# 3-3-9

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**Question:**
Draw an isosceles triangle ABC in which BC = 5.5cm and altitude AL = 5.3cm.
**Solution:**
The vertices of the above triangle are given by:

| **lengths** | **values** |
|---|---|
| **BC** | 5.5cm |
| **AL** | 5.3cm |

TABLE 1 0: values of lengths of triangle

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{0.3}$$

$$b = c \tag{0.4}$$

as AL perpendicularly bisect BC, triangle ALB is rightangled

$$\therefore AL^2 + \frac{a^2}{4} = b^2 \tag{0.5}$$

$$b = c = 5.97 \tag{0.6}$$

$$\cos B = \frac{a}{2b} = 0.46 \tag{0.7}$$

Where a,b,c are BC,AB,AC respectively and B is the angle formed by the side AB and BC.

$$\begin{aligned} a + b + c &= K \\ -a + b\cos(C) + c\cos(B) &= 0 \\ b\sin(C) - c\sin(B) &= 0 \end{aligned}$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

as the given triangle is iscosceles $\angle B = \angle C$
We can find all the side lengths by solving the above matrix equation where $x = \frac{a}{K}$, $y = \frac{b}{K}$, and $z = \frac{c}{K}$.
the augmented matrix will be

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \cos B & \cos B & 0 \\ 0 & \sin C & -\sin C & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow \frac{R_3}{\sin C}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \cos B & \cos B & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 + \cos B & 1 + \cos B & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{1+\cos B}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \frac{1}{1+\cos B} \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1+\cos B} \\ 0 & 1 & 1 & \frac{1}{1+\cos B} \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$\xleftrightarrow{R_3 \leftarrow -R_3 + R_2} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1+\cos B} \\ 0 & 1 & 1 & \frac{1}{1+\cos B} \\ 0 & 0 & 2 & \frac{1}{1+\cos B} \end{pmatrix}$$

$$\xleftrightarrow{R_3 \leftarrow \frac{R_3}{2}} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1+\cos B} \\ 0 & 1 & 1 & \frac{1}{1+\cos B} \\ 0 & 0 & 1 & \frac{1}{2(1+\cos B)} \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_3} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1+\cos B} \\ 0 & 1 & 0 & \frac{1}{2(1+\cos B)} \\ 0 & 0 & 1 & \frac{1}{2(1+\cos B)} \end{pmatrix}$$

The values of $x$,$y$,$z$ are

$$\frac{a}{K} = 1 - \frac{1}{1 + \cos B} \tag{0.8}$$

$$\frac{b}{K} = \frac{1}{2\,(1 + \cos B)} \tag{0.9}$$

$$\frac{c}{K} = \frac{1}{2\,(1 + \cos B)} \tag{0.10}$$

Substituting the values of a,b,c in 0.1, 0.2 and 0.3. Gives the coordinates.

$$\mathbf{A} = \begin{pmatrix} 2.75 \\ 5.3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix}$$

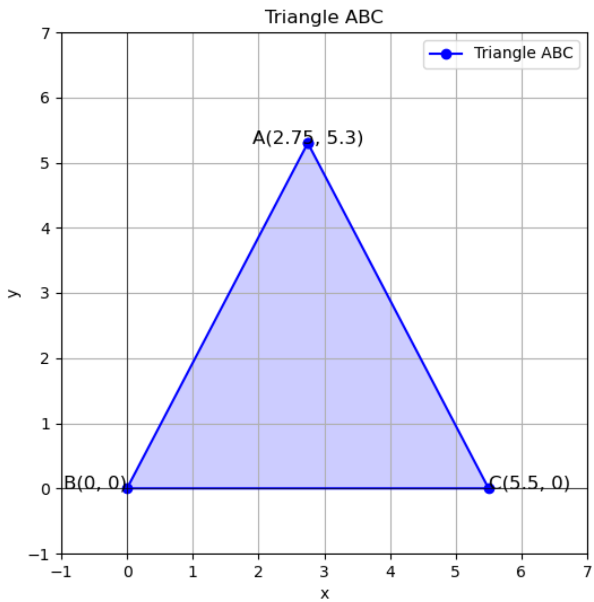


Fig. 0.1: plot for isosceles triangle