Lecture 17

Econometrics-I

30.03.2021

Multiple linear regression model.

$$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \cdots + \beta_k x_{ik} + u_i$$

Test $H_0: \beta_3 = c$ (a constant)

$H_a: \beta_3 \neq c$

two-sided test

$$t = \frac{\hat{\beta}_{3,ols} - c}{s.e(\hat{\beta}_{3\,ols})} \sim t_{N-k}$$

where N: # obs.

K: # regressors

Inference: $\alpha = 5\%$ or 0.05

① if $t > t^*_{N-k,\,0.025}$ then reject H_0.

$\Rightarrow \beta_3$ is statistically different than c @ 5% significance level or @ 95% confidence.

② if $t \leq t^*_{N-k,\,0.025}$ then fail to reject H_0.

$\Rightarrow \beta_3$ is statistically NOT different from c @ 5% significance

Instead, if we were to test:

one-sided test. $\begin{cases} H_0 : \beta_3 = c \\ H_a : \beta_3 > c \end{cases}$ OR $H_0 : \beta_3 = c$, $H_a : \beta_3 < c$

$$t = \frac{\hat{\beta}_3 - c}{s.e.(\hat{\beta}_3)} \sim t_{N-k}.$$

Inference : $\alpha = 5\%$; or 0.05

① If $t > t^*_{N-k,\,0.05}$ then reject H_0.

$\Rightarrow \beta_3$ is statiscally different from c @ 5% significance.

② If $t \leq t^*_{N-k,\,0.05}$ then fail to reject H_0.

$\Rightarrow \beta_3$ is NOT statistically diff from c @ 5%. significance or 95% confidence.

* Testing a linear restriction on model coefficients

Example

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i$$

$$H_0: \ \beta_1 - 2\beta_3 = 6$$

$$H_a: \ \beta_1 - 2\beta_3 \neq 6$$

$$\underline{\hat{\beta}}_{OLS} = \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix} = (X'X)^{-1} X' \underline{Y}$$

$$\underbrace{V(\hat{\beta}_{OLS})}_{K \times K} = \underbrace{\begin{bmatrix} V(\hat{\beta}_1) & Cov(\hat{\beta}_1, \hat{\beta}_2) & Cov(\hat{\beta}_1, \hat{\beta}_3) \\ Cov(\hat{\beta}_2, \hat{\beta}_1) & V(\hat{\beta}_2) & Cov(\hat{\beta}_2, \hat{\beta}_3) \\ & & V(\hat{\beta}_3) \end{bmatrix}}_{3 \times 3}$$

3x3 for our example

variance-covariance matrix for OLS coefficients

$$= \sigma^2 (X'X)^{-1}$$ (see your lecture notes).

Define : $\beta_1 - 2\beta_3 = \gamma$

Test $H_0 : \gamma = 6$
$H_a : \gamma \neq 6$

$$t_\gamma = \frac{\hat{\gamma} - 6}{s.e.(\hat{\gamma})} \sim t_{N-K}.$$

$$t_\gamma = \frac{\hat{\gamma} - 6}{s.e.(\hat{\gamma})} = \frac{\hat{\beta}_1 - 2\hat{\beta}_3 - 6}{s.e.(\hat{\beta}_1 - 2\hat{\beta}_3)} \sim t_{N-K}$$

Note : we use $\hat{\sigma}^2$ instead of σ^2.

$$s.e.(\hat{\beta}_1 - 2\hat{\beta}_3) = \sqrt{var(\hat{\beta}_1 - 2\hat{\beta}_3)}$$

$$= \underbrace{\sqrt{v(\hat{\beta}_1) + 4v(\hat{\beta}_3) - 4Cov(\hat{\beta}_1, \hat{\beta}_3)}}_{\text{to be evaluated using data}}$$

Beyond this point, inference follows like before.

Also, see that when we defined

$$\underset{1\times 1}{\hat{\gamma}} = \underbrace{\hat{\beta}_1 - 2\hat{\beta}_3}_{\text{scalar entity}} = \underset{1\times 3}{\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{bmatrix}}$$

$$= \underline{a}' \underline{\hat{\beta}}$$

$$\underset{1\times 3}{\underline{a}'} = [1\ 0\ -2] \quad ; \quad \underset{3\times 1}{\underline{a}} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\underset{1\times 1}{\hat{\gamma}} = \underset{1\times 3}{\underline{a}'} \underset{3\times 1}{\underline{\hat{\beta}}_{ols}}$$

$$\underbrace{V(\hat{\gamma})}_{1\times 1} = \underbrace{\underbrace{\underset{1\times 3}{\underline{a}'} \underset{3\times 3}{V(\underline{\hat{\beta}}_{ols})}}_{1\times 3} \underset{3\times 1}{\underline{a}}}_{1\times 1}$$

[refer to notes for MLRM].

scalar entities on L.H.S.

