SECTION-A

Q1. $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$

ORDER = 2

DEGREE = 1

Q2. $f(x) = x+7$; $g(x) = x-7$

$fog(x) = f(g(x))$
$= f(x-7)$
$= (x-7)+7$
$= x \quad \forall x \in R$

$\frac{d}{dx} fog(x) = \frac{d}{dx}(x) = 1$

$\boxed{\frac{d}{dx} fog(x) = 1}$

Q3.

$$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix} + \begin{bmatrix}y & 0\\1 & 2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix} = \begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

comparing corresponding elements of each matrix,

$$2+y=5 \qquad 2x+2=8$$

$$\boxed{y=3} \qquad \boxed{x=3}$$

$$x-y = 3-3$$

$$\boxed{x-y=0}$$

Q5.

Q4

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

where $\vec{a}$ = position vector of pt on line
$\vec{b}$ = parallel vector to line

$$\boxed{\vec{r} = (3\hat{i}+4\hat{j}+5\hat{k}) + \lambda(2\hat{i}+2\hat{j}-3\hat{k})}$$

Also $\vec{r} = (3+2\lambda)\hat{i} + (4+2\lambda)\hat{j} + (5-3\lambda)\hat{k}$

Q5. $a^* b = ab + 1$

i) For all $(a, b) \in (R \times R)$

$\Rightarrow ab + 1 \in R$ [$\because$ If $a, b \in R \Rightarrow ab \in R \Rightarrow (ab+1) \in R$] [$\because$ Multiplica-tion is binary. operation]

$\therefore$ a R b relates to a unique element in R and hence is a binary operation from $R \times R$ to R

ii) For binary operation to be associative $(a^* b)^* c = a^*(b^* c)$

LHS $= (a^* b)^* c$

$= (ab+1)^* c$

$= (ab+1)c + 1$

$= abc + c + 1$

RHS $= a^*(b^* c)$

$= a^*(bc+1)$

$= a(bc+1) + 1$

$= abc + a + 1$

LHS $\neq$ RHS

$\therefore$ It is NOT ASSOCIATIVE

Q6.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = AA$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$A^2 - 5A = A^2 + (-5)A$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + (-5)\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -10 \\ -5 & 5 & 0 \end{bmatrix}$$

Q7.

$$A^2 - 5A = A^2 + (-5A)$$

$$= \begin{bmatrix} +5 & -4 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$$

Q7. $I = \int \underset{\mathrm{I}}{1} \cdot \underset{\mathrm{II}}{\sin^{-1}(2x)}\, dx$

$I = [\sin^{-1}(2x)](x) - \int \frac{(x)(2)}{\sqrt{1-4x^2}}\, dx$ [INTEGRATION by parts]

$I = x\sin^{-1}(2x) - \frac{1}{2}\int \frac{2x}{\sqrt{(\frac{1}{2})^2 - x^2}}\, dx$.

$I = x\sin^{-1}(2x) + \frac{1}{4}\int \frac{(-8x)}{\sqrt{1-4x^2}}\, dx = x\sin^{-1}(2x) + \frac{1}{4} I_1$ where $I_1 = \int \frac{-8x}{\sqrt{1-4x^2}}\, dx$

Now, $I_1 = \int \frac{dt}{\sqrt{t}}$ let $1-4x^2 = t$

$-8x\,dx = dt$

$= 2\sqrt{t} + C$

$= 2\sqrt{1-4x^2} + C$

$$I = x\sin^{-1}(2x) + \frac{1}{4}\left(2\sqrt{1-4x^2}\right) + C$$

$$\boxed{I = x\sin^{-1}(2x) + \frac{\sqrt{1-4x^2}}{2} + C}$$

Q8

$$y = e^{2x}(a+bx) \quad \text{—— (1)}$$

Diff. both sides w.r.t. x.

$$\frac{dy}{dx} = e^{2x}(b) + (a+bx)(e^{2x})(2)$$

$$\Rightarrow \frac{dy}{dx} = e^{2x}(b+2a+2bx) \quad \text{—— (2)}$$

Diff. both sides w.r.t. x.

$$\frac{d^2y}{dx^2} = e^{2x}(2b) + (b+2a+2bx)(e^{2x})(2)$$

$$= e^{2x}(4b+4a+4bx)$$

$$\frac{d^2y}{dx^2} = 2e^{2x}(2b+2a+2bx) \quad \text{—— (3)}$$

Subtract $2\times$ eqn (2) from eqn (3)

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x}(4b+4a+4bx) - e^{2x}(2b+4a+4bx)$$

$$= e^{2x}(2b)$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2(e^{2x})(b) \quad \text{—— (4)}$$

In eq^n (3)

$$\frac{d^2y}{dx^2} = e^{2x}(4b + 4a + 4bx)$$

$$= 2\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx}\right) + 4e^{2x}(a+bx) \qquad [\text{From (4)}]$$

$$\frac{d^2y}{dx^2} = 2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y \qquad [\text{From (1)}]$$

$\therefore$ The required differential eq^n is

$$\boxed{\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0} \Rightarrow y'' - 4y' + 4y = 0.$$

Where $y'' = d^2y/dx^2$, $y' = dy/dx$, $\cancel{y = e^2}$

P.T.O

Q9 $\sum_{i=0}^{2} P(X_i) = 1$ $\left[\because \sum_{i=0}^{n} P(X_i) = 1\right]$

[Sum of all probabilities = 1] ~~disjoint~~ exhaustive & exclusive

$\Rightarrow k + 2k + 3k = 1$

$\Rightarrow 6k = 1$

$\boxed{k = \frac{1}{6}}$

Q11

Q10 ~~P(A~~ $P(A) = \frac{3}{6} = \frac{1}{2}$ $\left[\because P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}\right]$

$P(B) = \frac{3}{6} = \frac{1}{2}$

[⇒ sample space $S = \{1r, 2r, 3r, 4g, 5g, 6g\}$
$A = \{2r, 4g, 6g\}$ $A \cap B = \{2r\}$
$B = \{1r, 2r, 3r\}$]

$P(A \cap B) = P(\text{no. which is even and red}) = \frac{1}{6}$ $\left[\because \text{No. which is even and red} = \{2r\}\right]$

$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Q12

$P(A \cap B) \neq P(A) \cdot P(B)$

$\therefore$ Events ~~are not~~ NOT INDEPENDENT.

