Drivers, Riders and Service Providers: The Impact of the Sharing Economy on Mobility

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1 INTRODUCTION

Ride sharing denotes the practice of sharing a car such that more than one person travels in a car during a journey. Sharing rides was traditionally restricted to family members and close friends or long-distance journeys scheduled well before the intended time of departure. Only the emergence of mobile computing technologies and GPS location services in combination with electronic payments and online reputation systems provided for the technological cornerstones to make on-demand short-distance ride sharing among strangers viable. Typically individuals enter their trip details on an online platform[1] which then facilitates the matching of riders with cars and drivers - and within minutes the individual's trip commences. Such platforms must attract both, supply and demand for rides.

In populated urban areas, ride sharing platforms offer an alternative mode of transportation in addition to established ways such as public transportation services or driving your own car. As such, ride sharing may significantly impact traffic volume, ownership and the population's welfare in terms of mobility. For example, [1, 3] argue that ride sharing has the potential to significantly reduce congestion and bridge the gap between public and private transportation. While at first glance it seems reasonable that filling up one empty seat in a car will take another single person vehicle off the road, the reasoning becomes more intricate once monetary incentives are added to the equation. On the one hand, sharing a ride should involve the payment of a rental fee to incentivise drivers to offer empty seats on the platform. On the other hand that same fee might encourage individuals to provide additional seats on the platform and seek to make a profit through ride sharing, potentially taking up driving on a full-time basis as an alternative form of employment. Hence, although ride sharing is heralded as a more sustainable and environmentally friendly form of urban mobility - better utilization of society's assets enabled by sharing reduces the need to buy or own for an individual - the economic opportunities provided by the sharing platforms might result in more complex outcomes. Additional income from shareable assets impacts individual ownership decisions and the availability of the sharing platform itself might lead to the emergence of professional service providers who seek to capitalize on the opportunities provided by the platform.

In this paper, we introduce a model in which a population of individuals heterogeneous in salary and utility for private transportation interacts with a ride sharing platform. Individuals may act on the supply and demand side of the platform. They may supply seats to the platform by renting out empty seats in their cars on a casual or full-time basis or use the platform to find a ride. Individuals are assumed rational and optimize their long term average costs, deciding on whether and how to interact with the platform and whether or not to own a personal car. Collective decision making is modeled as an anonymous non-atomic game. We study equilibria of this game dependent on model parameters including seat rental, usage and ownership costs for cars and the distribution of income and utility in the population. We identify a total of 7 different equilibrium types which may significantly alter how the ride sharing platform affects individual mobility.

Being able to predict and understand the behavior of populations when introducing new technologies which affect basic everyday services like short distance mobility will be a key challenge for regulatory authorities around the globe. It is an ongoing debate where the difference between carpooling and running an illegal taxi enterprise lies. For example in Singapore, recent regulation allows commuters to carpool if the service is on a non-profit basis and the trips would have been undertaken by the driver independent of the amount of people willing to share with her, if any at all.[2] In this paper, we study a platform in which every individual may supply empty seats to the rest of the population and consider any registration fees negligible in the individuals' payoffs. The absence of any such costs in our model allows casual and professional ride sharing services to coexist, raising several important questions: Which type of supply will be dominating? How will welfare be affected if participation on the supply side is restricted by law? More generally, how will people fulfill their transportation needs? Compared to a population without access to the platform, how would congestion and car ownership change? Will the increased occupation of cars reduce congestion? Or is the platform taking people off public transport by reducing the price of private transportation and thus making private transportation accessible for a larger population segment? How would a revenue maximizing platform differ from a welfare maximizing platform and how does this affect mobility? And lastly, how do ownership and usage costs affect ownership and congestion? The remainder of this document and the extended version [2] is dedicated to addressing the above questions.

2 THE MODEL

2.1 Agents' description

An individual alternates between two states, "non-transport" and "transport." Whenever an individual enters the transport state, (which may be triggered by the need to perform some other economic activity such as going to work or shopping at some other location),

[1] We call the business enabling on demand ride sharing a platform.

[2] http://statutes.agc.gov.sg

she seeks access to transportation. During the non-transport state an individual is free to pursue her own interests. In our model that amounts to pursuing an activity which generates income. We assume the transport state lasts for a period of time with mean duration $1/\lambda_t$ (measured in units of time) and the non-transport state lasts for a period of time with mean duration $1/\lambda_n$. [3] Furthermore we standardize time such that each individual takes on average one unit of time to visit both states, that is $1/\lambda_t + 1/\lambda_n = 1$. In other words each individual visits each state once per unit of time and hence does one personal trip per unit of time. This implies $\lambda_t, \lambda_n > 1$ and specifying one determines the other parameter. We treat λ_t as the free parameter.

