

Bits, Bytes and Integers – Part 1

15-213/18-213/15-513: Introduction to Computer Systems
2nd Lecture, Sept. 1, 2016

Today's Instructor:

Phil Gibbons

Announcements

- Recitations are on Mondays, but next Monday (9/5) is Labor Day, so recitations are cancelled
- We will schedule a Linux Boot Camp some time next week
- Lab 1 is now available via Autolab. Those of you who do not yet have Autolab accounts can get a copy of the documentation and the supplied files from the schedule web page. You can work on this lab using one of the class (Shark) machines, or one of the Andrew Linux machines.

Autolab accounts

- **Students enrolled 10am on Mon, Aug 29 have Autolab accounts**
- **You must be enrolled to get an account**
 - Autolab is not tied in to the Hub's rosters
 - If you add in, contact 15-213-staff@cs.cmu.edu for an account
 - We will update the autolab accounts once a day, so check back in 24 hours
- **For those who are waiting to add in, the first lab (datalab) is available on the Schedule page of the course Web site.**

Waitlist questions

- 15-213: Catherine Fichtner (cathyf@cs.cmu.edu)
- 18-213: Zara Collier (zcollier@andrew.cmu.edu)
- 15-513: Catherine Fichtner (cathyf@cs.cmu.edu)

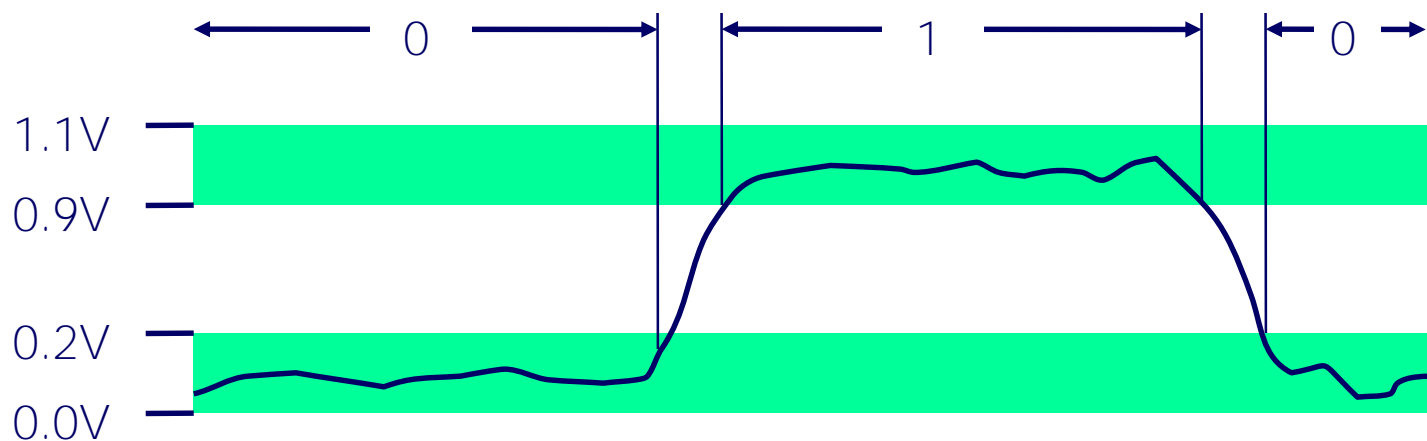
- Please don't contact the instructors with waitlist questions.

Today: Bits, Bytes, and Integers

- **Representing information as bits**
- **Bit-level manipulations**
- **Integers**
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
 - Summary
- **Representations in memory, pointers, strings**

Everything is bits

- Each bit is 0 or 1
- By encoding/interpreting sets of bits in various ways
 - Computers determine what to do (instructions)
 - ... and represent and manipulate numbers, sets, strings, etc...
- Why bits? Electronic Implementation
 - Easy to store with bistable elements
 - Reliably transmitted on noisy and inaccurate wires



For example, can count in binary

■ Base 2 Number Representation

- Represent 15213_{10} as 11101101101101_2
- Represent 1.20_{10} as $1.0011001100110011[0011]..._2$
- Represent 1.5213×10^4 as $1.1101101101101_2 \times 2^{13}$

