<u>Approach Description</u>

To reach goal state efficiently, A* algorithm has been used with the aid of two types of heuristics – Manhattan Distance and Number of Linear Conflicts along with judicious choice of data structures.

A set of frontier nodes is maintained with the aid of a heap. At every iteration a node from frontier set is expanded. The chosen node is the one with minimum sum of the following 2 things:
1) Actual path length to reach this node.

2) Heuristic approximation of length to reach goal from this node.

Heuristics

- <u>Manhattan Distance</u>
  It estimates the cost of reaching goal state from concerned state as the sum of Manhattan Distances between current position of each tile and final position of each tile.

- <u>Number of Linear Conflicts</u>
  Whenever two tiles that appear in a row (or column) and are supposed to be present in same row (or column) in the final state as well, 2 is added to the heuristic function. (Proof of correctness in the next slide)

<u>Important Note</u>: While approximating the heuristic for any particular state, the contribution of tile with zero was omitted, this significantly reduced the running time of code.

<u>Proof of Optimality</u>

Heuristics

- <u>Manhattan Distance</u>

  The Manhattan Distance of a tile's position in current state to its position in final state can be thought of as number of moves a tile would have to make to reach its final position if multiple tiles were allowed to be present at the same position, however since this is not the case, it is definitely an under-estimation of the movements the tile would have actually done to reach the final state.

- <u>Number of Linear Conflicts</u>

  A linear conflict happens in a row when two tiles that are present in the row, and belong to the same row in goal state are in inverted sequence relative to the goal state. For every inversion, we would need at least two extra moves to correct the sequence of nodes. Hence number of horizontal and vertical conflicts have been counted and 2 moves have been added for each conflict.

Since both heuristics underestimate the number of moves required to go to goal state, they are both admissible, which would ensure optimality.

Omission of Contribution of "ZERO" Tile

When the sum of Manhattan Distances corresponding to each tile was done to make a heuristic approximation, the Manhattan Distance between final and current state for the tile with value 0 wasn't taken into account. The intuition behind doing so was that if we have managed to place all tiles other than zero on their correct position, then automatically the tile with value zero would be in correct position as well. With same reasoning, the contribution of zero was omitted when counting for number of conflicts was done.