1-1.2-18

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Question:
If the origin is the centroid of the triangle PQR with vertices $\mathbf{P}(2a, 2, 6)$, $\mathbf{Q}(-4, 3b, -10)$ and $\mathbf{R}(8, 14, 2c)$, then find the values of a, b and c.
Solution:
proof:

points	values
P	$(2a, 2, 6)$
Q	$(-4, 3b, -10)$
R	$(8, 14, 2c)$
G	$(0, 0, 0)$

TABLE 1 0: values of the geometrical points in given question

Proof of the Centroid Formula using Matrix Notation

Given a triangle with vertices A(x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3), the centroid $\left(x_g, y_g, z_g\right)$ of the triangle is defined as the point where the three medians of the triangle intersect. The centroid can also be found as the average of the coordinates of the vertices.

Step 1: Representing the Vertices as Column Vectors

The coordinates of the vertices A , B , and C can be represented as column vectors:

Step 2: Centroid as a Column Vector

The centroid G$\left(x_g, y_g, z_g\right)$ can be represented as a column vector:

$$\mathbf{G} = \begin{bmatrix} x_g \\ y_g \\ z_g \end{bmatrix}$$

Step 3: Centroid Formula in Matrix Notation

The centroid (G) is the average of the coordinates of the vertices (A) , (B) , and (C):

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C})$$

Step 4: Matrix Addition and Scalar Multiplication

First, add the vectors **A**, **B**, and **C**:

$$\mathbf{A}+\mathbf{B}+\mathbf{C}=\begin{bmatrix}x_1\\y_1\\z_1\end{bmatrix}+\begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix}+\begin{bmatrix}x_3\\y_3\\z_3\end{bmatrix}=\begin{bmatrix}x_1+x_2+x_3\\y_1+y_2+y_3\\z_1+z_2+z_3\end{bmatrix}$$

Next, multiply by the scalar $\frac{1}{3}$:

$$\mathbf{G}=\frac{1}{3}\begin{bmatrix}x_1+x_2+x_3\\y_1+y_2+y_3\\z_1+z_2+z_3\end{bmatrix}=\begin{bmatrix}\frac{x_1+x_2+x_3}{3}\\\frac{y_1+y_2+y_3}{3}\\\frac{z_1+z_2+z_3}{3}\end{bmatrix}$$

Conclusion

Thus, the coordinates of the centroid $\mathrm{G}\left(x_g, y_g, z_g\right)$ are given by:

$$x_g=\frac{x_1+x_2+x_3}{3},\quad y_g=\frac{y_1+y_2+y_3}{3},\quad z_g=\frac{z_1+z_2+z_3}{3}$$

This proves the centroid formula using matrix notation.

$$\mathbf{P}\left(x_1, y_1, z_1\right)=(2a, 4, 6) \tag{0.1}$$

$$\mathbf{Q}\left(x_2, y_2, z_2\right)=(-4, 3b, 10) \tag{0.2}$$

$$\mathbf{R}\left(x_3, y_3, z_3\right)=(8, 14, 2c) \tag{0.3}$$

Given that, the centroid of the triangle **PQR** is origin$(0,0,0)$.
Centroid(G).

$$let\,the\,matrix\;S=\begin{bmatrix}2a&4&6\\-4&3b&10\\8&14&2c\end{bmatrix}$$

$$The\,matrix\;G=\frac{1}{3}\begin{bmatrix}1&1&1\end{bmatrix}S$$

after the matrix multiplication

$$G=\frac{1}{3}\begin{bmatrix}2a-4+8&4+3b+14&6+10+2c\end{bmatrix}$$

and given that

$$G=\begin{bmatrix}0&0&0\end{bmatrix}$$

on comparing we get that

$$2a-4+8=0$$

$$a=-2 \tag{0.4}$$

$$4+3b+14=0$$

$$b = -6 \tag{0.5}$$

$$6 + 10 + 2c = 0$$

$$c = -8 \tag{0.6}$$

$\therefore$ the values of a,b,c are -2,-6,-8 respectively.

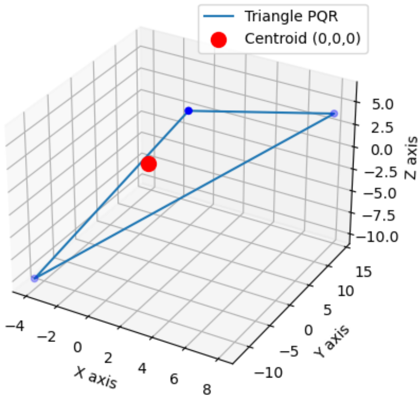

Fig. 0.1: plot for triangle