NAME: ROHAN NANABHAU PAWAR

STD: CDAC NOIDA PG-DAI

YOUVA

SUBJECT: Engg. Mathematics

INDEX

* Day - 1^{2} *

25th April, 2022
Monday.

* Vectors *

* Definition :-

i) Vector is 1D data array.

ii) Vector can have more than one entries

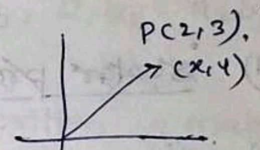
iii) Vector can be verticle (column) or horizontal (row)

iv) Vector have magnitude and direction.

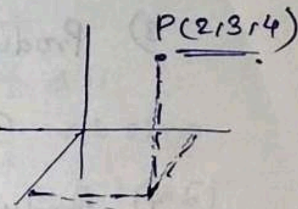
v) Number of enteries defines the dimensionality.

Ex - Row vector, $a = [2, 4, 6, 7]$

Coloumn vector, $b = \begin{bmatrix} 1 \\ 5 \\ 6 \\ 7 \end{bmatrix}$



2-D vector.



3D-Vector

* Vector Algebra :-

$a = [3, 2, 4]$, $b = [1, -2, 4]$

1) Addition: $a+b = [3\ 2\ 4] + [1, -2, 4] = [4, 0, 8]$

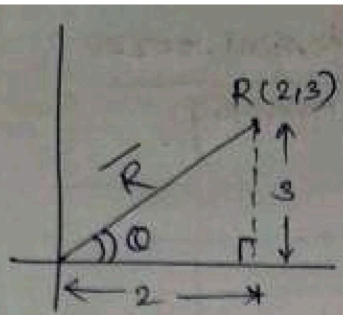
2) Subtraction: $a-b = [3, 2, 4] + [1, -2, 4] = [2, 4, 0]$

3) Multiplication with scalar: $5 \times a = 5 \times [3, 2, 4]$
$= [15, 10, 20]$

4) Basic algebra involve term by term operation.

5) Multiplication with scalar only changes the magnitude
Direction of vector will be the same.

6) Two types of multiplication among vectors:
Scalar & Vector Product



$R = [2, 3]$

Magnitude (R) = $\sqrt{2^2 + 3^2} = \sqrt{13}$

Direction (R) = $\tan\theta = \frac{y}{x}$

$\tan\theta = 3/2$

$\theta = \tan^{-1}(3/2)$

* Vector multiplication :-

1) Scalar product :-

$a = [a_1, a_2, a_3]$, $b = [b_1, b_2, b_3]$

1) 'Scalar product' also known as dot/Inner product.

2) It's result is scalar hence called scalar product

3) Product can be calculated as.

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

4) Product of two vectors can be written as.

Row vector : $a \times b^t$ column vector : $a^t \times b$

* Significance of Scalar product :

1) Also given as : $a \cdot b = |a||b| \cos\theta$

II) for orthogonal vectors, inner product is zero.

III) $a \cdot b = |a||b| \cos\theta = |a||b| \cos 90^\circ = 0$

III) Cosine of angle tells the cosine Similarity

iv) value close to ±1 - very similar or negatively similar.
value close to 0 - no similarity.

v) scalar product tells the projection of one vector on other.

2) Vector Product :-

Also given as $\bar{a} \times \bar{b} = |a| \cdot |b| \cdot \sin\theta$.

$$a = [a_1, a_2, a_3], \ b = [b_1, b_2, b_3]$$

i) Also known as cross product.

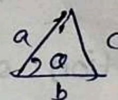
ii) Result in a vector hence called 'vector product'.

iii) Product can be calculated as.

$$a \times b = [a_2 b_3 - a_3 b_2, \ a_3 b_1 - a_1 b_3, \ a_1 b_2 - a_2 b_1]$$

iv) Product of two vectors can be written in matrix form also.

$$a \times b = \begin{bmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$



Area of Δ is $= \frac{1}{2} ab \sin\theta$

$= \frac{1}{2} |\bar{a} \times \bar{b}|$

* Normal and Orthonormal Vector :-

i) Normal vectors are normalised.

ii) Their magnitude will be "unity"

For vector

$$a = [a_1, a_2, a_3, \ldots]$$

$$|a| = \sqrt{a_1^2 + a_2^2 + a_3^2 + \ldots} = 1$$

Normalised Vector

$$\hat{b} = \frac{\bar{b}}{|b|}$$

iii) Can be used to define direction.

iv) Inner product betn two normal vectors will be betn -1 to 1

* Orthonormal Vectors :-

i) Orthonormal sets of vector: Normal and orthogonal

ii) Their inner product will be "Zero"

iii) Orthonormal vectors can be used as basis set of vectors.

iv) The most common example are $\hat{i}, \hat{j}, \hat{k}$.

* Vector Space :-

A vector space is collection of object called vectors, which may be added together and multiplied by numbers (scalars) results in vector in same space

Ex:- Consider R^n where $n=2$ i.e, 2D space.

i) Addition of two vectors : $c = a \pm b$

ii) Multiplication with scalars : $d = k \times a$

iii) Both vector c and d also in R^n space.

iv) R^n is Vector space in 2D

* Questions - 1 *

1) find the normalised form (Unit vector) of $2i+3j+k$

→ Given, say $a = [2i+3j+k]$

Unit vector of a = $\hat{a} = \frac{\bar{a}}{|a|} = \frac{2i+3j+k}{\sqrt{2^2+3^2+1^2}}$

$= \frac{2i+3j+k}{\sqrt{14}}$

2) Will the two vectors whose magnitude is same will be same vector ? justify your answer with example.

→ No. their direction will be different

Ex. $a = 2i+3j$ $\quad |a| = \sqrt{13}$

$b = 3j+2i$ $\quad |b| = \sqrt{13}$.

Magnitude of both vector is same but their direction will be different

$\therefore \quad \bar{a} \neq \bar{b}$

3) show that $\frac{[1,-1]}{\sqrt{2}}$ is a unit vector

→ $a = [1,-1]$ $\quad |a| = \sqrt{1+1} = \sqrt{2}$.

$\therefore$ Given, $\frac{[1,-1]}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$

Hence, It is a unit vector

4) Find the vector parallel to the vector $[1,-2]$ and has magnitude 10 units.

→ Given = $\bar{A} = [1,-2]$, $|\bar{B}| = 10$

$\bar{B}$ can be written as , $\bar{B} = \hat{A} \cdot |B|$

$\hat{A} = \frac{[1,-2]}{\sqrt{5}} = \frac{1}{\sqrt{5}}[1,-2]$

$\therefore \quad \bar{B} = \frac{1}{\sqrt{5}}[1,-2] \cdot 10 = \frac{10}{\sqrt{5}}[1,-2] = \frac{2\times\sqrt{5}\times\sqrt{5}}{\sqrt{5}}[1,-2]$

$\therefore \quad \bar{B} = 2\sqrt{5}\,[1,-2]$

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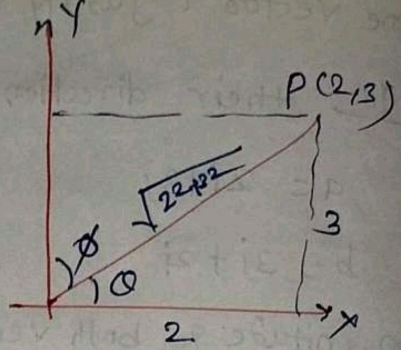
* Direction ratio & Direction Cosines :-

$A = 2i + 3j$

Direction ratio :

Direction ratio of vector A can be given as. $(2:3)$



Direction cosine :-

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypt}} = \dfrac{2}{\sqrt{13}}$, $\cos\phi = \dfrac{3}{\sqrt{13}}$

lly for 3D

if $a = li + mj + nk$

$\therefore \dfrac{l}{\sqrt{l^2+m^2+n^2}}, \dfrac{m}{\sqrt{l^2+m^2+n^2}}, \dfrac{n}{\sqrt{l^2+m^2+n^2}}$.

5) Find the Direction ratios and direction cosines of the vector. $a = i + j - 2k$.

$\rightarrow$ Direction Ratio is $(1:1:-2)$

Direction cosines, $\sqrt{1^2+1^2+2^2} = \sqrt{6}$. ; $\dfrac{1}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}, \dfrac{-2}{\sqrt{6}}$

6) Show the vectors $2i - 3j + 4k$ & $4i + 6j - 8k$ are collinear

$\rightarrow$

Way-1)

$A = 2i - 3j + 4k$

$B = -4i + 6j - 8k$

$\therefore B = -2 \cdot \bar{A}$

Hence their direction is same means they are collinear

Way-2) If two vectors are collinear their cross product is 0.

$\therefore A \times B = 0$ $\begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ -4 & 6 & -8 \end{vmatrix}$

$(24-24) - (16-16) + (12-12)$

$\therefore 0 - 0 + 0$

$= 0$

RHS = LHS Hence proved.

They are collinear.

* Questions - 2 *

1) find x, if for a unit vector a, $(x-a).(x+a)=12$

→ $a=1$

$(x-1).(x+1)=12$

$x^2-1=12$

$x^2=12+1=13$

$x=\sqrt{13}$

2) If $a=[1,-7,7]$ & $b=[3,-2,2]$ find $a \times b$ and $|a \times b|$.

→ $\bar{a}=[1,-7,7]$, $\bar{b}=[3,-2,2]$

$$\bar{a}\times\bar{b} = \begin{bmatrix} i & j & k \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}$$

$i(-14+14) - j(2-21) + k(-2+21)$

$0i-19j+19k$

$\bar{a}\times\bar{b} = [0,-19,19]$

$|\bar{a}\times\bar{b}| = \sqrt{19^2+19^2} = \sqrt{361+361} = \sqrt{722}$

$= \sqrt{2\times361} = 2\sqrt{19}$

3) Find λ and μ, if $[2,6,27]\times[1,\lambda,\mu]=0$

→ Given $[2,6,27]\times[1,\lambda,\mu]=0$

Hence they are collinear.

So, We can write

$\frac{2}{1}=\frac{6}{\lambda}=\frac{27}{\mu}$ $\therefore \frac{2}{1}=\frac{6}{\lambda}$ & $\frac{2}{1}=\frac{27}{\mu}$

$\boxed{\lambda=3}$ $\boxed{\mu=\frac{27}{2}}$

~~4) Show that the vector [2,3,6], [3,6,2]~~ mutually ⊥er.

→ Resultant of two vectors of cross product is ⊥er to the plane containing the parent vector.

