

Assignment 22

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14. Let Γ be the positively oriented circle $x^2 + y^2 = 9$ in the xy -plane. If

$$\oint_{\Gamma} (3y + e^{x \sin x} dx + (7x + \sqrt{e^y + 2})) dy = \alpha\pi,$$

where α is a real constant then α is equal to.....

15. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0, x > 0.$$

Let $W(y_1, y_2)(x)$ denote the Wronskian of $y_1(x)$ and $y_2(x)$ at x . If $W(y_1, y_2)(1) = 1$ then $W(y_1, y_2)(2)$ is equal to.....

16. Let $A = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 1 & 2 & 5 & -5 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}$. Then the sum of the geometric multiplicities of the distinct eigenvalues of A is equal to

17. In a cosmopolitan city, the population comprises of 30% female and 70% male. Suppose that 5% of female and 30% of male in the population are foreigners. A person is selected at random from this population. Given that the selected person is a foreigner, the probability that the person is a female is (round off to three decimal places).

18. Let $f : (0, \infty) \rightarrow \mathcal{R}$ be the continuous function such that $f(x) = 2 + \frac{g(x)}{x}$ for all $x > 0$, where $g(x) = \int_1^x f(t) dt$ for all $x \geq 0$. Then $f(2)$ is equal to

- (A) $2 + \ln 2$
- (B) $2 - \ln 2$
- (C) $2 + \ln 4$
- (D) $2 - \ln 4$

19. Let A and B be $n \times n$ matrices with real entries. Consider the following statements:

P: If A is symmetric then $\text{rank}(A) = \text{Number of nonzero eigenvalues (counting multiplicity) of } A$. Q: If $AB = 0$ then $\text{rank}(A) + \text{rank}(B) \leq n$.

Then

- (A) both P and Q are TRUE
- (B) P is TRUE and Q is FALSE
- (C) Q is TRUE and P is FALSE
- (D) both P and Q are FALSE

20. Let $f : \mathcal{R}^2 \rightarrow \mathcal{R}$ be given by $f(x, y) = 4xy - 2x^2 - y^4 + 1$. The number of critical points where f has local maximum is equal to

21. If the quadrature rule

$$\int_{-1}^1 f(x) dx \approx f(\alpha) + \gamma f(\beta),$$

where α, β and γ are real constants, is exact for all polynomials for degree ≤ 3 , then $\gamma + 3(\alpha^2 + \beta^2) + (\alpha^3 + \beta^3)$ is equal to

22. A heavy horizontal cylinder of diameter D supports a mass of liquid having density ρ as shown in the figure. Find out the vertical component of force exerted by the liquid per unit length of the cylinder if g is the acceleration due to gravity.



Fig. 0.1

- (A) $\frac{\pi D^2}{4} \rho g$
- (B) $\frac{\pi D^2}{8} \rho g$
- (C) $\frac{\pi D^2}{2} \rho g$
- (D) $\frac{\pi D^2}{3} \rho g$

23. The figure shows the developing zone and the fully developed region in a pipe flow where the steady flow takes place from left to right. The wall shear stress in the sections A, B, C, and D are given by $\tau_A, \tau_B, \tau_C, \tau_D$, respectively. Select the correct statement.

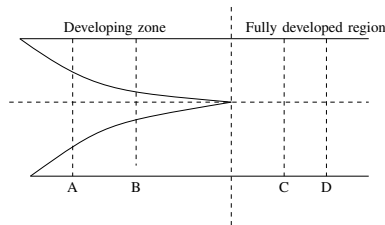


Fig. 0.2

- (A) $\tau_A > \tau_B$
- (B) $\tau_B > \tau_A$
- (C) $\tau_C > \tau_B$
- (D) $\tau_C > \tau_D$

24. The left hand column lists some non-dimensional numbers and the right hand column lists some physical phenomena. Indicate the correct combination

- (A) 1-iii, 2-i, 3-ii, 4-iv
- (B) 1-i, 2-ii, 3-iv, 4-iii
- (C) 1-iv, 2-iii, 3-iv, 4-iii

1. Reynolds number	i. Wave drag
2. Froude number	ii. compressible flow
3. Mach number	iii. Viscous drag
4. Weber number	iv. Spray formation

TABLE 0

(D) 2-iv, 1-iii, 3-ii, 4-i

25. As temperature increases

- (A) the dynamic viscosity of a gas increases.
- (B) the dynamic viscosity of a liquid decreases.
- (C) the dynamic viscosity of a liquid does not change.
- (D) the dynamic viscosity of a gas decreases.

26. Which of the following statement(s) regarding a venturimeter is/are correct?

- (A) In the direction of flow, it consists of a converging section, a throat, and a diverging section
- (B) In the direction of flow, it consists of a diverging section, a throat, and a converging section.
- (C) It is used for flow measurement at a very low Reynolds number.
- (D) Pressure tapings are provided just upstream of the venturimeter and at the throat.