7-7.2-22

AI24BTECH11008- Sarvajith

Question:
Find the equation of the circle having (1, -2) as its centre and passing through the intersection of 3x + y = 14, 2x + 5y = 18.
Solution:

varaibles	**values**
centre	$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$
line 1	$3x + y = 14$
line 2	$2x + 5y = 18$

TABLE 1 0: given values

Intersection point of the 2 linear equations $3x + y = 14$ and $2x + 5y = 18$ is given by

$$\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 18 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 & 14 \\ 2 & 5 & 18 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 3R_2 - 2R_1} \begin{pmatrix} 3 & 1 & 14 \\ 0 & 13 & 26 \end{pmatrix}$$
$$\xleftrightarrow{R_2 \leftarrow R_2/13} \begin{pmatrix} 3 & 1 & 14 \\ 0 & 1 & 2 \end{pmatrix}$$
$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 3 & 0 & 12 \\ 0 & 1 & 2 \end{pmatrix}$$
$$\xleftrightarrow{R_1 \leftarrow R_1/3} \begin{pmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \end{pmatrix}$$

$\therefore$ the intersecting point of the 2 lines is A(4,2).

$$r = \|A - C\| = \sqrt{(A - C)^T (A - C)}$$

Where A(4, 2) is point of intersection and point on the circle with the centre C(1, −2).

$$r = \sqrt{\begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix}} = 5$$

equation of a conic is given by $x^T V x + 2u^T x + f = 0$
for a circle

$$V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$u = \begin{pmatrix} -h \\ -k \end{pmatrix},$$
$$f = ||u||^2 - r^2$$

substituting the above values in the equation we get

$$x^T x + 2 \begin{pmatrix} -1 & 2 \end{pmatrix} x - 20 = 0$$

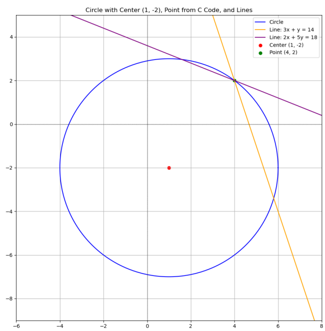

Fig. 0.1: plot for circle