Encoding Byte Values

■ Byte = 8 bits

- Binary 00000000_2 to 11111111_2
- Decimal: 0_{10} to 255_{10}
- Hexadecimal 00_{16} to FF_{16}
 - Base 16 number representation
 - Use characters '0' to '9' and 'A' to 'F'
 - Write $FA1D37B_{16}$ in C as
 - `0xFA1D37B`
 - `0xfa1d37b`

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

15213: 0011 1011 0110 1101
 3 B 6 D

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
<code>char</code>	1	1	1
<code>short</code>	2	2	2
<code>int</code>	4	4	4
<code>long</code>	4	8	8
<code>float</code>	4	4	4
<code>double</code>	8	8	8
<code>pointer</code>	4	8	8

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Boolean Algebra

■ Developed by George Boole in 19th Century

- Algebraic representation of logic
 - Encode “True” as 1 and “False” as 0

And

- $A \& B = 1$ when both $A=1$ and $B=1$

$\&$	0	1
0	0	0
1	0	1

Or

- $A | B = 1$ when either $A=1$ or $B=1$

$ $	0	1
0	0	1
1	1	1

Not

- $\sim A = 1$ when $A=0$

\sim	
0	1
1	0

Exclusive-Or (Xor)

- $A \wedge B = 1$ when either $A=1$ or $B=1$, but not both

\wedge	0	1
0	0	1
1	1	0

General Boolean Algebras

■ Operate on Bit Vectors

- Operations applied bitwise

01101001	01101001	01101001	
& 01010101	01010101	^ 01010101	~ 01010101
<u> </u>	<u> </u>	<u> </u>	<u> </u>
01000001	01111101	00111100	10101010

■ All of the Properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

■ Representation

- Width w bit vector represents subsets of $\{0, \dots, w-1\}$
- $a_j = 1$ if $j \in A$

▪ 01101001 $\{0, 3, 5, 6\}$

▪ 76543210

▪ 01010101 $\{0, 2, 4, 6\}$

▪ 76543210

■ Operations

- | | | |
|----------------------------|----------|------------------------|
| ▪ & Intersection | 01000001 | $\{0, 6\}$ |
| ▪ Union | 01111101 | $\{0, 2, 3, 4, 5, 6\}$ |
| ▪ ^ Symmetric difference | 00111100 | $\{2, 3, 4, 5\}$ |
| ▪ ~ Complement | 10101010 | $\{1, 3, 5, 7\}$ |

Bit-Level Operations in C

■ Operations `&`, `|`, `~`, `^` Available in C

- Apply to any “integral” data type
 - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

■ Examples (Char data type)

- `~0x41` →
- `~0x00` →
- `0x69 & 0x55` →
- `0x69 | 0x55` →

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit-Level Operations in C

■ Operations $\&$, $|$, \sim , \wedge Available in C

- Apply to any “integral” data type
 - long, int, short, char, unsigned
- View arguments as bit vectors
- Arguments applied bit-wise

■ Examples (Char data type)

- $\sim 0x41 \rightarrow 0xBE$
 - $\sim 0100\ 0001_2 \rightarrow 1011\ 1110_2$
- $\sim 0x00 \rightarrow 0xFF$
 - $\sim 0000\ 0000_2 \rightarrow 1111\ 1111_2$
- $0x69 \& 0x55 \rightarrow 0x41$
 - $0110\ 1001_2 \& 0101\ 0101_2 \rightarrow 0100\ 0001_2$
- $0x69 | 0x55 \rightarrow 0x7D$
 - $0110\ 1001_2 | 0101\ 0101_2 \rightarrow 0111\ 1101_2$

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
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A	10	1010
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Contrast: Logic Operations in C

■ Contrast to Bit-Level Operators

■ Logic Operations: `&`, `||`, `!`

- View 0 as “False”

- Anything nonzero

- Always

- Early

■ Example

- `!0x41` →

- `!0x00` →

- `!!0x41` → `0x01`

- `0x69 && 0x55` → `0x01`

- `0x69 || 0x55` → `0x01`

- `p && *p` (avoids null pointer access)

**Watch out for `&&` vs. `&` (and `||` vs. `|`)...
one of the more common oopsies in
C programming**

Shift Operations

- **Left Shift:** $x \ll y$
 - Shift bit-vector x left y positions
 - Throw away extra bits on left
 - Fill with 0's on right
- **Right Shift:** $x \gg y$
 - Shift bit-vector x right y positions
 - Throw away extra bits on right
 - Logical shift
 - Fill with 0's on left
 - Arithmetic shift
 - Replicate most significant bit on left
- **Undefined Behavior**
 - Shift amount < 0 or \geq word size

