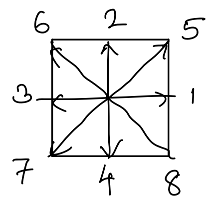
From lattice Boltzmann equation to Navier-Stokes + Continuity equations.



we are given the D2Q9 lattice

D2 : two dimensions
Q9 : Nine possible velocities

D2Q9 lattice

Given the lattice, we have to figure out lattice weights ω_i and lattice speed of sound c_s

Left-right and top-bottom symmetry requires

$\omega_1 = \omega_2 = \omega_3 = \omega_4$ (Subscript 1-4 stands for the direction)

Symmetry at $\frac{\pi}{4} \pm n\frac{\pi}{2}$ angles ($n = 0, -1, 1, 2$)

requires $\omega_5 = \omega_6 = \omega_7 = \omega_8$

Thus we have three types of weights : $\omega_0, \omega_1, \omega_5$
one way to say is lattice weights in a given direction depend upon their distance from center. Greater the distance from the center, lower should be the lattice weight ω_i.

we expect $\omega_0 > \omega_1 > \omega_5$

Equivalent way is to say that higher the lattice velocity, lower the lattice weight

$|\vec{c_0}| < |\vec{c_1}| < |\vec{c_5}|$.

The second way is closer to the symmetry arguments we make in the following

The equilibrium distribution f^{eq} is (in two dimensions)

$$f^{eq}(\vec{x},\vec{\xi},t) = \frac{\rho}{2\pi c_s^2} e^{-\frac{(\vec{\xi}-\vec{u})^2}{2c_s^2}}$$

Note that the Maxwell-Boltzmann distribution does not depend upon time and position but only on the relative velocity $\vec{v} = \vec{\xi} - \vec{u}$ ($\vec{u}$ is large (macro) scale fluid velocity and $\vec{\xi}$ is particle velocity).

The discrete equilibrium distribution function

$$f_i^{eq} = \rho w_i \left(1 + \frac{\vec{c_i}.\vec{u}}{c_s^2} + \frac{(\vec{c_i}.\vec{u})^2}{2c_s^4} - \frac{\vec{u}^2}{2c_s^2}\right)$$

was obtained as truncated series where only terms less than or equal to second order in fluid velocity $\vec{u}^2$ by expanding the exponential function with a small parameter $\left|\frac{u}{c_s}\right|$

Consider equilibrium distribution functions in discrete and continuous form with $\vec{u} = 0$

$$f_i^{eq}(u=0) = \rho w_i$$

$$f^{eq}(u=0,\vec{\xi}) = \frac{\rho}{2\pi c_s^2} e^{-\frac{\xi^2}{2c_s^2}}$$

Lattice weights w_i and lattice speed of sound c_s are determined by demanding that lattice velocity moments are are identical upto fourth order.

$$\int f^{eq}(u=0,\vec{\xi})\, \xi_x^m \xi_y^n \, d\xi_x d\xi_y = \sum f_i^{eq} c_{ix}^m c_{iy}^n$$

where $m+n \leq 4$. $m \geq 0$ and $n \geq 0$

ideally, one would like to every possible velocity moment but not possible on discrete level.

$m+n=0$

$$\int f^{eq}(u=0)\, d\xi_x\, d\xi_y = \rho = \sum_i f_i^{eq}(u=0) = \rho(\omega_0 + 4\omega_1 + 4\omega_5) \quad \text{--- (1)}$$

$m=2, n=0$

$$\int f^{eq}(u=0)\, \xi_x^2\, d\xi_x\, d\xi_y = \rho c_s^2 = \sum_i f_i^{eq}(u=0)\, c_{ix}^2 = \rho(2\omega_1 + 4\omega_5)\left(\frac{\Delta x}{\Delta t}\right)^2 \quad \text{--- (2)}$$

$m=2, n=2$

$$\int f^{eq}(u=0)\, \xi_x^2 \xi_y^2\, d\xi_x\, d\xi_y = \rho c_s^4 = \sum_i f_i^{eq}(u=0)\, c_{ix}^2 c_{iy}^2 = \rho(4\omega_5)\left(\frac{\Delta x}{\Delta t}\right)^4$$

