

If we want to find intersection of some array,  
it is in less than  $O(n)$

1.  $[2, 3] \rightarrow 1100$

2.  $[0, 1, 2] \rightarrow 0111$

3.  $[1, 3] \rightarrow 1001$

To find intersection, take AND.

11 00

01 11

0100  $\times 2^{\text{nd}}$  fruit is common

→ We cannot use for more fruits as int,  
long long has limit

no. of workers  $\leq 5000$ . They can work for max 30 days.  
Find intersection of working days. two workers  
that are best for job. You need to assign an  
important project to 2 workers. Find max.  
intersection days

```
A int n; cin >> n;
vector<int> masks(n, 0);
for (int i = 0; i < n; ++i) {
    int num_workers;
    cin >> num_workers; int mask = 0;
    for (int j = 0; j < num_workers; ++j)
```

```
    int day; cin >> day;
    masks[i] |= (1 << day);
}
```

P.T.O

```
int n; cin >> n; int max_intersection = 0;
vector<int> masks(n, 0);
for (int i = 0; i < n; i++)
```

```
{
    int num_workers; cin >> num_workers;
    int mask = 0;
    for (int j = 0; j < num_workers; ++j)
```

```
{
    int day; cin >> day;
    mask = (mask | (1 << day)); // to set the bits
}
```

```
masks[i] = mask;
}
```

```
for (int i = 0; i < n; ++i)
```

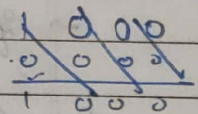
```
{
    for (int j = 0; j < n; ++j)
```

```
{
    int intersection = (masks[i] & masks[j]);
```

```
    int common_days = __builtin_popcount(intersection);
```

```
    max_intersection = max(max_intersection, common_days);
}
```

```
}
```



1. `--builtin-clz(a)` → leading zeros

2. `--builtin-ctz(a)` → trailing zeros

3. XOR 8 → N 8  
 7 → 0 0  
 6 → N+1 7  
 5 → 1 1  
 4 → N 4  
 3 → 0 0  
 2 → N+1 3  
 1 → 1 1

Given X; it behaves like  $(X-1) \% 4 + 1$

$$1^2 \wedge 2^2 \wedge 3^2 \wedge 4^2 = 4$$

$$1^2 \wedge 2^2 \wedge 3^2 = 0$$

$$1^2 \wedge 2^2 = 3$$

$$1^2 \wedge 2^2 = 3$$

4. to get bitcount  $l = (\text{int}) \log_2(\text{num}) + 1;$



$\{4, 3, 4, 4, 4, 5, 5, 3, 3\}$

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pti;

~~Property used~~ .. Property used  $\rightarrow x \wedge x = 0$

set union  $A \cup B$

set intersection  $A \cap B$

set subtraction  $A \setminus B$

set negation  $\sim A$

set bit  $A \mid = 1 \ll \text{bit}$

clear bit  $A \& = \sim (1 \ll \text{bit})$

XOR  $\rightarrow$  1 if both bits are diff.

$x \& (1 \ll i) \rightarrow$  if  $i^{\text{th}}$  bit of  $x$  is set

0 if not set  $\rightarrow$  !0 if set

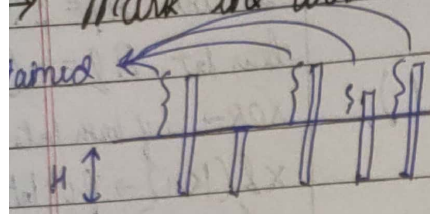
Hash Maps

Insertion  
Deletion  
Searching

O(1)

## Advanced Binary Search

→ Mark the Woodcutter



He needs atleast  $M$  wood  
Find min.  $H$  at which he cuts.

