Assignment-I

1. Show that he modulus of the difference of two complex numbers can never be less than the difference of their moduli.
2. Prove that
$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + |z_2|^2.$$
Interpret the result geometrically and deduce that
$$|\alpha + \sqrt{\alpha^2 - \beta^2}| + |\alpha - \sqrt{\alpha^2 - \beta^2}| = |\alpha + \beta| + |\alpha - \beta|.$$
3. Show that the triangle whose vertices are the points z_1, z_2 and z_3 is equilateral if
$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1.$$
4. Find the radius of convergence of the power series :
   (a) $f(z) = \sum \dfrac{z^n}{n!}$.
   (b) $f(z) = \sum \left(1 + \dfrac{1}{n}\right)^{n^2} z^n$.
5. Find the domain of convergence of the following series :
   (a) $\sum \left(\frac{iz-1}{2+i}\right)^n$.
   (b) $\sum 2^{-n} z^{2n}$.
6. Find all values of z such that $e^z = 1 + i$.
7. Find the principal value of i^i.
8. Prove that $\operatorname{Re} z > 0$ and $|z - 1| < |z + 1|$ are equivalent statements.
9. Find the modulus and argument of the complex number
$$(1 + \cos\alpha) + i \sin\alpha, \quad 0 < \alpha < \pi/2.$$
10. Find the complex number z if $\arg(z + 1) = \pi/6$ and $\arg(z - 1) = 2\pi/3$.

Answers :

4. (a) $R = \infty$, (b) $R = 1/e$.
5. (a) $|z+i| < \sqrt{5}$, (b) $|z| < \sqrt{2}$.
6. $z = \frac{1}{2}\ln 2 + i\left(2n + \frac{1}{4}\right)\pi \quad \text{where} \quad n \in I$.
7. $e^{-\pi/2}$.
9. $2\cos(\alpha/2)$ and $\alpha/2$.
10. $z = (1+\sqrt{3}i)/2$.