Unit 4

Electricity and Magnetism

<u>Syllabus:</u>

- Laws and applications of electrostatics and magnetostatics (3 hrs.),
- Maxwell’s equations and applications (2 hrs.),
- Introduction to waveguides (1 hrs.)

<u>Electrostatics</u>

Introduction

The systematic study in which the force, field, potential of electric charges, which are at rest (or doesn’t depend on the motion of the charges) are done is called electrostatics and a systematic study in which the force, field, potential of steady current is done is called magneto-statics.In this unit we study some fundamental laws of electrostatics and magneto-statistics with some application of it.

<u>Coulomb’s law</u>

Like charges REPEL and unlike charges ATTRACT each other. This repulsion or attraction creates an electrical force. This electrical force can be calculated with Coulomb’s law. Coulomb’s law is valid for two stationary point charges (size of the charge is very small as compared to there their separation).

Statement of Coulomb’s Law

Coulomb's law states that the electrical force between two point charges is directly proportional to the product of the quantity of charge on the objects and inversely proportional to the square of the separation distance between the two objects. In equation form, Coulomb's law can be stated as

$$F = \frac{k \bullet Q_1 \bullet Q_2}{d^2}$$

Where, $\mathbf{Q_1}$ represents the quantity of charge on object 1 (in Coulombs), $\mathbf{Q_2}$ represents the quantity of charge on object 2 (in Coulombs), and **d** represents the distance of separation between the two objects (in meters). The symbol **k** is a proportionality constant known as the Coulomb's law constant. The value of this constant is dependent upon the medium that the charged objects are immersed in. In the case of air, the value is approximately equal to $1/4\pi\varepsilon_0$

(= 9.0 x 10^9 N • m^2 / C^2). If the charged objects are present in water, the value of **k** can be reduced by as much as a factor of 80.

Force is a VECTOR quantity. So here the force must be accompanied with direction. So,

- Force on Q2 is $\overline{F2} = \frac{Q_1 Q_2 K}{d_{12}^2}\overline{a_{12}}$
- Force on Q1 is $\overline{F1} = \frac{Q_1 Q_2 K}{d_{12}^2}\overline{a_{21}}$

If there are more than two point charges, then each will exert force on the other, then the net force on any charge can be obtained by the **principle of super position**.

Steps to solve Problems on Coulomb's law:

1. Obtain the position vectors of the points where the charges are located.
2. Obtain the unit vector along the straight line joining the charges. The direction is towards the charge on which the force exerted is to be calculated.
3. Using Coulomb's law, express the force exerted in the vector form.
4. If there are more charges, repeat step 1 to 3 for each charge exerting a force on the charge under consideration.
5. Using the principle of superposition, the vector sum of all the forces calculated earlier is the resultant force, exerted on the charge under consideration.

Problem 1. Two like and equal charges are at a distance of d= 5 cm, and exert a force of F = 9 × 10^{-3} N on each other (a) Find the magnitude of each charge. (b) What is the direction of the electrostatic force between them?

Ans: $\mathbf{F} = \mathbf{k}\,\frac{\boldsymbol{q_1 \times q_2}}{\boldsymbol{r^2}} = \frac{\mathbf{9 \times 10^9}[\frac{Nm^2}{C^2}]\times \boldsymbol{q^2}}{(\mathbf{0.05})^2[\boldsymbol{m^2}]} = 9 \times 10^{-3}\mathrm{N}$After solving further we get, $\mathrm{q} = 5 \times 10^{-8}$ C. The direction of the Coulomb force depends on the sign of the charges.

Two like charges repel and two unlike ones attract each other.

Since two charges have the same signs so the electric force between them is repulsive.

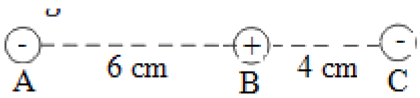
Problem 2. A point charge of 4 µC is 3 cm apart from the charge -1 µC. Find the magnitude of the Columbic force. Is the force attractive or repulsive?

$$\mathbf{F} = \mathbf{k}\,\frac{\boldsymbol{q_1 \times q_2}}{\boldsymbol{r^2}} = \frac{\mathbf{9} \times \mathbf{10^9}[\frac{\boldsymbol{Nm^2}}{\boldsymbol{C^2}}] \times \mathbf{4} \times \mathbf{10^{-6}}[\boldsymbol{C}] \times -\mathbf{1} \times \mathbf{10^{-6}}[\boldsymbol{C}]}{(\mathbf{0.03})^2[\boldsymbol{m^2}]} \approx \mathbf{40}\ \boldsymbol{N}$$

Since the charges have opposite signs so the electric force between them is attractive.

Problem 3. Three charged particles are arranged in a line as shown in figure below. Charge A = -5 μC, charge B = +10 μC and charge C = -12 μC. Calculate the net electrostatic force on particle B due to the other two charges.

Solution,



Given,
Charge A (q_A) = -5 μC = -5 x 10^{-6} C
Charge B (q_B) = +10 μC = +10 x 10^{-6} C
Charge C (q_C) = -12 μC = -12 x 10^{-6} C
k = 9 x 10^9 Nm^2C^{-2}
The distance between particle A and B (r_{AB}) = 6 cm = 0.06 m = 6 x 10^{-2} m
The distance between particle B and C (r_{BC}) = 4 cm = 0.04 m = 4 x 10^{-2} m

We need to find the magnitude and the direction of net electrostatic force on charge B. The net force on particle B is the vector sum of the force FBA exerted on particle B by particle A and the force FBC exerted on particle B by particle C.

The force F_{BA} exerted on particle B by particle A

$$F_{AB}=k\frac{q_A q_B}{r_{AB}^2}$$

$$F_{AB}=9x10^9\frac{(5x10^{-6})(10x10^{-6})}{(6x10^{-2})^2}=\frac{(9x10^9)(50x10^{-12})}{36x10^{-4}}$$

$$F_{AB}=\frac{450x10^{-3}}{36x10^{-4}}=(12.5)(10^{-3})(10^4)=12.5x10^1$$

$$F_{AB}=125 \quad \text{Newton}$$

The direction of the electrostatic force points to particle A (point to left).

The force F_{BC} exerted on particle B by particle A

$$F_{BC}=k\frac{q_B q_C}{r_{BC}^2}$$

$$F_{AB}=9x10^9\frac{(10x10^{-6})(12x10^{-6})}{(4x10^{-2})^2}=\frac{(9x10^9)(120x10^{-12})}{16x10^{-4}}$$

$$F_{AB}=\frac{1080x10^{-3}}{16x10^{-4}}=(67.5)(10^{-3})(10^4)=67.5x10^1$$

$$F_{AB}=675\,N$$

The direction of the electrostatic force points to particle C (point to right).

The net electrostatic force on particle B is
F_B = F_{AB} – F_{BC} = 675 N – 125 N = 550 Newton.The direction of the net electrostatic force on particle B points to particle C (points to the right).

