Assignment 9

AI24BTECH11008- Sarvajith

49. The Lagrangian of a particle of mass m moving in one dimension is $L = exp\,(\alpha t)\,[\frac{m\dot{x}^2}{2} - \frac{kx^2}{2}]$, where α and k are positive constants. The equation of motion of the particle is (2009)
   (A) $\ddot{x} + \alpha\dot{x} = 0$
   (B) $\ddot{x} + \frac{k}{m}x = 0$
   (C) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$
   (D) $\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$
50. Two monochromatic waves having frequencies ω and $\omega + \triangle\omega\,(\Delta\omega <<< \omega)$ and corresonding wavelength λ and $\lambda - \Delta\lambda\,(\Delta\lambda <<< \lambda)$ of same polarization, travelling along x-axis are superimposed on each other. The phase velocity and group velocity of the resultant wave are respectively given by (2009)
   (A) $\frac{\omega\lambda}{2\pi}, \frac{\Delta\lambda^2}{2\pi\Delta\lambda}$
   (B) $\omega\lambda, \frac{\Delta\lambda^2}{\Delta\lambda}$
   (C) $\frac{\omega\Delta\lambda}{2\pi}, \frac{\Delta\lambda}{2\pi\Delta\lambda}$
   (D) $\omega\Delta\lambda, \omega\Delta\lambda$

   Common data questions Common data questions 51 and 52 Consider a two level quantum system with energies $\epsilon_1 = 0and\epsilon_2 = \epsilon$
51. The Helmholtz free energy of the system is given by (2009)
   (A) $-k_BT\,ln\left(1 + e^{\frac{-\epsilon}{k_BT}}\right)$
   (B) $k_BT\,ln\left(1 + e^{\frac{-\epsilon}{k_BT}}\right)$
   (C) $\frac{3}{2}k_BT$
   (D) $\epsilon - k_BT$
52. The specific heat of the system is given by (2009)
   (A) $\frac{\epsilon}{k_BT}\frac{e^{\frac{-\epsilon}{k_BT}}}{\left(1+e^{\frac{-\epsilon}{k_BT}}\right)^2}$
   (B) $\frac{\epsilon^2}{k_BT^2}\frac{e^{\frac{-\epsilon}{k_BT}}}{\left(1+e^{\frac{-\epsilon}{k_BT}}\right)}$
   (C) $-\frac{\epsilon^2 e^{\frac{-\epsilon}{k_BT}}}{\left(1+e^{\frac{-\epsilon}{k_BT}}\right)^2}$
   (D) $\frac{\epsilon^2}{k_BT^2}\frac{e^{\frac{-\epsilon}{k_BT}}}{\left(1+e^{\frac{-\epsilon}{k_BT}}\right)^2}$

   Common data questions 53 and 54 A free particle of mass m moves along the x-direction. At t = 0, the normalized wave function of the particle is given by $\psi\,(x, 0) = \frac{1}{(2\pi\alpha)^{1/4}}exp-\frac{x^2}{4\alpha^2} + ix$, where α is a real constant
53. The expectation value of the momentum, in this state is (2009)
   (A) $\hbar\alpha$

(B) $\hbar\sqrt{\alpha}$
(C) α
(D) $\frac{\hbar}{\sqrt{\alpha}}$

54. The expectation value of the particle energy is (2009)
(A) $\frac{\hbar^2}{2m}\frac{1}{2\alpha^{3/2}}$
(B) $\frac{\hbar^2}{2m}\alpha^2$
(C) $\frac{\hbar^2}{2m}\frac{4\alpha^2+1}{4\alpha^{3/2}}$
(D) $\frac{\hbar^2}{8m\alpha^{3/2}}$

Common data questions 55 and 56 Consider the Zeeman splitting of a single electron system for the 3d to 3p electric dipole transition

55. The Zeeman spectrum is (2009)
(A) Randomly polarized
(B) only π polarized
(C) only σ polarized
(D) both π and σ polarized

56. The fine structure line having the longest wavelength will split into (2009)
(A) 17 components
(B) 10 components
(C) 8 components
(D) 4 components

Linked Answer Questions Statement for Linked Answer Questions 57 ans 58: The primitive translation vectors of the face centered cubic (fcc) lattice are

$$\hat{a}_1 = \frac{a}{2}\left(\hat{j}+\hat{k}\right); \hat{a}_2 = \frac{a}{2}\left(\hat{i}+\hat{k}\right); \hat{a}_1 = \frac{a}{2}\left(\hat{j}+\hat{i}\right)$$

57. The primitive transition vectors of the fccreciprocal lattice are (2009)
(A) $\hat{b}_1 = \frac{2\pi}{a}\left(\hat{j}+\hat{k}-\hat{i}\right); \hat{b}_2 = \frac{2\pi}{a}\left(-\hat{j}+\hat{k}+\hat{i}\right); \hat{b}_3 = \frac{2\pi}{a}\left(\hat{j}-\hat{k}+\hat{i}\right)$
(B) $\hat{b}_1 = \frac{\pi}{a}\left(\hat{j}+\hat{k}-\hat{i}\right); \hat{b}_2 = \frac{\pi}{a}\left(-\hat{j}+\hat{k}+\hat{i}\right); \hat{b}_3 = \frac{\pi}{a}\left(\hat{j}-\hat{k}+\hat{i}\right)$
(C) $\hat{b}_1 = \frac{\pi}{2a}\left(\hat{j}+\hat{k}-\hat{i}\right); \hat{b}_2 = \frac{\pi}{2a}\left(-\hat{j}+\hat{k}+\hat{i}\right); \hat{b}_3 = \frac{\pi}{2a}\left(\hat{j}-\hat{k}+\hat{i}\right)$
(D) $\hat{b}_1 = \frac{3\pi}{a}\left(\hat{j}+\hat{k}-\hat{i}\right); \hat{b}_2 = \frac{3\pi}{a}\left(-\hat{j}+\hat{k}+\hat{i}\right); \hat{b}_3 = \frac{3\pi}{a}\left(\hat{j}-\hat{k}+\hat{i}\right)$

58. The volume of the primitive cell of the fcc reciprocal lattice is (2009)
(A) $4\left(\frac{\pi}{a}\right)^3$
(B) $4\left(\frac{2\pi}{a}\right)^3$
(C) $4\left(\frac{\pi}{2a}\right)^3$
(D) $4\left(\frac{3\pi}{a}\right)^3$

Statement for Linked Answer Questions 59 and 60: The Karnaugh map of logic circuit shown is below

	$\bar{R}$	R
$\bar{P}\bar{Q}$	1	1
$\bar{P}$Q	1	
PQ		
$P\bar{Q}$	1	1

Fig. 0.1: 1

59. The minimized logic expression for the above map is (2009)
    (A) $Y = \bar{P}\bar{R} + \bar{Q}$
    (B) $Y = \bar{Q}.PR$
    (C) $Y = PR + \bar{Q}$
    (D) $Y = \bar{P}\bar{R}.Q$
60. The corresponding logic implementation using gates is given as: (2009)

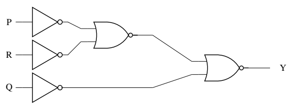

Fig. 0.2: option1

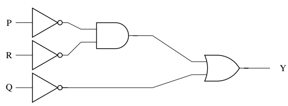

Fig. 0.3: option2

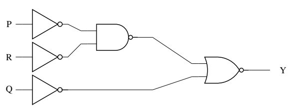

Fig. 0.4: option3

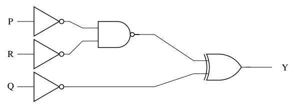

Fig. 0.5: option4