The transport state. A ride can be obtained through different means, which influence an individual's payoff. Namely, an individual may fulfill her transportation needs using one of three options:

- Public transport: An individual may choose to use public transport which we assume is always available.
- Drive: An individual may use her own car to drive the journey. While doing so the individual may offer to take others along, with the platform providing her with potential riders.
- Ride share: An individual may seek a ride in someone else's car, with the platform providing her with potential willing drivers. This is only possible for a fraction $p \in [0,1]$ of her ride requests. For a fraction $1-p$ of her requests she must resort to one of the other two options.

We require that every individual satisfies her need for a trip. Hence if a rental request is not successfully matched by the platform, the individual may either choose to drive her own car if she owns one, or use public transport instead. Individuals derive utility $\rho > 0$ from using a car (either a car an individual owns or one driven by someone else). Individuals are heterogeneous in their utility ρ. Without loss of generality, we assume that the utility an individual derives from using public transport is zero.

The non-transport state. While in the non-transport state individuals may pursue work to increase their payoff. In our model they may choose among two options:

- Work for a wage $v > 0$.
- Offer seats to the platform to attract fare-paying riders although the trip is of no personal value to the driver.

Hence an individual may either choose a guaranteed wage v, which is heterogeneous in the population, or rely on a platform-dependent income that is available to all individuals but affected by the population's collective behavior.

We call $\chi = (\rho, v)$ an individual's type and assume types are distributed according to a measure M with unit mass and strictly positive density on $X = (0,\infty)^2$.

2.2 The platform

We assume there exists a platform operated by a third party which matches supply and demand for seats. Every individual may use the platform free of charge (our analysis can be easily extended to the case where the platform extracts a commission on revenue). There are two ways to interact with the platform: Individuals may either request a seat/ride on the platform or, whenever an individual is driving her own car, she may put the remaining seats in her car up for rent at the platform. The former we call seat demand while the latter we call seat supply. Every car is assumed capable of carrying, in addition to the driver, a maximum of $k > 0$ riders per trip. Whenever the platform manages to find a rider for an empty seat, a rental price $r > 0$, specified by the platform, gets transferred from the rider to the driver of the car.

The capability of the platform to successfully match riders with seats depends on the current levels of supply and demand. Let α denote the rate of cars starting trips (measured in number of cars per unit time) and hence denote $k\alpha$ the rate at which cars offer seats to the platform (measured in terms of number of seats per unit time). Denote β the rate of seat requests (also measured in terms of number of seats per unit time). To account for imbalances in seat supply and demand, we define matching functions p and $\bar{p}$ dependent on α and β, where p determines the fraction of seat requests that the platform is able to match and $\bar{p}$ the fraction of seats sold (of those put up for sale) per trip per car. We require $p, \bar{p} : [0,\infty)^2 \to [0,1]$. In addition we require $p\beta \le k\alpha$ and $\bar{p}k\alpha \le \beta$, that is the number of seats sold through the platform is bounded by $k\alpha \wedge \beta$, where $a \wedge b := \min\{a, b\}$. For simplicity we assume the upper bound is achieved and model the matching with the two elementary functions below.

- *Demand:* Fraction of seat requests matched.

$$p(\alpha,\beta) := \frac{k\alpha}{\beta} \wedge 1$$

- *Supply:* Fraction of seats sold during ride:

$$\bar{p}(\alpha,\beta) := \frac{\beta}{k\alpha} \wedge 1$$

The exact formula we use to model matching in the platform is not essential to most of our results.

2.3 Strategies

Given the different decisions an individual may make as described in section 2.1, we construct five different strategies that reflect the decision choices. Namely, we allow individuals to choose among the five strategies in $\Sigma = \{A, D, O, U, S\}$.

- [A] The abstinent: An individual who chooses this strategy generates income during non-transport time, does not own a car and whenever she needs to take a ride she chooses public transport.
- [D] The driver: An individual who chooses this strategy generates income during non-transport time, owns a car and drives her own car to satisfy her own need for a ride. Empty seats in her car are offered on the platform. Any seat rental fee paid to the driver reduces her trips costs. Motivated by that we call a driver's sharing behavior and seat supply cost driven.
- [O] The optimist: An individual who chooses this strategy generates income during non-transport time, owns a car and "optimistically" tries to obtain a ride through the platform, but if the ride request is unsuccessful she uses her own car to drive and consequently offers empty seats on

[3] Our results would not change if we introduced one more state of fixed duration where there is no economic activity (e.g., sleep).

the platform. An optimist always uses private transportation but prefers riding with someone else over driving her own car. We call seat supply generated by optimists utility driven.
- [S] The service provider: An individual who chooses this strategy uses her non-transport time to drive a car on the road and provide seat capacity to the system irrespective of her own need for a ride. In her transport state she is equivalent to a driver: she still offers seats to the platform and earns utility ρ for the trip. The service provider owns a car but does not earn a salary. We call seat supply generated by service providers profit driven since the rental fees paid through the platform replace their traditional income and driving is done purely for net profit instead of gaining also utility from each ride as in the case of drivers.
- [U] The user: An individual who chooses this strategy generates income during non-transport time, does not own a car and, whenever she needs to take a ride, will try to obtain a seat through the sharing platform. If that request is unsuccessful she takes the outside option.

