Here are **50 multiple-choice questions (MCQs)** across all difficulty levels (easy, medium, hard) based on the topic **Eigenvalues and Eigenvectors**. These are designed to cover theory, computation, and conceptual understanding.

🟢 Easy Level (1–20)

1. **What is an eigenvector?**
   A) A vector that changes direction under a transformation
   B) A vector that changes magnitude but not direction under a transformation
   C) A vector that remains unchanged under all transformations
   D) None of the above
   Ans: B

2. **Which of the following equations defines an eigenvalue problem?**
   A) Ax=xAx = x
   B) Ax=λxAx = \lambda x
   C) A=xλA = x\lambda
   D) A+x=λA + x = \lambda
   Ans: B

3. **The scalar λ\lambda in Ax=λxAx = \lambda x is called:**
   A) Determinant
   B) Eigenvector
   C) Eigenvalue
   D) Cofactor
   Ans: C

4. **If xx is an eigenvector of matrix AA, then:**
   A) Ax=λAAx = \lambda A
   B) Ax=λxAx = \lambda x
   C) Ax=xλAx = x \lambda
   D) A=λxA = \lambda x
   Ans: B

5. **Eigenvectors are always:**
   A) Unique
   B) Orthogonal
   C) Non-zero
   D) Zero
   Ans: C

6. **The eigenvalues of a diagonal matrix are:**
   A) All zeros
   B) All ones
   C) The diagonal entries

D) Cannot be determined
Ans: C

7. **If AA is a 3x3 matrix, then how many eigenvalues does it have (counting multiplicities)?**
   A) 0
   B) 1
   C) 2
   D) 3
   Ans: D

8. **For a scalar multiple of the identity matrix A=kIA = kI, all eigenvalues are:**
   A) 0
   B) 1
   C) k
   D) Unknown
   Ans: C

9. **Which matrix always has real eigenvalues?**
   A) Any matrix
   B) Symmetric matrix
   C) Diagonalizable matrix
   D) Nilpotent matrix
   Ans: B

10. **The eigenvalues of a triangular matrix are:**
    A) On the main diagonal
    B) Zeros
    C) Ones
    D) All the same
    Ans: A

11. **What is the determinant of a matrix equal to in terms of its eigenvalues?**
    A) Sum of eigenvalues
    B) Product of eigenvalues
    C) Trace
    D) Rank
    Ans: B

12. **The trace of a matrix is equal to:**
    A) Product of eigenvalues
    B) Sum of eigenvalues
    C) Determinant
    D) Inverse
    Ans: B

13. **Eigenvectors of a symmetric matrix are:**
    A) Complex

B) Parallel
C) Orthogonal
D) Random
Ans: C

14. **If a matrix has distinct eigenvalues, it is:**
A) Singular
B) Non-diagonalizable
C) Diagonalizable
D) Identity
Ans: C

15. **Eigenvalues can be:**
A) Only real
B) Only positive
C) Real or complex
D) Integer only
Ans: C

16. **What is the geometric multiplicity of an eigenvalue?**
A) Number of times it appears in the characteristic equation
B) Number of linearly independent eigenvectors associated with it
C) Its value
D) Its power
Ans: B

17. **Which of the following is always true?**
A) All matrices have eigenvectors
B) All matrices are diagonalizable
C) Every square matrix has at least one eigenvalue (in complex field)
D) A matrix can have more eigenvectors than its order
Ans: C

18. **If λ\lambda is an eigenvalue of AA, then λ2\lambda^2 is an eigenvalue of:**
A) AA
B) A2A^2
C) A−1A^{-1}
D) None
Ans: B

19. **The characteristic equation is found by solving:**
A) A=0A = 0
B) ∣A−λI∣=0|A - \lambda I| = 0
C) ∣A+λI∣=0|A + \lambda I| = 0
D) A−λI=0A - \lambda I = 0
Ans: B

20. **If all eigenvalues of a matrix are 0, the matrix is:**
    A) Full-rank
    B) Invertible
    C) Nilpotent
    D) Symmetric
    Ans: C

🟡 Medium Level (21–40)

21. **Which condition implies that a matrix is diagonalizable?**
    A) Repeated eigenvalues
    B) Geometric multiplicity < algebraic multiplicity
    C) Existence of n linearly independent eigenvectors
    D) Singular matrix
    Ans: C

22. **Let AA be a 2x2 matrix with eigenvalues 2 and 3. Then trace of AA is:**
    A) 1
    B) 5
    C) 6
    D) 0
    Ans: B

23. **What is the eigenvalue of the identity matrix?**
    A) 0
    B) 1
    C) 2
    D) None
    Ans: B

24. **If AA is an invertible matrix, then:**
    A) 0 is its eigenvalue
    B) Its eigenvalues are all non-zero
    C) All eigenvalues are 1
    D) It has no eigenvalues
    Ans: B

25. **The inverse of a matrix AA has eigenvalues:**
    A) Same as AA
    B) Reciprocals of AA's eigenvalues
    C) Negatives of AA's eigenvalues
    D) Square roots
    Ans: B

