MATGEO - 7-7.2-22

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Question

Find the equation of the circle having (1, -2) as its centre and passing through the intersection of $3x + y = 14, 2x + 5y = 18$.

Terms Used

Table: Terms used

varaibles	**values**
centre	$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
line 1	$3x + y = 14$
line 2	$2x + 5y = 18$

Solution

Intersection point of the 2 linear equations $3x + y = 14$ and $2x + 5y = 18$ is given by

$$\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 18 \end{pmatrix}$$

Row Reduction

$$\begin{bmatrix} 3 & 1 & 14 \\ 2 & 5 & 18 \end{bmatrix} \longleftrightarrow [R_2 \leftarrow 3R_2 - 2R_1] \begin{bmatrix} 3 & 1 & 14 \\ 0 & 13 & 26 \end{bmatrix}$$

$$\longleftrightarrow [R_2 \leftarrow R_2/13] \begin{bmatrix} 3 & 1 & 14 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\longleftrightarrow [R_1 \leftarrow R_1 - R_2] \begin{bmatrix} 3 & 0 & 12 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\longleftrightarrow [R_1 \leftarrow R_1/3] \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$\therefore$ the intersecting point of the 2 lines is A(4,2).

Solution

$$r = \|A - C\| = \sqrt{(A - C)^T (A - C)}$$

Where A$(4, 2)$ is point of intersection and point on the circle with the centre C$(1, -2)$.

$$r = \sqrt{\begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}} = 5$$

equation of a conic is given by $x^T V x + 2u^T x + f = 0$
for a circle

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$u = \begin{pmatrix} -h \\ -k \end{pmatrix},$$

$$f = \|u\|^2 - r^2$$

substituting the above values in the equation we get

$$x^T x + 2\begin{pmatrix}-1 & 2\end{pmatrix} x - 20 = 0$$

C code for finding I

```c
#include <stdio.h>

void sol(int m, int a[m], int b[m], int x[m-1]){
    int p,q;
    p = (a[1]*b[2]-a[2]*b[1])/(a[0]*b[1]-a[1]*b[0]);
    q = (a[2]*b[0] - a[0]*b[2])/(a[0]*b[1] - a[1]*b[0]);
    x[0] = p;
    x[1] = q;
}
```

Python code for graph I

```
        import ctypes
import numpy as np
import matplotlib.pyplot as plt
# Load the shared library
solution_lib = ctypes.CDLL('./libsolution.so')

# Set the argument types for the 'sol' function

solution_lib.sol.argtypes =
[ctypes.c_int, ctypes.POINTER(ctypes.c_int),ctypes.POINTER(ctypes.c_int), ctypes.POINTER(ctypes.c_int)]

# Define input arrays
m = 3
a = (ctypes.c_int * m)(3, 1, -14)  # int a[3] = {3, 1, -14}
b = (ctypes.c_int * m)(2, 5, -18)  # int b[3] = {2, 5, -18}
x = (ctypes.c_int * (m - 1))()  # int x[2];
```

Python code for graph I

```
# Call the C function
solution_lib.sol(m, a, b, x)

# Retrieve the point from x array
point_x = x[0]
point_y = x[1]

# Print the point
print(f"Point on the circle: ({point_x}, {point_y})")
# Define the center of the circle
center = (1, -2)
# Calculate the radius
radius = np.sqrt((point_x - center[0])**2 +                    (point_y - center[1])**2)
```

Python code for graph I

```
# Create circle data with more points
theta = np.linspace(0, 2 * np.pi, 1000)
circle_x = center[0] + radius * np.cos(theta)
circle_y = center[1] + radius * np.sin(theta)

# Create line data for 3x + y = 14
x_line1 = np.linspace(center[0] - radius - 2,center[0] + radius + 2, 100)
y_line1 = 14 - 3 * x_line1  # Rearranged: y = 14 - 3x

# Create line data for 2x + 5y = 18
x_line2 = np.linspace(center[0] - radius - 2, center[0] + radius + 2, 100)
y_line2 = (18 - 2 * x_line2) / 5  # Rearranged: y = (18 - 2x) / 5

# Plot the circle and lines
plt.figure(figsize=(8, 8))
plt.plot(circle_x, circle_y, label='Circle', color='blue')
```

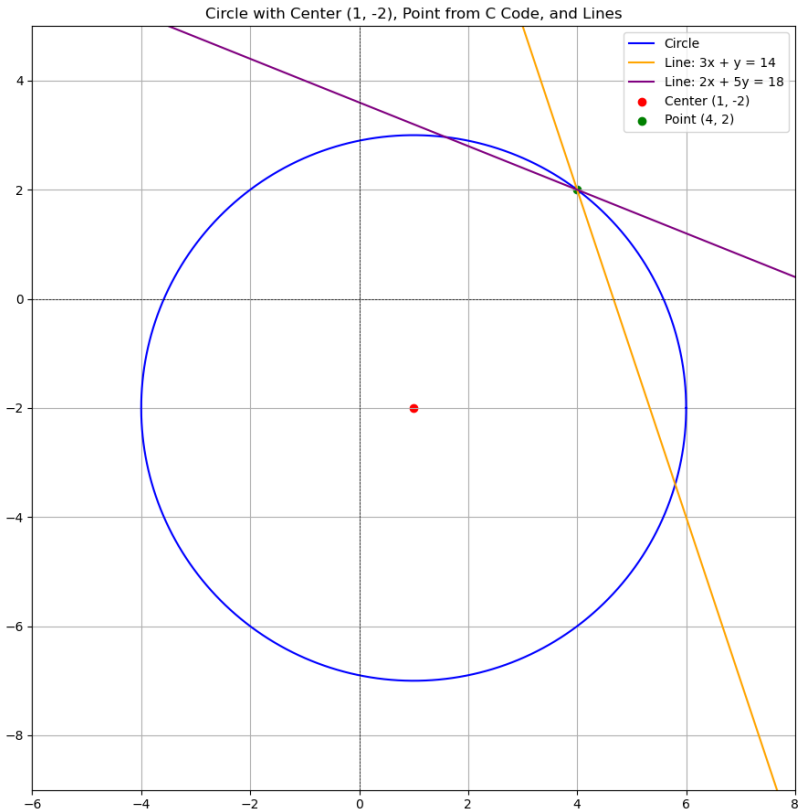
Python code for graph I

```
plt.plot(x_line1, y_line1,label='Line: 3x + y = 14',color='orange')
plt.plot(x_line2, y_line2, label='Line: 2x + 5y = 18',color='purple')
plt.scatter(*center, color='red', label='Center (1, -2)')
plt.scatter(point_x,point_y,color='green', label=f'Point ({point_x}, {point_y})')

# Set appropriate limits for better view
plt.xlim(center[0]-radius-2,center[0]+radius+2)
plt.ylim(center[1]-radius-2,center[1]+radius+2)

plt.axhline(0,color='black',linewidth=0.5,ls='--')
plt.axvline(0,color='black',linewidth=0.5,ls='--')
plt.grid()
plt.gca().set_aspect('equal', adjustable='box')
plt.title()('Circle with Center (1, -2),Point from C Code, and Lines')
plt.legend()
plt.show()
```

Figure


Circle with Center (1, -2), Point from C Code, and Lines
Circle
Line: 3x + y = 14
Line: 2x + 5y = 18
Center (1, -2)
Point (4, 2)
4
2
0
−2
−4
−6
−8
−6
−4
−2
0
2
4
6
8