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Range-based DCC models for covariance and value-at-risk forecasting☆

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ABSTRACT

The dynamic conditional correlation (DCC) model by Engle (2002) is one of the most popular multivariate volatility models. This model is based solely on closing prices. It has been documented in the literature that the high and low prices of a given day can be used to obtain an efficient volatility estimation. We therefore suggest a model that incorporates high and low prices into the DCC framework. We conduct an empirical evaluation of this model on three datasets: currencies, stocks, and commodity exchange traded funds. Regardless of whether we consider in-sample fit, covariance forecasts or value-at-risk forecasts, our model outperforms not only the standard DCC model, but also an alternative range-based DCC model.

1. Introduction

Models that can describe the dynamic properties of two or more asset returns play an important role in financial econometrics. Multivariate volatility models have been used to understand and predict the temporal dependence in second order moments of asset returns. These models can explain how covariances change over time and therefore describe temporal dependencies among assets. Such relations are vital in many financial applications, such as asset pricing, portfolio optimization, risk management, the estimation of systemic risk in banking, value-at-risk estimation, asset allocation and many others.

One of the most popular multivariate volatility models is the dynamic conditional correlation (DCC) model introduced independently by Engle (2002) and Tse and Tsui (2002). The latter representation however has attracted considerably less interest in the literature. The advantages of the DCC model are the positive definiteness of the conditional covariance martices and the ability to describe time-varying conditional correlations and covariances in a parsimonious way. The parameters of the DCC model can be estimated in two stages, which makes this approach relatively simple and possible to apply even for very large portfolios. The DCC model has become extremely popular and has been widely applied and modified (e.g. Heaney and Srinanthakumar, 2012; Lehkonen and Heimonen, 2014; Bouri et al., 2017; Bernardi and Catania, 2018; Dark, 2018; Karanasos et al., 2018).

Most volatility models are return-based models, i.e. they are estimated on returns, which are calculated based only on closing prices. Meanwhile, the use of daily low and high prices leads to more accurate estimates and forecasts of variances (see e.g. Chou, 2005; Brandt and Jones, 2006; Lin et al., 2012; Fiszeder and Perczak, 2016; Molnár, 2016) and covariances (see e.g. Chou et al., 2009; Fiszeder, 2018). Daily low and high prices are almost always available alongside closing prices in financial series. Therefore, making use of them in volatility models is very important from a practical viewpoint. DCC models formulated with the usage of

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low and high prices have already been proposed in the literature, including the range-based DCC by Chou et al. (2009) and the range-based regime-switching DCC by Su and Wu (2014). These models, however, are based on modelling the time evolution of price range and it is not possible to compare them directly with the return-based DCC model. We propose a DCC model constructed using the Range-GARCH model by Molnár (2016), which is formulated with the usage of low and high prices but also based on returns calculated from closing prices.

Our contribution is threefold. First, we construct a new specification of the DCC model based on the Range-GARCH model by Molnár (2016), which we refer to as the DCC-Range-GARCH model (denoted by DCC-RGARCH). The model itself is very similar to the DCC model by Engle (2002). Squared errors in the univariate GARCH model are replaced by the Parkinson (1980) volatility estimator, but the parametrization of the covariance matrix remains the same. Second, we show using low and high prices in the formulation of the DCC model improves the estimation of the covariance matrix of returns and increases the accuracy of covariance and VaR forecasts based on this model, compared with the standard DCC model based on closing prices. Since both models, DCC and DCC-RGARCH, share the same structure in the correlation component, achieving more precise volatility estimates improves the covariance forecasts. Third, we demonstrate that covariance forecasts based on our proposed model are more accurate than those obtained using the range-based DCC model by Chou et al. (2009). That is an important conclusion, because the range-based DCC model is also formulated using low and high prices and is the main competitor for the DCC-RGARCH model in this class of models.

The rest of the paper is organized in the following way. Section 2 provides a description of applied models and methods. Section 3 presents data: three currency pairs -EUR/USD, USD/JPY and GBP/USD, three commodity exchange traded funds (ETFs) - United States Oil Fund, United States Natural Gas Fund and Energy Select Sector SPDR Fund and five U.S. stocks - Amazon, Apple, Goldman Sachs, Google and IBM. In Section 4.1 the parameters of the return-based DCC, range-based DCC and DCC-RGARCH models are estimated and compared. Section 4.2 evaluates the forecasts of the variance of returns from the GARCH, CARR and RGARCH models. In Section 4.3 the accuracy of covariance forecasts based on the DCC-GARCH and DCC-CARR models is compared with the forecasts from the DCC-RGARCH model. Section 4.4 evaluates the VaR forecasts based on all considered DCC models. Section 5 concludes.

2. Theoretical background

2.1. The DCC-GARCH model

In this paper we extend the DCC model by Engle (2002) by introducing the range (the difference between low and high prices) to the model. First, we present the standard DCC model based on closing prices. In order to better distinguish this model from its competitors used in the paper, which are based on different univariate models, we will refer to it as the DCC-GARCH model.

Let us assume that $\boldsymbol{\epsilon}_t$ ($N \times 1$ vector) is the innovation process for the conditional mean (or in a particular case the multivariate return process) and can be written as:

$$\boldsymbol{\varepsilon}_t \big| \psi_{t-1} \sim Normal\ (0, \mathbf{cov}_t), \tag{1}$$

where ψ_{t-1} is the set of all information available at time $t-1$, *Normal* is the multivariate normal distribution and $\mathbf{cov}_t$ is the $N \times N$ symmetric conditional covariance matrix.

The DCC(P, Q)-GARCH(p, q) model by Engle (2002) can be presented as:

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{2}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}, \tag{3}$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^{Q} \zeta_i - \sum_{j=1}^{P} \theta_j \right) \mathbf{S} + \sum_{i=1}^{Q} \zeta_i (\mathbf{z}_{t-i} \mathbf{z}'_{t-i}) + \sum_{j=1}^{P} \theta_j \mathbf{Q}_{t-j}, \tag{4}$$

where $\mathbf{D}_t = \mathrm{diag}(h_{1t}^{1/2}, h_{2t}^{1/2} \dots, h_{Nt}^{1/2})$, conditional variances h_{kt} (for $k = 1, 2, \dots, N$) are described as univariate GARCH models (Eqs. (5)–(6)), $\mathbf{z}_t$ is the standardized $N \times 1$ residual vector assumed to be serially independently distributed given as $\mathbf{z}_t = \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t$, $\mathbf{cor}_t$ is the time varying $N \times N$ conditional correlation matrix of $\mathbf{z}_t$, $\mathbf{S}$ is the unconditional $N \times N$ covariance matrix of $\mathbf{z}_t$ (according to Engle, 2002) and $\mathbf{Q}_t^*$ is the diagonal $N \times N$ matrix composed of the square root of the diagonal elements of $\mathbf{Q}_t$. The parameters ζ_i (for $i = 1, 2, \dots, Q$), θ_j(for $j = 1, 2, \dots, P$) are nonnegative and satisfy the condition $\sum_{i=1}^{Q} \zeta_i + \sum_{j=1}^{P} \theta_j < 1$.

The univariate GARCH(p, q) model applied in the DCC-GARCH model can be written as:

$$\varepsilon_{kt} \big| \psi_{t-1} \sim Normal\ (0, h_{kt}), \quad k = 1, 2, \dots, N, \tag{5}$$

$$h_{kt} = \alpha_{k0} + \sum_{i=1}^{q} \alpha_{ki} \varepsilon_{k\,t-i}^2 + \sum_{j=1}^{p} \beta_{kj} h_{k\,t-j}, \tag{6}$$

where $\alpha_{k0} > 0, \alpha_{ki} \geq 0, \beta_{kj} \geq 0$ (for $k = 1, 2, \dots, N;\ i = 1, 2, \dots, q;\ j = 1, 2, \dots, p$), weaker conditions for non-negativity of the conditional variance can be assumed (see Nelson and Cao, 1992). The requirement for covariance stationarity of ε_{kt} is $\sum_{i=1}^{q} \alpha_{ki} + \sum_{j=1}^{p} \beta_{kj} < 1$.

A nice feature of the DCC-GARCH model is that its parameters can be estimated by the quasi-maximum likelihood method using a two-stage approach (see Engle and Sheppard, 2001). Let the parameters of the model $\boldsymbol{\Theta}$ be written in two groups $\boldsymbol{\Theta}' = (\boldsymbol{\Theta}'_1, \boldsymbol{\Theta}'_2)$, where $\mathbf{\Theta}_1$ is the vector of parameters of conditional means and variances and $\mathbf{\Theta}_2$ is the vector of parameters of the correlation part of the model. The log-likelihood function can be written as the sum of two parts:

$$L(\mathbf{\Theta}) = L_{Vol}(\mathbf{\Theta}_1) + L_{Corr}(\mathbf{\Theta}_2 | \mathbf{\Theta}_1), \tag{7}$$

where $L_{Vol}(\mathbf{\Theta}_1)$ represents the volatility part:

$$L_{Vol}(\mathbf{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(N\ln(2\pi) + ln\left|\mathbf{D}_t\right|^2 + \boldsymbol{\varepsilon}'_t\mathbf{D}_t^{-2}\boldsymbol{\varepsilon}_t\right), \tag{8}$$

while $L_{Corr}(\mathbf{\Theta}_2 | \mathbf{\Theta}_1)$ can be viewed as the correlation component:

$$L_{Corr}(\mathbf{\Theta}_2 | \mathbf{\Theta}_1) = -\frac{1}{2}\sum_{t=1}^{n}\left(\ln\left|\mathbf{cor}_t\right| + \mathbf{z}'_t\mathbf{cor}_t^{-1}\mathbf{z}_t - \mathbf{z}'_t\mathbf{z}_t\right). \tag{9}$$

$L_{Vol}(\mathbf{\Theta}_1)$ can be written as the sum of log-likelihood functions of N univariate GARCH models:

$$L_{Vol}(\mathbf{\Theta}_1) = -\frac{1}{2}\sum_{k=1}^{N}\left(n\ln(2\pi) + \sum_{t=1}^{n}\left(ln(h_{kt}) + \frac{\epsilon_{kt}^2}{h_{kt}}\right)\right). \tag{10}$$

This means that in the first stage the parameters of univariate GARCH models can be estimated separately for each of the assets and the estimates of h_{kt} can be obtained. In the second stage residuals transformed by their estimated standard deviations are used to estimate the parameters of the correlation part ($\mathbf{\Theta}_2$) conditioning on the parameters estimated in the first stage ($\hat{\mathbf{\Theta}}_1$).

