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CY-A

Date

Linear Algebra HW # 03.

Q1. $A = \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ -3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}_{5\times5}$

$R_3 - 2R_1 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 0 & 3 & -6 & 0 & 2 \\ -3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$ $R_4 + 3(R_1) \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & -15 & 6 & 12 & 8 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$

$R_5 + 2R_1 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & -15 & 6 & 12 & 8 \\ 0 & 3 & 6 & 0 & -3 \end{bmatrix}$ $R_5 - R_2 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & -15 & 6 & 12 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2/3 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 3 & -6 & 0 & 2 \\ 0 & -15 & 6 & 12 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 - 3R_2 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & -15 & 6 & 12 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_4 + 15R_2 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & 36 & 12 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $R_4 + 3R_3 \rightarrow \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & -12 & 0 & 5 \\ 0 & 0 & 0 & 12 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

i) $\text{Col } A = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -3 \\ -6 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 4 \\ 6 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 4 \\ 5 \\ -5 \end{bmatrix} \right\}$

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Dimension of Col A = 4 (no. of pivot columns of A)

Basis for Col A = $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -3 \\ -6 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \\ 6 \\ -4 \end{bmatrix} \right\}$

ii) $r_1 = 1, -3, 2, 2, 1$

$r_2 = 0, 3, 6, 0, -3$

$r_3 = 2, -3, -2, 4, 4$

$r_4 = 3, -6, 0, 6, 5$

$r_5 = -2, 9, 2, -4, 5$

Row space: Span $\{r_1, r_2, r_3, r_4, r_5\}$

Basis for row A:

$\{(1, -3, 2, 2, 1), (0, 1, 2, 0, -1), (0, 0, -12, 0, 5), (0, 0, 0, 12, 0)\}$

Dimension of Row A = Dimension of Col A = 4

iii) Null space:

$x_1 - 3x_2 + 2x_3 + 2x_4 + x_5 = 0$

$x_4 + 2x_3 - x_5 = 0$

$-12x_3 + 5x_5 = 0$

$12x_4 + 0x_5 = 0$

x_5 is free variable

$x_4 = -\frac{2}{3}x_5$

$x_3 = \frac{5}{12}x_5$

$x_2 = x_5 - \frac{5}{12}x_5 = \frac{7}{12}x_5$

$x_1 = 3x_2 - 2x_3 - 2x_4 - x_5$

$x_1 = 3\left[\frac{7}{12}\right]x_5 - 2\left[\frac{5}{12}\right]x_5 - 2\left[\frac{-2}{3}x_5\right] - x_5$

$= \frac{7}{4}x_5 - \frac{5}{6}x_5 + \frac{4}{3}x_5 - x_5$

$x_1 = \frac{5}{4}x_5$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5/4\,x_5 \\ 7/12\,x_5 \\ 5/12\,x_5 \\ -2/3\,x_5 \\ x_5 \end{bmatrix} = x_5 \begin{bmatrix} \frac{5}{4} \\ \frac{7}{12} \\ \frac{5}{12} \\ -\frac{2}{3} \\ 1 \end{bmatrix}$

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null space : $\left\{ \begin{bmatrix} \frac{5}{4} \\ \frac{7}{12} \\ \frac{5}{12} \\ -\frac{2}{3} \\ 1 \end{bmatrix} \right\}$

Dimension = 1 (no. of free variables in $Ax=0$)

$Ax = 0$ only zero when $x = 0$.

So no non-trivial solutions give zero vector

iv) $A^T = \begin{bmatrix} 1 & 0 & 2 & 3 & -2 \\ -3 & 3 & -3 & -6 & 9 \\ 2 & 6 & -2 & 0 & 2 \\ 2 & 0 & 4 & 6 & -4 \\ 1 & -3 & 4 & 5 & -5 \end{bmatrix}$

$\sim \begin{bmatrix} \textcircled{1} & 0 & 0 & 1 & -2 \\ 0 & \textcircled{1} & 0 & 0 & 1 \\ 0 & 0 & \textcircled{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (done on rough pages)

$x_1 + x_4 - 2x_5 = 0.$

$x_2 + x_5 = 0$

$x_3 + x_4 = 0$

x_4 and x_5 are free variables

$x_1 = 2x_5 - x_4$

$x_2 = x_5$.

$x_3 = -x_4$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_5 - x_4 \\ -x_5 \\ -x_4 \\ x_4 \\ x_5 \end{bmatrix} = x_4 \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

left null space : span $\left\{ \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

Basis : $\left\{ \begin{bmatrix} -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

Dimension = nullity (A^T)

$= m - \text{rank}(A) = 2$