26. **The matrix A=[01−2−3]A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} has eigenvalues:**
    A) Real and equal
    B) Real and distinct
    C) Complex conjugates
    D) Both zero
    Ans: B

27. **A matrix with eigenvalue zero is always:**
    A) Invertible
    B) Orthogonal
    C) Singular
    D) Diagonal
    Ans: C

28. **What is the main use of eigenvalues in differential equations?**
    A) To invert matrices
    B) To classify equilibrium points
    C) To find roots
    D) To compute limits
    Ans: B

29. **For the matrix AA, det(A−λI)=0\det(A - \lambda I) = 0 gives:**
    A) Eigenvectors
    B) Eigenvalues
    C) Inverse
    D) Norm
    Ans: B

30. **Which of the following has all zero eigenvalues?**
    A) Identity matrix
    B) Diagonal matrix
    C) Nilpotent matrix
    D) Symmetric matrix
    Ans: C

31. **In PCA, the principal components are:**
    A) Eigenvectors of the data covariance matrix
    B) Diagonal entries of data
    C) Inverses of original data
    D) Noise
    Ans: A

32. **Eigen decomposition fails when:**
    A) Matrix is symmetric
    B) Matrix is not square
    C) Matrix is diagonal
    D) Matrix has repeated eigenvalues

Ans: B

33. **If AA is orthogonal, then its eigenvalues satisfy:**
    A) Modulus = 1
    B) Always real
    C) Always complex
    D) All are 0
    Ans: A

34. **Which of the following matrices has only real eigenvalues?**
    A) Symmetric matrix
    B) Skew-symmetric
    C) Complex matrix
    D) Orthogonal
    Ans: A

35. **Algebraic multiplicity of an eigenvalue is:**
    A) Max power of eigenvalue in minimal polynomial
    B) Its geometric multiplicity
    C) Number of times it appears in characteristic polynomial
    D) Dimension of vector space
    Ans: C

36. **Which is true for eigenvectors v1v_1 and v2v_2 of the same eigenvalue λ\\lambda?**
    A) They must be orthogonal
    B) They must be identical
    C) Any linear combination is also an eigenvector
    D) Their dot product is always zero
    Ans: C

37. **What are eigenvalues used for in graph theory?**
    A) Plotting nodes
    B) Analyzing adjacency matrix
    C) Coloring
    D) Removing edges
    Ans: B

38. **If a matrix has one non-zero eigenvalue and all others zero, then it is:**
    A) Full rank
    B) Singular
    C) Diagonal
    D) Unitary
    Ans: B

39. **A matrix is diagonalizable if:**
    A) All eigenvalues are equal
    B) It has a full set of eigenvectors

C) It is symmetric
D) All entries are non-zero
Ans: B

40. **The matrix exponential eAte^{At} uses:**
A) Determinants
B) Eigenvalues
C) Trace
D) Cofactors
Ans: B

🔴 Hard Level (41–50)

41. **If AA has eigenvalues λ1,λ2\lambda_1, \lambda_2, then AkA^k has eigenvalues:**
A) kλik\lambda_i
B) λik\lambda_i^k
C) 1/λik1/\lambda_i^k
D) Same as AA
Ans: B

42. **If a matrix is not diagonalizable, it is called:**
A) Orthogonal
B) Unitary
C) Defective
D) Singular
Ans: C

43. **Which of the following is true for a normal matrix AA?**
A) AT=AA^T = A
B) A∗A=AA∗A^*A = AA^*
C) A−1=AA^{-1} = A
D) A=A2A = A^2
Ans: B

44. **If AA is 2x2 with eigenvalues 3 and 5, and corresponding eigenvectors v1,v2v_1, v_2, then:**
A) A(v1+v2)=8(v1+v2)A(v_1 + v_2) = 8(v_1 + v_2)
B) A(v1+v2)=3v1+5v2A(v_1 + v_2) = 3v_1 + 5v_2
C) A(v1+v2)=15(v1+v2)A(v_1 + v_2) = 15(v_1 + v_2)
D) Cannot determine
Ans: B

45. **What is the minimal polynomial of a diagonalizable matrix with distinct eigenvalues?**

A) Degree 1
B) Product of distinct linear factors
C) Irreducible polynomial
D) Zero
Ans: B

46. **Which matrix operation is preserved under similarity transformation P−1APP^{-1}AP?**
A) Rank
B) Determinant
C) Eigenvalues
D) All of the above
Ans: D

47. **Which matrix cannot be diagonalized over the reals?**
A) Identity
B) Rotation matrix with angle π/2\pi/2
C) Symmetric
D) Zero matrix
Ans: B

48. **Jordan form is used when a matrix:**
A) Has complex eigenvalues
B) Is not diagonalizable
C) Is diagonal
D) Is invertible
Ans: B

49. **If eigenvalues of a matrix are complex, the matrix:**
A) Cannot be real
B) Can still be real
C) Must be diagonal
D) Cannot be orthogonal
Ans: B

50. **Which matrix transformation preserves eigenvectors but changes eigenvalues?**
A) Multiplying matrix by scalar
B) Adding identity
C) Matrix transpose
D) Rotation
Ans: A

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