Tutorial 1

Second order linear differential equations with variable coefficients, Method of variation of parameters

Q1. Solve the following linear Differential Equation using operator method:

1. $\left(D^2+4D+4\right)y=\sin x$ [Ans: $y=\left(c_1+c_2x\right)e^{-2x}+\dfrac{3\sin x-4\cos x}{25}$
2. $\left(D^2+4\right)y=5x^2+\sin 2x$ [Ans: $y=(C_1\cos 2x+C_2\sin 2x)+\frac{5}{8}\left(2x^2-1\right)-\frac{x\cos 2x}{4}$]
3. $\left(D^2+3D+2\right)y=x+x^2$ [Ans: $y=c_1e^{-x}+c_2e^{-2x}+\dfrac{x^2}{2}-x+1$]
4. $\left(D^2-D\right)y=2x+1+4\cos x+2e^x$ [Ans: $y=c_1+c_2e^{-x}-xe^x-x^2-3x-2\sin x-2\cos x$]
5. $\left(D^2-1\right)y=e^x\left(1+x\right)^2$ [Ans: $y=c_1e^x+c_2e^{-x}+\dfrac{xe^x}{12}\left(3+3x+2x^2\right)$]
6. $\left(D^2+D-6\right)y=e^{2x}\sin 3x$ [Ans: $y=c_1e^{2x}+c_2e^{-3x}-\dfrac{e^{2x}}{102}\left(5\cos 3x+3\sin 3x\right)$]

Q.2 Solve by using method of undetermined coefficients:

1. $\left(D^2+2D+4\right)y=2x^2+3e^{-x}$

2. $\left(D^2+1\right)y=\sin x$

3. $\left(D^2+1\right)y=2\cos x$

4. $\left(D^2-5D+6\right)y=e^{3x}+\sin x$

5. $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+3y=x^3+\cos x$

6. $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}=e^x\sin x$

Q3.Solve the following by method of variation of parameters :

1. $(D^2+1)y=\tan x$ $\left[\textbf{Ans.:}\ y=C_1\cos x+C_2\sin x-\cos x\left[\log(\sec x+\tan x)\right]\right]$
2. $(D^2+1)y=\operatorname{cosec} x\cot x$ [Ans.: $C_1\sin x+C_2\cos x-\cos x\log(\sin x)-\cos x-x\sin x$]
3. $\dfrac{d^2y}{dx^2}+y=\dfrac{1}{1+\sin x}$ $\left[\textbf{Ans.:}\ y=C_1\cos x+C_2\sin x-1-x\cos x+\sin x\log(1+\sin x)+\sin x\right]$
4. $(D^2+1)y=3x-8\cot x$ $\left[\textbf{Ans.:}\ y=C_1\cos x+C_2\sin x-8\sin x\log(\operatorname{cosec} x-\cot x)+3x\right]$
5. $(D^2-1)y=\dfrac{2}{1+e^x}$ [Ans.: $y=C_1e^x+C_2e^{-x}-1+(e^x-e^{-x})\log(1+e^x)$]

Tutorial 2

Cauchy-Euler equation; Power series solutions, Legendre polynomials

Q1. Solve the following linear Differential equation:

1. $x^2\dfrac{d^2y}{dx^2}+2x.\dfrac{dy}{dx}-20y=(x+1)^2$ $\left[y=C_1x^4+C_2x^{-5}-\dfrac{1}{14}x^2-\dfrac{1}{9}x-\dfrac{1}{20}\right]$
2. $x^2\dfrac{d^2y}{dx^2}-2x.\dfrac{dy}{dx}-4y=x^2+2\log x$ $\left[y=C_1x^4+\dfrac{C_2}{x}-\dfrac{1}{6}x^2-\dfrac{1}{2}\left[\log x-\dfrac{3}{4}\right]\right]$
3. $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+5y=x^2\sin(\log x)$ $\left[y=x^2\left[C_1\sin(\log x)+C_2\cos(\log x)\right]-\dfrac{x^2}{2}\log x\cos(\log x)\right]$

4. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos\{\log(x)\}+x\sin\{\log(x)\}$

$\left[\textbf{Ans.:}\ y=x\left[C_1\cos(\sqrt{3}\log x)+C_2\sin(\sqrt{3}\log x)\right]+\frac{x}{2}\sin(\log x)+\frac{1}{13}[3\cos(\log x)-2\sin(\log x)]\right]$

5. $x^2\frac{d^2y}{dx^2}+x.\frac{dy}{dx}-4y=x^3$ $\left[\textbf{Ans.:}\ y=C_1x^2+C_2\left(\frac{1}{x^2}\right)+\frac{1}{5}x^3\right]$

Q2. Find the power series solution in powers of x of the differential equation

1. $xy'-(x+2)y-2x^2-2x=0$ Ans: $y=2x+c_2x^2e^x$
2. $y''+xy'+(x^2+2)y=0$ Ans: $y=c_0\left[1-x^2+\frac{1}{4}x^4+\ldots\ldots\right]+c_1\left[x-\frac{1}{2}x^3+\frac{3}{40}x^5+\ldots\ldots\right]$
3. $(x^2-1)y''+3xy'+xy=0\quad with\ y(0)=4,\ y'(0)=6$ Ans: $y=4+6x+\frac{11}{3}x^3+\frac{1}{2}x^4+\frac{11}{4}x^5\ldots.$