from eq^n ①.

$$= (2x + y - 2x,\ 3x + 4(y - 2x))$$

$$= (y,\ 4y - 5x)$$

Unit -3

Differential Equations Mathematics - II

	order	degree
dy/dx	1	1
d^2y/dx^2	2	1
$(dy/dx)^2$	1	2
$(d^3y/dx^3)^2$	3	2.

ODE (Ordinary differential equation) -

an eq^n contains only ordinary derivatives of one or more dependent variables of single independent variable.

Eg: $dy/dx + 5y = e^x$, $(dx/dt) + (dy/dt) = 2x + y$.

PDE (Partial Differential equation) -

An eq^n contains partial derivatives of one or more dependent variables of two or more independent variables.

eg: $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial t^2} = 2\dfrac{\partial u}{\partial t}$, $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

Exact Differential eq^n

1st order DE $\underline{M(x,y)dx + N(x,y)dy = 0}$

is called an exact DE if.

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$$\boxed{\int M\,dx \text{ (treating } y \text{ as constant)} + \int N \text{ (free from } x) \, dy = c}$$

Q1) $\underbrace{\left[4x^3y^3 + \frac{1}{x}\right]}_{M} dx + \underbrace{\left[3x^4y^2 - \frac{1}{y}\right]}_{N} dy = 0.$

① Check exactness →

$$\frac{\partial M}{\partial y}_{x \to const} = \frac{\partial}{\partial y}\left[4x^3y^3 + \frac{1}{x}\right] \quad x \to \text{constant}$$

$$= 4x^3 \times 3y^2 + 0 = 12x^3y^2$$

$$\frac{\partial N}{\partial x}_{y \to const} = \frac{\partial}{\partial x}\left[3x^4y^2 - \frac{1}{y}\right] \quad y \to \text{constant}$$

$$= 12x^3y^2.$$

$$\Rightarrow \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \quad \therefore eq^n \text{ is exact.}$$

② $\int \frac{dx}{x} = \log x.$

① $\int x^n dx = \frac{x^{n+1}}{n+1}$

② Solutⁿ → $\int_{y \to const} M\,dx + \int_{x \to const} N\,(\text{free from } x)\,\partial y = c$

$$\int_{y \to const.} \left(4x^3y^3 + \frac{1}{x}\right) dx + \int_{x \to \cancel{const}\ \text{free from } x} \left(3x^4y^2 - \frac{1}{y}\right) dy.$$

y → constant

$\int \left(0 - \frac{1}{y}\right)$

$$\cancel{4}y^3 \frac{x^4}{\cancel{4}} + \log x - \log y = c$$

$$\Rightarrow \boxed{y^3x^4 + \log\left(\frac{x}{y}\right) = c}$$

Q2) $\cos x\,(\cos x - \underset{M}{\sin a \sin y})\,dx + \cos y\,(\cos y - \underset{N}{\sin a \sin x})\,dy = 0$

① Check for exactness →

$$\frac{\partial M}{\partial y}_{x \to const.} = \frac{\partial}{\partial y}\left[\cos^2 x - \sin a \sin y \cos x\right]_{x\, const.}$$

$$= \left[0 - \sin a \cos y \cos x\right].$$

$$\frac{\partial N}{\partial x}_{y \to const} = \frac{\partial}{\partial x}\left[\cos^2 y - \sin a \sin x \cos y\right]_{y \to const.}$$

$$= \left[0 - \sin a \cos x \cos y\right]$$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ∴ it is exact.

③ $\cos^2 x = \frac{\cos 2x + 1}{2}$ ④ $\int \cos ax\,dx = +\frac{\sin ax}{a} + c$

⑥ $\int dx = x$ ⑤ $\int \sin ax\,dx = -\frac{\cos ax}{a} + c$

② solution →

$$\int_{y=\text{const}} M\,dx + \int N(\text{free from } x)\,dy = c$$

$$\int_{y=\text{const.}} (\cos^2 x - \sin a \cos x \sin y)\,dx + \int (\cos^2 y - \sin a \sin x \cos y)\,dy$$

$$\int_{y=\text{const.}} \left(\frac{\cos 2x}{2} + \frac{1}{2} - \sin a \cos x \sin y\right) dx + \int \cos^2 y\,dy$$

(sirf free from x wala term lenge)

$$= \frac{1}{2}\frac{\sin 2x}{2} + \frac{1}{2}x - \sin a \sin y \sin x + \int \frac{\cos 2y + 1}{2}\,dy.$$

$$= \frac{1}{2}\frac{\sin 2x}{2} + \frac{1}{2}x - \sin a \sin y \sin x + \frac{1}{2}\frac{\sin 2y}{2} + \frac{1}{2}y$$

$$\Rightarrow \frac{\sin 2x}{4} + \frac{x}{2} - \sin a \sin y \sin x + \frac{\sin 2y}{4} + \frac{y}{2} = c$$

ans

Q3) $(\underbrace{x^2 + y^2 + 2x}_{M})dx + \underbrace{2y}_{N}\,dy = 0.$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x};$$

$$\frac{\partial M}{\partial y} = 2y, \quad \frac{\partial N}{\partial x} = 0$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$ equation is not exact.

Way to convert eq^n into exact eq^n

• Rule 1: if $\boxed{\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = g(x)}$

is a function of x.

alone then, $\boxed{\mu = e^{\int g(x).dx}}$

is an integrating factor of the given D.E.

Eg. for Rule 1: (Q3) $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = g(x)$

$\dfrac{2y - 0}{2y} = 1$ only function of x.

$IF = \mu = e^{\int g(x).dx} = e^{\int 1.dx} = e^{x}$.

to convert non-exact to exact eq^n.

multiply by IF, in Q3.

$$\underbrace{(x^2+y^2+2x)e^x}_{M} dx + \underbrace{2y e^x}_{N} dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial (x^2+y^2+2x)e^x}{\partial y} = 2ye^x.$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2y.e^x) = 2y.e^x.$$

eq^n is exact.

$$\int (x^2e^x + y^2e^x + 2xe^x)\,dx.$$

$$= \int \underset{I}{x^2} . e^x dx + y^2 \int_{II} e^x dx + 2\int \underset{III}{x . e^x} dx.$$

$$= x^2 . e^x - 2xe^x + 2e^x + y^2e^x + 2[xe^x - 1e^x] = c.$$

(1, 0, 0) (0, 1, 0) (0, 0, 1)

Rule 2: if $\boxed{\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{-M} = h(y)}$

is a function of y.

alone, then $\boxed{\mu = e^{\int h(y)dy}}$

is an integrating factor of the given DE.

