Assignment-3

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16: A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y+z = 5$ and $y^{-1}+z^{-1} = 5/6, y > z$. Then the number of odd divisors of n, including 1, is:

a. 11 c. 12
b. 6x d. 6

17: Let $f(x) = \sin^{-1}(x)$ and $g(x) = \frac{x^2-x-2}{2x^2-x-6}$. If $g(2) = \lim_{x\to 2} g(x)$, then the domain of the function f*o*g is:

a. $(-\infty,-2]\cup[-4/3,\infty]$ c. $(-\infty,-2]\cup[-1,\infty]$
b. $(-\infty,-1]\cup[2,\infty]$ d. $(-\infty,-2]\cup[-3/2,\infty]$

18: If the mirror image of the point $(1,3,5)$ with respect to the plane $4x-5y+2z = 8$ is (α,β,γ), then $5(\alpha+\beta+\gamma)$:

a. 47 c. 43
b. 39 d. 41

19: Let $f(x) = \int_0^x e^t f(t)\,dt + e^x$ be a differentiable function for all $x \in \mathbf{R}$. Then f(x) equals:

a. $2e^{e^x-1} - 1$ c. $2e^{e^x} - 1$
b. e^{e^x-1} d. $e^{e^x} - 1$

20: The triangle of the maximum area that can be inscribed in a given circle of radius 'r' is:

a. A right-angle triangle having two of its sides of length 2r and r.
b. An equilateral triangle of height 2r/3.
c. Isosceles triangle with base equal to 2r.
d. An equilateral triangle having each of length $\sqrt{3}r$

I. Section-B

1: The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is .....

2: Let α and β be two real numbers such that $\alpha+\beta = 1$ and $\alpha\beta = -1$. Let $P_n = \alpha^n + \beta^n$, $P_{n-1} = 11$and$P_{n+1} = 29$ for some integer n = 1. Then the value of P_n^2 is .....

3: Let $X_1, X_2, \ldots, X_{18}$ be eighteen observation such that $\sum_{i=1}^{18}(X_i-\alpha) = 36$ and $\sum_{i=1}^{18}(X_i-\beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha-\beta|$ is .....

4: In $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for $m,n \geq 1$ and $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = \alpha I_{m,n}, \alpha \in R$, then α is .....

5: Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is .....

6: If the matrix $\begin{pmatrix}1&0&0\\0&2&0\\3&0&-1\end{pmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{pmatrix}1&0&0\\0&4&0\\0&0&1\end{pmatrix}$ for some real numbers α and β, then $\beta-\alpha$ is equal to?

7: If the arithmetic mean and the geometric mean of the p^{th} and q^{th} terms of the sequence -16,8,-4,2,... satisfy the equation $4x^2 - 9x + 5 = 0$, then p+q is equal to?

8: Let the normals at all the points on a given curve pass through a fixed point (a,b). If the curve passes through $(3,-3)$ and $\left(4,-2\sqrt{2}\right)$, and given that $a - 2\sqrt{2}b = 3$, then $\left(a^2+b^2+ab\right)$ is equal to?

9: Let z be those complex number which satisfies $|z+5| \leq 4$ and $z(i+1) + \bar{z}(1-i) \geq -10, i = \sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $\alpha+\beta$ is?

10: Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4+10x^3+10x^2+10x+10$ lie in the interval $(a, a+1)$, then $|a|$ is equal to?