25)

a. formula de Rodrigues: $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^n)$

$$L_2(x) = \frac{e^x}{2!}\cdot\frac{d^2}{dx^2}(e^{-x}x^2)$$
$$= \frac{e^x}{2}\frac{d}{dx}(-e^{-x}x^2 + 2e^{-x}x)$$
$$= \frac{\cancel{e^x}}{2}(\cancel{e^{-x}}x^2 - 2\cancel{e^{-x}}x - 2\cancel{e^{-x}}x + 2\cancel{e^{-x}})$$
$$= \frac{1}{2}(x^2 - 4x + 2)$$

b. $L_2(x) = 0$

$\frac{1}{2}(x^2-4x+2) = 0$ $\qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$x_0 = 2-\sqrt{2} \approx 0.58578$
$x_1 = 2+\sqrt{2} \approx 3.41421$

c. $$w_0 = \int_0^\infty \sigma(x)\left(\frac{x-x_1}{x_0-x_1}\right)dx = \int_0^\infty e^{-x}\left(\frac{x-2-\sqrt{2}}{2-\sqrt{2}-2-\sqrt{2}}\right)dx$$
$$= \int_0^\infty e^{-x}\left(\frac{x-2-\sqrt{2}}{-2\sqrt{2}}\right)dx = -\frac{1}{2\sqrt{2}}\int_0^\infty e^{-x}(x-2-\sqrt{2})\,dx$$
$$= -\frac{1}{2\sqrt{2}}\int_0^\infty e^{-x}x\,dx + \underbrace{\frac{\cancel{2}}{\cancel{2}\sqrt{2}}\int_0^\infty e^{-x}dx + \frac{\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}}\int_0^\infty e^{-x}dx}$$

$\int u\,dv = uv - \int v\,du$

$u = x$
$du = dx$
$dv = e^{-x}dx$
$v = -e^{-x}$

$$= -\frac{1}{2\sqrt{2}}\left(-xe^{-x}\right)\Big|_0^\infty - \frac{1}{2\sqrt{2}}\int_0^\infty e^{-x}dx$$
$$= -\frac{1}{2\sqrt{2}}\cancel{\lim_{x\to\infty}(-xe^{-x})} + \frac{1}{2\sqrt{2}}e^{-x}\Big|_0^\infty$$
$$= \frac{1}{2\sqrt{2}}\cancel{\lim_{x\to\infty}(e^{-x})} - \frac{1}{2\sqrt{2}}$$

$u = -x$
$du = -dx$

$$= -\frac{1}{\sqrt{2}}\int_0^\infty e^{u}du - \frac{1}{2}\int_0^\infty e^{u}du$$
$$= -\frac{1}{\sqrt{2}}e^{-x}\Big|_0^\infty - \frac{1}{2}e^{-x}\Big|_0^\infty$$
$$= \cancel{\lim_{x\to\infty}\left(-\frac{e^{-x}}{\sqrt{2}}\right)} + \frac{1}{\sqrt{2}} - \cancel{\lim_{x\to\infty}\left(\frac{e^{-x}}{2}\right)} + \frac{1}{2}$$
$$= \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$w_0 = -\frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{2} \approx 0.85355$$

$$\omega_1 = \int_0^\infty e^{-x}\left(\frac{x-2+\sqrt{2}}{2+\sqrt{2}-2+\sqrt{2}}\right)dx = \int_0^\infty e^{-x}\left(\frac{x-2+\sqrt{2}}{2\sqrt{2}}\right)dx$$

$$= \frac{1}{2\sqrt{2}}\int_0^\infty e^{-x}x\,dx + \frac{1}{2\sqrt{2}}\int_0^\infty e^{-x}(-2+\sqrt{2})\,dx$$

$$= \cancel{\frac{1}{2\sqrt{2}}(-xe^{-x})}\Big|_0^\infty{}^{0} + \frac{1}{2\sqrt{2}}\int_0^\infty e^{-x}dx + \frac{-2+\sqrt{2}}{2\sqrt{2}}(-e^{-x})\Big|_0^\infty$$

$$= \frac{1}{2\sqrt{2}}(-e^{-x})\Big|_0^\infty + \frac{-2+\sqrt{2}}{2\sqrt{2}} = +\frac{1}{2\sqrt{2}} + \frac{-2+\sqrt{2}}{2\sqrt{2}} = \cancel{\frac{-1+2-\sqrt{2}}{2\sqrt{2}}} = \cancel{\frac{1-\sqrt{2}}{2\sqrt{2}}} \approx 0,1464466$$

$$\frac{1-2+\sqrt{2}}{2\sqrt{2}} = \frac{-1+\sqrt{2}}{2\sqrt{2}} = \frac{2-\sqrt{2}}{4}$$

d) $\displaystyle\int_0^\infty e^{-x}f(x)\,dx \approx \sum_{i=0}^{n}\omega_i f(x_i)$

$$\Rightarrow \int_0^\infty e^{-x}x^3\,dx \approx \sum_{i=0}^{1}\omega_i f(x_i) = \omega_0 x_0^3 + \omega_1 x_1^3$$

$$= \left(-\frac{1}{2\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{2}\right)(2-\sqrt{2})^3 + \left(\frac{2-\sqrt{2}}{4}\right)(2+\sqrt{2})^3$$

$$= 3\cancel{-2\sqrt{2}} + 3\cancel{+2\sqrt{2}} = 6.$$

$$\int_0^\infty e^{-x}x^{4-1}\,dx = \Gamma(4) = (4-1)! = 3! = 6.$$