DSP assignment.

Q2

1. $x[n] = 2(0.5)^n u(n+2)$

$$X(e^{jw}) = \sum_{-\infty}^{\infty} 2(0.5)^n u(n+2) e^{-jwn}$$

↳ will change limits to $-2 \to \infty$

$$= 2\left[(0.5)^{-2} e^{jw2} + (0.5)^{-1} e^{jw} + \underbrace{\sum_{0}^{\infty} (0.5)^n e^{-jwn}}_{\text{infinite GP}}\right]$$

$$= 8e^{2jw} + 4e^{jw} + \frac{1}{1 - 0.5e^{-jw}}$$

$$= 8e^{2jw} + \frac{4e^{jw} - 2 + 1}{1 - 0.5e^{-jw}}$$

$$= \frac{8e^{2jw} - \cancel{4e^{jw}} + \cancel{4e^{jw}} - 1}{1 - 0.5e^{-jw}}$$

$$= \frac{8e^{2jw} - 1}{1 - 0.5e^{-jw}}$$

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② $(0.6)^{|n|}\left[u(n+10) - u(n-11)\right] = x(n)$

$$X(e^{jw}) = \sum_n x(n)\underbrace{e^{-jwn}}_{= t}$$

$$= \sum_n (0.6)^{|n|}\left(u(n+10) - u(n-11)\right)t^{-n}$$

$$= (0.6)^{10}t^{10} + (0.6)^{9}t^{9} + \ldots$$
$$(0.6)^{0}t^{0} + (0.6)^{1}t^{-1} + \ldots (0.6)^{10}t^{-10}$$

$$\frac{(0.6t)^{10}\left(1-(0.6t)^{-10}\right)}{1-(0.6t)^{-1}} + \frac{1\left(1-0.6t^{-1}\right)^{11}}{1-(0.6t^{-1})}$$

$$= \frac{(0.6e^{jw})^{10}\left(1-(0.6e^{jw})^{-10}\right)}{1-(0.6e^{jw})^{-1}}$$
$$+ \frac{\left(1-(0.6e^{-jw})^{11}\right)}{\left(1-(0.6e^{-jw})\right)}$$

Q 3) $x(n) = n(0.9)^{(n)}u(n+3)$

$$X(e^{jw}) = \sum_n^{\infty} n(0.9)^n u(n+3)\, \underbrace{e^{-jwn}}_{\to t^{-n}}$$

$$X(t) = \sum n(0.9)^n u(n+3)t^{-n}$$

$$-3(0.9)^{-3}t^{3} + -2(0.9)^{-2}t^{+2}$$
$$\ldots\ n(0.9)^n t^{-n}$$

Sum of agp $\quad a = -3,\ d = 1,\ \gamma = (0.9t^{-1})$

[sum of agp

$$= \frac{a}{1-r} + \frac{dr}{(1-r)^2} \quad a = -3, \quad d = 1, \quad r = (0.9 t^{-1})$$

$$= \frac{-3}{1-0.9t^{-1}} + \frac{0.9\, t^{-1}}{(1-(0.9\, t^{-1}))^2}$$

$$= \frac{-3}{1-0.9e^{-j\omega}} + \frac{0.9\, e^{-j\omega}}{(1-0.9e^{-j\omega})^2}$$

4) $(n+3)(0.8)^{n-1} \, u(n-2) = x(n)$

$$X(e^{j\omega}) = \sum_{n}^{\infty} x(n) e^{-j\omega n}.$$

$$= X(t) \quad |\text{let}|$$

$$5(0.8)^1 t^{-2} + 6(0.8)^2 t^{-3} + 7(0.8)^3 t^{-4} \text{ --- } \infty$$

$a = 5, d = 1 \quad r = (0.8) t^{-1}$

$$\frac{5}{1-0.8\, t^{-1}} + \frac{(0.8)\, t^{-1}}{(1-0.8t^{-1})^2}$$

$$\Rightarrow \frac{5}{1-0.8e^{-j\omega}} + \frac{0.8\, e^{-j\omega}}{(1-0.8e^{-j\omega})^2}$$

