3-Shaxsiy topshiriqlar

6-7-8 mavzular bo‘yicha.

1-teorema (Kroneker-Kapelli teoremasi). Chiziqli tenglamalar sistemasi birgalikda bo‘lishi uchun uning A asosiy matritsasi va kengaytirilgan $(A|B)$ matritsalarining ranglari teng bo‘lishi zarur va yetarli.

1). $rangA \neq rangB$ bo‘lsa, tenglamalar sistemasi birgalikda emas;

2). $rangA = rangB = r = n$ bo‘lsa, tenglamalar sistemasi yagona yechimga ega;

3). $rangA = rang\ B = r < n$ bo‘lsa, tenglamalar sistemasi cheksiz ko‘p yechimga ega.

Chiziqli algebraik tenglamalar sistemasini yechimning Kramer usuli.

Determinantlarni chiziqli tenglamalar sistemasini yechishga tatbiqi bo‘lgan *Kramer(determinant) usuli* bilan tanishamiz. Aytaylik, bizga n ta noma’lumli n ta chiziqli tenglamalar sistemasi berilgan bo‘lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots\dots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots\dots + a_{nn}x_n = b_n \end{cases} \qquad (1)$$

Bu yerda $x_1, x_2, ..., x_n$ $-$noma’lumlar, $a_{11}, a_{12}, ..., a_{nn}$ $-$koeffitsientlar, $b_1, b_2, ..., b_n$ $-$ozod sonlar.

Teorema *Agar (1)- tenglamalar sistemasining asosiy determinanti* $(\Delta \neq 0)$ *noldan farqli bo‘lsa, u holda sistema yagona yechimga ega bo‘ladi va u quyidagi formulalardan topiladi .*

$$\Delta \neq 0\,, \quad x_1 = \frac{\Delta_{x_1}}{\Delta}, \quad x_2 = \frac{\Delta_{x_2}}{\Delta}, \ldots, \; x_n = \frac{\Delta_{x_n}}{\Delta} \tag{2}$$

Bu *Kramer* formulasidan iborat. Bu yerda $\Delta \neq 0$ ga bosh determinant, $\Delta_{x_1}, \Delta_{x_2}, \Delta_{x_3}, \ldots, \Delta_{x_n}$ larga *yordamchi determinantlar* deyiladi. Soddalik uchun uch noma'lumli, uchta chiziqli tenglamalar sistemasini qaraymiz:

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases} \tag{3}$$

uch noma'lumli uchta chiziqli tenglamalar sistemasini yechishda dastlab bosh (asosiy) determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \tag{4}$$

topiladi. $\Delta \neq 0$ bo'lsin. Undan so'ng yordamchi determinantlar hisoblanadi (bunda bosh determinantning ustun elementlari mos ravsihda ozod hadlar bilan almashtiriladi):

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} \tag{5}$$

Noma'lumlar quyidagi formulalar yordamida hisoblanadi:

$$x = \frac{\Delta_x}{\Delta}, \qquad y = \frac{\Delta_y}{\Delta}, \qquad z = \frac{\Delta_z}{\Delta} \tag{6}$$

1-misol.

Ushbu sistemani Kramer usulida yeching:

$$\begin{cases} x + 5y - z = 3, \\ 2x + 4y - 3z = 2, \\ 3x - y - 3z = -7 \end{cases}$$

►Quyidagi determinantlarni tuzamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 5 & -1 \\ 2 & 4 & -3 \\ 3 & -1 & -3 \end{vmatrix} = -16; \qquad \Delta_x = \begin{vmatrix} 3 & 5 & -1 \\ 2 & 4 & -3 \\ -7 & -1 & -3 \end{vmatrix} = 64;$$

$$\Delta_y = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 2 & -3 \\ 3 & -7 & -3 \end{vmatrix} = -16; \qquad \Delta_z = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 4 & 2 \\ 3 & -1 & -7 \end{vmatrix} = 32.$$

Bundan, $x = \dfrac{64}{-16} = -4,\quad y = \dfrac{-16}{-16} = 1,\quad z = \dfrac{32}{-16} = -2.$ ◄

Agar bosh determinant nolga teng bo‘lsa, tenglamalar sistemasi yechimga ega bo‘lmaydi yoki cheksiz ko‘p yechimga ega bo‘ladi. Ya’ni,

1) $\Delta = 0$ bo‘lib, Δ_x, Δ_y, Δ_z lardan kamida bittasi nolga teng bo‘lmasa, (3) tengamalar system asi yechimga ega bo‘lmaydi,

2) $\Delta = 0$ bo‘lib, $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$ bo‘lsa, sistema cheksiz ko‘p yechimga ega bo‘ladi.

2-misol.

Ushbu sistemani Kramer usulida yeching:

$$\begin{cases} x + 2y - 3z = 7 \\ 2x + y - 2z = 9 \\ 3x \qquad - z = 10 \end{cases}$$

► Bosh determinantini hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 1 & -2 \\ 3 & 0 & -1 \end{vmatrix} = -1 - 12 + 0 + 9 - 0 + 4 = 0.$$

Yordamchi determinantlarni hisoblaymiz:

$$\Delta_x = \begin{vmatrix} 7 & 2 & -3 \\ 9 & 1 & -2 \\ 10 & 0 & -1 \end{vmatrix} = -7-40+0+30-0+18=1.$$

$\Delta = 0$ bo‘lib, $\Delta_x = 1 \neq 0$ bo‘lgani uchun berilgan tenglamalar sistemasi yechimga ega emas.◄

3-misol.

