1-1.2-18

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Question:
If the origin is the centroid of the triangle PQR with vertices
P $\begin{pmatrix} 2a \\ 2 \\ 6 \end{pmatrix}$
Q $\begin{pmatrix} -4 \\ 3b \\ -10 \end{pmatrix}$
R $\begin{pmatrix} 8 \\ 14 \\ 2c \end{pmatrix}$
then find the values of a, b and c.
Solution:
proof:

points	values
P	$\begin{pmatrix} 2a \\ 2 \\ 6 \end{pmatrix}$
Q	$\begin{pmatrix} -4 \\ 3b \\ -10 \end{pmatrix}$
R	$\begin{pmatrix} 8 \\ 14 \\ 2c \end{pmatrix}$
G	$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

TABLE 1 0: values of the geometrical points in given question

Proof of the Centroid Formula using Matrix Notation

Triangle with vertices A$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$, B$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$, and C$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$, the centroid G$\begin{pmatrix} x_g \\ y_g \\ z_g \end{pmatrix}$

The centroid (G) is the average of the coordinates of the vertices (A) , (B) , and (C):

$$\mathbf{G} = \frac{1}{3}\left(\mathbf{A} + \mathbf{B} + \mathbf{C}\right)$$

Matrix Addition and Scalar Multiplication First, add the vectors ($\mathbf{A}$), ($\mathbf{B}$), and ($\mathbf{C}$):

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} + \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + x_3 \\ y_1 + y_2 + y_3 \\ z_1 + z_2 + z_3 \end{pmatrix}$$

Next, multiply by the scalar $\left(\frac{1}{3}\right)$:

$$\mathbf{G} = \frac{1}{3}\begin{pmatrix} x_1 + x_2 + x_3 \\ y_1 + y_2 + y_3 \\ z_1 + z_2 + z_3 \end{pmatrix} = \begin{pmatrix} \frac{x_1+x_2+x_3}{3} \\ \frac{y_1+y_2+y_3}{3} \\ \frac{z_1+z_2+z_3}{3} \end{pmatrix}$$

Conclusion

Thus, the coordinates of the centroid $G\left(x_g, y_g, z_g\right)$ are given by:

$$x_g = \frac{x_1 + x_2 + x_3}{3}, \quad y_g = \frac{y_1 + y_2 + y_3}{3}, \quad z_g = \frac{z_1 + z_2 + z_3}{3}$$

This proves the centroid formula using matrix notation.

$$\mathbf{P}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 2a \\ 4 \\ 6 \end{pmatrix} \tag{0.1}$$

$$\mathbf{Q}\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 10 \end{pmatrix} \tag{0.2}$$

$$\mathbf{R}\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} 8 \\ 14 \\ 2c \end{pmatrix} \tag{0.3}$$

Given that, the centroid of the triangle **PQR** is origin$\left(0, 0, 0\right)$.
Centroid(G).
From the proof

$$\mathbf{G} = \frac{1}{3}\left(\mathbf{P} + \mathbf{Q} + \mathbf{R}\right)$$

$$G = \left(\begin{pmatrix} 2a \\ 4 \\ 6 \end{pmatrix} + \begin{pmatrix} -4 \\ 3 \\ 10 \end{pmatrix} + \begin{pmatrix} 8 \\ 14 \\ 2c \end{pmatrix}\right)\frac{1}{3}$$

$$G = \begin{pmatrix} 2a - 4 + 8 \\ 4 + 3 + 14 \\ 6 + 10 + 2c \end{pmatrix}\frac{1}{3}$$

$$G = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

on comparing we get that

$$2a - 4 + 8 = 0$$
$$a = -2 \tag{0.4}$$
$$4 + 3b + 14 = 0$$
$$b = -6 \tag{0.5}$$
$$6 + 10 + 2c = 0$$
$$c = -8 \tag{0.6}$$

$\therefore$ the values of a,b,c are -2,-6,-8 respectively.

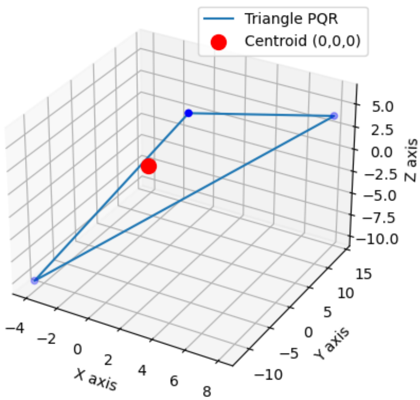

Fig. 0.1: plot for triangle