$m=4, n=0$

$$\int f^{eq}(u=0)\, \xi_x^4\, d\xi_x\, d\xi_y = 3\rho c_s^2 = \sum_i f_i^{eq}(u=0)\, c_{ix}^4 = \rho(2\omega_1 + 4\omega_5)\left(\frac{\Delta x}{\Delta t}\right)^4$$

$$\omega_0 = \frac{4}{9}, \quad \omega_1 = \frac{1}{9}, \quad \omega_5 = \frac{1}{36}, \quad c_s^2 = \frac{1}{3}\frac{\Delta x^2}{\Delta t^2}$$

for other values m and n ($m=3, n=0$; $m=1, n=2$) do not yield any useful relation

Equilibrium distribution function is

$$f_i^{eq} = \rho\omega_i\left(1 + \frac{\vec{c_i}\cdot\vec{u}}{c_s^2} + \frac{(\vec{c_i}\cdot\vec{u})^2}{2c_s^4} - \frac{u^2}{2c_s^2}\right)$$

For the Chapman-Enskog analysis, we need lattice velocity moments from zero to third order of the equilibrium distribution function.

Few useful relations

$$\sum f_i^{eq} = \rho$$

$$\sum f_i^{eq} c_{i\alpha} = \rho u_\alpha$$

$$\sum f_i^{eq} c_{i\alpha} c_{i\beta} = \rho u_\alpha u_\beta + \rho c_s^2 \delta_{\alpha\beta}$$

$$\sum f_i^{eq} c_{i\alpha} c_{i\beta} c_{i\nu} = u_\alpha \delta_{\nu\beta} + u_\beta \delta_{\nu\alpha} + u_\nu \delta_{\alpha\beta}$$

Relations are shown in tutorial

Chapman Enskog Analysis (Section 4.1 : Textbook)

fluid populations are expanded as a series

$$f = f^{(0)} + \varepsilon f_1^{(1)} + \varepsilon^2 f_2^{(2)} + O(\varepsilon^3) \quad -(1)$$

Where ε is a small parameter. for the moment we do not define meaning of the small parameter.

time derivative is decomposed in two parts

$$\frac{\partial}{\partial t} = \varepsilon \frac{\partial}{\partial t^{(1)}} + \varepsilon^2 \frac{\partial}{\partial t^{(2)}} = \varepsilon \partial_t^{(1)} + \varepsilon^2 \partial_t^{(2)} \quad -(2)$$

and spatial derivative is

$$\partial_\alpha = \varepsilon \partial_\alpha^{(1)} \quad -(3)$$

The discrete Boltzmann equation (without a force term)

$$\frac{\Delta f_i}{\Delta t} = \frac{f_i(\vec{x}+\vec{c}_i\Delta t, t+\Delta t) - f_i(\vec{x},t)}{\Delta t} = -\frac{1}{\tau}(f_i - f_i^{eq})$$

$$= \frac{1}{\Delta t}\left[\Delta t(\partial_t + c_{i\alpha}\partial_\alpha) f_i + \frac{(\Delta t)^2}{2!}(\partial_t + c_{i\alpha}\partial_\alpha)^2 f_i + O(\Delta t^3)\right] = -\frac{1}{\tau}(f_i - f_i^{eq})$$

$$\Delta t(\partial_t + c_{i\alpha}\partial_\alpha) f_i + \frac{(\Delta t)^2}{2}(\partial_t + c_{i\alpha}\partial_\alpha)^2 f_i = -\frac{\Delta t}{\tau}(f_i - f_i^{eq}) \quad -(4)$$

operate by $\Delta t(\partial_t + c_{i\alpha}\partial_\alpha)$ on both sides and denoting $f^{neq} = f_i - f_i^{eq}$

$$(\Delta t)^2(\partial_t + c_{i\alpha}\partial_\alpha)^2 f_i + O(\Delta t^3) = -\frac{\Delta t^2}{\tau}(\partial_t + c_{i\alpha}\partial_\alpha) f^{neq}$$

$$(\partial_t + c_{i\alpha}\partial_\alpha)^2 f_i = -\frac{1}{\tau}(\partial_t + c_{i\alpha}\partial_\alpha) f^{neq} \quad -(5)$$

