SCHOOL OF COMPUTATION, INFORMATION AND TECHNOLOGY - INFORMATICS

TECHNISCHE UNIVERSITÄT MÜNCHEN

Bachelor's Thesis in Informatics

Tuning Linear Programming Solvers for Query Optimization

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Tuning Linear Programming Solvers for Query Optimization

Anpassung von Linear Programming Solvern für Anfrageoptimierung

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I confirm that this bachelor's thesis in informatics is my own work and I have documented all sources and material used.

Munich, 15/10/2023 Sarra Ben Mohamed

Acknowledgments

Abstract

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1 Introduction

Our aim with this project is to investigate and compare different methods and techniques to solve small linear programming problems representing the problem of cardinality estimation. Our goal is to estimate realistic and useful upper bounds on query sizes. Studies have shown that cardinality estimation is the major root of many issues in query optimization[Ngo22]. And yet, theoretical upper bounds that are way too large would be useless since we want practical estimation to choose the best from data plans to run efficient queries. For this purpouse, we will introduce a formal description of the cardinality estimation problem, represent it in the form of a packing linear programming problem with the intention of maximizing the size of the query under some constraints. The result is hundreds of relatively small LP that we collect in datasets and solve them with different methods and algorithms. We then draw conclusions based on the results of our experiments, benchmarks and the previous work done on similar packing LP problems. This should guide us into constructing a thorough analysis of the particularities of these LP problems, what's unique about their structure and if their solution process is following any patterns. We then discuss and draw hypotheses on the ways this analysis can be exploited to further optimize the solution process: which methods or combination of methods deliver the best time and memory complexity.

2 Related work

2.1 Background

or Fundamentals: here we lay down the knowledge the reader needs to understand my contribution, and I will mostly present the definition of mathematical and algorithmic concepts needed to build the hypotheses and experiments that follow. First, let's say some words about cardinality estimation. It is an important process in the pipeline of query execution. In fact, the query optimizor selects which query plan to execute after computing and comparing the cardinality (number of tuples in the output) of each of these plans. This is crucial in the way most of today's DBSMs work. The SQL Server Query Optimizer [Mic23] for instance is a cost-based Query Optimizer. This means that it selects query plans that have the lowest estimated processing cost to execute. The Query Optimizer determines the cost of executing a query plan based on two main factors, the cardinality of the plan, as previously defined, and the cost model of the algorithm dictated by the operators used in the query. The first factor, cardinality, is used as an input parameter of the second factor, the cost model. Therefore, improved cardinality leads to better estimated costs and, in turn, faster execution plans. The goodness of the estimate is dependent on its precision. We want a tight upper bound estimate with as little constraints as possible and with the least assumptions.

2.1.1 Cardinality Estimation

As we previously elaborated, to be able to compute reliable and "good" cardinality extimates of query plans leads to faster execution. This is the desirable outcome. We want to implement a cardinality estimator in the form of an LP solver, with the functionality of maximising the cost function which represents how large the output size can possibly be. We want this LP solver to be as efficient as possible from the viewpoint of memory and time complexity. We will generally be looking at a metric called Query-per-Hour Performance Metric (QphH@Size). For the purpouse of defining how our dataset is generated let us consider the problem of upper-bounding the cardinality, or the output size of a join query Q. Now we start with the worst-case join size: The reason for this work is defining the problem of upper bounding a multi-join query size as a packing linear programming problem. To illustrate the main ideas, we

start with a simple example where the query is a join between two relations

$$Q(a, b, c) = R(a, b) \wedge S(b, c)$$

where we denote the sizes of the relations as $|R|$ and $|S|$ respectively. It is easy to see that the largest possible output is $N1 \cdot N2$, which occurs when the join behaves like a cartesian product. So, this is the worst-case upper bound. Now the maximum sizes of these relations (i.e. the number of tuples, in our case pairs) depend on the variables a, b and c, their respective types and domains. It can also be affected by te nature of the data or business rules. We start with the inequality 2.1. Applying the natural logarithm to both sides yields 2.2. We then rename the variables, simplifying the inequality to 2.3. Normalizing by dividing both sides by r', we obtain 2.4. This leads us to the objective function for our packing LP problem.

$$|a| \cdot |b| \leq |R| \tag{2.1}$$

$$\ln |a| + \ln |b| \leq \ln |R| \tag{2.2}$$

$$a' + b' \leq r' \tag{2.3}$$

$$\frac{1}{r'}a' + \frac{1}{r'}b' \leq 1 \tag{2.4}$$

$$\text{maximize } a' + b' + c' + d' \quad \text{s.t.} \quad \frac{1}{r'}a' + \frac{1}{r'}b' \leq 1 \tag{2.5}$$

And in this simple abstracted way we get a sample packing LP from our dataset.

2.1.2 Linear Programming

The LP problem class that we are dealing with is called the packing LP problem. Additionally it is a special instance where:

- c, the vector of the variable coefficients in the objective function, is a vector of all ones
- b , or the right hand side vector, is a vector of all ones

Our specific problem is then expressed as follows:

$$\begin{aligned} \text{Maximize} \quad & \sum_{j=1}^{n} x_j \\ \text{subject to} \quad & \\ & \sum_{j=1}^{n} a_{ij} x_j \leq 1, && i = 1, \ldots, m \\ & x_j \geq 0, && j = 1, \ldots, n \end{aligned} \tag{2.6}$$

- x_j is the j^{th} decision variable.
- m is the number of constraints.
- n is the number of variables.

This specific class of LPs has a simple structure that we can exploit to further optimize our implementation.

