Propositional Logic– Solution

1) Translate the following Propositional Logic to English sentences. Let:

*• E*=Liron is eating

*• H*=Liron is hungry

(a) *E ⇒ ¬H*

Answer: If Liron is eating, then Liron is not hungry

(b) *E ∧ ¬H*

Answer: Liron is eating and not hungry

(c) *¬*(*H ⇒ ¬E*)

Answer: Liron is hungry and eating

2) Translate the following English sentences to Propositional Logic. Propositions: (R)aining, Liron is (S)ick, Liron is (H)ungry, Liron is (HA)appy, Liron owns a (C)at, Liron owns a (D)og

(a) It is raining if and only if Liron is sick

Answer: *R ⇔ S*

(b) If Liron is sick then it is raining, and vice versa

Answer: (*S ⇒ R*) *∧* (*R ⇒ S*) (which is equivalent to *R ⇔ S*)

(c) It is raining is equivalent to Liron is sick

Answer: *R ⇔ S*

(d) Liron is hungry but happy

Answer: *H ∧ HA*

(e) Liron either owns a cat or a dog

Answer: (*C ∧ ¬D*) *∨* (*¬C ∧ D*)

3) Which of the following propositions are tautologies? Which are contradictions? Why?

(a) Three is a prime number.

Answer: neither a tautology nor a contradiction

(b) It is raining or it is not raining.

Answer: tautology

(c) It is raining (*P*) and it is not raining (*¬P*).

Answer: contradiction

Example reasoning:

All rows in the truth table evaluate to false.

| P | *P ∧ ¬P* |
| --- | --- |
| t  f | f  f |

4) Which of the following propositions are tautologies? Why?

(a) *P*

Answer: not a tautology

(b) *P ⇒ P*

Answer: tautology

(c) (*P ⇒ P*) *⇒ P*

Answer: not a tautology

Example reasoning:

Not all rows in the truth table evaluate to true.

| P | P *⇒* P | (P *⇒* P) *⇒* P |
| --- | --- | --- |
| t  f | t  t | t  f |

(d) *P ⇒* (*P ⇒ P*)

Answer: tautology

5) Which of the two following propositions are equivalent in the sense that one can always be substituted for the other one in any proposition without changing its truth value? Why?

(a) first proposition:*P ⇒ Q* second proposition: *¬P ∨ Q*

Answer: yes

Example reasoning:

All rows in the truth table evaluate to the same truth value.

| P | Q | P *⇒* Q | *¬* P *∨* Q |
| --- | --- | --- | --- |
| t  t  f  f | t  f  t  f | t  f  t  t | t  f  t  t |

(b) first proposition: *¬P* second proposition: *P ⇒ F alse*

Answer: yes

(c) first proposition: *¬P* second proposition: *F alse ⇒ P*

Answer: no

(d) first proposition: *¬P* second proposition: *¬P ∨ Q*

Answer: no

6) Is it possible that

(a) (*KB |*= *S*) and (*¬KB |*= *S*)

Answer: Yes. For example, if *S ≡ TRUE*, then any interpretation that satisfies *KB* or *¬KB* also satisfies *S*.

(b) (*KB |*= *S*) and (*KB |*= *¬S*)

Answer: Yes. For example, if *KB ≡ FALSE*, then *KB* entails any sentence, including *S* and *¬S*.

(c) (*KB |*= *S*) and (*KB 6|*= *S*)

Answer: No. Either all the interpretations that satisfy *KB* also satisfy *S* (*KB |*= *S*), or there is an interpretation that satisfies *KB* but not *S* (*KB 6|*= *S*). Both cannot be true at the same time.

(d) (*KB |*= *S*) and (*KB 6|*= *¬S*)

Answer: Yes. For example, if *KB ≡ TRUE* and *S ≡ TRUE*, then *TRUE |*= *TRUE* and *TRUE 6|*= *FALSE*.

(e) (*KB 6|*= *S*) and (*KB 6|*= *¬S*)

Answer: Yes. For example, if *KB ≡ TRUE*, then it cannot entail a sentence *S* unless *S* is a tautology. So, if we pick *S ≡ P*, where *P* is a propositional symbol, then *TRUE 6|*= *P* and *TRUE 6|*= *¬P*.

(f) (*KB 6|*= *S*) and (*¬KB 6|*= *S*)

Answer: Yes. For example, if *KB ≡ P* and *S ≡ Q*, where *P* and *Q* are propositional symbols, then *P 6|*= *Q* and *¬P 6|*= *Q*.

If so, provide an example. If not, explain why it is impossible. 7) Prove that *P ∧ Q |*= *P ∨ Q*.

Answer:

| *P* | *Q* | *P ∧ Q* | *P ∨ Q* |
| --- | --- | --- | --- |
| t  t  f  f | t  f  t  f | t  f  f  f | t  t  t  f |

Since every interpretation that satisfies *P ∧ Q* also satisfies *P ∨ Q*, it holds that *P ∧ Q |*= *P ∨ Q*.

8) Consider the following popular puzzle. When asked for the ages of her three chil dren, Mrs. Baker says that Alice is her youngest child if Bill is not her youngest child, and that Alice is not her youngest child if Carl is not her youngest child. Write down a knowledge base that describes this riddle and the necessary back ground knowledge that only one of the three children can be her youngest child. Show with resolution that Bill is her youngest child.

Answer:

Let the propositions *A*, *B* and *C* denote that Mrs. Baker’s youngest child is Alice, Bill and Carl, respectively. We have the following clauses for the background knowledge:

1 *A ∨ B ∨ C* (One child has to be the youngest.)

2 *¬A ∨ ¬B* (Alice and Bill cannot both be the youngest.)

3 *¬A ∨ ¬C*

4 *¬B ∨ ¬C*

The following clauses represent the information from Mrs. Baker:

5 *B ∨ A* (Alice is her youngest child if Bill is not her youngest child. That is, *¬B ⇒ A*.)

6 *C ∨ ¬A* (Alice is not her youngest child if Carl is not her youngest child. That is, *¬C ⇒ ¬A*.)

We want to show that Bill is the youngest child. Negating this, we get the following clause:

7 *¬B* (Assume that Bill is not the youngest child.)

We use resolution to derive the empty clause as follows:

8 (from 5,7) *A*

9 (from 3,6) *¬A*

10 (from 8,9) *⊥*