Using (5) in (4)

$$\Delta t(\partial_t + c_{i\alpha}\partial_\alpha) f_i - \frac{\Delta t^2}{2\tau}(\partial_t + c_{i\alpha}\partial_\alpha) f^{neq} = -\frac{\Delta t}{\tau} f^{neq}$$

$$(\partial_t + c_{i\alpha}\partial_\alpha) f_i - \frac{\Delta t}{2\tau}(\partial_t + c_{i\alpha}\partial_\alpha) f^{neq} = -\frac{1}{\tau} f^{neq} \quad -(6)$$

Use (1), (2) and (3) in (6)

$$\left[\varepsilon\partial_t^{(1)} + \varepsilon^2\partial_t^{(2)} + \varepsilon c_{i\alpha}\partial_\alpha\right]\left[f_i^{(0)} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} - \frac{\Delta t}{2\tau}\left(\varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)}\right)\right] = -\frac{1}{\tau}\left[\varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)}\right] \quad -(7)$$

equating terms of order ε

$$(\partial_t^{(1)} + c_{i\alpha}\partial_\alpha) f_i^{(0)} = -\frac{f_i^{(1)}}{\tau} \quad -(8)$$

equating terms of second order ε

$$\left(1 - \frac{\Delta t}{2\tau}\right)(\partial_t^{(1)} + c_{i\alpha}\partial_\alpha) f_i^{(1)} + \partial_t^{(2)} f_i^{(0)} = -\frac{f_i^{(2)}}{\tau} \quad -(9)$$

Take zeroth and first moment of eq(8) and sum over velocity space

$$\sum_i (\partial_t^{(1)} + c_{i\alpha}\partial_\alpha) f_i^{(0)} = -\frac{1}{\tau}\sum_i f_i^{(1)} = 0 \quad (10a)$$

$$\partial_t^{(1)}\rho + \partial_\alpha(\rho u_\alpha) = 0 \quad -(10)$$

$$\sum_i (\partial_t^{(1)} + c_{i\alpha}\partial_\alpha) f_i^{(0)} c_{i\beta} = -\frac{1}{\tau}\sum_i f_i^{(1)} c_{i\beta} = 0$$

$$\left[\partial_t^{(1)}(\rho u_\beta) + \partial_\alpha(\Pi_{\alpha\beta})\right] = 0 \quad (11)$$

$$\Pi_{\alpha\beta} = \sum_i f_i^{eq} c_{i\alpha} c_{i\beta} = \rho u_\alpha u_\beta + \rho c_s^2 \delta_{\alpha\beta} \quad -(12)$$

Tutorial

1) $$I(0) = \frac{\rho}{2\pi \varsigma^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{(\xi_x^2+\xi_y^2)}{2\varsigma^2}} \, d\xi_x \, d\xi_y = 1$$ is given. and

define $$I(n) = \frac{\rho}{2\pi \varsigma^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{(\xi_x^2+\xi_y^2)}{2\varsigma^2}} \xi_x^n \, d\xi_x \, d\xi_y$$

show that $I(n+2) = (n+1)\varsigma^2 I(n)$ and hence fin $I(n)$

Note that
$$\frac{d}{d\xi_x}\left[\xi_x^{n+1} e^{-\frac{(\xi_x^2+\xi_y^2)}{2\varsigma^2}}\right] = (n+1)\xi_x^n e^{\left[-\frac{(\xi_x^2+\xi_y^2)}{2\varsigma^2}\right]} - \frac{\xi_x^{n+2}}{\varsigma^2} e^{-\frac{(\xi_x^2+\xi_y^2)}{2\varsigma^2}}$$

Integrate both sides in $(-\infty < \xi_x < \infty)$ and $(-\infty < \xi_y < \infty)$

$$\xi_x^{n+1} e^{-\frac{(\xi_x^2+\xi_y^2)}{2\varsigma^2}} \Big|_{-\infty}^{\infty} = (n+1) I(n) - \frac{I(n+2)}{\varsigma^2} = 0$$

$$I(n+2) = (n+1)\varsigma^2 I(n)$$
$$I(n) = (n-1)\varsigma^2 I(n-2)$$
$$I(n-2) = (n-3)\varsigma^2 I(n-4)$$

$$I(2) = 1\varsigma^2 I(0)$$

$$I(n+2) = 1.3.5.\ldots\ldots(n+1)\varsigma^2 I(0) = 1.3.5.\ldots\ldots(n+1)\varsigma^{n+2}$$

Thus $I(2) = \varsigma^2$, $I(4) = 3\varsigma^4$, $I(6) = 15\varsigma^6$, ...