Q4) $(\underbrace{y^4 + 2y}_{M})dx + (\underbrace{xy^3 - 4x + 2y^4}_{N})dy = 0$

$\dfrac{\partial M}{\partial y} = 4y^3 + 2$, $\dfrac{\partial N}{\partial x} = y^3 - 4$

Rule 2: $\dfrac{4y^3 + 2 - y^3 + 4}{-y^4 - 2y} = \dfrac{3y^3 + 6}{-y^4 - 2y} = \dfrac{3\cancel{(y^3+2)}}{-y\cancel{(y^3+2)}} = \dfrac{-3}{y}$

$IF = \mu = e^{-3\log y} = e^{(\log y)^{-3}} = y^{-3} = \dfrac{1}{y^3}$

multiply eq^n by IF

$(y^4 + 2y)\dfrac{1}{y^3}\, dx + (3y^3 - 4x + 2y^4)\dfrac{1}{y^3}\, dy = 0$

$\left(y + \dfrac{2}{y^2}\right)dx + \left(3 - \dfrac{4x}{y^3} + 2y\right)dy = 0$

$\dfrac{\partial M}{\partial y} = \dfrac{-4}{y^3}$, $\dfrac{\partial N}{\partial x} = \dfrac{-4}{y^3}$

$\therefore$ eq^n is exact

$\int \left(y + \dfrac{2}{y^2}\right)dx + \int \left(3 - \dfrac{4x}{y^3} + 2y\right) \cdot dy = 0$

$\Rightarrow xy + \frac{2x}{y^2} + 3y + \frac{8x}{y^2} + y^2 = c$ ans

* Rule 3: If the eq^n is of the form

$$\boxed{y f(xy)\,dx + x g(xy)\,dy = 0}$$

then,

$$\boxed{\mu = IF = \frac{1}{Mx - Ny}}$$

IF = integrating factor.

is an integrating factor of the given D.E.

Q5) $y(\sin xy + xy\cos xy)\,dx + x(xy\cos xy - \sin xy)\,dy = 0$

$(y\sin xy + xy^2\cos xy)\,dx + (x^2y\cos xy - x\sin xy)\,dy$

$\frac{\partial M}{\partial y} = \sin xy + xy\cos xy + 2xy\cos xy - x^2y^2\sin xy$

$\frac{\partial M}{\partial y} = \sin xy + 3xy\cos xy - x^2y^2\sin xy$

$\frac{\partial N}{\partial x} = 2xy\cos xy - x^2y^2\sin xy - \sin xy - xy\cos xy$

$= -\sin xy + xy\cos xy - x^2y^2\sin xy$

eq^n is not exact

applying rule 3

$\mu = \frac{1}{xy\sin xy + \cancel{x^2y^2\cos xy} - \cancel{x^2y^2\cos xy} + xy\sin xy}$

$\mu = \frac{1}{2xy\sin xy}$

check for exactness after multiplying by IF

$$\left(\frac{y \sin xy}{2xy \sin xy} + \frac{xy^2 \cos xy}{2xy \sin xy}\right)dx + \left(\frac{x^2 y \cos xy}{2xy \sin xy} - \frac{x \sin xy}{2xy \sin xy}\right)dy = 0$$

$$\int\left(\frac{1}{2x} + \frac{y \cot xy}{2}\right)dx + \int\left(\frac{x}{2}\cot xy - \frac{1}{2y}\right)dy = 0$$

$$\frac{\log x}{2} + \frac{\log \sin xy}{2} + \frac{\log \sin xy}{2} - \frac{\log y}{2} = c \quad \underline{\underline{\text{ans}}}$$

homogenous eq^n → degree of all terms to be same

* Rule 4 :

if eq^n is homogeneous.

then

$$\boxed{IF = \frac{1}{Mx + Ny}}$$

then

$$\mu = IF = \frac{1}{Mx+Ny}.$$

Q6) $(\underbrace{3xy - 2ay^2}_{M})\,dx + (\underbrace{x^2 + 2axy}_{N})\,dy = 0.$

$\frac{\partial M}{\partial y} = (3x - 4ay)$, $\frac{\partial N}{\partial x} = 2x + 2ay.$

$\because \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ $\therefore$ non-exact. eq^ns.

$\because$ eq^n is homogenous of degree 2.

$\therefore$ by rule 4.

$$\mu = \frac{1}{Mx+Ny} = \frac{1}{3x^2y - \cancel{2axy^2} + x^2y + \cancel{2axy^2}}$$

$$= \frac{1}{4x^2y}$$

multiply the given eq^n by μ.

$$\frac{(3xy - 2ay^2)}{4x^2y}dx + \frac{(x^2 - 2axy)}{4x^2y}dy = 0.$$

$$\cancel{\frac{3x-2ay}{4x(x-ay)}dx + \frac{(x-2ay)}{4y(x-ay)}dy = 0}$$

$$\cancel{\int M\,dx = \int \frac{3x-2ay}{4x(x-ay)}dx = \int \frac{3x-2ay}{4x^2-4xay}dx}$$

~~Let $4x^2-4xay=t$~~ $\frac{(3x-2ay)}{4x^2}dx + \frac{(x-2ay)}{4xy}dy = 0$

Q) $\underbrace{(x^4+y^4)}_{M}dx - \underbrace{xy^3}_{N}dy = 0$

$$\frac{\partial M}{\partial x} = 4x^3 + y^4 \quad , \quad \frac{\partial N}{\partial y} = -3y^2x.$$

$\because \frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$ $\therefore$ non exact.

as eq^n is homog. $\therefore$ by rule 4.

$$\mu = \frac{1}{Mx+Ny} = \frac{1}{x^5 + \cancel{y^4x} - \cancel{xy^4}} = \frac{1}{x^5}$$

$\therefore$ multiply in eq^n by IF.

$$\frac{x^4+y^4}{x^5}\,dx - \frac{xy^3\,dy}{x^5} = 0$$

$$\left(\frac{1}{x} + \frac{y^4}{x^5}\right)dx - \frac{y^3}{x^4}\,dy = 0.$$

$$\int M\,dx = \int \frac{1}{x}\,dx + \int \frac{y^4}{x^5}\,dx. \qquad \frac{x^{-5+1}}{-3+1}$$

$$= \log x + \frac{y^4}{(-4x^4)}$$

$$= \log x - \frac{y^4}{4x^4}$$

$$\int N\,dy = -\int \frac{y^3}{x^4}\,dy \pm \cancel{\frac{y^4}{4x^4}} = 0$$

$\Rightarrow$ $\int M\,dx + \int N(\text{free from } x)\,dy = c.$

$$\log x - \frac{y^4}{4x^4} + 0 = c.$$

Q) $(\underbrace{x^2 - xy + y^2}_{M})\,dx - \underbrace{xy}_{N}\,dy = 0.$

$$\frac{\partial M}{\partial x} = 2x - y + y^2, \quad \frac{\partial N}{\partial y} = -y.$$

$\because \dfrac{\partial M}{\partial x} \neq \dfrac{\partial N}{\partial y}$ $\therefore$ non exact.

