Sarita Sangvez
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CY - A.

Linear Algebra. Date / /

Homework 08

Q.

1. $\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} \right\}$

$\quad u_1 \qquad u_2 \qquad u_3.$

$u_1 \cdot u_2 = 0.$ for both vectors to be orthogonal

$u_1^T u_2 = \begin{bmatrix} -1 & 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ -8 \\ -2 \\ 4 \end{bmatrix} = -6 - 24 - 2 - 4 = -36.$

Since u_1 and u_2 are not orthogonal to one another, the basis is not orthogonal basis.

2. $v_1 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

projection of v_1 onto v_2:

$$\text{proj}_{v_2} v_1 = \frac{v_1 \cdot v_2}{v_2 \cdot v_2} v_2.$$

$v_1 \cdot v_2 = -36.$

$v_2 \cdot v_2 = -1^2 + 3^2 + 1^2 + 1^2 = 12.$

so $\text{proj}_{v_2} v_1 = -\frac{36}{12} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ -9 \\ -3 \\ -3 \end{bmatrix}$ orthogonal projection

$z = v_1 - \text{proj}_{v_2} v_1$

$= \begin{bmatrix} 6 \\ -8 \\ -2 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ -9 \\ -3 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$

orthogonal component.

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3. $u_1 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}$, $u_3 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix}$

$v_1 = u_1$

Projection of u_2 onto v_1.

$$\text{proj}_{v_1} u_2 = \frac{u_2 \cdot v_1}{v_1 \cdot v_1} v_1$$

$$u_2 \cdot v_1 = [\,6\ -8\ -2\ -4\,] \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$= -6 - 24 - 2 - 4 = -36.$$

$v_1 \cdot v_1 = 12.$

so $\text{proj}_{v_1} u_2 = \begin{bmatrix} 3 \\ -9 \\ -3 \\ -3 \end{bmatrix}$

$$z = v_2 = u_2 - \text{proj}_{v_1} u_2 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} - \begin{bmatrix} 3 \\ -9 \\ -3 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$w = \{ v_1, v_2 \}$

$v_3 = u_3 - \text{proj}_{w} u_3$.

$$= \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \left[\text{proj}_{v_1} u_3 + \text{proj}_{v_2} u_3 . \right]$$

$$= \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \left[\frac{u_3 \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{u_3 \cdot v_2}{v_2 \cdot v_2} v_2 \right]$$

$$= \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \left[\frac{6}{12} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} + \frac{5}{2} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right]$$

$$v_3 = \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$$

so $w = \left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$

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4. $v_1 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$

unit vector : $\hat{u}_1 = \dfrac{\vec{u_1}}{\|u_1\|}$

$\|u_1\| = \sqrt{(-1)^2 + (3) + (1) + (1)} = \sqrt{12}.$

$\hat{u}_1 = 1/\sqrt{12} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{12} \\ 3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \end{bmatrix}$

$\|u_2\| = \sqrt{12}$

$\hat{u}_2 = \dfrac{1}{\sqrt{12}} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \\ -1/\sqrt{12} \end{bmatrix}$

$\|u_3\| = \sqrt{12}$

$\hat{u}_3 = \dfrac{1}{\sqrt{12}} \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{12} \\ -1/\sqrt{12} \\ 3/\sqrt{12} \\ -1/\sqrt{12} \end{bmatrix}$

$W = \{ \hat{u}_1 , \hat{u}_2 , \hat{u}_3 \}$

5. $a_1 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$ $a_2 = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}$ $a_3 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ 3 \end{bmatrix}$

$v_1 = a_1.$

$a_2^T a_1 = [6 \ -8 \ -2 \ -4] \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

$-6 - 24 - 2 - 4$

$= -36$

$\text{proj}_{a_1} a_2 = \dfrac{a_2 \cdot a_1}{a_1 \cdot a_1} a_1$

$[-1 \ 3 \ 1 \ 1] \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

$1 + 9 + 1 + 1 = 12$

$= \dfrac{-\cancel{36}\,3}{1\,\cancel{12}} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}$