Although there are 3 modes of transportation (public transport, drive, ride share) and a decision among two choices how to spend the non-transport time, other feasible strategies are strictly dominated by the ones above once we account for associated costs. For example, the strategy of owning a car (which incurs a positive cost) but always using public transport is strictly dominated by strategy A in terms of payoff. Similarly, a cost sensitive individual that drives her own car will always put the additional seats up for rent at the platform as we do not model any inconvenience associated with sharing a ride. A positive seat rental fee r will induce this behavior. In that sense the strategy set Σ is exhaustive for our purposes.

We denote by μ_σ the fraction of individuals playing strategy σ and by $\mu = (\mu_A, \mu_D, \mu_O, \mu_S, \mu_U)$ the corresponding vector. In section 2.2 we specified the matching functions depending on supply and demand for seats. Having defined the behavior of each strategy and specifying an aggregate strategy distribution μ allows us to compute supply and demand rates and thus the matching functions p and $\bar{p}$ as functions of μ. To do so, we examine the contributions per strategy. Drivers offer one ride per unit time, service providers offer rides at rate λ_t, whereas optimists offer rides at rate $1-p(\alpha,\beta)$. Both optimists and users generate seat demand at unit rate. This yields a system of equations

$$\begin{aligned}\alpha(\mu) &= \mu_D + \lambda_t\mu_S + (1 - p(\alpha(\mu),\beta(\mu)))\,\mu_O,\\ \beta(\mu) &= \mu_U + \mu_O,\\ p(\alpha(\mu),\beta(\mu)) &= \frac{k\alpha(\mu)}{\beta(\mu)} \wedge 1.\end{aligned}$$

Solving this system of equations for $\alpha(\mu), \beta(\mu)$ yields the matching functions

$$\begin{aligned}p(\mu) &= \frac{k\mu_O + k\lambda_t\mu_S + k\mu_D}{(k+1)\mu_O + \mu_U} \wedge 1,\\ \bar{p}(\mu) &= \frac{\mu_O + \mu_U}{k\lambda_t\mu_S + k\mu_D} \wedge 1.\end{aligned} \tag{1}$$

2.4 Payoffs

Each individual performs one trip per unit time. If it is performed using private transportation the individual obtains utility ρ per unit time. Otherwise zero utility is obtained. Additionally all individuals not playing strategy S earn a wage at rate ν while in the non-transport state which corresponds to an income of ν/λ_n per unit time. Individuals that use the platform to find a seat pay r per ride to the driver who is offering the seat as compensation for the service. Those who offer rides, pay a cost c per trip, which incorporates any variable costs such as fuel costs, wear and tear to the car and road tolls, among others. Additionally, in case supply exceeds demand, we assume the rental paid by riders is shared uniformly among all seat suppliers. Finally, everyone who owns a car, that is individuals who choose strategies D,O or S, pays a cost ω per unit time to cover the cost of ownership.

For completeness we list the five payoff functions below. Each strategy's payoff only depends on an individual's type $\chi = (\rho, \nu) \in (0,\infty)^2$ and the aggregate distribution of strategies played μ.

$$\begin{aligned}\pi_A(\chi,\mu) &= \nu/\lambda_n\\ \pi_D(\chi,\mu) &= \rho + k\bar{p}(\mu)r - c - \omega + \nu/\lambda_n\\ \pi_O(\chi,\mu) &= \rho + \nu/\lambda_n - \omega - p(\mu)r + (1-p(\mu))(k\bar{p}(\mu)r - c)\\ \pi_S(\chi,\mu) &= \rho - \omega + \lambda_t(k\bar{p}(\mu)r - c)\\ \pi_U(\chi,\mu) &= \nu/\lambda_n + p(\mu)(\rho - r).\end{aligned}$$

Together with the formulae for $p(\mu)$ and $\bar{p}(\mu)$ in (1) the payoff functions are fully characterized and we have defined the game $G = ((X, M), \Sigma, \pi)$.

3 EQUILIBRIUM ANALYSIS

In this section we study equilibria of the game G. We mandate that in equilibrium every individual of type $\chi \in X$ plays a strategy $\sigma(\chi)$ which represents her (weakly) best choice among the strategies defined in Section 2.3. This motivates our equilibrium definition.

Definition 3.1. A *partition (of the space X)* is a collection of sets $P = \{P_\eta\}_{\eta\in N}$ such that each set P_η is measurable, the sets are mutually disjoint, and $X = \bigcup_{\eta\in N} P_\eta$.

