

# 1-1.2-18

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## Question:

If the origin is the centroid of the triangle PQR with vertices  $\mathbf{P}(2a, 2, 6)$ ,  $\mathbf{Q}(-4, 3b, -10)$  and  $\mathbf{R}(8, 14, 2c)$ , then find the values of a, b and c.

## Solution:

### proof:

if there are 3 points which form a triangle in 3 dimensional axis then, their centroid can be calculated using matrices by :

- 1) First form a 3 x 3 matrix with the values of points as each row and name it as S

$$S = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

- 2) then take another 3 x 1 matrix I which is filled with  $\frac{1}{3}$

$$I = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

- 3) now perform matrix multiplication  $I \times S$ , which results in an 3 x 1 matrix G, which will have a the elements corresponding to centroid elements

$$G = \begin{bmatrix} \frac{x_1+x_2+x_3}{3} & \frac{y_1+y_2+y_3}{3} & \frac{z_1+z_2+z_3}{3} \end{bmatrix}$$

- 4) this indicates that centroid can be calculated using this method.

$$\mathbf{P}(x_1, y_1, z_1) = (2a, 4, 6) \quad (4.1)$$

$$\mathbf{Q}(x_2, y_2, z_2) = (-4, 3b, 10) \quad (4.2)$$

$$\mathbf{R}(x_3, y_3, z_3) = (8, 14, 2c) \quad (4.3)$$

Given that, the centroid of the triangle **PQR** is origin(0,0,0).  
Centroid(G).

$$\text{let the matrix } S = \begin{bmatrix} 2a & 4 & 6 \\ -4 & 3b & 10 \\ 8 & 14 & 2c \end{bmatrix}$$

$$\text{The matrix } G = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} S$$

after the matrix multiplication

$$G = \frac{1}{3} \begin{bmatrix} 2a - 4 + 8 & 4 + 3b + 14 & 6 + 10 + 2c \end{bmatrix}$$

and given that

$$G = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

on comparing we get that

$$2a - 4 + 8 = 0$$

$$a = -2 \tag{4.4}$$

$$4 + 3b + 14 = 0$$

$$b = -6 \tag{4.5}$$

$$6 + 10 + 2c = 0$$

$$c = -8 \tag{4.6}$$

$\therefore$  the values of a,b,c are -2,-6,-8 respectively.