EE2703 : ASSIGNMENT 7

Sarthak Vora (EE19B140)

April 29, 2021

1 Introduction

This assignment involves the analysis of filters using **laplace transforms**. Python's symbolic solving library, `sympy` is a tool we use in the process to handle our requirements in solving Modified Nodal Analysis equations. Besides this the library also includes useful classes to handle the simulation and response to inputs.

Coupled with scipy's `signal` module, we are able to analyse both High pass and low pass filters, both second order, realised using a single op amp.

2 Low-Pass Filter

In this section, we analyse the following circuit:

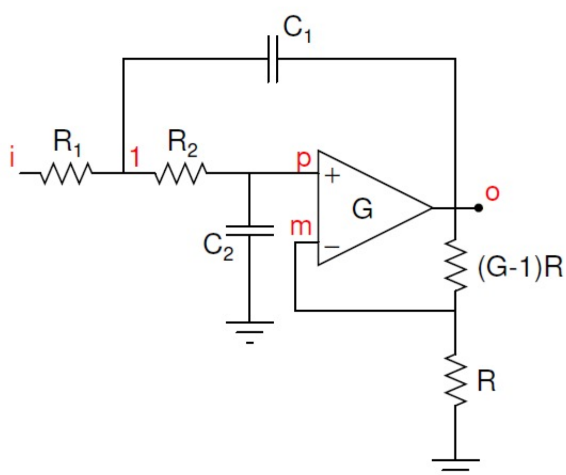


Figure 1: Op-Amp based Low-Pass Filter

Writing KCL equations (in s-domain) of the nodes marked on the figure, we get the following matrix:

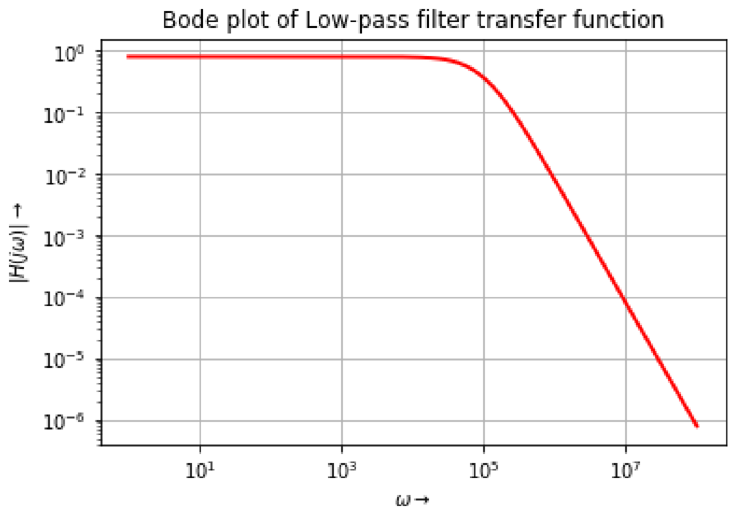
$$\begin{pmatrix} 0 & 0 & 1 & \frac{-1}{G} \\ \frac{-1}{1+sR_2C_2} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -\frac{1}{R_1}-\frac{1}{R_2}-sC_1 & \frac{1}{R_2} & 0 & sC_1 \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_p(s) \\ V_m(s) \\ V_o(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{-V_i(s)}{R_1} \end{pmatrix} \tag{1}$$

`Sympy` allows us to create matrices with symbolic entries, and also perform mathematical operations on them, as if they were `numpy` arrays. In the above circuit, the values of R_1, R_2, C_1, C_2 are $10k\Omega$, $10k\Omega$, 10pF, 10pF respectively.

Solving for $V_o(s)$, (with the above given values) we get:

$$V_o(s) = \frac{-0.0001586 \cdot V_i(s)}{2 \times 10^{-14}s^2 + 4.414 \times 10^{-9}s + 0.0002} \tag{2}$$

The magnitude **Bode plot** of Low-pass filter transfer function is given below:

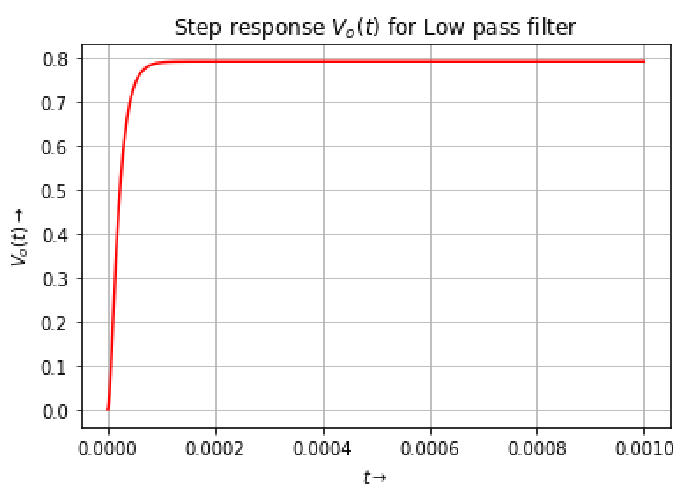


2.1 Task 1

- Obtain the Transfer function of the network and determine the Laplace transform of impulse response. ($V_i(t) = \delta(t)$ whose $Laplace\ Transform$ is $\mathcal{V}_i(s) = \frac{1}{s}$).
- To observe this behaviour we give **unit step** as input and analyse the output. The unit step is given as -

$$V_i(t) = u(t)\ Volts \tag{3}$$

- The plot for the same is shown below -



2.1.1 Observation

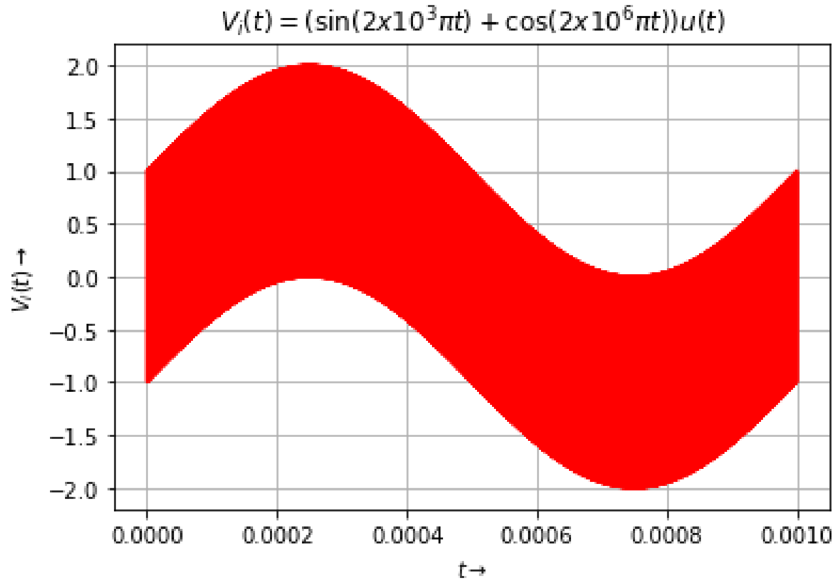
- As we can see from the Bode plot above, the circuit is a low pass filter with **bandwidth** $0<\omega<10^4$.
- Hence, circuit will only pass input with frequencies which are in range of bandwidth and **attenuates** other frequencies largely since its second order filter with **-40dB/dec** drop in gain.
- The step response plot, $V_o(t)$ increases quickly from 0 to 0.8 and settles at 0.8 after some time. Since the network is a low-pass filter, the output must be dominated by **DC gain** at steady state.
- The **Quality factor** of the system is $Q = 0.453.. < \frac{1}{\sqrt{2}}$ which implies that the gain of the system never **exceeds** DC Gain and always less than that. So with this, we see that unit step response is always less that the DC Gain of 0.8 which is obtained by putting $s = 0$ in $\mathcal{V}_o(s)$.