Argument x	01100010
$\ll 3$	00010000
<u>Log. $\gg 2$</u>	00011000
<u>Arith. $\gg 2$</u>	00011000

Argument x	10100010
$\ll 3$	00010000
Log. $\gg 2$	00101000
Arith. $\gg 2$	11101000

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Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

```
short int x = 15213;
short int y = -15213;
```

Sign Bit



■ C short 2 bytes long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
y	-15213	C4 93	11000100 10010011

■ Sign Bit

- For 2's complement, most significant bit indicates sign
 - 0 for nonnegative
 - 1 for negative

Two-complement: Simple Example

$$\begin{array}{rcccccc}
 & & -16 & 8 & 4 & 2 & 1 \\
 10 = & 0 & 1 & 0 & 1 & 0 &
 \end{array}
 \qquad
 8+2 = 10$$

$$\begin{array}{rcccccc}
 & & -16 & 8 & 4 & 2 & 1 \\
 -10 = & 1 & 0 & 1 & 1 & 0 &
 \end{array}
 \qquad
 -16+4+2 = -10$$

Two-complement Encoding Example (Cont.)

$x =$ 15213: 00111011 01101101
 $y =$ -15213: 11000100 10010011

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum	15213		-15213	

Numeric Ranges

■ Unsigned Values

- $UMin = 0$
000...0
- $UMax = 2^w - 1$
111...1

■ Two's Complement Values

- $TMin = -2^{w-1}$
100...0
- $TMax = 2^{w-1} - 1$
011...1
- Minus 1
111...1

Values for $W = 16$

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

■ Observations

- $|TMin| = TMax + 1$
 - Asymmetric range
- $UMax = 2 * TMax + 1$

■ C Programming

- `#include <limits.h>`
- Declares constants, e.g.,
 - `ULONG_MAX`
 - `LONG_MAX`
 - `LONG_MIN`
- Values platform specific

Unsigned & Signed Numeric Values

X	$B2U(X)$	$B2T(X)$
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	-7
1010	10	-6
1011	11	-5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

■ Equivalence

- Same encodings for nonnegative values

■ Uniqueness

- Every bit pattern represents unique integer value
- Each representable integer has unique bit encoding