$\because$ eq^n is homog. of degree 2 $\therefore$ by rule 4

$$\mu = \frac{1}{Mx+Ny} = \frac{1}{x^3 - x^2y + \cancel{xy^2} - \cancel{xy^2}} = \frac{1}{x^2(x-y)}.$$

∴ multiply IF in eqⁿ.

$$\frac{x^2-xy+y^2}{x^2(x-y)}\,dx - \frac{xy}{x^2(x-y)}\,dy = 0$$

~~Let $x^3-x^2y=t$~~

~~$\frac{dt}{dx}=3x^2-2xy$~~

$$\int M\,dx = \int \frac{x^2-xy+y^2}{x^3-x^2y}\,dx.$$

$$\int \frac{x\cancel{(x-y)}}{x^2\cancel{(x-y)}} + \frac{y^2}{x^2(x-y)}\,dx$$

$$\int \frac{1}{x}\,dx + y^2\int \frac{dx}{x^3-x^2y}$$

$$\log x + y^2\int \frac{1}{x^2(x-y)}\,dx$$

$$= \int \frac{x^2-y(x-y)}{x^2(x-y)}\,dx.$$

$$+\,y^2\int \frac{}{x^2} + \frac{}{x-y}\,dx$$

$$= \int \frac{x^2}{x^2(x-y)}\,dx - \frac{y\cancel{(x-y)}}{x^2\cancel{(x-y)}}\,dx$$

$$= \int \frac{dx}{x-y} - \frac{y}{x^2}\,dx$$

Linear D.E

\# order = 1 = deg.

$$\frac{dy}{dx} + Py = Q.$$

$P, Q \longrightarrow$ functⁿs of x

single term y.

$$IF = e^{\int P dx}.$$

$$y \times IF = \int Q . IF\, dx + c.$$

swap x & y.

$$\frac{dx}{dy} + Px = R.$$

P, R $\longrightarrow$ functions of y.

single term of x.

$$IF = e^{\int P dy}.$$

$$\boxed{x\, IF = \int Q\, IF\, dy + c}$$ ans.

Q) $\frac{dy}{dx} + 2y \tan x = \sin x$ at $y = 0$, $x = \frac{\pi}{3}$.

1. order = deg = 1.
2. single term y.

$$\frac{dy}{dx} + Py = Q.$$

$$\frac{dy}{dx} + 2y \tan x = \sin x.$$

3. $P = 2\tan x$, $Q = \sin x$.
4. $IF = e^{\int P dx} = e^{\int 2\tan x\, dx}$

$$IF = e^{2\log \sec x} = e^{\log \sec^2 x} = \sec^2 x.$$

⑤ complete solutⁿ :-

$$y \cdot If = \int Q \cdot If \, dx + c.$$

$$y \sec^2 x = \int \sin x \sec^2 x \, dx + c.$$

$$y \sec^2 x = \int \frac{\sin x}{\cos^2 x} \, dx + c.$$

Let $\cos x = t \Rightarrow -\sin x \, dx = dt$

$$y \sec^2 x = \int -\frac{dt}{t^2} + c = -\frac{t^{-1}}{-1} + c$$

$$y \sec^2 x = \frac{1}{t} + c = \frac{1}{\cos x} + c \quad \text{ans.}$$

$$y \sec^2 x = \sec x + c \quad \underline{\text{ans}}$$

Q. $dx + x\,dy = e^{-y} \sec^2 y \, dy.$

① order = deg = 1

② single term of x.

③ $dx + x\,dy = e^{-y} \sec^2 y \, dy.$

÷ by dy.

$$\frac{dx}{dy} + x = e^{-y} \sec^2 y.$$

$P = 1, \quad Q = e^{-y} \sec^2 y.$

④ $If = e^{\int P dy} = e^{\int 1 dy} = e^{y}.$

(5) complete solutⁿ -

$$x \cdot IF = \int Q \, IF \, dy + c.$$

$$x. e^y = \int \cancel{e^{-y}} \sec^2 y \, \cancel{e^y} \, dy + c$$

$$\boxed{x \cdot e^y = \tan y + c}$$ ans.

* <u>Bernoulli's Differential Equation</u>

standard form:

$$\frac{dy}{dx} + Py = Qy^n.$$

Q) $$x\frac{dy}{dx} + y = x^3y^6$$

$\div$ eqⁿ by x $\Rightarrow \frac{dy}{dx} + \frac{y}{x} = x^2y^6$.

$\div$ by y^6

$$\frac{1}{y^6}\frac{dy}{dx} + \frac{1}{xy^5} = x^2$$

let $\frac{1}{y^5} = t$. $\Rightarrow y^{-5} = t$

$$\Rightarrow -5y^{-6}\,dy = dt.$$

$$\Rightarrow \frac{1}{y^6}dy = \frac{-1}{5}dt.$$

$$\Rightarrow \frac{-1}{5}\frac{dt}{dx} + \frac{t}{x} = x^2$$

$\times$ by -5

$$\Rightarrow \frac{dt}{dx} - \frac{5}{x}t = -5x^2.$$

P Q

$IF = e^{\int -\frac{5}{x} dx} = e^{-5\log x} = e^{\log x^{-5}} = x^{-5}$

complete soln.

$t \cdot IF = \int \rho \, IF \, dx + c$

$t \, x^{-5} = \int -5x^2 x^{-5} \, dx + c$

$y^{-5} x^{-5} = -5\int x^{-3} dx + c = -5 \frac{x^{-3+1}}{-3+1} + c$

$y^{-5} x^{-5} = \frac{5}{2} \frac{1}{x^2} + c$ ans.

DE higher order and 1st degree →

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + a_3 \frac{d^{n-3} y}{dx^{n-3}} + \cdots$$

$$\cdots a_n y = X$$

Eg: Higher order with const coeff.

$$5\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 20y = 16 \log x.$$

operator D

$D \equiv \frac{d}{dx}$, $D^2 = \frac{d^2}{dx^2}$, $D^3 = \frac{d^3}{dx^3}$ ......

Standard form –

$f(D)y = X$

Eg: $(D^2 + 2D - 1)y = x^2 \sin x$

general or complete solution -

$$y = CF + PI$$

Complementary function → Particular Integral.

① Eg. Q. $(D^2 + 3D + 2)y = e^{2x}$.

$$f(D).\,y = X$$

for CF.

Auxilliary Eqns. $y(m) = 0$.

$m^2 + 3m + 2 = 0 \Rightarrow m = -1, -2$.

→ real & different

Case I $\boxed{CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}}$

$CF = c_1 e^{-x} + c_2 e^{-2x}$.