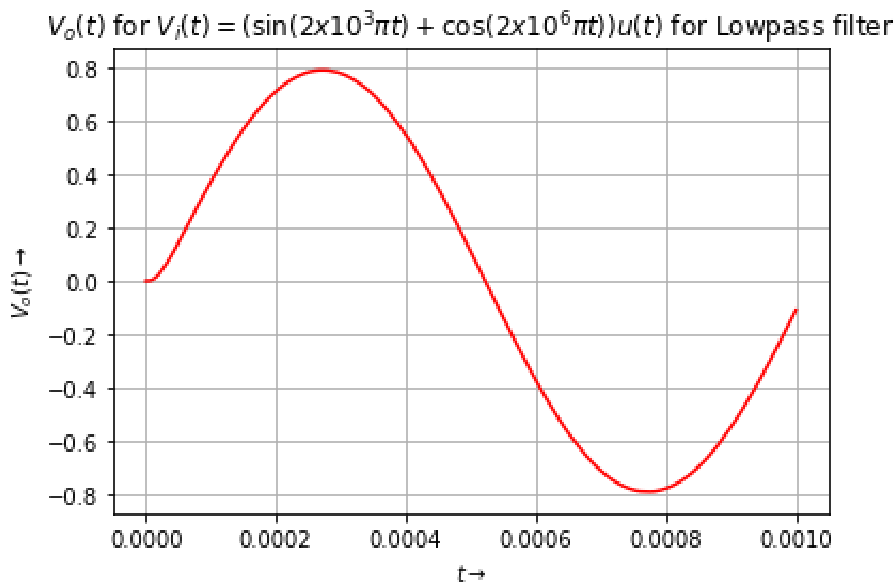
2.2 Task 2

- Obtain and analyse the response of a low-pass filter for sinusoid with a low frequency and high frequency component of $\omega_1 = 2000\pi\ rads^{-1}\ and\ \omega_2 = 2 * 10^6\pi\ rads^{-1}$.

$$V_i(t) = (\ \sin(2000\pi t) + \cos(2 * 10^6\pi t)\ )u_o(t)\ Volts \tag{4}$$

- The Plot for input voltage, $V_i(t)$ and output voltage, $V_o(t)$ is shown below -





2.2.1 Observation

- From the plot, we can see that the circuit will only pass input with frequencies which are in range of its **bandwidth**. But its not an ideal low pass filter since gain doesn't drop abruptly at 10^4.

- Hence, the output $V_o(t)$ will mainly consist of $\sin(2000\pi t)$ component as the **higher frequency** components are largely attenuated (second order filter, hence gain drops by 40dB/dec).

- Since the output is largely a function, we can observe that $V_o(t)$ starts from $t = 0$.

3 High-Pass Filter

We shall now look at a slightly modified version of the above circuit.

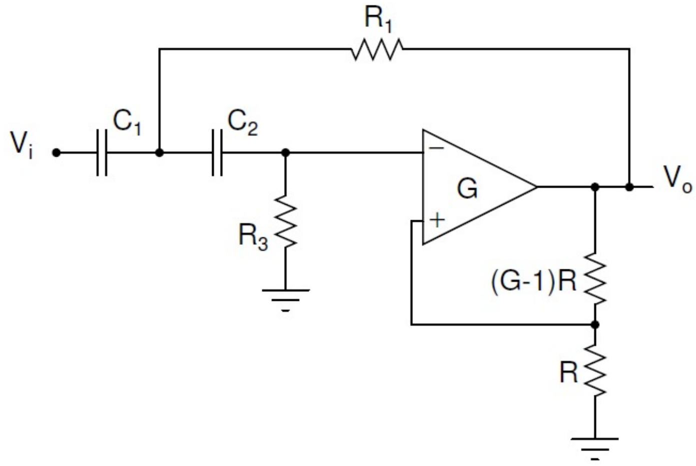


Figure 2: Op-amp based High-Pass Filter

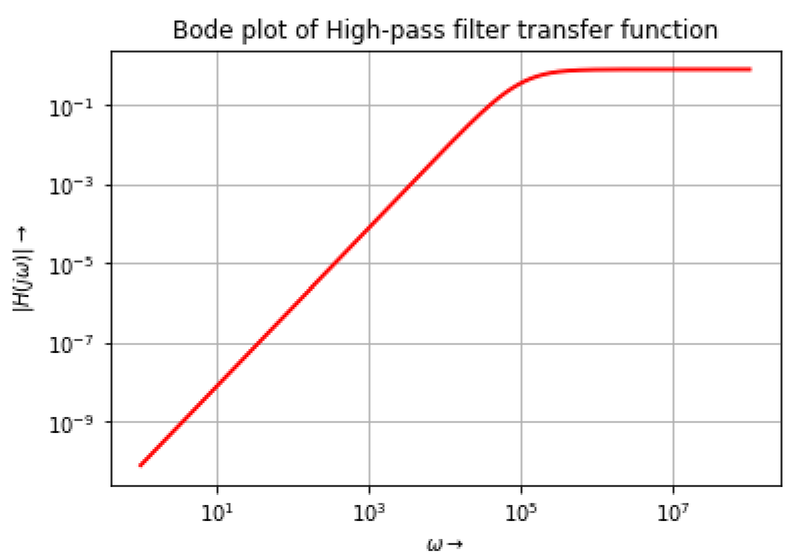
Performing a similar procedure like before, we get the KCL matrix as:

$$\begin{pmatrix} 0 & 0 & 1 & \frac{-1}{G} \\ \frac{sR_3C_2}{1+sR_3C_2} & 0 & -1 & 0 \\ 0 & -G & G & 1 \\ -\frac{1}{R_1} - sC_2 - sC_1 & 0 & sC_2 & \frac{1}{R_1} \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_p(s) \\ V_m(s) \\ V_o(s) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -sC_1V_i(s) \end{pmatrix} \tag{5}$$

Solving it for $V_o(s)$, we get,

$$V_o(s) = \frac{1.586 \times 10^{-14} s^2 \cdot V_i(s)}{2 \times 10^{-14} s^2 + 4.414 \times 10^{-9} s + 0.0002} \tag{6}$$

The **Bode magnitude plot** of High-pass filter transfer function is shown below:



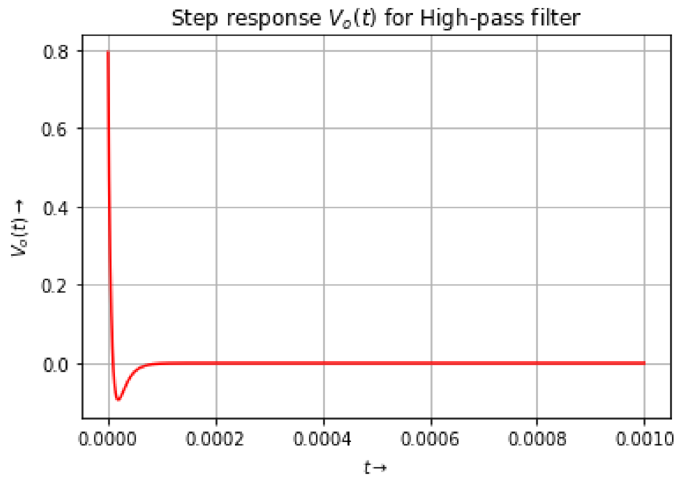
3.1 Task 3

- Obtain the Transfer function of the**High-pass filter network** and determine the Laplace transform of impulse response. ($V_i(t) = \delta(t)$ whose $Laplace\ Transform$ is $\mathcal{V}_i(s) = \frac{1}{s}$).

- To observe this behaviour we give **unit step** as input and analyse the output. The unit step is given as -

$$V_i(t) = u(t)\ Volts \tag{7}$$

- The plot for the same is shown below -



3.1.1 Observation

- According to the Bode magnitude plot, the circuit is a High-pass filter with bandwidth $\omega > 10^5$. So the circuit will only pass input with frequencies which are in the **range of bandwidth**.

- The step response plot, $V_o(t)$ decreases quickly from 0.8 to 0 and settles at 0 after some time. But interestingly it **crosses zero and goes negative** for some period of time.

- Since the network is a High-pass filter, the output must **not allow DC** at steady state and also the input is unit step which is constant for $t > 0$. So at steady state, $V_0(t)$ should be zero. Also, since its a Highpass filter, its $Mean = 0$ which means $\int_0^\infty V_o(t)dt = 0$.

