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23i - 2088
CY - A.

Date 28 - 08 - 24.

LA Homework 01.

Q1.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \xleftrightarrow{R_1 + R_2} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\xleftrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \xleftrightarrow{2R_3} \begin{bmatrix} 3 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & 2 & 0 \end{bmatrix}$$

$$\xleftrightarrow{R_2 + 2R_3} \begin{bmatrix} 3 & 1 & -1 \\ 3 & 5 & 1 \\ 2 & 2 & 0 \end{bmatrix} = B.$$

Q2.

$$\begin{bmatrix} 1 & 3 & -1 & a \\ 1 & 1 & 2 & b \\ 0 & 2 & -3 & c \end{bmatrix} \xleftrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 2 & -3 & c \end{bmatrix}$$

$$\xleftrightarrow{R_3 + R_2} \begin{bmatrix} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 0 & 0 & c+(b-a) \end{bmatrix}$$

$c + b - a = 0$

$c = a - b.$

This condition must be satisfied for system to be consistent.

Q3.

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \end{array}$$
$$\begin{bmatrix} 1/2 & 1 & 1 & -6 & 0 & 2 \\ 1/6 & 1/2 & 0 & -3 & 1 & -1 \\ 1/3 & 0 & -2 & 0 & -4 & 8 \end{bmatrix}$$

$$\xleftrightarrow{2R_1} \begin{bmatrix} 2 & 2 & -2 & -12 & 0 & 4 \\ 1/6 & 1/2 & 0 & -3 & 1 & -1 \\ 1/3 & 0 & -2 & 0 & -4 & 8 \end{bmatrix}$$

$$\xleftrightarrow{R_2 - \frac{1}{2}R_3} \begin{bmatrix} 2 & 2 & -2 & -12 & 0 & 4 \\ 1/6 - 1/6 & 1/2 - 0 & 0-(-1) & -3-0 & 1-(-2) & -1+4 \\ 1/3 & 0 & -2 & 0 & -4 & 8 \end{bmatrix}$$

Date

$$\begin{bmatrix} 1 & 2 & -2 & -12 & 0 & 4 \\ 0 & 1/2 & 1 & -3 & 3 & -5 \\ 1/3 & 0 & -2 & 0 & -4 & 8 \end{bmatrix}$$

$$\xleftrightarrow{R_3 - 1/3 R_1} \begin{bmatrix} 1 & 2 & -2 & -12 & 0 & 4 \\ 0 & 1/2 & 1 & -3 & 3 & -5 \\ 0 & 0-\frac{2}{3} & -2+\frac{2}{3} & 0+4 & -4-0 & 8-4/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -2 & 12 & 0 & 4 \\ 0 & 1/2 & 1 & -3 & 3 & -5 \\ 0 & -2/3 & -4/3 & 4 & -4 & 20/3 \end{bmatrix}$$

$$\xleftrightarrow{2R_2} \begin{bmatrix} 1 & 2 & -2 & 12 & 0 & 4 \\ 0 & 1 & 2 & -6 & 6 & -10 \\ 0 & -2/3 & -4/3 & 4 & -4 & 20/3 \end{bmatrix}$$

$$\xleftrightarrow{R_3 + \frac{2}{3}R_2} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \end{array} \\ \begin{bmatrix} 1 & 2 & -2 & -12 & 0 & 4 \\ 0 & 1 & 2 & -6 & 6 & -10 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x_1 + 2x_2 - 2x_3 - 12x_4 = 4$ — (1)

$x_2 + 2x_3 - 6x_4 + 6x_5 = -10$ — (2).

$x_2 = -2x_3 + 6x_4 - 6x_5 - 10.$ — (3)

Substitute (3) in (1).

$x_1 + 2(-2x_3 + 6x_4 - 6x_5 - 10) - 2x_3 - 12x_4 = 4.$

$x_1 - 4x_3 + 12x_4 - 12x_5 - 20 - 2x_3 - 12x_4 = 4$

$x_1 - 6x_3 - 12x_5 = 24.$

$x_1 = 24 + 6x_2 + 12x_5.$

∴ The system has infinite many solutions.