CS 771A: Introduction to Machine Learning				**Quiz 3** (16 Oct 2019)
Name				**30 marks**
Roll No		Dept.		Page **1** of **2**

Instructions:

1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ. We will entertain no requests for leniency.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. Write **T** or **F** for True/False (write **only in the box on the right hand side**) (**8x2=16 marks**)

1	Suppose a matrix $A \in \mathbb{R}^{d\times d}$ is invertible. Then the linear map corresponding to A can never have two or more distinct elements in its kernel	
2	For every set of three vectors $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^2$ that lie on a line, all linear combinations of those three vectors must lie on that same line.	
3	Every linear transformation $f: \mathbb{R}^d \to \mathbb{R}^k$ must map the origin to the origin i.e. $f(\mathbf{0}) = \mathbf{0}$	
4	Every convex combination of two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^2$ is necessarily an affine combination as well.	
5	Given an orthonormal matrix $A \in \mathbb{R}^{d\times d}$, an eigen-decomposition of A will always exist although an singular value decomposition for A may or may not exist	
6	Suppose we have $k < d$ vecs $\mathbf{x}^1, \ldots, \mathbf{x}^k \in \mathbb{R}^d$ and $\mathbf{u} \in \mathbb{R}^d$ s.t. $\mathbf{u} \in \text{span}(\mathbf{x}^1, \ldots, \mathbf{x}^k)$. Then for any other vector $\mathbf{v} \in \mathbb{R}^d$, we must have $\mathbf{u} \in \text{span}(\mathbf{x}^1, \ldots, \mathbf{x}^k, \mathbf{v})$ as well	
7	A square symmetric matrix that is positive definite (i.e. all eigenvalues strictly greater than 0) is always invertible	
8	If a vector $\mathbf{a} \in \mathbb{R}^2$ is linearly dependent on $\mathbf{p}, \mathbf{q} \in \mathbb{R}^2$ with $\mathbf{a} = m \cdot \mathbf{p} + n \cdot \mathbf{q}$ for some $m \neq 0, n \neq 0$, then the vector $\mathbf{p}$ is linearly dependent on $\mathbf{a}, \mathbf{q}$ as well	

Q2. Fill the circle (**don't tick**) next to all the correct options (**many may be correct).**(**3+2=5 marks**)

2.1 Suppose $A \in \mathbb{R}^{d\times d}$ is a square symmetric positive definite matrix with all eigenvalues strictly greater than zero. Which all of the following statements is true?

A	We must have $\text{trace}(A) > 0$	○
B	The function $f(\mathbf{x}) = \mathbf{x}^\top A\mathbf{x}$ has a unique minimum	○
C	The matrix $A + I_d$ must be positive definite as well	○
D	The matrix $A - I_d$ must be positive definite as well	○

2.2 Let X, Y be two r.v. (not necessarily independent) with same support $S_X = \{1,2\} = S_Y$. Let $P \in \mathbb{R}^{2\times 2}$ with $P_{ij} = \mathbb{P}[X = i, Y = j], i, j \in \{1,2\}$ encode the joint PMF where P_{ij} is the element in the i^{th} row and j^{th} column. Let $\mathbf{1} = [1,1] \in \mathbb{R}^2$ denote the all ones vector. Which of the following is true?

A	$P\mathbf{1}$ gives marginal PMF of X	○
B	$P^\top\mathbf{1}$ gives marginal PMF of X	○
C	$P\mathbf{1}$ gives marginal PMF of Y	○
D	$P^\top\mathbf{1}$ gives marginal PMF of Y	○

Q3 Consider the vector $\mathbf{x} = [1,2,3,4,5] \in \mathbb{R}^5$. Think of $X = \mathbf{x}^\top \in \mathbb{R}^{1\times 5}$ as a 1×5 matrix with one row and five columns. First of all write down the rank of X. Then write down the thin SVD of X i.e. find $U \in \mathbb{R}^{1\times r}, \Sigma \in \mathbb{R}^{r\times r}, V \in \mathbb{R}^{5\times r}$, such that $X = U\Sigma V^\top$ where $r = \mathrm{rank}(X)$.**(1+3 = 4 marks)**

Q4. The following matrix is known to have rank one and trace equal to 10. Fill in the missing entries of the matrix. Write only inside the boxes. Scribbles outside won't be graded. (**5 marks**)

$$\begin{bmatrix} \square & -3 & 1 \\ -8 & \square & -2 \\ \square & \square & \square \end{bmatrix}$$

------------------------------------ END OF QUIZ ------------------------------------

ROUGH WORK

Nothing written here will get graded