Q11. $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \vec{a}.(\vec{b} \times \vec{c})$$

$$= \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = 2\hat{i} + 3\hat{j} + \hat{k}$
$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \hat{i} - 2\hat{j} + \hat{k}$
$\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k} = -3\hat{i} + \hat{j} + 2\hat{k}$

$$= \begin{bmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{bmatrix}$$

$$= 2(-4-1) - 3(2+3) + 1(1-6)$$

$$= -10 - 15 - 5$$

$$\therefore \boxed{[\vec{a}\ \vec{b}\ \vec{c}] = -30}$$

Q12. $I = \int \dfrac{\tan^2 x \, . \sec^2 x}{1 - \tan^6 x} \, dx$

let $\tan^3 x = t$

$3\tan^2 x \sec^2 x \, dx = dt$

Q13.

$$\cancel{I = \int \frac{t^2}{1-t^6}\,dt} = \cancel{\int \frac{t^2}{1-(t^3)^2}\,dt}$$

~~let $t^3 = m$~~

~~$3t^2 dt$~~

$$I = \frac{1}{3}\int \frac{dt}{(1)^2 - t^2}$$

$$= \frac{1}{3}\cdot\frac{1}{2.(1)}\log\left|\frac{1+t}{1-t}\right| + C \qquad \left[\because \int \frac{dx}{a^2-x^2} = \log\left|\frac{a+x}{a-x}\right| + c\right]$$

$$\boxed{I = \frac{1}{6}\log\left|\frac{1+\tan^3 x}{1-\tan^3 x}\right| + c}$$

SECTION-C

P.T.O

Q13. $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$

$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$, $AB < 1$

$\Rightarrow \tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$ and $(2x)(3x) < 1$

$6x^2 < 1$

$-\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$

taking tan on both side

$\Rightarrow \frac{5x}{1-6x^2} = 1$

$\Rightarrow 6x^2 + 5x - 1 = 0$

$\Rightarrow 6x^2 + 6x - x - 1 = 0$

$\Rightarrow (6x-1)(x+1) = 0$

$\Rightarrow x = \frac{1}{6}$ or $x = -1$

$\therefore x = -1$ is not a solution $\left[\because -1 \notin \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)\right]$

$\Rightarrow \boxed{x = \frac{1}{6}}$ is ~~giv~~ solⁿ.

VERIFICATION

For $x = 1/6$

LHS: $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{5}{5}\right) = \pi/4 = $ RHS.

LHS = RHS

$\Rightarrow x = 1/6$ is a sol^n

For $(x) = -1$

$\tan^{-1}(-2) - \tan^{-1}(-3) = \frac{\pi}{2} - \cot^{-1}(-2) - \frac{\pi}{2} + \cot^{-1}(-3)$

$= \pi - \cot^{-1}(3) - \pi + \cot^{-1}(2)$

$= \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}\right) = \tan^{-1}\left(\frac{1}{7}\right) \neq$ RHS

LHS $\neq$ RHS

$\therefore x = -1$ is NOT sol^n

Q14. $\log(x^2+y^2) = 2\tan^{-1}\frac{y}{x}$.

Diff. both sides w.r.t x.

$\frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right) = \frac{2}{1+\frac{y^2}{x^2}} \cdot \frac{d}{dx}\left(\frac{y}{x}\right)$ $\left[\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}; \frac{d}{dx}(\log x) = \frac{1}{x}\right]$

$\Rightarrow \frac{x + yy'}{x^2+y^2} = \frac{x^2}{x^2+y^2}\frac{(xy'-y)}{x^2}$ [where $y' = \frac{dy}{dx}$]

$\Rightarrow x + yy' = xy' - y$

$\Rightarrow x + y = y'(x-y)$

Q15

$$\Rightarrow \quad y' = \frac{x+y}{x-y}$$

$$\boxed{\frac{dy}{dx} = \frac{x+y}{x-y}}$$

Hence proved.

Q15 $\quad I = \int \frac{3x+5}{x^2+3x-18}\,dx$

$$3x+5 = a\left[\frac{d}{dx}(x^2+3x-18)\right] + b$$

$$= a(2x+3) + b$$

comparing coefficients on both sides,

$3 = 2a \qquad\qquad 3a+b = 5$

$\boxed{a = \frac{3}{2}} \qquad\qquad b = 5 - 3\left(\frac{3}{2}\right) = \frac{1}{2} \quad \Rightarrow \boxed{b = \frac{1}{2}}$

P.T.O

$$I = \int \frac{\frac{3}{2}(2x+3) + \frac{1}{2}}{x^2+3x-18}\,dx$$

$$= \frac{3}{2}\int \frac{(2x+3)}{x^2+3x-18}\,dx + \frac{1}{2}\int \frac{dx}{x^2+3x-18}$$

$$= \frac{3}{2} I_1 + \frac{1}{2} I_2 .$$

$$I_1 = \int \frac{2x+3}{x^2+3x-18}\,dx \qquad \cancel{I_2 = \int}$$

let $x^2+3x-18 = t$

$(2x+3)\,dx = dt$

$$I_1 = \int \frac{dt}{t} = \log|t| + c_1$$

$$I_1 = \log|x^2+3x-18| + c_1$$

Q16

Ans, $$I_2 = \int \frac{dx}{x^2+3x-18} = \int \frac{dx}{x^2+3x+\left(\frac{3}{2}\right)^2-18-\left(\frac{3}{2}\right)^2}$$

$$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2-18-\frac{9}{4}} = \int \frac{dx}{\left(x+\frac{3}{2}\right)^2-\frac{81}{4}}$$

$$I_1 = -\int_a^0 f(a-t)\,dt$$

$$= \int_0^a f(a-t)\,dt \qquad \left[\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right]$$

$$I_1 = \int_0^a f(a-x)\,dx \qquad \left[\because \int_a^b f(t)\,dt = \int_a^b f(x)\,dx\right]$$