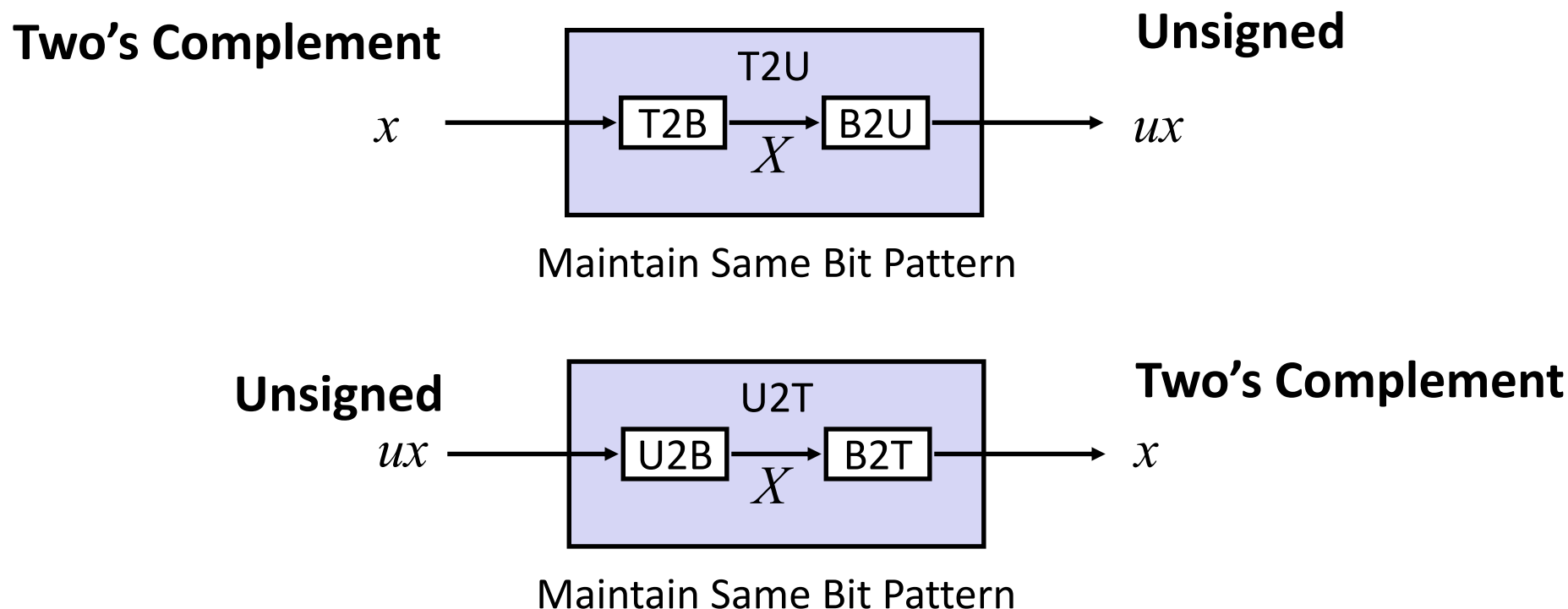
■ \Rightarrow Can Invert Mappings

- $U2B(x) = B2U^{-1}(x)$
 - Bit pattern for unsigned integer
- $T2B(x) = B2T^{-1}(x)$
 - Bit pattern for two's comp integer

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Mapping Between Signed & Unsigned



- Mappings between unsigned and two's complement numbers:
Keep bit representations and reinterpret

Mapping Signed \leftrightarrow Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

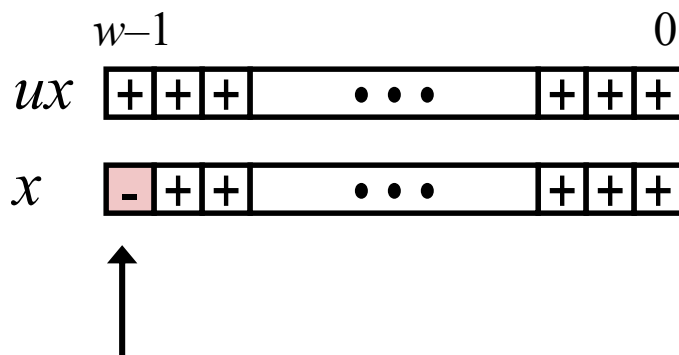
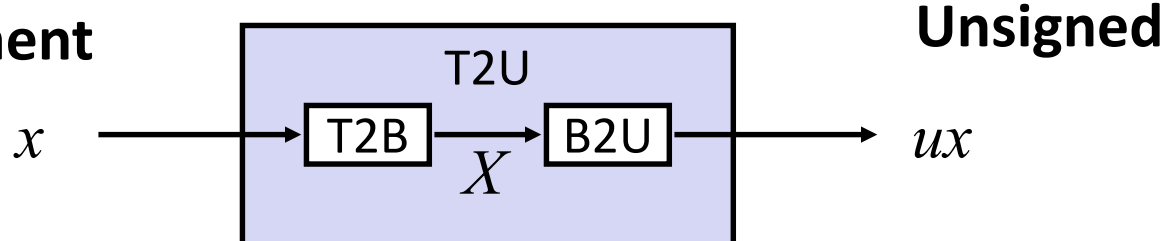
\rightarrow **T2U** \rightarrow
 \leftarrow **U2T** \leftarrow

Mapping Signed \leftrightarrow Unsigned

Bits	Signed		Unsigned
0000	0	\longleftrightarrow =	0
0001	1		1
0010	2		2
0011	3		3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8	\longleftrightarrow +/- 16	8
1001	-7		9
1010	-6		10
1011	-5		11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