Case II : roots are real and equal

say $m_1 = m_2 = m$

$$\boxed{CF = (c_1 + c_2 x) e^{mx}}$$

Case III : roots are imaginary, $m = a \pm ib$.

$$\boxed{CF = e^{ax}(c_1 \cos bx + c_2 \sin bx)}$$

Q) $(D^3 - 2D^2 - 5D + 6)y = e^{3x} + 8$.

for CF, AE $f(m) = 0$

$\Rightarrow m^3 - 2m^2 - 5m + 6 = 0$

[illegible] $\boxed{m = -2, 1, 3}$

<u>Case I</u> roots are real and different

$$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

$$CF = c_1 e^{-2x} + c_2 e^{1x} + c_3^{3x}.$$

- suppose $m = 11, 11, 13$ (mix of case I & II)

  then,

$$CF = (c_1 + c_2 x) e^{11x} + c_3 e^{13x}.$$

$m = 7, 4 \pm 3i$ (mix of case I & III).

$$CF = c_1 e^{7x} + e^{4x}(c_2 \cos 3x + c_3 \sin 3x)$$

<u>PI</u> operator method :

$$PI = \frac{1}{f(D)} \cdot X.$$

Case I : if $X = e^{ax}$ then.

$$PI = \frac{1}{f(D)} \cdot e^{ax} = \frac{1}{f(a)} \cdot e^{ax}, \text{ where } f(a) \neq 0.$$

$$PI = \frac{1}{f(D)} \cdot e^{ax} = x \cdot \frac{1}{f'(a)} \cdot e^{ax}, \text{ where } f(a) = 0,$$

in (1) Eq!

$$PI = \frac{1}{D^2 + 3D + 2} \cdot e^{2x}$$

replace D by a i.e. power of exponential.

$a = 2$

$$PI = \frac{1}{(2)^2+3(2)+2} \cdot e^{2x} = \frac{1}{12} \cdot e^{2x}$$

$$y = CF + PI$$

$$y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{12} \cdot e^{2x} \quad \underline{\underline{\text{ans}}}$$

Q2) $PI = \frac{1}{f(D)} \cdot X = \frac{1}{D^3-2D^2-5D+6} \cdot (e^{3x}+8)$

$$= \frac{1}{D^3-2D^2-5D+6} e^{3x} + \frac{1}{D^3-2D^2-5D+6} 8 \cdot e^{0x}.$$

$a = 0$, $D \to a$,

$$PI = \frac{1}{(3)^3-2(3)^2-5(3)+6} e^{3x} + \frac{1 \times 8}{6} a^{0x}.$$

$$= \frac{1}{27-18-15+6.} e^{3x} + \frac{8}{6.}$$

$$= \frac{e^{3x}}{0.} + \frac{\cancel{8}\,4}{\cancel{6}\,3} = \frac{4}{3}$$

not applicable. $\because$ denom $= 0$

27
15
12
6.
18
-18
0.

use $PI = \frac{1}{f(D)} e^{ax} = x \frac{1}{f'(a)} \cdot e^{ax}$,

$$= \frac{x e^{3x}}{3D^2-4D-5} + \frac{8}{6.}$$

$$= \frac{xe^{3x}}{27-12-5} + \frac{4}{3}$$

$$= \frac{x e^{3x}}{10} + \frac{4}{3}$$

$y = CF + PI$

$= c_1 e^{-2x} + c_2 e^{x}$

$+ c_3 e^{3x} + \frac{xe^{3x}}{10} + \frac{4}{3}$

case II : if $x = \cos ax$ or $\sin ax$, then

1) $PI = \frac{1}{f(D^2)} \cdot \cos ax / \sin ax = \frac{1}{f(-a^2)} \cdot \cos ax/\sin ax$.

where $f(-a^2) \neq 0$.

2) $PI = \frac{1}{D^2+a^2} \cdot \cos ax = \frac{x}{2a} \cdot \sin ax$, where

$f(-a^2) = 0$.

3) $PI = \frac{1}{D^2+a^2} \cdot \sin ax = \frac{-x}{2a} \cos ax$, where

$f(-a^2) = 0$.

Q) $(D^2 - 5D + 6)\, y = \sin 3x$

$f(D) \cdot y = X$

to find out CF

AE $\quad f(m) = 0$

$m^2 - 5m + 6 = 0 \quad \Rightarrow \quad m = 2, 3$.

$CF = c_1 e^{2x} + c_2 e^{3x}$

to find out PI

$PI = \frac{1}{f(D)} \cdot X = \frac{1}{D^2 - 5D + 6} \sin 3x$.

$D^2 \rightarrow -a^2 \rightarrow -3^2 \rightarrow -9$.

$PI = \frac{1}{-9-5D+6} \cdot \sin 3x = \frac{1}{-5D+3} \sin 3x$

$= \frac{1}{-(5D+3)} \times \frac{(5D-3)}{(5D-3)} \times \sin 3x$

$$= \frac{-(5D-3)}{25.D^2-9} \cdot \sin 3x \quad = \frac{-(5D-3)}{25(-9)-9} \cdot \sin 3x$$

$$= \frac{\cancel{-}(5D-3)}{\cancel{-}234} \sin 3x = (5D-3) \sin 3x.$$

$$= \frac{1}{234}\left[\left(5\frac{d}{dx} - 3\right)\sin 3x\right]$$

$$= \frac{1}{234}\left[\frac{5d}{dx} \sin 3x - 3 \sin 3x\right]$$

$$= \frac{1}{234}\left[15\cos 3x - 3 \sin 3x\right]$$

$y = CF + PI$

Q) $(D^2-2D+1)y = \cos 2x.$

to find out C. F.

$f(m) = 0.$

$m^2-2m+1 = 0 \Rightarrow m = 1, 1$

$CF = (C_1 + C_2 x)e^x$

$$PI = \frac{1}{D^2-2D+1} \cdot \cos 2x$$

$D^2 \longrightarrow -a^2 \rightarrow -2^2 \longrightarrow -4$

$$PI = \frac{1}{-4-2D+1} \cdot \cos 2x = \frac{1}{-2D-3} \cdot \cos 2x.$$

$$PI = \frac{-1}{2D+3} \times \frac{2D-3}{2D-3} \cdot \cos 2x$$

$$= \frac{-(2D-3)}{4D^2-9} \cdot \cos 2x. \quad = \frac{-(2D-3)}{4(-4)-9} \cdot \cos 2x$$

$$= \frac{(2D-3)}{25}\cos 2x = \frac{1}{25}(2D\cos 2x - 3\cos 2x)$$

$$= \frac{1}{25}\left(2\frac{d}{dx}\cos 2x - 3\cos 2x\right) = \frac{1}{25}(-4\sin 2x - 3\cos 2x)$$

complete soltⁿ is $y = CF + PI$.

$$y = (c_1 + c_2 x)e^{x} + \frac{1}{25}(-4\sin 2x - 3\cos 2x) \quad \underline{\text{ans}}$$

9) $(D^2+9)y = \sin 3x$

CF $\quad m^2+9=0 \Rightarrow m = \pm 3i$

$$CF = e^{0x}(c_1\cos 3x + c_2\sin 3x).$$

$D^2 \to -a^2 \to -9.$

$$PI = \frac{1}{D^2+9}\cdot\sin 3x = \frac{1}{0}\cdot\sin 3x$$

not applicable

$$PI = \frac{-x}{2\times 3}\cos 3x \Rightarrow \boxed{PI = \frac{-x}{6}\cos 3x}$$