Definition 3.2. We call an *equilibrium* a partition $P = \{P_\sigma\}_{\sigma\in\Sigma}$ of X, such that for all individuals contained in a partition set P_{σ_0} the (weakly) best strategy choice is σ_0:

$$\forall \chi \in P_{\sigma_0}: \quad \pi_{\sigma_0}(\chi,\mu) \geq \pi_\sigma(\chi,\mu) \quad \forall \sigma \in \Sigma,$$

where μ_σ is given by $\mu_\sigma = M(P_\sigma) = \int_{P_\sigma} \mathrm{d}M(\chi)$, the measure of the set P_σ. We write $\mu(P) := (M(P_\sigma))_{\sigma\in\Sigma}$.

The remainder of this section is devoted to characterizing equilibria as defined above. We begin by stating a general result for equilibrium partitions. For two measurable sets $B_1, B_2 \subseteq X$ we denote their symmetric difference by $B_2 \ominus B_1 = B_1 \ominus B_2 = (B_2 \backslash B_1) \cup (B_1 \backslash B_2)$.

Definition 3.3. A measurable set $B \subset X$ is called *nearly convex (with respect to M)* if there exists a convex set $\bar{B} \subseteq X$ such that $M(\bar{B} \ominus B) = 0$. A collection of disjoint measurable sets $B_1, \ldots, B_n \subseteq X$ is *jointly nearly convex (with respect to M)* if their union $\bigcup_{k=1}^n B_k$ is nearly convex.

Lemma 3.4. *Assume $|\Sigma| < \infty$. Furthermore assume that $\pi_\sigma(\chi,\mu)$ is an affine function in χ for all strategies $\sigma \in \Sigma$ and aggregate strategy distributions $\mu \in \Delta^{|\Sigma|-1}$. Denote P an equilibrium partition in the game and $\mu(P)$ the induced distribution of strategies played. Take a partition set $P_\sigma \in P$. Then either P_σ is nearly convex or it is a member of a jointly nearly convex collection of partition sets, i.e., there exists a subset of other strategies $\Sigma^\star \subseteq \Sigma$ with $\sigma \in \Sigma^\star$ such that the payoff functions are identical for all $\sigma^\star \in \Sigma^\star$, (that is $\pi_{s_1}(\chi,\mu(P)) = \pi_{s_2}(\chi,\mu(P)), \forall \chi \in X, s_1, s_2 \in \Sigma^\star$,) and $\{P_{\sigma^\star}\}_{\sigma^\star \in \Sigma^\star}$ are jointly nearly convex.*

This already gives us a glimpse of how an equilibrium partition looks like in case it exists. Since all five payoff functions we consider are affine functions of the types, we expect to partition the space of types X into up to five distinct convex regions. The convexity in particular implies that the indifference curves between the partition sets are simple lines and the partition sets essentially polygonal. In particular, it is possible to characterize equilibria by specifying only a few points.

Lemma 3.4 essentially follows from the fact that two affine functions in one dimension can intersect at most once unless they are identical. The first part of the lemma deals with non-identical functions, yielding convexity. The second part of the lemma is aimed at the two functions being identical. If this happens, considering two strategies that lead to the identical payoffs will give a combined surface that satisfies the convexity condition. Take for example strategies D and U when seat supply exceeds demand, i.e. $p = 1$. The payoffs satisfy $\pi_U(\chi,\mu) - \pi_D(\chi,\mu) = \gamma$, where γ is a constant for all $\chi \in X$. In case $\mu_D, \mu_U > 0$ in equilibrium, implying that there are non-negligible fractions of drivers and users present in the equilibrium, the constant γ must be zero. Thus, rearranging individuals in $P_D \cup P_U$ while ensuring μ_D, μ_U stay constant would yield another equilibrium partition. We do not intend to differentiate equilibria with this level of detail and consequently call such equilibria equivalent[4].

Definition 3.5. Two equilibria P^1 and P^2 are called equivalent if they induce the same strategy distributions $\mu(P^1) = \mu(P^2)$ and the payoff for each individual is identical under both partitions. That is,

$$\forall \chi \in X: \quad \pi_{\sigma(\chi,P^1)}(\chi,\mu) = \pi_{\sigma(\chi,P^2)}(\chi,\mu),$$

where $\sigma(\chi,P)$ denotes the strategy σ assigned to an individual of type χ under the partition P.

The definition above calls two equilibria equivalent if every individual enjoys the same payoff under both partitions, although the actual strategy assignment might be different for a non-negligible region in X. In the following we characterize possible equilibrium partitions modulo the equivalence relation of Definition 3.5.