(2) $$\frac{\partial^2}{\partial \xi_x \partial \xi_y}\left(\frac{\xi_x \xi_y}{2\pi\theta} \cdot \exp\left[-\left(\frac{\xi_x^2+\xi_y^2}{2c_s^2}\right)\right]\right)$$

$$= \frac{1}{2\pi c_s^6}\left[\exp\left(\frac{-(\xi_x^2+\xi_y^2)}{2c_s^2}\right)\left(-\xi_x^2+c_s^2\right)\left(-\xi_y^2+c_s^2\right)\right] \quad -(1)$$

is given. Find $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\xi_x^2 \xi_y^2}{2\pi c_s^2} \exp\left[-\frac{(\xi_x^2+\xi_y^2)}{2c_s^2}\right] d\xi_x d\xi_y$

integrate (1) over ξ_x, ξ_y

$$0 = \frac{1}{2\pi c_s^2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp{-\frac{(\xi_x^2+\xi_y^2)}{2c_s^2}}\, \xi_x^2 \xi_y^2\, d\xi_x d\xi_y - 2c_s^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp{-\frac{(\xi_x^2+\xi_y^2)}{2c_s^2}}\, \xi_x^2\, d\xi_x d\xi_y\right.$$

$$\left. + c_s^4\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp{-\frac{(\xi_x^2+\xi_y^2)}{2c_s^2}}\, d\xi_x d\xi_y\right]$$

$$0 = I - 2c_s^2 \cdot c_s^2 \rho + c_s^4 \rho$$

$$I = c_s^4 \rho$$

Thus

$\int f^{eq} \xi_\alpha \xi_\beta \xi_\nu \xi_\delta \, d\xi_x d\xi_y$ is non-zero only when (a) $\alpha = \beta = \nu = \delta$

or (b) two indices are pairwise same

$$\int f^{eq} \xi_\alpha \xi_\beta \xi_\nu \xi_\delta \, d\xi_x d\xi_y = c_s^4\left(\delta_{\alpha\beta}\delta_{\nu\delta} + \delta_{\alpha\nu}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\nu}\right)$$

That is Continuous / discrete equilibrium are matched upto fourth orders in moments.

② Maxwell - Boltzmann equilibrium distribution is given

$$f^{eq}(x,\vec{\xi},\vec{u}) = \frac{\rho}{2\pi c_s^2} \exp -\left[\frac{(u_x-\xi_x)^2 + (u_y-\xi_y)^2}{2c_s^2}\right]$$

where $\vec{u} = (u_x, u_y)$ is mean fluid velocity

Find second moment of the f^{eq} with respect to particle velocity $\vec{\xi}$ $\iint f^{eq}\, \xi_x \xi_y \, d\xi_x d\xi_y$

$$I = \frac{\rho}{2\pi c_s^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[\frac{-(u_x-\xi_x)^2 + (u_y-\xi_y)^2}{2c_s^2}\right] \xi_x \xi_y \, d\xi_x d\xi_y$$

Change variables $n_x = \xi_x - u_x$, $n_y = \xi_y - u_y$

$$I = \frac{\rho}{2\pi c_s^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{(n_x^2+n_y^2)}{2c_s^2}\right] (u_x+n_x)(u_y+n_y)\, dn_x dn_y$$

$$= \frac{\rho}{2\pi c_s^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{(n_x^2+n_y^2)}{2c_s^2}\right] (\overbrace{u_x u_y}^{\text{Constant}} + \overbrace{u_x n_y + u_y n_x}^{\text{odd}} + \overbrace{n_x n_y}^{\text{even if } n_x = n_y})\, dn_x dn_y$$

$$= \frac{\rho u_x u_y}{2\pi c_s^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{(n_x^2+n_y^2)}{2c_s^2}\right] dn_x dn_y + \frac{\rho u_x}{2\pi c_s^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(\frac{-(n_x^2+n_y^2)}{2c_s^2}\right) n_y \, dn_x dn_y$$