2.2. The CARR model

The second benchmark to compare with our new model is the range-based DCC model. This is based on the CARR model by Chou (2005), which we describe now.

Let assume that H_t and L_t are high and low prices over a fixed period such as day, week or month and the observed price range is given as $R_t = ln\left(H_t\right) - ln\left(L_t\right)$. The CARR$(p,q)$ model can be described as:

$$R_t = \lambda_t u_t, \tag{11}$$

$$u_t | \psi_{t-1} \sim \exp\left(1, \xi_t\right), \tag{12}$$

$$\lambda_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i R_{t-i} + \sum_{j=1}^{p}\beta_j\lambda_{t-j}, \tag{13}$$

where λ_t is the conditional mean of the range and u_t is the disturbance term.
The exponential distribution is a natural choice for the conditional distribution of u_t because it takes positive values. To ensure the positivity of λ_t the parameters of the CARR model have to meet conditions analogous to those in the GARCH model (see Nelson and Cao, 1992). The process is covariance stationary if the following condition is met:

$$\sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j < 1. \tag{14}$$

It is worth emphasizing that the CARR model describes the dynamics of the conditional mean of the price range, not the conditional variance of returns as in the case of the GARCH model.

The parameters of the CARR model can be estimated by the quasi-maximum likelihood method. The log-likelihood function can be written as:

$$L(\boldsymbol{\varsigma}) = -\sum_{t=1}^{n}\left(\ln\lambda_t + \frac{R_t}{\lambda_t}\right), \tag{15}$$

where $\boldsymbol{\varsigma}$ is a vector containing unknown parameters of the model. The estimators obtained by the quasi-maximum likelihood method are consistent (see Engle and Russell, 1998; Engle, 2002; Chou, 2005).

2.3. The DCC-CARR model

In this paper the new DCC-RGARCH model is compared not only with the DCC-GARCH model, formulated on closing prices, but also with the range-based DCC model which, like the proposed model, is formulated using low and high prices. Chou et al. (2009) combined the CARR model by Chou (2005) with the DCC model by Engle (2002) to propose the range-based DCC model, which we refer to as the DCC-CARR model in this paper. The CARR model describes the dynamics of the conditional mean of the price range, and so in order to estimate values of the conditional standard deviation of returns the conditional price range has to be scaled according to the formula: $\lambda_{kt}^* = \mathrm{adj}_k\lambda_{kt}$ for $k = 1, 2, \ldots, N$, where $\mathrm{adj}_k = \overline{\sigma}_k / \overline{\lambda}_k$. The scaling factor adj_k is estimated as the quotient of unconditional standard deviation of returns by the sample mean of the conditional range.

The DCC(P,Q)-CARR(p,q) model can be expressed as:

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim Normal\ (0, \mathbf{cov}_t), \tag{16}$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{17}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}, \tag{18}$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^{Q} \zeta_i - \sum_{j=1}^{P} \theta_j\right) \mathbf{S} + \sum_{i=1}^{Q} \zeta_i (\mathbf{z}_{t-i}^{CARR} (\mathbf{z}_{t-i}^{CARR})') + \sum_{j=1}^{P} \theta_j \mathbf{Q}_{t-j}, \tag{19}$$

where $\mathbf{D}_t = \mathrm{diag}\left(\lambda_{1t}^*, \lambda_{2t}^*, \ldots, \lambda_{Nt}^*\right)$, $\mathbf{z}_t^{CARR}$ is the standardized $N \times 1$ residual vector which contains the standardized residuals z_{kt}^{CARR} calculated from the CARR model (Eqs. (11)–(13)) as $z_{kt}^{\text{CARR}} = \varepsilon_{kt} / \lambda_{kt}^*$, the other variables are defined in the same way as in the DCC-GARCH model.

The parameters of the DCC-CARR model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as the sum of two parts, the volatility part and the correlation part:

$$L^{DCC-CARR}(\mathbf{\Theta}) = L_{Vol}^{DCC-CARR}(\mathbf{\Theta}_1) + L_{Corr}^{DCC-CARR}(\mathbf{\Theta}_2 | \mathbf{\Theta}_1), \tag{20}$$

$$L_{Vol}^{DCC-CARR}(\mathbf{\Theta}_1) = -\frac{1}{2} \sum_{k=1}^{N} \left(n \ln(2\pi) + \sum_{t=1}^{n} \left(2 ln(\lambda_{kt}^*) + \frac{\epsilon_{kt}^2}{\lambda_{kt}^{*2}} \right) \right) \tag{21}$$

$$L_{Corr}^{DCC-CARR}(\mathbf{\Theta}_2 | \mathbf{\Theta}_1) = -\frac{1}{2} \sum_{t=1}^{n} \left(\ln |\mathbf{cor}_t| + (\mathbf{z}_t^{CARR})' \mathbf{cor}_t^{-1} \mathbf{z}_t^{CARR} - (\mathbf{z}_t^{CARR})' \mathbf{z}_t^{CARR} \right). \tag{22}$$

This means that in the first stage the parameters of the CARR models can be estimated separately for each of the assets. In the second stage the standardized residuals z_{kt}^{CARR} are used to maximize Eq. (22) in order to estimate the parameters of the correlation component.

2.4. The Range-GARCH model

In the new specification of the DCC-RGARCH model we use the Range-GARCH model introduced by Molnár (2016). The RGARCH(p, q) model can be formulated as:

$$\varepsilon_t | \psi_{t-1} \sim Normal\ (0, h_t), \tag{23}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \sigma_{Pt-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}, \tag{24}$$

where σ_{Pt}^2 is the Parkinson (1980) estimator calculated as $\sigma_{Pt}^2 = [ln(H_t/L_t)]^2/(4 \ln 2)$.
In this formulation other variance estimators based on low, high and opening or closing prices, like the Garman and Klass (1980) or Rogers and Satchell (1991) estimators, can be applied instead of the Parkinson estimator. For an overview of range-based volatility estimators see Molnár (2012), Fiszeder and Perczak (2013).

To ensure the positivity of h_t the parameters of the RGARCH model must meet conditions analogous to those in the GARCH model (see Nelson and Cao, 1992). The RGARCH process is covariance stationary if the following condition is met:

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1. \tag{25}$$

It is worth emphasizing that the RGARCH model describes the dynamics of the conditional variance of returns, not the conditional mean of the price range, as in the case of the CARR model. The parameters of the RGARCH model can be estimated by the quasi-maximum likelihood method and the likelihood function is the same as in the return-based GARCH model.

2.5. The DCC-Range-GARCH model

In this subsection we introduce our new DCC-Range-GARCH model (denoted by DCC-RGARCH). The DCC(P, Q)-RGARCH(p, q) model can be presented as:

$$\boldsymbol{\varepsilon}_t | \psi_{t-1} \sim Normal\ (0, \mathbf{cov}_t), \tag{26}$$

$$\mathbf{cov}_t = \mathbf{D}_t \mathbf{cor}_t \mathbf{D}_t, \tag{27}$$

$$\mathbf{cor}_t = \mathbf{Q}_t^{*-1} \mathbf{Q}_t \mathbf{Q}_t^{*-1}, \tag{28}$$

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^{Q} \zeta_i - \sum_{j=1}^{P} \theta_j\right) \mathbf{S} + \sum_{i=1}^{Q} \zeta_i (\mathbf{z}_{t-i}^{RGARCH} (\mathbf{z}_{t-i}^{RGARCH})') + \sum_{j=1}^{P} \theta_j \mathbf{Q}_{t-j}, \tag{29}$$

where $\mathbf{D}_t = \operatorname{diag}\left((h_{1t}^{RGARCH})^{1/2}, (h_{2t}^{RGARCH})^{1/2} \ldots, (h_{Nt}^{RGARCH})^{1/2}\right)$, conditional variances h_{kt}^{RGARCH} (for $k = 1, 2, \ldots, N$) are described as for the RGARCH model (Eqs. (23)–(24)), $\mathbf{z}_t^{RGARCH}$ is the standardized $N \times 1$ residual vector which contains the standardized residuals z_{kt}^{RGARCH} calculated from the RGARCH model as $z_{kt}^{\mathrm{RGARCH}} = \varepsilon_{kt} / \left(h_{kt}^{RGARCH}\right)^{1/2}$, the other variables are defined in the same way as in the DCC-GARCH model.

The parameters of the DCC-R-GARCH model can be estimated by the quasi-maximum likelihood method using a two-stage approach. The log-likelihood function can be written as the sum of two parts, the volatility part and the correlation part:

$$L^{DCC-RGARCH}(\boldsymbol{\Theta}) = L_{Vol}^{DCC-RGARCH}(\boldsymbol{\Theta}_1) + L_{Corr}^{DCC-RGARCH}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1), \tag{30}$$

$$L_{Vol}^{DCC-RGARCH}(\boldsymbol{\Theta}_1) = -\frac{1}{2} \sum_{k=1}^{N} \left(n \ln(2\pi) + \sum_{t=1}^{n} \left(ln(h_{kt}) + \frac{\epsilon_{kt}^2}{h_{kt}} \right) \right) \tag{31}$$

$$\begin{aligned} L_{Corr}^{DCC-RGARCH}(\boldsymbol{\Theta}_2 | \boldsymbol{\Theta}_1) = -\frac{1}{2} \sum_{t=1}^{n} \Big(& \ln |\mathbf{cor}_t| + (\boldsymbol{z}_t^{RGARCH})' \mathbf{cor}_t^{-1} \boldsymbol{z}_t^{RGARCH} \\ & -(\boldsymbol{z}_t^{RGARCH})' \boldsymbol{z}_t^{RGARCH} \Big), \end{aligned} \tag{32}$$

This means that in the first stage the parameters of univariate RGARCH models can be estimated separately for each of the assets. In the second stage the standardized residuals z_{kt}^{RGARCH} are used to maximize Eq. (32) in order to estimate the parameters of the correlation component.

