QUIZ-2

Total Marks: 20

Q Show that:

$$Var(\hat{\beta}_{0,OLS}) = \frac{\sigma^2}{N}\left(\frac{\sum_{i=1}^{N} x_i^2}{\sum_{i=1}^{N}(x_i-\bar{x})^2}\right)$$

A Note that subscript x reflects that x is GIVEN.

i.e; $V_x(\hat{\beta}_{0,OLS}) = V(\hat{\beta}_{0,OLS}|x)$.

$$V_x(\hat{\beta}_{0,OLS}) = V_x(\bar{y} - \hat{\beta}_1\bar{x})$$

$$= \underbrace{V_x(\bar{y})}_{\text{I}} + \underbrace{\bar{x}^2 V_x(\hat{\beta}_1)}_{\text{II}} - \underbrace{2\bar{x}\, cov(\hat{\beta}_1, \bar{y})}_{\text{III}}$$

$(\because V(ax) = a^2V(x))$

$$\text{I} = V_x\left(\frac{\sum_{i=1}^{N} y_i}{N}\right)$$

7 marks

$$= \frac{1}{N^2}\sum_{i=1}^{N} V_x(y_i).$$

$$= \frac{1}{N^2}\sum_{i=1}^{N}\sigma^2. \qquad (\because V(y_i) = \sigma^2 \;\forall i)$$

$$= \frac{1}{N^2}(N\sigma^2)$$

$$= \sigma^2/N.$$

II $= \bar{x}^2 \cdot V_x(\hat{\beta}_1)$.

6 marks

$= \bar{x}^2 \left(\dfrac{\sigma^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2} \right)$ (from class notes).

7 marks

III $= 2\bar{x} \operatorname{cov}(\hat{\beta}_1, \bar{y})$.

$= 2\bar{x}\left(E\left[\left(\hat{\beta}_{1,OLS} - E(\hat{\beta}_{1,OLS}) \right) \left(\bar{y} - E(\bar{y}) \right) \right] \right)$

$\boxed{\because \operatorname{cov}(X,Y) = E[(X-E(X))(Y-E(Y))]}$

$= 2\bar{x}\, E\left[\left(\hat{\beta}_{1,OLS} - \beta_1 \right) \underbrace{(\bar{y} - \bar{y})}_{0} \right] = 0$

($\because \hat{\beta}_{1,OLS}$ is unbiased. & expectation of a constant is the constant itself.)

So, $V(\hat{\beta}_{0,OLS}) =$ I + II − III.

$= \dfrac{\sigma^2}{N} + \dfrac{\bar{x}^2 \sigma^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$.

$= \dfrac{\sigma^2}{N \sum_{i=1}^{N} (x_i - \bar{x})^2} \left(\sum_{i=1}^{N} (x_i - \bar{x})^2 + N\bar{x}^2 \right)$

$$= \frac{\sigma^2}{N} \frac{1}{\sum_{i=1}^{N}(x_i - \bar{x})^2} \left(\underbrace{\sum_i x_i^2 - \cancel{N\bar{x}^2}} + \cancel{N\bar{x}^2} \right)$$

[from Ⓐ in Quiz-1 soln (8)]

$$\boxed{V_x(\hat{\beta}_{0,OLS}) = \frac{\sigma^2}{N} \frac{\sum_{i=1}^{N} x_i^2}{\sum_{i=1}^{N}(x_i - \bar{x})^2}}$$