Ushbu sistemani Kramer usulida yeching:

$$\begin{cases} x-2y+z=5 \\ 2x \quad\;\; -z=3 \\ 3x+2y-3z=1 \end{cases}$$

►Quyidagi determinantlarni tuzamiz va hisoblaymiz:

$$\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ 3 & 2 & -3 \end{vmatrix} = 0+6+4-0+2-12=0,$$

$$\Delta_x = \begin{vmatrix} 5 & -2 & 1 \\ 3 & 0 & -1 \\ 1 & 2 & -3 \end{vmatrix} = 0+2+6-0+10-18=0,$$

$$\Delta_y = \begin{vmatrix} 1 & 5 & 1 \\ 2 & 3 & -1 \\ 3 & 1 & -3 \end{vmatrix} = -9-15+2-9+1+30=0,$$

$$\Delta_z = \begin{vmatrix} 1 & -2 & 5 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0-18+20-0-6+4=0.$$

$\Delta = 0$ bo‘lib, $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$ bo‘lgani uchun sistema cheksiz ko‘p yechimga ega bo‘ladi.

Bu holda 2 ta tenglamani qoldirib, erkli noma'lum, masalan, z ni tenlikning o'ng tomoniga o'tkazamiz:

$$\begin{cases} x-2y+z=5 \\ 2x \quad\quad -z=3 \end{cases} \Leftrightarrow \begin{cases} x-2y=5-z \\ 2x \quad\quad =3+z \end{cases}$$

Hosil bo'lgan ikki noma'lumli tenglamalar sistemasini yana Kramer usulida yechamiz.

$$\Delta=\begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix}=4,\ \Delta_x=\begin{vmatrix} 5-z & -2 \\ 3+z & 0 \end{vmatrix}=6+2z,\ \Delta_y=\begin{vmatrix} 1 & 5-z \\ 2 & 3+z \end{vmatrix}=-7+3z.$$

Demak,

tenglamalar sistemasining umumiy yechimi: $\left\{\dfrac{z+3}{2};\ \dfrac{3z-7}{4};\ z\right\}$.◄

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1- Vazifa.

Chiziqli algyebraik tenglamalar sistemasi birgalikda ekanligini tekshiring. Agar birgalikda bo'lsa, uni Kramer formulalari bo'yicha eching:

1. $\begin{cases} 2x_1+x_2+3x_3=7, \\ 2x_1+3x_2+x_3=1, \\ 3x_1+2x_2+x_3=6; \end{cases}$

2. $\begin{cases} 2x_1-x_2+2x_3=3, \\ x_1+x_2+2x_3=-4, \\ 4x_1+x_2+4x_3=-3; \end{cases}$

3. $\begin{cases} 3x_1-x_2+x_3=12, \\ x_1+2x_2+4x_3=6, \\ 5x_1+x_2+2x_3=3; \end{cases}$

4. $\begin{cases} 2x_1-x_2+3x_3=-4, \\ x_1+3x_2-x_3=11, \\ x_1-2x_2+2x_3=-7; \end{cases}$

5. $\begin{cases} 3x_1-2x_2+4x_3=12, \\ 3x_1+4x_2-2x_3=6, \\ 2x_1-x_2-x_3=-9; \end{cases}$

6. $\begin{cases} 8x_1+3x_2-6x_3=-4, \\ x_1+x_2-x_3=2, \\ 4x_1+x_2-3x_3=-5; \end{cases}$

7. $\begin{cases} 4x_1+x_2-3x_3=9, \\ x_1-x_2-x_3=-2, \\ 8x_1+3x_2-6x_3=0; \end{cases}$

8. $\begin{cases} 2x_1+3x_2+4x_3=33, \\ 7x_1-5x_2 \quad\quad =24, \\ 4x_1 \quad\quad +x_3=39; \end{cases}$

9. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 12, \\ 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7; \end{cases}$ 10. $\begin{cases} x_1 + 4x_2 - x_3 = 6, \\ 5x_2 + 4x_3 = -20, \\ 3x_1 - 2x_2 + 5x_3 = -22; \end{cases}$

11. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 21, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 - x_2 - x_3 = 10; \end{cases}$ 12. $\begin{cases} 3x_1 - 2x_2 - 5x_3 = 5, \\ 2x_1 + 3x_2 - 4x_3 = 12, \\ x_1 - 2x_2 + 3x_3 = -1; \end{cases}$

13. $\begin{cases} 4x_1 + x_2 + 4x_3 = 19, \\ 2x_1 - x_2 + 2x_3 = 11, \\ x_1 + x_2 + 2x_3 = 8; \end{cases}$ 14. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 6, \\ x_1 + x_2 + 2x_3 = 4; \end{cases}$

15. $\begin{cases} 2x_1 - x_2 + 2x_3 = 8, \\ x_1 + x_2 + 2x_3 = 11, \\ 4x_1 + x_2 + 4x_3 = 22; \end{cases}$ 16. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15; \end{cases}$

17. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + 5x_2 + x_3 = -3; \end{cases}$ 18. $\begin{cases} -3x_1 + 5x_2 + 6x_3 = -8, \\ 3x_1 + x_2 + x_3 = -4, \\ x_1 - 4x_2 - 2x_3 = -9; \end{cases}$

19. $\begin{cases} 3x_1 + x_2 + x_3 = -4, \\ -3x_1 + 5x_2 + 6x_3 = 36, \\ x_1 - 4x_2 - 2x_3 = -19; \end{cases}$ 20. $\begin{cases} 3x_1 - x_2 + x_3 = 11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16; \end{cases}$

21. $\begin{cases} x_1 - 3x_2 - 7x_3 = 0 \\ x_1 + 2x_2 + 4x_3 = 6, \\ 4x_1 - x_2 - 2x_3 = -3; \end{cases}$ 22. $\begin{cases} 3x_1 + 2x_2 + 2x_3 = 7, \\ x_1 + 3x_2 - x_3 = 11, \\ 3x_1 + 4x_2 = 15; \end{cases}$