Relation between Signed & Unsigned

Two's Complement

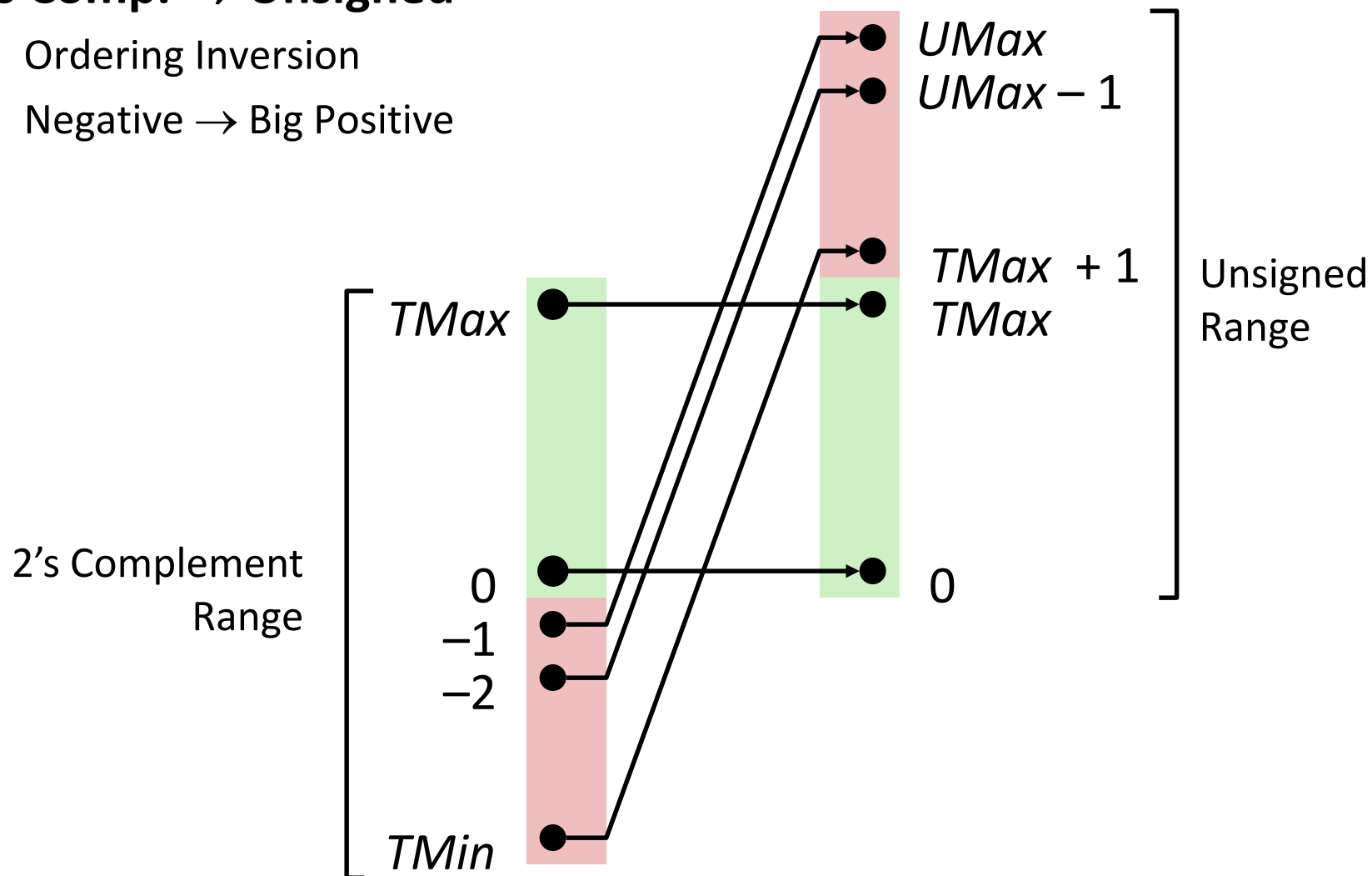


Large negative weight
becomes
Large positive weight

Conversion Visualized

- 2's Comp. → Unsigned

- Ordering Inversion
- Negative \rightarrow Big Positive



Signed vs. Unsigned in C

■ Constants

- By default are considered to be signed integers
- Unsigned if have “U” as suffix

`0U, 4294967259U`

■ Casting

- Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

- Implicit casting also occurs via assignments and procedure calls

```
tx = ux;                int fun(unsigned u);
uy = ty;                uy = fun(tx);
```

Casting Surprises

■ Expression Evaluation

- If there is a mix of unsigned and signed in single expression,
signed values implicitly cast to unsigned
- Including comparison operations $<$, $>$, $==$, $<=$, $>=$
- Examples for $W = 32$: **TMIN = -2,147,483,648**, **TMAX = 2,147,483,647**

■ Constant ₁	Constant ₂	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Unsigned vs. Signed: Easy to Make Mistakes

```
unsigned i;  
for (i = cnt-2; i >= 0; i--)  
    a[i] += a[i+1];
```

- Can be very subtle

```
#define DELTA sizeof(int)  
int i;  
for (i = CNT; i-DELTA >= 0; i-= DELTA)  
    . . .
```

Summary

Casting Signed \leftrightarrow Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w
- Expression containing signed and unsigned int
 - `int` is cast to `unsigned`!!

