Unit V
Partial Differential Equations

Tutorial Problems

Form the partial differential equations by eliminating the arbitrary constants from the following:

1. $z=\left(x^2+a\right)\left(y^2+b\right)$
   [**Ans.** $pq=4xyz$]
2. $2z=\left(ax+y\right)^2+b$
   [**Ans.** $px+qy=q^2$]
3. $ax^2+by^2+cz^2=1$
   [**Ans.** $z(xp+yq)=1-z^2$]

Form the partial differential equations by eliminating the arbitrary functions from the following:

1. $z=F\left(x^2-y^2\right)$
   [**Ans.** $py+qx=0$]
2. $z=x+y+f(xy)$
   [**Ans.** $px-qy=x-y$]
3. $z=f\left(\frac{xy}{z}\right)$
   [**Ans.** $px=qy$]

Solutions of Partial Differential Equations by the Method of Direct Integration

1. $\frac{\partial^2 z}{\partial x\partial y}=\cos x\cos y$ [**Ans.** $z=\sin x\sin y+f(x)+\phi(y)$]
2. Solve $\frac{\partial^2 z}{\partial x^2}+z=0$, given that when $x=0,\ z=e^y$ *and* $\frac{\partial z}{\partial x}=1$
   [**Ans.** $z=e^y\cos y+\sin x$]
3. Solve $\frac{\partial^2 z}{\partial y^2}=z$, given that when $y=0,\ z=e^x$ *and* $\frac{\partial z}{\partial y}=e^{-x}$
   [**Ans.** $z=e^y\cosh x+e^{-y}\sinh x$]
4. Solve $\frac{\partial^2 z}{\partial x^2}=z$, given that when $x=0,\ z=e^y$ *and* $\frac{\partial z}{\partial x}=e^{-y}$

[**Ans.** $z = e^{x}\cosh y + e^{-x}\sinh y$]

Separation of variables method

1. Solve by the method of separation of variables $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$

   [**Ans.** $z = \left[Ae^{\left(1+\sqrt{1+k}\right)}x + Be^{\left(1-\sqrt{1+k}\right)}x \right] e^{-ky}$]

2. Solve by the method of separation of variables $\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$, *where* $u(0, y) = 8e^{-3y}$

   [**Ans.** $u = 8e^{-12-3y}$]

3. Solve by the method of separation of variables $3\dfrac{\partial u}{\partial x} + 2\dfrac{\partial u}{\partial y} = 0$, *where* $u(x, 0) = 4e^{-x}$

   [**Ans.** $u = 3e^{-5x-3y} + 2e^{-3x-2y}$]