Electric field

Electric field is an electric property associated with each point in space when charge is present in any form. The magnitude and direction of the electric field are expressed by the value of *E,* called electric field strength or electric field intensity or simply the electric field. It is defined as the force experienced by a unit positive charge when placed in an electric field.

For example, a charge Q_2 is placed in a region where a charged body Q_1 is present. The force experienced by the charge Q_2 due to Q_1 is given by Coulomb's lawas,

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\overline{a_{12}}$$

Thus force per unit charge can be written as,

$$\bar{E} = \frac{\overline{F_2}}{Q2} = \frac{Q_1}{4\pi\varepsilon_0 R_{12}^2}\overline{a_{12}}$$

This force exerted per unit charge is called electric field intensity or electric field strength. It is a vector quantity and is directed along a segment from the charge Q_1 to the position of any other charge. It is denoted by E.

Like Coulomb's law obeys the superposition principle, the electric field intensity also obeys the superposition principle of the charges.

Unit of $\bar{E}$ is N/C (Newtons/Coulomb) or V/m (Volts per meter).

By using the Concept of Coulomb's law and electric field, one can calculate the electric field in specific cases like

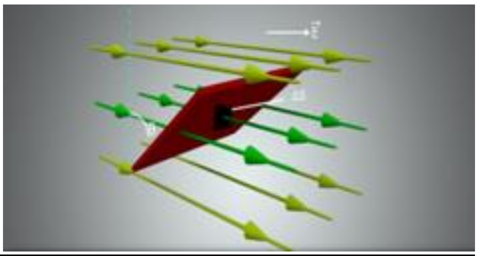
1. Field due to a linear distribution of charge.
2. Field due to a uniformly charged ring at an axial point.
3. Field due to a uniformly charged disk.
4. Field due to an electric dipole.

Definition of electrostatic potential:

When an electric charge is moved towards a like charge or away from an unlike charge, work is done against the electric forces by the external agency that moves the charge. As result, the electric charge acquires potential energy. If the charge is released, work is done by the field and charge accelerates. It means that its potential energy is converted into kinetic energy. In case of electric field the work done on a charge can be expressed in terms of the potential energy of the charge.

Electric flux

In electromagnetism, **electric flux** is the measure of flow of the **electric** field through a given area. **Electric flux** is proportional to the number of **electric** field lines going through a normally perpendicular surface.



Let us consider a uniform electric field as shown in above figure. Let the electric force lines penetrate a plane rectangular surface of area, S, which is perpendicular to the field. The number of field lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of field lines penetrating the area S is proportional to the product of *ES*. The electric flux is defined as the product of the magnitude of the electric field and surface area, S, perpendicular to the field.

Φ=ES

When the surface is not perpendicular to the field lines, then the component of E along the normal to the surface is to be multiplied by the area. Thus

Φ=(E**cos θ**)S

We may express the above relation as the scalar product of vectors E and S, as

Φ=E.S

In more general situation, the surface is of arbitrary shape. To know the flux passing through the surface, we have to divide the surface into a large number of small elements, each of area *dS*. The area *dS* of the surface element is defined as a vector whose magnitude represents the area of the element and the direction is indicated by the outward normal to the element surface. The electric flux through this small element is

$$\Delta\emptyset = E_i . \Delta S_i$$

By adding the contributions of all elements, we obtain the total flux through the entire open surface S is given by

$$\emptyset = \int_S E.dS$$

And if the surface is closed, we call it a closed surface integral and denote it by a circle on integration. Thus,

$$\emptyset = \oint_S E.dS$$

Problem 4. A uniform electric field with a magnitude of E = 400 N/C is incident on a plane surface of area S = 10 m^2 and makes an angle of θ = 30^0 with it. Find the electric flux through this surface?

Solution: Electric flux is defined as the amount of electric field passing through a surface of area S normally with formula

Φ=(E cos θ)S

= 400 × 10 (cos30^0) = 2000 Nm^2 / C

<u>Divergence and Curl</u>

Divergence and curl are two measurements of vector fields that are very useful in a variety of applications.

Both are most easily understood by thinking of the vector field as representing a flow of a liquid or gas; that is, each vector in the vector field should be interpreted as a velocity vector.

Roughly speaking, divergence measures the tendency of the fluid to collect or disperse at a point, and curl measures the tendency of the fluid to swirl around the point.

Divergence is a scalar, that is, a single number, while curl is itself a vector. The magnitude of the curl measures how much the fluid is swirling, the direction indicates the axis around which it tends to swirl.

Divergence is an operation on a vector field that tells us how the field behaves toward or away from a point. Locally, the divergence of a vector field F in R^2 or R^3 at a particular point P is a measure of the "outflowing-ness" of the vector field at P. If F represents the velocity of a fluid, then the divergence of F at P measures the net rate of change with respect to time of the amount of fluid flowing away from P (the tendency of the fluid to flow "out of" P). In particular, if the amount of fluid flowing into P is the same as the amount flowing out, then the divergence at P is zero.

Note that the divergence of a vector field is not a vector field, but a scalar function.

Del Operator

$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla f = \hat{\imath}\frac{\partial f}{\partial x} + \hat{\jmath}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

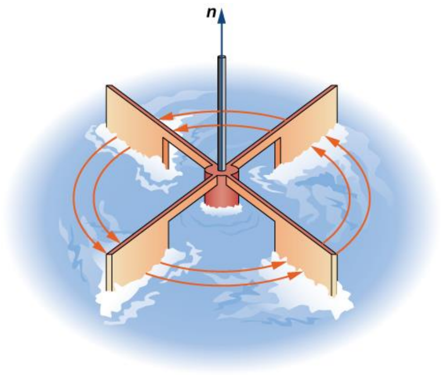
$f = 5\ (say)$

$$\nabla f = \hat{\imath}\frac{\partial(5)}{\partial x} + \hat{\jmath}\frac{\partial(5)}{\partial y} + \hat{k}\frac{\partial(5)}{\partial z}$$

$$\nabla f = 0$$

<u>Curl</u>

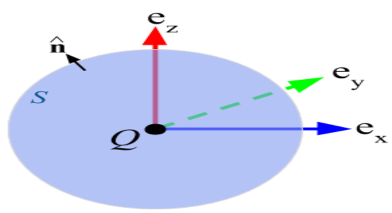
The second operation on a vector field that we examine is the curl, which measures the extent of rotation of the field about a point. Suppose that F represents the velocity field of a fluid. Then, the curl of F at point P is a vector that measures the tendency of particles near P to rotate about the axis that points in the direction of this vector. The magnitude of the curl vector is a measure of how quickly the particles rotate around this axis. In other words, the curl at a point is a measure of the vector field's "spin" at that point. Visually, imagine placing a paddlewheel into a fluid at P, with the axis of the paddlewheel aligned with the curl vector (Figure below). The curl measures the tendency of the paddlewheel to rotate.