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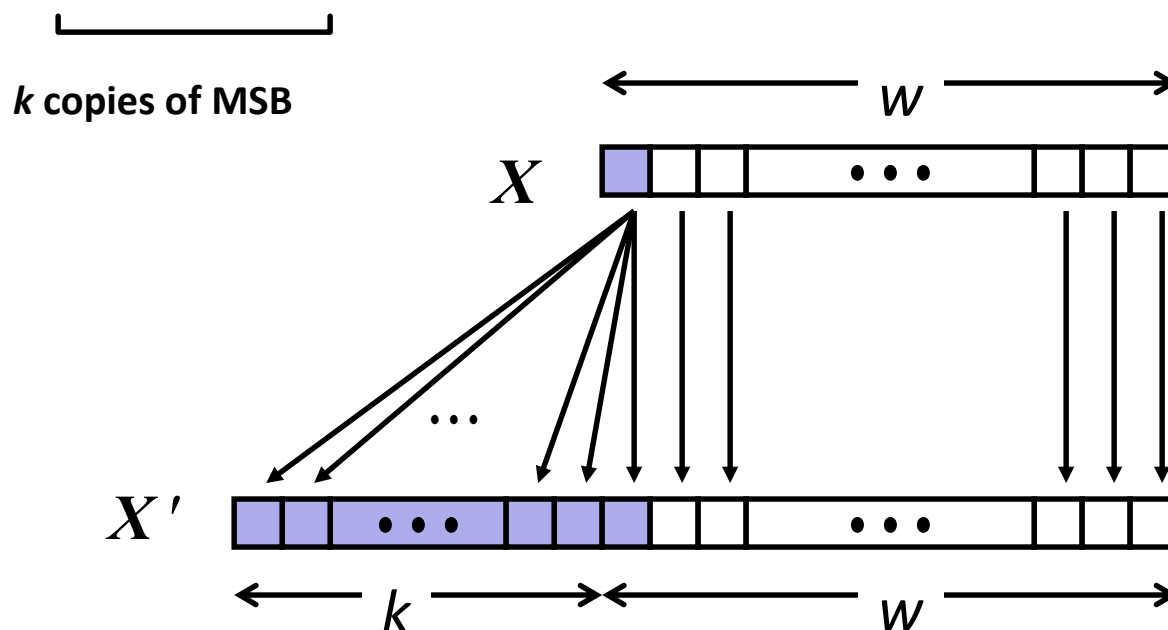
Sign Extension

■ Task:

- Given w -bit signed integer x
- Convert it to $w+k$ -bit integer with same value

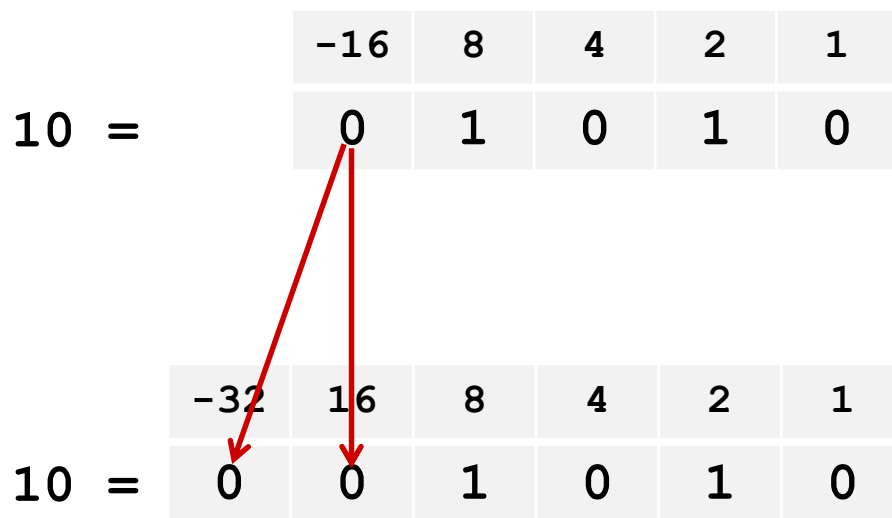
■ Rule:

- Make k copies of sign bit:
- $X' = \underbrace{x_{w-1}, \dots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2}, \dots, x_0$

