Unit 2
Tutorial Problems

Based on Linear transformations (maps), Matrix associated with a linear map.

Show that the following mappings are linear transformations:

1. $T:\mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (x+y, x)$
2. $T:\mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (y, x)$
3. $T:\mathbb{R}^3 \to \mathbb{R}^2, T(x,y,z) = (x-y, x+z)$
4. $T:\mathbb{R}^2 \to \mathbb{R}^3, T(x,y) = (x-y, 2x+3y, 3x+2y)$
5. $T:\mathbb{R}^3 \to \mathbb{R}^3, T(x,y) = (x-y, y-z, z-x)$
6. $T:\mathbb{R}^3 \to \mathbb{R}^3, T(x,y,z) = (a,b,c)\bullet(x,y,z)$, where (a,b,c) is a fixed vector in $\mathbb{R}^3$.
7. $T:M_{22} \to M_{22}$ defined by, $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & 0 \\ 0 & c+d \end{pmatrix}$.
8. $T:M_{33} \to M_{33}$ defined by, $T(A) = A^T$.
9. Let P_n denote the set of all polynomials of degree at most n. Let $T:P_1 \to P_2$, $T\left(p(x)\right) = x\,p(x)$.

Show that the following maps are not linear:

10. $T:\mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (x+1, y)$
11. $T:\mathbb{R} \to \mathbb{R}, T(x) = |x|$
12. Let A be a 3×3 matrix. Let $T:\mathbb{R}^3 \to \mathbb{R}^3, T(X) = AX + \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a fixed non zero vector in $\mathbb{R}^3$.
13. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear map such that $T(1,0) = (1,1)$, $T(0,1) = (-1,2)$. Let S be a square whose corners are at $(0,0), (1,0), (1,1)$ and $(0,1)$. Show that the image of S under T is a parallelogram.
14. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear map such that $T(1,0) = u$, $T(0,1) = v$, where u and v are two linearly independent vectors. Describe the image under T of the rectangle whose corners are at $(0,0), (0,1), (3,1)$ and $(3,0)$.

    Ans: parallelogram.
15. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a linear map such that $T(3,1) = (1,2)$, $T(-1,0) = (1,1)$. Compute $T(1,0)$.

$Ans: (3,4)$

16. Let $T:\mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (2x-3y, -x+y)$. Find the matrix associated with T with respect to the standard bases.

$Ans: \begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$

17. Let $T:\mathbb{R}^3 \to \mathbb{R}^2, T(x,y,z) = (2x-z, y+2z)$. Find the matrix M associated with T with respect to the standard bases. Verify that $M\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = T(2,-1,3)$.

$Ans: \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$

18. Let $T:\mathbb{R}^2 \to \mathbb{R}^3, T(x,y) = (x-y, 2x+3y, 3x+2y)$. Find the matrix associated with T with respect to the bases : $\{(1,0),(1,1)\}$ of $\mathbb{R}^2$ and the standard basis of $\mathbb{R}^3$.

$Ans: \begin{pmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 5 \end{pmatrix}$

19. Let $T:\mathbb{R}^3 \to \mathbb{R}^2, T(x,y,z) = (x+y+z, x-2y+3z)$. Let $B = \{e_3, e_2, e_1\}$ and $C = \{e_2, e_1\}$ be bases for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. Find the matrix M with respect to B and C. Verify that $M\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = T(1,2,3)$.

$Ans: \begin{pmatrix} 3 & -2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

20. Let $T:P_3 \to P_4$ be the linear transformation given by $T(p(x)) = (2+3x)p(x)$. Find the matrix of T relative to the standard bases of P_3 and P_4 .

$Ans: \begin{pmatrix} 2 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 \end{pmatrix}$

21. Find the matrix associated with T , where T is reflection about $y-axis$.

$Ans: \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.

Based on range and kernel of a linear map, rank and nullity, Composition of linear maps& Inverse of a linear transformation:

1. Find the range and kernel of $T:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x,y,z)=(x-y,x+z)$.

$Ans: Ker\,T=\{(-z,-z,z)\mid z\in\mathbb{R}\}, Range=\{(r,s)\mid r,s\in\mathbb{R}\}$

2. Find the range and kernel of $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z)=(x-y,x+y,2x-3y)$.

$Ans: Ker\,T=\{(0,0,z)\mid z\in\mathbb{R}\}, Range=\left\{\left(r,s,\frac{5r-s}{2}\right)\mid r,s\in\mathbb{R}\right\}$

3. Find the range and kernel of $T:M_{22}\to M_{22}$ defined by, $T\begin{pmatrix} a & b \\ c & d\end{pmatrix}=\begin{pmatrix} a-b & 0 \\ 0 & c-d\end{pmatrix}$.