3. Data

We apply the proposed model and its competitors to three different sets of data: three currency rates, three commodity exchange traded funds and five stocks. The currency rates are the three most heavily traded currency pairs in the Forex market, namely: EUR/USD, USD/JPY and GBP/USD.

The second set are three exchange-traded funds (ETF) listed on the New York Stock Exchange Arca, namely (the names given in the brackets will be used later in tables): the United States Oil Fund (Oil), the United States Natural Gas Fund (Natural Gas) and the Energy Select Sector SPDR Fund (Energy). Commodity exchange traded funds provide investors with the convenience of commodity exposure without a commodity futures account. The first two ETFs offer exposure to a single commodity (oil/gas), whereas the third ETF tracks the price and performance of the Standard and Poor's Energy Select Sector Index.

The third set of data consists of five selected U.S. stocks, namely: Amazon, Apple, Goldman Sachs, Google and IBM. Since there are many stocks that could be chosen for this purpose, we decided to follow CBOE and select the stocks for which CBOE calculates implied volatility indices (even though implied volatility indices are not used in this paper).

We evaluate the models considered for daily data in the nine-year period from January 2, 2008, to December 30, 2016. This is a relatively long period, which includes both very volatile periods – the collapse of Lehman Brothers, the worst phase of the global financial crisis, the European sovereign debt crisis and Brexit – but also tranquil periods with low volatility.

The descriptive statistics for the percentage returns calculated as $r_t = 100 \ln(p_t/p_{t-1})$, where p_t is the closing price at time t, are presented in Table 1. The means of returns are positive for stocks and the Energy Select Sector SPDR Fund and negative for currencies and the other ETFs. The standard deviation of returns is significantly lower for currencies. Most distributions of returns are asymmetric, and all display high leptokurtosis.

4. Results

We consider three DCC models in the analysis:

(1) The DCC-GARCH model by Engle (2002) summarized by Eqs. (1)–(6), where parameters are estimated based only on closing prices.

(2) The DCC-CARR model by Chou et al. (2009), see Eqs. (16)–(19). In this specification the CARR model (Eqs. (11)–(13)) is applied in the DCC model instead of the univariate GARCH model.

(3) The proposed DCC-RGARCH model summarized by Eqs. (26)–(29). In this specification the RGARCH model described by Eqs. (23)–(24) is applied in the DCC model instead of the univariate GARCH model.

We also consider a DCC model using two asymmetric GARCH models, i.e. the EGARCH (Nelson, 1991) and GJR (Glosten et al., 1993) models, instead of the standard GARCH model. These models are able to capture often-reported asymmetric responses to positive and negative shocks in the conditional variance. However we find that covariance forecasts based on the DCC-EGARCH and DCC-GJR models are not significantly better than forecasts from the DCC-GARCH model for any of the currencies and ETFs considered, or for most stocks (the results are given in Tables A.1 and A.2 in the Appendix), and so we do not extend our models to describe the effect of asymmetry in variance.

The considered exchange rates, ETFs and stocks are not cointegrated (according to the Johansen test). Mean equations for returns are very simple: each mean equation is a constant, because in our data the sample return of any asset is not dependent on its own past returns nor on the past returns of other assets.

We first compare the fit of the models estimated on the whole sample of data, and then compare the forecasts from these models. We analyse forecasts of variances and forecasts of covariances separately, because models for variances already exist whereas forecasting covariances is our main contribution.

Table 1
Summary statistics of daily returns.

Assets	Mean × 10^2	Minimum	Maximum	Standard deviation	Skewness	Excess kurtosis
Currency rates						
EUR/USD	−1.401	−2.554	3.503	0.657	0.116*	4.825*
JPY/USD	−0.198	−5.448	3.779	0.692	−0.008	7.670*
GBP/USD	−2.037	−8.322	2.870	0.641	−1.245*	17.043*
Exchange-traded funds						
Oil	−8.234	−11.439	9.199	2.286	−0.133*	5.256*
Natural Gas	−15.146	−9.745	13.942	2.651	0.172*	4.173*
Energy	0.463	−19.033	18.051	1.965	−0.408*	15.390*
Stocks						
Amazon	9.220	−13.640	23.768	2.482	0.548*	11.837*
Apple	6.631	−19.128	12.577	2.039	−0.499*	10.454*
Goldman Sachs	0.984	−22.022	23.245	2.538	0.054	18.504*
Google	3.658	−10.271	18.231	1.894	0.752*	14.891*
IBM	2.801	−8.799	11.035	1.443	−0.215*	8.928*

The sample period is January 2, 2008, to December 30, 2016.
*Indicates that the null hypothesis (the skewness or excess kurtosis is equal to zero) was rejected at the 10% significance level.

4.1. In-sample comparison of models

The parameters of the considered models are estimated using the quasi-maximum likelihood method. The results of the estimation are presented in Tables 2–4 separately for exchange rates, ETFs and stocks.

The estimation of parameters for the GARCH, R-GARCH and CARR models is based on different kinds of data: on closing prices for the first two models[1] and on range data for the third model. However, for the DCC-CARR, which uses the CARR model, it is possible to calculate the likelihood function based on the scaled conditional price range according to formula (21). Thanks to this, it is possible to evaluate all the DCC models based on the whole likelihood function, including both the volatility and correlation parts. In order to assess whether the differences between values of likelihood function are statistically significant, we apply the Rivers and Vuong (2002) and Clarke (2007) tests for non-nested model selection. The values of the likelihood function are higher for the DCC-RGARCH model than for the benchmark DCC-GARCH model for all analysed data sets, which means that the DCC-RGARCH model better describes the considered time series. The results for the DCC-CARR model are ambiguous and depend on the type of test applied.

The application of range data changes the parameter estimates for the considered models significantly. Specifically, the estimates of the parameters α_{k1} are much higher and the estimates of the parameters β_{k1} much lower in the CARR and RGARCH models compared with the GARCH model. This is important in terms of both modelling and forecasting volatility, because for the CARR and RGARCH models the shocks in the previous period have a stronger impact on the current volatility than the impact you observe for the GARCH model. Thus models formulated with range data respond more quickly to changing market conditions. Slow response to abrupt changes in the market is widely cited as one of the greatest weaknesses of GARCH-type models formulated based on closing prices (e.g. Andersen et al., 2003; Hansen et al., 2012).

Direct comparison of the parameters of the CARR model with the parameters of the GARCH and RGARCH models is, however, difficult, because they describe different measures of volatility. The CARR model describes the dynamics of the conditional mean of the price range, while the GARCH and RGARCH models describe the conditional variance of returns.

One can also notice that the sum of the estimates of the parameters α_{k1} and β_{k1} in the RGARCH model is higher than one for ETFs and stocks. However, this does not mean that the analysed processes are covariance non-stationary. It results from the fact that the Parkinson estimator underestimates the volatility of returns in the presence of opening jumps (such jumps do not occur in the Forex market since it does not close overnight), causing an increase in the estimate of the parameter α_{k1} (see Molnár, 2016).

On the other hand, there are no considerable differences between the considered models in the estimates of parameters for the correlation component. Thus, the main differences in the behaviour of the time-varying covariances from those models results from the usage of the different standardized residuals z_{kt}, z_{kt}^{CARR} and z_{kt}^{RGARCH} in Eqs. (4), (19) and (29) of the DCC-GARCH, DCC-CARR and DCC-RGARCH models, respectively.

4.2. Comparison of variance forecasts

In this section we compare the forecasting performance of the three univariate models, which are used in the DCC models. We formulate out-of-sample one-day-ahead forecasts of variance based on the GARCH, CARR and RGARCH models, where parameters are estimated separately each day based on a rolling sample of a fixed size of 500 (approximately a two-year period; the first

[1] In the R-GARCH model, the Parkinson estimator with the high-low range is used as an explanatory variable but the likelihood function is formulated based on closing prices.

Table 2
Results of parameter estimation for currency rates.

Parameter	DCC-GARCH		DCC-CARR		DCC-RGARCH	
	Estimate	Std. error	Estimate	Std. error	Estimate	Std. error
γ_{10}	−0.011	0.011	–	–	−0.019	0.011
α_{10}	0.001	0.001	0.007	0.003	0.002	0.002
α_{11}	0.037	0.006	0.093	0.012	0.052	0.013
β_{11}	0.960	0.006	0.901	0.013	0.943	0.015
γ_{20}	−0.015	0.013	–	–	−0.005	0.012
α_{20}	0.006	0.004	0.019	0.007	0.010	0.006
α_{21}	0.055	0.019	0.134	0.023	0.133	0.040
β_{21}	0.933	0.024	0.847	0.029	0.843	0.046
γ_{30}	−0.009	0.011	–	–	−0.016	0.010
α_{30}	0.003	0.002	0.006	0.003	0.004	0.002
α_{31}	0.076	0.030	0.110	0.014	0.116	0.049
β_{31}	0.921	0.026	0.883	0.014	0.871	0.041
ζ_1	0.044	0.006	0.048	0.007	0.044	0.006
θ_1	0.922	0.011	0.923	0.012	0.921	0.011
ln L	−5694.139		−5649.297		−5648.297	
Rivers–Vuong	–		2.796 (0.003)		2.563 (0.005)	
Clarke	–		−2.028 (0.979)		6.414 (0.000)	

The sample period is January 2, 2008, to December 30, 2016, the parameters γ_{10}, γ_{20}, γ_{30} are constants, α_{k0} α_{k1} β_{k1} are the parameters of the univariate GARCH model (Eq. (6)), the CARR model (Eq. (13)) and the RGARCH model (Eq. (24)), $k = 1, 2, 3$ for EUR/USD, JPY/USD and GBP/USD, respectively, ζ_1, θ_1 are the parameters of the correlation part (Eqs. (4), (19) and (29) for the DCC-GARCH, DCC-CARR and DCC-RGARCH models, respectively), ln L is the logarithm of the likelihood function, the Rivers–Vuong and Clarke are test statistics for model selection, where comparisons are made with the DCC-GARCH model, p-values are given in brackets. A low p-value means that the indicated model is superior to the benchmark DCC-GARCH model.