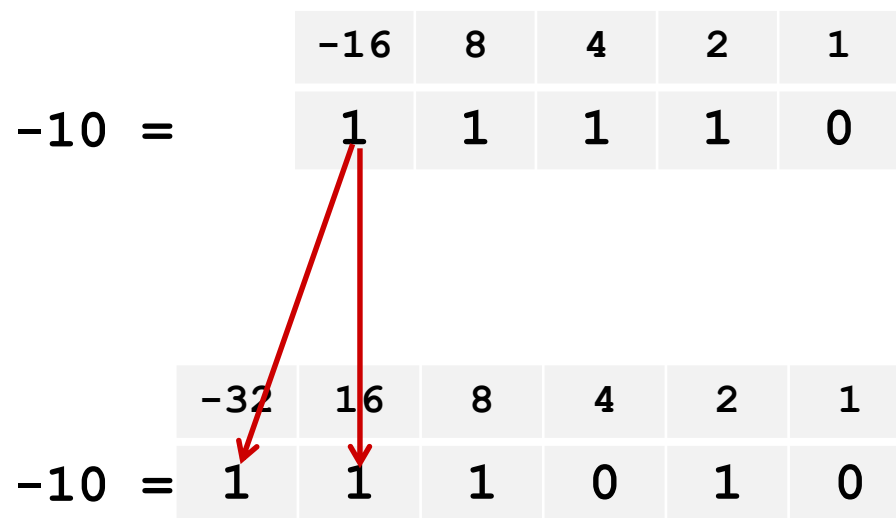


Sign Extension: Simple Example

Positive number



Negative number



Larger Sign Extension Example

```
short int x = 15213;
int      ix = (int) x;
short int y = -15213;
int      iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

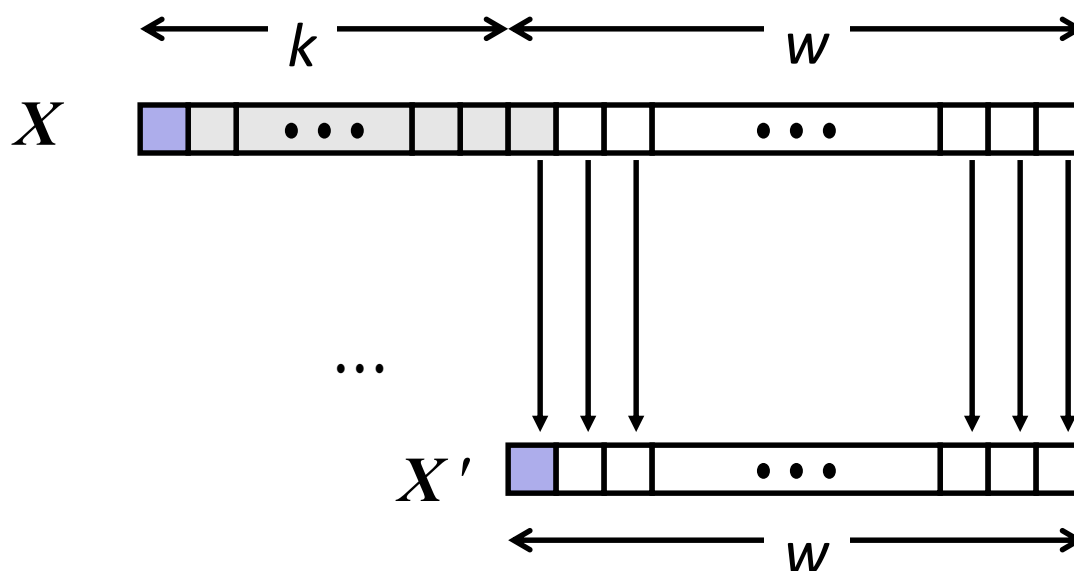
Truncation

■ Task:

- Given $k+w$ -bit signed or unsigned integer X
- Convert it to w -bit integer X' with same value for “small enough” X

■ Rule:

- Drop top k bits:
- $X' = x_{w-1}, x_{w-2}, \dots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

$$2 \bmod 16 = 2$$

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$-6 \bmod 16 = 26U \bmod 16 = 10U = -6$$

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

$$10 \bmod 16 = 10U \bmod 16 = 10U = -6$$

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

$$-10 \bmod 16 = 22U \bmod 16 = 6U = 6$$

Summary:

Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small numbers yields expected behavior

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