23. $\begin{cases} x_1 - x_2 + 5x_3 = 21, \\ x_1 + 5x_2 - x_3 = 15, \\ 2x_1 - x_2 - x_3 = -9; \end{cases}$ 24. $\begin{cases} 6x_1 + x_2 - 4x_3 = -8, \\ x_1 + x_2 - x_3 = 2, \\ 4x_1 + x_2 - 3x_3 = -5; \end{cases}$

25. $\begin{cases} x_1 - x_2 - x_3 = -2, \\ 4x_1 + x_2 - 3x_3 = 9, \\ 4x_1 + 2x_2 - 3x_3 = -9; \end{cases}$ 26. $\begin{cases} 2x_1 - 3x_2 - 3x_3 = 6, \\ 5x_1 - 8x_2 - 4x_3 = -9, \\ 4x_1 + x_3 = 39; \end{cases}$

27. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 12, \\ 7x_1 - 8x_2 + 4x_3 = 38, \\ 4x_1 + x_3 = -7; \end{cases}$ 28. $\begin{cases} 7x_2 + 4x_3 = 20, \\ x_1 - x_2 - 5x_3 = 26, \\ x_1 - 3x_2 + 3x_3 = -14; \end{cases}$

29. $\begin{cases} x_1 - x_2 + 5x_3 = 11, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 + 5x_2 + 4x_3 = 10; \end{cases}$ 30. $\begin{cases} x_1 + x_2 + 7x_3 = 5, \\ x_1 - 9x_2 + 13x_3 = -15, \\ x_1 - 2x_2 + 3x_3 = -39. \end{cases}$

Chiziqli algebraik tenglamalar sistemasini yechishning matritsa usuli.

Ushbu n noma'lumli n ta chiziqli algebraik tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2, \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n. \end{cases} \tag{7}$$

tenglamalar sistemada quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \; X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \; B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

Bu yerda, $A-$noma'lumlar oldida turgan koeffitsiyentlardan tuzilgan matritsa; $X-$noma'lumlardan tuzilgan matritsa; $B-$ozod hadlardan tuzilgan matritsa.

U holda (1) tenglamalar sistemasini

$$AX = B \tag{8}$$

ko'rinishda ifodalash mumkin.

Faraz qilamiz, $\det|A| \neq 0$ bo'lsin. U holda A matritsa uchun A^{-1} teskari matritsa mavjud. $AX = B$ tenglikning har ikkala tomonini A^{-1} ga chapdan ko'paytiramiz:

$$A^{-1}AX = A^{-1}B, \qquad EX = A^{-1}B, \qquad X = A^{-1}B.$$

Hosil bo'lgan $X = A^{-1}B$ ifoda chiziqli tenglamalar sistemasini matritsalar usuli bilan yechish formulasidan iborat.

4-misol. Chiziqli tenglamalar sistemasini matritsalar usuli bilan yeching:

$$\begin{cases} 2x_1 + 2x_2 - 3x_3 = 5, \\ x_1 - x_2 + x_3 = 0, \\ 3x_1 + x_2 + x_3 = 2. \end{cases}$$

Yechish. A, X, B matritsalarni tuzib olamiz:

$$A=\begin{pmatrix} 2 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{pmatrix},\ X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\ B=\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

Bundan, $\det|A|=-12\neq 0.$ Teskari matritsani topamiz:

$$A_{11}=\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix}=-2,\quad A_{12}=\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix}=2,\quad A_{13}=\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}=4,$$

$$A_{21}=\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}=-5,\quad A_{22}=\begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}=11,\quad A_{23}=\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix}=4,$$

$$A_{31}=\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix}=-1,\quad A_{11}=\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}=-5,\quad A_{33}=\begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix}=-4,$$

$$A^{-1}=-\frac{1}{12}\begin{pmatrix} -2 & -5 & -1 \\ 2 & 11 & -5 \\ 4 & 4 & -4 \end{pmatrix}.$$

Bundan:

$$X=A^{-1}B=-\frac{1}{12}\begin{pmatrix} -2 & -5 & -1 \\ 2 & 11 & -5 \\ 4 & 4 & -4 \end{pmatrix}\cdot\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}=-\frac{1}{12}\begin{pmatrix} -10+0-2 \\ 10+0-10 \\ 20+0-8 \end{pmatrix}=-\frac{1}{12}\begin{pmatrix} -12 \\ 0 \\ 12 \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Demak, $x_1=1,\ x_2=0,\ x_3=-1$ yoki $X=(1;0;-1)^t$.

5-misol. Quyidagi tenglamani yeching:

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}X\begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix}=\begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}$$

Yechish. Tenglamaga quyidagi belgilashlarni kiritamiz:

$$A=\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix},\ B=\begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix},\ C_1=\begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}.$$

U holda berilgan tenglama

$$A\cdot X\cdot B=C$$

ko‘rinishni oladi.

Agar AXB ifodaning chap tomondan A^{-1} va o‘ng tomondan B^{-1} ga ko‘paytirsak, hamda $A^{-1}A=E$, $EX=X$, $BB^{-1}=E$ va $XE=X$ ekanligini hisobga olsak quyidagi yechimga ega bo‘lamiz:

$$X=A^{-1}CB^{-1}=-\frac{1}{2}\begin{pmatrix}1 & -1\\ -2 & 0\end{pmatrix}\cdot\begin{pmatrix}4 & 0\\ 5 & 8\end{pmatrix}\cdot\begin{pmatrix}-2 & 1\\ 1 & -\frac{1}{3}\end{pmatrix}=$$

$$=-\frac{1}{2}\begin{pmatrix}-1 & -8\\ -8 & 0\end{pmatrix}\cdot\begin{pmatrix}-2 & 1\\ 1 & -\frac{1}{3}\end{pmatrix}=\begin{pmatrix}3 & -\frac{5}{6}\\ -8 & 4\end{pmatrix}.$$

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2-Vazifa.

