Assignment-II

1. Use the Cauchy-Riemann equations to prove that $f(z) = |z|^2$ is analytic nowhere.
2. Find the image of the infinite strip $1/4 \leq y \leq 1/2$ under the transformation $w = 1/z$.
3. Discuss the transformation $w = e^z$.
4. Explain the difference between an analytic function and differentiable function.
5. Determine the analytic function $f(z) = u + iv$ if $v = \ln[(x-1)^2 + (y-2)^2]$.
6. Use the Cauchy-Riemann equations to prove that $f(z) = 1/z$, $z \neq 0$ is analytic at all points except at the origin.
7. If $f(z) = u + iv$ is an analytic function of z, and $u - v = (x-y)(x^2 + 4xy + y^2)$, then find $f(z)$.
8. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied there.
9. Prove that $f(z) = \sin z$ is analytic everywhere.
10. If $f(z)$ is an analytic function of z, then show that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2.$$

Answers :

2. $u^2 + v^2 \leq -4v \leq 2(u^2 + v^2)$
5. $f(z) = 2i \ln\{z - 1 - 2i\} + c$.
7. $f(z) = 3x^2y - y^3 + i(3xy^2 - x^3) + c$.