Assignment 11

AI24BTECH11008- Sarvajith

14. The linear Programming Problem:
    Maximize $z = x_1 + x_2$
    subject to (2011)

$$x_1 + 2x_2 \leq 20$$
$$x_1 + x_2 \leq 15$$
$$x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

   (A) has exactly one optimum solution
   (B) has more than one optimum solution
   (C) has unbounded solution
   (D) has no solution

15. Consider the Primal Linear Programming Problem:

$$\text{P:} \begin{cases} \text{Maximize} z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ \text{subject to} \\ a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \\ ... \\ ... \\ ... \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \\ x_j \geq 0, j = 1, \ldots, n \end{cases}$$

The Dual of P is

$$\text{D:} \begin{cases} \text{Minimize} z' = b_1w_1 + b_2w_2 + \ldots + b_mw_n \\ \text{subject to} \\ a_{11}w_1 + a_{21}w_2 + \ldots + a_{m1}w_m \geq c_1 \\ a_{12}w_1 + a_{22}w_2 + \ldots + a_{m2}w_m \geq c_2 \\ ... \\ ... \\ ... \\ a_{1n}w_1 + a_{2n}w_2 + \ldots + a_{mn}w_m \geq c_n \\ w_i \geq 0, i = 1, \ldots, m \end{cases}$$ (2011)

   (A) $N_g << N_e$
   (B) $N_g >> N_e$

(C) $N_g \approx N_e \approx \frac{N}{2}$
(D) $N_g - N_e \approx \frac{N}{2}$

16. The number of irreducible quadratic polynomials over the field of 2 elements F_2 is (2011)
   (A) 0
   (B) 1
   (C) 2
   (D) 3
17. The number of elements in the conjugacy class of the 3-cycle(234) in symmetric group S_6 is (2011)
   (A) 20
   (B) 40
   (C) 120
   (D) 216
18. The initial value Problem

$$x\frac{dy}{dx} = y + x^2, x > 0; y(0) = 0,$$

   has (2011)
   (A) infinitely many solutions
   (B) exactly two solutions
   (C) a unique solution
   (D) no solution
19. The subspace $P = \{(x, y, z) \in R^3 : z = x^2 + y^2 + 1\}$ is (2011)
   (A) comapct and connected
   (B) compact but not connected
   (C) not compact but connected
   (D) neither compact nor connected
20. Let $P = (0, 1); Q = [0, 1); U = (0, 1], T = R\text{and}A = \{P, Q, U, S, T\}$. The equivalence relation 'homeomorphism' induces which one of the following as the partition of A? (2011)
   (A) {P,Q,U,S},{T}
   (B) {P,T},{Q},{U},{S}
   (C) {P,T},{Q,U,S}
   (D) {P,T},{Q,U},{S}
21. Let $x = (x_1, x_2, \ldots) \in l^4, x \neq 0$. For which one of the following values of p, the series $\sum_{i=1}^{\infty} x_i y_i$ converges for every $y = (y_1, y_2, \ldots) \in l^p$? (2011)
   (A) 1
   (B) 2
   (C) 3
   (D) 4
22. Let H be a complex Hilbert space and H^* be its dual. The mapping $\phi : H \to H^*$ defined by $\phi(y) f_y \text{where} f_y(x) = \langle x, y \rangle$ is (2011)

(A) not linear but onto
(B) both linear and onto
(C) linear but not onto
(D) neither linear nor onto

23. A horizontal lever is in static equilibrium under the application of vertical forces F_1 at a distance l_1 from the fulcrum and F_2 at a distance l_2 from the fulcrum. The equilibrium for the above quantities can be obtained if (2011)

(A) $F_1L_1 = 2F_2L_2$
(B) $2F_1L_1 = F_2L_2$
(C) $F_1L_1 = F_2L_2$
(D) $F_1L_1 < F_2L_2$

24. Assume F to be a twice continuously differentiable function. Let $J(y)$ be a functional of the form

$$\int_0^1 F(x, y')\,dx, 0 \leq x \leq 1$$

defined on the set of all continuously differentiable functions y on [0,1] satisfying $y(0) = a, y(1) = b$. For some arbitrary constant c, a necessary condition for y to be an extremum of J is (2011)

(A) $\frac{\partial F}{\partial x} = c$
(B) $\frac{\partial F}{\partial y'} = c$
(C) $\frac{\partial F}{\partial y} = c$
(D) $\frac{\partial F}{\partial x} = 0$

25. The eigenvalue λ of the following Fredholm integral equation

$$y(x) = \lambda \int_0^1 x^2 t y(t)\,dt$$

is (2011)

(A) -2
(B) 2
(C) 4
(D) -4

Q.26-Q.55 carry two marks each.

26. The application of Gram-Schmidt process of orthonormalization to

$$u_1 = (1,1,0), u_2 = (1,0,0), u_3 = (1,1,1)$$

yields (2011)

(A) $\frac{1}{\sqrt{2}}(1,1,0), (1,0,0), (0,0,1)$
(B) $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(1,-1,0), \frac{1}{\sqrt{2}}(1,1,1)$
(C) $(0,1,0), (1,0,0), (0,0,1)$
(D) $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(1,-1,0), (0,0,1)$