3.1 Characterization of equilibria

We present a complete characterization of the equilibria of the game G for a measure M that has a strictly positive density on $X = \mathbb{R}^2_+$. When we claim that a partition does not contain strategy σ, it is implied that $\mu_\sigma = 0$ and without loss of generality we omit the corresponding partition set P_σ in the discussion.

[4] See for example equilibria of type III below

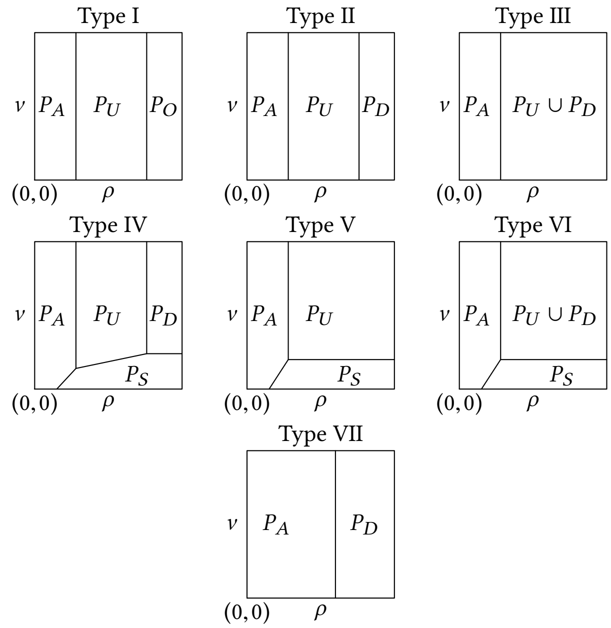

Figure 1: Illustrations of the seven partition types defined in Appendix A.

We define seven types of partitions and show in Theorem 3.6 below how the rental price r in relation to the other parameters determines the type of the equilibrium partition. The type of a partition describes its geometrical properties up to at most five real valued parameters. In particular, the type of a partition specifies which strategies are contained in the partition. Figure 1 provides graphical illustrations of all partition types. Additionally, we provide explicit algebraic definitions in Appendix A below.

Theorem 3.6. *For every set of parameters $c, r, k, \omega > 0$, $\lambda_t > 1$ such that $r \neq \frac{c}{k+1}$ [5], there exists a unique equilibrium modulo equivalence (as defined in 3.5) in the game $G = ((X, M), \Sigma, \pi)$. The equilibrium partitions are classified as follows.*

- *If $\omega \le c/k$, only equilibria of types I, II, III, VII are possible. The equilibrium is*
  - *of Type I and $p < 1$ if $r \in [0, \frac{c}{k+1})$,*
  - *of Type II and $p < 1$ if $r \in (\frac{c}{k+1}, \frac{\omega+c}{k+1})$,*
  - *of Type III and $p = 1$ if $r \in [\frac{\omega+c}{k+1}, \omega + c)$,*
  - *of Type VII with no sharing taking place if $r \in [\omega+c, \infty)$.*

  This holds independent of M.
- *If $\omega > c/k$, we observe either six or all seven different types of equilibria. The equilibrium is*
  - *of Type I and $p < 1$ if $r \in [0, \frac{c}{k+1})$,*
  - *of Type II and $p < 1$ if $r \in (\frac{c}{k+1}, \frac{c}{k}]$,*
  - *of Type IV and $p < 1$ or[6] Type V and $p = 1$ if $r \in (\frac{c}{k}, \frac{\omega+c}{k+1})$,*

[5] In case the ride fee is exactly $\frac{c}{k+1}$ there exist multiple equilibria in which drivers and optimists have identical payoffs as functions of χ. In this situation we can find equilibria for every $\delta > 0$ such that $\mu_O/\mu_D = \delta$. Such equilibria need not be equivalent as per Definition 3.5.

[6] Here and in the remainder of Theorem 3.6 we use "or" in its exclusive sense.

– *of Type V and* $p = 1$ *or Type VI and* $p = 1$ *if* $r \in [\frac{\omega+c}{k+1}, \omega)$,
– *of Type III and* $p = 1$ *if* $r \in [\omega, \omega + c)$,
– *of Type VII with no sharing taking place if* $r \in [\omega+c, \infty)$.

When $r \in (\frac{c}{k}, \omega)$*, the measure* M *determines whether there are any equilibria of type V. In case the ride fee* r *is outside this range, the type of equilibrium is independent of the population measure. For each type, the exact equilibrium partition can be computed by solving a fixed point equation.*

As long as $c/k < r < \omega$ service providers will always be part of the equilibrium since M is assumed to have positive density on X. Consequently we call parameter combinations such that $c/k < r < \omega$ the "profit regime". Only in this regime can individuals make a profit (net of utility) in equilibrium by using the platform. In addition, in case $\omega \leq c/k$ or $r \notin (c/k, \omega)$, the distribution of the wage v is irrelevant for the formation of equilibria. The marginal distribution of M with respect to ρ contains all necessary information to determine the equilibrium.