If $F = \langle F_1, F_2, F_3 \rangle$ is a vector field, then the curl of F is defined by

$$\text{Curl}=\nabla \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

Gauss's law of electrostatics in free space

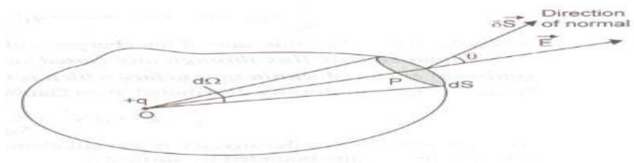


Gauss's law for the electric field describes the static electric field generated by a distribution of electric charges. **It states that the electric flux through any closed surface is equal to the $1/\varepsilon_0$ times the magnitude of the total electric charge enclosed by this surface.** By convention, a positive electric charge generates a positive electric field (coming out of the charge).

Concept of solid angle: In geometry, a **solid angle** (symbol: Ω) is a measure of the amount of the field of view from some particular point that a given object covers. That is, it is a measure of how large the object appears to an observer looking from that point.

Proof of Integral form of Gauss's Law in free space:

Let us consider a close surface S surrounding a charge "q", as shown in below figure.



Consider a small area $d\vec{S}$ of the surface surrounding the point P. Then the electric flux through $d\vec{S}$ is given by

$$d\phi = \vec{E}\,.\,d\vec{S}$$

But the electric field strength at P, $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}.\frac{q\vec{r}}{r^3}$

Since unit vector $\hat{r} = \frac{\vec{r}}{\hat{r}}$

$$d\phi = \vec{E}\,.\,d\vec{S} = \frac{1}{4\pi\varepsilon_0}q\frac{\vec{r}.\,d\vec{S}}{r^3} = \frac{q}{4\pi\varepsilon_0}\,d\Omega$$

where $d\Omega = \frac{\vec{r}.\,d\vec{S}}{r^3} = \frac{dS\cos\theta}{r^2}$ is the solid angle subtended by area dS at point O. Here θ is the angle between $d\vec{S}$ and $\vec{E}$

Hence electric flux through whole of closed surface.

$$\phi = \oint \vec{E} \bullet d\vec{S} = \frac{q}{4\pi\varepsilon_0} \times \oint d\Omega$$

But $\oint d\Omega$ is the solid angle due to the entire closed surface S at an internal point O $= 4\pi$

$$\therefore \phi = \frac{q}{4\pi\varepsilon_0}.4\pi = \frac{1}{\varepsilon_0}q$$

If there are several charges, $+q_1, +q_2, q_3, \ldots\ldots\ldots -q_1', -q_2', -q_3' \ldots\ldots$ inside the closed surface, each will contribute to the total electric flux. For positive charges the flux will be outward and hence positive; for negative charges the flux will be inward and negative. Therefore, the total electric flux in such a case

$$= \frac{1}{\varepsilon_0}q_1 + \frac{1}{\varepsilon_0}q_2 + \frac{1}{\varepsilon_0}q_3 \ldots\ldots - \frac{1}{\varepsilon_0}q_1' - \frac{1}{\varepsilon_0}q_2' - \frac{1}{\varepsilon_0}q_3' \ldots\ldots$$

$$= \frac{1}{\varepsilon_0}(q_1 + q_2 + q_3 + \ldots\ldots - q_1' - q_2' - q_3' \ldots\ldots) = \frac{1}{\varepsilon_0}\sum q$$

where $\sum q$ is the algebraic sum of the charges within the closed surface.

Hence total electric flux through any closed surface is equal to $1/\varepsilon_0$ times the total charge (in coulomb) enclosed within the surface which is Gauss's law.

Differential form of Gauss's Law in free space:

As shown above, the integral form of Gauss's law relates the net flux out of a finite volume to the net amount of charge enclosed in that volume. In contrast to above equation, the differential form of the Gauss theorem establishes the relation between the volume charge density and the charge in the field intensity E in the vicinity of a given point in space. The integrals will then transform to differentials in the limit as the volume goes to zero, and we will obtain a point relationship of Gauss's law involving derivatives only. The differential form of Gauss's law is more general and will be n=very useful since derivatives are easier to calculate compared to integrals.Let us represent the charge q in the above volume V enclosed by a closed surface S as $q_{int} = \langle \rho \rangle V$, where $\langle \rho \rangle$ is the volume charge density, averaged over the volume V. Using this into equation flux and dividing the equation with V, we obtain

$$\frac{1}{V}\oint E.ds = \frac{q}{\varepsilon_0}$$

We now make the volume V tend to zero, then $\langle q \rangle$ will tend to the value of q at the given point of the field. When V tends to zero, the quantity on the L.H.S. in above equation is called the divergence of the field E. By definition

$$\nabla.E = \lim_{V\to 0}\frac{1}{V}\oint_s \ E.ds$$

Consequently, the above relation transforms into

$$\nabla.E = \frac{q}{\varepsilon_0}$$

Above equation is the Gauss theorem expressed in the differential form.

Gauss's law of electrostatics in a dielectric medium

Dielectric (broadly, insulators) do not contain free electrons that can move over considerable distance, so no conduction of current is possible. However, when an external electric field is applied to the dielectric, the positive nuclei in each molecule are displaced along the field direction and the bound electrons are displaced in the opposite direction. The bound electrons are not mobile but are elastically bound to the molecule. Therefore, they can move only with in the electrically neutral molecules through a very small distance. This displacement of charges is known as polarization of the dielectric. As a result of polarization the electric field E acting in a dielectric is the superposition of the field E_0 of the extraneous charges and the field E' of bound charges. Thus,

$E = E_0 + E'$

Since the sources of an electric field E are all electric charges –free and bound, we can write Gauss theorem for the **E** as

$\oint \varepsilon_0 E\, ds \;=\; (q + q')_{\text{int}}$ (A)

Where q and q' are free and bound charges enclosed by the surface S. We now express q' in terms of P, the polarization vector as

$\oint P\, ds = -q'_{int}$ (B)

Using equation,A and B, we get

$$\oint (\varepsilon_0 E + P) ds = q_{int}$$

Let us define D = $(\varepsilon_0 E + P)$ auxiliary vector. Then Gauss's theorem can be reframed in terms of D can be written as

$$\oint Dds = q_{int}$$

This Auxiliary Vector D is called as dielectric displacement.

For normal conductors (without dielectric), P = 0, therefore the Gauss's theorem hold as it is explained in the previous case.

So the revised form with displacement vector of Gauss's law in its integral and differential form can be written as

$$\oint Dds = q_{int}$$

And

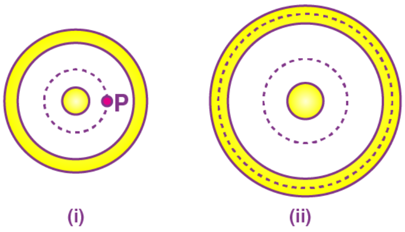
$$\nabla . D = q$$

This is also known as the Maxwell's first equation in its integral and differential form.

