Name : Sarita Sangrez Roll No. 23i-2088

Section : CY-A

Assignment 02

Q1.

a) $P(A) = 0.4$, $P(B) = 0.37$.

$P(A \cap B) = 0.10$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.67$

c) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.37} = 0.27$

d) $P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)}$

$P(A \cap B^c) = P(A) - P(A \cap B)$

$= 0.3$

$P(B^c) = 1 - 0.37 = 0.63$

So $P(A|B^c) = 0.3/0.63 = 0.47$.

e) $P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{P(A \cup B)^c}{1 - P(B)}$

$= \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - 0.67}{0.63} = 0.52$

Q2. T O M O R R O W

a) $\frac{8!}{2!\,3!} = 3360.$

b) R TO M O O W R

$\frac{6!}{3!} = 120$

c) P(at least one 'O' and atleast one 'R')

Total ways

$= 8C4 = 70.$

case w/o O : $5C4 = 5$

case w/o R : $6C4 = 15$

case w/o O and R : not possible

$70 - (5+15) = 50.$

So $P() = \frac{50}{70} = \frac{5}{7}$

000 T M W R.

So fg : $4! = 24.$

And $(R \ldots R) - (O's\ fg)$

$= 120 - 24 = 96$

Q3. bag — 2R, 3G, 4B — 3 balls drawn.

a) $P(\text{diff color}) = \frac{2C1 \times 3C1 \times 4C1}{9C3} = \frac{2}{7}$

b) P(same color)

All red not possible

All green : $3C3$

All black : $4C3$.

So $P(S.C) = \frac{3C3 + 4C3}{9C3} = \frac{5}{84}$

c) Case 1 : 2R + 1G

$2C2 \times 3C1 = 3$.

Case 2 : 2R + 1B

$2C2 \times 4C1 = 4$

Case 3 : 2G + 1R

$3C2 \times 2C1 = 6$.

Case 4 : 2G + 1B

$3C2 \times 4C1 = 12$

Case 5 : 2B + 1R

$4C2 \times 2C1 = 12$

Case 6 : 2B + 1G

$4C2 \times 3C1 = 18$

So $P = \frac{3+4+6+12+12+18}{84} = \frac{55}{84}$

Q4. $P(A) = 0.7$, $P(B \cup A) = 0.9$, $P(A \cap B) = 0.3$.

a) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= 0.7 - 0.3 = 0.4$

b) $P(B \cap A^c) = P(B) - P(A \cap B)$

$= 0.5 - 0.3 = 0.2$

$P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$= 0.9 - 0.7 + 0.3$

$= 0.5$

c) $P(A^c \cap B^c) = P(A \cup B)^c$

$= 1 - P(A \cup B) = 1 - 0.9 = 0.1$

d) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B)$ → $P(B) - P(A \cap B)$

$= 0.3 + 0.5 - (0.5 - 0.3)$

$= 0.6$

e) $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B) = 1 - 0.3 = 0.7$

Q5. Total outcomes : $4^2 = 16$.

	1	2	3	4
1				
2				
3				
4				

a) P(at least one die = 3)

= { (1,3), (2,3), (3,1), (3,2), (3,3), (3,4), (4,3) }.

P(score even) = { (1,3), (3,1), (3,3) }

$$P(\text{even from 3} \mid 3) = \frac{P(\text{even})}{P(3)} = \frac{3}{7} = 0.42$$

b) P(score is even) = { (1,1), (1,3), (2,2), (2,4), (3,1), (3,3), (4,2), (4,4) }

$$\frac{P(\text{even from 3})}{P(\text{even from total})} \quad \frac{3}{8} = 0.37$$

Q6.

M1 0.4

M2 0.5

M3 0.6

a) $P(M_1 \cap M_2 \cap M_3)$

$= 0.4 \times 0.5 \times 0.6 = 0.12$

b) atleast one hits

1 - P(None hit)

$1 - P(M_1^c \cap M_2^c \cap M_3^c)$

$1 - (0.6)(0.5)(0.4) = 0.88$

c) At most one hit (0 or 1 hit)

case 1: M_1 hit

$P(M_1 \cap M_2^c \cap M_3^c) = 0.4 \times 0.5 \times 0.4 = 0.08$

case 2: M_2 hit

$0.6 \times 0.5 \times 0.4 = 0.12$

case 3: M_3 hit

$0.6 \times 0.5 \times 0.6 = 0.18$

case 4: None

$0.6 \times 0.5 \times 0.4 = 0.12.$

$0.08 + 0.12 + 0.18 + 0.12$

$= 0.5.$

d) exactly 1.

$0.08 + 0.12 + 0.18 = 0.38$

e) exactly 2.

case 1: M_1 M_2

$0.4 \times 0.5 \times 0.4 = 0.08$

case 2: M_1 M_3

$0.4 \times 0.5 \times 0.6 = 0.12$

case 3: M_2 M_3

$0.6 \times 0.5 \times 0.6 = 0.18.$

$0.08 + 0.12 + 0.18$

$= 0.38$

Q7. C = clean $P(C) = 0.44$

F = Fill $P(F) = 0.24$

E = extract $P(E) = 0.21$

$P(C \cap E) = 0.11$, $P(F \cap E) = 0.07$, $P(C \cap F \cap E) = 0.03$.

$P(C \cup F \cup E) = P(C) + P(F) + P(E) - P(C \cap F) - P(C \cap F) - P(F \cap E) + P(C \cap F \cap E)$

$= 0.44 + 0.24 + 0.21 - 0.08 - 0.11 - 0.07 + 0.03$

$= 0.66$.

Q8. $P(H) = 0.21$ $P(W) = 0.28$ $P(H \cap W) = 0.15$

a) 1 will vote.

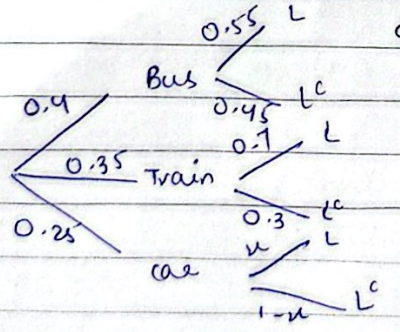
$P(H \cup W) = 0.21 + 0.28 - 0.15 = 0.34$

b) $P(W \mid H) = \frac{P(W \cap H)}{P(H)} = \frac{0.15}{0.21} = \frac{5}{7}$

c) $P(H \mid W^c) = \frac{P(H \cap W^c)}{P(W^c)} = \frac{P(H) - P(H \cap W)}{1 - P(W)}$

$= \frac{0.21 - 0.15}{1 - 0.28} = \frac{0.06}{0.72} = 1/12$

Q9.



a) $P(L^c) = 0.48$

$P(L) = 1 - 0.48$

$= 0.52$.

$0.48 = P(L^c \mid B) \cdot P(B) + P(L^c \mid T) \cdot P(T) + P(L^c \mid C) \cdot P(C)$

$0.48 = 0.45 \times 0.4 + 0.3 \times 0.35 + 0.25(1-x)$

$1 - x = \frac{0.195}{0.25} = 0.78$

$x = 0.22$

b) $P(T \mid L) = \frac{P(T \cap L)}{P(L)} = \frac{0.35 \times 0.7}{0.52}$

$= 0.471$