Table 3
Results of parameter estimation for exchange-traded funds.

Parameter	DCC-GARCH		DCC-CARR		DCC-RGARCH	
	Estimate	Std. error	Estimate	Std. error	Estimate	Std. error
γ_{10}	−0.127	0.051	–	–	−0.127	0.051
α_{10}	0.090	0.034	0.049	0.015	0.111	0.052
α_{11}	0.056	0.009	0.096	0.011	0.154	0.026
β_{11}	0.932	0.011	0.887	0.014	0.897	0.018
γ_{20}	−0.017	0.036	–	–	−0.055	0.035
α_{20}	0.020	0.011	0.017	0.007	0.031	0.022
α_{21}	0.065	0.014	0.140	0.017	0.236	0.069
β_{21}	0.933	0.014	0.854	0.019	0.864	0.039
γ_{30}	0.058	0.026	–	–	0.016	0.026
α_{30}	0.024	0.009	0.048	0.013	0.019	0.016
α_{31}	0.090	0.015	0.256	0.023	0.382	0.075
β_{31}	0.904	0.015	0.719	0.026	0.748	0.047
ζ_1	0.014	0.003	0.017	0.003	0.013	0.003
θ_1	0.980	0.004	0.980	0.004	0.982	0.005
ln L	−13 419.952		−13 445.131		−13 358.665	
Rivers-Vuong	–		−0.553 (0.710)		3.143 (0.001)	
Clarke	–		−11.344 (1.000)		4.117 (0.000)	

The sample period is January 2, 2008, to December 30, 2016, the parameters γ_{10}, γ_{20}, γ_{30} are constants, α_{k0} α_{k1} β_{k1} are the parameters of the univariate GARCH model (Eq. (6)), the CARR model (Eq. (13)) and the RGARCH model (Eq. (24)), $k = 1, 2, 3$ for Natural Gas, Oil and Energy, respectively, ζ_1, θ_1 are the parameters of the correlation part (Eqs. (4), (19) and (29) for the DCC-GARCH, DCC-CARR and DCC-RGARCH models, respectively). Ln L is the logarithm of the likelihood function, the Rivers–Vuong and Clarke are test statistics for model selection, where comparisons are made with the DCC-GARCH model, p-values are given in brackets. A low p-value means that the indicated model is superior to the benchmark DCC-GARCH model.

in-sample period is from January 2, 2008 to December 31, 2009). We evaluate forecasts for the seven-year period from January 4, 2010, to December 30, 2016.

The sum of squares of 15-min returns (the realized variance) is used as a proxy of the daily variance. The forecasts from the models are evaluated based on two primary measures, namely, the mean squared error (MSE) and the mean absolute error (MAE). In order to evaluate the statistical significance of the results the Diebold–Mariano test (Diebold and Mariano, 1995) corrected for small-sample bias (Harvey et al., 1997) is applied.

A pairwise comparison is performed and the results for the RGARCH model are presented with respect to the two benchmarks: first the GARCH model and second the CARR model. The GARCH and CARR models are the most popular univariate volatility

Table 4
Results of parameter estimation for stocks.

Parameter	DCC-GARCH		DCC-CARR		DCC-RGARCH	
	Estimate	Std. error	Estimate	Std. error	Estimate	Std. error
γ_{10}	0.118	0.045	–	–	0.119	0.043
α_{10}	0.023	0.039	0.057	0.019	0.400	0.138
α_{11}	0.014	0.010	0.187	0.024	0.396	0.087
β_{11}	0.982	0.016	0.793	0.028	0.684	0.071
γ_{20}	0.155	0.038	–	–	0.086	0.034
α_{20}	0.132	0.042	0.128	0.035	0.189	0.069
α_{21}	0.098	0.024	0.264	0.038	0.250	0.055
β_{21}	0.868	0.027	0.679	0.051	0.783	0.046
γ_{30}	0.053	0.037	–	–	0.029	0.033
α_{30}	0.062	0.047	0.064	0.017	0.063	0.036
α_{31}	0.115	0.063	0.241	0.030	0.243	0.054
β_{31}	0.879	0.062	0.734	0.034	0.821	0.037
γ_{40}	0.055	0.035	–	–	0.057	0.032
α_{40}	0.127	0.085	0.081	0.018	0.339	0.140
α_{41}	0.083	0.060	0.240	0.025	0.595	0.193
β_{41}	0.885	0.070	0.720	0.031	0.556	0.143
γ_{50}	0.036	0.026	–	–	0.019	0.024
α_{50}	0.126	0.041	0.061	0.032	0.134	0.033
α_{51}	0.124	0.037	0.221	0.048	0.404	0.065
β_{51}	0.814	0.046	0.741	0.058	0.663	0.049
ζ_1	0.003	0.001	0.006	0.002	0.003	0.000
θ_1	0.993	0.003	0.989	0.004	0.991	0.001
ln L	−21 205.733		−21 055.408		−20 920.942	
Rivers-Vuong	–		2.538 (0.006)		4.910 (0.000)	
Clarke	–		−3.255 (0.999)		12.497 (0.000)	

The sample period is January 2, 2008, to December 30, 2016, the parameters γ_{10}, γ_{20}, γ_{30} are constants, α_{k0} α_{k1} β_{k1} are the parameters of the univariate GARCH model (Eq. (6)), the CARR model (Eq. (13)) and the RGARCH model (Eq. (24)), $k = 1, 2, 3, 4, 5$ for Amazon, Apple, Goldman Sachs, Google and IBM, respectively, ζ_1, θ_1 are the parameters of the correlation part (Eqs. (4), (19) and (29) for the DCC-GARCH, DCC-CARR and DCC-RGARCH models, respectively), ln L is the logarithm of the likelihood function, the Rivers–Vuong and Clarke are test statistics for model selection, where comparisons are made with the DCC-GARCH model, p-values are given in brackets. A low p-value means that the indicated model is superior to the benchmark DCC-GARCH model.

models formulated based on returns constructed on closing prices and price range, respectively. The forecasting performance results are presented in Tables 5 and 6 for the MSE and MAE criteria, respectively.

According to the MSE criterion, the forecasts of variance from the RGARCH model are more accurate for currencies and the Energy Select Sector SPDR Fund. For the other ETFs and stocks, the results are mixed. However, there are large outliers in the data set, which affect the MSE measure. Such outliers are present for ETFs and stocks (see e.g. minimum and maximum returns in Table 1). A quite different picture emerges from the MAE criterion. According to this measure the best forecasts are formulated based on the RGARCH (except Amazon and Apple stocks) and, in almost all cases, the higher forecasting accuracy of this model is statistically significant at the 10% significance level (the exceptions are the GBP/USD currency pair and Google's stock with respect to the CARR benchmark model). The CARR and RGARCH models' forecasting superiority over the GARCH model has already been documented by Chou (2005) and Molnár (2016), respectively. Higher forecast accuracy based on the RGARCH model in comparison to the CARR model has not previously been demonstrated in the literature.

In order to check the robustness of the results, we also consider 5-min returns instead of 15-min returns and three additional evaluation measures (the coefficient of determination, the logarithmic loss function and the linear exponential loss function). The results for the MSE and MAE criteria for 5-min returns are presented in Table A.3 in Appendix. The conclusions are very similar to those presented for 15-min returns.

The first additional measure is the coefficient of determination from the Mincer–Zarnowitz regression. A proxy of volatility is regressed on a constant and the forecast of volatility. It is a very simple and popular way to evaluate the forecasting performance of volatility models (see e.g. Poon and Granger, 2003). The values of the coefficient of determination for the competing models are presented in Table 7. These results are in accordance with those for the MSE measure.

To reduce the impact of outliers, we also use the logarithmic loss function. This is calculated similarly to the MSE measure, but the logarithm of a volatility proxy and the logarithm of the volatility forecast are applied (see Pagan and Schwert, 1990). The estimates of the logarithmic loss function are given in Table 8. These results are very similar to those for the MAE criterion and indicate that the forecasts from the RGARCH model are superior.

Additionally, we apply a linear exponential loss function (LINEX). For the positive coefficient a of the LINEX, the function is approximately linear for over-prediction errors and exponential for under-prediction errors. This means that under-prediction errors have a higher impact on the loss function than over-prediction errors. For the negative coefficient a the situation is exactly the opposite. The values of the LINEX function for $a = -1$ and $a = 1$ are presented in the Appendix in Tables A.4 and A.5 respectively. The results for all currency rates indicate that the variance forecasts based on the RGARCH model are more accurate than the

Table 5
Evaluation of variance forecasts: the MSE criterion.

Assets	GARCH	CARR	RGARCH	GARCH vs. RGARCH	CARR vs. RGARCH
	MSE			P-value of DM test	
Currency rates					
EUR/USD	0.112	0.120	**0.098**	0.010	0.004
GBP/USD	0.811	1.134	**0.560**	0.062	0.197
JPY/USD	0.426	0.485	**0.330**	0.022	0.049
Exchange-traded funds					
Energy	9.133	9.493	**7.558**	0.019	0.004
Oil	**14.049**	19.470	15.005	0.973	0.000
Natural Gas	**22.402**	26.507	23.383	0.960	0.000
Stocks					
Amazon	**164.230**	183.148	181.768	0.978	0.313
Apple	122.262	**94.508**	98.246	0.177	0.857
Goldman Sachs	11.917	11.986	**11.365**	0.264	0.172
Google	**50.899**	58.700	58.730	0.760	0.521
IBM	**11.586**	13.727	13.208	0.834	0.069

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 15-min returns. The lowest values of MSE are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the two benchmarks: the GARCH and CARR models. A p-value lower than the significance level means that the forecasts of variance from the RGARCH model are more accurate than the forecasts from a benchmark model (here GARCH or CARR).

