Lecture 15

Econometrics-I

23.03.2021.

- **Brief recap:**

Hypothesis testing and statistical inference.

Some useful distribution

* Standard Normal distribution:

$$\underbrace{X \sim N(0,1)}$$

** If we have a sample from $X \sim N(\mu, \sigma^2)$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{N}\right)$$

$\rightarrow$ size of sample.

$$z = \frac{\bar{X} - \mu}{\sigma^2/N} \sim N(0,1)$$

*** Chi-squared distribution

If $X_i \overset{iid}{\sim} N(0,1)$

$$Y = \sum_{i=1}^{d} X_i^2 \sim \chi^2_d$$

Chi-squared distributed with "d" degrees of freedom

A result:

If $Y_i \overset{iid}{\sim} N(\mu, \sigma^2)$

then $\sum_{i=1}^{N} \left(\frac{Y_i - \bar{Y}}{\sigma} \right)^2 \sim \chi^2_{N-1}$

By def^n, we know that $\sum_{i=1}^{N} \left(\frac{Y_i - \mu}{\sigma} \right)^2 \sim \chi^2_N$

**** t - distribution / Student's t-distribution

If

(a) $X \sim N(0,1)$

(b) $Z \sim \chi^2_d$

(c) X and Z are independent.

then

$$t = \frac{X}{\sqrt{Z/d}} \sim t_d$$

$\rightarrow$ $X \overset{iid}{\sim} N(\mu, \sigma^2) \Rightarrow \frac{X-\mu}{\sigma} \overset{iid}{\sim} N(0,1)$

$$Y = \sum_{i=1}^{N} \left(\frac{X-\bar{X}}{\sigma}\right)^2 \sim \chi^2_{N-1}$$

$$\frac{(X-\mu)/\sigma}{\sqrt{\sum_{i=1}^{N}\left(\frac{X_i-\bar{X}}{\sigma}\right)^2 / N-1}} \sim t_{N-1}$$

***** F-distribution

Let X and Z be independent χ^2 r.v's with N_1 and N_2 degrees of freedom, respectively, then

$$\frac{X/N_1}{Z/N_2} \sim F^{N_1}_{N_2} \text{ or } F_{N_1, N_2}$$

F-statistic can never by $(-)$ve.

$$\frac{MSE}{MSR} \sim F_{K-1, N-K}$$

Revisit the ANOVA

Source	SS	df	MS	F
Model	SSE $\sum_{i=1}^{N}(\hat{y}_i - \bar{y})^2$	$K-1$	MSE $\dfrac{\sum_{i=1}^{N}(\hat{y}_i - \bar{y})^2}{K-1}$	$\dfrac{MSE}{MSR} \sim F_{K-1, N-K}$
Residual	SSR $\sum_{i=1}^{N} \hat{u}_i^2$	$N-K$	MSR $\sum_{i=1}^{N} \hat{u}_i^2 / N-K$	
Total	SST $\sum_{i=1}^{N}(y_i - \bar{y})^2$	$N-1$		

Inference

H_0 : Null hypothesis

H_a : Alternative hypothesis.

[eg. Mean height of an Indian male is 7 ft.]

TRUTH → corresponds to the population

	H_0 is true	H_0 is false.
Reject H_0	Type-I Error	✓
Fail to reject H_0	✓	Type-II Error

Decision or Inference → corresponds to the sample

$Pr(\text{type-I error}) \equiv \alpha$, called as the significance level of a statistical test.

$Pr(\text{type-I error}) \equiv \underset{\sim}{\alpha}$: called as the significance level of the statistical test.

depends on the decision of the analyst.

$\Rightarrow$ α is a "tolerance level" set by the analyst of the chance of commiting the type-I error.

$\Rightarrow$ α is set before performing inference to avoid subjective bias of the analyst.

$\Rightarrow$ Careful about the borderline cases.

