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Assignment # 2 (BS-CS: Section CY-A, B)
(CS-1005 Discrete Structures – FALL-2024)

Due Date and Time: Wednesday, 13th November, 2024 (11:59 pm) Marks: 100

Instructions:

- *Late assignment will not be accepted*
- *Only the clear scan (using, e.g., CamScanner) copy of handwritten attempt will be graded, i.e., printed attempts will not be graded.*
- *Only the attempts submitted on **Google classroom (till the due date & time) will be considered**, i.e., the submissions that will be slided beneath instructors' office doors or submitted via email or elsewhere will not be graded.*
- *You should **handover the same hard copy of the assignment to your TA on the same or next day only (The exact time will be decided by your TA).***
- *There will be no credit if the given requirements are changed.*
- *Your solution will be evaluated in comparison with the best solution.*
- *Whenever a calculation is involved, your solution should show complete steps and a final answer. There will be significant marks for the correct final answer (as far as assignments are concerned).*
- *You must write your roll number, name, and section (of your Discrete Course only) on your submitted attempt.*

Question 1: [Marks 15]

(a) **What is the truth value of each of the following wffs in the interpretation where the domain consists of the integers, $O(x)$ is "x is odd", $L(x)$ is "x < 10", and $G(x)$ is "x > 9"?**

(i) $\exists x\, O(x)$

There exists an integer x that is odd.
Since $x \in Z$, truth value is True.

(ii) $\forall x\, L(x) \rightarrow O(x)$

For all integers, if $x < 10$, then x is odd.

Counter example : $x = 2$ (even) and less than 10

So truth value is false

(iii) $\exists x\, L(x) \wedge G(x)$

$x < 10$ and $x > 9$. where $x \in Z$.

There exists no such integer

Truth value is false

(iv) $\forall x\, L(x) \vee G(x)$

For all x, $x < 10$ or $x > 9$ (either)

Truth value is True

(b) **Find the truth value of the following predicates where domains of x and y, D_x and D_y are given below. Also mention the interpretation for each: Let $D_x = D_y = \{1,2,3,4,5,6\}$. Define the predicate $P(x,y)$ as**

$$P(x,y) := (y \geq x) \vee (x + y > 6)$$

(i) $\exists x \forall y\, P(x,y)$

For all y, there exists atleast one x such that $y \geq x$

or $x + y > 6$.

for $x = 1$, $y \geq 1$, for $y \in D_y$

true for every y $= \{1,2,3,4,5,6\}$

Truth value : $\underline{\text{true}}$

?

(ii) $\exists y \forall \underline{c}\, P(y,x)$ atleast

There exists, one y for all c such that

$x \geq y$ or $y+x > 6$

(iii) $\forall x \exists y\, P(x,y)$

For all x, there exists a y such that $\because$ $y \geq x$

or $x+y > 6$.

for $y = 1$, $y \geq x$ is true for $x \in D_x$
$= \{1, 2, 3, 4, 5, 6$

So true value : $\underline{\text{True}}$.

(iv) $\exists x\, P(x,y)$

There exists an x such that $y \geq x$ or $x+y > 6$.

For $x = 1$, $y \geq x$ for $y \in \{1, 2, 3, 4, 5, 6\}$

Truth value : $\underline{\text{True}}$

(c) Let $P(x)$, $Q(x)$, $R(x)$ and $S(x)$ denote the following predicates with domain Z:

$P(x)$: $x \leq 0$,
$Q(x)$: $x^2 = 1$,
$R(x)$: x is odd, $S(x)$: x
$= x + 1$.

$S(x)$: $x = x+1$

(i) **For each predicate, determine its truth set.**

$P(x) : x \leq 0.$

. Truth set : $\{ x \in Z \mid x \leq 0 \} = \{ \dots -3, -2, -1, 0 \}$

$Q(x) : x^2 = 1$ as $x^2 = 1$ when $x = -1$ or $x = 1$.

Truth set : $\{ -1, 1 \}$

$R(x)$: x is odd

Truth set : $\{ x \in \mid x$ is odd $\} = \{ \dots 5, -3, -1, 1, 3, 5 \dots \}$

$S(x) : x = x+1$

Truth set : $\{ \}$ or ϕ

(ii) **Determine whether each of the following statements is true or false. Mention the interpretations for each if false.**

(a) $\forall x\, P(x) \rightarrow Q(x)$

For all x, if $P(x)$ true then $Q(x)$ true.