LHS = RHS

Hence proved

$$I = \int_0^\pi \frac{x \sin x}{1+\cos^2 x}\,dx \quad \text{---(1)}$$

$$I = \int_0^\pi \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}\,dx \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$I = \int_0^\pi \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx \quad \text{---(2)}$$

Q17

Adding eqn (1), (2)

$$2I = \int_0^\pi \frac{\pi \sin x}{1+\cos^2 x}\,dx$$

$$I_2 = \int \frac{dx}{\left(x+\frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2}$$

$$= \frac{1}{2 \cdot \frac{9}{2}} \log \left| \frac{\left(x+\frac{3}{2}\right) - \frac{9}{2}}{\left(x+\frac{3}{2}\right) + \frac{9}{2}} \right| + c_2$$

$$= \frac{1}{9} \log \left| \frac{x-3}{x+6} \right| + c_2$$

$$I = \frac{3}{2} \log |x^2+3x-18| + c_1 + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + c_2.$$

$$I = \frac{3}{2} \log |x^2+3x-18| + \frac{1}{18} \log \left| \frac{x-3}{x+6} \right| + c$$ where $c = c_1 + c_2 =$ constt.

Q16 T.P: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

LHS : $I_1 = \int_0^a f(x)\,dx$

let $x = a-t$ When $x=0$ $t=a$

$dx = -dt$ $x=a$ $t=0$

$$\Rightarrow 2I = \pi\int_0^{\pi} \frac{\sin x \, dx}{1+\cos^2 x}$$

let $\cos x = t$ when $x = 0$ $t = 1$

$-\sin x \, dx = dt$ $x = \pi$ $t = -1$

$$\Rightarrow I = \frac{\pi}{2}\int_1^{-1} \frac{-dt}{1+t^2}$$

$$= -\frac{\pi}{2}\left[\tan^{-1} t\right]_1^{-1} = -\frac{\pi}{2}\left[-\frac{\pi}{4} - \frac{\pi}{4}\right] = \frac{\pi}{2}\cdot\frac{\pi}{2}$$

$$\boxed{I = \frac{\pi^2}{4}}$$

$$\therefore \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} = \frac{\pi^2}{4}$$

Q17

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$

$\vec{B} = 2\hat{i} + 5\hat{j} + 0\hat{k}$

$\vec{C} = 3\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{D} = \hat{i} - 6\hat{j} - \hat{k}$

Q.18.

$\vec{AB}$ = Position vector of $\vec{B}$ − Position vector of $\vec{A}$

$\vec{m} = \hat{i} + 4\hat{j} - \hat{k}$

$\vec{CD}$ = Position vector of $\vec{D}$ − Position vector of $\vec{C}$

$\vec{n} = -2\hat{i} - 8\hat{j} + 2\hat{k}$

Angle b/w $\vec{AB}$ and $\vec{CD} = \theta$

$$\cos\theta = \frac{\vec{m}.\vec{n}}{|\vec{m}|\,|\vec{n}|}$$

$$= \frac{m_1n_1 + m_2n_2 + m_3n_3}{\sqrt{m_1^2+m_2^2+m_3^2}\,\sqrt{n_1^2+n_2^2+n_3^2}}$$

$$= \frac{-2-32-2}{\sqrt{1+16+1}\,\sqrt{4+64+4}}$$

$$= \frac{-36}{\sqrt{18}\,\sqrt{72}} = \frac{-36}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{18\times2}$$

$$= \frac{-36}{36} = -1$$

$\theta = \cos^{-1}(-1) \Rightarrow \boxed{\theta = \pi}$

∴ Since, $\vec{AB}$ and $\vec{CD}$ are antiparallel,

they ARE COLLINEAR.

Q18.

$$\Delta = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix},$$

$$\Delta = \begin{vmatrix} a+b & a+b & -(a+b) \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$ $R_1 \to R_1 + R_2$

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$ Taking $(a+b)$ common from R_1

$$= (a+b)\begin{vmatrix} 1 & 1 & -1 \\ -(b+c) & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix}$$ $R_2 \to R_2 + R_3$

$$= (a+b)(b+c)\begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix}$$ Taking $(b+c)$ common from R_2

$$= (a+b)(b+c)\begin{vmatrix} 0 & 0 & -1 \\ 0 & 2 & 1 \\ a+c & b+c & a+b+c \end{vmatrix}$$ $C_1 \to C_1 + C_3$

$C_2 \to C_2 + C_3$

$$\Delta = (a+b)(b+c)(c+a) \begin{vmatrix} 0 & 0 & -1 \\ 0 & 2 & 1 \\ 1 & b+c & a+b+c \end{vmatrix}$$

Taking (a+c) common from C_1

Expanding along C_1

$\Delta = (a+b)(b+c)(c+a)(2)$

$\Delta = 2(a+b)(b+c)(c+a)$

Hence proved.

