

3-3-9

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Question:

Draw an isosceles triangle ABC in which BC = 5.5cm and altitude AL = 5.3cm.

Solution:

The vertices of the above triangle are given by:

| lengths | values |
|---------|--------|
| BC | 5.5cm |
| AL | 5.3cm |

TABLE 1 0: values of lengths of triangle

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (0.1)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.2)$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (0.3)$$

Where a,b,c are BC,AB,AC respectively and B is the angle formed by the side AB and BC.

AL bisects BC, then 2 right angled triangle ALB and ALC are formed.

As triangle ABL is a right angled triangle, and K is sum of b and c, B is given by

$$\tan B = \frac{5.3}{2.75}$$

$$\cos B = \frac{1}{\sqrt{1 + \tan^2 B}}$$

$$\cos B = 0.46$$

$$\sin B = 0.87$$

$$a + b + c = K$$

$$-a + b \cos(C) + c \cos(B) = 0$$

$$b \sin(C) - c \sin(B) = 0$$

It results in the following matrix equation

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos(C) & \cos(B) \\ 0 & \sin(C) & -\sin(B) \end{pmatrix} \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = K \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

as the given triangle is isosceles $\angle B = \angle C$

We can find all the side lengths by solving the above matrix equation where $x = \frac{a}{K}$, $y = \frac{b}{K}$, and $z = \frac{c}{K}$.

the augmented matrix will be

$$\begin{aligned} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \cos B & \cos B & 0 \\ 0 & \sin C & -\sin C & 0 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow \frac{R_3}{\sin C}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & \cos B & \cos B & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 + \cos B & 1 + \cos B & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow \frac{R_2}{1 + \cos B}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & \frac{1}{1 + \cos B} \\ 0 & 1 & -1 & 0 \end{pmatrix} \\ & \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1 + \cos B} \\ 0 & 1 & 1 & \frac{1}{1 + \cos B} \\ 0 & 1 & -1 & 0 \end{pmatrix} \\ & \xleftrightarrow{R_3 \leftarrow -R_3 + R_2} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1 + \cos B} \\ 0 & 1 & 1 & \frac{1}{1 + \cos B} \\ 0 & 0 & 1 & \frac{1}{1 + \cos B} \end{pmatrix} \\ & \xleftrightarrow{R_2 \leftarrow R_2 - R_3} \begin{pmatrix} 1 & 0 & 0 & 1 - \frac{1}{1 + \cos B} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{1 + \cos B} \end{pmatrix} \end{aligned}$$

The values of x, y, z are

$$\frac{a}{K} = 1 - \frac{1}{1 + \cos B} \quad (0.4)$$

$$\frac{b}{K} = 0 \quad (0.5)$$

$$\frac{c}{K} = \frac{1}{1 + \cos B} \quad (0.6)$$

as a is already given, by substituting in 0.4, 0.5, 0.6 gives K, c, b . substituting the above values in 0.1, 0.3 gives the coordinates as

$$\mathbf{A} = \begin{pmatrix} 2.75 \\ 5.3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 5.5 \\ 0 \end{pmatrix}$$

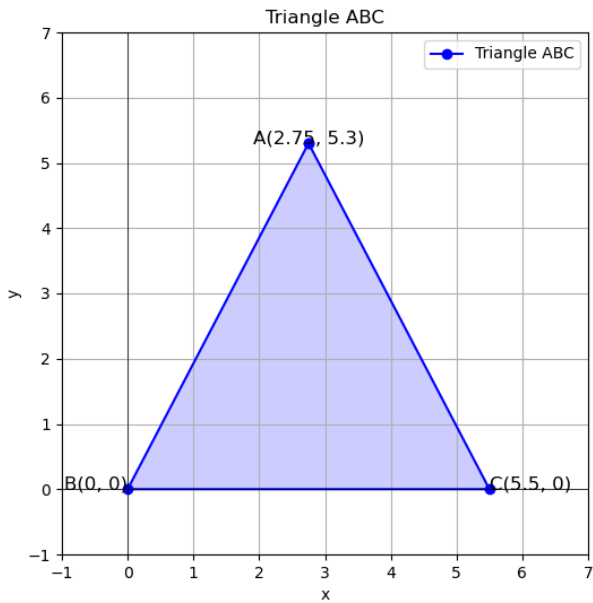


Fig. 0.1: plot for isosceles triangle