But when $x = -2$, then $Q(x)$ is false

so truth value : False

(b) $\forall x\, Q(x) \rightarrow R(x)$

For all x, if $Q(x)$ is true then $R(x)$ is true. If $x = 3$, $Q(x)$ is false but $R(x)$ is still true, so

truth value : ..... True.

(c) $\forall x\, R(x) \rightarrow S(x)$

If $R(x)$ is true for all x, then $S(x)$ must be true for all x.

$S(x)$ is false for all integers

so truth value, ...

False

(d) $\forall x\, S(x) \to R(x)$

If S(x) is true for all x, then R(x) is true for all x.

S(x) is never true for any x but R(x) is true for all odd x

so truth value : True

(e) $\exists x\, Q(x) \wedge \neg R(x)$

There exists some integer x such that Q(x) is true and R(x) is false. Q(x) true when x = 1 or -1 and R(1) or R(-1) is also true.

so no such x exists.

Truth value: False.

Question 2 [Marks 10]

Translation:

Consider the following murder mystery.

Express each of the clues above in predicate logic. You may use types in your formalization so that every quantified variable has an associated type. Be sure to give the intended meaning of each of the constants and predicates that you use.

a : aunt agatha b : Beatrice

c : charles

w : wisteria Lodge.

(a) Someone who lives at Wisteria Lodge murdered Aunt Agatha.

$\exists x\, (L(x,w) \wedge M(x))$

(b) Aunt Agatha, Beatrice, and Charles live at Wisteria Lodge and nobody else lives there.

$L(a,w) \wedge L(b,w) \wedge L(c,w)$

OR $\forall x\, (L(x,w) \to x \in (a,b,c))$

(c) Beatrice is the only person Aunt Agatha doesn't hate.

For all x, if Aunt Agatha hates x, then x is not b. n = x is b.

so $\forall x\, (H(a,x) \to \sim n)$

(d) Aunt Agatha hates no-one that Charles hates.

For all x, if charles hates x, then Aunt does not hate x.

$\forall x\, (H(c,x) \to \sim H(a,x))$

M(x) : x murdered Aunt Agatha

L(x,y) : x lives at location y

H(x,y) : x hates y

R(x,y) : x is richer than y

K(x) : x is murderer.

(e) **Beatrice hates everyone unless that person is richer than Aunt Agatha.**

For all x, if x is not richer than A.A, then Beatrice hates x.

$$\forall (\sim R(x,a) \rightarrow H(b,x))$$

(f) **Beatrice hates everyone Aunt Agatha hates.**

For all x, if A.A hates x, then Beatrice hates x.

$$\forall x (H(a,x) \rightarrow H(b,x))$$

(g) **Aunt Agatha and Beatrice are not the same person.**

$$a \neq b.$$

(h) **Everyone has someone that they don't hate.**

For every person x, there exists a person y such that x does not hate y.

$$\forall x \, \exists y \, \sim H(x,y)$$

(i) **A murder victim is always hated by their murderer.**

For all x, if x is victim, then x is hated by murderer y

$V(x)$ = x is victim

$$\forall x \, (V(x) \rightarrow \exists y (K(y) \wedge H(y,x))$$

y murderer x and hated x

(j) **A murderer is never richer than his victim.**

For all x, if x is murderer, then x can not be richer than his victim v

$$\forall x \, (K(x) \rightarrow \sim R(x,v))$$

Question 3 [Marks 10]

Translation

Translate the following predicate logic formulas into English sentences. Where the domain consists of all integers.

(a) $\forall m \forall n(((m < 0) \wedge (n < 0)) \rightarrow (mn > 0))$

For all m and n, if m is less than zero and n is less than zero, then product of m and n is more than zero.

(b) $\forall m \forall n(((m > 0) \wedge (n > 0)) \rightarrow (\frac{m+n}{2} > 0))$

For all m and n, if m is more than zero and n is more than zero, then sum of m and n divided by 2 is more than 0.

(c) $\exists m \exists n((m < 0) \wedge (n < 0) \wedge \neg(m - n < 0))$

There exists an m and n such that m is less than 0 and n is less than 0 and difference of m and n is not less than zero.

(d) $\forall m \forall n(|m + n| \leq |m| + |n|)$

For all m and n, absolute value of sum of m and n is less than or equal to the sum of absolute values of m and n.

Question 4 [Marks 15]

a) Are the following arguments valid? If valid then also write the name of the law that implies on them. If superman were able and willing to prevent evil, he would do so. $a \wedge w$

Is superman being unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. $\sim a \rightarrow i$ $\sim w \rightarrow m$

Superman does not prevent evil. $\sim w$

If superman exists, he is neither impotent nor malevolent.