Q19 $y = \sin t$ $\qquad x = \cos t + \log(\tan(t/2))$

Diff. w.r.t t

$\boxed{\frac{dy}{dt} = \cos t}$ —(1)

Diff w.r.t ~~dt~~ t

$\frac{d^2y}{dt^2} = -\sin t$

$\left.\frac{d^2y}{dt^2}\right]_{t=\pi/4} = -\sin\frac{\pi}{4}$

$\boxed{\left.\frac{d^2y}{dt^2}\right]_{t=\pi/4} = -\frac{1}{\sqrt{2}}}$

Diff. w.r.t t

$\frac{dx}{dt} = -\sin t + \frac{1}{\tan t/2} \cdot \sec^2 t/2 \cdot \frac{1}{2}$

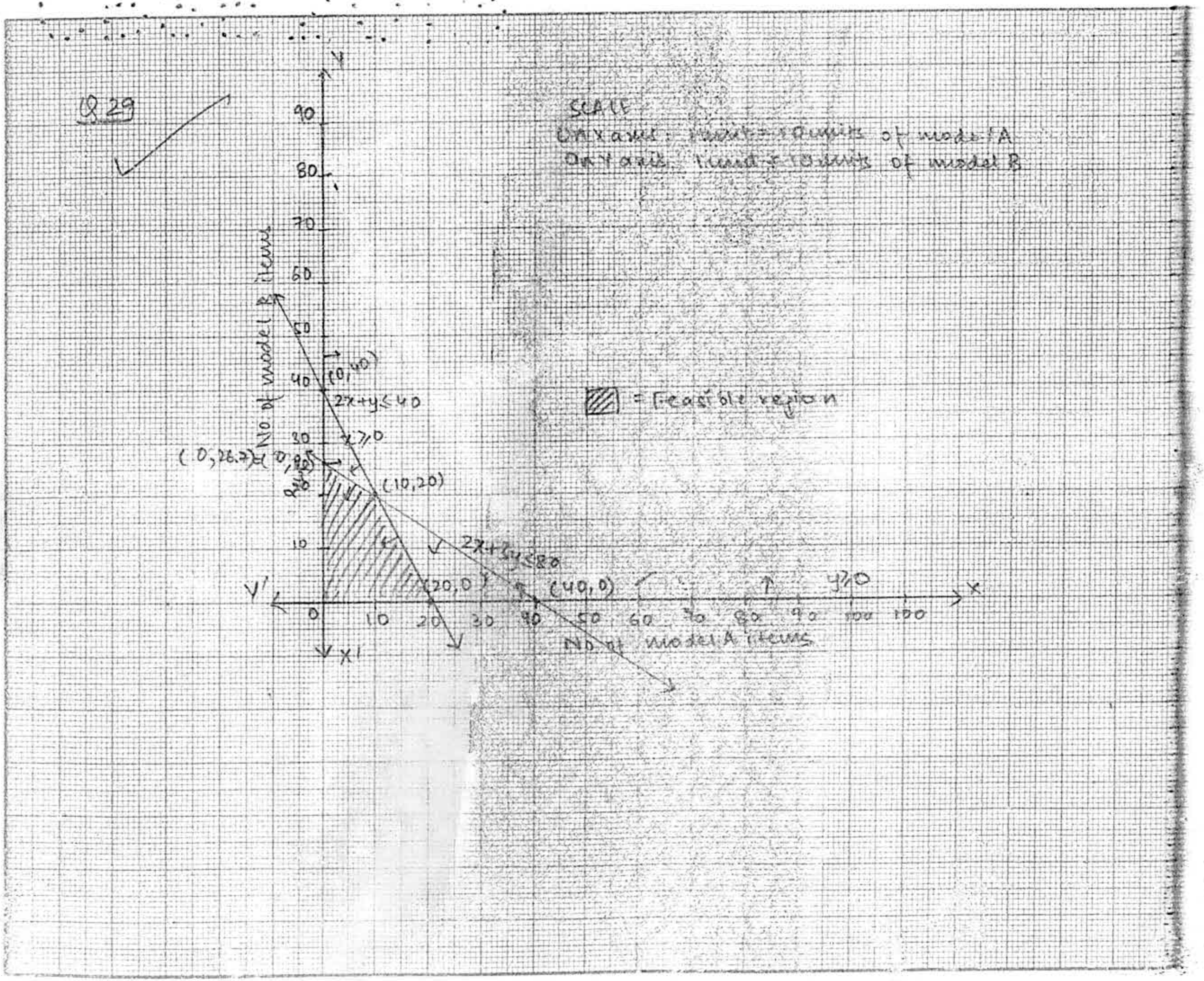
$= -\sin t + \frac{\cos t/2}{2\sin t/2 \cos^2 t/2}$

