

Assignment 9

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49. The Lagrangian of a particle of mass m moving in one dimension is $L = \exp(\alpha t) \left[\frac{m\dot{x}^2}{2} - \frac{kx^2}{2} \right]$, where α and k are positive constants. The equation of motion of the particle is
- (A) $\ddot{x} + \alpha\dot{x} = 0$
 (B) $\ddot{x} + \frac{k}{m}x = 0$
 (C) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$
 (D) $\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$
50. Two monochromatic waves having frequencies ω and $\omega + \Delta\omega$ ($\Delta\omega \ll \omega$) and corresponding wavelength λ and $\lambda - \Delta\lambda$ ($\Delta\lambda \ll \lambda$) of same polarization, travelling along x-axis are superimposed on each other. The phase velocity and group velocity of the resultant wave are respectively given by
- (A) $\frac{\omega\lambda}{2\pi}, \frac{\Delta\lambda^2}{2\pi\Delta\lambda}$
 (B) $\omega\lambda, \frac{\Delta\lambda}{2\pi}$
 (C) $\frac{\omega\Delta\lambda}{2\pi}, \frac{\Delta\lambda}{2\pi\Delta\lambda}$
 (D) $\omega\Delta\lambda, \omega\Delta\lambda$

Common data questions Common data questions 51 and 52 Consider a two level quantum system with energies $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon$

51. The Helmholtz free energy of the system is given by

- (A) $-k_B T \ln \left(1 + e^{\frac{-\epsilon}{k_B T}} \right)$
 (B) $k_B T \ln \left(1 + e^{\frac{-\epsilon}{k_B T}} \right)$
 (C) $\frac{3}{2} k_B T$
 (D) $\epsilon - k_B T$

52. The specific heat of the system is given by

- (A) $\frac{\epsilon}{k_B T} \frac{e^{\frac{-\epsilon}{k_B T}}}{\left(1 + e^{\frac{-\epsilon}{k_B T}} \right)^2}$
 (B) $\frac{\epsilon^2}{k_B T^2} \frac{e^{\frac{-\epsilon}{k_B T}}}{\left(1 + e^{\frac{-\epsilon}{k_B T}} \right)}$
 (C) $-\frac{\epsilon^2 e^{\frac{-\epsilon}{k_B T}}}{\left(1 + e^{\frac{-\epsilon}{k_B T}} \right)^2}$
 (D) $\frac{\epsilon^2}{k_B T^2} \frac{e^{\frac{-\epsilon}{k_B T}}}{\left(1 + e^{\frac{-\epsilon}{k_B T}} \right)^2}$

Common data questions 53 and 54 A free particle of mass m moves along the x-direction. At $t = 0$, the normalized wave function of the particle is given by $\psi(x, 0) = \frac{1}{(2\pi\alpha)^{1/4}} \exp\left(-\frac{x^2}{4\alpha^2} + ix\right)$, where α is a real constant

53. The expectation value of the momentum, in this state is

- (A) $\hbar\alpha$

- (B) $\hbar \sqrt{\alpha}$
 (C) α
 (D) $\frac{\hbar}{\sqrt{\alpha}}$

54. The expectation value of the particle energy is

- (A) $\frac{\hbar^2}{2m} \frac{1}{2\alpha^{3/2}}$
 (B) $\frac{\hbar^2}{2m} \alpha^2$
 (C) $\frac{\hbar^2}{2m} \frac{4\alpha^2+1}{4\alpha^{3/2}}$
 (D) $\frac{\hbar^2}{8m\alpha^{3/2}}$

Common data questions 55 and 56 Consider the Zeeman splitting of a single electron system for the 3d to 3p electric dipole transition

55. The Zeeman spectrum is

- (A) Randomly polarized
 (B) only π polarized
 (C) only σ polarized
 (D) both π and σ polarized

56. The fine structure line having the longest wavelength will split into

- (A) 17 components
 (B) 10 components
 (C) 8 components
 (D) 4 components

Linked Answer Questions Statement for Linked Answer Questions 57 and 58: The primitive translation vectors of the face centered cubic (fcc) lattice are

$$\hat{a}_1 = \frac{a}{2} (\hat{j} + \hat{k}); \hat{a}_2 = \frac{a}{2} (\hat{i} + \hat{k}); \hat{a}_3 = \frac{a}{2} (\hat{j} + \hat{i})$$

57. The primitive translation vectors of the fcc reciprocal lattice are

- (A) $\hat{b}_1 = \frac{2\pi}{a} (\hat{j} + \hat{k} - \hat{i}); \hat{b}_2 = \frac{2\pi}{a} (-\hat{j} + \hat{k} + \hat{i}); \hat{b}_3 = \frac{2\pi}{a} (\hat{j} - \hat{k} + \hat{i})$
 (B) $\hat{b}_1 = \frac{\pi}{a} (\hat{j} + \hat{k} - \hat{i}); \hat{b}_2 = \frac{\pi}{a} (-\hat{j} + \hat{k} + \hat{i}); \hat{b}_3 = \frac{\pi}{a} (\hat{j} - \hat{k} + \hat{i})$
 (C) $\hat{b}_1 = \frac{\pi}{2a} (\hat{j} + \hat{k} - \hat{i}); \hat{b}_2 = \frac{\pi}{2a} (-\hat{j} + \hat{k} + \hat{i}); \hat{b}_3 = \frac{\pi}{2a} (\hat{j} - \hat{k} + \hat{i})$
 (D) $\hat{b}_1 = \frac{3\pi}{a} (\hat{j} + \hat{k} - \hat{i}); \hat{b}_2 = \frac{3\pi}{a} (-\hat{j} + \hat{k} + \hat{i}); \hat{b}_3 = \frac{3\pi}{a} (\hat{j} - \hat{k} + \hat{i})$

58. The volume of the primitive cell of the fcc reciprocal lattice is

- (A) $4 \left(\frac{\pi}{a} \right)^3$
 (B) $4 \left(\frac{2\pi}{a} \right)^3$
 (C) $4 \left(\frac{\pi}{2a} \right)^3$
 (D) $4 \left(\frac{3\pi}{a} \right)^3$

Statement for Linked Answer Questions 59 and 60: The Karnaugh map of logic circuit shown is below

	\bar{R}	R
$\bar{P}\bar{Q}$	1	1
$\bar{P}Q$	1	
PQ		
$P\bar{Q}$	1	1

Fig. 0.1: 1

59. The minimized logic expression for the above map is

- (A) $Y = \bar{P}R + \bar{Q}$
- (B) $Y = \bar{Q}.PR$
- (C) $Y = PR + \bar{Q}$
- (D) $Y = \bar{P}R.Q$

60. The corresponding logic implementation using gates is given as:

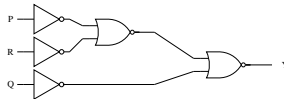


Fig. 0.2: option1

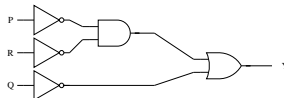


Fig. 0.3: option2

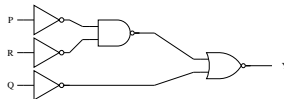


Fig. 0.4: option3

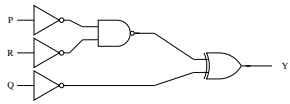


Fig. 0.5: option4