$$\underbrace{\int_0^T \eta\, \partial_x u_0(t,1)\, \partial_x \tilde{u}(t,1)\, dt + \int_0^1 y_0\, \tilde{e}_1 - \int_0^1 y_1\, \tilde{e}_0}_{\left(\nabla J(e_0, e_1),\, (e_0, \tilde{e}_1) \right)} = 0$$

$$\left\langle \Lambda(e_0, e_1),\, (\tilde{e}_0, \tilde{e}_1) \right\rangle$$

$$\underbrace{\left\langle \Lambda(e_0, e_1),\, (\tilde{e}_0, \tilde{e}_1) \right\rangle}_{} = \int_0^1 y_1\, \tilde{e}_0 - \int_0^1 y_0\, \tilde{e}_1$$

$$\int_0^1 \underbrace{(\Lambda e_0)_1}_{} \tilde{e}_0 + \int_0^1 (\Lambda e_0)_2\, \tilde{e}_1 = \int_0^1 y_1 \tilde{e}_0 - \int y_0\, \tilde{e}_1$$

$$\begin{cases} (\Lambda e_0)_1 = y_1 & (\tilde{e}_1 = 0) \\ (\Lambda e_0)_2 = -y_0 \end{cases}$$

Initialisation ordre 2

$$y(\Delta t, x_j) = \underbrace{y(0, x_j)}_{y_0(x_j)} + \Delta t \underbrace{\frac{\partial y}{\partial t}(0, x_j)}_{y_1(x_j)} + O(\Delta t^2)$$

$$\frac{\partial y}{\partial t}(0, x_j) = \frac{y(\Delta t, x_j) - y(0, x_j)}{\Delta t} + O(\Delta t)$$

$$\Rightarrow \quad y_j^1 = y_j^0 + \Delta t \; y_1(x_j)$$

$$y(\Delta t, x_j) = \underbrace{y(0, x_j)}_{y_0(x_j)} + \Delta t \underbrace{\frac{\partial y}{\partial t}(0, x_j)}_{y_1(x_j)} + \frac{\Delta t^2}{2} \frac{\partial^2 y}{\partial t^2}(0, x_j) + O(\Delta t^3)$$

$$\frac{\partial^2 y}{\partial t^2}(t, x) = \frac{\partial^2 y}{\partial x^2} \qquad (\text{equation des ondes})$$

$$y(\Delta t, x_j) = y_0(x_j) + \Delta t \; y_1(x_j) + \frac{\Delta t^2}{2} \frac{\partial^2 y}{\partial x^2}(0, x_j)$$

$$= \frac{u(x_{j+1}, 0) - 2u(x_j, 0) + y(x_{j-1}, 0)}{\Delta x^2} + O(\Delta x^2)$$

$$\boxed{y_j^1 = y_0(x_j) + \Delta t \; y_1(x_j) + \frac{\Delta t^2}{2\Delta x^2}\left(u_{j+1}^0 - 2u_j^0 + u_{j-1}^0\right)}$$

2) gradient conjugué. $\Lambda : (H_0^1 \times L^2 \to L^2 \times H^{-1})$

$$\boxed{\Lambda e = f}$$

$E = (e_0, e_1)$ $\Lambda e = ((\Lambda e)_0, (\Lambda e)_1$

E_0 donné $(E_0 = (e_0, e_1))$

$R_0 = f_{mod} - \underbrace{\Lambda E_0}_{(mod\, 0)}$ appel de la fonction Λ

$P_0 = R_0$

$k=0$

norme à spécifier

while ($k <$ maxiter , $\|R\| >$ tolérance)

$T_k = \Lambda P_{mod}$ il faut calculer ΛP (appeler la fonction Λ)

$$\boxed{a_k = \|R_k\|^2 / \langle \Lambda P_{mod}, P \rangle}$$

$E_{k+1} = E_k + a_k \times T P_k$

$R_{k+1} = R_k - a_k \underbrace{\Lambda P_k}_{T}$

$\beta = \dfrac{\|R_{k+1}\|^2}{\|R_k\|^2}$

$P_{k+1} = R_{k+1} + \beta \times P_k$

end

préconditionnement

$$\Lambda e = f \quad / \quad \boxed{\underbrace{\Pi \Lambda}_{\Lambda_{mod}} e = \underbrace{\Pi f}}$$

$$\Lambda(e_0, e_1) = \Big(\overbrace{-\partial_t y(0,x)}^{\text{pas très régulier}},\ y(0,x)\Big)$$

On résoud :

$$\Pi = \begin{pmatrix} -\Delta^{(-1)} & 0 \\ 0 & I \end{pmatrix}$$

$$(*) \begin{cases} -\underbrace{\varphi''}_{\in H_0^1} = -\partial_t y(0,x) \\ \varphi(0) = \varphi(1) = 0 \end{cases}$$

φ calculé en résolvant cette EDP

notation abusive

$$-\Delta \varphi = (-\partial_t y(0,x))$$

$$\varphi = (-\Delta)^{-1} (\partial_t y(0,x))$$

$$\Lambda_{mod}(e_0, e_1) \longrightarrow (\varphi, y(0,x))$$

Λ_{mod} : 1) on applique Λ normalement

2) on résoud $(*)$ (par différence finie)

(f_0, f_1)

Πf : $\begin{cases} \varphi_0 \\ \\ f_1 \end{cases}$ où φ_0 $\begin{cases} -\varphi_0'' = f_0 \\ \varphi_0(0) = \varphi_0(1) = 0 \end{cases}$

$$\Lambda_{mod} \, e = \tilde{f}$$

$$\Lambda_{mod} : H_0^1 \times L^2 \longrightarrow H_0^1 \times L^2$$

$$\underbrace{E}_{(e_0, e_1)} \times H_0^1 \times L^2$$

$$\| E \|^2_{H_0^1 \times L^2} = \int_0^1 (e_0')^2 \, dx + \int_0^1 (e_1)^2 \, dx .$$

$$\boxed{a_k = \| R_k \|^2 / \langle \Lambda P, P \rangle}$$

$$\Lambda P = T$$

$$\| R_k \|^2 = \int_0^1 (r_{0}^{k\,\prime})^2 + \int_0^1 (r_1^k)^2 \, dx$$

$$\langle \underbrace{\Lambda P}_{T}, P \rangle = \int_0^1 (T_0)' \, P_0' \, dx + \int_0^1 T_1 \, P_1 \, dx$$

$\Rightarrow$ discrétiser les intégrales par la méthode des rectangles.

cas discret :

$$\int_0^1 (u'(x))^2 \, dx \simeq \sum_{j=0}^{N} \left(\frac{u_{j+1} - u_j}{h} \right)^2 h \, dx .$$

$$\int_0^1 u(x)^2 \, dx \simeq \sum_{j=0}^{N} (u_j)^2 \, h \, dx .$$