_2 = 5 \quad , \quad x_3 = 0$$

$$\therefore Z = 5x_1 + (-2)x_2 + 3x_3 = 28.33$$

\therefore Optimized Value = 28.33

Question - 3

maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4$

subject to constraint

$$4x_1 + 6x_2 + 5x_3 + 4x_4 \geq -20 \quad \text{--- (1)}$$

$$-3x_1 - 2x_2 + 4x_3 + x_4 \leq 10 \quad \text{--- (2)}$$

$$-8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20 \quad \text{--- (3)}$$

where x_1, x_2, x_3 and $x_4 \geq 0$

Soln

Converting into standard problem :-

maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4 + 0S_1 + 0S_2 + 0S_3$

subject to constraint :-

$$-4x_1 - 6x_2 - 5x_3 - 4x_4 + 1S_1 + 0S_2 + 0S_3 = 20$$

$$-3x_1 - 2x_2 + 4x_3 + x_4 + 0S_1 + 1S_2 + 0S_3 = 10$$

$$-8x_1 - 3x_2 + 3x_3 + 2x_4 + 0S_1 + 0S_2 + 1S_3 = 20$$

where

$x_1, x_2, x_3, x_4, S_1, S_2$ and $S_3 \geq 0$.

ITERATION - 1

C_{Bi}	C_i	4	1	3	5	0	0	0		
Basic variables		x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_{01}	Ratio
-M	S_1	-4	6	5	4	0	0	0	20	5 KR
0	S_2	-3	-2	4	1	0	1	0	10	10 KR
0	S_3	-8	-3	3	2	0	0	1	20	10 KR
Z_i		0	0	0	0	0	0	0	0	
$C_j - Z_i$		4	1	3	5	0	0	0	0	
		x	x	x	x	✓	✓	✓	✓	

Key element : 4

∴ entering Variable: x_4

leaving Variable: S_1

ITERATION - 2

$C_B i$	C_i	4	1	3	5	0	0	0	
Basic variables	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_{01}	Ratio
-5	x_4	-1	1.25	1.25	1	0.25	0	0	5
0	S_2	-2	-3.5	2.75	0	-0.25	1	0	5
0	S_3	-6	-6	0.5	0	-0.5	0	1	10
Z_i		-5	7.5	6.25	5	7.25	0	0	
$(C_j - Z_i)$		9	-6.5	-3.75	0	-1.25	0	0	
	X	✓	✓	✓	✓	✓	✓	✓	

All ratios values are negative

variable x_1 should enter into the basis, but all the coefficient in the x_1 column are negative. So, x_1 cannot be entered into basic variables

hence, the solution to given problem is unbounded.

Question - 4

Maximize $Z = 4x_1 + 5x_2 + 2x_3$.

Subject to constraints,

$$2x_1 + x_2 + x_3 \leq 10$$

$$x_1 + 3x_2 + x_3 \leq 12$$

$$x_1 + x_2 + x_3 \geq 6$$

where

$$x_1, x_2 \text{ and } x_3 \geq 0.$$

Sol'n Converting into standard problem.

maximize

$$Z = 4x_1 + 5x_2 + 2x_3 + 0S_1 + 0S_2 - 1R_1$$

Subject to Constraints

$$2x_1 + x_2 + x_3 + 1S_1 + 0S_2 = 10$$

$$x_1 + 3x_2 + x_3 + 0S_1 + 1S_2 = 12$$

$$x_1 + x_2 + x_3 + 1R_1 = 6$$

where x_1, x_2, x_3, S_1, S_2 and $R_1 \geq 0$.

ITERATION - 1

C_{Bi}	C_j	4	5	2	0	0	$-M$		
Basic Variables		x_1	x_2	x_3	S_1	S_2	R_1	Slack	Ratio
0	S_1	2	1	1	1	0	6	10	10
0	S_2	1	3	1	0	1	8	12	4 KR
$-M$	R_1	1	1	1	0	0	1	6	6
Z_j		$-M$	$-M$	$-M$	0	0	$-M$		
$C_j - Z_j$		$4+M$	$5+M$	$2+M$	0	0	0	0	
		X	X	X	✓	✓	✓		
									NC

i.e. key element = 3

Entering variable = x_2

leaving variable = S_2 $x_2 \geq 0$

$$\therefore Z = 4x_1 + 5x_2 + 2x_3 + 8S_1 + 12S_2 \geq 28.33$$

Optimized Value = 28.33

ITERATION - 2

C_B	C_j	4	5	2	0	0	-M		
Basic Variables		x_1	x_2	x_3	S_1	S_2	R_1	Sol	Ratio
0	$-S_1$	1.67	0	0.67	1	-0.33	0	6	3.6
5	x_2	0.33	1	0.33	0	0.33	0	4	12
-M	R_1	0.67	0	0.67	0	-0.33	1	2	3
Z_i		-1.67M = 0.67	5 +1.67	-0.67M +1.67	0 +1.67	0.33M -1.67	-M 0		
$C_j - C_B$		0.67M -2.33	0	0.67M +0.33	0	-0.33M -1.67	0		
		X	N	X	V	V	V		
								KR	