$a \times b = c$

$\therefore$ a & b are parent vector of c.

i.e means $c \perp a$ also $c \perp b$

~~a = [3,2,~~

4) Show that the vector [2,3,6], [3,-6,2], [6,2,-3] are mutually ⊥er

→ $a = [2,3,6]$, $b = [3,-6,2]$, $c = [6,2,-3]$

$a \cdot b = [2\ 3\ 6] \cdot [3\ -6\ 2]$

$= 6 - 18 + 12 = 0$

$b \cdot c = [3\ -6\ 2] \cdot [6\ 2\ -3]$

$= 18 - 12 - 6 = 0$

$a \cdot c = [2\ 3\ 6] \cdot [6\ 2\ -3]$

$= 12 + 6 - 18 = 0$

i.e, $a \cdot b = b \cdot c = a \cdot c = 0$.

Hence, they are ⊥er to each other.

5) Find a unit vector ⊥er to each of the vectors (a-b) and (a+b), where $a = [3,2,2]$ & $b = [1,2,-2]$

→ $c = a - b = [3\ 2\ 2] - [1,2,-2] = [2\ 0\ 4]$

$d = a + b = [3,2,2] + [1,2,-2]$

$= [4,4,0]$

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$$c \times d = \begin{vmatrix} i & j & k \\ 2 & 0 & 4 \\ 4 & 4 & 0 \end{vmatrix} \Rightarrow -16i + 16j + 8k$$

$$\widehat{c \times d} = \frac{\overline{c \times d}}{|c \times d|} = \frac{-16i+16j+8k}{\sqrt{16^2+16^2+8^2}} = \frac{8(-2i+2j+k)}{\sqrt{256+256+64}}$$

$$= \frac{8(-2i+2j+k)}{\sqrt{576}} = \frac{\cancel{8}\,1(-2i+2j+k)}{\cancel{24}\,3}$$

$$= \frac{-2i+2j+k}{3}$$

6). Find the area of triangle with vertices.

$A = [1,1,2]$, $B = [2,3,5]$, $C = [1,5,5]$

$\rightarrow$

$$\overline{AB} = [2-1]i + (3-1)j + (5-2)k = [1,2,3]$$

$$\overline{BC} = [1-2]i + (5-3)j + (5-5)k = [-1,2,0]$$

$$\overline{AB} \times \overline{BC} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = (-6)i - (-3)j \quad (4)k$$

$$[-6,-3,4]$$

Magnitude

$$|D| = \sqrt{6^2+3^2+4^2} = \sqrt{36+9+16} = \sqrt{61}$$

$$\cancel{\hat{D} = \frac{D}{|D|} = \frac{\sqrt{61}}{\sqrt{}}}$$

$$\text{Area of triangle} = \frac{1}{2}[\overline{AB} \times \overline{BC}]$$

$$= \frac{1}{2}\sqrt{61}$$

$$= \frac{\sqrt{61}}{2}$$

* Normalise Vector :

$a = [2, 3, 4]$

$\therefore \bar{a} = 2i + 3j + 4K$

2, 3 & 4 are called component of vector.

i) When you try to project vector along x-axis. You will get the x component.

ii) When you want to project vector along y-axis. You will get the y component

iii) when you project vector along z-axis. You will get the ~~y~~ z component.

How can we project $\bar{a}$ vector on axis?

Dot product is used to get the projection of one vector on another vector.

Projection along x-axis $\rightarrow \langle \bar{a} \cdot i \rangle = (2i + 3j + 4k) \cdot i = 2(i \cdot i) = 2$

Projection along y-axis $\rightarrow \langle \bar{a} \cdot j \rangle = (2i + 3j + 4k) \cdot j = 3(j \cdot j) = 3$

Projection along z-axis $\rightarrow \langle \bar{a} \cdot k \rangle = (2i + 3j + 4k) \cdot k = 4(k \cdot k) = 4$

If you want to know the direction of vector along a particular direction, then used dot product.

for eg. $a = [2, 3, 4]$ $\quad b = [3, 4, 0]$

$a \cdot b = (2 \times 3) + (3 \times 4) + (4 \times 0)$

$= 6 + 12 + 0 = \underline{\underline{18}}$

If we take

$\langle \bar{a}, \bar{b} \rangle \Rightarrow$ It will give length of vector a along the direction of b.

Unit vector is also known as Normalise Vector.

Unit vector can be calculated as.

$$\hat{b} = \frac{\bar{b}}{|b|}$$

Unit vector is used to quantify the direction.

Unit Vector - $|\hat{a}| = 1$

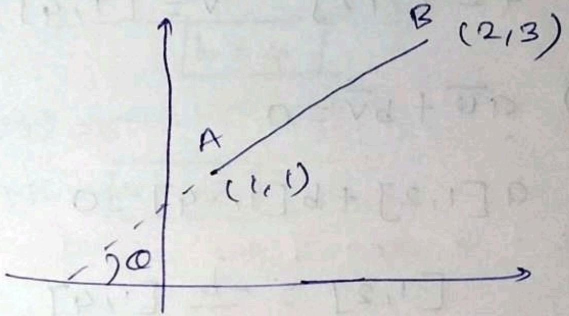
i.e, magnitude of Normalise/Unit vector is 1.

Ex. Find Unit vector of a, where $a = [2,3,4]$

$\rightarrow$ $$\hat{a} = \frac{\bar{a}}{|a|} = \frac{[2,3,4]}{\sqrt{4+9+16}} = \frac{1}{\sqrt{29}}[2,3,4]$$

Ex Find vector $\overline{AB}$.

$A = i+j \qquad B = 2i+3j$



$\rightarrow$

$\overline{AB} = (2-1)i + (3-1)j$

$= i + 2j$

$|AB| = \sqrt{1^2+2^2} = \sqrt{5}$

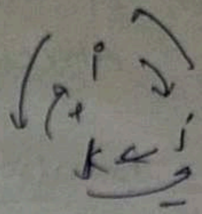
$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}(2/1) = \tan^{-1}(2)$

Q. What is the cross product.

$i \times j = k \qquad j \times i = -k$

$j \times k = i \qquad k \times j = -i$

$k \times i = j \qquad i \times k = -j$



* Day-3 *

27th April, 2022

Wednesday

* Linear combination :

* Two vector u and v are said to be linearly independent when

$$au + bv = 0$$

only when $a = b = 0$ otherwise they are said to be dependent

Ex- $\overline{u} = [1, 2] \quad \overline{v} = [1, 4] \quad , \overline{w} = [2, 4]$

i) $a\overline{u} + b\overline{v} = 0$

$a[1, 2] + b[1, -4] = 0$

$[1, 2] = \frac{-b}{a} [1, 4]$

Here, we can't find any value of $\frac{-b}{a}$, by putting that value we get LHS = RHS.

Hence u and v are independent vectors.

2) $a\overline{u}+b\overline{w}=0$

$a[1,2]+b[2,4]=0$

$[1,2] = \frac{-b}{a}[2,4] = \frac{-1}{2}[2,4] = -[1,2]$

This means $\boxed{a=2}$ & $\boxed{b=1}$.

But in this case, u and w are dependent vectors. Because vector w is 2 times of vector a

*) In n-D space, we can have maximum n number of linearly independent vectors.

Ex - $U=[1,2]$, $V=[2,3]$, $W=[5,6]$

$a=?$ $b=?$

$a\overline{u}+b\overline{v}=\overline{w}$

$a[1,2]+b[2,3]=[5,6]$

$\therefore$ $a+2b=5$ $\times 2$ $\quad$ $2a+4b=10$

$2a+3b=6$ $\quad$ $-2a-3b=-6$

$\boxed{b=4}$

put $b=4$, $a+2(4)=5$

$a+8=5$ $\boxed{a=-3}$

*) For. eg In 2D space, we can have utmost 2 linearly independent vector.

If third vector is present, then it is dependent on other two vectors. As shown in above example.

* Basis Vector :-

1) Basis Vector, as name suggested are the base or building blocks to represents all the possible vectors in n-D Vector space.

2) In n-D space, any set of n linearly independent vectors can be a set of basis vectors.

3) There can be infinite number of sets of basis Vector but in each set, number of basis vector is same which is equal to the dimensionality of the space.

4) A special set of linearly independent vector which are orthogonal also have special importance

5) Because of orthogonality, the computation become easy and cosine similarity them will be zero.

Ex.

$a\bar{u} + b\bar{v} = 0$

In this eqn u and v are Basis Vector

* Linear Transformation :-

i) Let X and Y be any two vector space then if for any vector in X - say x, if we can assign vector y in vector space Y, then y is called mapping of x.

ii) Generally such mapping is denoted by F.

iii) The y is called image of x under transformation F.

iv) The transformation F is called linear mapping if it preserve addition and scaling i.e.

Addition $\rightarrow$ $F(x_1+x_2) = F(x_1) + F(x_2)$

scaling $\rightarrow$ $F(cx) = c\,F(x)$

* Some Important Notes *

1) $\bar{a} \cdot \bar{b} = |a|\,|b| \cos\theta$

$\therefore$ θ is the angle between $\bar{a}$ & $\bar{b}$.

$\therefore$ $\cos 0^\circ, \cos 180^\circ = 1, -1$

$\cos 90^\circ, 270 = 0$

2) $|\bar{a} \times \bar{b}| = |a| \cdot |b| \cdot \sin\theta$

$\sin\theta$ is 0 - only when θ is 0° or 180°.

Direction can get from Right hand thumb Rule.

3) Vector Norms

i) It means generalised length or distance.

ii) There are three types of Norms.

a) l_1 norm or absolute norm.

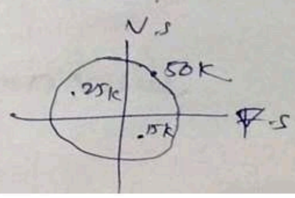
$$\|a_1\| = |a_1| + |a_2| + |a_3| + \dots$$

Ex - ① $a = [2, -3, 4, 1]$

$$l_1 = |a_1| + |a_2| + |a_3| + |a_4|$$

$$= |2| + |-3| + |4| + |1| = 2+3+4+1 = \underline{10}$$

② In an office, if want to find no. of employees whose salary is less than 50K, then we used l_1 norm.



b) l_2 norm or euclidean norm

$$\|a\|_2 = \sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots}$$

Ex. If we want to find the (Distance) location of employee's home from office then we have to use l_2 norm.

l_2 = True length / magnitude of vector

c) l_∞ Norm.