Therefore, superman does not exist. $e \rightarrow \sim(i \wedge m)$

$\therefore \sim e$

1. $a \wedge w \rightarrow p$
2. $\sim w \rightarrow m$
3. $\sim a \rightarrow i$
4. $\sim p$
5. $e \rightarrow (\sim i \wedge \sim m)$

valid

Modus Tollens

~~1. : $a \wedge w$ Simplification~~

~~$\therefore a$~~

~~3 & 4 : $\sim w \rightarrow m$~~

~~$\sim w$~~

~~$\therefore m$~~

3. $\sim a \rightarrow i$

$\underline{a}$ → from conclusion 1.

$\therefore \sim i$

so 5. $e \rightarrow \sim(i \wedge m)$ → False

$\underline{\sim\sim(i \wedge m)}$ → True

$\therefore \sim e$

Modus Tollens.

Done again !!

1. and 4. $a \wedge w \rightarrow p$ Modus Tollens

$\underline{\sim p}$

$\sim(a \wedge w)$ so $\sim a \vee \sim w$ - C1

2 & 3 $\sim a \rightarrow i$

$\underline{\sim a}$

$\therefore i$

or $\sim w \rightarrow m$

$\underline{\sim w}$

$\therefore m$

from C1 we say either one is true, so $i \vee m$

5. $e \rightarrow (\sim i \wedge \sim m)$

$\underline{i \vee m}$ (negation of conclusion)

$\therefore \sim e$

Modus Tollens

b) *You cannot be both happy and rich. Therefore, you are either not happy, or not rich. Now you do appear to be happy. Therefore, you must not be rich.*

valid

$\sim(h \wedge r)$

$\therefore \sim h$

Disjunctive syllogism.

$\sim h \vee \sim r$

$\underline{h}$

$\therefore \sim r$

c) *If interest rates fall, then the stock market will rise. If interest rates do not fall, then housing starts and consumer spending will fall. Now, consumer spending is not falling. So, it's true that housing starts are not falling or consumer spending is not falling; that is, it is false that housing starts and consumer spending are both falling. This means that interest rates are falling, so the stock market will rise.*

a) $i \to s$

b) $\sim i \to (h \wedge c)$

c) $\sim c$

d) $(\sim h \vee \sim c) \to T$

e) $(h \wedge c) \to F$

X
$$\left[\begin{array}{c} \sim i \to (h \wedge c) \\ \sim c \\ \hline \therefore\ i \end{array} \quad \begin{array}{c} \sim h \vee \sim c \\ \sim c \\ \hline \therefore\ h \end{array}\right]$$

valid $\quad h \wedge c \to$

~~Modus Tollens and disjunctive syllogism~~

b and c:

$$\begin{array}{c} \sim i \to (h \wedge c) \\ \sim c \\ \hline \therefore\ i \end{array}$$

Modus Tollens

e : $\sim(h \wedge c) \to F$

$$\overline{\therefore\ \sim(h \wedge c)}$$

or

$\sim h \vee \sim c$

a and b:

$$\begin{array}{c} i \to s \\ \sim i \\ \hline \therefore\ s \end{array}$$

Modus Ponens

$$\cancel{\begin{array}{c} \sim h \vee \sim c \to \\ \sim c \\ \hline \therefore\ h \end{array}} \qquad \begin{array}{c} h \wedge c \\ \hline \therefore\ h \end{array}$$

Simplification

housing is falling

so interest rate falls.

d) "If it is Wednesday, then the Smartmart will be crowded. It is Wednesday. Thus, the Smartmart is crowded."

valid

Modes Ponens.

$$\begin{array}{l} w \rightarrow c \\ \underline{w} \\ \therefore c \end{array}$$

e) "It is cloudy and drizzling now. Therefore, it is cloudy now."

valid

specialization/ rule .

simplification

$$\begin{array}{l} \underline{c \wedge d} \\ \therefore c \end{array}$$

Question 5 [Marks 10]

Write the negation statements of following quantifiers.

a) ∀ primes x, x is even

Negation: $\exists x$, x is odd.

b) a triangle y, the sum of angles y equals 90

Negation: $\forall$ triangle y , sum of angles $\neq 90$.
of y

$L(x)$: x is lawyer

c) No lawyers are honest : For all x, if x is lawyer, then x is NOT honest. $N(x)$

negation: $\exists x \, (L(x) \wedge \sim N(x))$

→ $\exists x$, x is lawyer and x is not honest.

d) 275 is not dividable by any integer between 1 and 13 : For all int n, if $1 \leq n \leq 13$, then 275 is not dividable by n.

negation $\exists x$, $1 \leq x \leq 13$ and 275 is dividable by n.

e) ∀ real a, if a>5 then a^2>125

$\exists a$, $a > 5$ and $a^2 < 125$.