$Ans: Ker\,T=\left\{\begin{pmatrix} a & a \\ c & c\end{pmatrix}\middle| a,c\in\mathbb{R}\right\}, Range=\left\{\begin{pmatrix} r & 0 \\ 0 & s\end{pmatrix}\middle| r,s\in\mathbb{R}\right\}$

4. Find rank and nullity of $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x,y,z)=(x-y,y-z,z-x)$ and verify rank – nullity theorem.

$Ans: Ker\,T=\{(x,x,x)\mid x\in\mathbb{R}\}, Range=\{(r,s,-r-s)\mid r,s\in\mathbb{R}\}, nullity=1, rank=2$

5. Find rank and nullity of $T:\mathbb{R}^2 \to \mathbb{R}$ defined by $T(x,y)=(2,-1)\bullet(x,y)$ and verify rank – nullity theorem.

$Ans: Ker\,T=\{(x,2x)\mid x\in\mathbb{R}\},\ Range=\mathbb{R}\ nullity=1, rank=1$

6. Let $T:P_n \to P_{n+1}$ be defined by

$$T\left(p_0+p_1x+p_2x^2+.....+p_nx^n\right)=p_0x+\frac{p_1}{2}x^2+\frac{p_2}{3}x^3+.....+\frac{p_n}{n+1}x^{n+1}$$

Verify rank- nullity theorem.

$Ans: Ker\,T=\{0\},\ Range=\{a_0+a_1x+a_2x^2+...+a_{n+1}x^{n+1}\mid a_0=0\}\ nullity=0, ra$

7. Let $T:\mathbb{R}^2\to\mathbb{R}^2$ be defined by $T(x,y)=(x+2y,2x-y)$ and $S:\mathbb{R}^2\to\mathbb{R}$ be defined by $S(a,b)=a-2b$. Find $S\circ T(x,y)$.

$Ans: S\circ T(x,y)=-3x+4y$

8. Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be defined by $T(x,y)=(x,x-y,y)$ and $S:\mathbb{R}^3\to\mathbb{R}^2$ be defined by $S(a,b,c)=(a-2b,a+c)$. Find $S\circ T(x,y)$. Find matrices representing all these transformations. Verify that the matrix representing $S\circ T$ is the product of matrices representing S and T.

$Ans: S\circ T(x,y)=(-x+2y,x+y), M_S=\begin{pmatrix}1 & -2 & 0\\ 1 & 0 & 1\end{pmatrix}, M_T=\begin{pmatrix}1 & 0\\ 1 & -1\\ 0 & 1\end{pmatrix}, M_{S\circ T}=\Big($

9. Let $S: P_1 \to P_2$ defined by $S(p(x)) = x\,p(x)$ and $T: P_2 \to P_1$ be the derivative operator. Find the composite transformation.
   $Ans: Let\ p(x)=ax+b.\quad S\circ T(x,y)=ax, T\circ S(x,y)=2ax+b$
10. Show that $\mathrm{T}: R^2 \to R^2$ be defined by $\mathrm{T}(x,y)=(x+y,x-y)$ is invertible.
11. Show that $\mathrm{T}: R^3 \to R^3$ be defined by $\mathrm{T}(x,y,z)=(x-y,x+z,x+y+2z)$ is invertible.
12. Determine whether $\mathrm{T}: R^3 \to R^3$ be defined by $\mathrm{T}(x,y,z)=(x+y+z,y,x+z)$ is invertible. Ans: T is not invertible.
13. Show that $\mathrm{T}: R^3 \to R^3$ defined by $\mathrm{T}(x,y,z)=(2x-y+z,x+y,3x+y+z)$ is invertible. Find the matrix associated with T^{-1}.
14. Show that $\mathrm{T}: R^3 \to R^3$ defined by $\mathrm{T}(x,y,z)=(3x-2z,y,3x+4y)$ is invertible.
15. Let $T: R^2 \to R^2$ for which $T(3,1)=(2,-4)$ and $T(1,1)=(0,2)$. Find an explicit formula for $T(x,y)$. Also find if T is invertible. If it is invertible, find the matrix associated with T^{-1}.

    Ans: $T(x,y)=(\mathrm{x}-y,5-3x)$. T is invertible, $T^{-1}(x,y)=\frac{1}{2}(5x+y,3x+y)$.