$= -\sin t + \frac{1}{\sin t}$ $\quad [\because 2\sin A\cos A = \sin 2A]$

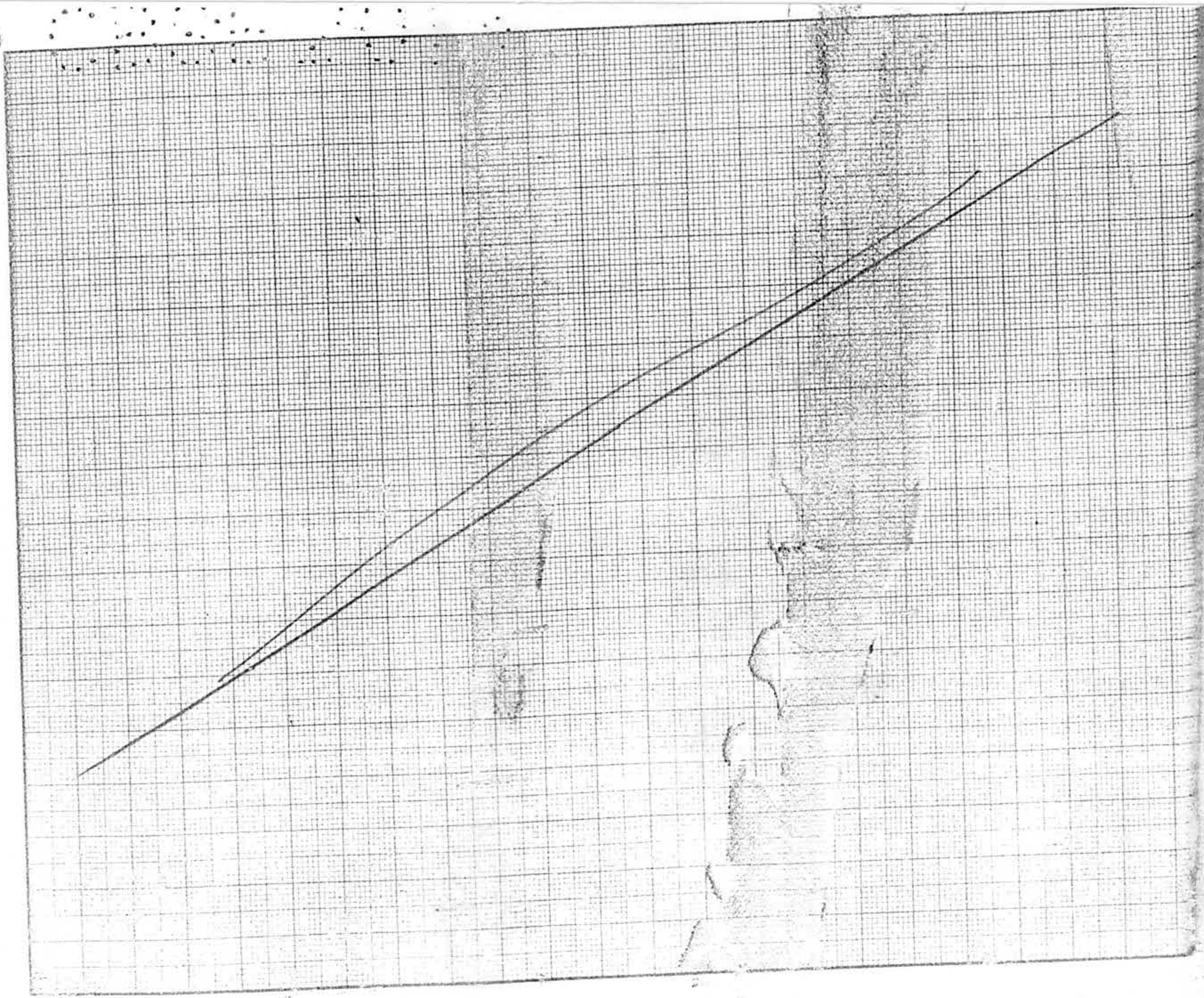
$\boxed{\frac{dx}{dt} = -\sin t + \frac{1}{\sin t}}$ $\Rightarrow \frac{dx}{dt} = \frac{\cos^2 t}{\sin t}$

(2)

P.T.O



Q 29
SCALE
= Feasible region
Y
X
Y'
X'
No. of model B items
No. of model A items
(0,40)
$2x+y\leq 40$
$x\geq 0$
(0, 26.7)
(10,20)
$2x+3y\leq 80$
(20,0)
(40,0)
$y\geq 0$



Q.19 Contd...

~~Dif~~ Divide eqn (1) by (2)

$$\frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t + \frac{1}{\sin t}}$$

$$\frac{dy}{dx} = \frac{\cos t . \sin t}{1 - \sin^2 t} = \frac{\sin t}{\cos t} = \tan t$$

Diff. w.r.t x.

$$\frac{d^2y}{dx^2} = \frac{d(\tan t)}{dt} \frac{dt}{dx}$$

$$= \sec^2 t . \frac{1}{dx/dt} = \frac{\sec^2 t}{-\sin t + \frac{1}{\sin t}}$$

$$= \frac{\sec^2 t . \sin t}{1 - \sin^2 t} = \frac{\sin t}{\cos^2 t . \cos^2 t} = \frac{\sin t}{\cos^4 t}$$

$$= \sin t . \sec^4 t$$

$$\left.\frac{d^2y}{dx^2}\right]_{t=\pi/4} = \frac{1}{\sqrt{2}} (\sqrt{2})^4 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\therefore \boxed{\left.\frac{d^2y}{dx^2}\right]_{t=\pi/4} = 2\sqrt{2}}$$

Q20. $R = \{(a,b) : a \leq b\}$

REFLEXIVE :

Every element $a \in \mathbb{R}$ is equal to itself

$\Rightarrow a = a$

$\Rightarrow a \leq a$ is true

$\therefore (a,a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of all Real nos

The relation is REFLEXIVE R = Relation.

TRANSITIVE :

For all $(a,b) \in R$ and $(b,c) \in R$

$a \leq b$ and $b \leq c$ where, $a, b, c \in \mathbb{R}$

$\Rightarrow a \leq c$

$\Rightarrow (a,c) \in R$

$\therefore$ The ~~set is tr~~ relation is TRANSITIVE

$\because$ for $(a,b), (b,c) \in R$, ~~$(a,c) \in R$~~ $(a,c) \in R$

Q21.

SYMMETRIC : For relation to be symmetric, for all $(a,b) \in R$, (b,a) should also exist in R

~~$a \leq b$~~ $\not\Rightarrow$ $a \leq b$

$b \nleq a$ → This relation is true only $a = b = 1$.

