Computation of the control in the case $\boxed{T=4}$

$$J(e^0, e^1) = \frac{1}{2}\int_0^T |u_x(1,t)|^2\, dt + \int_0^1 y^0 e_0 - \int y^1 e_1$$

$$y^0 = \sum_{k=1}^{+\infty} \hat{y}_k^0 \sin(k\pi x)$$

$$y^1 = \sum_{k=1}^{+\infty} \hat{y}_k^1 \sin(k\pi x)$$

$$\sum_{k=1}^{+\infty} (k\pi)^2 |\hat{y}_k|^0 + |\hat{y}_k^1|^2 < \infty$$

$$u_0 = \sum_{k\geq 1} a_k \sin(k\pi x)$$

$$u_1 = \sum_{k\geq 1} b_k \sin(k\pi x)$$

$$u(x,t) = \sum_{k\geq 1} \left(a_k \cos(k\pi t) + \frac{b_k}{k\pi}\sin(k\pi t)\right)\sin(k\pi x)$$

$$\partial_x u(x,t) = -\sum_{k\geq 1} (k\pi)\left(a_k \cos(k\pi t) + \frac{b_k}{k\pi}\sin(k\pi t)\right)\cos(k\pi x)$$

$$\partial_x u(x,1) = \sum_{k\geq 1} (-1)^k (k\pi)\left(a_k \cos(k\pi t) + \frac{b_k}{k\pi}\sin(k\pi t)\right)$$

$$\Rightarrow \quad \frac{1}{2}\int_0^T |u_x(1,t)|^2\,dt = \sum_{k=1}^{+\infty}\left(|a_k|^2 k^2\pi^2 + |b_k|^2\right)$$

$$\Rightarrow J(e_0,e_1) = \sum_{k=1}^{+\infty}\left(|a_k|^2 k^2\pi^2 + |b_k|^2\right) + \frac{1}{2}\left(\hat{y}_k^0 b_k - \hat{y}_k^1 a_k\right)$$

$$L_1 = -\underbrace{y(:,2)}$$

Le minimiseur de J

$$e_0^* = \sum_{k=1}^{+\infty} a_k^* \sin(k\pi x)$$

$$e_1^* = \sum_{k=1}^{+\infty} b_k^* \sin(k\pi x)$$

$$\Rightarrow \begin{cases} a_k = \dfrac{y_k^1}{4k^2\pi^2} \\[2ex] b_k = -\dfrac{\hat{y}_k^0}{4} \end{cases}$$

$\Lambda e = f \qquad \Lambda_{\text{exact}}$