Chiziqli algebraik tenglamalar sistemasi birgalikda ekanligini tekshiring. Agar birgalikda bo’lsa, uni matritsa usulida yeching.

1. $\begin{cases}2x_1+x_2+3x_3=7,\\ 2x_1+3x_2+x_3=1,\\ 3x_1+2x_2+x_3=6;\end{cases}$ 2. $\begin{cases}2x_1-x_2+2x_3=3,\\ x_1+x_2+2x_3=-4,\\ 4x_1+x_2+4x_3=-3;\end{cases}$

3. $\begin{cases}3x_1-x_2+x_3=12,\\ x_1+2x_2+4x_3=6,\\ 5x_1+x_2+2x_3=3;\end{cases}$ 4. $\begin{cases}2x_1-x_2+3x_3=-4,\\ x_1+3x_2-x_3=11,\\ x_1-2x_2+2x_3=-7;\end{cases}$

5. $\begin{cases}3x_1-2x_2+4x_3=12,\\ 3x_1+4x_2-2x_3=6,\\ 2x_1-x_2-x_3=-9;\end{cases}$ 6. $\begin{cases}8x_1+3x_2-6x_3=-4,\\ x_1+x_2-x_3=2,\\ 4x_1+x_2-3x_3=-5;\end{cases}$

7. $\begin{cases}4x_1+x_2-3x_3=9,\\ x_1-x_2-x_3=-2,\\ 8x_1+3x_2-6x_3=0;\end{cases}$ 8. $\begin{cases}2x_1+3x_2+4x_3=33,\\ 7x_1-5x_2=24,\\ 4x_1+x_3=39;\end{cases}$

9. $\begin{cases}2x_1+3x_2+4x_3=12,\\ 7x_1-5x_2+x_3=-33,\\ 4x_1+x_3=-7;\end{cases}$ 10. $\begin{cases}x_1+4x_2-x_3=6,\\ 5x_2+4x_3=-20,\\ 3x_1-2x_2+5x_3=-22;\end{cases}$

11. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 21, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 - x_2 - x_3 = 10; \end{cases}$

12. $\begin{cases} 3x_1 - 2x_2 - 5x_3 = 5, \\ 2x_1 + 3x_2 - 4x_3 = 12, \\ x_1 - 2x_2 + 3x_3 = -1; \end{cases}$

13. $\begin{cases} 4x_1 + x_2 + 4x_3 = 19, \\ 2x_1 - x_2 + 2x_3 = 11, \\ x_1 + x_2 + 2x_3 = 8; \end{cases}$

14. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 6, \\ x_1 + x_2 + 2x_3 = 4; \end{cases}$

15. $\begin{cases} 2x_1 - x_2 + 2x_3 = 8, \\ x_1 + x_2 + 2x_3 = 11, \\ 4x_1 + x_2 + 4x_3 = 22; \end{cases}$

16. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15; \end{cases}$

17. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + 5x_2 + x_3 = -3; \end{cases}$

18. $\begin{cases} -3x_1 + 5x_2 + 6x_3 = -8, \\ 3x_1 + x_2 + x_3 = -4, \\ x_1 - 4x_2 - 2x_3 = -9; \end{cases}$

19. $\begin{cases} 3x_1 + x_2 + x_3 = -4, \\ -3x_1 + 5x_2 + 6x_3 = 36, \\ x_1 - 4x_2 - 2x_3 = -19; \end{cases}$

20. $\begin{cases} 3x_1 - x_2 + x_3 = 11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16; \end{cases}$

21. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 21, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 - x_2 - x_3 = 10; \end{cases}$

22. $\begin{cases} 3x_1 - 2x_2 - 5x_3 = 5, \\ 2x_1 + 3x_2 - 4x_3 = 12, \\ x_1 - 2x_2 + 3x_3 = -1; \end{cases}$

23. $\begin{cases} 4x_1 + x_2 + 4x_3 = 19, \\ 2x_1 - x_2 + 2x_3 = 11, \\ x_1 + x_2 + 2x_3 = 8; \end{cases}$

24. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 6, \\ x_1 + x_2 + 2x_3 = 4; \end{cases}$

25. $\begin{cases} 2x_1 - x_2 + 2x_3 = 8, \\ x_1 + x_2 + 2x_3 = 11, \\ 4x_1 + x_2 + 4x_3 = 22; \end{cases}$

26. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15; \end{cases}$

27. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + 5x_2 + x_3 = -3; \end{cases}$

28. $\begin{cases} -3x_1 + 5x_2 + 6x_3 = -8, \\ 3x_1 + x_2 + x_3 = -4, \\ x_1 - 4x_2 - 2x_3 = -9; \end{cases}$

29. $\begin{cases} 3x_1 + x_2 + x_3 = -4, \\ -3x_1 + 5x_2 + 6x_3 = 36, \\ x_1 - 4x_2 - 2x_3 = -19; \end{cases}$

30. $\begin{cases} 3x_1 - x_2 + x_3 = 11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16; \end{cases}$

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Chiziqli algebraik tenglamalar sistemasini yechishning Gauss usuli.

6-misol. Chiziqli tenglamalar sistemasini Gauss usuli bilan yeching:

$$\begin{cases} 2x_1 - x_2 - 4x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = -9, \\ x_1 + 4x_2 - 2x_3 = 4. \end{cases}$$

Yechish. Chiziqli tenglamalar sistemasidagi noma'lumlarni ketma-ket yo'qotib yechimni topamiz:

$$\begin{cases} 2x_1 - x_2 - 4x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = -9, \\ x_1 + 4x_2 - 2x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 4x_2 - 2x_3 = 4, \\ 2x_1 - x_2 - 4x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = -9 \end{cases} \Rightarrow \begin{cases} x_1 + 4x_2 - 2x_3 = 4, \\ 9x_2 = 9, \\ 14x_2 - 7x_3 = 21. \end{cases}$$

$$\Rightarrow \begin{cases} x_1 + 4x_2 - 2x_3 = 4, \\ x_3 - 2x_2 = -3, \\ x_2 = 1. \end{cases}$$

$x_2 = 1$ qiymatni ikkinchi tenglamaga qo'yib, $x_3 = -1$ qiymatni hosil qilamiz. $x_2 = 1$ va $x_3 = -1$ qiymatlarni birinchi tenglamaga qo'yib $x_1 = -2$ qiymatni olamiz. Shunday qilib, sistema yagona $(-2;1;-1)$ yechimga ega.

