Assignment 6

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1 : If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{\pi}{4}}^{x} \left(4\sqrt{2}\sin t - 3\phi'(t)\right) dt, x > 0$, then $\phi'\left(\frac{\pi}{4}\right)$ is equal to

a. $\frac{8}{\sqrt{\pi}}$
b. $\frac{4}{6+\sqrt{\pi}}$
c. $\frac{8}{6+\sqrt{\pi}}$
d. $\frac{4}{6-\sqrt{\pi}}$

2 : If a point $P(\alpha,\beta,\gamma)$ satisfying $\begin{bmatrix}\alpha & \beta & \gamma\end{bmatrix}\begin{bmatrix}2 & 10 & 8\\ 9 & 3 & 8\\ 8 & 4 & 8\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\end{bmatrix}$ lies on the plane $2x + 4y + 3z = 5$ then $6\alpha + 9\beta + 7\gamma$ is equal to

a. -1
b. $\frac{-11}{5}$
c. $\frac{5}{4}$
d. 11

3 . Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$ the product a_1a_4 is minimum and the sum of its first n terms is zero, then n!-4$a_{n(n+2)}$ is equal to

a. 24
b. $\frac{33}{4}$
c. $\frac{381}{4}$
d. 9

4 : Let $(a,b) \subset (0,2\pi)$ be the largest interval for which$\sin^{-1}(\sin\theta) - \cos^{-1}(\sin\theta) > 0, \theta \in (0,2\pi)$holds. If$\alpha x^2 + \beta x + \sin^{-1}\left(x^2 - 6x + 10\right) + \cos^{-1} x^2 - 6x + 10 = 0$ and $\alpha - \beta = b - a$, then (α) is equal to:

a. $\frac{\pi}{48}$
b. $\frac{\pi}{16}$
c. $\frac{\pi}{8}$
d. $\frac{\pi}{12}$

5 : Let $y = y(x)$ be the solution of the differential equation $\left(3y^2 - 5x^2\right) y dx + 2x\left(x^2 - y^2\right) dy = 0$ such that y(1) = 1 then $\left|(y(2))^3 - 12y(2)\right|$ is equal to

a. $32\sqrt{2}$
b. 64
c. $16\sqrt{2}$
d. 32

6 : The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$

a. $(8, \infty)$

b. $(4, \infty)$
c. $(0, 4]$
d. $(2, 12]$

7 : If $S = \{(a, b) : a, b \in \mathbf{R} - \{0\}, 2\frac{a}{b} > 0\}$ And $T = \{(a, b) : a, b \in \mathbf{R}, a^2 - b^2 \in \mathbf{Z}\}$:

a. S is transitive but T is not
b. T is symmetric but S is not
c. Neither S nor T is transitive
d. Both S and T are symmetric

8 : The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0, x \in R$ has:

a. two solutions and both are negative
b. no solution
c. four solutions two of which are negative
d. two solutions and only one of them is negative

9 : The number of values of $r \in \{p, q, \neg p, \neg q\}$ for which $((p \wedge q) \rightarrow (r \vee q)) \wedge ((p \wedge r) \rightarrow q)$ is a tautology is:

a. 3
b. 2
c. 1
d. 4

10 : Let $f : \mathbf{R} - \{2, 6\} \rightarrow \mathbf{R}$ be a real-valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$ Then the range of f is

a. $(-\infty, -\frac{21}{4}] \cup [0, \infty)$
b. $(-\infty, -\frac{21}{4}] \cup (0, \infty)$
c. $(-\infty, -\frac{21}{4}] \cup [\frac{21}{4}, \infty)$
d. $(-\infty, -\frac{21}{4}] \cup [1, \infty)$

11 :$\lim_{x\to\infty} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$

a. is equal to 9
b. is equal to 27
c. does not exist
d. is equal to $\frac{27}{2}$

12 : Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is

a. 6
b. 4
c. 5
d. 7

13 : The absolute minimum values of the function $f(x) = \left|x^2 - x + 1\right| + [x^2 - x + 1]$, where [t] denotes the greatest integer function, in the interval [-1,2] is

a. $\frac{3}{4}$
b. $\frac{3}{2}$

c. $\frac{1}{2}$
d. $\frac{5}{4}$

14 : Let the plane $P : 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line $L : \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is

a. $\sqrt{14}$
b. $\frac{6}{\sqrt{14}}$
c. $\sqrt{\frac{2}{7}}$
d. $\sqrt{\frac{7}{2}}$

15 : The foot of the perpendicular from the origin O to a plane P which meets to coordinate axes at the points A, B, C is $(2, a, 4)$, $a \in N$.if the volume of the tetrahedron OABC is $144unit^3$, then which of the following points is NOT on P?

a. $(2, 2, 4)$
b. $(0, 4, 4)$
c. $(3, 0, 4)$
d. $(0, 6, 3)$