Based on Cayley-Hamilton Theorem

1. Apply Cayley-Hamilton theorem to $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$ and deduce that $A^8=625I$
2. If $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$, show that for every integer $n\ge 3$, $A^n=A^{n-2}+A^2-I$, hence find A^{50}. [Ans. $A^{50}=\begin{bmatrix}1 & 0 & 0\\ 25 & 1 & 0\\ 25 & 0 & 1\end{bmatrix}$]

Apply Cayley-Hamilton theorem to the following matrices and obtain the inverse

3. $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ [Ans. $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$]

4. $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ [Ans. $A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -1 & \frac{1}{2} & \frac{1}{2} \\ 2 & 0 & -1 \end{bmatrix}$]

5. $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ [Ans. $A^{-1} = \frac{1}{4}\begin{bmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{bmatrix}$]

6. $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ [Ans. $A^{-1} = -\frac{1}{2}\begin{bmatrix} 1 & -1 & -1 \\ -6 & 2 & 8 \\ 3 & -1 & -5 \end{bmatrix}$]

Find the characteristic equation of the matrix A and hence find A^{-1} and A^{4}.

7. $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$ [Ans. $A^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$, $A^{4} = \begin{bmatrix} 1 & 2 & 4 \\ 8 & 3 & -10 \\ -12 & -2 & 23 \end{bmatrix}$]

8. $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ [Ans. $A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}$, $A^{4} = \begin{bmatrix} 41 & 0 & -40 \\ 0 & 16 & 0 \\ -40 & 0 & 41 \end{bmatrix}$]

9. $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ [Ans. $A^{-1} = \frac{1}{4}\begin{bmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{bmatrix}$, $A^{4} = \begin{bmatrix} -88 & -168 & -264 \\ 192 & 416 & 144 \\ 56 & 72 & 472 \end{bmatrix}$]

10. Find the characteristic equation of the matrix A given below and hence, find the matrix represented by $A^{6} - 6A^{5} + 9A^{4} + 4A^{3} - 12A^{2} + 2A - I$, where

$$A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

[Ans. $\begin{bmatrix} 5 & 20 & 10 \\ -4 & -7 & -8 \\ 6 & 10 & 13 \end{bmatrix}$]

11. Find the characteristic equation of the matrix A given below and hence, find the matrix represented by $A^{9} - 6A^{8} + 10A^{7} - 3A^{6} + A + I$, where $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$.

[Ans. $\begin{bmatrix} 2 & 2 & 3 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{bmatrix}$]

Based on Eigenvalues and eigenvectors: Symmetric, skew-symmetric and orthogonal matrices

1. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

   [Ans. -1,-6 and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}$]

2. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

   [Ans. 2,3,5 and $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$]

3. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

   [Ans. 1,2,2 and $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

4. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

   [Ans. 1,1,1 and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$]

5. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

   [Ans. 5,-3,-3 and $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$]

6. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

   [Ans. -2,3,6 and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$]

7. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

   [Ans. 1,1,1 and $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$]

8. Find eigenvalues and eigenvectors of $A = \begin{bmatrix} 3 & -2 & -5 \\ 4 & -1 & -5 \\ -2 & -1 & -3 \end{bmatrix}$

[Ans. 5,2,2 and $\begin{bmatrix}3\\2\\4\end{bmatrix}, \begin{bmatrix}1\\3\\-1\end{bmatrix}$]

9. If $A=\begin{bmatrix}1 & 0\\ 2 & 4\end{bmatrix}$ then find eigen values of $4A^{-1}+3A+2I$. [Ans. 9, 15]

10. If $A=\begin{bmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{bmatrix}$ then find eigen values and eigen vectors of A^3+I.

[Ans: 2, 2, 126 and $\begin{bmatrix}2\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}$]

11. If $A=\begin{bmatrix}3 & 1 & -1\\ 2 & 2 & -1\\ 2 & 2 & 0\end{bmatrix}$ then find eigen values and eigen vectors of A^2.

[Ans: 2, 4, 4 and $\begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}1\\1\\-2\end{bmatrix}$]

12. If the product of two eigen values of $\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$ is 16, find the third eigen value.
[Ans. 2]

13. Determine the algebraic multiplicity and geometric multiplicity for $A = \begin{bmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 0 & 0 & 1\end{bmatrix}$.