Tenglamalar sistemasida noma'lumlar soni tenlamalar sonidan ko'p bo'lsa ham, ya'ni sistema birgalikda bo'lib aniq bo'lmasa ham uning yechimini Gauss usulida topish mumkin. Buni quyidagi misolda ko'rib chiqamiz.

7-misol. Quyidagi chiziqli tenglamalar sistemasini Gauss usuli bilan yeching:

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 4, \\ x_1 + x_2 + x_3 + x_4 = 10, \\ 7x_1 + 2x_2 + 8x_3 - 6x_4 = 44, \\ 5x_1 + 2x_2 + 5x_3 - 6x_4 = 30. \end{cases}$$

Yechish. Birinchi qadamda sistemadagi birinchi tenglamani o'zgarishsiz qoldirib, qolganlaridan ketma-ket x_1 noma'lumni yo'qotamiz, ikkinchi qadamda ikkinchi tenglamani qoldirib qolganlaridan x_2 noma'lumni yo'qotamiz, uchinchi qadamda uchinchi tenglamani qoldirib qolganlaridan x_3 noma'lumni yo'qotamiz. Soddalik uchun tenglamalar sistemasi o'rniga kengaytirilgan matritsa ustida ish olib boramiz:

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 1 & 1 & 1 & 1 & 10 \\ 7 & 2 & 8 & -6 & 44 \\ 5 & 2 & 5 & -6 & 30 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 9 & -6 & 1 & 16 \\ 0 & 7 & -5 & 1 & 10 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 0 & 3 & 16 & 22 \\ 0 & 0 & 3 & 16 & 22 \end{array}\right)$$

Hosil bo'lgan sistemada ikkita bir hil tenglamadan bittasini qoldirib, ikkinchisini tashlab yuboramiz. Shu yerda chiziqli tenglamalar sistemasini yechishning chapdan o'ngga qarab bosqichi tugadi. Tenglamalar soni noma'lumlar sonidan kichik. Endi x_4 erkli o'zgaruvchini o'ng tomonga o'tkazamiz. So'ngra o'ngdan chapga qarab harakat yordamida sistemaning barcha yechimlari topiladi.

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 4, \\ \quad 2x_2 - x_3 + 2x_4 = 6, \\ \quad\quad 3x_3 + 16x_4 = 22 \end{cases} \Rightarrow \begin{cases} x_1 = 8x_4 - 34/3 \\ x_2 = -(11x_4 + 2)/3 \\ x_3 = -(16x_4 - 22)/3 \end{cases} \Rightarrow$$

Javob: $\left(8x_4 - \frac{34}{3}; -\frac{11x_4 + 2}{3}; -\frac{16x_4 - 22}{3}; x_4\right)$, $x_4 \in R$.

Tenglamalar sistemasini yechishda Gauss – Jordan usuli

Tenglamalar sistemasini yechishda Gauss – Jordan usulining (Gauss usulining Jordan modifikatsiyasi) mazmun-mohiyati quyidagidan iborat: dastlabki normal ko'rinishda berilgan sistemaning kengaytirilgan $(A|B)$ matritsasi quriladi. Yuqorida keltirilgan sistemaning teng kuchliligini saqlovchi elementar almashtirishlar yordamida, kengaytirilgan matritsaning chap qismida birlik matritsa hosil qilinadi. Bunda birlik matritsadan o'ngda yechimlar ustuni hosil bo'ladi. Gauss - Jordan usulini quyidagicha sxematik ifodalash mumkin:

$$(A|B) \sim (E|X^*).$$

Chiziqli tenglamalar sistemasini yechish Gauss-Jordan usuli noma'lumlarni ketma-ket yo'qotishning Gauss strategiyasi va teskari matritsa qurishning Jordan taktikasiga asoslanadi. Teskari matritsa oshkor shaklda qurilmaydi, balki o'ng ustunda bir yo‘la teskari matritsaning ozod hadlar ustuniga ko‘paytmasi–yechimlar ustuni quriladi.

8-misol. Quyidagi chiziqli tenglamalar sistemasini Gauss-Jordan usulida yeching:

$$\begin{cases} x_1 + x_2 + 2x_3 + 3x_4 = 1, \\ 3x_1 - x_2 - x_3 - 2x_4 = -4, \\ 2x_1 + 3x_2 - x_3 - x_4 = -6, \\ x_1 + 2x_2 + 3x_3 - x_4 = -4. \end{cases}$$

Yechish. Chiziqli tenglamalar sistemasi koeffitsiyentlaridan kengaytirilgan matritsa tuzamiz. Tenglamalar ustida bajariladigan almashtirishlar yordamida asosiy matritsani quyidagicha birlik matritsaga keltirib javobni topamiz:

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 3 & -1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 & -6 \\ 1 & 2 & 3 & -1 & -4 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 4 & 7 & 11 & 7 \\ 0 & -1 & 5 & 7 & 8 \\ 0 & 1 & 1 & -4 & -5 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & -1 & 5 & 7 & 8 \\ 0 & 4 & 7 & 11 & 7 \end{array}\right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 7 & 6 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & 0 & 6 & 3 & 3 \\ 0 & 0 & 3 & 27 & 27 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 7 & 6 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 2 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -13 & -14 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 0 & -17 & -17 \end{array}\right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -13 & -14 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

