

Assignment 11

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14. The linear Programming Problem:

Maximize $z = x_1 + x_2$

subject to

$$x_1 + 2x_2 \leq 20$$

$$x_1 + x_2 \leq 15$$

$$x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

- (A) has exactly one optimum solution
- (B) has more than one optimum solution
- (C) has unbounded solution
- (D) has no solution

15. Consider the Primal Linear Programming Problem:

$$\text{P: } \left\{ \begin{array}{l} \text{Maximize } z = c_1x_1 + c_2x_2 + \dots + c_nx_n \\ \text{subject to} \\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2 \\ \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m \\ x_j \geq 0, j = 1, \dots, n \end{array} \right.$$

The Dual of P is

$$\text{D: } \left\{ \begin{array}{l} \text{Minimize } z' = b_1w_1 + b_2w_2 + \dots + b_mw_m \\ \text{subject to} \\ a_{11}w_1 + a_{21}w_2 + \dots + a_{m1}w_m \geq c_1 \\ a_{12}w_1 + a_{22}w_2 + \dots + a_{m2}w_m \geq c_2 \\ \dots \\ a_{1n}w_1 + a_{2n}w_2 + \dots + a_{mn}w_m \geq c_n \\ w_i \geq 0, i = 1, \dots, m \end{array} \right.$$

- (A) $N_g \ll N_e$
- (B) $N_g \gg N_e$

- (C) $N_g \approx N_e \approx \frac{N}{2}$
 (D) $N_g - N_e \approx \frac{N}{2}$

16. The number of irreducible quadratic polynomials over the field of 2 elements F_2 is

- (A) 0
 (B) 1
 (C) 2
 (D) 3

17. The number of elements in the conjugacy class of the 3-cycle(234) in symmetric group S_6 is

- (A) 20
 (B) 40
 (C) 120
 (D) 216

18. The initial value Problem

$$x \frac{dy}{dx} = y + x^2, x > 0; y(0) = 0,$$

has

- (A) infinitely many solutions
 (B) exactly two solutions
 (C) a unique solution
 (D) no solution

19. The subspace $P = \{(x, y, z) \in R^3 : z = x^2 + y^2 + 1\}$ is

- (A) compact and connected
 (B) compact but not connected
 (C) not compact but connected
 (D) neither compact nor connected

20. Let $P = (0, 1); Q = [0, 1]; U = (0, 1], T = \text{Rand } A = \{P, Q, U, S, T\}$. The equivalence relation 'homeomorphism' induces which one of the following as the partition of A?

- (A) $\{P, Q, U, S\}, \{T\}$
 (B) $\{P, T\}, \{Q\}, \{U\}, \{S\}$
 (C) $\{P, T\}, \{Q, U, S\}$
 (D) $\{P, T\}, \{Q, U\}, \{S\}$

21. Let $x = (x_1, x_2, \dots) \in l^4, x \neq 0$. For which one of the following values of p, the series $\sum_{i=1}^{\infty} x_i y_i$ converges for every $y = (y_1, y_2, \dots) \in l^p$?

- (A) 1
 (B) 2
 (C) 3
 (D) 4

22. Let H be a complex Hilbert space and H^* be its dual. The mapping $\phi : H \rightarrow H^*$ defined by $\phi(y) f_y$, where $f_y(x) = \langle x, y \rangle$ is

- (A) not linear but onto
 (B) both linear and onto

- (C) linear but not onto
 (D) neither linear nor onto
23. A horizontal lever is in static equilibrium under the application of vertical forces F_1 at a distance l_1 from the fulcrum and F_2 at a distance l_2 from the fulcrum. The equilibrium for the above quantities can be obtained if
- (A) $F_1 L_1 = 2 F_2 L_2$
 (B) $2 F_1 L_1 = F_2 L_2$
 (C) $F_1 L_1 = F_2 L_2$
 (D) $F_1 L_1 < F_2 L_2$
24. Assume F to be a twice continuously differentiable function. Let $J(y)$ be a functional of the form

$$\int_0^1 F(x, y') dx, 0 \leq x \leq 1$$

defined on the set of all continuously differentiable functions y on $[0, 1]$ satisfying $y(0) = a, y(1) = b$. For some arbitrary constant c , a necessary condition for y to be an extremum of J is

- (A) $\frac{\partial F}{\partial x} = c$
 (B) $\frac{\partial F}{\partial y'} = c$
 (C) $\frac{\partial F}{\partial y} = c$
 (D) $\frac{\partial F}{\partial x} = 0$
25. The eigenvalue λ of the following Fredholm integral equation

$$y(x) = \lambda \int_0^1 x^2 t y(t) dt$$

is

- (A) -2
 (B) 2
 (C) 4
 (D) -4
- Q.26-Q.55 carry two marks each.**
26. The application of Gram-Schmidt process of orthonormalization to

$$u_1 = (1, 1, 0), u_2 = (1, 0, 0), u_3 = (1, 1, 1)$$

yields

- (A) $\frac{1}{\sqrt{2}}(1, 1, 0), (1, 0, 0), (0, 0, 1)$
 (B) $\frac{1}{\sqrt{2}}(1, 1, 0), \frac{1}{\sqrt{2}}(1, -1, 0), \frac{1}{\sqrt{2}}(1, 1, 1)$
 (C) $(0, 1, 0), (1, 0, 0), (0, 0, 1)$
 (D) $\frac{1}{\sqrt{2}}(1, 1, 0), \frac{1}{\sqrt{2}}(1, -1, 0), (0, 0, 1)$