

R.M.K. COLLEGE OF ENGINEERING AND TECHNOLOGY

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22MA401 – Probability and Statistics

UNIT III – TESTING OF HYPOTHESIS**PART – A****1. Distinguish between parameters and statistics.**

Solution : Statistical constants of the population such as mean (μ) and S.D (σ) are usually known as parameters.

Statistical measures computed from sample observations alone such as mean (\bar{x}) and S.D (s) are usually known as statistics.

2. Explain statistical hypothesis.

Solution: Assumptions made about a parameter of a statistical population are called statistical hypothesis.

3. What are Type I and Type II errors?

Solution: Type I Error: Rejecting a null hypothesis when it is true.

Type II Error: Accepting a null hypothesis when it is wrong.

4. Critical values in hypothesis testing – Explain.

Solution: The value that separates the rejection region from the acceptance region is called the critical value.

5. What do you mean by a Null Hypothesis?

Solution: A definite statement about population parameters is called a Null hypothesis. Null hypothesis is a hypothesis of no difference. It is denoted by H_0 .

A statement which is complementary to the null hypothesis is called an Alternative hypothesis. It is denoted by H_1 .

6. What do you mean by test of hypothesis – two tailed test?

Solution: A test of any statistical hypothesis where the alternative hypothesis is two tailed, is called two-tailed test.

Eg: $H_0 : \mu = \mu_0$, $H_1 : \mu \neq \mu_0$

7. Define the term “Degrees of freedom”.

Solution: Degrees of freedom is the number of independent observations in a set. If ‘n’ is the sample size then the degrees of freedom for variance is n-1

8. Write down the confidence limits for the mean at 1%, 5% levels of large sample.

Solution: $\bar{x} \pm 2.58 \frac{\sigma}{\sqrt{n}}$, $\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

9. Write down the test statistic for single mean for large sample.

Solution: $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$, where \bar{x} - Sample mean μ - Population mean,

σ - Population S.D. and n - Sample size.

10. The mean value of a random sample of 60 items was found to be 145 with a S.D of 40. Find the 95% confidence limits for the population mean.

Solution: 95% confidence limits for the population mean is

$$\left(\bar{x} - 1.96 \frac{s}{\sqrt{n}}, \bar{x} + 1.96 \frac{s}{\sqrt{n}} \right) \quad (\text{i.e.}) \quad \left(145 - \frac{(1.96)(40)}{\sqrt{60}}, 145 + \frac{(1.96)(40)}{\sqrt{60}} \right) \quad (\text{i.e.}) \quad (134.9, 155.1).$$

11. The wages of a factory’s workers are assumed to be normally distributed with mean μ and variance Rs. 25. A random sample of 36 workers gives the total wages equal to Rs. 1800. Test the hypothesis $\mu = 52$, against the alternative $\mu = 49$ at 1% level of significance.

Solution: Given $n = 36$, $\bar{x} = \frac{1800}{36} = 50$, $\mu = 52$ & $\sigma = 5$.

Let $H_0 : \mu = 52$ and $H_1 : \mu = 49 < 52$ [left tailed test]

Test statistic is $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{50 - 52}{5 / \sqrt{36}} = -2.4$. At 1 % level, the table value of z

\therefore calculated value of z > table value of z. Hence H_0 is rejected at 1% level of significance.

∴ The average $\mu = 49$ is accepted.

12. State the applications of t – distribution.

Solution : t - distribution is used

- (i) to test if the sample mean \bar{x} differs significantly from the population mean (μ),
- (ii) to test the significance of the difference between two sample means,
- (iii) to test the significance of an observed sample correlation coefficient and sample regression coefficient.
- (iv) to test the significance of observed partial and multiple correlation coefficients.

13. Write the applications of F-test.

Solution : F – test is used to test,

- (i) whether the two independent samples have been drawn from the normal populations with the same variance σ^2 .
- (ii) whether the two independent estimates of population variance differ significantly or not.