In equilibria of type I the sharing economy is completely utility driven, optimists are the only seat suppliers. Ride sharing is cheaper than the incurred usage costs in a fully occupied car. This results in high seat demand. Seat supply only exists because some individuals value private transportation high enough to ensure the availability of private transportation when the platform cannot provide a ride. This is only possible through car ownership.

Equilibria of types II and III display a completely cost driven sharing economy since drivers are the only seat suppliers. Rental income is used to subsidize usage and ownership costs. A driver cannot make a profit in equilibrium (net of utility)[7] because otherwise driving would strictly dominate the strategy of a user. Thus there would be no rental income paid by anyone - a contradiction.

Equilibria of types IV, V or VI only appear in the profit regime. In case the equilibrium is of type IV or VI, both, drivers and service providers, offer seats on the platform. Depending on the measure M, either could be the dominant contributors. Consider a population measure which concentrates most of its mass around a high wage v such that all other costs incurred through ride sharing are negligible in comparison. Drivers would be the majority of seat suppliers in such a society and equilibria of type V cannot emerge. But in case the population measure is such that there exists a sizable population with little or no income from wage, service providers will dominate the ride sharing market. We observe a completely profit-driven sharing economy - modeled as equilibria of type V - in which the population separates in three different categories: those who have too little value for the service and do not participate, a low income population share which gives up their wage to pursue full time work on the platform and is numerous enough to fully cover the transportation demand of platform customers, and a population segment of high income users which do not need to own a car to have full access to private transportation services (get a ride in a car every time they need to). We observe the paradoxical situation, that those with low income v own a relatively expensive good, while those with high income do not. Ownership and usage are almost completely decoupled.

[7]We define profit as seat rental income minus costs. That is, profit equals payoff minus any terms containing utility or income.

3.2 The traditional economy

To study the impact of introducing sharing economy concepts into a society we define a benchmark game which corresponds to a society in which the platform does not exist. Consider the game $\bar{G}$ with $\bar{\Sigma} = \{A, D\}$ but everything else is exactly as in game G. We only remove the possibility of playing strategies O, S and U. This describes a situation in which the platform is unavailable (and hence drivers cannot find any riders). Equilibria of this game are unique and easily characterized as $P_A = \{\chi \in X : \rho < \omega + c\}, P_D = X \backslash P_A$, that is, only individuals whose valuation for private transportation is high enough own and use a car. The resulting equilibrium is a partition of type VII. Note that these are equivalent to the equilibria of the original game G in case the rental fee exceeds the sum of ownership and usage fees, that is $r \geq \omega + c$. We refer to the platform-less setting as the traditional economy.

4 THE IMPACT ON MOBILITY

We define four performance metrics. Let P be an equilibrium partition and denote μ the associated aggregate distribution of strategies played.

- Congestion: the traffic volume or average number of cars on the road per unit time.
$$\Gamma(\mu) := \mu_S + \mu_D/\lambda_t + (1 - p(\mu))\mu_O/\lambda_t.$$
- Ownership: the total amount of cars owned by the population.
$$\Omega(\mu) := \mu_D + \mu_S + \mu_O.$$

Note that both performance metrics are invariant under equivalent equilibria. Theorem 3.6 thus implies that by restricting the game parameters to $Z := \{(r, \omega, c, k, \lambda_t) \in \mathbb{R}_+^4 \times (1, \infty) : r \neq \frac{c}{k+1}\}$ we may define the performance metrics as functions of the underlying game parameters instead of the equilibrium partitions. With slight abuse of notation the same symbols are used for these functions. Theorem 3.6 then yields the following corollary.

Corollary 4.1. *Γ and Ω are continuous functions on Z.*

Not all performance measures necessarily need be discontinuous at $r = \frac{c}{k+1}$. For the remainder of this section we fix $m(\chi) = \mathbb{1}_{[[0,1]^2]}(\chi)$, that is we consider a population measure where utility and income are uniformly distributed and independent of each other. We do so mostly for computational reasons.

In Figure 2 we compute the two performance metrics as functions of the rental price r for four different combinations of ownership and usage costs ($(\omega, c) \in \{(0.1, 0.4), (0.1, 0.7), (0.4, 0.1), (0.7, 0.1)\}$) and fixed $k = 2, \lambda_t = 6$. We do not vary the maximum number of riders per trip k or the maximum number of trips per unit time λ_t as these parameters do not influence the type of equilibrium. The combinations of ownership and usage fees we chose are qualitatively exhaustive to describe the phenomena occurring in our model. The black vertical lines mark a shift in equilibrium type with the Roman numbers at the bottom indicating the respective type in between two vertical lines. Equilibria of type VII occur when $r \geq \omega + c$, hence we may truncate the plot when this threshold on r has been reached as neither performance metric will be affected by further increase in r. Additionally, for $r \geq \omega + c$ the values of the performance metrics correspond to the respective values in the

