CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz I** (22 Jan 2020)
Name				**30 marks**
Roll No		Dept.		Page **1** of **2**

Instructions:
1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ. We will entertain no requests for leniency.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. Write **T** or **F** for True/False (write **only in the box on the right hand side**) (**5x2=10 marks**)

1	If a set $\mathcal{C} \subset \mathbb{R}^2$ is convex, then all subsets of $\mathcal{C}$ must be convex sets as well i.e. if $\mathcal{C}' \subseteq \mathcal{C}$ then $\mathcal{C}'$ must be convex too	
2	Let $f: \mathbb{R} \to \mathbb{R}$ be a doubly differentiable function (i.e. first and second derivatives exist). If $f'(x^0) > 0$ at $x^0 \in \mathbb{R}$, then it may be possible that $f''(x^0) < 0$	
3	For a binary classification problem with feature vectors in $\mathbb{R}^5$, a linear model will in general have smaller size than an LwP model (if using one prototype per class)	
4	Suppose $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function but neither convex nor concave. Then it must be the case that f has strictly more than one local optima	
5	There can exist a training dataset for a binary classification problem on which the 1NN (one nearest neighbour) algorithm has a linear decision boundary	

Q2. (Squared Hinge Loss) Let $\mathbf{a} \in \mathbb{R}^d$ and $b \in \mathbb{R}$ be constants. For $\mathbf{x} \in \mathbb{R}^d$, define the function $f(\mathbf{x}) = ([1 - b \cdot \mathbf{a}^\top \mathbf{x}]_+)^2$. Find $\nabla f(\mathbf{x})$ and briefly show all major steps in your derivation (**5 marks**)

Q3. (Model Exfiltration) Ms M has a secret function $f: \mathbb{R}^4 \to \mathbb{R}$. We know that f thresholds one of the 4 coordinates of the input at a value i.e. for all $\mathbf{x} \in \mathbb{R}^4$, $f(\mathbf{x}) = \mathbf{x}_j - c$ where $j \in \{1,2,3,4\}$ and $c \in \mathbb{R}$. E.g. if $j = 2, c = 1.5$, then for $\mathbf{x} = (1,5,2,2)$, $f(\mathbf{x}) = 5 - 1.5 = 3.5$. I want to steal Ms M's model and find out what value of j, c is she using. I can send Ms M any number of inputs $\mathbf{x}^1, \mathbf{x}^2, \ldots \in \mathbb{R}^4$ and she will return $f(\mathbf{x}^1), f(\mathbf{x}^2), \ldots \in \mathbb{R}$ back to me. Design an algorithm below that asks Ms M function values on one or more 4D vectors and uses her responses to find the value of j and c being used. You must give explicit descriptions of the 4D vectors you are querying Ms M (e.g. you may say that you wish to query only three vectors $(1, -0.5, 1, -3), (8, -0.1, 1, 5)$ and $(9, 1, 5, 6)$). The fewer vectors you query Ms M to correctly find j, c, the more marks you will get. (**10 marks**)

Q4. (Placement Woes) Ms M wants to set up a shop at a point in $\mathbb{R}^2$. The consumer density is such that if she opens shop at a point $(x, y) \in \mathbb{R}^2$, her daily income will be $2x + 4y$. However, since her supplier is situated at $(2, -1)$, she will incur a daily cost of $(x-2)^2 + (y+1)^2$ for transporting goods from her supplier to her shop. Where should Ms M open her shop to get the maximum daily profit and what is that maximum value of profit? Give a brief derivation below. **(5 marks)**