

centroid
EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

Sarvajith Guddety

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1-2-18. If the origin is the centroid of the triangle PQR with vertices $\mathbf{P}(2a, 4, 6)$, $\mathbf{Q}(-4, 3b, 10)$ and $\mathbf{R}(8, 14, 2c)$, then find the values of a, b and c .

sol:

$$\begin{aligned}\mathbf{P}(x_1, y_1, z_1) &= (2a, 4, 6) \\ \mathbf{Q}(x_2, y_2, z_2) &= (-4, 3b, 10) \\ \mathbf{R}(x_3, y_3, z_3) &= (8, 14, 2c)\end{aligned}$$

Given that, the centroid of the triangle PQR is origin $(0, 0, 0)$.
Centroid(G).

$$\text{let the matrix } S = \begin{bmatrix} 2a & 4 & 6 \\ -4 & 3b & 10 \\ 8 & 14 & 2c \end{bmatrix}$$

$$\text{The matrix } G = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} S$$

after the matrix multiplication

$$G = \frac{1}{3} \begin{bmatrix} 2a - 4 + 8 & 4 + 3b + 14 & 6 + 10 + 2c \end{bmatrix}$$

and given that

$$G = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

on comparing we get that

- $2a - 4 + 8 = 0$

- $4 + 3b + 14 = 0$

- $6 + 10 + 2c = 0$

- $a = -2$

- $b = -6$

- $c = -8$

∴ the values of a,b,c are -2,-6,-8 respectively.