7-7.2-22

AI24BTECH11008- Sarvajith

Question:

Find the equation of the circle having (1, -2) as its centre and passing through the intersection of 3x + y = 14, 2x + 5y = 18.

Solution:

Intersection point of the 2 linear equations $3x + y = 14$ and $2x + 5y = 18$ is given by

varaibles	values
centre	$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
line 1	$3x + y = 14$
line 2	$2x + 5y = 18$

TABLE 1 0: given values

$$\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 14 \\ 2 & 5 & 18 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 1 & 14 \\ 0 & 13 & 26 \end{pmatrix}$$

$$\xleftrightarrow{R_2 \leftarrow R_2/13} \begin{pmatrix} 3 & 1 & 14 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 3 & 0 & 12 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\xleftrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

$\therefore$ the intersecting point of the 2 lines is (4,2).

$$\text{Radius r} = \sqrt{(4-1)^2 + (2+2)^2} = 5$$

$$\text{equation of a conic} = x^T V x + 2u^T x + f$$

$$\text{for a circle} V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$u = \begin{pmatrix} -h \\ -k \end{pmatrix},$$

$$f = h^2 + k^2 - r^2$$

substituting the above values in the equation we get

$$x^T x + 2\begin{pmatrix} -1 & 2 \end{pmatrix} x - 20 = 0$$

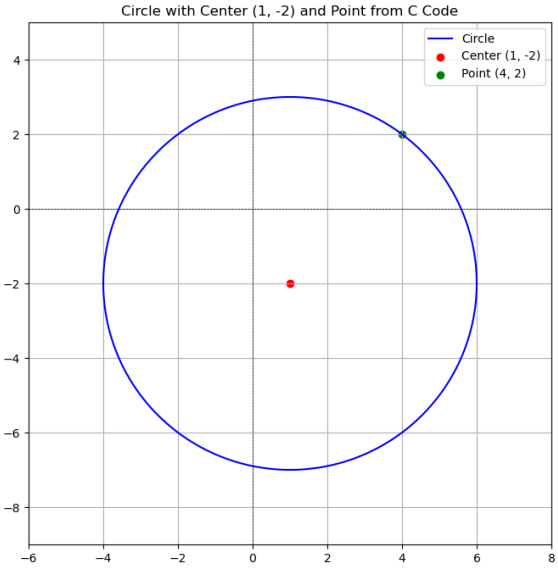

Fig. 0.1: plot for circle