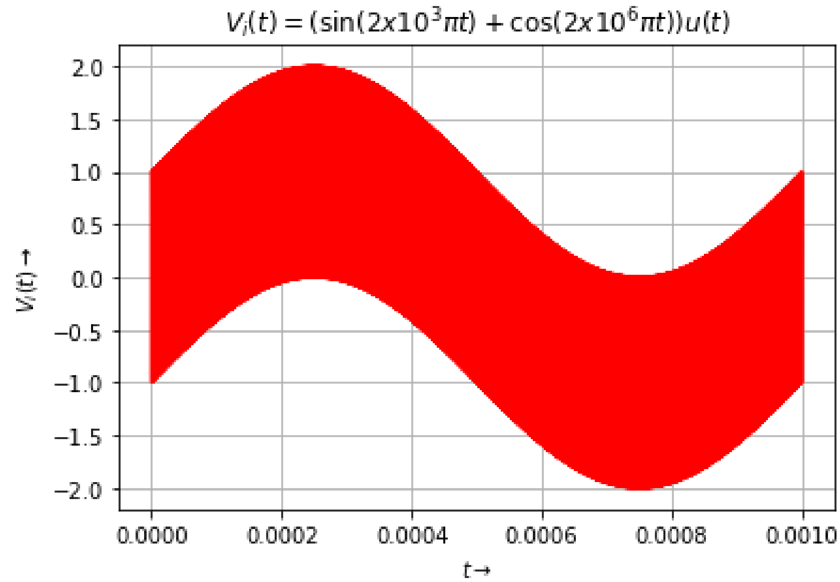
- Hence, we observe that the voltage becomes negative to make the average as **zero**.

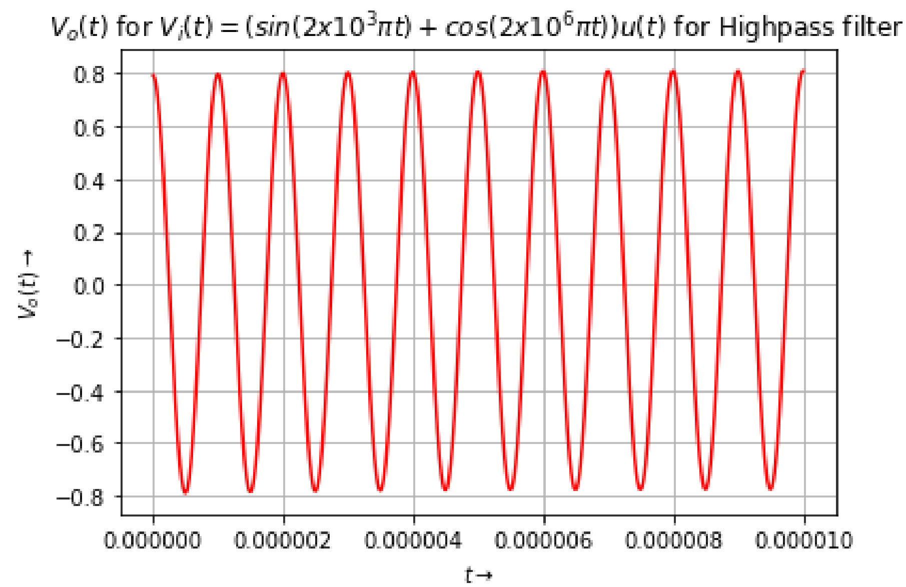
3.2 Task 4

- Obtain and analyse the response of a High-pass filter for sinusoid with a low frequency and high frequency component of $\omega_1 = 2000\pi \ rads^{-1} \ and \ \omega_2 = 2 * 10^6\pi \ rads^{-1}$.

$$V_i(t) = (\ \sin(2000\pi t) + \cos(2 * 10^6\pi t)\ )u_o(t)\ Volts \tag{8}$$

- The Plot for input voltage, $V_i(t)$ and output voltage, $V_o(t)$ is shown below -





3.2.1 Observation

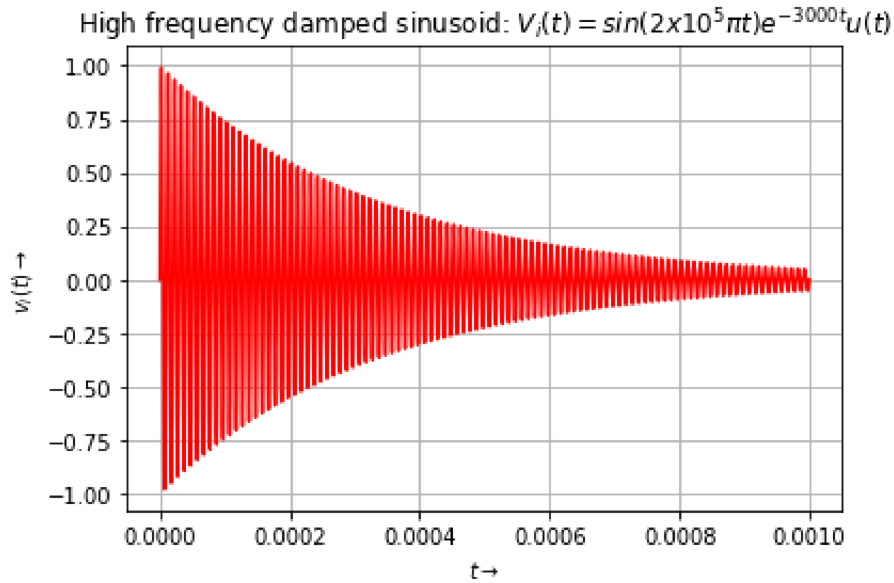
- As we can see from the plot of output voltage $V_o(t)$, the curve is a **sinusoid** with a very **high** frequency.
- This is expected, since the high-pass filter **retains** the frequency components greater that 10^4 rad/s. Hence, the lower frequency components are **attenuated** and high frequency components are passed through.