Physical significance of Maxwell's first equation is that the electric flux through a closed surface is proportional to the charged enclosed. It means that a charge distribution generates a steady electric field.

Problem 5. A charge of 4×10^{-8} C is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of radius 5 cm.

- Find the electric field at a point 2 cm away from the centre.
- A charge of 6×10^{-8}C is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.



(a) Let us consider the figure (i).

Suppose, we have to find the field at point P. Let's draw a concentric spherical surface through P. All the points on this surface are equivalent and by symmetry, the field at all these points will be equal in magnitude and radial in direction.

The flux through this surface = $4\pi r^2 E$.

Where, r (radius of the sphere) = 2 cm = 2×10^{-2} m.

From Gauss law, this flux is equal to the charge q contained inside the surface divided by ε_0. Thus,

$$4\pi r^2 E = q/\varepsilon_0$$

Or $$E = q/4\pi\varepsilon_0 x^2$$

$$= [(9 \times 10^9 [Nm^2/C^2]) \times [(4 \times 10^{-8}[C])] /(4 \times 10^{-4} m^2)]$$

$$= 9 \times 10^5 N C^{-1}.$$

(b) Let us consider the figure (ii).

Take the Gaussian surface through the material of the hollow sphere. As the electric field in a conducting material is zero, the flux through this Gaussian surface is zero.

Using Gauss law, the total charge enclosed must be zero. Hence, the charge on the inner surface of the hollow sphere is 4×10^{-8}C.

But the total charge given to this hollow sphere is 6×10^{-8} C. Hence, the charge on the outer surface will be 10×10^{-8} C.

<u>Magnetostatics</u>

Static magnetic fields are produced by permanent magnets and steady currents flowing in conductors. Magnetostatics deals with magnetic fields produced by steady currents.

Magnetic field

A steady current *I* flowing in a conductor produces a magnetic field around it. The region around a current carrying conductor or a permanent magnet where magnetic effects are experienced is called a magnetic field. A magnetic field is schematically represented by magnetic lines of force, which are also known as field lines or lines of magnetic induction. A magnetic field is described by magnetic induction (or magnetic flux density), *B*.

The magnetic induction or flux density is defined as the magnetic lines of force (flux) passing normally through a unit area of cross-section at that point. It is denoted by the symbol B and is expressed in Wb/m^2.

Thus $B = \frac{Magnetic\ flux}{Area} = \frac{\varphi}{A}$

(Magnetic flux is defined as the total number of lines of force emanating from the north pole.)

Therefore, the magnetic flux is given by $\phi = BA$. Let θ be the angle between the normal to the area and the direction of magnetic field. Then,

$\phi = BA \cos \theta = B.A$

The magnetic flux through any surface may also be given by the surface integral of the normal component of B. Thus,

$\phi = \int_S^0 \boldsymbol{B}.d\boldsymbol{s}$ where ds is the elemental surface.

Biot- Savart Law Let us consider some conductor of arbitrary shape carrying static current I. Let small element AB of length dl produce magnetic field dB at point P. Let r be the distance of P from the current element $I\ dl$ and θ be the angle between dl and r. According to Biot-Savart law, the magnitude of magnetic field at point P will be given as

$dB \propto I$ (1)

$dB \propto dl$ (2)

$dB \propto Sin\ \theta$ (3)

$dB \propto \frac{1}{r^2}$ (4)

$dB \propto \frac{Idl\ Sin\ \theta}{r^2}$ (5)

$B = k\ \frac{Idl\ Sin\ \theta}{r^2}$ (6)

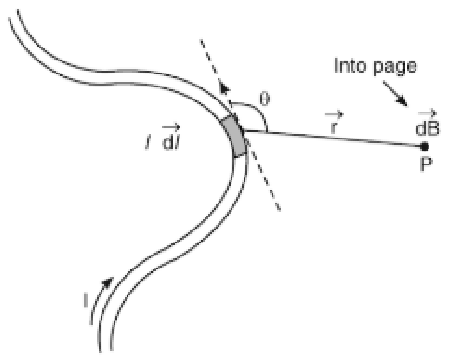


Fig.1.

where k is constant of proportionality. The value of k depends on medium in which conductor is situated and the system of units adopted. The SI units, its value for the free space is

$$k = \frac{\mu_o}{4\pi} \text{ where } \mu_o = 4\pi \times 10^{-7} \text{ Tm/A} \quad (7)$$

$$dB = \frac{\mu_o}{4\pi} \frac{Idl\ Sin\theta}{r^2} \quad (8)$$

The Biot-Savart law holds only for steady current. The current element $I\ dl$ is the source of the static magnetic field, just as q is the source of the static electric field. The above law is written in the vector form as

$$dB = \frac{\mu_o}{4\pi} \frac{\boldsymbol{dl \times r}}{r^3} \qquad (9)$$

The direction of the magnetic field is given by the right hand thumb rule. The direction of dB is into the plane of the paper.

The total magnetic field at P due to the conductor is obtained by summing up the contribution of all current element.

$$B = \int dB = \int \frac{\mu_o}{4\pi} \frac{\boldsymbol{dl \times r}}{r^3} \qquad (10)$$

Ampere's Law

Ampere's law states that the line integral of the tangential component of the magnetic field over any closed path is equal to the amount of the current enclosed by the loop. Thus,

$$\oint B.dl = \mu_o I \qquad (11)$$

Both Ampere's law and the Biot- Savart law are relations between a current distribution and the magnetic field that it generates. We can apply Biot-Savart law to calculate the magnetic field caused by any current distribution. On the other hand, Ampere's law allows us to calculate magnetic field with ease in case of symmetry.

Let us consider an infinite long wire along the z-axis carrying a current I. The magnetic flux density due to this wire is directed everywhere circular to the wire and its magnitude is dependent only on the distance from the wire. Let us consider a circular path C of radius r in the plane normal to the wire and centred at the wire. The current enclosed by an arbitrary closed path C is given by the surface integral of the current density over any surface S bounded by the closed path C.

The total current flowing through the surface area S is given by,

$$I = \int J.ds \qquad (12)$$

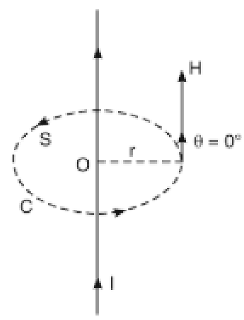


Fig. 2.

Multiplying above equation by μ_o

$$\mu_o I = \mu_o \int J.ds \qquad (13)$$

whereas J is the volume current density

and substituting equation (1), we get

$$\oint B.dl = \mu_o \int J.ds \qquad (14)$$

This is known as *Ampere's circuital law*.

