SVKM's NarseeMonjee Institute of Management Studies
Mukesh Patel School of Technology Management & Engineering

Unit III
First order ordinary Differential Equations

Tutorial No. 1
Exact Differential Equation, Non Exact Differential equation reducible to Exact form

Solve the following Differential equations:

1. $(y^2e^{xy^2}+4x^3)\,dx+(2xye^{xy^2}-3y^2)\,dy=0$ **(ans:** $e^{xy^2}+x^4-y^3=c$**)**

2. $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$ given that $y(0)=4$ **(ans:** $x+ye^{\frac{x}{y}}=4$**)**

3. $\left(2xy\cos x^2-2xy+1\right)dx+\left(\sin x^2-x^2\right)dy=0$ **(ans:** $y\sin x^2-x^2y+x=c$**)**

4. $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}\left(x+xy^2\right)dy=0$ **(ans:** $x^6+3x^4y+x^4y^3=c$**)**

5. $(2x\log x-xy)dy+2ydx=0$ **(ans:** $2y\log x-\frac{y^2}{2}=c$**)**

6. $(xy^2-e^{\frac{1}{x^3}})dx-x^2ydy=0$ **(ans:** $\frac{e^{\frac{1}{x^3}}}{3}-\frac{y^2}{2x^2}=c$**)**

7. $\left(6x^2+4y^3+12y\right)dx+3x\left(1+y^2\right)dy=0$ **(ans:** $x^6+3x^4y+x^4y^3=c$**)**

8. $\left(2xy^4e^y+2xy^3+y\right)dx+\left(x^2y^4e^y-x^2y^2-3x\right)dy=0$ **(ans:** $x^2e^y+\frac{x^2}{y}+\frac{x}{y^3}=c$**)**

9. $y\left(xy+e^x\right)dx-e^x\,dy=0$ **(ans:** $\frac{x^2}{2}+\frac{e^x}{y}=c$**)**

10. $(3x^2y^4+2xy)dx+(2x^3y^3-x^2)dy=0$ **(ans:** $x^2+x^3y^3=cy$**)**

11. $y(x^2y+e^x)dx-e^xdy=0$ **(ans:** $\frac{x^3}{3}+\frac{e^x}{y}=c$**)**

12. $y(1+xy)dx+x(1+xy+x^2y^2)dy=0$ **(ans:** $\frac{1}{2x^2y^2}+\frac{1}{xy}-\log y=c$**)**

13. $y(xy+2x^2y^2)dx+x(xy+x^2y^2)dy=0$ **(ans:** $\log\left(x^2y\right)-\frac{1}{xy}=c$**)**

14. $y(x+y)dx-x\left(y-x\right)dy=0$ **(ans:** $\frac{1}{2}\log\left(xy\right)-\frac{y}{2x}=c$**)**

15. $(x^2-xy+y^2)dx-xy\,dy=0$ **(ans:** $\log\left(x-y\right)+\frac{y}{x}=c$**)**

16. $(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$ **(ans:** $\frac{x^3}{cy^2} = e^{\frac{-y}{x}}$ **)**

Tutorial No. 2

Linear Differential Equations,Bernoulli's Equation, Orthogonal trajectories:

1. $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$ **(ans:** $xy\sec x = \tan x + c$ **)**

2. $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$ **(ans:** $y = \sqrt{1-x^2} + c\left(1-x^2\right)$ **)**

3. $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$ **(ans:** $y = x^3 + \frac{cx^2}{x-1}$ **)**

4. $(1+x+xy^2)dy + (y+y^3)dx = 0$ **(ans:** $xy + \tan^{-1} y = c$ **)**

5. $\frac{dy}{dx} + (2x\tan^{-1} y - x^3)(1+y^2) = 0$ **(ans:** $\tan^{-1} y = \frac{\left(x^2-1\right)}{2} + ce^{-x^2}$ **)**

6. $\frac{dy}{dx} = xy + y^2e^{(-x^2/2)}.\log x$ **(ans:** $\frac{-e^{x^2/2}}{y} = x\log x - x + c$ **)**

7. $\frac{dy}{dx} + \left(\frac{x}{1-x^2}\right)y = x\sqrt{y}$ **(ans:** $\sqrt{y} + \frac{1}{3}\left(1-x^2\right) = c\left(1-x^2\right)^{\frac{1}{4}}$ **)**

8. $xy\left(1+xy^2\right)\frac{dy}{dx} = 1$ **(ans:** $\frac{1}{x} = 2 - y^2 - ce^{-\frac{y^2}{2}}$ **)**

9. $4x^2y\frac{dy}{dx} = 3x\left(3y^2+2\right) + \left(3y^2+2\right)^3$ **(ans:** $\frac{-1}{\left(3y^2+2\right)^2} = \frac{3}{7}x^{-1} + cx^{-9/2}$ **)**

10. $4xy\frac{dy}{dx} = (y^2+3) + x^3(y^2+3)^3$ **(ans:** $\frac{-x}{(y^2+3)^2} = \frac{x^4}{4} + c$ **)**

11. Find the orthogonal trajectories of each of the following family of curves:

a. $x - 4y = c$

b. $x^2 + y^2 = c^2$

c. $x^2 - y^2 = c$

d. $y^2 = cx^3$

e. $y = c\left(\sec x + \tan x\right)$

f. $y^2 = \frac{x^3}{a-x}$

g. $y = cx^2$

12. Given $x^2 + 3y^2 = cy$ find that member of the orthogonal trajectories which passes through point (1,2).

13. Find the constant e such that $y^3 = c_1x$ and $x^2 + ey^2 = c_2$ are orthogonal to each other.