9-misol. Tenglamalar sistemasini Gauss – Jordan usulida yeching:

$$\begin{cases} 5x_1 + 2x_2 + 3x_3 + 3x_4 = 1, \\ 2x_1 - 2x_2 + 5x_3 + 2x_4 = 4, \\ 3x_1 + 4x_2 + 2x_3 + 2x_4 = -2. \end{cases}$$

Yechish. Berilgan sistemada kengaytirilgan matritsani ajratib olamiz:

$$\left(\begin{array}{cccc|c} 5 & 2 & 3 & 3 & 1 \\ 2 & -2 & 5 & 2 & 4 \\ 3 & 4 & 2 & 2 & -2 \end{array}\right)$$

va unga Gauss – Jordan usulini tatbiq etamiz:

$$\left(\begin{array}{cccc|c} 1 & 2/5 & 3/5 & 3/5 & 1/5 \\ 0 & -14/5 & 19/5 & 4/5 & 18/5 \\ 0 & 14/5 & 1/5 & 1/5 & -13/5 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 8/7 & 5/7 & 5/7 \\ 0 & 1 & -19/14 & -2/7 & -9/7 \\ 0 & 0 & 4 & 1 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3/7 & 3/7 \\ 0 & 1 & 0 & 3/56 & -53/56 \\ 0 & 0 & 1 & 1/4 & 1/4 \end{array}\right)$$

Sistema trapetsiyasimon ko‘rinishiga keldi:

$$\begin{cases} x_1 + \frac{3}{7}x_4 = \frac{3}{7} \\ x_2 + \frac{3}{56}x_4 = -\frac{53}{56} \\ x_3 + \frac{1}{4}x_4 = \frac{1}{4} \end{cases}$$

Bu yerda x_1, x_2 va x_3 o‘zgaruvchilarni bazis sifatida qabul qilamiz, chunki ular oldidagi koeffitsiyentlardan tuzilgan determinant $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0$. Bu determinant oxirgi sistemaning koeffitsiyentlaridan tuzilgan asosiy matritsaning ham bazis minori bo‘ladi. Erkli o‘zgaruvchi bo‘lib x_4 xizmat qiladi.

Oxirgi sistemadan quyidagi yechimga

$$x_1 = \frac{3}{7} - \frac{3}{7}x_4,$$
$$x_2 = -\frac{53}{56} - \frac{3}{56}x_4,$$
$$x_3 = \frac{1}{4} - \frac{1}{4}x_4.$$

ega bo‘lamiz. Shunday qilib, berilgan sistemaning umumiy X yechimini

$$X=\begin{pmatrix} \frac{3}{7}-\frac{3}{7}x_4 \\ -\frac{53}{56}-\frac{3}{56}x_4 \\ \frac{1}{4}-\frac{1}{4}x_4 \\ x_4 \end{pmatrix}$$

ko‘rinishda tasvirlash mumkin.

Agar $x_4=2$, deb olsak, u holda berilgan sistemaning

$$X_1=\begin{pmatrix} -\frac{3}{7} \\ -\frac{59}{56} \\ -\frac{1}{4} \\ 2 \end{pmatrix}$$

ko‘rinishdagi xususiy yechimini topamiz.

Agar $x_4=0$ ni olsak berilgan sistemaning quyidagi bazis yechimiga ega bo‘lamiz:

$$X_b=\begin{pmatrix} \frac{3}{7} \\ -\frac{53}{56} \\ \frac{1}{4} \\ 0 \end{pmatrix}.$$

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3 – vazifa.

Chiziqli algebraik tenglamalar sistemasi birgalikda ekanligini tekshiring. Agar birgalikda bo’lsa, uni Gauss va Gauss- Jordan usullarida yeching.

1. $\begin{cases} 2x_1 + x_2 + 3x_3 = 7, \\ 2x_1 + 3x_2 + x_3 = 1, \\ 3x_1 + 2x_2 + x_3 = 6; \end{cases}$

2. $\begin{cases} 2x_1 - x_2 + 2x_3 = 3, \\ x_1 + x_2 + 2x_3 = -4, \\ 4x_1 + x_2 + 4x_3 = -3; \end{cases}$

3. $\begin{cases} 3x_1 - x_2 + x_3 = 12, \\ x_1 + 2x_2 + 4x_3 = 6, \\ 5x_1 + x_2 + 2x_3 = 3; \end{cases}$

4. $\begin{cases} 2x_1 - x_2 + 3x_3 = -4, \\ x_1 + 3x_2 - x_3 = 11, \\ x_1 - 2x_2 + 2x_3 = -7; \end{cases}$

5. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 12, \\ 3x_1 + 4x_2 - 2x_3 = 6, \\ 2x_1 - x_2 - x_3 = -9; \end{cases}$

6. $\begin{cases} 8x_1 + 3x_2 - 6x_3 = -4, \\ x_1 + x_2 - x_3 = 2, \\ 4x_1 + x_2 - 3x_3 = -5; \end{cases}$

7. $\begin{cases} 4x_1 + x_2 - 3x_3 = 9, \\ x_1 - x_2 - x_3 = -2, \\ 8x_1 + 3x_2 - 6x_3 = 0; \end{cases}$

8. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 33, \\ 7x_1 - 5x_2 = 24, \\ 4x_1 + x_3 = 39; \end{cases}$

9. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 12, \\ 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7; \end{cases}$

10. $\begin{cases} x_1 + 4x_2 - x_3 = 6, \\ 5x_2 + 4x_3 = -20, \\ 3x_1 - 2x_2 + 5x_3 = -22; \end{cases}$

11. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 21, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 - x_2 - x_3 = 10; \end{cases}$