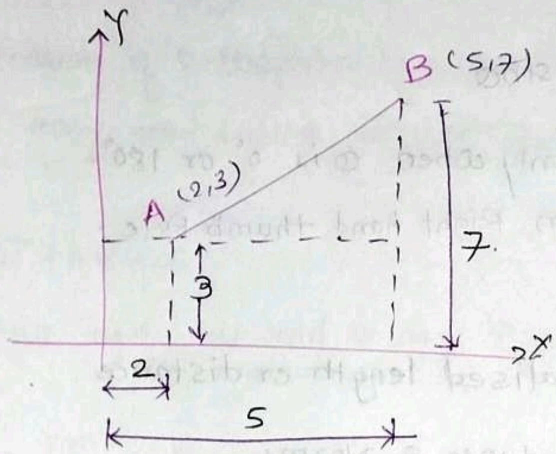
$$\|a\|_\infty = \max_j |a_j|$$

It used to find the maximum value from the no. of components.

Q. Find the distance between two points?

or.

What is the length of vector AB?



Distance bet A & B $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-2)^2 + (7-3)^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= \underline{\underline{5}}$

4) Orthonormal :-

i) Set of unit vector which is perpendicular to each other.

ii) Dot product / Inner product is used to check the perpendicularity of vectors.

iii) Cross product is used to check collinearity of vectors.

i.e, means.

If dot product/ Inner product of two vector ~~are~~ is zero, then they are perpendicular.

If cross product of two vector is zero then they are collinear.

5) If 2 or 3 vectors are orthogonal, means they are linearly independent.

But vice a versa not possible

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* Day-3 * (Afternoon) 27th April, 2022

* MATRIX *

* Definition :-

i) A 2D rectangular array of number.

ii) Numbers in matrix called entries.

iii) Horizontal orientation called row

iv) Vertical orientation called column

v) Any entries defined by row and column value

* Matrix Representation :-

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

No of rows $= m$

No of column $= n$

Rectangular matrix i.e, $m \neq n$

Otherwise square matrix i.e, $m = n$

* Matrix Algebra :-

* Addition or Subtraction :-

When two matrix has same size i.e, same rows and same columns corresponding term will get added.

Ex - A and B can added but not A and C or B and C

as.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 2 & 3 \\ 1 & -4 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 11 & -12 \\ -8 & 9 \end{bmatrix}$

2×3 2×3 2×2

1) The sum of A and B is

$$A + B = \begin{bmatrix} 3 & 4 & 6 \\ 3 & -3 & 7 \end{bmatrix}$$

2) The Subtraction of ~~A from~~

$$A - B = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 5 & 11 \end{bmatrix}$$

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* Matrix multiplication / Division by Scalar :-

i) Each element of matrix get multiplied by same scalar.

ii) No change in the number of row and column in the resulting matrix.

Ex. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 9 \end{bmatrix}$ is multiplied by scalar $C=5$

$$CA = \begin{bmatrix} 5 & 10 & 15 \\ 10 & 5 & 45 \end{bmatrix}$$

Therefore, a_{ik} become ca_{ik}

Matrix Multiplication :-

i) Product of two matrices $C = AB$ for matrix A and B is possible when number of column (n) in A is equal to number of row (r) in i.e, $r = n$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}_{2 \times 3}$$

ii) Let a_{ij} and b_{ij} be the elements of matrix A and B respectively, then elements C is c_{ij} will be given as

$$c_{ij} = \sum_{1=1}^{n} a_{i1} b_{1j} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

iii) The size of resultant matrix c is number of rows of A x number columns of B.

iv) Matrix multiplication isn't commutative.

* Matrix Transpose :-

Transpose of matrix A with size m×n, represented as A^T change the size to n×m and make the row as column and vice versa.

Ex. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

* Conjugate of Matrix :-

Complex conjugate of matrix A, represented as A^*, replace all the elements by their complex conjugate.

conjugate of real number is the same but conjugate of complex number, their is sign change.

for. Ex

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $A^* = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 1+2i & 3+4i \\ 5+6i & 7+8i \end{bmatrix}$ $B^* = \begin{bmatrix} 1-2i & 3-4i \\ 5-6i & 7-8i \end{bmatrix}$

* Hadamard Product :-

Hadamard product of two ~~vector~~/ matrix is just like the similar to the matrix Addition.

It's a element by element product.

It's named after french mathematician Jacques Hadamard.

for eq. $A = \begin{bmatrix} 3 & 5 & 7 \\ 4 & 9 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 6 & 3 \\ 0 & 2 & 9 \end{bmatrix}$

$$A \otimes B = \begin{bmatrix} 3\times1 & 5\times6 & 7\times3 \\ 4\times0 & 9\times2 & 8\times9 \end{bmatrix} = \begin{bmatrix} 3 & 30 & 21 \\ 0 & 18 & 72 \end{bmatrix}$$

It is denoted by $\otimes$.

Hadamard product is used in image compression techniques such as JPEG. It is also known as "Schur Product" after German mathematician, "Issai Schur"

Hadmard product is used in LSTM (Long short term memory) cells & Recurrent Neural Network (RNN)

* Some Matrix Properties / Rule :-

1) $(A^T)^T = A$

2) $(A+B)^T = A^T + B^T$

3) $(CA)^T = CA^T$

4) $(AB)^T = B^T A^T$

5) $(AB)^* = A^* B^*$

6) $(AB)^\dagger = B^\dagger A^\dagger$

It is combination & Transpose of conjugate.

* special matrix

1) Symmetric matrix $\rightarrow A^T = A$

2) Anti or skew symmetric matrix $\rightarrow A^T = -A$

3) Orthogonal Matrix $\rightarrow A * A^T = I \Rightarrow A^T = A^{-1}$

4) Unitary Matrix $\rightarrow A * A^\dagger = I \Rightarrow A^\dagger = A^{-1}$

* Special ~~structure matrix~~ :-

1) Upper Diagonal matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

2) Lower Diagonal matrix

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}$$

3) Diagonal matrix

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

* Problem Set *

1) Recall the rotation matrix you have encountered. Lets there is two consecutive rotation by angle α and β takes place. show that the two consecutive rotation is equivalent to one single rotation by angle $(\alpha+\beta)$

$\Longrightarrow$

We know that

Rotational matrix can be given as

$$R = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$$

$$\sin(A+B) = \sin A \cancel{\sin B} \cos B + \cos A \cancel{\cos B} \sin B$$

$$\sin(A-B) = \sin A \cdot \cancel{\sin} \cos B - \cos A \cdot \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

For α angle, rotational matrix given as.

$$R_1(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

for β angle

$$R_2(\beta) = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

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$$R_1(\alpha) \cdot R_2(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta & \cos\alpha \cdot \sin\beta + \sin\alpha \cdot \cos\beta \\ -\sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta & -\sin\alpha \cdot \sin\beta + \cos\alpha \cdot \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta & \sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta \\ -(\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta) & \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

For angle $(\alpha+\beta)$, Rotational matrix can be given as

$$R_3(\alpha+\beta) = \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

$$\therefore \quad R(\alpha+\beta) = R(\alpha) \cdot R(\beta)$$

$$LHS = RHS$$

2) Does the order of rotation matters in final output?

→ The order of rotation does not matter in this perticular example [i.e problem 1 of rotational matrix]

3) For what value of k, the vectors $[2,4,1,k]^T$ and $[k,2,4,5]^T$ are orthogonal?

$\rightarrow$

$$A = [2,4,1,k]^T, \quad B = [k,2,4,5]^T$$

$$A \cdot B = \begin{bmatrix} 2 \\ 4 \\ 1 \\ k \end{bmatrix} \cdot \begin{bmatrix} k \\ 2 \\ 4 \\ 5 \end{bmatrix} = 0$$

$$\therefore \quad 2k + 8 + 4 + 5k = 0$$

$$7k + 12 = 0$$

$$\boxed{k = -12/7}$$

4) For what value of k, the vectors $[8,4,1,k]^T$ and $[2,-4,k,k]^T$ are orthogonal?

$\rightarrow$

$$A = [8,4,1,k]^T \quad B = [2,-4,k,k]^T$$

$$A \cdot B = 0$$

$$\begin{bmatrix} 8 \\ 4 \\ 1 \\ k \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -4 \\ k \\ k \end{bmatrix} = 0 \quad \Rightarrow \quad 16 - 16 + k + k^2 = 0$$

$$\therefore \quad k^2 + k = 0$$

$$k(k+1) = 0$$

$$\therefore \boxed{k = 0} \ \& \ \boxed{k = -1}$$

5) Write the vector $v = [1,-2,5]$ as a linear combination of the vectors $u_1 = [1,1,1]$, $u_2 = [1,2,3]$, $u_3 = [2,-1,1]$

$\rightarrow$

$$a u_1 + b u_2 + c u_3 = v$$

$$a \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} \quad -①$$

From 1, we get 3 eqⁿ.

$a+b+2c=1$ —— 2

$a+2b-c=-2$ —— 3

$a+3b+c=5$ —— 4.