\therefore key element = 0.67

\therefore Entering Variable = x_1

leaving variable = R_1

ITERATION - 3

C_B	C_j	Z_1	Z_2	Z_3	S_1	S_2	R_1	Sol	P_{ij}
	Basic Variables	x_1	x_2	x_3	S_1	S_2	R_1		
0	S_1	1 0 0	-1 1 0	0.5 -0.5	-2.49	1			
5	x_2	0 1 0	1 0 0	0 0.5	-0.49	3			
4	x_1	0 1 0	0 1 0	0 -0.5	1.49	3			
Z_i		4 5 4	0 0 0	0.5 0.5	3.5				
$C_j - Z_j$		0 0 -2	0 0 -0.5	-M -3.5					
		✓ ✓ ✓ ✓ ✓ ✓							

$\therefore C_j - Z_j \leq 0$ optimality reached

$$\therefore x_1 = 3, x_2 = 3, x_3 = 0$$

$$Z = 4 \times 3 + 5 \times 3 + 2 \times 0 = 27$$

\therefore optimized value = 27

Optimal Value = 27.33

Question - 5

$$\text{maximize } Z = 4x_1 + 5x_2 - 3x_3$$

Subject to constraints,

$$x_1 + x_2 + x_3 = 10$$

$$x_1 - x_2 \geq 1$$

$$2x_1 + 3x_2 + x_3 \leq 40$$

where

$$x_1, x_2 \text{ and } x_3 \geq 0$$

Soln = Converting problem into standard form.

$$\text{maximize } Z = 4x_1 + 5x_2 - 3x_3 + 0S_1 + 0S_2 - MR_1 - MR_2$$

Subject to constraints

$$x_1 + x_2 + x_3 + 1R_1 = 10$$

$$x_1 - x_2 - 1S_1 + 1R_2 = 1$$

$$2x_1 + 3x_2 + x_3 + 1S_2 = 40$$

where

$$x_1, x_2, x_3, S_1, S_2, R_1 \text{ and } R_2 \geq 0$$

ITERATION - 1

C_B	C_i	4	5	-3	0	0	-M	-M		
Basic Variables		x_1	x_2	x_3	s_1	s_2	R_1	R_2	Soln. Ratio	
-M	R_1	1	1	1	0	0	1	0	10	10
-M	R_2	1	-1	0	-1	0	0	1	1	Def.
0	s_2	2	3	1	0	1	0	0	40	20
-Z _i		-2M	0	-M	M	0	-M	-M		
C_{j-Z_i}		2M+5	5	+M	-M	0	0	0		
		X	X	X	✓	✓	✓	✓		
		K _c								

\therefore key element = 1

entering element = x_1

leaving element = R_2 .

ITERATION - 2

C_B	C_i	4	5	-3	0	0	1	-M - M		
Basic Variables		x_1	x_2	x_3	S_1	S_2	R_1	R_2	Soln.	Ratio
-M	R_1	0	2	1	1	0	1	-1	9	4.5
A	S_1	1	-1	0	1	0	0	1	1	M
0	S_2	0	5	1	2	1	0	-2	3.8	7.6
-Z _i		4	-2M	-M	-M+4	0	-M	M+4		
$G - Z_i$		0	2M+9	M-3	M+4	0	0	-2M		
		X	X	X						
		ke	ke	ke						

\therefore key element = 2

entering element = x_2

leaving element = R_1

ITERATION - 3

C_B	C_j	4	5	-3	0	0	-M	-M	
Basic Variables	x_1	x_2	x_3	s_1	s_2	R_1	R_2	B_{all}	
5	x_2	10	1	0.5	6.5	0	0.5	-0.5	4.5
4	x_1	15	0	0.5	-6.5	0	6.5	0.5	5.5
0	s_2	0	0	-1.5	-0.5	1	-26.5	0.5	15.5
Z_i		4	5	4.5	0.5	0	4.5	-0.5	
$C_j - Z_j$		0	0	-7.5	-0.5	0	-M	-M	+0.5
		✓	✓	✓	✓	✓	✓	✓	

$\therefore C_j - Z_j \leq 0$ optimality reached

$$\therefore x_1 = 5.5, x_2 = 4.5, x_3 = 0$$

$$Z = 4 \times 5.5 + 5 \times 4.5 + (-3) \times 0 = 44.5$$

\therefore Optimized Value = 44.5