Table 6
Evaluation of variance forecasts: the MAE criterion.

Assets	GARCH	CARR	RGARCH	GARCH vs. RGARCH	CARR vs. RGARCH
	MAE			P-value of DM test	
Currency rates					
EUR/USD	0.166	0.169	**0.155**	0.000	0.000
GBP/USD	0.167	0.161	**0.147**	0.000	0.165
JPY/USD	0.230	0.228	**0.206**	0.000	0.000
Exchange-traded funds					
Energy	1.190	1.292	**1.040**	0.000	0.000
Oil	2.213	2.485	**2.137**	0.002	0.000
Natural Gas	3.233	3.527	**3.198**	0.095	0.000
Stocks					
Amazon	3.704	**3.265**	3.322	0.000	0.974
Apple	2.410	**2.220**	2.263	0.011	0.915
Goldman Sachs	1.752	1.854	**1.682**	0.015	0.000
Google	2.001	1.861	**1.844**	0.013	0.172
IBM	1.064	1.043	**1.007**	0.003	0.000

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 15-min returns. The lowest values of MAE are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the two benchmarks: the GARCH and CARR models. A p-value lower than the significance level means that the forecasts of variance from the RGARCH model are more accurate than the forecasts from a benchmark model (here GARCH or CARR).

forecasts from the competing models. The outcomes for other assets are ambiguous, but they depend heavily on outliers. When the highest 1% of values are excluded, the values of the LINEX loss function are much smaller and more often indicate the RGARCH model as the best forecasting model.

4.3. Comparison of covariance forecasts

In this section, we compare out-of-sample one-day-ahead forecasts of covariance from the DCC-GARCH and DCC-CARR models with the forecasts from the DCC-RGARCH model. We use the same estimation and forecasting samples as for variances in Section 4.2. The sum of products of 15-min returns (the realized covariance) is employed as a proxy of the daily covariance for the evaluation of the forecasts. We use the same evaluation measures as in the previous section. We perform a pairwise comparison by the Diebold–Mariano test for the DCC-RGARCH model with respect to the two benchmarks: first the DCC-GARCH model and second the DCC-CARR model.

The forecasting performance results for the covariance of returns are presented in Tables 9 and 10 for the MSE and MAE criteria, respectively. For all analysed relations except the one between the United States Oil and United States Natural Gas Funds based

Table 7
Evaluation of variance forecasts: the coefficient of determination.

Assets	GARCH	CARR	RGARCH
Currency rates			
EUR/USD	0.254	0.217	**0.355**
GBP/USD	0.305	0.034	**0.513**
JPY/USD	0.200	0.080	**0.417**
Exchange-traded funds			
Energy	0.318	0.290	**0.453**
Oil	**0.405**	0.315	0.372
Natural Gas	**0.253**	0.138	0.216
Stocks			
Amazon	**0.307**	0.084	0.100
Apple	0.089	**0.395**	0.302
Goldman Sachs	0.380	0.390	**0.391**
Google	**0.244**	0.141	0.149
IBM	**0.378**	0.128	0.154

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 15-min returns. The highest values of R^2 are marked in bold.

Table 8
Evaluation of variance forecasts: the logarithmic loss function.

Assets	GARCH	CARR	RGARCH
Currency rates			
EUR/USD	0.326	0.332	**0.294**
GBP/USD	0.249	0.262	**0.208**
JPY/USD	0.487	0.479	**0.404**
Exchange-traded funds			
Energy	0.485	0.552	**0.357**
Oil	0.617	0.631	**0.532**
Natural Gas	0.561	0.641	**0.546**
Stocks			
Amazon	1.039	**0.750**	0.770
Apple	0.974	**0.815**	0.867
Goldman Sachs	0.594	0.605	**0.557**
Google	0.883	**0.723**	0.742
IBM	0.735	0.681	**0.628**

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 15-min returns. The lowest values of the logarithmic loss function are marked in bold.

on the MSE measure, the lowest values of loss functions are found for the DCC-RGARCH model. In most cases, this model's higher forecasting accuracy is statistically significant.[2] In the MAE measure less weight is assigned to outliers and the results for this measure clearly indicate that the DCC-RGARCH model is the best forecasting model.

The forecasts formulated based on the DCC-RGARCH are more precise than the forecasts from both the benchmark models. The first benchmark, DCC-GARCH, is based on returns formulated on the closing prices. This result shows that the application of range data in the standard univariate GARCH model increases the accuracy of covariance forecasts based on the DCC model. The second benchmark, DCC-CARR, is based on range data. This means that the way in which range data is utilized in the univariate volatility model is decisive in determining the forecasting accuracy of the DCC model. Since both benchmarks, i.e. the DCC-GARCH and DCC-CARR models, share the same structure in the correlation component as the DCC-RGARCH model, our results clearly show that more precise volatility estimates improve covariance forecasts.

The DCC-CARR model, which can be treated as the main benchmark model for models constructed based on range data, was not only inferior to the DCC-RGARCH model for most assets, but also inferior to the DCC-GARCH model for currencies and ETFs.

To check the robustness of the results, we also consider 5-min returns instead of 15-min returns and two other loss functions (the coefficient of determination and the LINEX loss function). The results for the MSE and MAE criteria for 5-min returns are presented in Table A.6 in Appendix. The outcomes are very similar to those presented for 15-min returns.

[2] Under the MSE criterion the difference between the loss function of the DCC-RGARCH model and the benchmark model is not statistically significant for EUR/USD-GBP/USD, JPY/USD-GBP/USD, Apple-IBM (with both benchmark models), Oil-Energy (with the DCC-GARCH benchmark) and Amazon-Apple, Amazon-Goldman Sachs, Apple-Google (with the DCC-CARR benchmark). Under the MAE measure there are only two relations for which there is no evidence to reject the null hypothesis of equal predictive ability. These are JPY/USD-GBP/USD (with both benchmark models) and Amazon-Apple (with the DCC-CARR benchmark).

Table 9
Evaluation of covariance forecasts: the MSE criterion.

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH vs. DCC-RGARCH	DCC-CARR vs. DCC-RGARCH
	MSE			P-value of DM test	
Currency rates					
EUR/USD-JPY/USD	0.654	0.748	**0.561**	0.044	0.044
EUR/USD-GBP/USD	0.698	0.941	**0.508**	0.149	0.167
JPY/USD-GBP/USD	1.334	2.075	**1.016**	0.233	0.193
Exchange-traded funds					
Oil-Natural Gas	**62.800**	64.876	63.108	0.750	0.000
Oil-Energy	61.59	103.049	**60.347**	0.182	0.000
Natural Gas-Energy	30.734	81.546	**30.230**	0.036	0.000
Stocks					
Amazon-Apple	198.390	191.357	**172.797**	0.038	0.109
Amazon-Goldman Sachs	70.799	73.782	**70.702**	0.008	0.186
Amazon-Google	169.973	160.102	**148.522**	0.001	0.088
Amazon-IBM	48.763	45.834	**42.501**	0.005	0.088
Apple-Goldman Sachs	99.443	99.274	**87.165**	0.023	0.048
Apple-Google	268.149	265.364	**227.639**	0.097	0.113
Apple-IBM	148.476	145.042	**114.888**	0.122	0.145
Goldman Sachs-Google	66.727	63.389	**56.076**	0.000	0.032
Goldman Sachs-IBM	41.518	40.048	**36.398**	0.000	0.006
Google-IBM	56.165	53.489	**47.270**	0.021	0.076

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 15-min returns. The lowest values of MSE are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the two benchmarks: the DCC-GARCH and DCC-CARR models. A p-value lower than the significance level means that the forecasts of covariance from the DCC-RGARCH model are more accurate than the forecasts from a benchmark model (here DCC-GARCH or DCC-CARR).

Table 10
Evaluation of covariance forecasts: the MAE criterion.

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH vs. DCC-RGARCH	DCC-CARR vs. DCC-RGARCH
	MAE			P-value of DM test	
Currency rates					
EUR/USD-JPY/USD	0.106	0.112	**0.104**	0.002	0.000
EUR/USD-GBP/USD	0.098	0.099	**0.092**	0.000	0.006
JPY/USD-GBP/USD	0.088	0.092	**0.086**	0.166	0.120
Exchange-traded funds					
Oil-Natural Gas	1.453	1.478	**1.446**	0.028	0.000
Oil-Energy	1.291	1.494	**1.204**	0.000	0.000
Natural Gas-Energy	1.046	1.997	**1.024**	0.000	0.000
Stocks					
Amazon-Apple	1.299	1.157	**1.142**	0.000	0.103
Amazon-Goldman Sachs	1.210	1.134	**1.106**	0.000	0.001
Amazon-Google	1.506	1.299	**1.255**	0.000	0.001
Amazon-IBM	0.875	0.793	**0.765**	0.000	0.000
Apple-Goldman Sachs	1.060	1.037	**0.978**	0.000	0.000
Apple-Google	1.157	1.065	**1.015**	0.000	0.000
Apple-IBM	0.820	0.767	**0.716**	0.000	0.000
Goldman Sachs-Google	1.093	1.050	**0.971**	0.000	0.000
Goldman Sachs-IBM	0.841	0.813	**0.752**	0.000	0.000
Google-IBM	0.743	0.689	**0.651**	0.000	0.000

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 15-min returns. The lowest values of MAE are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the two benchmarks: the DCC-GARCH and DCC-CARR models. A p-value lower than the significance level means that the forecasts of covariance from the DCC-RGARCH model are more accurate than the forecasts from a benchmark model (here DCC-GARCH or DCC-CARR).

Table 11 presents the coefficient of determination values from the Mincer–Zarnowitz regression. A proxy of covariance is regressed on a constant and the forecast of covariance. We are unable to calculate the logarithmic loss function (see Section 4.2) because some covariances are negative.

