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CY-A.

Date

Linear Algebra HW # 2.

Q1.

$$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & 7 \end{bmatrix} \xrightarrow{R_4 - 2R_1} \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 0 & 1 & 3 & 3 \end{bmatrix}$$

$$\xrightarrow{R_3 - 2R_2} \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 1 & 3 & 3 \end{bmatrix} \xrightarrow{R_4 - R_3} \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

$$\xrightarrow{R_1 - 4R_2} \begin{bmatrix} 1 & 0 & -11 & -18 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & 7 \end{bmatrix} \xrightarrow{R_3/15} \begin{bmatrix} 1 & 0 & -11 & -18 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 7 \end{bmatrix}$$

$$\xrightarrow{R_4 - 7R_3} \begin{bmatrix} 1 & 0 & -11 & 18 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

i) As there are 3 pivot positions in 3 rows so columns of A span $\mathbb{R}^3$.

ii) Out of 4 rows, pivot positions exists in only 3 rows. According to theorem for columns of A span $\mathbb{R}^4$, there should be pivot in each row. So columns of A don't span $\mathbb{R}^4$.

iii) As pivot doesn't exist in each row, all other conditions become automatically false. Hence, $Ax = b$ does not have a solution for b in $\mathbb{R}^4$.

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Q2. i) Augmented Matrix $\begin{bmatrix} 2 & 2 & 4 & 6 \\ -4 & -4 & -8 & -12 \\ 0 & -3 & 3 & 0 \end{bmatrix}$

$\xrightarrow{R_1/2} \begin{bmatrix} 1 & 1 & 2 & 3 \\ -4 & -4 & -8 & -12 \\ 0 & -3 & 3 & 0 \end{bmatrix} \xrightarrow{R_2+4R_1} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & 3 & 0 \end{bmatrix}$

$\xrightarrow{R_3 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2/-3} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

↓ non-pivot

$\xrightarrow{R_1-R_2} \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ x_3 becomes free variable

$x_1 + x_3 = 3$

$x_2 - x_3 = 0.$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 - x_3 \\ x_3 \\ x_3 \end{bmatrix} =$

$\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$. Solution set is a line through point $P(3,0,0)$ parallel to vector $v = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

ii) $\sim A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ x_3 is free variable

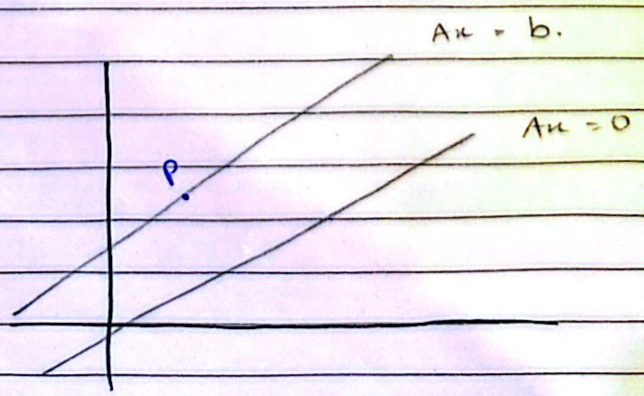
$x_1 + x_3 = 0$

$x_2 - x_3 = 0.$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow x_3\vec{u}.$

set of all scalar multiples of $\vec{u}$ because there is only one vector.

iii)



Q3 i) $\vec{w} = x_1 \vec{u} + x_2 \vec{v}$

$$\begin{bmatrix} -1/2 \\ -1/2 \\ h \end{bmatrix} = x_1 \begin{bmatrix} h \\ -1/2 \\ -1/2 \end{bmatrix} + x_2 \begin{bmatrix} -1/2 \\ h \\ -1/2 \end{bmatrix}$$

$-1/2 = hx_1 - 1/2 x_2$. — ①

$-1/2 = -1/2 x_1 + hx_2$. — ②

$h = -1/2 x_1 - 1/2 x_2$. — ③

$h = -1/2 (x_1 + x_2)$

$-2h = x_1 + x_2$

$x_1 = -2h - x_2$. (in first and second eq).

$-1/2 = h(-2h - x_2) - 1/2 (x_2)$

$-1/2 = -2h^2 - hx_2 - 1/2 x_2$ — ①.

$-1/2 = -1/2(-2h - x_2) + hx_2$.

$-1/2 = +\frac{h}{1\times 2} + \frac{x_2}{2} + \frac{hx_2}{1\times 2}$ — ②.

$$-1/2 = \frac{2h + x_2 + 2hx_2}{2}$$

$-1 = 2h + x_2 + 2hx_2$.

$-1 = x_2(1 + 2h) + 2h$.

$\frac{-1-2h}{1+2h} = x_2$. — put in eq ①.

$$-1/2 = -2h^2 - h\left[\frac{-1-2h}{1+2h}\right] - 1/2\left[\frac{-1-2h}{1+2h}\right]$$

$$-1/2 = -2h^2 + \frac{h + 2h^2}{1+2h} + \frac{1/2 + h}{1+2h}.$$

$$\frac{-1-2h}{2} = -2h^2(1+2h) + h + 2h^2 + \frac{1}{2} + h$$

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ii) Because w lies in the span of u,v when $x = 1, -1/2$ then w is also linearly dependent on u,v.

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Q4.

$$A = \begin{bmatrix} 0 & 6 & 4 \\ 3 & 0 & -7 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{bmatrix} \xrightarrow{R3 \leftrightarrow R1} \begin{bmatrix} 1 & 5 & 1 \\ 3 & 0 & -7 \\ 0 & 6 & 4 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\xrightarrow{R2-3R1} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -15 & -10 \\ 0 & 6 & 4 \\ -1 & 1 & 3 \end{bmatrix} \xrightarrow{R4+R1} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -15 & -10 \\ 0 & 6 & 4 \\ 0 & 6 & 4 \end{bmatrix}$$

$$\xrightarrow{R4-R3} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -15 & -10 \\ 0 & 6 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R2/-15} \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 2/3 \\ 0 & 6 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R1-5R2} \begin{bmatrix} 1 & 0 & -7/3 \\ 0 & 1 & 2/3 \\ 0 & 6 & 4 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R3-6R2} \begin{bmatrix} 1 & 0 & -7/3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- column 3 is non-pivot column → free variable → non-trivial solutions → linearly dep.

$$x_1 - \frac{7}{3}x_3 = 0.$$

$$x_2 + \frac{2}{3}x_3 = 0.$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7/3x_3 \\ -2/3x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 7/3 \\ -2/3 \\ 1 \end{bmatrix}$$

For linear dependence relation, let $x_3 = 3$.

$x_1 = 7$ and $x_2 = -2$.

So $7\vec{u} - 2\vec{v} + 3\vec{w} = 0.$