12. $\begin{cases} 3x_1 - 2x_2 - 5x_3 = 5, \\ 2x_1 + 3x_2 - 4x_3 = 12, \\ x_1 - 2x_2 + 3x_3 = -1; \end{cases}$

13. $\begin{cases} 4x_1 + x_2 + 4x_3 = 19, \\ 2x_1 - x_2 + 2x_3 = 11, \\ x_1 + x_2 + 2x_3 = 8; \end{cases}$

14. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 6, \\ x_1 + x_2 + 2x_3 = 4; \end{cases}$

15. $\begin{cases} 2x_1 - x_2 + 2x_3 = 8, \\ x_1 + x_2 + 2x_3 = 11, \\ 4x_1 + x_2 + 4x_3 = 22; \end{cases}$

16. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15; \end{cases}$

17. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + 5x_2 + x_3 = -3; \end{cases}$

18. $\begin{cases} -3x_1 + 5x_2 + 6x_3 = -8, \\ 3x_1 + x_2 + x_3 = -4, \\ x_1 - 4x_2 - 2x_3 = -9; \end{cases}$

19. $\begin{cases} 3x_1 + x_2 + x_3 = -4, \\ -3x_1 + 5x_2 + 6x_3 = 36, \\ x_1 - 4x_2 - 2x_3 = -19; \end{cases}$

20. $\begin{cases} 3x_1 - x_2 + x_3 = 11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16; \end{cases}$

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Bir jinsli chiziqli algebraik tenglamalar sistemasi. Fundamental yechimlar sistemasi

10. Berilgan bir jinsli sistema umumiy yechimini vektor shaklda yozing:

$$\begin{cases} 4x_1 + 7x_2 + 2x_3 + 3x_4 = 0 \\ x_1 + 3x_2 - x_3 + 2x_4 = 0 \\ 2x_1 + x_2 + 4x_3 - x_4 = 0 \end{cases}$$

Yechish:

$$\left(\begin{array}{cccc|c} 4 & 7 & 2 & 3 & 0 \\ 1 & 3 & -1 & 2 & 0 \\ 2 & 1 & 4 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & -5 & 6 & -5 & 0 \\ 1 & 3 & -1 & 2 & 0 \\ 0 & -5 & 6 & -5 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 1 & -1{,}2 & 1 & 0 \\ 1 & 0 & 2{,}6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$m=4,\ r=2$ bo‘lgani uchun $m-r=2$ ta chiziqli erkli $\vec{e_1}(1;0)$ va $\vec{e_2}(0;1)$ sistemani tanlaymiz. $\vec{e_1}(1;0)$ vektor koordinatalarini umumiy yechimning mos erkli noma’lumlari o‘rniga qo‘yib, bazis noma’lumlarni aniqlaymiz va $\vec{F_1}(-2,6;1,2;1;0)$ fundamental yechimni quramiz. $\vec{e_2}(0;1)$ vektor yordamida $\vec{F_2}(1;-1;0;1)$ fundamental yechimni quramiz. Boshqacha qilib aytganda kengaytirilgan matritsadagi koeffitsiyentlarni sistemaga qo‘yamiz:

$$\begin{cases} \quad x_2 - 1{,}2x_3 + x_4 = 0 \\ x_1 \quad\ \ 2{,}6x_3 - x_4 = 0 \end{cases} \qquad \begin{cases} x_1 = -2{,}6x_3 + x_4 \\ x_2 = 1{,}2x_3 - x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{cases}$$

Fundamental yechimlar $\vec{F_1}(-2,6;1,2;1;0)$ va $\vec{F_2}(1;-1;0;1)$ quriladi.

Sistema umumiy yechimi vektor shaklini yozamiz:

$$X = \lambda_1 F_1 + \lambda_2 F_2 = \lambda_1 \begin{pmatrix} -2{,}6 \\ 1{,}2 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

bu yerda λ_1 va λ_2 lar ixtiyoriy haqiqiy sonlar.

11. Berilgan bir jinsli bo‘lmagan sistema umumiy yechimini vektor shaklda yozing:

$$\begin{cases} 4x_1 + 7x_2 + 2x_3 + 3x_4 = 8 \\ x_1 + 3x_2 - x_3 + 2x_4 = 3 \\ 2x_1 + x_2 + 4x_3 - x_4 = 2 \end{cases}$$

Yechish:

$$\left(\begin{array}{cccc|c} 4 & 7 & 2 & 3 & 8 \\ 1 & 3 & -1 & 2 & 3 \\ 2 & 1 & 4 & -1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & -5 & 6 & -5 & -4 \\ 1 & 3 & -1 & 2 & 3 \\ 0 & -5 & 6 & -5 & -4 \end{array}\right) \sim \left(\begin{array}{cccc|c} 0 & 1 & -1{,}2 & 1 & 0{,}8 \\ 1 & 0 & 2{,}6 & -1 & 0{,}6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$F_0 = (0{,}6;\ 0{,}8;\ 0;\ 0)$ sistemaning xususiy yechimlaridan birini yozdik.

Sistema umumiy yechimi vektor shaklini yozamiz:

$$X = F_0 + \lambda_1 F_1 + \lambda_2 F_2 = \begin{pmatrix} 0{,}6 \\ 0{,}8 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} -2{,}6 \\ 1{,}2 \\ 1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

bu yerda λ_1 va λ_2 lar ixtiyoriy haqiqiy sonlar.

4 – vazifa.