10) solve:- $(D^4 - 2D^3 + D^2)y = 0$

$f(m) = 0$

$m^4 - 2m^3 + m^2 = 0$

$m^2(m^2 - 2m + 1) = 0$

$m^2 = 0, \quad \boxed{m = 1, 1}$

$m = 1, 1, 0, 0$

CF $\Rightarrow (c_1 + c_2 x)e^{x} + (c_3 + c_4 x)e^{0x}.$

$PI = 0 \quad \because RHS = 0$

2Q) $[(D-1)^4(D^2+2D+2)^2]y=0$

$f(m)=0$

$(m-1)^4(m^2+2m+2)^2=0.$

$m=1,1,1,1$, $\cancel{m^4+4m^2+4+4m^3+8m+4m^2=0}$

$\cancel{m^4+8m^2+4m^3+8m+4=0}$

$m^2+2m+2=0.$ $a=-1$, $b=+1$

$m=-1\pm i$, $-1\pm i$

$Cf = (c_1+c_2x+c_3x^2+c_4x^3)e^x + [((c_5+c_6x)\cos x+(c_7+c_8x)\sin x)e^{-x}]$

3Q) $(D^2-3D+2)y = e^x + \cos x.$

$f(m)=0$

$m^2-3m+2=0$

$m^2-2m-m+2=0 \Rightarrow m(m-2)-1(m-2)=0$

$\boxed{m=1,2}$

$Cf = c_1e^x + c_2e^{2x}$

$PI = \frac{1}{f(D)} \times X = \frac{1}{D^2-3D+2}(e^x+\cos x).$

$= \frac{1}{D^2-3D+2} \times e^x. + \frac{1}{D^2-3D+2} \times \cos x$

① $\frac{1}{f(D)} \cdot e^{ax} \longrightarrow D \longrightarrow a.$ $f(a)\neq 0.$

② $\frac{1}{f(D)} \cdot e^{ax} = x \cdot \frac{1}{f'(D)} \times e^{ax}$ $f(a)=0$

$\because$ denom. is 0 $\therefore$ by formula ②

(3) $\frac{1}{f(D)} \cos ax / \sin ax = D^2 \longrightarrow -a^2$.

$= x \cdot \frac{1}{2D-3} \cdot e^x + \frac{1}{-1-3D+2} \cos x$.

= put D as 1 + rationalise.

$= \frac{x}{-1} \cdot e^x \cdot + \frac{1}{1-3D} \times \frac{1+3D}{1+3D} \cdot \cos x$

$= -xe^x + \frac{1+3D}{1-9D^2} \cdot \cos x$.

$D^2 \to -a^2 \to -1$

$= -xe^x + \frac{(1+3D)}{10} \cos x$.

$D \to \frac{d}{dx}$

$PI = -xe^x + \frac{1}{10}[\cos x + 3(-\sin x)]$

$y = CF + PI$

$y = c_1 e^x + c_2 e^{2x} + \frac{1}{10}[\cos x - 3\sin x] - xe^x$ ans

Q4) $(D^2 - 6D + 9)y = e^{3x}(1+x)$

$f(m) = 0 \Rightarrow m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$\boxed{m = 3, 3}$

$CF = (c_1 + c_2 x) e^{3x}$

$PI = \frac{1}{f(D)} \times X = \frac{1}{D^2 - 6D + 9} \times e^{3x}(1+x)$

$= \frac{1}{D^2-6D+9} e^{3x} + \frac{1}{D^2-6D+9} e^{3x} x$

V.x.

by formula ② $\because f(a) = 0$

(4) $PI = \dfrac{1}{f(D)} \cdot e^{ax} \cdot V = e^{ax} \cdot \dfrac{1}{f(D+a)} \cdot V$

$$= \frac{x}{2D-6} \cdot e^{3x} + \left\{ x - \frac{2D-6}{D^2-6D+9} \right\} \frac{e^{3x}}{D^2-6D+9}$$

by 1st term

$$= \frac{x^2}{2} e^{3x} + \left[x - \frac{(2D-6)}{D^2-6D+9} \right] \cdot \frac{x^2}{2} e^{3x}$$

(5) $PI = \dfrac{1}{f(D)} \cdot x \cdot V = \left\{ x - \dfrac{1}{f(D)} \cdot f'(D) \right\} \dfrac{1}{f(D)} \cdot V$

$$= \frac{x^2}{2} e^{3x} + \frac{x^3}{2} e^{3x} - \frac{(2D-6)}{D^2-6D+9} \cdot \frac{x^2}{2} e^{3x}$$

$$= \frac{x^2}{2} e^{3x} + \frac{x^3}{2} e^{3x} - \frac{1}{D^2-6D+9} \left[2D\left(\frac{x^2}{2} e^{3x}\right) - \frac{6}{2} x^2 e^{3x} \right]$$

by formula (4)

$$= \frac{x^2 e^{3x}}{2} + e^{3x} \cdot \frac{1}{(D+3)^2 - 6(D+3) + 9} \cdot x$$

$$= \frac{x^2 e^{3x}}{2} + e^{3x} \cdot \frac{1}{D^2} \cdot x \qquad \left\{ \frac{1}{D} = \int \right\}$$

$$= \frac{x^2 e^{3x}}{2} + e^{3x} \iint x \cdot dx \cdot dx$$

$$= \frac{x^2 e^{3x}}{2} + e^{3x} \frac{x^3}{6}$$

(5) formula not to do.

Case III : If $X = x^m$ then.

$$PI = \frac{1}{f(D)} \cdot x^m = \left[1 + \phi(D) \right]^{-1} \cdot x^m$$

$PI = \dfrac{1}{f(D)} \cdot X$,

also-

$[1+x]^{-1} = 1 - x + x^2 - x^3 + \ldots$

$[1-x]^{-1} = 1 + x + x^2 + x^3 + \ldots$

$(1+x)^n = 1 + nx + n\frac{(n-1)}{2}x^2 + - - - - - - - -$

Q1) $(D^3 - 3D + 2)y = x.$

$f(m) = 0 \quad \Rightarrow \quad m^3 - 3m + 2 = 0$

$1 - 3(1) + 2 = 0$

$$\begin{array}{r} m^2 + m - 2 \\ m-1 \enclose{longdiv}{m^3 - 3m + 2.} \\ -\cancel{m^3} \ominus m^2 \\ \cancel{m^2} - 3m + 2 \\ -\cancel{m^2} \ominus m \\ \hline -2m + 2 \\ -2m + 2 \\ \hline 0 \end{array}$$

$m^2 + m - 2$

$m^2 + 2m - m - 2$

$m(m+2) - 1(m+2)$

$(m-1)(m+2)$

$m = 1, -2$

$\boxed{m = 1, 1, -2}$

$C.F = (C_1 + C_2 x)e^x + C_3 e^{-2x}$

$PI = \frac{1}{D^3 - 3D + 2} . x. = \frac{1}{2\left(\frac{D^3}{2} - \frac{3D}{2} + 1\right)} . x$

