Assignment 7

AI24BTECH11008- Sarvajith

1. If

$$\int_{0}^{\frac{\pi}{\alpha}}\int_{x}^{\frac{\pi}{\alpha}}\frac{siny}{y}dydx=\frac{1}{2}$$

for some $\alpha \geq 1$, then the value of α is ....

2. Three fair dice are rolled simultaneously. The probability of getting a sum of 5 is
   (A) $\frac{1}{108}$
   (B) $\frac{1}{72}$
   (C) $\frac{1}{54}$
   (D) $\frac{1}{36}$

3. Suppose α, β, γ and δ are constants such that

$$p\left(x\right)=\delta+\gamma\left(x+1\right)+\beta x\left(x+1\right)+\alpha x\left(x+1\right)\left(x-1\right)$$

is the interpolating polynomial for the data $\left(-1,-3\right),\left(0,1\right),\left(1,-1\right),\left(2,-3\right)$. Then the value of $\gamma-\beta$ is ....

4. Consider the ordinary differential equation

$$y''+\alpha y'+\beta y=0$$

, where α and β are constants. If $y\left(x\right)=xe^{x}$ is a solution of the above equation, then the value of $\beta-\alpha$ is ....

5. Consider the system of linear equations

$$\begin{aligned} 2x_2+x_3&=0,\\ -2x_1-x_3&=0,\\ -x_1+x_2&=1 \end{aligned}$$

. The above system has
   (A) a unique solution
   (B) infinite number of solutions
   (C) no solution
   (D) only 2 distinct solutions

6. Let C be a simple closed curve enclosing the region R in the xy-plane. Let C be oreinted counterclockwise. If the value of the integral

$$\oint_{C}\left(y+e^{x^{2}}\right)dx+\left(3x+cosy\right)dy$$

is 16, then the area of R is ....

7. Consider the ordinary differential equation

$$x^{2}y''+xy'-y=x, x>0$$

. In terms of arbitrary constants c_1 and c_2, te general solution of the above equation is

(A) $y(x) = c_1 x + c_2 x^{-1} + x^3$
(B) $y(x) = c_1 x + c_2 x^{-1} + \frac{1}{2}x$
(C) $y(x) = c_1 x + c_2 x^{-1} + \frac{1}{2}x \ln x$
(D) $y(x) = c_1 x + c_2 + x^{-1}$

8. Let $f : R \to R$ and $g : R \to R$ be defined by

$$f(x) = \begin{cases} x(sinx)\, cos\frac{1}{x} & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$g(x) = \begin{cases} xcos\frac{1}{x}, & x \neq 0 \\ 0 & x \neq 0 \end{cases}$$

where R denotes the set of real numbers. Then at x=0,

(A) f is differntiable but g is not differentiable
(B) f is not differentiable but g is differentiable
(C) both f and g are differentiable
(D) neither f nor g is differentiable

9. If $u(x,t) = g(t)\, sinx$ is the solution of the wave equation

$$u_{tt} = u_{xx}, t > 0, 0 < x < \pi$$

with the initial conditions

$$u(x,0) = 2sinx, u_t(x,0) = 0, 0 \leq x \leq \pi$$

The boundary conditions

$$u(0,t) = u(\pi,t) = 0, t \geq 0$$

then the value of $g\left(\frac{\pi}{3}\right)$ is ....

10. Let

$$I = \int_0^1 \frac{1}{1+t}dt + \frac{\pi i}{2}\int_0^1 \frac{e^{\frac{i\pi t}{2}}}{1+e^{\frac{i\pi t}{2}}}dt - i\int_0^1 \frac{1}{1+it}dt$$

, where t is real variable and $i = \sqrt{-1}$. The value of I is ....

11. Let

$$a_k = 2^{-k}k^4 sink$$

and

$$b_k = 2^{-k^2}ksin^2 k$$

for k=1,2..... then

(A) $\sum_{k=1}^{\infty} a_k$ converges but $\sum_{k=1}^{\infty} b_k$ does NOT converge
(B) $\sum_{k=1}^{\infty} a_k$ does NOT converges but $\sum_{k=1}^{\infty} b_k$ converges
(C) both $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ converge
(D) neither $\sum_{k=1}^{\infty} a_k$ nor $\sum_{k=1}^{\infty} b_k$ converges

12. In a given flow field, the velocity vector in Cartesian coordinate system is given as:

$$\overrightarrow{V} = \left(x^2 + y^2 + z^2\right)\hat{i} + \left(xy + yz + y^2\right)\hat{j} + \left(xz - z^2\right)\hat{k}$$

What is the volume dilation rate of the fluid at a point where x=1, y=2 and z=3?

(A) 6
(B) 5
(C) 10
(D) 0

13. A steady, incompressible, two-dimensional velociy fluid in Cartesian coordinate system is represented by the following expression.

$$\overrightarrow{V} = (0.7 + 0.4x)\,\hat{i} + (1.20.4y)\,\hat{j}$$

The coordinates of the point (x, y) in the flow field having "zero" velocity is,

(A) $(1.75, -3)$
(B) $(-1.75, 3)$
(C) $(1.75, 3)$
(D) $(-1.75, -3)$