$$+ \frac{\rho u_y}{2\pi c_s^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(\frac{-(n_x^2+n_y^2)}{2c_s^2}\right) n_x \, dn_x dn_y + \frac{\rho}{2\pi c_s^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[\frac{-(n_x^2+n_y^2)}{2c_s^2}\right] n_x n_y \, dn_x dn_y$$

$$= \rho u_x u_y + \rho c_s^2 \delta_{xy}$$

The result is a second order tensor (two Components need to be specified)

③ show that

$$\sum_i w_i c_{i\alpha} c_{i\beta} = c_s^2 \delta_{\alpha\beta}$$

LHS is a second order tensor. two components (α, β) can take values x and y. Consider two cases ⓐ $\alpha = \beta$ ⓑ $\alpha \neq \beta$

when $\alpha \neq \beta$, for every direction i there is opposite direction $\bar{i}$ such that $c_{i\beta} = - c_{\bar{i}\beta}$.

$$\sum_i w_i c_{i\alpha} c_{i\beta} = 0$$

when $\alpha = \beta$

$$\sum_i w_i c_{i\alpha}^2 = 0 + 2w_1 + 4w_5 = \frac{2}{9} + \frac{4}{36} = \frac{1}{3} = c_s^2$$

④ Show that

$$\sum_i w_i \frac{\vec{c_i} \cdot \vec{u}}{c_s^2} c_{i\alpha} c_{i\beta} = 0$$

$$\frac{1}{c_s^2} \sum_\nu \sum_i w_i c_{i\nu} u_\nu c_{i\alpha} c_{i\beta} = 0$$

LHS is again a component of second order tensor.

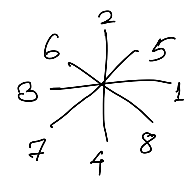
for every β, $\alpha = \nu$ or $\alpha \neq \nu$. In both cases, the term is odd in β.

$$\frac{1}{c_s^2} \sum_\nu \sum_i (w_i u_\nu c_{i\nu} c_{i\alpha}) c_{i\beta} = 0$$

(5) Show that $\sum_i \omega_i \frac{(\vec{c_i}\cdot\vec{u})^2}{2c_s^4} c_{i\alpha}c_{i\beta} - \frac{\vec{u}^2}{2c_s^2}\sum \omega_i c_{i\alpha}c_{i\beta} = u_\alpha u_\beta$

Perform summation over index.

first take $\alpha=\beta=x$



$$\frac{1}{2c_s^4}\Big[\omega_1(u_x^2)+\omega_2(0)+\omega_3 u_x^2+\omega_4(0)+\omega_5(u_x+u_y)^2+\omega_6(u_x+u_y)^2 + \omega_7(u_x+u_y)^2+\omega_8(u_x-u_y)^2\Big] - \frac{(u_x^2+u_y^2)}{2}\cdot\frac{3}{1}\cdot\frac{1}{3}\,\delta_{xx}$$

$$= \frac{1}{2c_s^4}\left[\frac{2}{9}u_x^2\right] + \frac{1}{2c_s^4}\left[\frac{4}{36}\right](u_x^2+u_y^2) - \frac{u_x^2+u_y^2}{2}$$

$$= u_x^2 + \frac{u_x^2+u_y^2}{2} - \left(\frac{u_x^2+u_y^2}{2}\right)$$

$$= u_x^2 \quad (\text{when } \alpha=\beta=x)$$

take $\alpha=x,\ \beta=y$

$$\frac{1}{2c_s^4}\Big[\omega_5(u_x+u_y)^2(1) + \omega_6(-u_x+u_y)^2(-1) + \omega_7(u_x+u_y)^2(1) + \omega_8(u_x-u_y)^2(-1)\Big] - \frac{(u_x^2+u_y^2)}{2}\,3\,\delta_{xy}$$

$$= \frac{1}{2}\,9\cdot\frac{1}{36}\,8\,u_x u_y = u_x u_y \quad (\text{when } \alpha=x,\ \beta=y)$$