Ampere's Circuital Law in differential form

If we now shrink the path C to a very small size ΔC so that the surface area bounded by it becomes very small, ΔS we can write equation as

$$\oint_{\Delta C} B.dl = \mu_o \int_{\Delta S} J.ds \qquad (15)$$

Since the surface area ΔS IS very small we can consider the current density to be uniform over the surface so that

$$\int_{\Delta S} J.ds \approx J.\Delta S \qquad (16)$$

The relation becomes exact in the limit ΔS → 0. Dividing both the sides of equation by ΔS and letting ΔS → 0, we have

$$\lim_{\Delta S \to 0} \frac{\oint_{\Delta C} B.dl}{\Delta S} = \lim_{\Delta S \to 0} \frac{\mu_o \int_{\Delta S} J.ds}{\Delta S} \qquad (17)$$

$$= \mu_o \lim_{\Delta S \to 0} \frac{\int_{\Delta S} J.\Delta S}{\Delta S} \qquad (18)$$

$$= \mu_o \boldsymbol{J.n} \qquad (19)$$

Now, the curl of *B* is defined as the vector having the magnitude given by the maximum value of the quality on the left side equation. We note that this maximum value occurs for an orientation of *ΔS* for which the direction of its normal coincides with the direction of *J* and it is equal to μ_o times the magnitude of *J*. Thus,

$$|\nabla \times B| = maximum\ value\ of \left(\lim_{\Delta S \to 0} \frac{\oint_{\Delta C} B.dl}{\Delta S} \right) = \mu_o |J| \qquad (20)$$

$$\nabla \times B = \mu_o J \qquad (21)$$

This is *Ampere's circuital law in differential form*.

Gauss's law for magnetism

Just as in the case of electrostatics, the magnetic flux through an element of area ds is given by the dot product of B with ds. For an arbitrary surface ds bounded by a closed contour S, total magnetic flux passing through the surface is given by

$$\phi = \oint B.ds \tag{22}$$

The lines of vector B have neither beginning nor ending. The number of lines emerging from any volume bounded by a closed surface S is always equal to the number of lines entering the volume. Hence, the flux of B through any closed surface is equal to zero. Thus,

$$\oint B.ds = 0 \tag{23}$$

Dividing both sides of the above equation by an incremental volume Δυ over which the surface is to be considered closed, we get

$$\frac{\oint B.ds}{\Delta\upsilon} = 0 \tag{24}$$

The limit of the left side of the equation, as $\Delta\upsilon \rightarrow 0$, is the divergence of the vector B.

$$\nabla.B = 0 \tag{25}$$

Its integral form is $\oint_s \; B.ds = 0$

This is the Second Maxwell's equation.

Physical significance of this equation is - The total magnetic flux through a closed surface is zero. It implies that magnetic poles do not exist separately in the way as electric charges do. In other words, magnetic monopoles do not exist.

Equation of continuity

Electric charge can neither be created nor destroyed. Therefore, the net charge in an isolated system remains constant. This is known as the principle of conservation of charge. This principle implies that the time rate of increase (decrease) of charge within a closed volume equals the net rate of flow of a charge into (or, out of) the volume. This statement of conservation of charge is expressed by the equation of continuity.

The integral form of continuity equation is given by

$$\oint_S^0 \boldsymbol{J}.dS \; = \; -\frac{dq}{dt}$$

The differential form of continuity equation is

$\nabla . \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$

The physical significance of equation of continuity is that the amount of electric charge in any volume of space can only change by the amount of electric current flowing into or out of that volume through its boundaries.

Maxwell's Equations

Maxwell's equations represent one of the most elegant and concise ways to state the fundamentals of electricity and magnetism. These equations describe how electric and magnetic fields propagate, interact, and how they are influenced by objects.

The four Maxwell's equations in differential form are

$$\nabla . E = \frac{\rho}{\epsilon_0} \qquad (1) \qquad \text{Gauss' law}$$

$$\nabla . B = 0 \qquad (2) \qquad \text{Magnetic monopoles}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad (3) \qquad \text{Faraday's law}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \qquad (4) \qquad \text{Ampere-Maxwell law}$$

The same equations in the integral form are expressed as

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{d\mathrm{t}}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_{\mathrm{E}}}{d\mathrm{t}} + \mu_0 i_{enc}$$

The word statement of the significance of the above four Maxwell's equations are

1. The total electric displacement through any surface enclosing a volume is equal to the total charge within the volume. [Gauss law of electrostatics]
2. The net magnetic flux through any closed surface s zero. Magnetic monopoles do not exist. [Gauss law of magnetostatics]
3. The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path. [Faraday's law]
4. The magnetomotive force around a closed path is equal to the conduction current plus time derivative of the electric displacement through any surface bounded by the path. [Ampere-Maxwell law]

Displacement Current

Ampere’s law implies that a magnetic field can be produced only by a flow of charges. Ampere’s law was established as result of large number of careful experiments done on steady situation. Maxwell showed that we run into difficulty when apply the ampere’s law for time-varying situations such as charge building up on the plates of a capacitor. He showed that we have to include another current, called displacement currant, which also can produce time-varying magnetic field. The need for displacement current can be well understood when current flow through a capacitor is considered.

To see how magnetic fields can be created by a time-varying electric field, consider a capacitor which is being charged. During the charging process, the electric field strength increases with time as more charge is accumulated on the plates. The conduction current that carries the charges also produces a magnetic field. In order to apply Ampere’s law to calculate this field, let us choose curve C shown in the Figure below to be the Amperian loop

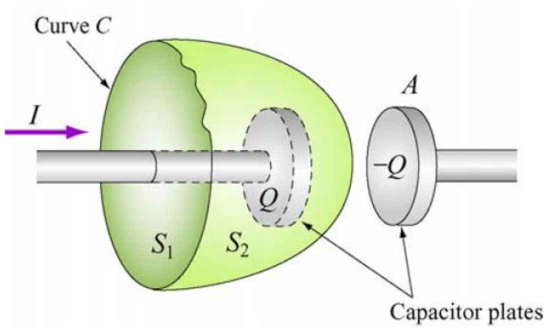


Fig. Surfaces S1 and S2 bound by curve C.

If the surface bounded by the path is the flat surface S1, then the enclosed current is $I_{enc} = I$.

$$I = \oint_C^o \boldsymbol{H}.dl = \oint_{S1}^0 \boldsymbol{J}.dS \qquad \text{(A)}$$

On the other hand, if we choose S2 to be the surface bounded by the curve, then $I_{enc} = 0$ since no current passes through S2.

$$I = \oint_C^o \boldsymbol{H}.dl = \oint_{S2}^0 \boldsymbol{J}.dS = 0 \qquad \text{(B)}$$

It can be seen that equation A and B contradict each other. Therefore, it appears that Ampere’s equation A requires modification.

