Assignment 3

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Question 16: A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = 5/6, y > z$. Then the number of odd divisors of n, including 1, is:

a. 11
b. 6x
c. 12
d. 6

Question 17: Let $f(x) = \sin^{-1}(x)$ and $g(x) = [x^2 - x - 2]/[2x^2 - x - 6]$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function fog is:

a. $(-\infty, -2] \cup [-4/3, \infty]$
b. $(-\infty, -1] \cup [2, \infty]$
c. $(-\infty, -2] \cup [-1, \infty]$
d. $(-\infty, -2] \cup [-3/2, \infty]$

Question 18: If the mirror image of the point $(1, 3, 5)$ with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$:

a. 47
b. 39
c. 43
d. 41

Question 19: Let $f(x) = \int_0^x e^t f(t)\, dt + e^x$ be a differentiable function for all $x \in \mathbf{R}$. Then f(x) equals:

a. $2e^{e^x - 1} - 1$
b. $e^{e^x - 1}$
c. $2e^{e^x} - 1$
d. $e^{e^x} - 1$

Question 20: The triangle of the maximum area that can be inscribed in a given circle of radius 'r' is:

a. A right-angle triangle having two of its sides of length 2r and r.
b. An equilateral triangle of height 2r/3.
c. Isosceles triangle with base equal to 2r.
d. An equilateral triangle having each of length $\sqrt{3}r$

1 Section-B

Question 1: The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ....

Question 2: Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $P_n = \alpha^n + \beta^n$, $P_{n-1} = 11 \text{and} P_{n+1} = 29$ for some integer n = 1. Then the value of P_n^2 is ....

Question 3: Let $X_1, X_2, \ldots, X_{18}$ be eighteen observation such that $\sum_{i=1}^{18} (X_i - \alpha) = 36$ and $\sum_{i=1}^{18} (X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ....

Question 4: In $I_{m,n} = \int_0^1 x^{m-1} (1-x)^{n-1}\, dx$, for $m, n \geq 1$ and $\int_0^1 [x^{m-1} + x^{n-1}]/[(1+x)^{m+n}]dx = \alpha I_{m,n}, \alpha \in R$, then α is ....

Question 5: Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ....

Question 6: If the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{pmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to?

Question 7: If the arithmetic mean and the geometric mean of the p^{th} and q^{th} terms of the sequence -16,8,-4,2,... satisfy the equation $4x^2 - 9x + 5 = 0$, then p+q is equal to?

Question 8: Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through $(3, -3)$ and $\left(4, -2\sqrt{2}\right)$, and given that $a - 2\sqrt{2}b = 3$, then $\left(a^2 + b^2 + ab\right)$ is equal to?

Question 9: Let z be those complex number which satisfies $|z + 5| \leq 4$ and $z(i+1) + \bar{z}(1-i) \geq -10, i = \sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $\alpha + \beta$ is?

Question 10: Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a+1)$, then $|a|$ is equal to?