In general,

$$T_{\alpha\beta} = u_\alpha u_\beta$$

(6) $f_i^{eq} = \rho w_i \left(1 + \frac{\vec{c_i}\cdot\vec{u}}{c_s^2} + \frac{(\vec{c_i}\cdot\vec{u})^2}{2c_s^4} - \frac{\vec{u}^2}{2c_s^2}\right)$

Using examples (5) + (4) + (3)

show that $\sum_i f_i^{eq} c_{i\alpha} c_{i\beta} = \rho u_\alpha u_\beta + \rho c_s^2 \delta_{\alpha\beta}$

(7) Show that

$$\sum_i w_i \frac{\vec{c_i}\cdot\vec{u}}{c_s^2} c_{i\alpha} c_{i\beta} c_{i\nu} = (u_\alpha \delta_{\beta\nu} + u_\beta \delta_{\alpha\nu} + u_\nu \delta_{\alpha\beta}) c_s^2$$

LHS is a Component of a third order tensor

$$\sum_i \frac{w_i}{c_s^2} (c_{ix} u_x + c_{iy} u_y) c_{i\alpha} c_{i\beta} c_{i\nu}$$

Lets say α, β, ν are all different from x, meaning $\alpha = \beta = \nu = y$

then

$$\sum_i \frac{w_i}{c_s^2} c_{ix} u_x c_{iy}^3 = 0$$

thus for u_x to survive, one of the index or all three indices α, β, ν should be x

$$\sum \frac{w_i}{c_s^2} c_{ix} u_x c_{ix} c_{iy}^2 = 3u_x (4w_5) = u_x c_s^2 \quad (\alpha = x,\ \beta = \nu = y \neq x)$$

$$\sum \frac{w_i}{c_s^2} c_{ix} u_x c_{ix}^3 = 3u_x [2w_1 + 4w_5] = 3u_x c_s^2 \quad (\alpha = \beta = \nu = x)$$

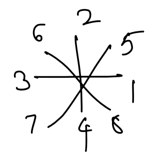
These are the only cases that agree with RHS

(8) Show that (for D2Q9 lattice), where $\vec{F}$ is a constant vector

(a) $\sum_i w_i \frac{\vec{c_i} \cdot \vec{F}}{c_s^2} = 0$

– follows from anti-symmetry that for every direction i, there is opposite one $\bar{i}$ such that $\vec{c_i} = -\vec{c_{\bar{i}}}$

(b) $\sum_i w_i \left(\frac{\vec{c_i} \cdot \vec{F}}{c_s^2} \right) \vec{c_i} = \vec{F} = (F_x, F_y)$



$$w_0(0) + \frac{1}{9}\frac{1}{c_s^2}\left[F_x(1,0) + F_y(0,1) + (-F_x)(-1,0) - F_y(0,-1) \right]$$

$$+ \frac{1}{36}\frac{1}{c_s^2}\left[(F_x+F_y)(1,1) + (-F_x+F_y)(-1,1) + (-F_x-F_y)(-1,-1) + (F_x-F_y)(1,-1) \right]$$

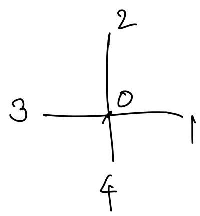
$$= \frac{2}{9}\frac{1}{c_s^2}(F_x, F_y) + \frac{4}{36}\frac{1}{c_s^2}(F_x, F_y) = (F_x, F_y) = \vec{F}$$

(9) Take the collision operator $\Omega(f) \equiv 0$ instead of $\Omega(f) = -\frac{1}{\tau}(f_i - f_i^{eq})$

Do equations obtained at the ε order in Chapman-Enskog analysis change? Discuss implications

(10) D2Q9 is isotropic upto fourth (even fifth as odd orders are always satisfied).

Check upto what order D2Q5 lattice is isotropic



there are two types of weight ω_0, ω_1 and lattice speed of sound for the lattice c_s^2 are unknowns

11 Consider only collision process without the propagation operation

$$f^{*}(x,t) = \frac{\Delta t}{\tau} f_i^{eq}(\vec{x},t) + \left(1 - \frac{\Delta t}{\tau}\right) f_i(\vec{x},t)$$

Determine the stability condition for the collision process.