14. Write down any two uses of χ^2 - test.

Solution: The following are the important applications of χ^2 distribution

- i. χ^2 distribution is used to test the goodness of fit.
- ii. It is also used to test the independence of attributes.

PART – B

1. The heights of college students in a city are normally distributed with S.D 6 cms. A sample of 1000 students has mean height 158 cms. Test the hypothesis that the mean height of college students in the city is 160 cms.

Ans: H_0 rejected.

2. The mean weight obtained from a random sample of size 100 is 64 gms. The S.D of the weight distribution of the population is 3 gms. Test the statement that the mean weight of the population is 67 gms at 5% level of significance. Also set up 95 confidence limits of the mean weight of the population.

Ans: H_0 rejected. Limits: 63.22, 64.77.

3. An examination was given to two classes consisting of 40 and 50 students respectively. In the 1st class the mean mark was 74 with a S.D of 8, while in the 2nd class the mean mark was 78 with a S.D of 7. Is there a significant difference between the performance of the two classes at the levels of 0.05, 0.01 ?

Ans: H_0 accepted at 1% and is rejected at 5%.

4. A sample of heights of 6400 soldiers has a mean of 67.85 inches and a S.D of 2.56 inches. While another sample of heights of sailors has a mean of 68.55 inches with a S.D of 2.52 inches. Do the data indicate that the sailors are on the average taller than soldiers?

Ans: H_0 rejected.

5. Ten individuals are chosen at random from a normal population of students and their marks are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, and 71. In the light of these data discuss the suggestion that the mean mark of the population of students is 66.

Ans: H_0 accepted.

6. A group of boys and girls were given an intelligence test. The mean scores, standard deviation and numbers in each group are as follows.

	Mean	S.D	Size
Boys	124	12	18
Girls	121	10	14

Is there a significant difference between means of the samples at 5% level?

Ans : $t = 0.75$, H_0 accepted.

7. Two independent samples of sizes 7 and 6 had the following values:

Sample A	28	30	32	33	31	29	34
Sample B	29	30	30	24	27	28	

Examine whether the samples have been drawn from normal populations having same variance.
Ans: F = 1.116, H₀ accepted.

8. The two random samples reveal the following data:

Sample	Size	Mean	Variance
I	16	440	40
II	25	460	42

Test whether the samples have come from the same normal populations.
Ans: F = 1.025, t = -9.49, H₀ rejected.

9. The table below gives the number of accidents that took place in an industry during the various days of the week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	14	18	12	11	15	14

Ans : $\chi^2 = 2.143$, H₀ accepted.

10. 4 coins were tossed at a time and this operation is repeated 160 times. It is found that 4 heads occur 6 times, 3 heads occur 43 times, 2 heads occur 69 times and one head occurs 34 times. Discuss whether the coins may be regarded as unbiased?
Ans: H₀ accepted.

11. In the accounting department of a bank, 100 accounts are selected at random and estimated for errors .The following results were obtained.

No. of errors	0	1	2	3	4	5	6
No. of accounts	35	40	19	2	0	2	2

Does this information verify that the errors are distributed according to the Poisson law.
Ans: H₀ accepted.

12. Among 64 offspring’s of a certain cross between guinea pigs, 34 were red, 10 were black and 20 were white. According to the genetic model, these numbers should be in the ratio 9 : 3 : 4. Do the data consistent with the model at 5 % level?
Ans : $\chi^2 = 1.44$, H₀ accepted.

13. Can vaccination be regarded as preventive measure of small pox as evidenced by the following data of 1482 persons exposed to small pox in a locality? 368 in all were attacked of these 1482 persons and 343 were vaccinated and of these only 35 were attacked.
Ans: $\chi^2 = 48.226$, H₀ rejected.

14. Calculate the expected frequencies for the following data presuming the two attributes, condition of home and condition of child are independent

		Condition of home	
Condition of child		Clean	Dirty
	Clean	70	50
	Family Clean	80	20
	Dirty	35	45

Use χ^2 at 5% level of significance to test whether the two attributes are independent.
Ans : $\chi^2 = 69.64$, H₀ rejected.