5) $x(n) = 4(0.7)^n \cos(0.25\pi n)u(n)$

$$X(e^{j\omega}) = X(z) = \sum_{n}^{\infty} x(n) z^{-n}$$

$$\sum_{n=0}^{\infty} \cancel{4}^{2}(0.7)^n z^{-n} \left[\frac{e^{j0.25\pi n} + e^{-j0.25\pi n}}{\cancel{2}}\right]$$

$$= \sum 2(0.7)^n z^{-n} e^{(j0.25\pi n)}$$

$$+$$

$$\sum 2(0.7)^n z^{-n} e^{-(j0.25\pi n)}$$

$$= \frac{2}{1-(0.7)z^{-1}e^{0.25\pi j}} + \frac{2}{1-(-0.7)z^{-1}e^{-0.25\pi j}}$$

$$= 2\left[\frac{1}{1-(0.7e^{0.25\pi j}-j\omega)} + \frac{1}{(1-(-0.7)e^{(0.25\pi j+j\omega)})}\right]$$

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Q 5 $X(e^{j\omega})$ can be decomposed into

$X_o(e^{j\omega})$; $X_e(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) + X^*(e^{-j\omega})]$

$X_o(e^{j\omega}) = \frac{1}{2}[X(e^{j\omega}) - X^*(e^{-j\omega})]$.

$$\mathcal{F}^{-1}[X_e(e^{j\omega})] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_e(e^{j\omega})\, e^{jn\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2}[X(e^{j\omega}) + X^*(e^{-j\omega})]\, e^{jn\omega}\, d\omega$$

$$= \frac{1}{2}[x[n] + x^*(-n)] = x_R(n)$$

$$\mathcal{F}^{-1}[X_o(e^{j\omega})] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_o(e^{j\omega})\, e^{jn\omega}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2}[X(e^{j\omega}) - X^*(e^{-j\omega})]\, e^{jn\omega}\, d\omega$$

$$= \frac{1}{2}[x[n] - x^*(-n)] = x_I(n)$$

Hence proven.

(6) $x[n] = (0.9)^n \cos\left(\frac{n\pi}{4}\right) u(n)$

$$y[n] = \begin{cases} x[n/2] & n = 0, \pm 2, \pm 4 \ldots \\ 0 & \text{else} \end{cases}$$

(1) using z transform.

$$Y(z) = \sum_{-\infty}^{\infty} y(n) z^{-n}$$

$$= \sum_{-\infty}^{\infty} x(\underbrace{n/2}_{m}) z^{-n} \qquad n = 0, \pm 2, \pm$$

$$= \sum_{-\infty}^{\infty} x[n] z^{-2m} \qquad m = 0, \pm 1, \pm 2 \ldots$$

$$= \sum_{m=-\infty}^{\infty} x[m] (z^2)^{-m}$$

$$= X(z^2) \quad (\text{Hence Proved})$$

2) $Y(z) = X(z^2)$

$$X(z) = Z\left[0.9^n \cos\left(\frac{n\pi}{4}\right) u[n]\right]$$

$$(r^n \cos w_0 n) u[n] \rightarrow \frac{1 - (r \cos w_0) z^{-1}}{1 - (2r \cos w_0) z^{-1} + r^2 z^{-2}} \quad |z| > r$$

$$x(z) = \frac{1 - [0.9 \cos[\pi/4]] z^{-1}}{1 - 2[0.9 \cos(\frac{\pi}{4})] z^{-1} + (0.9)^2 z^{-2}} \quad |z| > 0.9$$

$$= \frac{1 - 0.636396 z^{-1}}{1 - 1.272792 z^{-1} + 0.81 z^{-2}} \quad |z| > 0.9$$

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Hence $Y(z) = \dfrac{1 - 0.64\, z^{-2}}{1 - 1.27\, z^{-2} + 0.81\, z^{-4}}$ $\quad |z| > \sqrt{0.4}$