Assignment 24

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27. Let

$$\frac{z}{1-z-z^2} = \sum_{n=0}^{\infty} a_n z^n, a_n \in R$$

for all z in some neighbourhood of 0 in C.
Then the value of $a_6 + a_5$ is equal to.........

28. Let $p(x) = x^3 - 2x + 2$. If $q(x)$ is the interpolating polynomial of degree less than or equal to 4 for the data in the table **polynomial data**
then the value of $\frac{d^4q}{dx^4}$ at x=0 is ....

29. For a fixed $c \in R$, let $\alpha = \int_0^2 \left(9x^2 - 5cx^4\right) dx$.
If the value of $\int_0^2 \left(9x^2 - 5cx^4\right) dx$ obtained by using the Trapezoidal rule is equal to α, then the value of c is ....(rounded off to 2 decimal places).

30. If for some $\alpha \in R$

$$\int_1^4 \int_{-x}^{x} \frac{1}{x^2+y^2} dydx = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \int_{sec\theta}^{\alpha sec\theta} \frac{1}{r} drd\theta,$$

then the value of α equals...........

31. Let S be the portion of the plane $z = 2x + 2y - 100$ which lies inside the cylinder $x^2 + y^2 = 1$. If the surface area of S is $\alpha\pi$, theen the value of α is equal to ....

32. Let $L^2[-1,1] = \{f : [-1,1] \to R :$ f is Lebesgue measureable and $\int_{-1}^{1} |f(x)|^2 dx < \infty\}$ and the norm $||f||_2 = \left(\int_{-1}^{1} |f(x)|^2 dx\right)^{\frac{1}{2}}$ for $f \in L^2[-1,1]$.
Let $F : \left(L^2[-1,1], ||.||_2\right) \to R$ be denoted by

$$F(f) = \int_{-1}^{1} f(x) x^2 dx \text{ for all } f \in L^2[-1,1].$$

If $||F||$ denotes the norm of the linear functional F, then $5\,||F||^2$ is equal to

33. Let $y(t)$ be the solution of the initial value problem

$$y\text{''} + 4y = \begin{cases} t & 0 \le t \le 2, \\ 2 & 2 < t < \infty \end{cases} and y(0) = y'(0) = 0.$$

If $\alpha = y\left(\frac{\pi}{2}\right)$ then the value of $\frac{4}{\pi}\alpha$ is ....(rounded off to 2 decimal places).

x	-2	-1	0	1	3
$q(x)$	$p(-2)$	$p(-1)$	2.5	$p(1)$	$p(3)$

TABLE 0: polynomial data

34. Consider R^4 with the inner product $< x, y >= \sum_{i=1}^{4} x_i y_i$, for $x = (x_1, x_2, x_3, x_4)$ and $y = (y_1, y_2, y_3, y_4)$.
Let $M = \{(x_1, x_2, x_3, x_4) \in R^4 : x_1 = x_3\}$ and $M^\perp$ denote the orthogonal complement of M. The dimension of $M^\perp$ is equal to ....

35. Let $M = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. If $6M^{-1} = M^2 - 6M + \alpha I$ for some $\alpha \in R$, then the value of α is equal to ....

36. Let $GL_2(C)$ denote the group of 2×2 invertible complex matrices with usual matrix multiplication. For $S, T \in GL_2(C)$, $< S, T >$ denotes the subgroup generated by S and T. Let $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in GL_2(C)$and G_1, G_2, G_3be three subgroups of $GL_2(C)$ given by

$$G_1 =< S, T_1 >, \text{ where } T_1 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$G_2 =< S, T_2 >, \text{ where } T_2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

$$G_1 =< S, T_3 >, \text{ where } T_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Let $Z(G_i)$ denote the center of G_i for i = 1,2,3.
Which of the following statements is correct?

(A) G_1 is isomorphic to G_3
(B) $Z(G_1)$ is isomorphic to $Z(G_2)$
(C) $Z(G_3) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(D) $Z(G_2)$ is isomorphic to $Z(G_3)$

37. Let $l^2 = \{(x_1, x_2, x_3, ...) : x_n \in R$ for all $n \in N$ and $\sum_{n=1}^{\infty} x_n^2 < \infty\}$.
For a sequence $(x_1, x_2, x_3, ...) \in l^2$, define $\|(x_1, x_2, x_3, ...)\|_2 = \left(\sum_{n=1}^{\infty} x_n^2\right)^{\frac{1}{2}}$.Let $S : \left(l^2, \|.\|_2\right) \to \left(l^2, \|.\|_2\right)$ and $T : \left(l^2, \|.\|_2\right) \to \left(l^2, \|.\|_2\right)$ be defined by

$$S(x_1, x_2, x_3, ...) = (y_1, y_2, y_3, ...), \text{ where } y_n = \begin{cases} 0 & n = 1 \\ x_{n-1} & n \geq 2 \end{cases}$$

$$T(x_1, x_2, x_3, \dots) = (y_1, y_2, y_3, \dots), \quad \text{where } y_n = \begin{cases} 0, & \text{if } n \text{ is odd} \\ x_n, & \text{if } n \text{ is even} \end{cases}$$

(A) S is a compact linear map and T is NOT a compact linear map
(B) S is NOT a compact linear map and T is a compact linear map
(C) both S and T are compact linear maps
(D) NEITHER S NOR T is a compact linear map

38. Let

$$c_{00} = \{(x_1, x_2, x_3, \dots) : x_i \in \mathbb{R}, i \in \mathbb{N}, x_i \neq 0 \text{ only for finitely many indices } i\}.$$

For $(x_1, x_2, x_3, \dots) \in c_{00}$, let

$$|(x_1, x_2, x_3, \dots)|_\infty = \sup\{|x_i| : i \in \mathbb{N}\}.$$

Define $F, G : (c_{00}, \|\cdot\|_\infty) \to (c_{00}, \|\cdot\|_\infty)$ by

$$F\left((x_1, x_2, \dots, x_n, \dots)\right) = \left(\left(1 + \frac{1}{1}\right)x_1, \left(2 + \frac{1}{2}\right)x_2, \dots, \left(n + \frac{1}{n}\right)x_n, \dots\right),$$

$$G\left((x_1, x_2, \dots, x_n, \dots)\right) = \left(\frac{x_1}{1 + \frac{1}{1}}, \frac{x_2}{2 + \frac{1}{2}}, \dots, \frac{x_n}{n + \frac{1}{n}}, \dots\right),$$

for all $(x_1, x_2, \dots, x_n, \dots) \in c_{00}$

(A) F is continuous but G is NOT continuous
(B) F is NOT continuous but G is continuous
(C) both F and G are continuous
(D) NEITHER F NOR G is continuous

39. Consider the Cauchy problem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u;$$

$$u = f(t) \text{ on the initial curve } \Gamma = (t, t), \quad t > 0.$$

Consider the following statements:

- P: If $f(t) = 2t + 1$, then there exists a unique solution to the Cauchy problem in a neighbourhood of Γ.
- Q: If $f(t) = 2t - 1$, then there exist infinitely many solutions to the Cauchy problem in a neighbourhood of Γ.

(A) both P and Q are TRUE
(B) P is FALSE and Q is TRUE
(C) P is TRUE and Q is FALSE
(D) both P and Q are FALSE