Assignment-4

Sarvajith-AI24BTECH11008

16: If P and Q are two statements, then which of the following compound statements is a tautology?

a. $((P \to Q) \wedge \neg Q) \to P$
b. $((P \to Q) \wedge \neg Q) \to \neg P$
c. $((P \to Q) \wedge \neg Q)$
d. $((P \to Q) \wedge \neg Q) \to \neg Q$

17: Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P\left(4, \sqrt{6}\right)$ meet the x-axis at Q and the latus rectum at $R(x_1, y_1)$, where $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to:

a. $\sqrt{6} - 1$
b. $4\sqrt{6} - 1$
c. $4\sqrt{6}$
d. $\frac{7}{\sqrt{6}} - 2$

18: Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & x < 0 \\ b, & x = 0 \\ \frac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}}, & x > 0 \end{cases}$$

If f is continuous at x = 0, then the value of a+b is equal to

a. -2
b. $\frac{-2}{5}$
c. $\frac{-3}{2}$
d. -3

19: Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)\, e^{x^2/2} - x\right)$, $0 < x < 2.1$, with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x=1 is equal to

a. $\frac{e^{5/2}}{\left(1+e^2\right)^2}$
b. $\frac{5e^{1/2}}{\left(e^2\right)+1}$
c. $\frac{-2e^2}{\left(1+e^2\right)^2}$
d. $\frac{-e^{3/2}}{\left(e^2+1\right)^2}$

20: Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by a tangent is minimum is equal to:

a. $\frac{\pi}{8}$
b. $\frac{\pi}{6}$
c. $\frac{\pi}{3}$
d. $\frac{\pi}{4}$

I. Section-B

1: Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $(|5\alpha|)$ is equal to?

2: $\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right) = \alpha\,(11!)$ Then the value of α is equal to:

3: The term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$, $x \neq 1$ is equal to?

4: Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10} (22 + 3k)^n\, C_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$,then $\alpha + \beta$ is equal to?

5: Let $P(x)$ be a real polynomial of degree 3 which vanishes at $x = -3$. Let $P(x)$ have local minima at $x = 1$, local maxima at $x = -1$, and $\int_{-1}^{1} P(x)\, dx = 18$,then the sum of all the coefficients of the polynomial $P(x)$ is equal to?

6: Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\mathbf{r} \cdot (2\mathbf{i} - \mathbf{j} + \mathbf{k}) - b = 0$ be $(-3, 5, 2)$, Then, the value of $|a + b|$ is equal to?

7: If $f(x)$ and gx are two polynomials such that the polynomial $P(x) = f\left(x^3\right) + xg\left(x^3\right)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to?

8: Let I be an identity matrix of order 2×2 and

$$P = \begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$$

$P_n = 5I - 8P$ Then the value of $n \in \mathbb{N}$ for which $P_n = 5I - 8P$ is equal to?

9: Let $f : \mathbb{R} \to \mathbb{R}$ satisfy the equation $f(x+y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$ and $f(x) \neq 0$ for any $x \in \mathbb{R}$. If the function f is differentiable at $x = 0$ and $f'(0) = 3$, then $\lim_{h\to 0} \frac{1}{h}[f(h) - 1]$ is equal to?

10: Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{x^2 - y^2}\, dx, \quad x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^{\pi}$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$,then the value of $10(\alpha + \beta)$ is equal to?