From 2 & 3

$a+b+2c=1$

$-\ a+2b-c=-2$

$-b+3c=3$ —— 5

from eqⁿ 3 & 4

$\cancel{a}+2b-c=-2$

$-\ \cancel{a}+3b+c=5$

$-b-2c=-7$

$b+2c=7$ —— 6

From 5 & 6

$-\cancel{b}+3c=3$

$\cancel{b}+2c=7$

$5c=10$

$\boxed{c=2}$

put $c=2$ in 6

$b+2(2)=7$

$b+4=7$

$\boxed{b=3}$

Put $b=3$ & $c=2$ in 2

$a+3+2(2)=1$

$a+3+4=1$

$a=1-7$

$\boxed{a=-6}$

6 Supercom Ltd produces two computer models PC1086 and PC1186. The matrix A show the cost per computer (in thousand of dollars) and B the production figures for the year 2010 (in multiple of 10,000 units). Find a matrix C that shows the shareholder the cost per quarter (in millions of dollars) for raw material, labour and miscellaneous.

$$A = \begin{array}{c} \begin{array}{cc} PC1086 & PC1186 \end{array} \\ \begin{bmatrix} 1.2 & 1.6 \\ 0.3 & 0.4 \\ 0.5 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \text{Raw component} \\ \text{Labour} \\ \text{Miscellaneous.} \end{array} \qquad B = \begin{array}{c} \text{Quarter} \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 3 & 8 & 6 & 9 \\ 6 & 2 & 4 & 3 \end{bmatrix} \end{array} \begin{array}{l} PC1086 \\ PC1186 \end{array}$$

$\Rightarrow$ For solving this we want to find cross product.

$$A \times B = \begin{bmatrix} 1.2 & 1.6 \\ 0.3 & 0.4 \\ 0.5 & 0.6 \end{bmatrix} \begin{bmatrix} 3 & 8 & 6 & 9 \\ 6 & 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1.2\times3)+(1.6\times6) & (1.2\times8)+(1.6\times2) & (1.2\times6)+(1.6\times4) & (1.2\times9)+(1.6\times3) \\ (0.3\times3)+(0.4\times6) & (0.3\times8)+(0.4\times2) & (0.3\times6)+(0.4\times4) & (0.3\times9)+(0.4\times3) \\ (0.5\times3)+(0.6\times6) & (0.5\times8)+(0.6\times2) & (0.5\times6)+(0.6\times4) & (0.5\times9)+(0.6\times3) \end{bmatrix}$$

$$= \begin{bmatrix} 3.6+9.6 & 9.6+3.2 & 7.2+6.4 & 10.8+4.8 \\ 0.9+2.4 & 2.4+0.8 & 2.4+1.6 & 2.7+1.2 \\ 1.5+3.6 & 4+1.2 & 3+2.4 & 4.5+1.8 \end{bmatrix}$$

$$= \begin{array}{c} \text{Quarter} \\ \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 13.2 & 12.8 & 13.6 & 15.6 \\ 3.3 & 3.2 & 4 & 3.9 \\ 5.1 & 5.2 & 5.4 & 6.3 \end{bmatrix} \end{array} \begin{array}{l} \text{Raw component} \\ \text{Labour} \\ \text{Miscellaneous.} \end{array}$$

7) Suppose that in weight-watching program; a person of 185 lb burns 350 cal/hr in walking (3 mph), 500 in bicycling (13 mph), and 950 in jogging (5.5 mph).

Bill, weighing 185 lb, plans to exercise according to the matrix shown. Verify calculations.
(W = Walking, B = Bicycling, J = jogging)

$$
\begin{array}{c c}
 & \begin{array}{ccc} W & B & J \end{array} \\
\begin{array}{c} \text{mon} \\ \text{wed} \\ \text{fri} \\ \text{sat} \end{array} & \begin{bmatrix} 1.0 & 0 & 0.5 \\ 1.0 & 1.0 & 0.5 \\ 1.0 & 0 & 0.5 \\ 2.0 & 1.5 & 1.0 \end{bmatrix}
\end{array}
\begin{bmatrix} 350 \\ 500 \\ 950 \end{bmatrix} =
\begin{array}{cc}
\begin{bmatrix} 825 \\ 1325 \\ 1000 \\ 2400 \end{bmatrix} & \begin{array}{c} \text{mon} \\ \text{wed} \\ \text{fri} \\ \text{sat} \end{array}
\end{array}
$$

→ we have to prove, LHS = RHS.

$$
\begin{bmatrix} 1 & 0 & 0.5 \\ 1 & 1.0 & 0.5 \\ 1 & 0 & 0.5 \\ 2 & 1.5 & 1 \end{bmatrix}
\begin{bmatrix} 350 \\ 500 \\ 950 \end{bmatrix}
$$

$$
\begin{bmatrix} 350 \times 1 + 0 + 0.5 \times 950 \\ 350 \times 1 + 1 \times 500 + 0.5 \times 950 \\ 350 \times 1 + 0 + 0.5 \times 950 \\ 2 \times 350 + 1.5 \times 500 + 1 \times 950 \end{bmatrix} = \begin{bmatrix} 825 \\ 1325 \\ 1000 \\ 2400 \end{bmatrix}
$$

Hence, LHS = RHS.

* Trace of Matrix :-

1) Sum of the diagonal elements of matrix.

$$Tr(A) = \sum_{i=1}^{n} a_{ij}$$

2) The trace of a sum of two matrices is equal to the sum of their traces.

$$Tr(A+B) = Tr(A) + Tr(B)$$

3) Trace of transpose of matrix is same as the trace of matrix

$$Tr(A^T) = Tr(A)$$

4) Trace of product of two matrix is same as the trace of product in reverse order

$$Tr(AB) = Tr(BA)$$

5) for eg. Trace of Matrix is defined as the sum of all the diagonal elements.

It is denoted by Tr

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 8 \\ 2 & 3 & 1 \end{bmatrix}$$

$$Tr(A) = 2+7+1 = 10$$

* Determinants :-

I) A scalar number attached to matrix

II) Defined for square matrix say A and represented as det (A) or |A|

III) The determinant of matrix A and its transpose A^T has same value.

IV) The determinant of inverse matrix is inverse of determinant of matrix.

i.e.

$$\det(A^{-1}) = 1/\det(A)$$

v) Determinant of product of two matrix is equal to the product of determinant of two matrix i.e,

$$\det(A \times B) = \det(A) \times \det(B)$$

vi) If matrix A has a size $n \times n$ and c is a constant, then

$$\det(cA) = c^n \det(A)$$

For. eg :-

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\det(A) = 1(-1+9) - 3(-3+6) + 2(-9+2)$$

$$= \cancel{8+3}\ 8 - 9 - 14 = \underline{\underline{-15}}.$$

* Invertible Matrix :-

i) A square matrix A is said to be invertible or non-singular if there exists a matrix B such that

$$AB = BA = I \Rightarrow A = B^{-1}$$

ii) Only those matrix can be inverted whose determinants are non-zero.

iii) The inverse of product of two or more matrix will be product of inverse of each matrix in reverse order i.e.

$$(A_1 A_2 A_3 \dots A_n)^{-1} = A_n^{-1} A_{n-1}^{-1} \dots A_2^{-1} A_1^{-1}$$

* Inverse of Matrix :-

1) Only square matrix which is full rank can be inverted.

ii) Several methods available to calculate inverse of a matrix

iii) Method of adjoint, Gauss-Jordan, partioning method.

iv) In method of adjoint.

Find all the cofactors of matrix and create adjoint matrix. ~~as shown i.e,~~

v) Inverse of matrix will be given as

$$\boxed{inv(A) = A^{-1} = \frac{adj(A)}{det(A)}}$$

For eg. Find invers of matrix A

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

$$det(A) = 1(4) - 1(0-1) + 0 = 5$$

Finding the cofactors.

$$C_{11} = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix} = 4 - 0 = 4 \qquad C_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = -1$$

$$C_{13} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} = 0 - 2 = -2$$

$C_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$= 2$

$C_{22} = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = 2$

$C_{23} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = -1$

$C_{31} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = 1$

$C_{32} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$

$C_{33} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = 2.$

<u>cofactor matrix</u>

$$C = \begin{bmatrix} 4 & -2 & 1 \\ 1 & 2 & -1 \\ -2 & 1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -2 & 1 \\ 1 & 2 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

$* \; AA^{-1} = I \rightarrow$ Show that

$$A^{-1} = \frac{1}{\det(A)} \times \text{adj of } \cancel{\text{cofactor}} \text{ matrix} \times \text{Matrix } A.$$

$$= \frac{1}{5}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 4 & -2 & 1 \\ 1 & 2 & -1 \\ -2 & 1 & 2 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} (1\times4)+(1\times1) & (-2\times1)+(1\times2) & (1\times1)+(1\times-1) \\ (2\times1)+(1\times-2) & (2\times2)+(1\times1) & (2\times-1)+(1\times2) \\ (1\times4)+(2\times-2) & (-2\times1)+(2\times1) & (1\times1)+(2\times2) \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

* Day - 5 *

29th April, 2022

Friday.

* Cramer's Rule :-

1) consider a system of equation represented as

$$AX = B$$

2) calculate various determinants.

$D = \det(A)$, $D_1 = \det(A_1)$, $D_2 = \det(A_2)$ --- etc.

3) $\det(A_i)$ is determinant of matrix created by replacing ith coloumn with vector B.

4) If $\det(A) = 0$ then we don't know whether or not the system has a solution.

5) If $\det(A) \neq 0$, then we can have unique solution.

6) The sol^n will be.

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad x_3 = \frac{D_3}{D}, \ldots$$

For eg: ① In 2D.

find x & y. If two eq^{ns} are

$$x + 2y = 5$$
$$x - y = 2$$

→ In matrix form, $AX = B$

$$\begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$\det A = -1 - 2 = \underline{\underline{-3}}$

$$D_1(A) = \begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix} = -5-4 = -9$$

$$D_2(A) = \begin{vmatrix} 1 & 5 \\ 1 & 2 \end{vmatrix} = 2-5 = -3$$

$$\therefore x = \frac{D_1(A)}{D}, \quad y = \frac{D_2(A)}{D} = \frac{-3}{-3} = 1$$

$$= \frac{-9}{-3} = 3 \qquad \therefore \boxed{x=3} \ \& \ \boxed{y=1}$$

② In 3D: Find x, y, z. If eq^{n}s are

$$x+y-2z=0$$
$$2x+y-3z=0$$
$$3x+y+2z=6$$

$$\Rightarrow A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 3 & 1 & 2 \end{bmatrix} \Rightarrow \text{Det}(A) = 1(2+3) - 1(4+9) - 2(2-3) = -6$$

$AX = B$

$$\therefore \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$$

$$D_1 = \begin{vmatrix} 0 & 1 & -2 \\ 0 & 1 & -3 \\ 6 & 1 & 2 \end{vmatrix} = 6(-3+2) = -6$$

$$D_2 = \begin{vmatrix} 1 & 0 & -2 \\ 2 & 0 & -3 \\ 3 & 6 & 2 \end{vmatrix} = -6(-3+4) = -6$$

$$D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 6 \end{vmatrix} = 6(1-2) = -6$$

$\therefore\ x = \frac{D_1}{D} = \frac{-6}{-6} = 1$ | $y = \frac{D_2}{D} = \frac{-6}{-6} = 1$ | $z = \frac{D_3}{D} = \frac{-6}{6} = 1$

$\boxed{x=1}$ $\boxed{y=1}$ $\boxed{z=1}$

* Linear Equations :-

1) Degenerate linear Equation :-

If all the coefficient are zero.

i.e. $0x_1 + 0x_2 + 0x_3 + \cdots = b$.

a) If $b \neq 0$, then eq^n has no sol^n

b) If $b = 0$, then vector $u = (k_1, k_2, \cdots)$ in K^n is a sol^n.