Table 11
Evaluation of covariance forecasts: the coefficient of determination.

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH
Currency rates			
EUR/USD-JPY/USD	0.224	0.098	**0.364**
EUR/USD-GBP/USD	0.320	0.097	**0.513**
JPY/USD-GBP/USD	0.370	0.016	**0.507**
Exchange-traded funds			
Oil-Natural Gas	**0.023**	0.005	0.016
Oil-Energy	0.373	0.083	**0.392**
Natural Gas-Energy	0.029	0.012	**0.031**
Stocks			
Amazon-Apple	0.033	0.070	**0.215**
Amazon-Goldman Sachs	0.054	0.103	**0.154**
Amazon-Google	0.050	0.115	**0.256**
Amazon-IBM	0.049	0.116	**0.251**
Apple-Goldman Sachs	0.070	0.076	**0.208**
Apple-Google	0.045	0.050	**0.279**
Apple-IBM	0.025	0.047	**0.404**
Goldman Sachs-Google	0.077	0.122	**0.248**
Goldman Sachs-IBM	0.084	0.119	**0.250**
Google-IBM	0.050	0.108	**0.327**

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance based on 15-min returns is used as a proxy of covariance. The highest values of R^2 are marked in bold.

For all covariances except the relation between the United States Oil and United States Natural Gas Funds the highest R^2 values are obtained for the DCC-RGARCH model. In most cases the superiority of this model is considerable.

We obtain different results for the asymmetric loss function LINEX. The values of the function for $a = -1$ and $a = 1$ are presented in the Appendix in Tables A.7 and A.8, respectively. The results for all relations between currencies rates indicate that the covariance forecasts based on the DCC-RGARCH model are more accurate than the forecasts from the competing DCC models. The outcomes for other assets are mixed but outliers have considerable influence on the evaluation. After excluding the highest 1% of values the results depend on the valuation of the over- and under-prediction errors. For $a = -1$, i.e. when over-prediction errors have a higher impact on the loss function, then the best forecasts are based on the DCC-RGARCH model, whereas for $a = 1$, i.e. when under-prediction errors have a greater influence on the LINEX, then the DCC-CARR is better according to this criterion.

4.4. Forecasting value-at-risk

Covariance forecasting is crucial for most multivariate financial applications, such as portfolio construction, valuation of assets, risk management and contagion effect. More accurate covariance forecasts give an advantage in various financial applications. That is why covariance forecasting, similarly like volatility forecasting, has not only statistical but also economic consequences.

In this subsection we apply the considered DCC models to one such application, namely the evaluation of risk, using the value-at-risk (VaR) measure. VaR was developed by financial practitioners as an easily interpretable number which encodes information about a portfolio's risk. Despite being a single number, VaR enables managers to interpret the cost of risk and allocate capital efficiently. We formulate daily forecasts of VaR for three separate portfolios of currency rates, commodity exchange traded funds and stocks. All the portfolios are constructed with equal weights. The same assets and forecasting period are assumed as in the analysis of variances and covariances in Sections 4.2 and 4.3 We construct VaR forecasts for the 95% and 99% confidence levels.

Our evaluation of the forecasts is based on two approaches: the first involves testing the competing VaR models for statistical accuracy, while the second pertains to measuring the loss to the economic agent as a result of using the model. We test the statistical adequacy of the forecasts based on: the unconditional coverage test by Kupiec (1995), the independence and conditional coverage tests by Christoffersen (1998), and the unconditional coverage, independence and conditional coverage tests by Candelon et al. (2011). The results of these tests for the 95% VaR forecasts are presented in Table 12 (the outcomes for the 99% confidence level are given in Table A.9 in the Appendix). The results for the Candelon et al. (2011) tests are presented for 5 moments, but we also obtained very similar results for 1, 2, 3, 4 and 6 moments.

We do not obtain fully satisfactory results for all portfolios for any of the models, but the outcomes depend heavily on the kind of assets and tests applied. The statistical test results do not differ sufficiently between the competing models to clearly indicate which is a better model.

In the second approach, we perform an economic evaluation of the models based on loss functions. We concentrate on firm loss functions. This approach emphasizes the role of the utility function of risk managers, who have to consider their firms' profitability, and therefore prefer smaller scaled VaR measures for efficient capital allocation. In order to assess whether the differences between loss functions are statistically significant, we apply the Diebold–Mariano test. The results for the 95% VaR forecasts are given in Table 13 (the outcomes for the 99% confidence level are presented in Table A.10 in the Appendix).

Table 12
Evaluation of 95% VaR forecasts: unconditional coverage and independence tests.

Statistic	DCC-GARCH		DCC-CARR		DCC-RGARCH	
	Value	P-value	Value	P-value	Value	P-value
Currency rates						
LR_{UC}	0.549	0.489	0.436	0.509	0.927	0.336
LR_{IND}	1.207	0.272	2.783	0.095	1.019	0.313
LR_{CC}	1.756	0.416	3.219	0.200	1.946	0.378
J_{UC}	0.417	0.532	0.557	0.494	0.772	0.353
J_{IND}	5.049	0.095	0.318	0.929	16.040	0.010
J_{CC}	7.944	0.082	0.736	0.905	37.134	0.010
Exchange-traded funds						
LR_{UC}	3.294	0.070	0.000	0.991	0.618	0.432
LR_{IND}	0.368	0.544	0.548	0.459	1.041	0.308
LR_{CC}	3.662	0.160	0.548	0.760	1.660	0.436
J_{UC}	3.288	0.063	0.010	0.924	0.478	0.486
J_{IND}	4.312	0.130	2.181	0.363	5.593	0.075
J_{CC}	10.308	0.048	2.192	0.567	6.879	0.105
Stocks						
LR_{UC}	2.202	0.139	20.416	0.000	4.869	0.027
LR_{IND}	0.002	0.968	1.433	0.231	0.251	0.616
LR_{CC}	2.203	0.332	21.850	0.000	5.120	0.077
J_{UC}	2.269	0.139	28.409	0.000	4.590	0.031
J_{IND}	3.307	0.202	69.453	0.002	6.289	0.060
J_{CC}	5.880	0.146	19970.690	0.000	11.226	0.036

The evaluation period is January 4, 2010, to December 30, 2016, LR_{UC} is the statistic for the Kupiec (1995) unconditional coverage test, LR_{IND} is the statistic for the Christoffersen (1998) independence test, LR_{CC} is the statistic for the Christoffersen (1998) conditional coverage test, J_{UC} is the statistic for the Candelon et al. (2011) unconditional coverage test, J_{IND} is the statistic for the Candelon et al. (2011) independence test for up to five lags, J_{CC} is the statistic for the Candelon et al. (2011) conditional coverage test with the number of moments fixed to 5, p-values for J_{UC}, J_{IND}, J_{CC} were corrected by Dufour's (2006) Monte Carlo procedure.

Table 13
Evaluation of 95% VaR forecasts: firm loss functions tests.

Loss function	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH vs. DCC-RGARCH	DCC-CARR vs. DCC-RGARCH
	Value of loss function × 10			P-value of DM test	
Currency rates					
FLF(STS)	0.371	0.394	**0.369**	0.158	0.014
FLF(C1)	5.991	6.091	**5.972**	0.033	0.000
FLF(C2)	3.079	3.204	**3.055**	0.127	0.000
FLF(C3)	7.189	7.203	**7.161**	0.153	0.032
Exchange-traded funds					
FLF(STS)	**1.500**	1.689	1.566	0.975	0.001
FLF(C1)	5.829	5.912	**5.804**	0.044	0.000
FLF(C2)	9.776	10.144	**9.627**	0.016	0.000
FLF(C3)	23.725	23.918	**22.428**	0.001	0.000
Stocks					
FLF(STS)	1.533	1.806	**1.480**	0.077	0.000
FLF(C1)	6.018	6.811	**5.956**	0.001	0.000
FLF(C2)	7.784	14.668	**7.472**	0.000	0.000
FLF(C3)	18.911	27.205	**18.612**	0.003	0.000

The evaluation period is January 4, 2010, to December 30, 2016, FLF(STS) is the loss function by Sarma et al. (2003), FLF(C1), FLF(C2), FLF(C3) are three loss functions by Caporin (2008). The lowest values of loss functions are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the two benchmarks: the DCC-GARCH and DCC-CARR models. A p-value lower than the significance level means that economic losses for the DCC-RGARCH model are lower than losses for a benchmark model (here DCC-GARCH or DCC-CARR).

For most of the considered loss functions, significantly more accurate VaR forecasts are constructed based on the DCC-RGARCH model than the DCC-GARCH or DCC-CARR models. This means that risk managers should prefer the DCC-RGARCH model for the estimation of their VaR forecasts. The results are very similar for both commonly employed confidence levels, 95% and 99%.

Table A.1
Evaluation of covariance forecasts based on DCC-EGARCH and DCC-GJR models: the MSE criterion.

Assets	DCC-GARCH	DCC-EGARCH	DCC-GJR	DCC-GARCH vs. DCC-EGARCH	DCC-GARCH vs. DCC-GJR
	MSE			P-value of DM test	
Currency rates					
EUR/USD-JPY/USD	**0.654**	0.756	0.755	0.953	0.946
EUR/USD-GBP/USD	**0.698**	0.939	0.923	0.809	0.797
JPY/USD-GBP/USD	**1.334**	2.033	2.052	0.812	0.819
Exchange-traded funds					
Oil-Natural Gas	**62.800**	64.495	64.842	0.997	1.000
Oil-Energy	**61.59**	67.994	67.968	0.999	0.995
Natural Gas-Energy	**30.734**	31.307	31.530	0.984	1.000
Stocks					
Amazon-Apple	**198.291**	198.909	200.043	0.682	0.792
Amazon-Goldman Sachs	77.756	76.526	**75.877**	0.086	0.067
Amazon-Google	**169.777**	175.503	175.194	0.926	0.914
Amazon-IBM	48.742	**47.569**	48.025	0.005	0.092
Apple-Goldman Sachs	**99.387**	103.087	102.637	0.989	0.989
Apple-Google	**267.998**	279.135	274.478	1.000	0.853
Apple-IBM	**148.393**	150.017	150.018	0.987	0.990
Goldman Sachs-Google	**66.695**	70.884	69.567	1.000	0.995
Goldman Sachs-IBM	**41.495**	41.751	41.582	0.789	0.654
Google-IBM	**56.133**	56.646	56.662	0.873	0.878

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 15-min returns. The lowest values of MSE are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the benchmark the DCC-GARCH model. A p-value lower than the significance level means that the forecasts of covariance from the DCC-EGARCH or DCC-GJR models are more accurate than the forecasts from the DCC-GARCH model.