We know that negative charge flows from the battery up to the plate P_1 of the capacitor. Similarly, negative charge flows from plate P2 to the battery. But there is no charge flow between the plates of the charging capacitor. However, there is a continuous current I flowing in the circuit. Although there is no current crossing the surface S2, there is certainly a changing electric field, because the capacitor is charging up as the current *I* flow in. Maxwell argues that this changing electric field constitute an effective current.

We note that surface S2 cuts only the electric field. In accordance with the Gauss's Theorem, the flux of vector D through a closed surface is

$$\oint \boldsymbol{D}.ds = q$$

Therefore, the current I is

$$I = \frac{dq}{dt} = \frac{\partial}{\partial t}\oint \boldsymbol{D}.ds = \oint \frac{\partial \boldsymbol{D}}{\partial t}.ds \qquad \text{(C)}$$

According to equation of continuity equation,

$$\oint_S^0 \boldsymbol{J}.dS = -\frac{dq}{dt} \qquad \text{(D)}$$

where J is the conduction current density.

Summing up the left and the right hand sides of Eqns. C and D separately, we obtain

$$\oint \left(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right) ds = 0$$

This equation is similar to the continuity equation for direct current. There is one more term $\frac{\partial D}{\partial t}$ whose dimensions are the same as for current density. Maxwell termed this term as the density of displacement current. Thus,

$$\boldsymbol{J_d} = \frac{\partial \boldsymbol{D}}{\partial t}$$

The sum of conduction and the displacement currents is called the total current. Its density is given by

$$\boldsymbol{J_T} = \boldsymbol{J} + \boldsymbol{J_d} = \mathbf{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

The theorem on circulation of vector H, which was established for direct currents, can be generalized for an arbitrary case in the following form,

$$\boxed{\oint \boldsymbol{H}.dl = \int \left(J + \frac{\partial \boldsymbol{D}}{\partial t}\right).ds \qquad \text{Integral form of Ampere's Maxwell Law}}$$

Converting the line integral into the surface integral using Stoke's Theorem. Thus we get,

$$\int (\boldsymbol{\nabla} \times \boldsymbol{H}).ds = \int \left[\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}\right].\mathrm{ds}$$

Equating the integrals, we get

$$\boxed{\boldsymbol{\nabla} \times \boldsymbol{H} = J + \frac{\partial \boldsymbol{D}}{\partial t} \qquad \text{Differential Form of Ampere's Maxwell Law}}$$

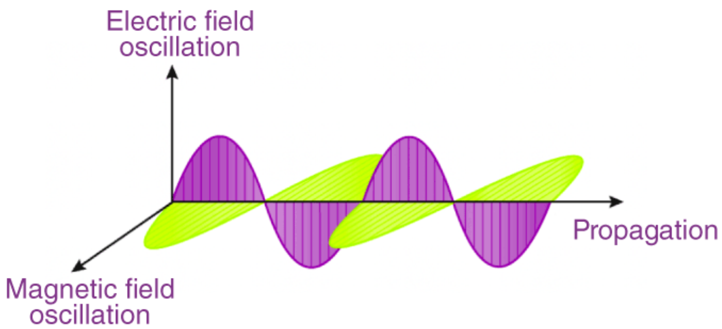
The Physical significance of these equations is that a changing electric field induces a magnetic field.

Applications of Maxwell's Equations

1. In calculation of speed of light and explanation of refraction of light using Maxwells equations.
2. MRI Scanning
3. In GPS for navigation
4. In defense
5. Wireless communication (Examples required)
6. The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors

Electromagnetic (EM) Waves

Electromagnetic waves are also known as EM waves. Electromagnetic radiations are composed of electromagnetic waves that are produced when an electric field comes in contact with the magnetic field. It can also be said that electromagnetic waves are the composition of oscillating electric and magnetic fields. Electromagnetic waves are solutions of Maxwell's equations, which are the fundamental equations of electrodynamics.



Electromagnetic waves are shown by a sinusoidal graph. It consists of time-varying electric and magnetic fields which are perpendicular to each other and are also perpendicular to the direction of propagation of waves. Electromagnetic waves are transverse in nature. In vacuum, the waves travel at a constant velocity of 3 x 10^8 m.s^{-1}. The detail mathematical derivation in the light of Maxwell's equations is given below.

The electromagnetic wave equation

Maxwell's equations in free space (i.e. where there is no charge or current) are given by

i. $\boldsymbol{\nabla . E = 0}$

ii. $\boldsymbol{\nabla . B = 0}$

iii. $\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\boldsymbol{\partial B}}{\boldsymbol{\partial t}}$

iv. $\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\boldsymbol{\partial E}}{\boldsymbol{\partial t}}$

Field vectors **E** and **B** are coupled with this set of equations. Now,

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\boldsymbol{\partial B}}{\boldsymbol{\partial t}} \qquad \text{using Eqn. (iii)}$$

Or $$\nabla \times (\nabla \times \boldsymbol{E}) = -(\nabla \times \frac{\partial \boldsymbol{B}}{\partial t})$$

Or $$\nabla(\nabla.\boldsymbol{E}) - \nabla^2 \boldsymbol{E} = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{B})$$

[Since ***A X (B X C) = (A.C)B – (A.B.)C***]

Or $$\nabla^2 \boldsymbol{E} = \frac{\partial}{\partial t}(\nabla \times \boldsymbol{B}) \qquad \text{Since} \nabla.\boldsymbol{E} = \boldsymbol{0}$$

Or $$\nabla^2 \boldsymbol{E} = \frac{\partial}{\partial t}(\mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}) \qquad \text{using Eqn. (iv)}$$

Or $$\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

Or $$\nabla^2 \boldsymbol{E} = \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} \quad \text{where } v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

---------------------------- (1)

This is the relation between space and time dependence of $\boldsymbol{E}$. Again,

$$\nabla \times (\nabla \times \boldsymbol{B}) = \nabla \times (\mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}) \qquad \text{using Eqn. (iv)}$$

Or $$\nabla(\nabla.\boldsymbol{B}) - \nabla^2 \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial(\nabla \times \boldsymbol{E})}{\partial t}$$

Or $$\nabla^2 \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{B}}{\partial t^2} \qquad \text{using Eqns. (ii) and (iii)}$$

Or $$\nabla^2 \boldsymbol{B} = \frac{1}{v^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} \quad \text{where } v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

---------------------------- (2)

This is the relation between space and time dependence of $\boldsymbol{B}$.

Eqns. (1) and (2) are wave equations for $\boldsymbol{E}$ and $\boldsymbol{B}$, respectively.