Native  $\Rightarrow$  ~~max~~  $H$  for obtained wood  $\geq M$

```
bool isPossible (int a[], int n, int k, int mid)
{
```

```
    int wood = 0;
    for (auto pts : a) if (pts >= mid) wood += (pts - mid);
    return wood >= k;
}
```

```
int main () {
```

```
    int n, k; cin >> n >> k; int ans = 0;
```

```
    int a[n] -> input int low = 0, high = INT_MAX, mid;
```

```
    while (low <= high)
```

```
    {
```

```
        mid = low + (high - low) / 2;
```

```
        if (isPossible (a, n, k, mid) == true)
```

```
        { ans = mid; low = mid + 1; }
```

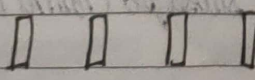
```
        else high = mid - 1; }
```

```
    cout << ans;
```

```
    return;
```

```
}
```

## Aggressive cows



→  $N$  stalls  
at  $x_0, x_1, x_2$  coordinate

$C$  cows to be placed such that min distance between them is as large as possible

Teacher's Signature





```
bool isPossible (int a[], int n, int c, int mid)
{
```

```
    int cows = 1; int last_pos = a[0];
```

```
    for (int i = 1; i < n; i++)
```

```
    { if (a[i] - last_pos >= mid) { cows++; last_pos = a[i]; }
```

```
        if (cows == c) return true; }
```

```
    } return false;
```

```
int main ()
```

```
{
```

```
    int n; cin >> n; int c; cin >> c; int a[n]; cin >> a
```

```
    sort (a, a+n);
```

```
    int low = 0, high = a[n-1], mid, ans = 0;
```

```
    while (low <= high)
```

```
    {
```

```
        mid = low + (high - low) / 2;
```

```
        if (isPossible (a, n, c, mid) == true) ans = mid, low = mid + 1;
```

```
        else
```

```
            high = mid - 1;
```

```
    }
```

```
    cout << ans << endl;
```

```
}
```

Segmented Sieve  $\Rightarrow$  If low and high are till  $10^{16}$  types

low = 6 high = 10

① Generate all primes from 0 to floor (high)  $\rightarrow 2, 3$

② Create array of size (h-l+1) i.e. (10-6+1) = 5

T	T	T	T	T
---	---	---	---	---

③ mark multiples of prime numbers generated

X	T	X	X	X
6	7	8	9	10

$\rightarrow$  7 is prime left in (6 to 10)

## Bit Manipulation

Q There is a Nbr number, and a  $k \& Z$  given such that  
 $Z = (X \ll 0) \oplus (X \ll 1) \oplus (X \ll 2) \dots \oplus (X \ll (k-1))$   
 Find  $X$ .

A  $X = 1010 \quad k = 3$

$$\begin{array}{r} 1010 \\ 1010 \\ 1010 \\ \hline 110110 \end{array}$$

$110110 \rightarrow Z$

$$Z[i] = x[i] \oplus \text{last } (k-1) \text{ bits of } x$$

$$\Rightarrow Z[i] \oplus \text{last } (k-1) \text{ bits of } x = x[i]$$

Q  $n + i = n \oplus i$  Find such #  $0 \leq i \leq n$

~~$\Rightarrow n \& i = 0$~~

A  $n + i = n \oplus i$

$\Rightarrow n \& i = 0$

$\Rightarrow n$  में जहाँ 0 है  $i$  में वहाँ 0/1 हो सकता है।

$x = \text{no. of } 0 \text{ in Binary rep. of } n$   
 Ans =  $2^x$

Q Given an array, Find XOR of all elements of all subarrays possible.

A If  $i^{\text{th}}$  index  $\Rightarrow$  its count in subarrays =  $(i+1) * (N-i)$   
 we can get count of all elements  
 if even count XOR = 0  
 if odd count XOR them once

$[3, 5, 8, 3] \rightarrow$  subarray

3	5	8	3
3, 5	5, 8	8, 3	
3, 5, 8		5, 8, 3	
			3, 5, 8, 3

count of 3 =  $(1) * (4) = 4$   
 $3 = (2) * (3) = 6$   
 $8 = (3) * (2) = 6$   
 $3 = (4) * (1) = 4$

Teacher's Signature



Q. Given array, find sum of  $\wedge$  of all 2 pairs possible.

A. 

2	3	5
---	---	---

 Ans =  $(2^3) + (2^5) + (3^5)$

We will go bit by bit

Suppose  $i^{\text{th}}$  bit of the numbers

- a numbers ~~at~~ have 0 ~~at~~ at the posn
- b numbers have 1 at the posn

Their contribution =  $(a * b)$

Why?  ${}^aC_1 \cdot {}^bC_1$  ways of choosing a 1 & 0 so that XOR is 1.

Only choosing such configuration helps as  $1^1 = 0^0 = 0$ .

∴ for each bit  $\Rightarrow a * b * \text{pow}(2, i)$