For eg: $\frac{1}{2} \leq 1 \Rightarrow \left(\frac{1}{2}, 1\right) \in R$

but $1 \nleq \frac{1}{2}$ ∴ $\left(1, \frac{1}{2}\right) \notin R$

∴ Relation is NOT SYMMETRIC

Q21

$y = \sqrt{3x-2}$

$\left.\frac{dy}{dx}\right]_{(x_1,y_1)} = \frac{3}{2\sqrt{3x_1-2}} = \frac{3}{2y_1} =$ slope of tangent = m

Acc. to ques, M = 2 [∵ Slope of $(4x-2y+5=0)$ is $m_1 = 2$. and ||el line have equal slope]

$\Rightarrow \frac{3}{2y_1} = 2 \Rightarrow \boxed{y_1 = \frac{3}{4}}$

$\Rightarrow \frac{3}{2\sqrt{3x_1-2}} = 2 \Rightarrow 3 = 4\sqrt{3x_1-2}$

$\Rightarrow 9 = 16(3x_1-2)$ squaring both side

$\Rightarrow 3x_1 - 2 = \frac{9}{16} \Rightarrow x_1 = \frac{41}{48} = \frac{41}{48} \Rightarrow \boxed{x_1 = \frac{41}{48}}$

$x_1 = \frac{41}{48}$

$\Rightarrow y_1 = \sqrt{3x_1 - 2} = \sqrt{3 \times \frac{41}{48} - 2} = \sqrt{\frac{41-32}{16}} = \pm\frac{3}{4} = \left|\frac{3}{4}\right| = \sqrt{\frac{9}{16}}$

At $\boxed{\left(\frac{41}{48}, \frac{3}{4}\right)}$ and $\left(\frac{41}{48}, \frac{-3}{4}\right)$, ~~slope of~~ tangent is parallel to given line.

Not possible

(But $\because y \not< 0 \Rightarrow y \geq 0$)

EQUATION OF TANGENT

For $x_1 = 41/48$, $y_1 = 3/4$

$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$y - \frac{3}{4} = 2x - \frac{41}{24}$

$2x - y - \frac{41}{24} + \frac{18}{24} = 0$

$\Rightarrow 2x - y - \frac{23}{24} = 0$

$\Rightarrow \boxed{48x - 24y - 23 = 0}$

For $x_1 = 41/48$, $y_2 = -3/4$

$y + \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$

$2x - y - \frac{41}{24} - \frac{18}{24} = 0$

$\boxed{48x - 24y - 59 = 0}$

NOT POSSIBLE

X

Slope of normal $= \dfrac{-1}{\left[\frac{dy}{dx}\right]_{(x_1,y_1)}} = \dfrac{-1}{2}$

EQUATION OF NORMAL :

At $\left(\frac{41}{48}, \frac{3}{4}\right)$

$y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow y - \frac{3}{4} = \frac{-x}{2} + \frac{41}{96}$

$\Rightarrow y + \frac{x}{2} - \frac{3}{4} - \frac{41}{96} = 0$

$\Rightarrow y + \frac{x}{2} + \left(\frac{-72 - 41}{96}\right) = 0$

$\Rightarrow \boxed{96y + 48x - 113 = 0}$

~~At~~ At $\left(\frac{41}{48}, \frac{-3}{4}\right)$

$y + \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$

$\Rightarrow y + \frac{3}{4} + \frac{x}{2} - \frac{41}{96} = 0$

$\Rightarrow y + \frac{x}{2} + \frac{72 - 41}{96} = 0$

$\Rightarrow \boxed{96y + 48x + 31 = 0}$

NOT POSSIBLE

$\frac{24}{72}$

P.T.O

Q22 $(1+x^2)\dfrac{dy}{dx} + 2xy = 4x^2$

$$\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$$

It is linear DE of form $\dfrac{dy}{dx} + Py = Q$

$P = \dfrac{2x}{1+x^2}$ $Q = \dfrac{4x^2}{1+x^2}$

$$I.F = e^{\int P\,dx} = e^{\ln|1+x^2|} = 1+x^2$$

Sol^n of DE :

$$y(1+x^2) = \int \frac{4x^2(\cancel{1+x^2})}{\cancel{1+x^2}}dx + C$$

$$= \int 4x^2\,dx + C$$

$$\boxed{y(1+x^2) = \frac{4x^3}{3} + c}$$

Q23.

$x=0, y=0$

$\therefore \; c=0$

$\Rightarrow \boxed{3y(1+x^2)=4x^3}$

Q23

line I

$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2}$

$\Rightarrow \frac{x-1}{-3} = \frac{y-2}{\lambda/7} = \frac{z-3}{2} = \beta$

line II

$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$

$\frac{x-1}{\frac{-3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} = \mu$

Direction ratios are $\underset{(a_1, a_2, a_3)}{\left(-3, \frac{\lambda}{7}, 2\right)}$ and $\underset{(b_1, b_2, b_3)}{\left(\frac{-3\lambda}{7}, 1, -5\right)}$ respectively

For lines to be $\perp$, $\cancel{\vec{a_1}.\vec{a_2}=0}$ $\vec{a_1}.\vec{b}=0$ where ($\vec{a}, \vec{b}$ are ||el vectors of lines)

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

$\frac{9\lambda}{7} + \frac{\lambda}{7} - 10 = 0$

$\frac{10\lambda}{7} = 10$

$\boxed{\lambda = 7}$

$\therefore$ For $\lambda=7$, lines are $\perp$.

Any point of line I is $(-3\beta+1,\ \beta+2,\ 2\beta+3)$ Q24

line II is $(-3\mu+1,\ \mu+5,\ -5\mu+6)$

For lines to intersect, they should be equal.