$= \frac{1}{2}\left[1 + \frac{D^3}{2} - \frac{3D}{2}\right]^{-1} . x.$

$[1 + x]^n \qquad \therefore n = -1, \quad x = \frac{D^3}{2} - \frac{3D}{2}.$

$\therefore$ by exponential expansion,

$= \left[1 + (-1)\left(\frac{D^3}{2} - \frac{3D}{2}\right) + \frac{(-1)(-1-1)}{2}\left[\frac{D^3}{2} - \frac{3D}{2}\right]^2 + - - - -\right]x$

$= x - \left(\frac{D^3}{2} - \frac{3D}{2}\right)x + \left[\frac{D^3}{2} - \frac{3D}{2}\right]^2 x + - - - \infty$

$$= \frac{1}{2}\left[\frac{1}{1} - 0 + \frac{3}{2}\right] = 5/4 \text{ ans}$$

Q2) $(D^2 - 4D + 4)y = x^2 + e^x + \cos 2x.$

$$m^2 - 4m + 4 = 0$$

$$m^2 - 2m - 2m + 4 = 0$$

$$m(m-2) - 2(m-2) = 0$$

$$m = 2, 2$$

$$CF = (C_1 + C_2 x)e^{2x}$$

$$PI = \frac{1}{D^2 - 4D + 4} \cdot (x^2 + e^x + \cos 2x)$$

$$= \frac{x^2}{D^2 - 4D + 4} + \frac{e^x}{D^2 - 4D + 4} + \frac{\cos 2x}{D^2 - 4D + 4.}$$

$D = a = 1.$ $\quad D^2 \to -a^2 \to -4$

$$= \frac{1}{(D-2)^2} x^2 + \frac{e^x}{(1-2)^2} + \frac{\cos 2x}{\cancel{-4} - 4D \cancel{+4}}$$

$$= \frac{1}{4\left(\frac{D}{2} - 1\right)^2} \cdot x^2 + e^x - \frac{1}{4D} \cos 2x$$

$\left\{\frac{1}{D} = \int\right\}$

$$= \frac{1}{4\left(1 - \frac{D}{2}\right)^2} x^2 + e^x - \frac{1}{4} \frac{\sin 2x}{2.}$$

$$= \frac{1}{4}\left[1 - \frac{D}{2}\right]^{-2} x^2 + e^x - \frac{1}{4} \frac{\sin 2x}{2}$$

$[1 + x]^{-n} \to x = -D/2, \quad n = -2$

$$= \frac{1}{4}\left[1 + (-2)\left(\frac{-D}{2}\right) + \frac{(-2)(-2-1)}{2}\left[\frac{-D}{2}\right]^2 + \cdots \infty\right] x^2$$

$$= \frac{1}{4}\left[1 + D + \frac{3}{4}D^2 + \cdots \infty\right] x^2$$

$$= \frac{1}{4}\left[x^2 + 2x + \frac{3}{4}\times 2 + \cdots\right] = \frac{1}{4}\left[x^2 + 2x + \frac{3}{2}\right]$$

ans

Method of Variation of parameters. 100%

$f(D)\, y = X.$

Step 1: $CF = c_1 y_1 + c_2 y_2$

Wronskian Determinant →

Step 2: $W = \begin{vmatrix} y_1 , & y_2 \\ y_1' , & y_2' \end{vmatrix}$

Step 3: $u = -\int y_2 \cdot \frac{X}{W} \cdot dx.$

Step 4: $v = \int y_1 \cdot \frac{X}{W}\, dx.$

Step 5: $PI = u y_1 + v y_2.$

Step 6: solution $y = Cf + PI.$

Q) $\frac{d^2y}{dx^2} + y = \sec x \cdot \tan x.$

$(D^2+1)\, y = \sec x \cdot \tan x.$

$m^2 + 1 = 0. \Rightarrow m = \pm i$

$a = 0, \; b = 1$

$CF = (c_1 \cos x + c_2 \sin x)\, e^{0x}.$

$= \underset{y_1}{c_1 \cos x} + \underset{y_2}{c_2 \sin x}$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x. \end{vmatrix} = \cos^2 x + \sin^2 x = 1.$$

$$u = -\int \sin x \frac{\sec x \tan x}{1.} dx.$$

$$= -\int \tan^2 x \, dx. \quad = -\int (\sec^2 x - 1) \, dx.$$

$$= -(\tan x - x) = \quad x - \tan x.$$

step4 : $v = \int \frac{y_1 X}{W} dx \quad = \int \cos x \frac{\sec x \tan x dx}{1.}$

$$v = \int \tan x \, dx.$$

$$v = \log \sec x$$

step 5 : $PI = uy_1 + vy_2$

$$= (x - \tan x) \cos x + \log \sec x \sin x.$$

step 6 : $y = CF + PI.$

ans $y = c_1 \cos x + c_2 \sin x + (x - \tan x) \cos x + \log \sec x . \sin x.$

Q) $(D^2 - 2D) y = e^x \sin x.$

$$m^2 - 2m = 0$$

$$m(m-2) = 0$$

$$m = 0, 2.$$

$$CF = c_1 e^{2x} + c_2 e^{0x}. \quad = c_1 \underset{y_1}{\underset{\uparrow}{e^{2x}}} + c_2 \underset{y_2}{\underset{\uparrow}{1.}}$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{2x} & 1 \\ 2e^{2x} & 0 \end{vmatrix} = -2e^{2x}.$$

$$u = -\int y_2 \frac{X}{W}\, dx = \int 1 \frac{e^x \sin x}{-2e^{2x}}\, dx.$$

$$= \frac{1}{2}\int e^{-x} \sin x\, dx.$$

$$\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx - b\sin bx\right]$$

$a=-1,\ b=1$

$$= \frac{1}{2}\left[\frac{e^{-1x}}{(-1)^2+(1)^2}\left(-1\cos x - \sin x\right)\right].$$

$$= \frac{1}{2}\frac{e^{-x}}{2}(-\cos x - \sin x).$$

$$v = \int \frac{y_1 X}{W}\, dx = \int \frac{\cancel{e^{2x}} \cdot e^x \sin x\, dx}{-2\cancel{e^{2x}}}$$

$$= -\frac{1}{2}\int e^x \sin x\, dx$$

$a=1,\ b=1$

$$= -\frac{1}{2}\frac{e^x}{1^2+1^2}\left[\cos x - \sin x\right] = -\frac{1}{4}e^x(\cos x - \sin x)$$

$$PI = -\frac{1}{4}e^{-x}(\cos x + \sin x)e^{2x} - \frac{1}{4}e^x(\cos x - \sin x)1.$$

$$y = CF + PI$$

$$= c_1 e^{2x} + c_2 - \frac{1}{4}e^{-x}(\cos x + \sin x)e^{2x} - \frac{1}{4}e^x(\cos x - \sin x)$$