2) Non-Degenerate linear Equation :-

In eq^n, if one or more coeff. of x are not zero, then it's called non-degenerate linear eq^n.

e.g. $0x_1 + 0x_2 + 3x_3 + 5x_4 = 5$

A) For One Variable :-

For linear eq^n $Ax = b$, the possibilities are

i) If $a \neq 0$ then $x = b/a$ is a unique sol^n

ii) If $a = 0$ and $b \neq 0$ then $ax = b$ has no sol^n.

iii) If $a = 0$ & $b = 0$ then every scalar K is the solution of $ax = b$

B) Two Variables :-

consider eqn in two variables.

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then have unique soln

ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then have no soln

iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then have infinite soln.

* System of linear Eqn :-

A set of Linear equation can be written in matrix form. consider

$$x_1 + x_2 + 4x_3 + 3x_4 = 5$$
$$2x_1 + 3x_2 + x_3 + 2x_4 = 1$$
$$x_1 + 2x_2 - 5x_3 + 4x_4 = 3$$

Two matrix can be defined as for this set

$$M = \left[\begin{array}{cccc|c} 1 & 1 & 4 & 3 & 5 \\ 2 & 3 & 1 & -2 & 1 \\ 1 & 2 & -5 & 4 & 3 \end{array}\right]$$

$$A = \begin{bmatrix} 1 & 1 & 4 & 3 \\ 2 & 3 & 1 & -2 \\ 1 & 2 & -5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$$

Augumented Matrix :-

i) The solⁿ to the system of linear equation can be identified using rank of ~~aq~~ augmented matrix and coefficient matrix.

ii) If the rank of $\underline{\underline{m}}$ is greater than rank of $\underline{A}$, there is no solⁿ to the system.

iii) If the rank of $\underline{\underline{m}}$ is the same as $\underline{A}$, then there will be atleast one solⁿ.

iv) If rank of both the matrix is equal to number of unknowns i.e. variable then system will have unique solⁿ.

* Day-6 * 2nd May, 2022 Monday

Rank of Matrix :-

The Rank of Matrix refers to number of linearly independent rows or column in the matrix.

Rank of matrix denoted as. $\underline{\rho(A)}$.

A matrix is said to be rank 'zero' when all of its elements become zero.

The rank of matrix is the dimensions of the vector space obtained by its column.

"The maximum number of linearly independent vector in matrix is equal to the no. of non-zero rows in row echelon matrix.

Therefore, to find the rank of matrix, we simply transform the matrix to its row echelon form and the number of non-zero rows.

i.e. We should try to make no of rows to 0. by operation.

If not possible then matrix is full rank.

& If we make the rows to 0. then rank will

No of rows - 0 value rows.

① What do you mean by rank of matrix?

→ Rank of matrix is the maximum no. of its linearly independent rows or columns.

② Can the rank of matrix exceed the no. of rows or columns?

→ The Rank of matrix can't exceed the number of its rows or columns.

③ How do you find the rank of matrix?

→ To find the rank of matrix, transform matrix into its echelon form. Then find the rank by the number of non-zero rows.

Ex - Find the Rank of Matrix A. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

→ $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - 3R_1$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -6 & -4 \end{bmatrix}$

3X3 matrix have Rank of matrix is 2

2X2 and 2x3 also have rank of matrix 2

$R_3 \rightarrow R_3 - 2R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Number of Non-zero rows = 2

$\therefore$ Rank of matrix A = 2

Notes:-

1) If Rank of M = Rank of A and no. of variable is equal to the rank.
Then there will be unique solⁿ.

2) If Rank of M = Rank of A and no. of variable and rank are not same (i.e less than no. of variable)
Then there will be infinite solⁿ

Examples of Augmented Matrix :-

① $4x - 6y = -11$

$-3x + 8y = 10$

$\rightarrow A = \begin{bmatrix} 4 & -6 \\ -3 & 8 \end{bmatrix} \Rightarrow$ Rank of A = 2

$M = \begin{bmatrix} 4 & -6 & -11 \\ -3 & 8 & 10 \end{bmatrix} \Rightarrow$ Rank of M = 2.

Hence there will be unique solⁿ.

Because rank = No of variables

Ex-2) $2x + y = 3$

$x + y/2 = 3/2$

→ $A = \begin{bmatrix} 2 & 1 \\ 1 & 1/2 \end{bmatrix} \Rightarrow \text{Rank}(A) = 2$

$M = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1/2 & 3/2 \end{bmatrix}$

$R_2 \to 2R_2 - R_1$

$M = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Rank}(M) = 1$

$\therefore$ Rank (M) < Rank (A).

$\therefore$ There is infinite soln to this.

Ex-3) $2x + y = 3$

$x + y/2 = 5$

→ $A = \begin{bmatrix} 2 & 1 \\ 1 & 1/2 \end{bmatrix} \Rightarrow R_2 \to 2R_2 - R_1 \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$

Rank (A) = 1

$M = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1/2 & 5 \end{bmatrix} \Rightarrow \text{Rank}(M) = 2$

$\because$ Rank (M) $\neq$ Rank (A)

$\therefore$ There is no solutions.

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* Eigen Vaule & Eigen Vectors *

Eigenvalues are associated with Eigenvectors in linear algebra.

Both the terms are used in the analysis of linear transformation.

* EigenValues :-

① Eigen-values are the special set of scalars associated with the system of linear equation.

② It is mostly used in matrix equation.

③ 'Eigen' is a german word that means proper or characteristics.

④ Therefore, the term eigenvalue can be termed as characteristic value, characteristic root, proper values or latent roots as well.

⑤ In simple words, the eigenvalues is a scalar that is used to transform the eigenvector. The basic eqn is.

$$AX = \lambda X \qquad X = \text{Eigenvector}$$
$$\Rightarrow (A - \lambda I)X = 0 \qquad \lambda = \text{Eigenvalue.}$$

The number or scalar value "λ" is an eigenvalue of A.

* Properties of Eigenvalues :-

1) Eigenvectors with distinct Eigenvalues are linearly independent.

11) Singular matrices have zero Eigenvalues.

iii) If A is square matrix, the $\lambda = 0$ isn't an eigenvalue of A.

iv) For scalar multiple of matrix :-

If A is a square matrix and λ is an eigenvalue of A. Then, $a\lambda$ is an eigenvalue of aA.

v) For matrix powers :-

If A is a square matrix and λ is an eigenvalue of A and $n \geq 0$ is an integer, the λ^n is an eigenvalue of A^n.

vi) For polynomials of matrix :-

If A is square matrix, λ is an eigenvalue of A and $p(x)$ is a polynomial in variable x, then $p(\lambda)$ is the eigenvalue of matrix $p(A)$.

vii) Inverse of matrix :-

If A is a square matrix, λ is an eigenvalue of A, then λ^{-1} is an eigenvalue of A^{-1}.

viii) Transpose matrix :-

If A is a square matrix, λ is an eigenvalue of A. then λ is an eigenvalue of A^t.

Examples :-

1) $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ Find the eigen value of matrix A.

→ For calculating this we used.

$(A - \lambda I)X = 0$ ⇒ This eq^n also called as characteristic eq^n

ie $A - \lambda I = 0$

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix} = 0$$

$\therefore (-5-\lambda)(-2-\lambda) - 4 = 0$

$\therefore 10 + 5\lambda + 2\lambda + \lambda^2 - 4 = 0$

$\therefore \lambda^2 + 7\lambda + 6 = 0$

$\therefore \lambda^2 + \lambda + 6\lambda + 6 = 0$

$\therefore \lambda(\lambda+1) + 6(\lambda+1) = 0$

$\therefore \lambda+1 = 0 \quad \lambda+6 = 0$

$\boxed{\lambda = -1} \quad \boxed{\lambda = -6}$

Note :- For $n \times n$ matrix, ~~quadratic~~ eqn. there are n eigenvalues.

2) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ find the eigen values.

$\rightarrow$

$A - \lambda I = 0$

$$\therefore \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = 0$$

$= -(2+\lambda)\left[(1-\lambda)(-\lambda)-12\right] - 2\left[-2\lambda-6\right] - 3\left[-4+(1-\lambda)\right]$

$= (2+\lambda)\left[(1-\lambda)\lambda+12\right] + 2(2\lambda+6) - 3(-4+1-\lambda)$

$= (2+\lambda)\left[\lambda-\lambda^2+12\right] + 4\lambda + 12 + 12 - 3 + 3\lambda$

$= 2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 4\lambda + 24 - 3 + 3\lambda$

$= \underline{-\lambda^3 - \lambda^2 + 21\lambda + 45}$ — ①

<u>solve eqⁿ ①</u>

try $\lambda = -3$ put in ①

$\Rightarrow -\cancel{27+9+63} + 27 + 9 - 63 + 45 = 0$

$0 = 0 \Rightarrow LHS = RHS.$

~~∴ One of eigen value is~~ $\boxed{\lambda = -3}$

$$\cancel{\begin{array}{c|cccc} -3 & -1 & -1 & 21 & 45 \\ & & -3 & & \\ \hline & 1 & & & \end{array}}$$

$\Rightarrow \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$

$$\begin{array}{c|cccc} -3 & 1 & 1 & -21 & -45 \\ & & -3 & 6 & 45 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$(\lambda+3)(\lambda^2 - 2\lambda - 15) = 0$

$\boxed{\lambda = -3}$ λ_1

$\rightarrow \lambda^2 - 2\lambda - 15 = 0$

$\lambda^2 + 3\lambda - 5\lambda - 15$

$\lambda(\lambda+3) - 5(\lambda+3) = 0 \Rightarrow (\lambda+3) = 0 \quad \lambda - 5 = 0$

$\lambda_2 \rightarrow \boxed{\lambda = -3} \quad \boxed{\lambda = 5} \leftarrow \lambda_3$

* Day-7 *

4th may, 2022
Wednesday.

* Recalling Linear transformation with Example :-

If there are two parallel vectors then their unit vector will be the same.

But if their magnitude is same then this property is not work because direction of vectors is different. i.e, they are not parallel.

Example :- $\bar{A} = 2i+3j \quad \bar{B} = 6i+9j$

$$\therefore \hat{A} = \frac{2i+3j}{\sqrt{4+9}} = \frac{2i+3j}{\sqrt{13}}$$

$$\hat{B} = \frac{6i+9j}{\sqrt{36+81}} = \frac{3(2i+3j)}{\sqrt{117}} = \frac{3(2i+3j)}{\sqrt{9\times 13}} = \frac{3}{3}\frac{(2i+3j)}{\sqrt{13}}$$

$$= \frac{2i+3j}{\sqrt{13}}.$$

$$\therefore \boxed{\hat{A} = \hat{B}}$$ Hence proved.