Table A.2
Evaluation of covariance forecasts based on DCC-EGARCH and DCC-GJR models: the MAE criterion.

Assets	DCC-GARCH	DCC-EGARCH	DCC-GJR	DCC-GARCH vs. DCC-EGARCH	DCC-GARCH vs. DCC-GJR
	MAE			P-value of DM test	
Currency rates					
EUR/USD-JPY/USD	**0.106**	0.113	0.111	1.000	1.000
EUR/USD-GBP/USD	**0.098**	0.107	0.100	1.000	0.801
JPY/USD-GBP/USD	**0.088**	0.090	0.089	0.754	0.639
Exchange-traded funds					
Oil-Natural Gas	**1.453**	1.466	1.468	0.992	1.000
Oil-Energy	**1.291**	1.297	1.302	0.650	0.734
Natural Gas-Energy	1.046	**1.043**	1.051	0.232	0.842
Stocks					
Amazon-Apple	1.299	**1.258**	1.289	0.000	0.134
Amazon-Goldman Sachs	1.209	1.134	**1.145**	0.000	0.000
Amazon-Google	1.506	1.333	**1.329**	0.000	0.000
Amazon-IBM	0.875	0.820	**0.829**	0.000	0.000
Apple-Goldman Sachs	**1.060**	1.115	1.158	1.000	1.000
Apple-Google	**1.157**	1.483	1.483	1.000	1.000
Apple-IBM	**0.820**	0.866	0.891	1.000	1.000
Goldman Sachs-Google	**1.093**	1.134	1.120	1.000	1.000
Goldman Sachs-IBM	0.841	0.828	**0.839**	0.005	0.355
Google-IBM	**0.742**	0.755	0.755	0.993	0.991

The realized covariance is used as a proxy of covariance and estimated as the sum of products of 15-min returns. The lowest values of MAE are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the benchmark DCC-GARCH model. A p-value lower than the significance level means that the forecasts of covariance from the DCC-EGARCH or DCC-GJR models are more accurate than the forecasts from the DCC-GARCH model.

5. Conclusion

The DCC-GARCH model is one of the most popular multivariate volatility models, due to its simplicity and ease of estimation. However, its parameters are usually estimated based only on closing prices, even though high and low prices contain more information about volatility. In this study, we have proposed a new specification of the DCC model called the DCC-Range-GARCH

Table A.3
Evaluation of variance forecasts: the MSE and MAE criteria for realized variance calculated based on 5-min returns.

Assets	GARCH	CARR	RGARCH	GARCH	CARR	RGARCH
	MSE			MAE		
Currency rates						
EUR/USD	0.106	0.111	**0.093**	0.157	0.158	**0.147**
GBP/USD	0.721	1.025	**0.486**	0.158	0.149	**0.139**
JPY/USD	0.411	0.462	**0.317**	0.219	0.215	**0.194**
Exchange-traded funds						
Energy	7.695	8.086	**6.581**	1.138	1.232	**0.991**
Oil	**13.154**	18.126	14.142	2.119	2.365	**2.043**
Natural Gas	**20.715**	24.516	21.849	3.061	3.350	**3.038**
Stocks						
Amazon	199.106	**192.202**	194.809	3.941	**2.347**	3.393
Apple	**96.345**	97.437	102.881	2.504	3.315	**2.404**
Goldman Sachs	18.464	17.595	**17.450**	1.901	1.953	**1.806**
Google	68.873	**59.054**	60.641	2.174	**1.898**	1.910
IBM	15.504	**14.592**	14.935	1.180	**1.078**	1.099

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 5-min returns. The lowest values of MSE and MAE are marked in bold.

Table A.4
Evaluation of variance forecasts: the LINEX function with a = −1.

Assets	GARCH	CARR	RGARCH	GARCH	CARR	RGARCH
	Full sample			Excluding 1% of upper outliers		
Currency rates						
EUR/USD	0.301	0.407	**0.210**	0.019	0.021	**0.018**
GBP/USD	1015.023	1.500e+04	**59.152**	0.013	0.015	**0.012**
JPY/USD	5.190e+07	8.880e+13	**1.012e+05**	0.036	0.033	**0.029**
Exchange-traded funds						
Energy	**4.842**	275.893	12.034	1.410	2.584	**0.872**
Oil	**7784.863**	3.710e+08	1.185e+04	77.673	148.632	**54.823**
Natural Gas	2379.619	3005.722	**2379.228**	**34.479**	1486.421	62.574
Stocks						
Amazon	698.416	**32.762**	62.284	2.094	1.801	**1.483**
Apple	**328.116**	4.120e+13	3.530e+16	13.742	**5.866**	7.255
Goldman Sachs	7.469e+05	**5.362e+04**	7.375e+04	**2.044**	3.335	2.046
Google	1.120e+18	52.643	**29.517**	1.625	1.297	**1.042**
IBM	1.396	**0.880**	1.176	0.375	0.334	**0.324**

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 15-min returns. The lowest values of the LINEX function are marked in bold.

Table A.5
Evaluation of variance forecasts: the LINEX function with a = 1.

Assets	GARCH	CARR	RGARCH	GARCH	CARR	RGARCH
	Full sample			Excluding 1% of upper outliers		
Currency rates						
EUR/USD	0.301	0.407	**0.210**	2,206e−02	2,283e−02	**2,031e−02**
GBP/USD	5,190e+07	8,881e+13	**1,012e+05**	1,504e−02	1,504e−02	**1,285e−02**
JPY/USD	1,015e+03	1,500e+04	**5,915e+01**	4,273e−02	4,170e−02	**3,617e−02**
Exchange-traded funds						
Energy	7,661e+24	6,502e+25	**3,173e+24**	101.585	26.510	**24.855**
Oil	**7,990e+16**	2,846e+17	1,817e+17	**2,463e+04**	8,599e+04	6,073e+04
Natural Gas	**1,206e+21**	1,793e+23	1,449e+23	2732.795	4001.910	**2715.055**

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Table A.5 (*continued*).

Assets	GARCH	CARR	RGARCH	GARCH	CARR	RGARCH
	Full sample			Excluding 1% of upper outliers		
Stocks						
Amazon	9,198e+135	4,589e+138	**3,725e+135**	2,250e+11	**1,790e+10**	1,416e+11
Apple	4,573e+128	**1,856e+95**	1,600e+97	**2,606e+07**	8,760e+07	1,921e+08
Goldman Sachs	8,917e+15	1,104e+16	**2,803e+14**	405.994	102.403	**243.711**
Google	6,214e+67	**4,470e+67**	8,280e+67	**3,076e+03**	3,737e+04	4,420e+04
IBM	**7,689e+22**	1,113e+30	8,391e+29	51.523	50.826	**41.546**

The evaluation period is January 4, 2010, to December 30, 2016, the realized variance is used as a proxy of variance and estimated as the sum of squares of 15-min returns. The lowest values of the LINEX function are marked in bold.

Table A.6
Evaluation of covariance forecasts: the MSE and MAE criteria for realized covariance calculated based on 5-min returns.

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH	DCC-CARR	DCC-RGARCH
	MSE			MAE		
Currency rates						
EUR/USD-JPY/USD	0.041	0.048	**0.035**	0.096	0.101	**0.093**
EUR/USD-GBP/USD	0.060	0.082	**0.043**	0.094	0.093	**0.087**
JPY/USD-GBP/USD	0.071	0.122	**0.055**	0.080	0.083	**0.078**
Exchange-traded funds						
Oil-Natural Gas	**5.940**	6.146	5.979	1.392	1.417	**1.383**
Oil-Energy	5.714	9.734	**5.640**	1.247	1.451	**1.161**
Natural Gas-Energy	2.923	8.124	**2.869**	1.002	2.005	**0.976**
Stocks						
Amazon-Apple	15.597	14.959	**13.280**	1.202	1.059	**1.051**
Amazon-Goldman Sachs	6.657	6.230	**5.994**	1.140	1.051	**1.034**
Amazon-Google	14.906	13.894	**12.957**	1.441	1.222	**1.193**
Amazon-IBM	4.315	4.038	**3.733**	0.826	0.739	**0.714**
Apple-Goldman Sachs	7.480	7.534	**6.677**	0.993	0.974	**0.917**
Apple-Google	17.530	17.337	**14.424**	1.075	0.990	**0.942**
Apple-IBM	11.066	10.786	**8.328**	0.781	0.728	**0.680**
Goldman Sachs-Google	5.318	4.966	**4.360**	1.027	0.980	**0.899**
Goldman Sachs-IBM	3.404	3.272	**2.999**	0.791	0.770	**0.709**
Google-IBM	4.630	4.360	**3.827**	0.705	0.649	**0.614**

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 5-min returns. The lowest values of MSE and MAE are marked in bold.

Table A.7
Evaluation of covariance forecasts: the LINEX function with a = −1.