$\text{proj}_{a_1} a_2 = \cancel{\begin{bmatrix} -6 \\ -18 \\ -6 \\ -6 \end{bmatrix}} = \begin{bmatrix} -2 \\ 6 \\ 2 \\ 2 \end{bmatrix}$

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$$V_2 = a_2 - \hat{a_2} = \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix} - \begin{bmatrix} -2 \\ -6 \\ -2 \\ -2 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 12 \\ 10 \\ 4 \\ 2 \end{bmatrix}$$

$$V_3 = a_3 - \operatorname{proj}_w a_3$$

$$= a_3 - \left[\frac{a_3 \cdot a_1}{a_1 \cdot a_1} a_1 + \frac{a_3 \cdot a_2}{a_2 \cdot a_2} a_2 \right]$$

$$= \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \left[\frac{\cancel{6}}{12} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} + \frac{5}{2} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} \right]$$

same as before (I just realised)

$$w = \left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$$

orthonormal basis $N = \left\{ \begin{bmatrix} -1/\sqrt{12} \\ 3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \end{bmatrix}, \begin{bmatrix} 3/\sqrt{12} \\ 1/\sqrt{12} \\ 1/\sqrt{12} \\ -1/\sqrt{12} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{12} \\ -1/\sqrt{12} \\ 3/\sqrt{12} \\ -1/\sqrt{12} \end{bmatrix} \right.$

$$Q = \begin{bmatrix} -1/\sqrt{12} & 3/\sqrt{12} & -1/\sqrt{12} \\ 3/\sqrt{12} & 1/\sqrt{12} & -1/\sqrt{12} \\ 1/\sqrt{12} & 1/\sqrt{12} & 3/\sqrt{12} \\ 1/\sqrt{12} & -1/\sqrt{12} & -1/\sqrt{12} \end{bmatrix}$$

$$Q^T = \begin{bmatrix} -1/\sqrt{12} & 3/\sqrt{12} & 1/\sqrt{12} & 1/\sqrt{12} \\ 3/\sqrt{12} & 1/\sqrt{12} & 1/\sqrt{12} & -1/\sqrt{12} \\ -1/\sqrt{12} & -1/\sqrt{12} & 3/\sqrt{12} & -1/\sqrt{12} \end{bmatrix}$$

$$R = Q^T A.$$

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$$R = \begin{bmatrix} -1/\sqrt{12} & 3/\sqrt{12} & 1/\sqrt{12} & 1/\sqrt{12} \\ 3/\sqrt{12} & 1/\sqrt{12} & 1/\sqrt{12} & -1/\sqrt{12} \\ -1/\sqrt{12} & -1/\sqrt{12} & 3/\sqrt{12} & -1/\sqrt{12} \end{bmatrix}_{3\times 4} \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}_{4\times 3}$$

$$\left[(-1/\sqrt{12} \times -1 + 3/\sqrt{12} \times 3 + 1/\sqrt{12} + 1/\sqrt{12}) + (-1/\sqrt{12} \times -1 + 3/\sqrt{12} \times 3 + 1/\sqrt{12} + 1/\sqrt{12}) + \right]$$

$$R = \frac{1}{\sqrt{12}} \begin{bmatrix} -1 & 3 & 1 & 1 \\ 3 & 1 & 1 & -1 \\ -1 & 1 & 3 & -1 \end{bmatrix}_{3\times 4} \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}_{4\times 3}$$

$$(-1 \times -1 + 3 \times 3 + 1 \times 1 + 1 \times 1) + (-1 \times 6 + 3 \times -8 + (-2) - 4)$$
$$+ (-1 \times 6 + 3 \times 3 + 6 - 3)$$

$$(3 \times -1 + 3 \times 3 + 1 - 1) + (3 \times 6 + (-8) - 2 + 4) + (-6 + 3 + 18 + 3$$

$$(+1 + 3 + 3 - 1) + (-6 + 3 + 18 + 3)$$

$$R = \frac{1}{\sqrt{12}} \begin{bmatrix} 12 & -36 & 6 \\ 0 & 12 & 30 \\ 6 & -16 & 18 \end{bmatrix}$$