$-3\beta+1 = -3\mu+1$ $\quad\quad$ $\mu+5=\beta+2$ $\quad\quad$ $2\beta+3 = -5\mu+6$

$3\mu-3\beta=0$ $\quad\quad$ $\mu-\beta+3=0$ — (3) $\quad\quad$ $2\beta+5\mu=3$ — (2)

$\mu=\beta$ — (1)

From (1), (2)

$7\mu=3$

$\boxed{\mu=3/7}$ $\boxed{\beta=3/7}$

In eqn(3)

$\frac{3}{7}-\frac{3}{7}+3 \neq 0$

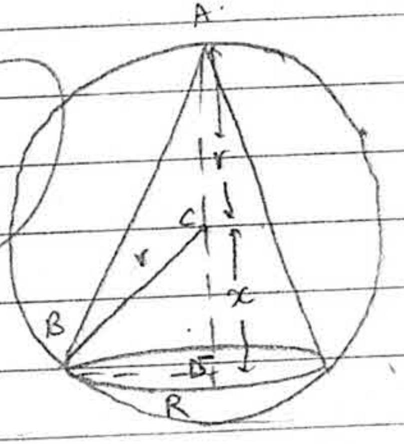
$\therefore$ Values donot satisfy eqn(3)

$\Rightarrow$ LINES doNOT intersect.

P.T.O

SECTION-D

Q24.



Consider a sphere of radius r.

Then, height of cone $= r + x = H$

Radius of cone $= \sqrt{r^2 - x^2} = R$

Volume of cone $= V = \frac{1}{3}\pi R^2 H$

$$V = \frac{1}{3}\pi (r^2 - x^2)(r + x)$$

$$= \frac{\pi}{3}(r^3 - rx^2 + xr^2 - x^3).$$

$$\frac{dV}{dx} = \frac{\pi}{3}\left(-2rx + r^2 - 3x^2\right)$$

For critical pt $\frac{dV}{dx} = 0$

$$r^2 - 2rx - 3x^2 = 0$$

$$r^2 - 3rx + rx - 3x^2 = 0$$

$$(r + x)(r - 3x) = 0$$

x cannot be $-$ve $\Rightarrow \boxed{\frac{r}{3} = x}$ is critical pt

$$\frac{d^2V}{dx^2} = \frac{\pi}{3}(-2r-6x)$$

$$\left.\frac{d^2V}{dx^2}\right]_{x=r/3} = \frac{\pi}{3}[-2r-2r] = \frac{-4r\pi}{3} < 0$$

Using Second Derivative test, $\therefore \boxed{x = \frac{r}{3}}$ is a ~~m~~ point of maxima

$H = r + x$

$$\boxed{H = \frac{4r}{3}}$$ Hence proved

Max volume of cone $= V_{max} = \frac{\pi}{3}(r+x)(r^2-x^2)$

$$= \frac{\pi}{3}\left(\frac{4r}{3}\right)\left(r^2-\frac{r^2}{9}\right)$$

$$= \frac{4\pi r^3 \cdot 8}{81} = \frac{32\pi r^3}{81} \text{ (unit)}^3$$

$$\boxed{V_{max} = \frac{32\pi r^3}{81} \text{ (unit)}^3}$$

Q25.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\text{adj} A = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 2 & +23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1)$$
$$= 0 - 6 + 5 = -1$$

$$\cancel{tA} \quad A^{-1} = \frac{1}{|A|} \text{adj} A = -1 \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Given Eq^n:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Q26.

$AX = B$

~~Pre~~ Premultiply A^{-1} on both sides

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\frac{x}{y}$ $\boxed{x = 1}$

$\boxed{y = 2}$

$\boxed{z = 3}$

Q26 R. Ex

Q26. A = ~~the~~ Event of choosing a defective item.

E_1 = Item produced by A

E_2 = Item produced by B

E_3 = Item produced by C.

$P(E_1) = \frac{50}{100} = \frac{1}{2}$, $P(E_2) = \frac{30}{100} = \frac{3}{10}$, $P(E_3) = \frac{20}{100} = \frac{1}{5}$

$P\left(\frac{A}{E_1}\right) = \frac{1}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{5}{100} = \frac{5}{100}$; $P\left(\frac{A}{E_3}\right) = \frac{7}{100}$.

Using Baye's Thm,

$$A\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3).P(A/E_3)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{100}}{\frac{1}{2} \times \frac{1}{100} + \frac{3}{10} \times \frac{5}{100} + \frac{1}{5} \times \frac{7}{100}}$$

$$= \frac{\frac{1}{200}}{\frac{1}{200} + \frac{3}{200} + \frac{7}{500}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{3}{2} + \frac{7}{5}} = \frac{\frac{1}{2}}{2 + \frac{7}{5}} = \frac{\frac{1}{2}}{\frac{17}{5}}$$

$\boxed{P(E_1/A) = \frac{5}{34}}$ = Probability that defective item was produced by A

Q27. let $\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}$ of the required plane

Eqⁿ of plane is $A(x-2) + B(y-2) + C(z+1) = 0$ —— (3)

The plane also passes through $(3,4,2)$ & $(7,0,6)$

$$A + 2B + 3C = 0 \quad \text{— (1)}$$

$$5A - 2B + 7C = 0 \quad \text{— (2)}$$

Adding (1), (2)

$$6A + 10C = 0$$

$$\boxed{A = -\frac{5C}{3}}$$

In eqn(2) $-\frac{25C}{3} + 7C = 2B$

$$-\frac{4C}{2\times 3} = B$$

$$\Rightarrow \boxed{B = -\frac{2C}{3}}$$

$$\vec{n}_1 = -\frac{5C}{3}\hat{i} - \frac{2C}{3}\hat{j} + C\hat{k} = -\frac{C}{3}(5\hat{i} + 2\hat{j} - 3\hat{k})$$

Q29. let x be no. of A models
y be no. of B models.

Objective $Z = 15x + 10y$ (Maximize)
Subject to constraints:

$x, y \geq 0$

$2x + y \leq 40$ (Skilled man working hrs)

$2x + 3y \leq 80$ (Semi-skilled man working hrs)

$2x + y \leq 40$

$2x + y = 40$

x	0	20
y	40	0

Zero test : TRUE

$2x + 3y \leq 80$

$2x + 3y = 80$

x	10	40
y	20	0

Zero test : TRUE

GRAPH: On graph paper.