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH	DCC-CARR	DCC-RGARCH
	Full sample			Excluding 1% of upper outliers		
Currency rates						
EUR/USD-JPY/USD	0.149	0.338	**0.076**	8,744e−03	6,985e−03	**6,085e−03**
EUR/USD-GBP/USD	0.020	0.019	**0.021**	8,755e−03	9,538e−03	**8,503e−03**
JPY/USD-GBP/USD	334.772	3,301e+04	**11.909**	5,310e−03	5,535e−03	**5,004e−03**
Exchange-traded funds						
Oil-Natural Gas	2,590e+06	**1,771e+06**	2,001e+06	8.122	8.769	**7.664**
Oil-Energy	2.190	**1.072**	1.585	1.368	0.870	**1.072**
Natural Gas-Energy	1716.904	3,315e+05	**1000.640**	2.278	107.617	**2.016**
Stocks						
Amazon-Apple	7075.788	**2292.080**	3985.507	0.722	**0.487**	0.518
Amazon-Goldman Sachs	5106.527	**2016.502**	5924.394	0.635	**0.556**	0.565
Amazon-Google	1,280e+08	1,570e+08	**1,170e+08**	1.008	0.670	**0.665**
Amazon-IBM	**51.210**	53.546	74.122	0.313	0.234	**0.230**

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Table A.7 (*continued*).

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH	DCC-CARR	DCC-RGARCH
	Full sample			Excluding 1% of upper outliers		
Apple-Goldman Sachs	2,750e+05	**2,350e+05**	2,446e+05	0.496	0.472	**0.428**
Apple-Google	**13.275**	111.506	1104.156	0.641	0.521	**0.493**
Apple-IBM	**1.060**	1.310	26.865	**0.258**	0.578	0.545
Goldman Sachs-Google	3,820e+08	6,970e+08	**3,480e+08**	0.582	0.578	**0.545**
Goldman Sachs-IBM	706.926	**523.911**	869.393	0.270	0.252	**0.235**
Google-IBM	1,367e+06	6,258e+05	**1,188e+06**	0.190	0.168	**0.162**

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 15-min returns. The lowest values of the LINEX function are marked in bold.

Table A.8
Evaluation of covariance forecasts: the LINEX function with a = 1.

Assets	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH	DCC-CARR	DCC-RGARCH
	Full sample			Excluding 1% of upper outliers		
Currency rates						
EUR/USD-JPY/USD	0.082	0.094	**0.053**	9,379e−03	**1,038e−02**	9,186e−03
EUR/USD-GBP/USD	2.832	33.415	**0.322**	7,551e−03	7,295e−03	**6,649e−03**
JPY/USD-GBP/USD	0.043	0.035	**0.100**	5,626e−03	5,913e−03	**5,303e−03**
Exchange-traded funds						
Oil-Natural Gas	**4,393e+06**	7,413e+06	8,555e+06	**7.022**	8.507	8.077
Oil-Energy	8,996e+12	2,484e+14	**7,500e+12**	68.685	1194.447	**54.811**
Natural Gas-Energy	3,420e+04	**4,503e+03**	4,924e+04	**1.060**	1.298	1.128
Stocks						
Amazon-Apple	**8,926e+54**	2,283e+55	1,786e+55	20.741	**12.284**	15.554
Amazon-Goldman Sachs	**3,554e+23**	6,687e+23	4,391e+23	16.859	**14.397**	23.327
Amazon-Google	**1,992e+33**	1,023e+34	6,252e+33	1217.696	**342.058**	744.859
Amazon-IBM	4,911e+16	5,291e+16	**4,548e+16**	5.500	**3.768**	5.002
Apple-Goldman Sachs	**1,213e+38**	4,439e+38	3,180e+38	3.709	**2.987**	3.258
Apple-Google	8,100e+11	**2,099e+10**	3,360e+10	976.587	1301.767	**935.519**
Apple-IBM	1,485e+07	**1,252e+07**	1,533e+07	5.479	**3.032**	3.542
Goldman Sachs-Google	9,756e+22	6,748e+23	**1,957e+09**	14.760	**6.429**	10.240
Goldman Sachs-IBM	**6,722e+12**	9,757e+12	7,831e+12	1.973	**1.700**	1.848
Google-IBM	7,565e+22	7,253e+22	**2,276e+16**	1.404	1.489	**1.322**

The evaluation period is January 4, 2010, to December 30, 2016, the realized covariance is used as a proxy of covariance and estimated as the sum of products of 15-min returns. The lowest values of the LINEX function are marked in bold.

model, which is a combination of the DCC model by Engle (2002) and the Range-GARCH model by Molnár (2016). The DCC-Range-GARCH model is very similar to the DCC model by Engle but it is based on a much more efficient volatility estimator formulated on the daily range, the log-difference between the high and low prices.

We have compared our DCC-Range-GARCH model to the DCC-GARCH model by Engle (2002) and the DCC-CARR model by Chou et al. (2009). All these three models are very similar in their correlation part, but differ in their specification for conditional variances. The DCC-GARCH model is based on the GARCH model, the DCC-Range-GARCH model is formulated on the Range-GARCH model and the DCC-CARR model is based on the CARR model by Chou (2005). We have evaluated these models on three data sets: currencies, exchange traded funds and stocks.

The univariate range-based models, CARR and Range-GARCH, had not been previously compared. We therefore first compare forecasting accuracy of these models. We found that the CARR model is outperformed by the Range-GARCH model. Surprisingly, the CARR model is often inferior even to the standard GARCH model, whereas the Range-GARCH model outperforms it in most cases. We then turned our attention to multivariate models and the comparison of covariance forecasts, which were the main focus of this paper. We found that the proposed DCC-Range-GARCH model is superior not only to the standard DCC-GARCH model but also to the DCC-CARR model.

Our results illustrate that the use of range data in the DCC model can improve the estimation of covariances of returns and increase the accuracy of covariance and VaR forecasts based on this model, compared with using closing prices only. Moreover, the way the range is utilized matters, as our proposed model outperforms the DCC-CARR model, which is also based on range. Therefore, other multivariate range-based volatility models such as the double smooth transition conditional correlation CARR model by Chou and Cai (2009), the range-based copula models by Chiang and Wang (2011) and Wu and Liang (2011) and the range-based regime-switching dynamic conditional correlation model by Su and Wu (2014) would probably also benefit from using the Range-GARCH model in place of the CARR specification.

Table A.9
Evaluation of 99% VaR forecasts: unconditional coverage and independence tests.

Statistic	DCC-GARCH		DCC-CARR		DCC-RGARCH	
	Value	P-value	Value	P-value	Value	P-value
Currency rates						
LR_{UC}	0.270	0.603	4.621	0.032	0.076	0.782
LR_{IND}	0.285	0.594	0.567	0.452	0.321	0.571
LR_{CC}	0.555	0.758	5.188	0.075	0.398	0.820
J_{UC}	0.080	0.746	4.093	0.042	0.001	0.911
J_{IND}	3.275	0.130	3.308	0.129	1.858	0.310
J_{CC}	3.728	0.253	6.720	0.087	2.012	0.520
Exchange-traded funds						
LR_{UC}	2.039	0.153	0.008	0.928	0.414	0.520
LR_{IND}	0.165	0.685	0.372	0.542	0.258	0.612
LR_{CC}	2.204	0.332	0.380	0.827	0.672	0.715
J_{UC}	1.658	0.206	0.101	0.739	0.166	0.652
J_{IND}	2.770	0.181	0.508	0.788	0.323	0.883
J_{CC}	16.695	0.019	0.545	0.990	0.559	0.915
Stocks						
LR_{UC}	26.337	0.000	1.514	0.219	28.175	0.000
LR_{IND}	0.721	0.396	1.050	0.306	0.631	0.427
LR_{CC}	27.059	0.000	2.564	0.278	28.806	0.000
J_{UC}	15.976	0.001	1.713	0.159	16.827	0.001
J_{IND}	8.888	0.013	83.216	0.000	5.052	0.055
J_{CC}	27.499	0.008	31.612	0.008	26.992	0.007

The evaluation period is January 4, 2010, to December 30, 2016, LR_{UC} is the statistic for the Kupiec (1995) unconditional coverage test, LR_{IND} is the statistic for the Christoffersen (1998) independence test, LR_{CC} is the statistic for the Christoffersen (1998) conditional coverage test, J_{UC} is the statistic for the Candelon et al. (2011) unconditional coverage test, J_{IND} is the statistic for the Candelon et al. (2011) independence test for up to five lags, J_{CC} is the statistic for the Candelon et al. (2011) conditional coverage test with the number of moments fixed to 5, p-values for J_{UC}, J_{IND}, J_{CC} were corrected by Dufour's (2006) Monte Carlo procedure.

Table A.10
Evaluation of 99% VaR forecasts: firm loss functions tests.

Loss function	DCC-GARCH	DCC-CARR	DCC-RGARCH	DCC-GARCH vs. DCC-RGARCH	DCC-CARR vs. DCC-RGARCH
	Value of loss function × 10^1			P-value of DM test	
Currency rates					
FLF(STS)	0.506	0.514	**0.502**	0.066	0.021
FLF(C1)	6.913	6.927	**6.900**	0.080	0.012
FLF(C2)	5.322	5.405	**5.280**	0.110	0.000
FLF(C3)	9.981	9.988	**9.939**	0.149	0.076
Exchange-traded funds					
FLF(STS)	**1.744**	1.849	1.751	0.599	0.001
FLF(C1)	6.865	6.862	**6.800**	0.000	0.000
FLF(C2)	17.239	17.239	**16.764**	0.000	0.000
FLF(C3)	33.010	33.260	**32.392**	0.000	0.000
Stocks					
FLF(STS)	1.474	2.029	**1.416**	0.014	0.000
FLF(C1)	6.811	7.535	**6.734**	0.000	0.000
FLF(C2)	13.532	23.767	**12.907**	0.000	0.000
FLF(C3)	26.455	37.688	**25.790**	0.000	0.000

The evaluation period is January 4, 2010, to December 30, 2016, FLF(STS) is the loss function by Sarma et al. (2003), FLF(C1), FLF(C2), FLF(C3) are three loss functions by Caporin (2008). The lowest values of loss functions are marked in bold. The p-values of the Diebold–Mariano test are presented for pairs of models with respect to the two benchmarks: the DCC-GARCH and DCC-CARR models. A p-value lower than the significance level means that economic losses for the DCC-RGARCH model are lower than those for a benchmark model (here DCC-GARCH or DCC-CARR).

Appendix

See Tables A.1–A.10.

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