Chiziqli tenglamalar sistemasining fundamental yechimlarini toping va umumiy yechimini yozing:

1. $\begin{cases} x_1 + x_2 + x_3 + x_4 - x_5 = 0 \\ x_1 + x_2 + 2x_3 + 2x_4 + x_5 = 0 \\ x_1 + x_2 + 3x_3 - 4x_4 - 3x_5 = 0 \end{cases}$ **2.** $\begin{cases} x_1 + 2x_2 - x_3 - x_4 = 0 \\ -x_1 - 4x_2 + 2x_3 + 2x_4 = 0 \\ 3x_1 + x_2 - 3x_3 - 3x_4 = 0 \end{cases}$

3. $\begin{cases} -3x_1 - 2x_2 + x_3 = 0 \\ -2x_1 + 2x_2 - 2x_3 = 0 \\ x_1 + 2x_2 - 3x_3 = 0 \end{cases}$ **4.** $\begin{cases} x_1 - x_2 + x_3 - 3x_4 = 0 \\ 2x_1 - 3x_2 + 5x_3 - x_4 = 0 \\ x_1 - x_2 + x_3 - 2x_4 = 0 \\ 5x_2 + x_3 = 0 \end{cases}$

5. $\begin{cases} x_1+2x_2+4x_3-3x_4=0 \\ 3x_1+5x_2+6x_3-4x_4=0 \\ 4x_1+5x_2-2x_3+3x_4=0 \\ -x_1+x_2+4x_3-9x_4=0 \end{cases}$ **6.** $\begin{cases} 8x-2y-2z=0 \\ x-y+3z=0 \\ -3y-4z=0 \end{cases}$

7. $\begin{cases} 3x+2y-6z=0 \\ 2x-y+3z=0 \\ x+y-5z=0 \end{cases}$ 8. $\begin{cases} x_1+x_2+x_3-2x_4=0 \\ x_1+x_2+2x_3+x_4=0 \\ 2x_1+2x_2+4x_3-3x_4=0 \end{cases}$

9. $\begin{cases} x_1+3x_2+5x_3-2x_4=3 \\ x_1+4x_2-2x_3+3x_4=2 \\ -x_1-2x_2-12x_3-7x_4=-4 \\ 3x_1+11x_2+x_3-4x_4=7 \end{cases}$ **10.** $\begin{cases} 3x_1+2x_2=4 \\ x_1-4x_2=-1 \\ 7x_1+10x_2=12 \\ 5x_1+6x_2=8 \\ 3x_1-16x_2=-5 \end{cases}$

11. $\begin{cases} x_1+5x_2+4x_3=1 \\ 2x_1+10x_2+8x_3=3 \\ 3x_1+15x_2+12x_3=5 \end{cases}$ **12.** $\begin{cases} x_1-3x_2+2x_3=-1 \\ x_1+9x_2+6x_3=3 \\ x_1+3x_2+4x_3=1 \end{cases}$

13. $\begin{cases} x_1+2x_2-4x_3=18 \\ 3x_1-x_2+4x_3=4 \\ 2x_1+x_2+5x_3=0 \end{cases}$ **14.** $\begin{cases} 2x_1+x_2+3x_3+x_4-x_5=5 \\ x_1-3x_2+5x_3+x_4+2x_5=4 \\ 2x_1+x_2+5x_3-3x_4+4x_5=0 \end{cases}$

15. $\begin{cases} x_1+2x_2-x_3+x_4=1 \\ 2x_1+4x_2-3x_3-x_4=2 \end{cases}$ **16.** $\begin{cases} x_1+2x_2-x_3-x_4-2x_5=1 \\ 2x_1+4x_2-3x_3+x_4+4x_5=5 \end{cases}$

17. $\begin{cases} 3x_1-3x_2+x_3+x_4+2x_5=1 \\ x_1-2x_2+3x_3-x_4+5x_5=2 \\ 2x_1+x_2+x_3+2x_4+2x_5=3 \end{cases}$ **18.** $\begin{cases} x_1+2x_2+3x_3+2x_4=14 \\ 3x_1+2x_2+x_3+x_4=10 \\ x_1+x_2+x_3-2x_4=6 \end{cases}$

19. $\begin{cases} 3x_1-5x_2+2x_3+4x_4=2 \\ 7x_1-4x_2+x_3+3x_4=5 \\ 5x_1+7x_2-4x_3-6x_4=3 \end{cases}$ **20.** $\begin{cases} x_1+2x_2-x_3-x_4=0 \\ -x_1-4x_2+2x_3+2x_4=0 \\ 3x_1+x_2-3x_3-3x_4=0 \end{cases}$

21. $\begin{cases} -3x_1-2x_2+x_3=0 \\ -2x_1+2x_2-2x_3=0 \\ x_1+2x_2-3x_3=0 \end{cases}$ **22.** $\begin{cases} x_1-x_2+x_3-3x_4=0 \\ 2x_1-3x_2+5x_3-x_4=0 \\ x_1-x_2+x_3-2x_4=0 \\ 5x_2+x_3=0 \end{cases}$

23. $\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0 \\ -x_1 + x_2 + 4x_3 - 9x_4 = 0 \end{cases}$ **24.** $\begin{cases} 8x - 2y - 2z = 0 \\ x - y + 3z = 0 \\ -3y - 4z = 0 \end{cases}$

25. $\begin{cases} 3x + 2y - 6z = 0 \\ 2x - y + 3z = 0 \\ x + y - 5z = 0 \end{cases}$ **26.** $\begin{cases} x_1 - x_2 + 3x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 - x_3 + x_4 = 0 \\ x_1 - x_2 + x_3 + x_4 = 0 \end{cases}$

27. $\begin{cases} x_1 - 2x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0 \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0 \end{cases}$ **28.** $\begin{cases} x_1 - x_2 + 3x_3 + 6x_4 = 1 \\ 3x_1 - 3x_2 - x_3 - x_4 = -4 \end{cases}$

29. $\begin{cases} 2x_1 - x_2 + x_3 + x_4 = 1 \\ x_2 - x_3 + x_4 = 2 \\ x_2 - 2x_3 + 3x_4 = 3 \end{cases}$ **30.** $\begin{cases} 8x_1 + 2x_2 - x_3 = 5 \\ 2x_1 - x_2 - x_3 = 4 \end{cases}$