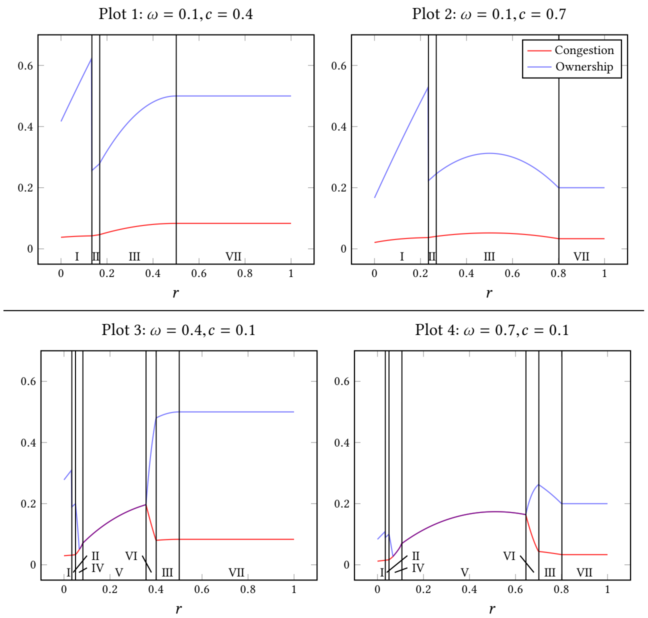

Figure 2: Equilibrium congestion and ownership levels as functions of the rental price $r \in [0, 1]$.

traditional economy as introduced in section 3.2. The top row plots' parameters satisfy $\omega < c/k$ and thus only allow for a utility or cost driven sharing economy, while the bottom two plots have $\omega > c/k$ and thus the profit regime is non-empty.

We now present some of our findings how the introduction of the platform impacts congestion and ownership. Namely, the introduction of the ride sharing platform can lead to either lower or higher ownership and lower or higher congestion - all four combinations are possible.

Lower congestion, lower ownership. If ownership was high in the traditional economy and thus many people already had access to private transportation, the sharing economy will decrease congestion (Plot 1). The higher occupation of cars (up to k passengers plus the driver) results in less vehicles being driven. Recall that for $r \geq \omega + c$ - a setting equivalent to the traditional economy - every car on the road is occupied by only a single individual with k spare seats.

Higher congestion, higher ownership. If ownership was low enough in the traditional economy, the sharing economy will increase congestion for a wide range of rental prices (Plot 2). The ability to share costs makes private transportation affordable for more people. The decreased price outweighs the increased efficiency in this scenario.

Lower congestion, higher ownership. The sharing economy may increase ownership and lower congestion at the same time. This is the case for very low rental prices inducing equilibria of type I. In such a setting, riding is significantly cheaper than driving (even with the additional rental income form co-riders). Hence (almost) everyone chooses riding as their first choice of transportation and the car merely serves as a back-up option in case the platform cannot provide for a ride. Consequently, not every owner will use her car once per unit time, leading to higher ownership (sharing reduces costs) but lower congestion (every trip transports $k + 1$ individuals).

Higher congestion, lower ownership. In the top two plots (1 and 2), where ownership costs are cheap compared to usage costs, drivers are the only car owners and hence ownership is simply a multiple of traffic volume, except for equilibria of type I. In the lower two plots (3 and 4) this is no longer true. When ownership costs are expensive compared to usage costs, service providers emerge for a range of rental prices. In such a setting, the platform is able set a (high) rental fee which results in an equilibrium with many service providers which contribute to traffic, although the fraction of occupied seats might be low. This is a situation where many individuals with low income own cars and compete for relatively few but high paying customers. (In our model a service provider is assumed to fully contribute to traffic independent of the value of $\bar{p}$, that is the fraction of seats occupied in her car.)

Additionally, when comparing the plots vertically (such that ownership levels in the traditional economy are the same in both settings), we observe that the ownership levels are actually lower when sharing is enabled in the lower two plots. This is again due to the profit driven behavior of service providers. Although the congestion is increased through service providers, the ownership is decreased. Since in equilibria of type V car ownership is no longer necessary to have full access to the transportation services, the ride demand is served by fewer but constantly driven cars. Since drivers only use their cars for $1/\lambda_t$ of time the ownership levels in the traditional economy may be significantly higher than in the sharing economy.

5 CONCLUDING REMARKS

In the extended version [2] we study a platform which optimizes either revenue or social welfare and chooses the rental fee r accordingly. With r chosen endogenously we study the impact of changes in ownership or usage costs in the sharing economy. For example, when cars are expensive, it is possible that an increase in ownership costs results in higher congestion, ownership and platform revenue if the platform optimizes revenue. Comparing a revenue- with a welfare-maximizing platform we find that when cars are cheap the two objectives may be aligned. When cars are expensive, a revenue maximizing platform tends to induce an equilibrium with strictly worse welfare and strictly higher congestion compared to the welfare optimum. This suggests that in such a setting, a monopolist platform would need to be regulated more strictly to avoid socially undesirable outcomes.

