1-1.7-5

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Question:

Find the value of p for which the points $A\begin{pmatrix} -5 \\ 1 \end{pmatrix}$, $B\begin{pmatrix} 1 \\ p \end{pmatrix}$, and $C\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ are colinear?

Solution:

Given,

points	values
A	$\begin{pmatrix} -5 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 1 \\ p \end{pmatrix}$
C	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

TABLE 1 0: values of the geometrical points in given question

if three points are collinear then their slope should be equal.

$$\text{slope of AB} = B - A$$
$$= \begin{pmatrix} 1 \\ p \end{pmatrix} - \begin{pmatrix} -5 \\ 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 + 5 \\ p - 1 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ p - 1 \end{pmatrix}$$
$$= 6 \begin{pmatrix} 1 \\ \frac{p-1}{6} \end{pmatrix}$$

$$\therefore \text{slope} = \frac{p - 1}{6}. \tag{0.1}$$

$$\text{slope of BC} = C - A$$

$$= \begin{pmatrix} 4 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ p \end{pmatrix}$$

$$= \begin{pmatrix} 4 - 1 \\ -2 - p \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -2 - p \end{pmatrix}$$

$$= 3 \begin{pmatrix} 1 \\ \frac{-2-p}{3} \end{pmatrix}$$

$$\therefore \text{slope} = \frac{-2 - p}{3}. \qquad (0.2)$$

slope AB = slope of BC

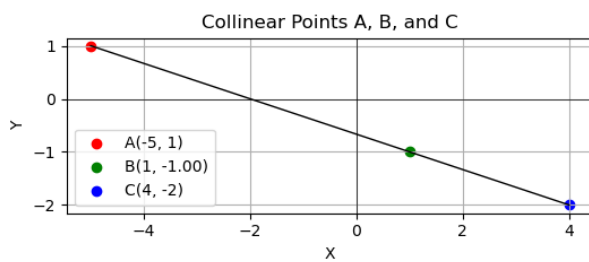$$\frac{p - 1}{6} = \frac{-p - 2}{3}$$

$$3\,(p - 1) = 6\,(-p - 2)$$

$$p - 1 = -2p - 4$$

$$p = -1$$

Fig. 0.1: plot for collinear points