Question 6: [10 Marks]

Prove the following statement for all non-negative integers x by using mathematical induction.

$x \geq 0$

$$\sum_{j=0}^{x} 2^j = 2^{x+1} - 1.$$

1. Basis step : for $j=0$ and $x=0$

$2^0 = 2^{1+0} - 1$

$1 = 2^1 - 1 = 1$

Basis step gives true value

2. Inductive hypothesis :

$x = k$ where $k \geq 0$

so, $\sum_{j=0}^{k} 2^j = 2^{k+1} - 1$ — ①

we need to prove that $\sum_{j=0}^{k+1} 2^j = 2^{(k+1)+1} - 1$ is true — ②

for $x = k+1$

we prove L.H.S : $\sum_{j=0}^{k+1} 2^j$ means sum of terms from $j=0$ to $j=k$ and $j=k+1$.

so $\sum_{j=0}^{k+1} 2^j$ can be written as

$\underbrace{\sum_{j=0}^{k} 2^j}_{①} + 2^{k+1}$ (split sum in 2 parts)

from equation ①

$2^{(k+1)} - 1 + 2^{k+1}$

$$2^{K+1} - 1 + 2^{K+1}$$

$a + a$

$2a$

$$2^{K+1} + 2^{K+1} - 1$$

$$2^1 \cdot (2^{K+1}) - 1$$

$$2^{(K+1)+1} - 1$$

$\hookrightarrow$ R.H.S

so, QED.

Question 7: [10 Marks].

Prove by contradiction, that if $\overbrace{a \text{ is rational and } b \text{ is irrational}}^{p}$, then $\underline{a + b \text{ is irrational}}_{q}$.

$p \rightarrow q$

$p \wedge \sim q$

Assumption: if a is rational and b is irrational, and $a+b$ is rational.

$a = p/q$, b = is irrational.

$a + b = m/n$.

$p/q + b = m/n$

$b = m/n \overset{\times q}{} - p/q \overset{\times n}{}$

$$b = \frac{mq - pn}{nq}$$

This contradicts our assumption that b is irrational, hence original statement is true.

Question 8: [10 Marks].

State the contrapositive statement and Proof of the conditional statement below: If $\underbrace{y^2 - 6y + 5 \text{ is even number}}_{p}$ then $\underbrace{y \text{ is odd}}_{q}$.

$p \to q$

$\sim q \to \sim p$: If y is even, then $y^2 - 6y + 5$ is odd.

So, let $y = 2m$ — ①

$y^2 - 6y + 5$ — ②

Put ① in ②

$(2m)^2 - 6(2m) + 5$

$4m^2 - 12m + 5$

$2\underbrace{(2m^2 - 6m)}_{r} + 5$

$2r + 5$ where $r \in Z$

$2\times0 + 5 = 5$

$2\times1 + 5 = 7$

$2\times2 + 5 = 9$

odd

Hence QED

Question 9: [10 Marks].

Prove the following by using proof of contradiction. There exist no integers x and y for which $18x + 6y = 1$.

Assumption : There exists x and y for which

$18x + 6y = 1.$

$6(3x + y) = 1.$

any integer e.g $r = 3x + y.$

$6 \cdot r = 1$ $\quad$ $r = 1/6$

$d/n = 6/1$ $\quad$ d r n

According to divisibility rule, if 6 / 1 , then there exists an integer r that satisfies the equation. However, 6.r will always be a multiple of 6 (6, 12, 18) $\neq 1$. So there is no such integer r. So 1 is not divible by 6. 1 is not a multiple of 6

Honor Policy

This assignment is an individual learning opportunity that will be evaluated based on your ability to think independently, work through a problem in a logical manner and implement a software program on your own. You may however discuss verbally or via email the general nature of the conceptual problem to be solved with your classmates or the course instructor, but you are to complete the actual writing for this assignment without resorting to help from any other person or other resources that are not authorized as part of this course. If in doubt, ask the course instructor. You may not use the Internet to search for solutions to the problem. Sign this page and attach it with your assignment.

I have read the above and I certify that I have followed the instructions and these submitted work is my own.

Signature