* Some matrix Properties :-

① All the matrix, satisfy their characteristic equation.

Instead of λ we put matrix A.

. eqn we from last example

$$-\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$$

$$-A^3 - A^2 + 21A + 45I = 0$$

This property is generally used to estimate matrix inverse

→ proof →

$$(-A^3 - A^2 + 21A + 45I) A^{-1} = 0 \times A^{-1}$$

$$\therefore -A^2 - A + 21I + 45A^{-1} = 0.$$

$$\therefore A^{-1} = \frac{1}{45}\left[A^2 + A - 21I\right] \quad (1)$$

$$\therefore A^2 = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4+3 & -4+2+6 & 6-12 \\ -4+2+6 & 4+1+12 & -6-6 \\ 2-4 & -2-2 & 3+12 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{bmatrix}$$

$\therefore$ Eqn (1) becomes

$$\frac{1}{45}\left\{\begin{bmatrix} 11 & 4 & -6 \\ 4 & 17 & -12 \\ -2 & -4 & 15 \end{bmatrix} + \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}\right\}$$

$$A^{-1} = \frac{1}{45}\begin{bmatrix} -12 & 6 & -9 \\ 6 & -3 & -18 \\ -3 & -6 & -6 \end{bmatrix}$$

check the A^{-1} is correct or not.

$\therefore$ $A \cdot A^{-1} = I$

$$\frac{1}{45}\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} -12 & 6 & -9 \\ 6 & -3 & -18 \\ -3 & -6 & -6 \end{bmatrix}$$

$$\therefore \frac{1}{45}\begin{bmatrix} 24+12+9 & -12-6+18 & 18-36+18 \\ -24+6+18 & 12-3+36 & -18-18+36 \\ 12-12 & -6+6 & 9+36 \end{bmatrix}$$

$$\therefore \frac{1}{45}\begin{bmatrix} 45 & 0 & 0 \\ 0 & 45 & 0 \\ 0 & 0 & 45 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

② Trace of matrix (A) = sum (λ) (i.e. $\sum \lambda_i$)

i.e where λ = Eigen values.

③ Determinant of matrix A is equal to the product of Eigen values.

i.e., $\det(A) = \prod \lambda_i$

where $\prod \Rightarrow$ used to show product.

$\lambda_i \Rightarrow$ Eigen values

(4) If det(A) = 0, then one of the eigen value is zero.

(5) For n×n matrix, Eigen value can be complex value.

∴ The complex eigen values comes in pair.

i.e if 2−3i is eigen value then 2+3i is also eigen value of that matrix

∴ The number of complex values will be even.

Eg. ① if there are 3 eigen values. then therefore out of 3 1 is real & 2 is imaginary.

② for 5×5 matrix,

	complex value	real value
	0	5
	2	3
	4	1

* Eigen Vector :-

Let suppose that A is n×n sq. matrix, and if X be a non-zero vector, "then product of matrix A and vector X is defined as "the product of scalar quantity λ and vector X".

i.e, $AX = \lambda X$ → It is also called Eigenvector Eq^n.

wher A = Matrix

λ = Scalar quantity. i.e, Eigenvalue

X = Vector i.e Eigen vector

* How to find Eigen vector ?

→ **Step 1)** → Find the eigen values of matrix A using the eqn $\det(A - \lambda I) = 0$

Step 2) → put values of λ in eqn $\underline{\underline{(A - \lambda I)X = 0}}$ ~~$AX = \lambda X$~~

~~we just solve AX we will get λX~~

~~$\boxed{LHS = RHS}$ - if vector is found correctly~~

Step 3) Above calculation will give eigen vector X associated with particular eigen value.

Step 4) Repeat step for further eigen values.

To check - Whether the found eigen vector is correct or not. just do multiply eigen vector with matrix A it will give result is $\underline{\lambda X}$

ie $\boxed{AX = \lambda X}$ ← This eqn will satisfy

* Types of Eigen Vectors :-

① **Left Eigen Vector** :- The left eigen vector is represented in the form of row vector which satisfy the following condn.

$$\underline{A X_L = \lambda X_L}$$

A = matrix λ = one of Eigenvalue

X_L is row vector of matrix i.e. $[x_1, x_2, \ldots x_n]$

② Right Eigen Vector :-

The right eigenvector is represented in the form of column vector which satisfy the following cond?

$AX_R = \lambda X_R$, where A = matrix λ = Eigenva

X_R = coloum vector of matrix i.e $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

* Eigen Vector Applications :-

1) Eigen vectors are used in physics in simple mode of oscillation.

2) In mathematics, eigenvector decomposition is widely used in order to solve the linear equation of first order, in ranking matrices, in different calculus etc

3) Eigen vector concept is widely used in quantum mechanics.

4) It is applicable in almost all the branches of engineering

Examples :-

1) $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$ find the eigen vector for $\lambda = -1, -6$

Ⓐ For $\lambda = -1$

→ $(A - \lambda I) X = 0$

$$\left\{ \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - (-1) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\left\{\begin{bmatrix}-5 & 2\\ 2 & -2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right\}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

$$\begin{bmatrix}-4 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0 \quad -①$$

From ① we get

$$-4x_1+2x_2=0 \quad -②$$

$$2x_1-x_2=0 \quad -③$$

∴ from ③, we get

$$x_2 = 2x_1.$$

∴ The eigen vector, X can be given as

$$X=\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}x_1\\ 2x_1\end{bmatrix}$$

if put $x_1 = 1$

$$\therefore \boxed{X=\begin{bmatrix}1\\ 2\end{bmatrix}}$$ → This is eigen vector corresponding to -1 value

Unit eigen vector corresponding to -1 value :-

$$\hat{X}=\frac{1}{\sqrt{5}}\begin{bmatrix}1\\ 2\end{bmatrix}$$

(B) Eigen vector $\lambda=-6$, $(A-\lambda I)X=0$

$$\left\{\begin{bmatrix}-5 & 2\\ 2 & -2\end{bmatrix}+\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}\right\}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=0$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \quad -(5)$$

From (5), we get.

$$x_1 + 2x_2 = 0 \quad -(6)$$

$$2x_1 + 4x_2 = 0. \quad -(7)$$

$\therefore$ From (6).

$$\underline{x_1 = -2x_2}.$$

$\therefore$ The eigenvector $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ if $x_2 = 1$.

Unit vector will be $\hat{X} = \dfrac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$.

(2) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ Find Eigenvector for $\lambda = 5, -3, -3$.

→ (A) For $\underline{\lambda = 5}$.

$$\left\{\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & 2 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}\right\}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0. \quad (1)$$

From (1),

$$-7x_1 + 2x_2 - 3x_3 = 0 \quad -(2)$$

$$2x_1 - 4x_2 - 6x_3 = 0 \quad -(3)$$

$$-x_1 - 2x_2 - 5x_3 = 0 \quad -(4)$$

Multiply eqn (4) × 2, & subtract from (5).

$$\cancel{2x_1} - 4x_2 - 6x_3 = 0$$
$$\cancel{-2x_1} - 4x_2 - 10x_3 = 0$$
$$-8x_2 - 16x_3 = 0$$

$$\boxed{x_2 = -2x_3}$$

Multiply eqn (1) × 2, subtract eqn (1) from (2).

$$-14x_1 + \cancel{4x_2} - 6x_3 = 0$$
$$2x_1 - \cancel{4x_2} - 6x_3 = 0$$
$$12x_1 - 12x_3 = 0$$

$$\therefore \boxed{x_1 = -x_3}$$

$\therefore$ The Eigen vector, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$ if $x_3 = 1$.

$$\therefore \underline{X_1} = -1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \text{ or } X = \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$$

B)

For $\underline{\lambda = -3}$

$$\left\{ \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad -(5)$$

from (5),

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad -(5)$$

From eq^n (5),

$$x_1 + 2x_2 - 3x_3 = 0 \quad -(6)$$

$$2x_1 + 4x_2 - 6x_3 = 0 \quad -(7)$$

$$-x_1 - 2x_2 + 3x_3 = 0 \quad -(8).$$

i.e (7) & (8)

As we can say that all these eq^{n}s are dependent on eq^n (6).

eq^n - (7) $\Rightarrow$ $2 \times$ eq^n (6).

eq^n - 8 $\Rightarrow$ $-1 \times$ eq^n (6).

In this case, we can do

$$x_1 = -2x_2 + 3x_3 \quad -(9) \text{ from } (6).$$

But we have repetative root i.e -3.

$\therefore$ for first -3, Assume $x_2 = 0$ & $x_3 = 1$

from eq^n (9), $x_1 = -2(0) + 3(1) = \underline{\underline{3}}$.

$\therefore$ Eigen vector $\underline{X_2} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$

& for second -3, Assume $x_2 = 1$ & $x_3 = 0$

$\therefore$ From eq^n (9). $x_1 = -2(1) + 3(0) = \underline{\underline{-2}}$

$\therefore$ Eigen vector is $\underline{X_3} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

③ $A = \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix}$ Find eigenvector & Also check whether it is Correct or Not, for $\underline{\underline{\lambda = 3}}$.

→

$$\left\{ \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 2 & -2 \\ 2 & 2 & 0 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad — ①$$

from ①

∴ $3x_1 + 2x_2 - 2x_3 = 0 \quad - ②$

$2x_1 + 2x_2 + 0x_3 = 0 \quad - ③$

$-2x_1 + 0x_2 + 4x_3 = 0 \quad - ④$

From ② & ③

$$3x_1 + \cancel{2x_2} - 2x_3 = 0$$
$$\underline{-\,2x_1 \pm \cancel{2x_2} + 0x_3 = 0}$$
$$x_1 - 2x_3 = 0$$

$$\boxed{x_1 = 2x_3}$$

from ③ & ④.

$$2\cancel{x_1} + 2x_2 + 0x_3 = 0$$
$$-2\cancel{x_1} + 0x_2 + 4x_3 = 0$$

$$2x_2 + 4x_3 = 0$$

$$x_2 = \frac{-4}{2} x_3$$

$$\boxed{x_2 = -2x_3}$$

$\therefore$ The eigen vector is $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ For $x_3 = 1$

Check Eigen Vector is correct or not

$$AX = \lambda X$$

$$\therefore \quad \lambda X = 3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \text{RHS}.$$

$$\text{LHS} = AX$$

$$= \begin{bmatrix} 6 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} (6\times2) + (2\times2) + (-2\times1) \\ (2\times2) + (5\times2) \\ (-2\times2) + (7\times1) \end{bmatrix}$$

$$= \begin{bmatrix} 12+4-2 \\ 4+10 \\ -4+7 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 3 \end{bmatrix} = 3\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \text{RHS}$$

$\therefore$ Hence, found ~~Eige~~ Eigen Vector is correct.

