## AI24BTECH11008- Sarvajith

## **Question:**

Find the equation of the circle having (1, -2) as its centre and passing through the intersection of 3x + y = 14, 2x + 5y = 18.

## **Solution:**

varaibles	values
centre	$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
line 1	3x + y = 14
line 2	2x + 5y = 18

TABLE 1 0: given values

Intersection point of the 2 linear equations 3x + y = 14 and 2x + 5y = 18 is given by

$$\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 14 \\ 2 & 5 & 18 \end{pmatrix} \xrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 1 & 14 \\ 0 & 13 & 26 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2/13} \begin{pmatrix} 3 & 1 & 14 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 3 & 0 & 12 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

 $\therefore$  the intersecting point of the 2 lines is A(4,2).

$$r = ||A - C|| = \sqrt{(A - C)^T (A - C)}$$

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Where A(4,2) is point of intersection and point on the circle with the centre C(1,-2).

$$r = \sqrt{\left(3 \quad 4\right)\left(\frac{3}{4}\right)} = 5$$

equation of a conic is given by  $x^T V x + 2u^T x + f = 0$  for a circle

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$u = \begin{pmatrix} -h \\ -k \end{pmatrix},$$
$$f = ||u||^2 - r^2$$

substituting the above values in the equation we get

$$x^{T}x + 2(-1 \quad 2)x - 20 = 0$$



Fig. 0.1: plot for circle