1-1.2-18

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Question:
If the origin is the centroid of the triangle PQR with vertices $\mathbf{P}\begin{pmatrix}2a\\2\\6\end{pmatrix}$, $\mathbf{Q}\begin{pmatrix}-4\\3b\\-10\end{pmatrix}$, $\mathbf{R}\begin{pmatrix}8\\14\\2c\end{pmatrix}$ then find the values of a, b and c.
Solution:
proof:

points	**values**
P	$\begin{pmatrix}2a\\2\\6\end{pmatrix}$
Q	$\begin{pmatrix}-4\\3b\\-10\end{pmatrix}$
R	$\begin{pmatrix}8\\14\\2c\end{pmatrix}$
G	$\begin{pmatrix}0\\0\\0\end{pmatrix}$

TABLE 1 0: values of the geometrical points in given question

The centroid (G) of a triangle with the vertices (P),(Q),(R) is given by

$$\mathbf{G} = \frac{1}{3}(\mathbf{P} + \mathbf{Q} + \mathbf{R}) \tag{0.1}$$

$$\mathbf{G} = \frac{1}{3}(\mathbf{P} + \mathbf{Q} + \mathbf{R}) \qquad = \frac{1}{3}\left(\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix} + \begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix} + \begin{pmatrix}x_3\\y_3\\z_3\end{pmatrix}\right) = \begin{pmatrix}x_1+x_2+x_3\\y_1+y_2+y_3\\z_1+z_2+z_3\end{pmatrix}$$

$$\mathbf{G} = \frac{1}{3}\begin{pmatrix}x_1+x_2+x_3\\y_1+y_2+y_3\\z_1+z_2+z_3\end{pmatrix} = \begin{pmatrix}\frac{x_1+x_2+x_3}{3}\\\frac{y_1+y_2+y_3}{3}\\\frac{z_1+z_2+z_3}{3}\end{pmatrix} \tag{0.2}$$

Given that, the centroid of the triangle **PQR** is origin $\begin{pmatrix}0\\0\\0\end{pmatrix}$.

From 0.2, substituting the values in the above proven formula:

$$G = \left(\begin{pmatrix}2a\\4\\6\end{pmatrix} + \begin{pmatrix}-4\\3\\10\end{pmatrix} + \begin{pmatrix}8\\14\\2c\end{pmatrix}\right)\frac{1}{3}$$

$$G = \begin{pmatrix}2a-4+8\\4+3+14\\6+10+2c\end{pmatrix}\frac{1}{3}$$

$$G = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

on comparing we get that

$$2a-4+8=0$$

$$a=-2 \tag{0.3}$$

$$4+3b+14=0$$

$$b=-6 \tag{0.4}$$

$$6+10+2c=0$$

$$c=-8 \tag{0.5}$$

$\therefore$ the values of a,b,c are -2,-6,-8 respectively.

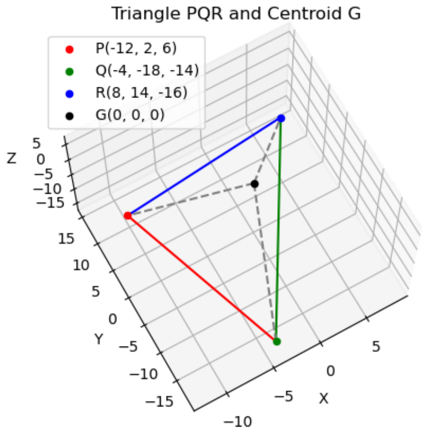

Fig. 0.1: plot for triangle