**Total Marks: 20**

# QUIZ - 1



Q Based on the 2 moment restrictions due to assumptions of the simple linear regression model; please show that :

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$$

A. A1: $E(u_i) = 0 \Rightarrow \frac{1}{N} \sum_{i=1}^{N} (y_i - \beta_0 - \beta_1 x_i) = 0 \mapsto$ 1.

A2: $E(u_i | x_i) = 0. \Rightarrow \frac{1}{N} \sum_{i=1}^{N} (y_i x_i - \beta_0 x_i - \beta_1 x_i^2) = 0 \mapsto$ 2.

2 equations

2 unknowns.

**15 marks**



From - 1 :

$$\frac{\sum_{i=1}^{N} y_i}{N} - \frac{\sum_{i=1}^{N} \hat{\beta}_0}{N} - \hat{\beta}_1 \sum_{i=1}^{N} x_i / N = 0.$$

$$\bar{y} - \frac{\cancel{N}\hat{\beta}_0}{\cancel{N}} - \hat{\beta}_1 \bar{x} = 0.$$

$$\therefore \boxed{\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}} \rightarrow 3.$$

From 2 :

$$\frac{\sum_{i=1}^{N} x_i y_i}{N} - \hat{\beta}_0 \frac{\sum_{i=1}^{N} x_i}{N} - \hat{\beta}_1 \frac{\sum_{i=1}^{N} x_i^2}{N} = 0$$

$$\frac{\sum_{i=1}^{N} x_i y_i}{N} - \hat{\beta}_0 \bar{x} - \hat{\beta}_1 \frac{\sum_{i=1}^{N} x_i^2}{N} = 0$$

Substitute 3 :

$$\frac{\sum_{i=1}^{N} x_i y_i}{N} - \bar{x}(\bar{y} - \hat{\beta}_1 \bar{x}) - \hat{\beta}_1 \frac{\sum_{i=1}^{N} x_i^2}{N} = 0$$

$$\frac{\sum_{i=1}^{N} x_i y_i}{N} - \bar{x}\bar{y} + \hat{\beta}_1 \bar{x}^2 - \hat{\beta}_1 \frac{\sum_{i=1}^{N} x_i^2}{N} = 0.$$

$$\sum_{i=1}^{N} x_i y_i - N\bar{x}\bar{y} = \hat{\beta}_1 \Big( \underbrace{\sum_{i=1}^{N} x_i^2 - N\bar{x}^2}_{N \cdot V(x)} \Big)$$

B

A

$$\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y}) = \hat{\beta}_1 \left( \sum_{i=1}^{N} (x_i - \bar{x})^2 \right)$$

$$\therefore \boxed{\hat{\beta}_1 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y - \bar{y})}{\sum_{i=1}^{N} (x_i - \bar{x})^2}}$$

shown.

A :

$$\because V(x_i) = \sum_{i=1}^{N} (x_i - \bar{x})^2$$

$$= \sum_{i=1}^{N} (x_i^2 + \bar{x}^2 - 2x_i\bar{x})$$

$$= \sum_{i=1}^{N} x_i^2 + N\bar{x}^2 - 2\bar{x}\sum_{i=1}^{N} x_i$$

$$= \sum_{i=1}^{N} x_i^2 + N\bar{x}^2 - 2N\bar{x}^2$$

$$= \sum_{i=1}^{N} x_i^2 - N\bar{x}^2$$

B :

$$\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})$$

$$= \sum_{i=1}^{N} (x_i y_i - \bar{y} x_i - \bar{x} y_i + \bar{x}\bar{y})$$

$$= \sum_{i=1}^{N} x_i y_i - \bar{y} \sum_{i=1}^{N} x_i - \bar{x} \sum_{i=1}^{N} y_i + N\bar{x}\bar{y}$$

$$= \sum_{i=1}^{N} x_i y_i - N\bar{y}\bar{x} - \cancel{N\bar{x}\bar{y}} + \cancel{N\bar{x}\bar{y}}$$

$$= \sum_{i=1}^{N} x_i y_i - N\bar{x}\bar{y}$$

**A & B: 2.5 marks each.**