Parameter estimation of Log-normal distribution using Maximum-likelihood methodology

Background

- The lognormal distribution is a two-parameter distribution that is useful in modeling continuous random variables which are greater than or equal to zero.
- Example scenarios in which the lognormal distribution is used include, among many others: in **medicine**, latent periods of infectious diseases; in **environmental science**, the distribution of particles, chemicals, and organisms in the environment; in **linguistics**, the number of letters per word and the number of words per sentence; and in **economics**, age of marriage, farm size, and income.
- The lognormal distribution is also useful in modeling data which would be considered normally distributed except for the fact that it may be more or less skewed (Limpert, Stahel, and Abbt 2001).
- A close relationship with the normal distribution: if ln(X) is normally distributed if X is lognormally distributed.
- The lognormal distribution finds its beginning in 1879. It was at this time that **F. Galton** noticed that if X_1, X_2, ..., X_n are independent positive random variables such that

  $$T_n = \prod_{i=1}^{n} X_i,$$

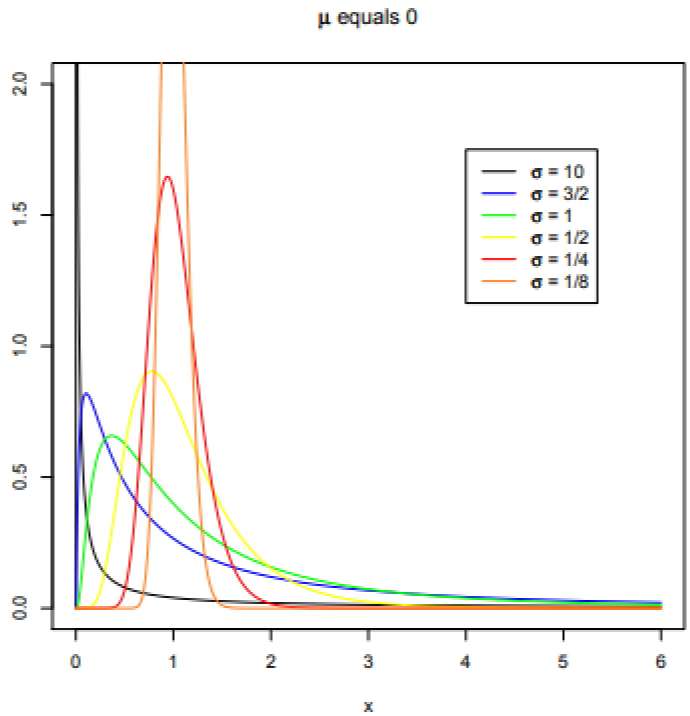
  then the log of their product is equivalent to the sum of their logs,

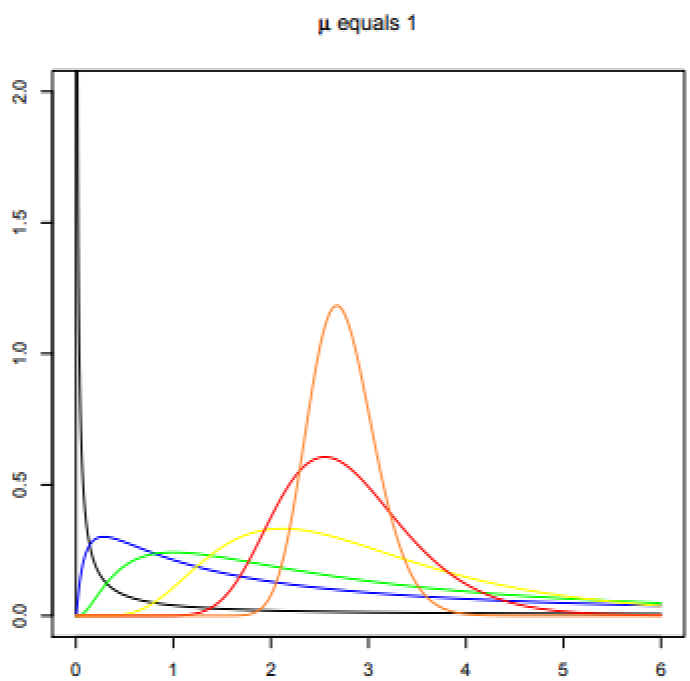
  $$\ln(T_n) = \sum_{i=1}^{n} \ln(X_i),$$

  Due to this fact, Galton concluded that the standardized distribution of $\ln(T_n)$ would tend to a unit normal distribution as n goes to infinity, such that the limiting distribution of T_n would tend to a two-parameter lognormal.
- Note that the lognormal is sometimes called the anti-lognormal distribution, because it is not the distribution of the logarithm of a normal variable, but is instead the anti-log of a normal variable (Brezina 1963; Johnson and Kotz 1970).
- The lognormal distribution probability density function is specified as

  $$f(X \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}\,X} \exp\left(-\frac{(\ln(X) - \mu)^2}{2\sigma^2} \right),$$

  where $X > 0, -\infty < \mu < \infty, \sigma > 0$.





Conduct the maximum-likelihood estimation of the two parameters, μ and σ^2, of the lognormal distribution. Please write all the steps clearly.