Eq^n of line BC:

$$(y-2) = \frac{5}{-2}(x-6)$$

$$\Rightarrow y-2 = -\frac{5}{2}x + 15$$

$$\Rightarrow \boxed{y_2 = -\frac{5}{2}x + 17}$$

Eq^n of line AC

$$(y-5) = \frac{3}{-4}(x-2)$$

$$y-5 = -\frac{3}{4}x + \frac{3}{2}$$

$$\boxed{y_3 = -\frac{3x}{4} + \frac{13}{2}}$$

Area of shaded region is required area A $= ar(ABDE) + ar(BCFD) - ar(ACFE)$

$$A = \int_2^4 y_1\,dx + \int_4^6 y_2\,dx - \int_2^6 y_3\,dx$$

$$= \int_2^4 (x+3)\,dx + \int_4^6 \left(-\frac{5x}{2}+17\right)dx - \int_2^6 \left(-\frac{3x}{4}+\frac{13}{2}\right)dx$$

$$A = \left[\frac{x^2}{2}+3x\right]_2^4 + \left[17x - \frac{5x^2}{4}\right]_4^6 - \left[\frac{13x}{2} - \frac{3x^2}{8}\right]_2^6$$

$$= \left[8+12-2-6\right] + \left[102-45-68+20\right] - \left[39-\frac{27}{2}-13+\frac{3}{2}\right]$$

$$= 12+9-14$$

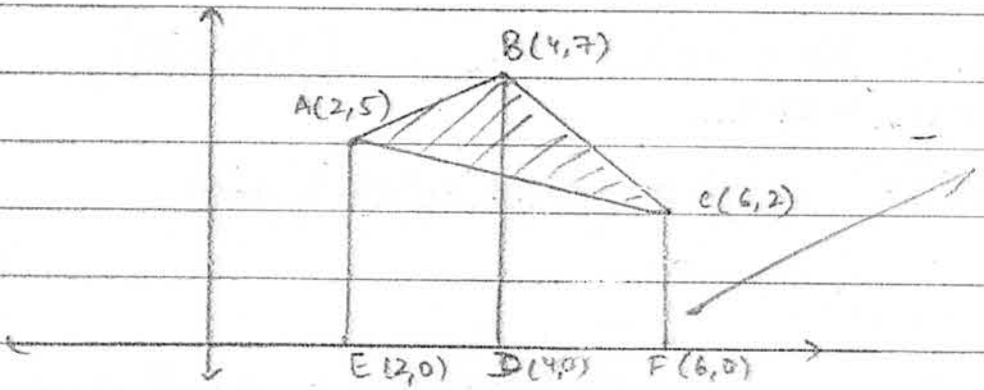
$$\boxed{A = 7 \text{ sq. units}}$$

$\therefore$ Eq^n of other plane is

$$\boxed{\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k}) = 23} \rightarrow \text{vector eq}^n \Rightarrow \boxed{\vec{r}\cdot(5\hat{i}+2\hat{j}-3\hat{k})-23=0}$$

$$5x+2y-3z=23 \rightarrow \text{cartesian eq}^n$$

Q28.



Eq^n of general line:

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Eq^n of line AB : ~~Eq^n of li~~

$$(y-5) = \frac{2}{2}(x-2)$$

$$\boxed{y = x+3}$$

Direction ratios of normal to the plane are

$$\left(\frac{-5c}{3}, \frac{-2c}{3}, c\right) \equiv \cancel{(5c, 2c, -3c)}\ (5, 2, -3)$$

$$\therefore\ \vec{n} = 5\hat{i} + 2\hat{j} - 3\hat{k}$$

Eq^n: of plane: $5(x-2) + 2(y-2) - 3(z+1) = 0$ [From(3)]

$$5x + 2y - 3z - 10 - 4 - 3 = 0$$

$\boxed{5x + 2y - 3z = 17}$ is cartesian eq^n of plane.

VECTOR EQN:

$\boxed{\vec{r}.\vec{n} = d} \Rightarrow \left[\vec{r} - (2\hat{i} + 2\hat{j} - \hat{k})\right].\vec{n} = 0$ [$\because (\vec{r} - \vec{a}).\vec{n} = 0$ is eq^n of plane]

$$\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) - (2\hat{i} + 2\hat{j} - \hat{k})(5\hat{i} + 2\hat{j} - 3\hat{k}) = 0$$

$\Rightarrow \boxed{\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17}$ is vector eq^n of plane.

For plane parallel to above plane, $\vec{n_2} = \vec{n} = 5\hat{i} + 2\hat{j} - 3\hat{k}$

$$(\vec{r} - \vec{a_1}).\vec{n} = 0$$

here $\vec{a_1} = 4\hat{i} + 3\hat{j} + \hat{k}$

$$\left[\vec{r} - (4\hat{i} + 3\hat{j} + \hat{k})\right].(5\hat{i} + 2\hat{j} - 3\hat{k}) = 0$$

$$\vec{r}(5\hat{i} + 2\hat{j} - 3\hat{k}) - (20 + 6 - 3) = 0$$

Corner pt	$Z = 15x + 10y$
$(0,0)$	$Z = 0$
$\cancel{(80/3, 0)}$ $(0, 80/3)$	$Z = \cancel{400}\ \cancel{0+}\ 800/3 \simeq 266.7 \simeq 267$
$(10, 20)$	$Z = 150 + 200 = 350$ (MAX)
$(20, 0)$	$Z = 300$

$\therefore$ No. of model A = 10

No. of model B = 20

Maximum profit = ₹ 350