* Day-8 * 5th may, 2022

Thursday

* Some properties Regarding Eigen Values & Eigenvectors :-

1) Eigenvector corresponding to different eigen value are linearly independent

But, for some eigenvalue corresponding eigenvector may or may not be linearly dependent and independent

2) Eigen values of matrix A and Eigen values of Transpose of matrix A are same.

$A = \lambda_1, \lambda_2 \ldots \quad A^T = \lambda_1, \lambda_2, \lambda_3 \ldots \lambda_n$.

3) Eigen values of matrix A and Eigenvalues of inverse of matrix A i. A^{-1} are not same.

i.. $A^{-1} = \lambda_1^{-1}, \lambda_2^{-1}, \lambda_3^{-1} \ldots \lambda_n^{-1}$

4) Eigenvalues of Harmesian matrix are all real.

there is no chance of complex eigenvalues for Harmesian matrix.

Harmesian matrix is that $\boxed{H^{\dagger} = H.}$

For example

$$H = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

Is H is Hermesian matrix? check the property ④.

→ Checking H is Hermesian matrix or not.

$$H = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}, \quad H^* = \begin{bmatrix} 0 & i & 0 \\ -i & 0 & i \\ 0 & -i & 0 \end{bmatrix}, \quad (H^*)^T = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

Eigenvalues

$$H - \lambda I = 0$$

$$\begin{bmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} -\lambda & i & 0 \\ i & -\lambda & -i \\ 0 & i & -\lambda \end{bmatrix}$$

$$\therefore \quad -\lambda(\lambda^2 + i^2) + i(-\lambda i) = 0$$

$$\therefore \quad -\lambda^3 - \lambda i^2 + \lambda i^2 = 0$$

$$\therefore \quad -\lambda^3 - 2\lambda i^2 = 0$$

$$\therefore \quad -\lambda^3 + 2\lambda = 0 \quad \text{—} \quad (i^2 = 1)$$

$$\lambda^3 - 2\lambda = 0.$$

$$\lambda(\lambda^2 - 2) = 0$$

$$\therefore \lambda = 0 \qquad \lambda^2 = 2$$

$$\boxed{\lambda = 0} \quad \boxed{\lambda = \pm\sqrt{2}}$$

Hence, proved All eigen values of Harmesian matrix are real.

⑤ Eigen values of symmetric matrix are real.

$\boxed{\text{Symmetric matrix is a special case of Harmesian matrix.}}$

⑥ Eigen vectors of Harmesian matrix are orthogonal set of vectors

Example.

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$ Find eigenvalues & Eigenvectors?

$$\therefore A - \lambda I = \begin{bmatrix} 1-\lambda & -1 & -1 \\ -1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{bmatrix}$$

$$\therefore (1-\lambda)\left[(1-\lambda)(1-\lambda) - 1^2\right] + 1\left[(-1)(1-\lambda) - 1^2\right]$$
$$-1\left[1^2 + (1-\lambda)\right]$$

$$\therefore (1-\lambda)(\cancel{1} - \lambda - \lambda + \lambda^2 - \cancel{1}) + 1\left[-1 + \lambda - 1\right]$$
$$-1\left[1 + 1 - \lambda\right]$$

$$\therefore -2\lambda + \lambda^2 + 2\lambda^2 - \lambda^3 - 2 + \lambda - 2 + \lambda = 0$$

$$\therefore \underline{-\lambda^3 + 3\lambda^2 - 4 = 0}$$

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For $\lambda = 2$.

$\therefore \lambda^3 - 3\lambda^2 + 4 = 0 \Rightarrow 2^3 - 3(2)^2 + 4 = 0$

$8 - 12 + 4 = 0 \Rightarrow \underline{0 = 0}$.

$\therefore \boxed{\lambda = 2}$ is first ~~value~~ eigen value.

Solve cubic eq^n

$$\begin{array}{c|cccc} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$\therefore$ eq^n we get, $\lambda^2 - \lambda - 2 = 0$

$\therefore \lambda^2 + \lambda - 2\lambda - 2 = 0$

$\therefore \lambda(\lambda+1) - 2(\lambda+1) = 0$

$\therefore \lambda + 1 = 0 \quad \lambda - 2 = 0$.

$\therefore \boxed{\lambda = -1}$ & $\boxed{\lambda = 2}$

$\therefore$ Three Eigen values are $\boxed{\lambda = 1, 2, 2}$

Now find Eigen Vectors.

As the Given matrix is symmetric, the eigen vectors are orthogonal to each other.

for $\lambda = -1$

$A - \lambda I$

$$\therefore \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} - (1)$$

From eq^n ①, we get

$$2x_1 - x_2 - x_3 = 0 \quad -②$$
$$-x_1 + 2x_2 - x_3 = 0 \quad -③$$
$$-x_1 - x_2 + 2x_3 = 0 \quad -④.$$

From ③ & ④:

$$\begin{array}{r} -\cancel{x_1} + 2x_2 - x_3 = 0 \\ -\cancel{x_1} \underset{+}{-} x_2 \underset{-}{+} 2x_3 = 0 \\ \hline 3x_2 - 3x_3 = 0 \end{array}$$

$$\therefore \boxed{x_2 = x_3}$$

multiply eq^n ④ by ② & ~~subtract~~ Add. ~~from~~ with ③

$$\begin{array}{r} -x_1 + \cancel{2x_2} - x_3 = 0 \\ -2x_1 - \cancel{2x_2} + 4x_3 = 0 \\ \hline -3x_1 + 3x_3 = 0 \end{array}$$

$$\boxed{x_1 = x_3}$$

$\therefore$ Eigen vectors for $\lambda = 1$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for $x_3 = 1$

For $\lambda = 2$

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad -⑤$$

From ⑤, we get only one eqn for all 3 rows,
i.e., $-x_1 - x_2 - x_3 = 0$

$$\therefore \quad x_3 = -(x_1 + x_2) \quad — ⑥$$

In this case, Assume value of x_1 is zero
i.e. $x_1 = 0$

then from ⑥, we get, $\boxed{x_3 = -x_2}$

$\therefore$ Eigen vectors are, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ x_2 \\ -x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ for $x_2 = 1$

As the vectors are orthogonal to each other, we can't solve by similar method for another $\lambda = 2$,
We have solve this by different method, as below.

We have 3 Eigen vectors which are

$V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad V_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

This get from $\lambda = 1$ $\quad$ this from $\lambda = 2$ $\quad$ find ?

As per the property of orthogonal vector, i.e dot product of two vector is zero, then solve

$$\langle V_1 \cdot V_3 \rangle = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow x_1 + x_2 + x_3 = 0 \quad — ⑦$$

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$$\langle v_2 \cdot v_3 \rangle = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow 0x_1 + x_2 - x_3 = 0$$

$$\therefore \boxed{x_2 = x_3}$$

Put $x_2 = x_3$ in eqn ⑦

$$\therefore x_1 + x_3 + x_3 = 0$$

$$\therefore x_1 + 2x_3 = 0 \Rightarrow \boxed{x_1 = -2x_3}$$

$\therefore$ The eigenvector is $V_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2x_3 \\ x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

For $x_3 = 1$

$\therefore$ All three eigen vectors are

For $\lambda = 1$ $V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ for $\lambda = 2$ $V_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, for $\lambda = 2$ $V_3 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

---

⑧ Eigen ~~Values~~ Vectors of a matrix are fixed.

⑨ For Skew Hermesian matrix, All the eigen values will be imaginary i.e. complex values.

Property of Skew Harmesian matrix is

$$\Rightarrow \boxed{H^{+} = -H}$$

* Corelation :-

Correlation refers to process for establishing between the two variables.

We can get general idea about whether or not two variables are related, is by plot them on " scatter plot "

Correlation in statistics :-

Method of correlation summarize the relationship between two variables in a single number called "correlation coefficient"

The correlation coeff is usually represented using the symbol r, and it ranges from -1 to 1

A correlation coeff quite close to 0, but either +ve or -ve, implies little or no relationship between the two variables.

A correlation coeff close to plus 1 means a positive relationship between two variables, with increase in one variable leeds to increase the other variable as well.

A correlation coeff close to -1 indicates a negative relationship between two variable, with an increase in one of the variables being associated with a decrease in other variable i.e, if one variable increases other decreases.

A correlation coeff can be produced for ordinal, interval, or ratio level variables., but has little meaning for variables which are measured on a scale which is no more than nominal.

For ordinal scales, the correlational coeff can be calcu-
-lated by using "Spearman's rho"

For interval or ratio, level scales, the most commonly used correlation coeff is "Pearson's r".

What does Correlation Measures?

In stat, correlation studies and measures the direction and extent of relationship among variables.

For ex - There exists a correlation between two variables x and y, which means the value of one variable is found to change in direction, the value of the other variable is found to change either in the same direction (i.e. positive change) or in opposite direction (i.e. -ve change).

Furthermore, if the correlation exists, it is linear i.e. we can represent the relative movement of the two variable by drawing the straight line or graph paper.

Note :- When r (i.e correlation coeff) is close to 0 this means that there is a little relationship between the variables.

* Scatter Diagram :-

A scatter diagram is a diagram that shows the values of two variables X and Y, along with the way in which these two variables relate to each other.

Later, when regression model is used, one variable is defined as independent variable and other as dependent variable.

In regression, the independent variable X is considered to have some effect or influence on the dependent variable Y.

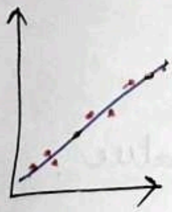
Types of Correlation :-

The scatter plot explains the correlation between the two attributes or variables. It represents how closely the two variables are connected. There can be three such situation to see the relation between the two variables.

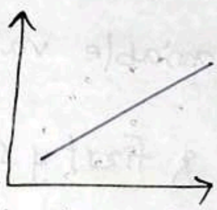
1) **Positive Correlation :-** When the values of two variables move in the same direction so that an increase/ decrease in the value of variable is followed by an increase/ decrease in the value of other variable

2) **Negative Correlation :-** When the values of two variables move in opposite direction so that an increase/ decrease in the value of one variable is followed by decrease/ incre. in the value of other variable.