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A APPENDIX

For $\chi = (\rho, \nu) \in X = \mathbb{R}_+^2$ we define seven different types of partitions.

(I) A partition of type I is of the form $P = \{P_A, P_O, P_U\}$ with

$$\begin{aligned} P_A &= \{\chi : \rho \in [0, \rho_1)\}, \\ P_U &= \{\chi : \rho \in [\rho_1, \rho_2)\}, \\ P_O &= \{\chi : \rho \in [\rho_2, \infty)\}, \end{aligned}$$

where $\rho_2 > \rho_1 > 0$. Such a partition only contains individuals playing strategies A, O and U. Wages ν are irrelevant and the space X is partitioned along vertical lines.

(II) A partition of type II is of the form $P = \{P_A, P_D, P_U\}$ with

$$\begin{aligned} P_A &= \{\chi : \rho \in [0, \rho_1)\}, \\ P_U &= \{\chi : \rho \in [\rho_1, \rho_2)\}, \\ P_D &= \{\chi : \rho \in [\rho_2, \infty)\}, \end{aligned}$$

where $\rho_2 > \rho_1 > 0$. Such a partition only contains individuals playing strategies A, D and U. Wages ν are irrelevant and the space X is partitioned along vertical lines.

(III) A partition of type III is of the form $P = \{P_A, P_D, P_U\}$ with

$$\begin{aligned} P_A &= \{\chi : \rho \in [0, \rho_1)\}, \\ P_U \cup P_D &= \{\chi : \rho \in [\rho_1, \infty)\}, \frac{|P_U|}{|P_D|} = \delta\}, \end{aligned}$$

where $\rho_1 > 0, \delta \in (0, k]$. In this partition only strategies A, D and U appear. In this equilibrium P_U and P_D are jointly nearly convex and hence we do not specify the exact location of the partition sets for strategies U and D, only their ratio δ. Again, wages ν are irrelevant.

(IV) A partition of type IV is of the form $P = \{P_A, P_D, P_S, P_U\}$ with

$$\begin{aligned} P_A &= \{\chi : \rho < \rho_1, \nu > [\nu_1 + \gamma(\rho - \rho_1)]^+\}, \\ P_D &= \{\chi : \rho \geq \rho_2, \nu > \nu_2\}, \\ P_U &= \{\chi : \rho \in [\rho_1, \rho_2), \nu > [\nu_1 + \frac{\nu_2 - \nu_1}{\rho_2 - \rho_1}(\rho - \rho_1)]^+\}, \\ P_S &= X \backslash \{P_A, P_D, P_U\}, \end{aligned}$$

where $\nu_2, \gamma, \rho_1 > 0,\ \nu_2 > \nu_1,\ \rho_2 > \rho_1$. Such partitions contain all strategies but strategy O. Note that we allow $\nu_1 < 0$.

(V) A partition of type V is of the form $P = \{P_A, P_S, P_U\}$ with

$$\begin{aligned} P_A &= \{\chi : \rho < \rho_1, \nu > [\nu_1 + \gamma(\rho - \rho_1)]^+\}, \\ P_U &= \{\chi : \rho \geq \rho_1, \nu > \nu_1\}, \\ P_S &= X \backslash \{P_A, P_U\} \end{aligned}$$

where $\rho_1, \nu_1, \gamma > 0$. Such partitions do not contain strategies O or D.

(VI) A partition of type VI is of the form $P = \{P_A, P_D, P_S, P_U\}$ with

$$\begin{aligned} P_A &= \{\chi : \rho < \rho_1, \nu > [\nu_1 + \gamma(\rho - \rho_1)]^+\}, \\ P_U \cup P_D &= \{\chi : \rho \geq \rho_1, \nu > \nu_1\}, \frac{|P_U|}{|P_D|} = \delta, \\ P_S &= X \backslash \{P_A, P_U, P_D\}, \end{aligned}$$

where $\gamma, \rho_1, \nu_1, \delta > 0$. Again we do not specify the exact form for the partition sets P_D and P_U, but only the ratio of their masses. Again, all strategies but strategy O are present such partitions.

(VII) A partition of type VII is of the form $P = \{P_A, P_D\}$ with

$$\begin{aligned} P_A &= \{\chi \in X : \rho < \rho_1\}, \\ P_D &= \{\chi \in X : \rho \geq \rho_1\}, \end{aligned}$$

where $\rho_1 > 0$. This type only contains strategies A and D separated by a single vertical line at ρ_1.