2.1.3 The Simplex Algorithm

The algorithm

In this subsection we will present the most widely used algorithm for solving LP problems. We have implemented our version of this algorithm in the C++ language and we use it, among others, to solve our dataset. To be approachable by the simplex algorithm, the LP 2.6 needs to be cast in a computational form, that fulfills the requirement of the constraint matrix having to have full row rank and only equality constraints are allowed. This is done by introducing slack variables. We now have what is called the Simplex Tableau. A simple such tableau would look like this.

	a	b	c	d	s_1	s_2	RHS
z	-1	-1	-1	-1	0	0	0
s_1	a_{11}	a_{12}	a_{13}	a_{14}	1	0	b_1
s_2	a_{21}	a_{22}	a_{23}	a_{24}	0	1	b_2

- feasible dictionaries?
- just write the algorithm in abstarct way, in the approach chapter write it in the specific way I implemented it (Bland's rule, zero tolerance, ...)
- the grand strategy of the simplex method is that of successive improvements
- decision variables vs. slack variables
- A maximization problem is optimized when the slack variables are "squeezed out," maximizing the true variables' effect on the objective function. Conversely, a minimization problem is optimized when the slack variables are "stuffed," minimizing the true variables' effect on the objective function.
- feasibility, boundedness,
- largest coefficient rule vs. largest increase rule.

- the problem of stalling, degeneracy
- Bland's rule guarantees termination.

The complexity

The Simplex algorithm for linear programming has an exponential worst-case time complexity, which we denote by $O(2^n)$. For packing linear programs, the worst-case time complexity of the Simplex algorithm remains exponential, even though there exists polynomial time implementations for it. [Sti10].
The simplex method is an active set method. Each step of the simplex method deactivates one box constraint and selects another one to be activated (general linear constraints are always satisfied). Typically for an active set method, O(N+M) steps are needed for an N-dimensional problem with M general linear constraints.

2.1.4 The Revised Simplex Algorithm

As explained in the book [Chv83]. Mention zero tolerances: A zero tolerance epsilon2 saefguards against divisions by extremely small numbers, which tend to produce the most dangerous rounding errors, and may even lead to degeneracy. diagonal entry in eta matrix should be fairly far from otherwise (in our experiment) degeneracy.

The product form update method

We will discuss the PFI, introduced by George Dantzig [DO54].

Data structures

Compressed Storage Formats: Eigen uses either the CSC or CSR format to store sparse matrices. These formats store the non-zero values, along with their corresponding row and column indices, in a compact way. This reduces memory usage and speeds up operations on sparse matrices

2.2 Previous Work

Here we will discuss alternative approaches that are superseded by my work.

2.2.1 Comparative studies of different update methods

We will focus on one study [HH15].

2.2.2 Other techniques

The primal simplex method starts from a trial point that is primal feasible and iterates until dual feasibility. The dual simplex method starts from a trial point that is dual feasible and iterates until primal feasibility. ALGLIB implements a three-phase dual simplex method with additional degeneracy-breaking perturbation:

- Forrest-Tomlin updates for faster LU refactorizations
- A bound flipping ratio test (also known as long dual step) for longer steps
- Dual steepest edge pricing for better selection of the leaving variable
- Shifting (dynamic perturbations applied to cost vector) for better stability

3 Tuning Linear Programming Solvers for Query Optimization

This is the body

3.1 Proposal

3.2 Experimental Design

3.2.1 Analysis of dataset properties

In this subsection we will conduct an analysis of our dataset properties. What are the particularites of the structure of these LP problems, is their any patterns in their solution process. This anaylsis is based on observing the statistical results we obtained from running different solvers on these problems. This will later provide us with insight regarding optimization of these problems. TPC-H is a Decision Support Benchmark The TPC-H is a decision support benchmark. It consists of a suite of business oriented ad-hoc queries and concurrent data modifications. The performance metric reported by TPC-H is called the TPC-H Composite Query-per-Hour Performance Metric (QphH@Size)

3.2.2 Dataset Structure

Our dataset stucture: as opposed to what the linear programming research has dealt with, which is very large problems, we are dealing with hundreds of small problems. These are represented in the revised simplex algorithm by sparse matrices but not as sparse as it would have been if the problem was large, small matrices that are not small enough to be dense. (they still have quite a number of non-zeroes).

3.3 Analysis

Some metrics:

- number of iterations

Algorithm 1 Tableau Simplex Algorithm

Input: Packing LP maximisation problem in computational form
Output: Optimal value z
Step 1: Pricing: Find pivot column, or entering variable using Bland's rule
    $enteringVars \leftarrow$ findPivotColumnCandidates()
    if no entering variable found **then**
        print "Optimal value reached."
        return z
    end if
    $pivotColumn \leftarrow enteringVars[0]$
Step 2: Find pivot row, or leaving variable using the ratio test
    $pivotRow \leftarrow$ findPivotRow($pivotColumn$)
    if no leaving variable **then**
        print "The given LP is unbounded."
        return ∞
    end if
Step 3: Update the tableau using pivotting and update the objective function value
    doPivotting($pivotRow, pivotColumn, z$)
Goto Step 1

- runtime
- number of loops ?
- for matrix : number of columns and rows, nonzeros and density

3.4 Results

All the following results have been obtained on a personal computer with AMD 4000 series RYZEN, 16GB RAM running Ubuntu. Using the following settings:

- Presolve techniques are not used
- scaling techniques are not used
- The computed optimal solutions have been validated using the scipy python library.

4 Evaluation

4.1 Setup

4.1.1 Evaluation metrics

4.1.2 Evaluation baselines

4.2 Results

4.3 Discussion

5 Conclusion

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