Assignment-2

1. Classify the PDE

$$x^2\frac{\partial^2 u}{\partial t^2} + 3\frac{\partial^2 u}{\partial x \partial t} + x\frac{\partial^2 u}{\partial x^2} + 17\frac{\partial u}{\partial t} = 100u$$

2. Solve the following equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variables.

Ans:- $z = \left[c_1 e^{(1+\sqrt{1+k})x} + c_2 e^{(1-\sqrt{1+k})x}\right] c_3 e^{-ky}$.

3. Solve the following equation by the method of separation of variables.

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$$

Given that $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ when $x = 0$

Ans:- $u = (1-e^{-t})\sin x$

4. A tightly strectched string with fixed end points $x=0$ and $x=l$ is intiailly in a position given $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x,t)$.

Ans $y = \frac{y_0}{4}\left(3\sin\frac{\pi x}{l}\cos\frac{c\pi t}{l} - \sin\frac{3\pi x}{l} \times \cos\frac{3c\pi t}{l}\right)$

5. A rectangular plate with insulated surface is 8cm wide and so long compared to its width that it may be considered infinite in length without

introducing an appreciable. of the tempearature along one short edge $y=0$ is given by

$$u(x,0) = 100 \sin \frac{\pi x}{8}, \quad 0 < x < 8$$

while the two long edges $x=0$ and $x=8$ as well as the other short edge are kept at $0^\circ C$. show that steady state temperature at any plate is given by

$$u(x,y) = 100 e^{\frac{\pi y}{8}} \sin \frac{\pi x}{8}$$