STDS22-Assignment2

Vladimir Ivanov, Zamira Kholmatova

March 2022

Task

In a research program on human health risk from recreational contact with water contaminated with pathogenic microbiological material, the National Institute of Water and Atmosphere (NIWA) instituted a study to determine the quality of NZ stream water at a variety of catchment types. This study is documented in McBride et al. (2002) where $n = 116$ one-liter water samples from sites identified as having a heavy environmental impact from birds (seagulls) and waterfowl. Out of these samples, $x = 17$ samples contained Giardia cysts. Let θ denote the true probability that a one-liter water sample from this type of site contains Giardia cysts.

1. What is the conditional distribution of X, the number of samples containing Giardia cysts, given θ?
2. Before the experiment, the NIWA scientists elicited that the expected value of θ is 0.2 with a standard deviation of 0.16. Determine the parameters α and β of a Beta prior distribution for θ with this prior mean and standard deviation. (Round α and β to the nearest integer).
3. Find the posterior distribution of θ and summarize it by its posterior mean and standard deviation.
4. Plot the prior, posterior and normalized likelihood.
5. Find the posterior probability that $\theta < 0.1$.
6. Find a central 95% posterior credible interval for θ.
7. Suppose that NIWA plans another study of $n^* = 50$ water samples as above. What is the posterior predictive probability that $x = 5$ of these contain Giardia cysts? Derive the formula for general n , x first.

   (Hint: You will need to construct a density function of beta distribution, take an integral substituting needed values.)
8. Test the hypothesis:

$H_0 : \theta \geq 0.2$

$H_1 : \theta < 0.2$

9. Test the same hypotheses in a Bayesian manner and interpret your results:

$H_0 : \theta \geq 0.2$

$H_1 : \theta < 0.2$