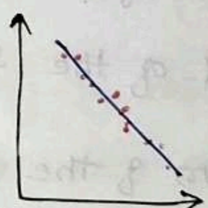
3) **No correlation :-** When there is no linear dependence or no relation between the two variables.



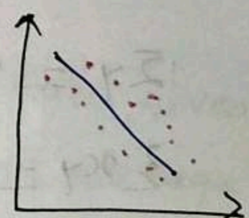
Strong +ve correlation.



Weak + correlation

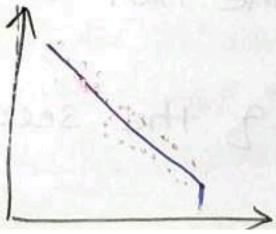


Strong -ve Correlation



Weak -ve correlation.

Moderate -ve Correlation.



No corelation

* Types of Correlation coeff formula :-

Correlation coeff shows the measure of correlation. To compare two datasets, we use the correlation formulas.

1) Pearson Correlation coeff formula :-

The most common formula is the Pearson Correlation coeff formula used for linear dependency between the data sets.

The value of coeff is lies betn -1 to +1. When coeff comes down to zero, then the data is not related. While, if we get value +1, the data is positively correlated and -1 ~~have~~ , data is negatively correlated.

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

Where, n = Quantity of Information

Σx = Total of the first variable value.

Σy = Total of the Second variable value.

Σxy = Sum of the product of first & second value.

Σx^2 = Sum of the squares of the first value.

Σy^2 = Sum of the squares of the second value

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2) Linear Correlation Coeff. formula :-

The formula for the linear correlation coeff is given by.

$$r_{xy} = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\sqrt{\left[n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

3) Sample Correlation coeff. formula :-

The formula is given by

$$r_{xy} = \frac{S_{xy}}{S_x S_y}$$

Where S_x & S_y → Are sample standard deviation.

S_{xy} → is the sample covariance.

4) Population Correlation coeff formula :-

The population correlation coeff uses σx and σy as the population standard deviations and σxy as the population covariance.

$$r_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}.$$

* Correlation Example :-

Year of Education and Age of Entry to labour force gives the number of years of formal education (X) and the age of entry into the labour force (Y), for 12 males from the Regina Labour Force Survey. Both variables are measured in years, a ratio level of measurement and the highest level of these males are likely to have completed their formal education.

Respondent Number.	Years Edu. x	Age of Entry into Labour Force Y	xy	x^2	y^2
1	10	16	160	100	256
2	12	17	204	144	289
3	15	18	270	225	324
4	8	15	120	64	225
5	20	18	360	400	324
6	17	22	374	289	484
7	12	19	228	144	361
8	15	22	330	225	484
9	12	18	216	144	324
10	10	15	150	100	225
11	8	18	144	64	324
12	10	16	160	100	256
	$\Sigma x = 149$	$\Sigma y = 214$	$\Sigma xy = 2716$	$\Sigma x^2 = 1999$	$\Sigma y^2 = 3876$

$n = 12$

→ $\Sigma x = 149$ $\quad \Sigma xy = 2716$ $\quad \Sigma x^2 = 1999$

$\Sigma y = 214$ $\quad \Sigma y^2 = 3876$

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{(12 \times 2716) - [149 \times 214]}{\sqrt{[(12 \times 1999) - (149)^2][(12 \times 3876) - (214)^2]}}$$

$$= \frac{32592 - 31886}{\sqrt{[23988 - 22201][46512 - 45796]}}$$

$$= \frac{706}{\sqrt{1787 \times 716}}$$

$$= \frac{706}{1131.146}$$

$$= \underline{\underline{0.624}}$$

* Covariance :-

Covariance is a measure of the relationship between two random variables and to what extent, they change together.

In other words, It defines the changes between the two variable. such that change in one variable is equal to change in another variable

This is a property of a function of maintaining its form when the variables are linearly transformed.

* Types :-

1) +ve covariance :-

If the covariance between two variable is +ve, that means, both the variable move in the same direction. Both variable show similar behaviour.

2) -Ve Covariance :-

If the covariance betn two variable is -Ve, that means both the variable move in opposite direction.

* **Population Covariance formula :-**

$$\text{cov}(x,y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{N}$$

* **Sample Covariance :-**

$$\text{cov}(x,y) = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

Where, x_i = data value of x

y_i = data value of y

$\bar{x}$ = mean of x.

$\bar{y}$ = mean of y

N = number of data values.

i) If $\text{cov}(x,y) > 0$, then we can say that the covariance for any two variables is +ve and both the variables move in the same direction.

ii) If $\text{cov}(x,y) < 0$, then we can say that the covariance for any two variables is -Ve and both the variables move in the opposite direction.

iii) If $\text{cov}(x,y) = 0$, then we can say that there is no relation between two variables.

Correlation coeff. formula :-

$$\text{Correlation}, \rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

Where,

$\rho(X,Y)$ = Correlation betn the variable X and Y.

$Cov(X,Y)$ = Covariance betn the variable X and Y.

σ_x = Standard deviation of the X-variable

σ_y = Standard deviation of the Y-variable

Covariance	Correlation
It is measure to show the extent to which given two random variables change with respect to each other.	1) It is a measure used to describe how strongly the given variables are related to each other.
It is a measure of correlation	2) It is defined as the scaled form of covariance.
The value of covariance lies betn $-\infty$ to ∞	3) The value of correlation lies betn -1 to 1
It indicates the direction of the linear relationship betn the given two variables.	4) It measures the direction and strength of the linear relationship betn the given two variables.

* Covariance Example :-

Calculate the coeff of covariance for following data.

X	2	8	18	20	28	30
Y	5	12	18	23	45	50

→ No. of observations = 6

mean of x = 17.67

mean of Y = 25.5

$$cov(X,Y) = \frac{1}{n}\left[(x_i - x)(y_i - y)\right]$$

$$\therefore = \frac{1}{6}\Big[(2-17.67)(5-25.5) + (8-17.67)(12-25.5) + (18-17.67)(18-25.5) + (20-17.67)(23-25.5) + (28-17.67)(45-25.5) + (30-17.67)(50-25.5)\Big]$$

$$= \underline{157.83}$$

Covariance describes the extent to which one variable is related to another whereas correlation states how strongly the given two random variables are related to each other.

* Day - 9 *

6th may, 2022
Friday

* Principal Component Analysis :- (PCA)

PCA is an unsupervised learning algorithm that is used for dimensionality reduction in machine learning. It is a statistical process that converts the observations of correlated features into a set of linearly uncorrelated features with the help of orthogonal transformation. This new transformed features are called "Principal Component"

PCA generally finds the lower dimensional surface to project the high-dimensional data.

Some real world applications of PCA are image processing, movie recommendation system, optimizing the power allocation in various communication channels.

PCA algorithm is based on some mathematical concept such as:

1) Variance and covariance
11) Eigenvalues and Eigenvectors.

Some common terms used in PCA algorithm:

1) Dimensionality :- It is the number of columns present in the dataset.

2) Correlation

3) Orthogonal :- It defines that variables aren't correlated to each other, and hence the correlation between the pair of variables is zero.

4) Eigenvectors.

5) Covariance matrix :- A matrix containing the covariance betn the pair of variables is called the "Covariance matrix"

Principal Components in PCA :-

I) The principal component must be the linear combination of the original features.

II) These components are orthogonal i.e., the correlation betn a pair of variable is zero.

Steps for PCA algorithm :-

1) Getting the datasheet :-

firstly, we need to take the input dataset and divide it into two subparts X and Y, where X is the training set, and Y is the validation set.

2) Representing data into a structure :-

such as we will represent the 2-D matrix of independent variable X. Here each row corresponds to the data items, and the column corresponds to the features. The number of columns is the dimension of ~~datasheet~~ dataset.

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3) Standardizing the data :-

Such as in particular column, the features with high variance are more important compared to the features with lower variance.

If the importance of feature is independent of the variance of the feature, then we will divide each data item in a column with the standard deviation of the column. ~~Here~~ we will name the matrix as Z.

4) Calculating the covariance of Z :-

To calculate the covariance of Z, we will take the matrix Z and will transpose it. After, transpose. we will multiply it by Z. The output matrix will be the covariance matrix of Z.

5) Calculating the Eigen vector & Eigen values.

6) Sorting the Eigen vectors. → ~~in decreasing order~~

Eigen values sort in decreasing order to largest to smallest.

Simultaneously, sort the eigen vector accordingly in matrix P of eigenvalues. The resultant matrix named as P*.

7) Calculating Principal components :

To do this, we will multiply the P* matrix to Z. In the resultant matrix Z*, each observation is the linear combination of original features. Each column of Z* matrix is independent of each other.

8) Remove less or unimportant Features from the new dataset :

It means, we will only keep the relevant or important Features in the new dataset, and unimportant Features will be removed out.

* **Applications :-**

PCA is mainly used as the dimensionality reduction technique in various AI applications such as computer vision, image comparision etc.

It can also be used for finding patterns if data has high dimensions.

Some fields where PCA is used are finance, data mining, Psychology etc.

PCA Example in Python :-

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from numpy.linalg import eig

df = pd.read_excel ('file path')
x = np.array (df['x'])
Y = np.array (df['Y'])

fig = plt.figure()
ax = fig.add_axes [0, 0, 1, 1]
```

```
ax.scatter(x,y)
c = np.corrcoef(x,y)
v, w = eig(c)

# percent of variance taken care by eigenvalue

var_percent = np.cumsum(v) / sum(v) * 100

# Estimating 1st principle

pc1 = x* W[0,0] + y*W[1,0]  # 1st principle component

pc2 = x* W[0,1] + y*W[1,1]  # 2nd principle component.

Fig = plt.figure()
ax = Fig.add_axes([0,0,1,1])
ax.scatter(pc1, pc2)
ax.scatter(x, y)
```

* Singular Value Decomposition :- (SVD)

The SVD of a matrix is a factorization of the matrix into three matrices. Thus, the SVD of matrix A can be expressed in terms of the factorization of A into the product of three matrices as

$$A = UDV^T$$

Here, columns U & V are orthonormal

D - Diagonal matrix with real +ve entries.

* Singular Value Decomposition Applications :-

1) SVD has some fascinating algebric characteristics and conveys relevant geometrical and theoretical insights regarding linear transformation.

2) SVD has some critical applications in data science too

3) mathematical applications ~~in data~~ of SVD involve calculating the matrix approximation, rank of matrix and so on.

4) The SVD also greatly useful in science & Engg.

5) It has some applications of statistics. ex- least sq. fitting of data and process control.