Example: Test $H_0: \mu_x = 0$

$H_a: \mu_x \neq 0$

Take a representative sample

$N = 22$

$\bar{X} = 2.73$

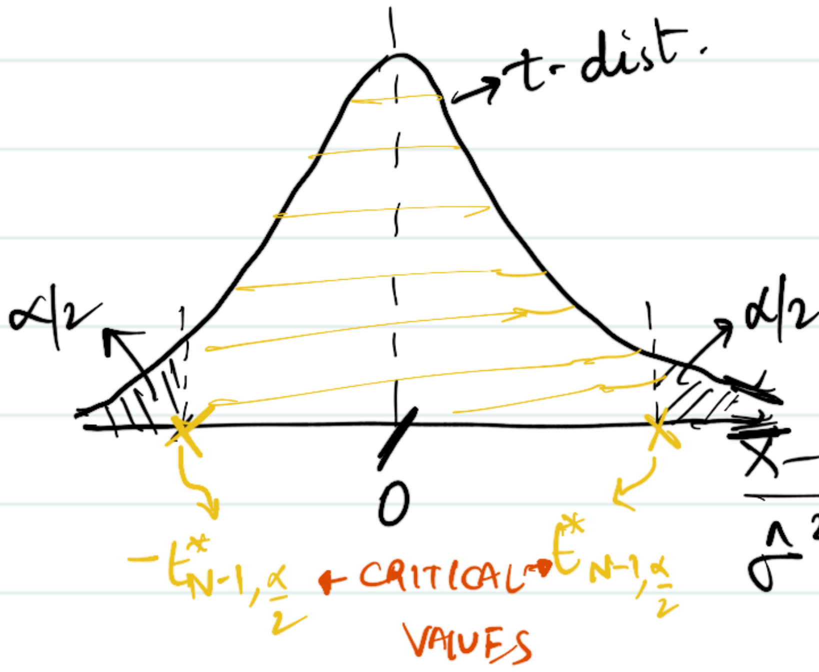
$$\hat{\sigma}_x^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})}{N-1} = 0.57.$$

$$\frac{\dfrac{\bar{X} - \mu_x}{\cancel{\sigma_x}/\sqrt{N}}}{\sqrt{\sum_{i=1}^{N}\left(\dfrac{x_i - \bar{x}}{\cancel{\sigma_x}}\right)^2 \Big/ N-1}} \sim t_{N-1} = t_{21}$$

$$\frac{\bar{X} - \mu_x}{\sqrt{\hat{\sigma}_x^2/N}} \sim t_{N-1} \underset{H_0}{\overset{\text{under}}{\equiv}} \frac{\bar{X}}{\sqrt{\hat{\sigma}_x^2/N}} \sim t_{N-1}$$

$$\boxed{\hat{\sigma}_x^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

we set the significance level of the test $\alpha = 5\%$



$$\frac{\bar{X}-\mu_x}{\hat{\sigma}^2/\sqrt{N}} \sim t_{N-1}$$

$$\Pr\left\{-t^*_{N-1,0.025} \le \frac{\bar{X}-\mu_x}{\sqrt{\hat{\sigma}_x^2/N}} \le t^*_{N-1,0.025}\right\} = 0.95$$

$$\Pr(\text{type-I error}) = \Pr\left\{\underbrace{\left|\frac{\bar{X}-\mu_x}{\hat{\sigma}_x/\sqrt{N}}\right|}_{t} > t^*_{N-1,\frac{\alpha}{2}}\right\} = 0.05$$

if $t > t^*_{N-1,\frac{\alpha}{2}}$ then reject the null Hypothesis (H_0)

⇒ we say μ_x is statistically different from zero.

if $t \le t^*_{N-1,\frac{\alpha}{2}}$ then we fail to reject the H_0

⇒ we say μ_x is not statistically different from zero.

$$Pr(\text{type-I error}) \equiv p\text{-value}$$

Inference

$\rightarrow$ If p-value $< \underbrace{0.05}_{=\alpha}$ then reject the null.

If p-value $\geq \underbrace{0.05}_{=\alpha}$ then ~~fail~~ fail to reject the H_0.