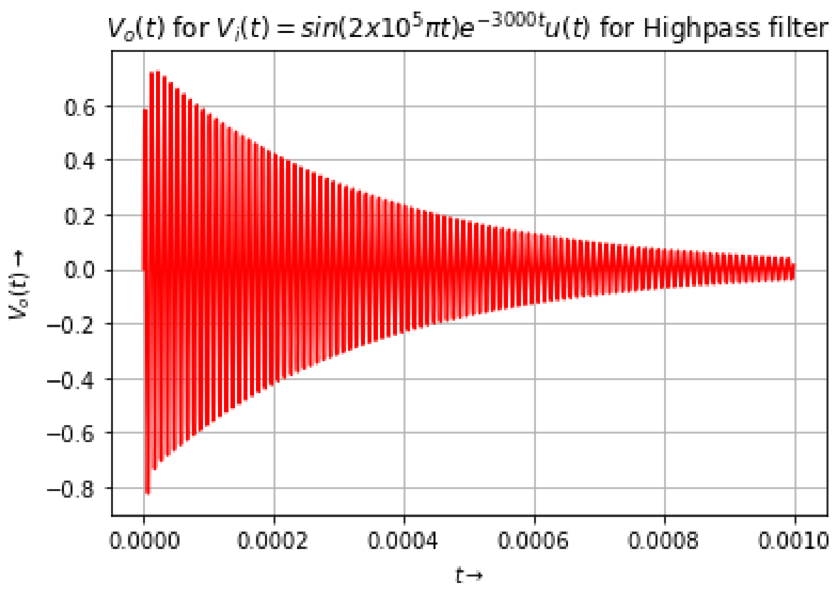
3.3 Task 5

- Obtain and analyse the response of a High-pass filter to a high frequency **damped** sinusoid given below -

$$f(t) = \sin(2 * 10^5 \pi t) * e^{-3000t} \tag{9}$$

- The Plot for input voltage, $V_i(t)$ and output voltage, $V_o(t)$ is shown below -





3.3.1 Observation

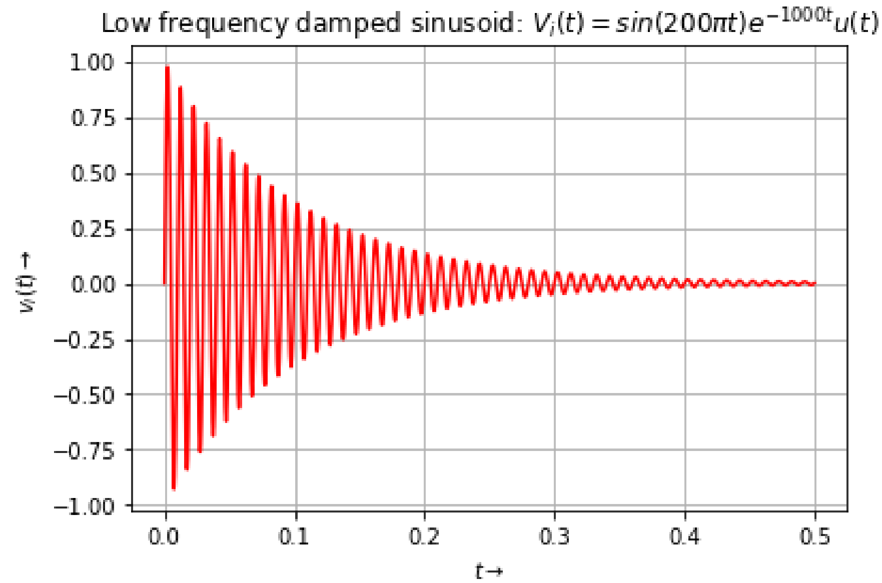
- It can be seen that the input voltage $V_i(t)$ has **high frequency** components in its sinusoidal decay. Hence, naturally the output response, $V_o(t)$ would be almost the same as the input signal $V_i(t)$ in case of a High-pass filter.
- This is because, the high-pass filter **passes** all the frequency components **greater** than 10^5rad/s. Since, the input frequency is just above 10^5rad/s, the output also has the same frequency.
- The minor **attenuation** of the the output response is due to **non-ideality** of the High-pass filter.