Here the velocity of propagation,

$$\boldsymbol{v} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{8.85\, x 10^{-12} x\, 4\pi\, x\, 10^{-7}\left[\frac{F}{m} x \frac{H}{m}\right]}} = 3\, x\, 10^8 \frac{m}{s}$$

= speed of light in free space = $\boldsymbol{c}$

Thus Maxwell's equations show that electromagnetic disturbances propagate with a velocity of light in vacuum. The implication is that light is an electromagnetic wave.

The simplest type of wave that is a solution of Eqns. (1) and (2) is a plane wave solution. Now the plane wave is one in which the field vector components are constant over all points in a plane which is normal to the direction of propagation. The field vector components that lie on the given plane are functions of the perpendicular distance of the plane from the origin and also time.

Now the wave equation for a wave propagating along *x*-direction is given by

$$\frac{\delta^2\psi}{\delta x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

Solution of the above equation is

$$\psi(x,t) = A\, e^{i(kx-\omega t)} + Be^{-i(kx-\omega t)}$$

$$= f\,(x\text{-}vt) + g\,(x + vt)$$

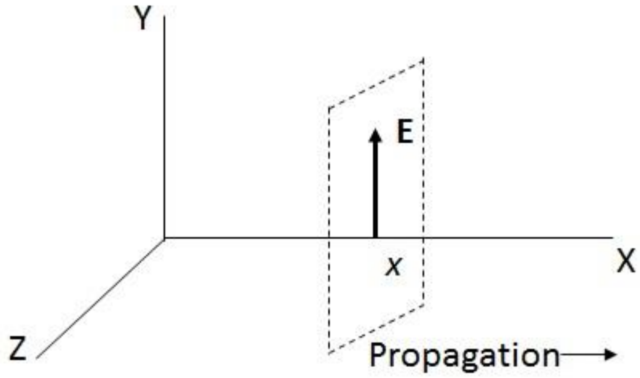
Where, $k = 2\pi/\lambda = \omega/v$, *k* being the propagation vector and *v* is the phase velocity.

We assume plane wave solutions for electromagnetic wave equations as

$$\boxed{\begin{aligned} \boldsymbol{E}(x,t) &= \boldsymbol{E}_0 e^{i(kx-\omega t)} \\ \boldsymbol{B}(x,t) &= \boldsymbol{B}_0 e^{i(kx-\omega t)} \end{aligned}}$$

--------------------- (3)

Where, $\boldsymbol{E_0}$ and $\boldsymbol{B_0}$ are complex amplitudes of the electric and magnetic fields, respectively.



E depends only on *x* and *t*, but not on *y* and *z*.

More general expressions are

$$\boldsymbol{E}(r,t) = \boldsymbol{E}_0 e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)} \text{ and } \boldsymbol{B}(r,t) = \boldsymbol{B}_0 e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$$ --------------------- (4)

Now,
$$E_x = \boldsymbol{E}_{0_x} e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$$

$$E_y = \boldsymbol{E}_{0_y} e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$$

$$E_z = \boldsymbol{E}_{0_z} e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$$

$$(\boldsymbol{\nabla}\times\boldsymbol{E})_{\boldsymbol{x}} = \text{x component of } \boldsymbol{\nabla}\times\boldsymbol{E}$$

We have,
$$\boldsymbol{\nabla}\times\boldsymbol{E} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \boldsymbol{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \boldsymbol{j}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \boldsymbol{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

Therefore,

$$(\boldsymbol{\nabla}\times\boldsymbol{E})_{\boldsymbol{x}}=\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=\frac{\partial(E_{0z}e^{i(k.r-\omega t)})}{\partial y}-\frac{\partial(E_{0y}e^{i(k.r-\omega t)})}{\partial z}$$

Again, $\frac{\partial E_z}{\partial y}=\frac{\partial(E_{0z}e^{i(k.r-\omega t)})}{\partial y}=\frac{\partial(E_{0z}e^{i\{(k_x x+k_y y+k_z z)-\omega t)\}})}{\partial y}=E_{0_z}e^{i(k.r-\omega t)}ik_y$

Similarly, $\frac{\partial E_y}{\partial y}=E_{0_y}e^{i(k.r-\omega t)}ik_z$

Thus, $(\boldsymbol{\nabla}\times\boldsymbol{E})_{\boldsymbol{x}}=(E_{0_z}k_y-E_{0_y}k_z)ie^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$

$$=i(E_zk_y-E_yk_z)$$

$$=\boldsymbol{i}(\boldsymbol{k}\times\boldsymbol{E})_{\boldsymbol{x}}$$

Therefore, $(\boldsymbol{\nabla}\times\boldsymbol{E})=\boldsymbol{i}(\boldsymbol{k}\times\boldsymbol{E})$

Again, we have assumed solution for **B** as,

$$\boldsymbol{B}(r,t)=\boldsymbol{B}_0e^{i(\boldsymbol{k}.r-\omega t)}$$

So, $\frac{\partial\boldsymbol{B}}{\partial t}=\boldsymbol{B}_0(-i\omega)e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}=-i\boldsymbol{B}_0\omega e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}=-i\omega\boldsymbol{B}$

Now, $\boldsymbol{\nabla}\times\boldsymbol{E}=-\frac{\partial\boldsymbol{B}}{\partial\boldsymbol{t}}$

Or $\boldsymbol{i}(\boldsymbol{k}\times\boldsymbol{E})=-(-i\omega\boldsymbol{B})$

Or $\boldsymbol{B}=\frac{1}{\omega}(\boldsymbol{k}\times\boldsymbol{E})$

Thus **B** is perpendicular to both **K** and **E**.

Again, in free space $\boldsymbol{\nabla}.\boldsymbol{E}=\boldsymbol{0}$

Therefore, $\left(\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}\right)=0$

With $E_x=\boldsymbol{E}_{0_x}e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$

So, $\frac{\partial E_x}{\partial x}=\frac{\partial[\boldsymbol{E}_{0x}e^{i\{(k_x x+k_y y+k_z z)-\omega t)\}}]}{\partial x}=\boldsymbol{E}_{0_x}e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}ikx=iK_xE_x$

Similarly, $\frac{\partial E_y}{\partial y}=iK_yE_y$ and $\frac{\partial E_z}{\partial z}=iK_zE_z$

Therefore, $\boldsymbol{\nabla}.\boldsymbol{E}=i(K_xE_x+K_yE_y+K_zE_z)=i(\boldsymbol{K}.\boldsymbol{E})$

Since $\boldsymbol{\nabla}.\boldsymbol{E}=0$, $\boldsymbol{K}.\boldsymbol{E}=0$

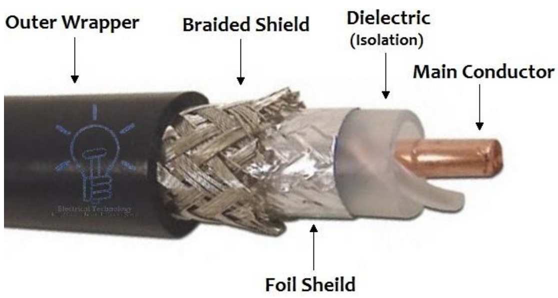
Therefore, $\boldsymbol{K}$ is perpendicular to $\boldsymbol{E}$.

