Assignment 5

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1 : $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$, is equal to

a. $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$
b. $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$
c. $\frac{1-x}{1+x} + \log_e(1-x)$
d. $\frac{1+x}{1-x} + \log_e(1-x)$

2 : If for $x, y \in \mathbf{R}, x > 0$, $y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} + \dots$ upto ∞ terms and $\frac{2+4+6+...+2y}{3+6+9+...+3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x, y) is equal to:

a. $\left(10^6, 6\right)$
b. $\left(10^4, 6\right)$
c. $\left(10^2, 3\right)$
d. $\left(10^6, 9\right)$

3 . Let A be a fixed point $(0, 6)$ and B be a moving point $(2t, 0)$. Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is

a. $3x^2 - 2y - 6 = 0$
b. $3x^2 + 2y - 6 = 0$
c. $2x^2 - 3y - 9 = 0$
d. $2x^2 - 3y - 9 = 0$

4 : If $\left(\sin^{-1} x\right)^2 - \left(\cos^{-1} x\right)^2 = a; 0 < x < 1, a \neq 0$ then the value of $2x^2 - 1$ is

a. $\cos\left(\frac{4a}{\pi}\right)$
b. $\sin\left(\frac{2a}{\pi}\right)$
c. $\cos\left(\frac{2a}{\pi}\right)$
d. $\sin\left(\frac{4a}{\pi}\right)$

5 : If the matrix

$$\begin{bmatrix} 0 & 2 \\ K & -1 \end{bmatrix}$$

satisfies $A\left(A^3 + 3I\right) = 2I$ then the value of K is

a. $\frac{1}{2}$
b. $\frac{-1}{2}$
c. -1
d. 1

6 :The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are 2, 3, -6 is :

a. 3

b. 5
c. 2
d. 1

7 : If $S = \{z \in \mathbf{C} : \frac{z-i}{z+2i} \in \mathbf{R}\}$ then:

a. S contains exactly two elements
b. S contains only one element
c. S is a circle in the complex plane
d. S is a straight line in the complex plane

8 : Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$$

such that y(0)=7, then $y(\pi)$is equal to

a. $2e^{\pi^2} + 5$
b. $e^{\pi^2} + 5$
c. $3e^{\pi^2} + 5$
d. $7e^{\pi^2} + 5$

9 : Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is :

a. $3x - y - 5z + 2 = 0$
b. $3x - 4z + 3 = 0$
c. $-x + 2y + 2z - 3 = 0$
d. $4x - y - 5z + 2 = 0$

10 : If $U_n = \left(1 + \frac{1}{n^2}\right)\left(1 + \frac{2^2}{n^2}\right)^2 \ldots \left(1 + \frac{n^2}{n^2}\right)^n$ then $\lim_{n\to\infty} (U_n)^{\frac{-4}{n^2}}$ is equal to

a. $\frac{e^2}{16}$
b. $\frac{4}{e}$
c. $\frac{16}{e^2}$
d. $\frac{4}{e^2}$

11 : The statement $(p \wedge (p \rightarrow) \wedge (q \rightarrow r)) \rightarrow r$ is

a. a tautology
b. equivalent to $p \rightarrow \neg r$
c. a fallacy
d. equivalent to $q \rightarrow \neg r$

12 : Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$ then:

a. $x^2 + 2xf(x) - 12 = 0$
b. $x^2 + xf(x) + 12 = 0$
c. $x^2 - 3xf(x) - 4 = 0$
d. $x^2 + 2xf(x) + 4 = 0$

13 : $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ is equal to

a. ${}^{40}C_{21}$
b. ${}^{40}C_{19}$

c. $^{40}C_{20}$
d. $^{41}C_{20}$

14 : A tangent and a normal are drawn at the point $P(2,-4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a,b)$ is a point such that AQBP is a square, then 2a + b is equal to :

a. -16
b. -18
c. -12
d. -20

15 : Let $\frac{\sin A}{\sin B} = \frac{\sin A-C}{\sin C-B}$, where A,B,C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a,b,c respectively then:

a. $b^2 - a^2 = a^2 + c^2$
b. b^2, c^2, a^2 are in A.P.
c. c^2, a^2, b^2 are in A.P.
d. a^2, b^2, c^2, are in A.P.