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DIP assignment

Q1 Biquadratic Interpolation

~~V(x,y)~~ : Let our pts be $(x_1, y_1), (x_2, y_2) \ldots (x_9, y_9)$ and the pixels in $v_1(x_1,y_1)$, $v_2(x_2, y_2)$ $v_9(x_9)$

$$X(x,y) = \Sigma_{i=0}^{2} \Sigma_{j=0}^{2} a_{ij} \, x_9^i y^j$$

$$\text{so } V(x,y) = a_{00} x^0 y^0 + a_{01} x^0 y^1 - - - a_{22} x^2 y^2$$

$$\text{so } V_1(x_1, y_1) = a_{00} x_1^0 y_1^0 + a_{01} x_1^0 y_1^1 - - a_{22} x_1^2 y_1^2$$

$$\vdots$$

$$V_9(x_9, y_9) = a_{00} x_9^0 y_9^0 + - - - - - a_{22} x_9^2 y_9^2$$

So, we can vectorize this expression

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_9 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & x_1 y_1 \\ x_2 & y_2 & x_2 y_2 \\ x_3 & y_3 & \vdots \\ \vdots & \vdots & \vdots \\ x_9 & y_9 & x_9 y_9 \end{bmatrix}$$

Expression in terms of matrices.

$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_9 \end{bmatrix}}_{V} = \underbrace{\begin{bmatrix} x_1^0 y_1^0 & x_1^0 y_1^1 & x_1^0 y_1^2 & - - - - & x_1^2 y_1^2 \\ \vdots & \vdots & \vdots & - - - & \vdots \\ \vdots & \vdots & \vdots & - - - & \vdots \\ x_9^0 y_9^0 & x_9^0 y_9^1 & x_9^0 y_9^2 & - - - - & x_9^2 y_9^2 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} a_{00} \\ a_{01} \\ \vdots \\ a_{22} \end{bmatrix}}_{A}$$

Expression for coefficient.

$$V = XA \Rightarrow A = X^{-1} V$$

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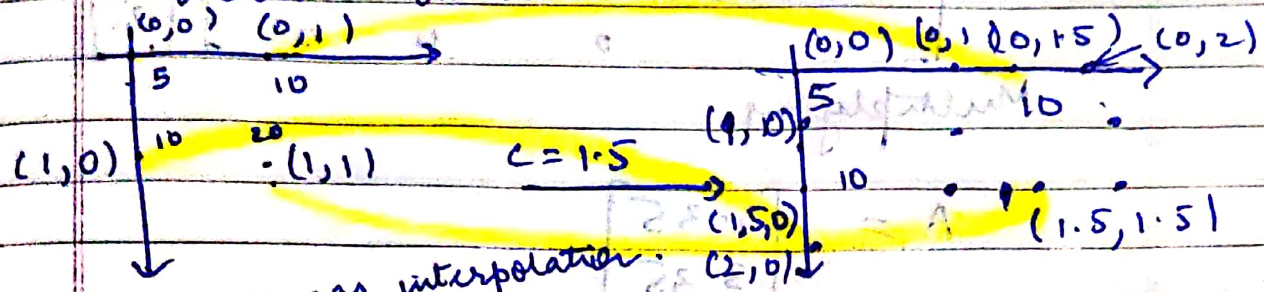
Q2 Given a 2×2 image $\begin{bmatrix} 5 & 10 \\ 10 & 20 \end{bmatrix}$, $c = 1.5$

So a 2×2 matrix will transform into a 3×3 matrix. as $2 \times 1.5 = 3$



For bilinear interpolation.

$\hookrightarrow V_{i,j} = ax_i + by_j + cx_iy_j + d.$

So we can say.

$(0,1) \longrightarrow (0,1.5)$
$(1,0) \longrightarrow (1.5,0)$
$(1,1) \longrightarrow (1.5,1.5)$

$V = XA$

$\Rightarrow X^{-1}V = A.$

$A = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ $X = \begin{bmatrix} x_1 & y_1 & x_1y_1 & 1 \\ x_2 & y_2 & x_2y_2 & 1 \\ x_3 & y_3 & x_3y_3 & 1 \\ x_4 & y_4 & x_4y_4 & 1 \end{bmatrix}$ and $V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$

$\Rightarrow X = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1.5 & 0 & 1 \\ 1.5 & 0 & 0 & 1 \\ 1.5 & 1.5 & 2.25 & 1 \end{bmatrix}$ $V = \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix}$

calculating inverse.

$X^{-1} = \begin{bmatrix} -0.67 & 0 & 0.67 & 0. \\ -0.67 & 0.67 & 0 & 0 \\ 0.44 & -0.44 & -0.44 & 0.44 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow A = X^{-1}V$

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$$>> A = \begin{bmatrix} -0.67 & 0 & 0.67 & 0 \\ -0.67 & 0.67 & 0 & 0 \\ 0.44 & -0.44 & -0.44 & 0.44 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \\ 10 \\ 20 \end{bmatrix}$$

Multiplying,

$$A = \begin{bmatrix} 3.35 \\ 3.35 \\ 2.2 \\ 5 \end{bmatrix}$$

So

$$V_{1,1} = 3.35 + 3.35 + 2.2 + 5$$
$$= 6.7 + 7.2 = \underline{13.9}$$

So pixel value @ (1,1) = 13.9

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