We can generalize the tests for linear restrictions:

Given:

MLRM $\Rightarrow$ K regressor $\Rightarrow$ K coefficients; N observation

Test:

$$H_0: \gamma_{11}\beta_1 + \gamma_{12}\beta_2 + \gamma_{13}\beta_3 + \dots + \gamma_{1K}\beta_K = \gamma_1$$

$$H_a: \gamma_{11}\beta_1 + \gamma_{12}\beta_2 + \gamma_{13}\beta_3 + \dots + \gamma_{1K}\beta_K \neq \gamma_1$$

Aside: A real world example on Hypothesis testing

Demand for sugar $\to S_i \qquad i = 1, 2, \dots, N$

$$S_i = \beta_0 + \beta_1 P_i^S + \beta_2 P_i^J + \beta_3 W_i + \beta_4 I_i + \beta_5 P_i^T + u_i$$

Various tests:

$H_0: \beta_1 = 0$	$H_0: \beta_2 = 0$	$H_0: \beta_5 = 0$	$H_0: \beta_1 = -\beta_2 \equiv \beta_1 + \beta_2 = 0$
$H_a: \beta_1 < 0$	$H_a: \beta_2 > 0$	$H_a: \beta_5 < 0$	$H_a: \beta_1 < -\beta_2 \equiv \beta_1 + \beta_2 < 0$
Demand theory	Demand theory	Demand theory	Assignment to a consultant by a retailer

Back to the notes.

MLRM : K-regressors ; ⇒ K-coefficients ; N observation

Test :

$$\left.\begin{aligned} H_0 &: r_{11}\beta_1 + r_{12}\beta_2 + r_{13}\beta_3 + \cdots + r_{1K}\beta_K = r_1 \\ H_a &: r_{11}\beta_1 + r_{12}\beta_2 + r_{13}\beta_3 + \cdots + r_{1K}\beta_K \neq r_1 \end{aligned}\right\}$$

Define: $$t = \frac{\hat{r}_1 - r_1}{s.e.(\hat{r}_1)} \sim t_{N-K}.$$

$$\underset{1\times1}{\hat{r}_1} = \underset{1\times K}{\begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1K} \end{bmatrix}} \underset{K\times1}{\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \\ \vdots \\ \hat{\beta}_K \end{bmatrix}} = \underbrace{\underset{1\times K}{\underline{a}'}\ \underset{K\times1}{\hat{\underline{\beta}}}}_{1\times1}$$

$$\underset{1\times1}{V(\hat{r}_1)} = \underset{1\times K}{\underline{a}'}\ \underset{K\times K}{V(\hat{\underline{\beta}}_{OLS})}\ \underset{K\times1}{\underline{a}}$$

→ Note that we use $\hat{\sigma}^2$ to evalue $V(\hat{\beta}_{OLS})$. Hence $\sqrt{V(\hat{r}_1)} \equiv s.e.(\hat{r}_1)$.

Next,

Testing a set of linear restrictions

Example 1

$H_0: \beta_1 = 0$ and $\beta_2 = 0$

$H_a: \beta_1 \neq 0$ OR $\beta_2 \neq 0$

joint testing

Q: How is above different than

$\tilde{H}_0: \beta_1 = 0$; $\tilde{\tilde{H}}_0: \beta_2 = 0$

$\tilde{H}_a: \beta_1 \neq 0$; $\tilde{\tilde{H}}_a: \beta_2 \neq 0$

?

fundamentally

joint probability

$Pr(\hat{\beta}_1 = 0$ and $\hat{\beta}_2 = 0)$ is different

from $Pr(\hat{\beta}_1 = 0)$ and $Pr(\hat{\beta}_2 = 0)$

marginal probabilities.

Example 2:

H_0: $\beta_1 = 0$ and $\beta_2 = 0$ and $\beta_3 = 0$ and ... and $\beta_k = 0$

H_a: $\beta_1 \neq 0$ OR $\beta_2 \neq 0$ OR $\beta_3 \neq 0$ OR ... OR $\beta_k \neq 0$

corresponds to

F-test. for which we defined a

F-statistic in the ANOVA Table.

$$\boxed{F = \frac{MSE}{MSR} = \frac{SSE/K-1}{SSR/N-K} \sim F_{K-1,\,N-K}}$$

Next lecture: Begin w/ the most general case of testing a set of linear restriction and then show