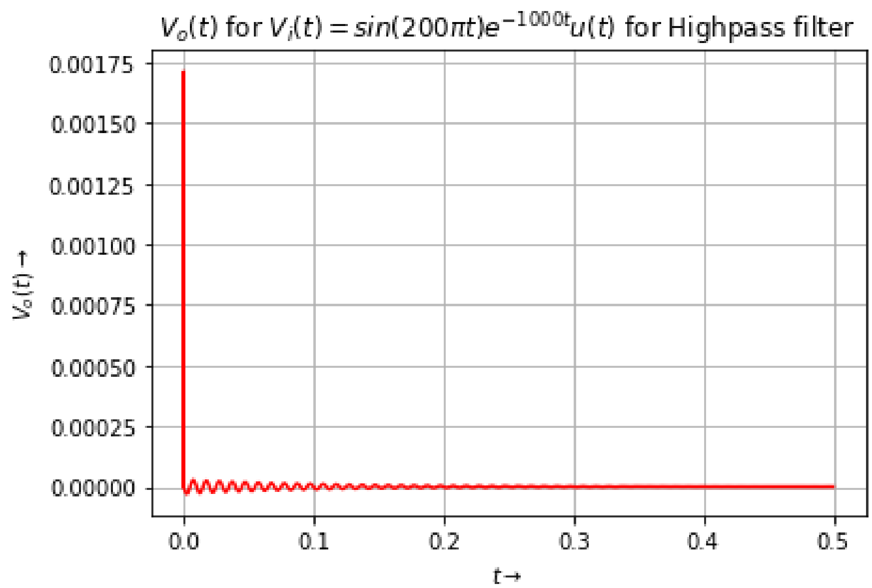
3.4 Task 6

- Obtain and analyse the response of a High-pass filter to a low frequency **damped** sinusoid given below -

$$f(t) = \sin(200\pi t) * e^{-1000t} \tag{10}$$

- The Plot for input voltage, $V_i(t)$ and output voltage, $V_o(t)$ is shown below -





3.4.1 Observation

- It can be seen that the input voltage $V_i(t)$ has **low frequency components** in its sinusoidal decay. Hence, naturally the output response, $V_o(t)$ would almost be **0**.
- This is because of the inherent property of circuit to act as high-pass filter. As a result, it attenuates any frequency **less** than 10^5rad/s. Since, the input has all frequency components greater than 10^5. The **output** is almost completely **attenuated**.
- The sudden change at $t = 0$ is because of the step input leading to very **high frequency** components which are passed by the filter at $t = 0$.

4 Conclusion

- The Bode plot magnitude response of Low-pass and High-pass filters was visualised and conclusions were made.
- The Low-pass filter allows sinusoids upto an angular frequency of 10^5rad/s (**cutoff frequency**). The High-pass filter allows **sinusoids** beyond an angular frequency of 10^5rad/s.
- The step response gets **attenuated** at $t = 0$ due to an **abrupt** change (high frequency components) in case of a Low-pass filter. However, at steady state, the DC component is **retained**.
- The high frequency components of **sum of sinusoids** gets filtered by Low-pass filters as it's frequency lies **below** the cutoff frequency. (10^5rad/s)
- The step response of High-pass filter is almost 0 everywhere as it gets attenuated completely by the High-pass filter. At $t = 0$, there is an abrupt jump due to presence of very **high frequency** components.
- The **high frequency damped** sinusoid passes almost **unaffected** through the High-pass filter. The **low frequency damped sinusoid** gets **attenuated** almost completely when it passes through the High-pass filter.