[Ans. For $\lambda = 1$, A.M.=2 & G.M.=2; For $\lambda = 3$, A.M.=1 & G.M.=1]

14. Find Eigenvalues and eigenvectors for following Symmetric matrix

a. $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 5\\ 0 & 5 & 4\end{bmatrix}$ [Ans. $\lambda=-1,1,9$ and $\begin{bmatrix}0\\-1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}$]

b. $A=\begin{bmatrix}4 & 2 & 4\\ 2 & 1 & 2\\ 4 & 2 & 4\end{bmatrix}$ [Ans. $\lambda=0,0,9$ and $\begin{bmatrix}-1\\2\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\2\end{bmatrix}$]

c. $A=\begin{bmatrix}3 & 2 & 4\\ 2 & 0 & 2\\ 4 & 2 & 3\end{bmatrix}$ [Ans. $\lambda=-1,-1,8$ and $\begin{bmatrix}-1\\2\\0\end{bmatrix}, \begin{bmatrix}-1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\1\\2\end{bmatrix}$]

15. Show that for given skew symmetric matrix $A = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$, the eigenvalues are purely imaginary.

16. Find eigenvalues of the skew symmetric matrix, $A = \begin{bmatrix} 0 & 9 & -12 \\ -9 & 0 & 20 \\ 12 & -20 & 0 \end{bmatrix}$.

[Ans. $0, 25i, -25i$]

17. Find eigenvalues of the orthogonal matrix, $A = \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$

[Ans. $-1, \frac{5+i\sqrt{11}}{6}, \frac{5-i\sqrt{11}}{6}$]

18. Find Eigen values and Eigen vectors for the following matrices:

a. $\begin{bmatrix} 9 & -1 \\ 5 & 7 \end{bmatrix}$ [Ans. $\lambda = 8+2i, 8-2i$ and $\begin{bmatrix} 1+2i \\ 5 \end{bmatrix}, \begin{bmatrix} 1-2i \\ 5 \end{bmatrix}$]

b. $\begin{bmatrix} -3 & -2 \\ 5 & -1 \end{bmatrix}$ [Ans. $\lambda = -2+3i, -2-3i$ and $\begin{bmatrix} 2 \\ -1-3i \end{bmatrix}, \begin{bmatrix} 2 \\ -1+3i \end{bmatrix}$]

c. $\begin{bmatrix} 6 & -13 \\ 1 & 0 \end{bmatrix}$ [Ans. $\lambda = 3+2i, 3-2i$ and $\begin{bmatrix} 3+2i \\ 1 \end{bmatrix}, \begin{bmatrix} 3-2i \\ 1 \end{bmatrix}$]

d. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & -3 & 0 \end{bmatrix}$ [Ans. $\lambda = 3, 3i, -3i$ and $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ i \end{bmatrix}, \begin{bmatrix} 0 \\ i \\ 1 \end{bmatrix}$]

19. Find Eigen values and Eigen vectors for the following orthogonal matrix

$\begin{bmatrix} 3/5 & -4/5 \\ 4/5 & 3/5 \end{bmatrix}$ [Ans. $\lambda = \frac{3+4i}{5}, \frac{3-4i}{5}$ and $\begin{bmatrix} i \\ 1 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix}$]

Based on Diagonalization of matrices & Similar Matrices, Diagonalisation

1. Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix.

[Ans. $M = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$]

2. Show that the matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix.

[Ans. $M = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$]

3. Show that the matrix $A=\begin{bmatrix}-2 & 2 & 3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix.

[Ans. $M=\begin{bmatrix}1 & -2 & 3\\ 2 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}, D=\begin{bmatrix}5 & 0 & 0\\ 0 & -3 & 0\\ 0 & 0 & -3\end{bmatrix}$]

4. Show that the matrix $A=\begin{bmatrix}-17 & 18 & -6\\ -18 & 19 & -6\\ -9 & 9 & 2\end{bmatrix}$ is diagonalisable. Find the transforming matrix and the diagonal matrix.

[Ans. $M=\begin{bmatrix}2 & 1 & -1\\ 2 & 1 & 0\\ 1 & 0 & 3\end{bmatrix}, D=\begin{bmatrix}-2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$]

5. Show that the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$ is not similar to a diagonal matrix.

6. Show that the matrix $A=\begin{bmatrix}1 & 3 & -2\\ 0 & 1 & 4\\ 0 & 0 & 1\end{bmatrix}$ is not similar to a diagonal matrix.

7. Find the symmetric matrix A having the Eigen values 0,3,15 with the corresponding Eigen vectors $X_1=[1,2,2]'$, $X_2=[-2,-1,2]'$ and X_3.

[Ans. $A=\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$]