q) $(D^2+1)y = x\sin x.$

$m^2+1 = 0 \Rightarrow m = \pm i$

$a = 0, \quad b = 1.$

$CF = (c_1 \cos x + c_2 \sin x) e^{0x}$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$u = -\int \frac{y_2 X}{W} dx = -\int \frac{\sin x \; x \sin x}{1} dx.$$

$$u = -\int x \sin^2 x \, dx.$$

use $\quad \sin^2 x = \frac{1 - \cos 2x}{2}$

$$u = -\int x \frac{(1 - \cos 2x)}{2} dx = -\frac{1}{2}\int x \, dx + \frac{1}{2}\int \underset{\mathrm{I}}{x} \underset{\mathrm{II}}{\cos 2x} \, dx$$

$$= -\frac{1}{4}x^2 + \frac{1}{2}\left[+x \frac{\sin 2x}{2} - 1\left(\frac{-\cos 2x}{4}\right)\right]$$

$$= \frac{-x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{4}$$

$$v = \int \frac{y_1 X}{W} dx = \int \frac{\cos x \sin x \, x}{1} dx$$

$$= \int x \sin x \cos x \, dx = \frac{1}{2}\int \underset{\mathrm{I}}{x} \underset{\mathrm{II}}{\sin 2x} \, dx$$

$$= \frac{1}{2}\left[x\left(-\frac{\cos 2x}{2}\right) - 1\left(-\frac{\sin 2x}{4}\right)\right]$$

Q) $(D^2+1)y = \tan x.$

$m^2+1 = 0 \Rightarrow m = \pm i \quad , a=0, b=1$

$CF = (c_1\cos x + c_2 \sin x)e^{0x}$

$= c_1\cos x + c_2 \sin x.$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x$
$= 1$

$u = -\int \frac{y_2 X}{W}\,dx = -\int \frac{\sin x \tan x}{1}\,dx.$

Q5) $(D^2+1)y = \operatorname{cosec} x \cot x.$

$m^2+1 = 0 \Rightarrow m = \pm i \quad a=0, b=1$

Power Series Solutions -

100%

Q1) $x(x+1)y' - (2x+1)y = 0.$ —— ①

$x(x+1)Dy - (2x+1)y = 0 \quad \to \therefore PI = 0$

$[x(x+1)D - (2x+1)]y = 0.$

$f(D).\,y = X.$

$D \to m.$

$x(x+1)m - (2x+1) = 0$

$mx^2 + xm - 2x - 1 = 0.$

not applicable.

$$y = \sum_0^{\infty} c_n (x - x_0)^n$$

If x_0 is not given, we assume $x_0 = 0$

$\therefore \quad y = \sum_{n=0}^{\infty} c_n x^n$

$y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + - - - - + \infty.$

our objective is to find out values $c_0, c_1, c_2, c_3 ....$

$y' = c_1 + 2c_2 x + 3c_3 x^2 + ----- \infty$

$y'' = 2c_2 + 6c_3 x + 12 c_4 x^2 + --- \infty$

~~continue of 1.~~

~~$x_0 = 0 \Rightarrow y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + ---- + \infty.$~~

put values of y & y' in (1).

$\Rightarrow x(x+1)(c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + ----)$
$- (2x+1)(c_0 + c_1 x + c_2 x^2 + --) = 0$

$\Rightarrow (x^2 + x)(c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + ---) - (2x+1)$
$(c_0 + c_1 x + c_2 x^2 + -) = 0$

$\rightarrow (c_1 x^2 + 2c_2 x^3 + 3c_3 x^4 + 4c_4 x^5 + 5c_5 x^6 + -----) + (c_1 x + 2c_2 x^2 + 3c_3 x^3 +$
$4c_4 x^4 + 5c_5 x^5 + ---) - [2c_0 x + 2c_1 x^2 + 2c_2 x^3 + 2c_3 x^4 +$
$2c_4 x^5 + ---] - [c_0 + c_1 x + c_2 x^2 + -] = 0$

constant terms $\rightarrow \quad -c_0 = 0 \quad \Rightarrow \boxed{c_0 = 0}$

coeff of $x \quad \rightarrow \quad c_1 - 2c_0 - c_1 = 0 \quad \Rightarrow \boxed{c_0 = 0} \quad c_1 = ?$

coeff. of $x^2 \longrightarrow c_1 + 2c_2 - 2c_1 - c_2 = 0$

$c_2 - c_1 = 0 \quad \Rightarrow \boxed{c_1 = c_2}$

coeff of $x^3 \rightarrow 2c_2 + 3c_3 - 2c_2 - c_3 = 0 \quad \Rightarrow \boxed{c_3 = 0}$

coeff of $x^4 \rightarrow$ $3c_3 + 4c_4 - 2c_3 - c_4 = 0 \Rightarrow \boxed{c_4 = 0}$

coeff of $x^5 \rightarrow$ $\boxed{c_5 = 0}$ and so on.

solutⁿ : $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$ ①

$y = c_1 x + c_1 x^2$

$\boxed{y = c_1 x(1 + x)}$ ans.

Q). $(1-x^2)y'' - 2xy' + 2y = 0$.

put value of y, y', y'' in given eqⁿ.

$(1-x^2)[2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + \cdots]$
$- 2x[c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + 5c_5 x^4 + \cdots]$
$+ 2[c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots] = 0$

$\Rightarrow [2c_2 + 6c_3 x + 12c_4 x^2 + 20c_5 x^3 + \cdots] - [2c_2 x^2 + 6c_3 x^3 + 12c_4 x^4 + 20c_5 x^5 + \cdots] - [2xc_1 + 4c_2 x^2 + 6c_3 x^3 + 8c_4 x^4 + 10c_5 x^5 + \cdots] + [2c_0 + 2c_1 x + 2c_2 x^2 + 2c_3 x^3 + 2c_4 x^4 + 2c_5 x^5 \cdots] = 0$

constant terms : $2c_2 + 2c_0 = 0 \Rightarrow \boxed{c_2 = -c_0}$

coeff of x ; $6c_3 - 2c_1 + 2c_1 = 0 \Rightarrow \boxed{c_3 = 0}$

coeff of x^2 : $12c_4 - \cancel{2c_2} - 4c_2 + \cancel{2c_2} = 0$

$\Rightarrow \boxed{c_4 = \frac{c_2}{3} = -\frac{c_0}{3}}$

coeff of $x^3 \rightarrow$ $20c_5 - 6c_3 - 6c_3 + 2c_3 = 0 \Rightarrow \boxed{c_5 = 0}$

coeff of $x^4 \longrightarrow 30C_6 - 12C_4 - 8C_4 + 2C_4 = 0$.