Thus we can conclude that $\boldsymbol{E}$ is perpendicular to the propagation vector $\boldsymbol{K}$. Similarly from $\boldsymbol{\nabla}.\boldsymbol{B}=\boldsymbol{0}$, we conclude B is perpendicular to K. Hence the inference is that $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{K}$ are mutually perpendicular to each other. So the electromagnetic wave is a transverse wave.

Wave propagation in guided media

Coaxial Cables: It is a transmission line that consists of a tube of electrically conducting material surrounding a central conductor held in place by insulators and that is used to transmit telegraph, telephone, television, and Internet signals. Coaxial cable consists of a center wire which is mostly copper wires that is surrounded by insulation, grounded shield and a braided wire. Some of the cables consist of both braided wire and aluminum foil. Coaxial cabling is primarily used by the cable television industry and is also used for computer networks such as Ethernet.

Many coaxial cables have outer insulating sheath which protects them from the external environment. The term coaxial (co-axial) comes from the inner conductor and shield sharing a geometric axis.



Wave Propagation in Bounded System: Wave Guide

A structure that can guide waves like EM waves, light waves or sound waves is called waveguide. The first waveguide was proposed by Thomson in 1893 and was experimentally verified by Lodge in1894.

For different types of the waves, there are different types of waveguides. For example, depending on the frequency of EM wave, the waveguide can be constructed from either conductive or dielectric material.

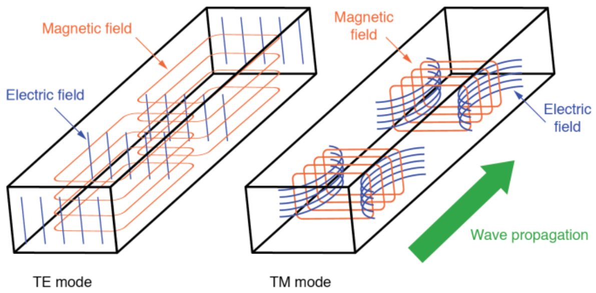
Electromagnetic Waveguide

The original meaning of waveguide is a hollow metal pipe used for guiding the waves. The EM waves in such waveguides may be imagined as waves travelling down the guide in a zig zag path as these waves are repeatedly reflected between opposite walls of the guide. The first mathematical analysis of the propagating modes within a hollow metal cylinder was performed by Rayleigh in 1897.

To function properly, a waveguide must have a certain minimum diameter relative to the wavelength of the signal. If the waveguide is too narrow or the frequency is too low, the electromagnetic field cannot propagate. There is a minimum frequency, known as the cut off frequency for the propagation of the wave. The cut off frequency is decided by the dimensions of the waveguide.

Rectangular waveguides

Rectangular waveguides are one of the earliest type of the transmission lines. They are used in many applications. A lot of components such as isolators, detectors, attenuators, couplers and slotted lines are available for various standard waveguide bands between 1 GHz to above 220 GHz. A rectangular waveguide supports TM and TE modes but not TEM waves because we cannot define a unique voltage since there is only one conductor in a rectangular waveguide. A rectangular waveguide cannot propagate below some certain frequency. This frequency is called the *cut-off frequency*.



Magnetic flux lines appear as continuous loops
Electric flux lines appear with beginning and end points

Modes in Waveguides

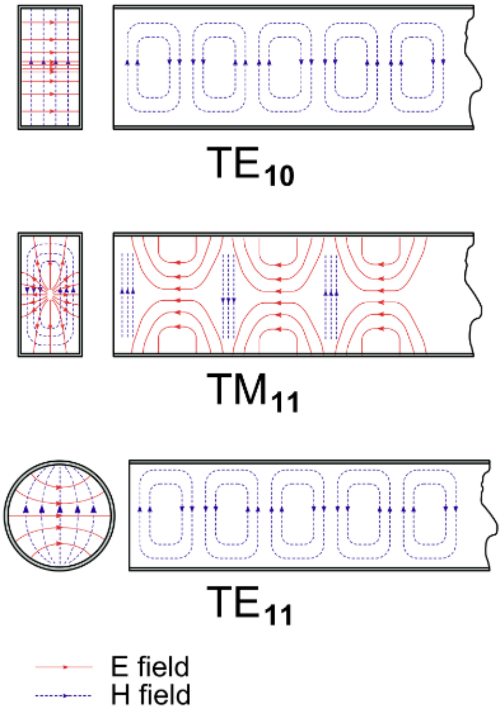
The propagation of the waveguide modes depends on the operating wavelength and polarization, and shape and size of the waveguide. The longitudinal mode can be realized in a cavity; the longitudinal mode is particularly standing wave pattern formed by the waves confined in the cavity. However, a number of transverse modes can be excited in the waveguide which are classified below.

1. **Transverse Electric (TE) modes:** These modes do not have electric field in the direction of propagation. So electric field vector is in transverse direction.
2. **Transverse Magnetic (TM) modes:** These modes do not have magnetic field in the direction of propagation. So magnetic field vector is in transverse direction.
3. **Transverse Electromagnetic (TEM) Modes:** These modes do not have electric and magnetic field in the direction of propagation. In hollow waveguides, TEM modes are not possible because as per Maxwell's equation the electric field then must have zero divergence, zero curl and be zero at the boundaries. This will result in a zero field. However, TEM modes can propagate in a coaxial cable.
4. **Hybrid Modes:** These modes have both electric and magnetic field components in the direction of propagation. The mode for which cut off frequency is the minimum is called the fundamental mode.

5. Hollow metallic waveguides filled with a homogeneous, isotropic material (usually air) support TE and TM modes but not the TEM mode. In coaxial cable energy is normally transported in the fundamental TEM mode. The TEM mode is also usually assumed for most other electrical conductor line formats as well. This is mostly an accurate assumption, but a major exception is microstrip which has a significant longitudinal component to the propagated wave due to the inhomogeneity at the

boundary of the dielectric substrate below the conductor and the air above it. In an optical fiber or other dielectric waveguide, modes are generally of the hybrid type.

6. In rectangular waveguides, rectangular mode numbers are designated by two suffix numbers attached to the mode type, such as TE_{mn} or TM_{mn}, where *m* is the number of half-wave patterns across the width of the waveguide and *n* is the number of half-wave patterns across the height of the waveguide. In circular waveguides, circular modes exist and here *m* is the number of full-wave patterns along the circumference and *n* is the number of half-wave patterns along the diameter.



Applications

Waveguides are widely used in the following:

- Optical fiber communication.
- Photonic integrated circuits.
- Maintaining high optical intensities in non-linear devices.
- As mode cleaners.
- Optical interferometers.