

# Assignment-4

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16: If P and Q are two statements, then which of the following compound statements is a tautology?

- $((P \rightarrow Q) \wedge \neg Q) \rightarrow P$
- $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg P$
- $((P \rightarrow Q) \wedge \neg Q)$
- $((P \rightarrow Q) \wedge \neg Q) \rightarrow \neg Q$

17: Consider a hyperbola  $H : x^2 - 2y^2 = 4$ . Let the tangent at a point  $P(4, \sqrt{6})$  meet the x-axis at Q and the latus rectum at  $R(x_1, y_1)$ , where  $x_1 > 0$ . If F is a focus of H which is nearer to the point P, then the area of  $\Delta QFR$  is equal to:

- $\sqrt{6} - 1$
- $4\sqrt{6} - 1$
- $4\sqrt{6}$
- $\frac{7}{\sqrt{6}} - 2$

18: Let  $f : R \rightarrow R$  be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & x < 0 \\ b, & x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & x > 0 \end{cases}$$

If f is continuous at  $x = 0$ , then the value of  $a+b$  is equal to

- 2
- $-\frac{2}{5}$
- $\frac{-3}{2}$
- 3

19: Let  $y = y(x)$  be the solution of the differential equation  $\frac{dy}{dx} = (y+1)((y+1)e^{x^2/2} - x)$ ,  $0 < x < 2.1$ , with  $y(2) = 0$ . Then the value of  $\frac{dy}{dx}$  at  $x=1$  is equal to

- $\frac{e^{5/2}}{(1+e^2)^2}$
- $\frac{5e^{1/2}}{(e^2)+1}$
- $\frac{-2e^2}{(1+e^2)^2}$
- $\frac{-e^{3/2}}{(e^2+1)^2}$

20: Let a tangent be drawn to the ellipse  $\frac{x^2}{27} + y^2 = 1$  at  $(3\sqrt{3}\cos\theta, \sin\theta)$  where  $\theta \in (0, \frac{\pi}{2})$ . Then the value of  $\theta$  such that the sum of intercepts on axes made by a tangent is minimum is equal to:

- $\frac{\pi}{8}$
- $\frac{\pi}{6}$
- $\frac{\pi}{3}$
- $\frac{\pi}{4}$

## I. SECTION-B

1: Let P be a plane containing the line  $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{7}$  and parallel to the line  $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$ . If the point  $(1, -1, \alpha)$  lies on the plane P, then the value of  $(|5\alpha|)$  is equal to?

2:  $\sum_{r=1}^{10} r! (r^3 + 6r^2 + 2r + 5) = \alpha (11!)$  Then the value of  $\alpha$  is equal to:

3: The term independent of  $x$  in the expansion of  $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ ,  $x \neq 1$  is equal to?

4: Let  ${}^nC_r$  denote the binomial coefficient of  $x^r$  in the expansion of  $(1+x)^n$ . If  $\sum_{k=0}^{10} (22+3k)^n C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$ , then  $\alpha + \beta$  is equal to?

5: Let  $P(x)$  be a real polynomial of degree 3 which vanishes at  $x = -3$ . Let  $P(x)$  have local minima at  $x = 1$ , local maxima at  $x = -1$ , and  $\int_{-1}^1 P(x) dx = 18$ , then the sum of all the coefficients of the polynomial  $P(x)$  is equal to?

6: Let the mirror image of the point  $(1, 3, a)$  with respect to the plane  $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) - b = 0$  be  $(-3, 5, 2)$ . Then, the value of  $|a + b|$  is equal to?

7: If  $f(x)$  and  $gx$  are two polynomials such that the polynomial  $P(x) = f(x^3) + xg(x^3)$  is divisible by  $x^2 + x + 1$ , then  $P(1)$  is equal to?

8: Let I be an identity matrix of order  $2 \times 2$  and

$$P = \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$$

$P_n = 5I - 8P$  Then the value of  $n \in \mathbb{N}$  for which  $P_n = 5I - 8P$  is equal to?

9: Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfy the equation  $f(x+y) = f(x) \cdot f(y)$  for all  $x, y \in \mathbb{R}$  and  $f(x) \neq 0$  for any  $x \in \mathbb{R}$ . If the function  $f$  is differentiable at  $x = 0$  and  $f'(0) = 3$ , then  $\lim_{h \rightarrow 0} \frac{1}{h} [f(h) - 1]$  is equal to?

10: Let  $y = y(x)$  be the solution of the differential equation  $xdy - ydx = \sqrt{x^2 - y^2} dx$ ,  $x \geq 1$ , with  $y(1) = 0$ . If the area bounded by the line  $x = 1$ ,  $x = e^\pi$ ,  $y = 0$  and  $y = y(x)$  is  $\alpha e^{2\pi} + \beta$ , then the value of  $10(\alpha + \beta)$  is equal to?