$30C_6 = 18C_4 \rightarrow C_6 = \frac{C_2}{5} = -\frac{C_0}{5}$

odd zero. $\boxed{C_7 = 0}$

$\therefore \quad C_8 = -\frac{C_0}{7}, \quad C_9 = 0.$

solution : $y = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + \cdots$

$y = c_0 + c_1 x - c_0 x^2 + 0 - \frac{c_0}{3} x^4 + 0 - \frac{c_0}{5} x^6 + \cdots$

$y = c_1 x + c_0 \left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} + \cdots\right)$ ans

Q3) $xy' - (x+2)y - 2x^2 - 2x = 0$

$x(c_1 + 2c_2 x + 3c_3 x^2 + 4c_4 x^3 + \cdots) - (x+2)$.
$(c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots)$

Q4) $xy'' + y' + 2y = 0$, $y(1) = 2$, $y'(1) = 4$
given $x_0 = 1$

$\therefore \quad y = \sum_{n=0}^{\infty} c_n (x - x_0)^n = \sum_{n=0}^{\infty} c_n (x-1)^n = \sum_{n=0}^{\infty} c_n t^n.$

Let $x - 1 = t \Rightarrow x = 1 + t$.

now eq^n becomes $(1+t)y'' + y' + 2y = 0$

$(1+t)[2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3 + \cdots] +$
$[c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4 + \cdots] +$
$2[c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + \cdots] = 0$

$\Rightarrow [2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3 + \cdots] + [2c_2 t + 6c_3 t^2 + 12c_4 t^3 + 20c_5 t^4 + \cdots]$

$+ [c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4 + \cdots] + [2c_0 + 2c_1 t + 2c_2 t^2 + 2c_3 t^3 + \cdots] = 0$

const. terms $\rightarrow 2c_2 + c_1 + 2c_0 = 0$.

coeff of $t \rightarrow 6c_3 + 2c_2 + 2c_2 + 2c_1 = 0$

$\Rightarrow 4c_2 + 2c_1 + 6c_3 = 0$

$t^2 \rightarrow 12c_4 + 6c_3 + 3c_3 + 2c_2 = 0$

$t^3 \rightarrow 20c_5 + 12c_4 + 4c_4 + 2c_2 = 0$.

$t^4 \rightarrow 20c_5 + 5c_5 + 30c_6 + 2c_4 = 0$.

on solving: $c_0 = 2$, $c_1 = 4$, $c_2 = -4$, $c_3 = \frac{4}{3}$.

Method of undetermined Coefficients

Q) $(D^2 + 2D + 4)y = 2x^2 + 3e^{-x}$

$\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$

$y'' + 2y' + 4y = 2x^2 + 3e^{-x}$.

CF $\rightarrow$ $m^2 + 2m + 4 = 0$.

$m = -1 \pm \sqrt{3}\, i$

CF $= e^{ax}(c_1 \cos bx + c_2 \sin bx)$, $a = -1$,

CF $= e^{-x}(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x)$. $b = \sqrt{3}$

PI $\rightarrow$. $y = a_1 x^2 + a_2 x + a_3 + a_4 e^{-x}$

$y' = 2a_1 x + a_2 - a_4 e^{-x}$

$y'' = 2a_1 + a_4 e^{-x}$

$y'' + 2y' + 4y = 2x^2 + 3e^{-x}$.

$2a_1 + a_4 e^{-x} + 4a_1 x + 2a_2 - 2a_4 e^{-x} + 4a_1 x^2 + 4a_2 x + 4a_3 + 4a_4 e^{-x} = 2x^2 + 3e^{-x}$, const term. $2a_1 + 2a_2 + 4a_3 = 0$

coeff of $x \Rightarrow 4a_1 + 4a_2 = 0$

of $x^2 \Rightarrow 4a_1 = 2 \Rightarrow \boxed{a_1 = 1/2}$

$e^{-x} \Rightarrow a_4 - 2a_4 + 4a_4 = 3$

$\boxed{a_2 = -1/2}$

$\therefore \boxed{a_3 = 0} \quad \boxed{a_4 = 1}$

$PI = y = a_1x^2 + a_2x + a_3 + a_4e^{-x}$

$\boxed{y = \frac{1}{2}x^2 - \frac{1}{2}x + e^{-x}.}$ ans.

Solut$^n \rightarrow$ CF + PI.

ans $S = e^{-x}(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x) + \frac{1}{2}x^2 - \frac{1}{2}x + e^{-x}$

Q) $y'' - 3y' + 2y = 4x^2.$

$(D^2 - 2D + 2)y = 4x^2$

$m^2 - 2m + 2 = 0$

$m^2 - m - m + 2 = 0$

$m(m-1) - 1(m - 2) = 0$

$(m-1)(m-2) = 0$

$\boxed{m = 1, 2}$

$CF = c_1e^x + c_2e^{2x}$

$PI \rightarrow \quad y = a_1x^2 + a_2x + a_3$

$y' = 2a_1x + a_2$

$y'' = 2a_1$

$2a_1 - 6a_1x - 3a_2 + 2a_1x^2 + 2a_2x + 2a_3 = 4x^2.$

Const. $\quad 2a_1 - 3a_2 + 2a_3 = 0$

$x \quad -6a_1 + 2a_2 = 0$

$x^2 \quad 2a_1 = 4$

$\boxed{a_1 = 2}$

$2a_2 = 12$

$\boxed{a_2 = 6}$

$4 - 18 + 2a_3 = 0$

$\boxed{a_3 = 7}$

PI $\Rightarrow$ $y = 2x^2 + 6x + 7$.

Solut^n $\quad y = c_1e^x + c_2e^{2x} + 2x^2 + 6x + 7$

9). $y'' + 6y' + 5y = 2e^x + 10e^{5x}$

$m^2 + 6m + 5 = 0$.

$m^2 + 5m + m + 5 = 0$

$m(m+5) + 1(m+5) = 0$

$(m+1)(m+5) = 0 \quad \Rightarrow \quad m = -1, -5$.

$CF = c_1e^{-x} + c_2e^{-5x}$

PI $\Rightarrow$ $y = a_1e^x + a_2e^{5x}$.

$y' = a_1e^x + 5a_2e^{5x}$

$y'' = a_1e^x + 25a_2e^{5x}$

$a_1e^x + 25a_2e^{5x} + 6a_1e^x + 30a_2e^{5x} + 5a_1e^x + 5a_2e^{5x}$
$= 2e^x + 10e^{5x}$.

$a_1 + 6a_1 + 5a_1 = 2$.

$12a_1 = 2 \quad \Rightarrow \quad \boxed{a_1 = \frac{1}{6}}$

$$25a_2 + 30a_2 + 5a_2 = 10$$

$$6a_2 = 10 \rightarrow \boxed{a_2 = 1/6}$$

$$PI = \frac{1}{6}e^{x} + \frac{1}{6}e^{5x}$$

~~Solut~~^{n} $\rightarrow$ $c_1 e^{-x} + c_2 e^{-5x} + \frac{1}{6}e^{x} + \frac{1}{6}e^{5x}$ ans.