

Преобразование Лапласа. Оригинал и изображение

Laplas almashtirishlari. Asl va tasvir.

№	Original $f(t)$	Tasvir $F(t)$	№	Original $f(t)$	Tasvir $F(t)$
1	1	$1/p$	6	$e^{\alpha t} \cdot \cos \beta t$	$\frac{p - \alpha}{(p - \alpha)^2 + \beta^2}$
2	$\frac{t^n}{n!}$	$\frac{1}{p^{n+1}}$	7	$e^{\alpha t} \cdot \sin \beta t$	$\frac{\beta}{(p - \alpha)^2 + \beta^2}$
3	$e^{\alpha t}$	$\frac{1}{p - \alpha}$	8	$\frac{t^n}{n!} \cdot e^{\alpha t}$	$\frac{1}{(p - \alpha)^{n+1}}$
4	$\cos \beta t$	$\frac{p}{p^2 + \beta^2}$	9	$t \cdot \cos \beta t$	$\frac{p^2 - \beta^2}{(p^2 + \beta^2)^2}$
5	$\sin \beta t$	$\frac{\beta}{p^2 + \beta^2}$	10	$t \cdot \sin \beta t$	$\frac{2p\beta}{(p^2 + \beta^2)^2}$

1. Berilgan funksiyaning tasvirini toping

1. Найти изображение функции

1. $f(t) = 11t$
2. $f(t) = 2 \sin t + 3 \cos t$
3. $f(t) = t + 2e^{-t}$
4. $f(t) = 2 \sin t + 3 \operatorname{sh} t$
5. $f(t) = t + 2e^t$
6. $f(t) = e^{at}$
7. $f(t) = \sin 4t$
8. $f(t) = \cos \omega t$
9. $f(t) = \operatorname{sh} 2t$
10. $f(t) = \sin^2 t$
11. $f(t) = \sin mt \cos nt$
12. $f(t) = \cos^2 t$
13. $f(t) = \sin mt \sin nt$
14. $f(t) = \cos t$
15. $f(t) = \sin 3t$
16. $f(t) = \cos 6t$
17. $f(t) = e^{3t}$
18. $f(t) = \operatorname{sh} t$
19. $f(t) = \operatorname{ch} 3t$
20. $f(t) = e^{-3t} \sin t$
21. $f(t) = e^{2t} \cos t$
22. $f(t) = t^3$

$$23. \quad f(t) = t^2$$

$$24. \quad f(t) = t^2$$

$$25. \quad f(t) = \cos 2t$$

$$26. \quad f(t) = \sin 5t$$

$$27. \quad f(t) = e^{-5t}$$

$$28. \quad f(t) = \operatorname{sh} 5t$$

$$29. \quad f(t) = \cos 3t$$

$$30. \quad f(t) = \sin 4t$$

2. Найти изображение функции

2. . Berilgan funksiyaning tasvirini toping

$$1. \quad f(t) = e^t \cos t$$

$$2. \quad f(t) = \sin^4 t$$

$$3. \quad f(t) = \cos 2t \sin 3t$$

$$4. \quad f(t) = \cos^3 t$$

$$5. \quad f(t) = \sin^3 t$$

$$6. \quad f(t) = t \sin \omega t$$

$$7. \quad f(t) = \cos^4 t$$

$$8. \quad f(t) = t \cos \omega t$$

$$9. \quad f(t) = te^t$$

$$10. \quad f(t) = t^2 \cos t$$

$$11. \quad f(t) = t(e^t \operatorname{ch} t)$$

$$12. \quad f(t) = (t + 1) \sin 2t$$

$$13. \quad f(t) = t \operatorname{sh} t$$

$$14. \quad f(t) = \int_0^t \sin \tau d\tau$$

$$15. \quad f(t) = \int_0^t (\tau + 1) \cos \omega \tau d\tau$$

$$16. \quad f(t) = \int_0^t \operatorname{ch} \omega \tau d\tau$$

$$17. \quad f(t) = \int_0^t \tau^2 e^{-\tau} d\tau$$

$$18. \quad f(t) = \frac{e^t - 1}{t}$$

$$19. \quad f(t) = \frac{1 - e^{-t}}{t}$$

$$20. \quad f(t) = \frac{\sin^2 t}{t}$$

$$21. \quad f(t) = \frac{1 - \cos t}{t}$$

$$22. \quad f(t) = \frac{\cos t - \cos 2t}{t}$$

$$23. \quad f(t) = \frac{e^t - 1 - t}{t}$$

$$24. \quad f(t) = \frac{e^t - e^{-t}}{t}$$

$$25. \quad f(t) = e^{2t} \sin t$$

$$26. \quad f(t) = e^t \cos nt$$

$$27. \quad f(t) = e^{-t} t^3$$

$$28. \quad f(t) = e^{-t} \operatorname{sh} t$$

$$29. \quad f(t) = te^t \cos t$$

$$30. \quad f(t) = e^{3t} \sin^2 t$$

Asl va tasvirning asosiy xossalari.

Основные свойства оригинала и изображения

№:	Tasviri: $F(p) = \int_0^{+\infty} e^{-pt} f(t) dt$	Asli: $f(t)$
1.	$\frac{1}{p}$	1

2.	$\frac{1}{p - \alpha}$	$e^{\alpha t}$
3.	$\frac{\alpha}{p^2 + \alpha^2}$	$\sin \alpha t$
4.	$\frac{p}{p^2 + \alpha^2}$	$\cos \alpha t$
5.	$\frac{\alpha}{p^2 - \alpha^2}$	$\text{sh } \alpha t$
6.	$\frac{p}{p^2 - \alpha^2}$	$\text{ch } \alpha t$
7.	$\frac{\beta}{(p - \alpha)^2 + \beta^2}$	$e^{\alpha t} \sin \beta t$
8.	$\frac{p - \alpha}{(p - \alpha)^2 + \beta^2}$	$e^{\alpha t} \cos \beta t$
9.	$\frac{n!}{p^{n+1}}$	t^n
10.	$\frac{2\alpha p}{(p^2 + \alpha^2)^2}$	$t \sin \alpha t$
11.	$\frac{p^2 - \alpha^2}{(p^2 + \alpha^2)^2}$	$t \cos \alpha t$
12.	$\frac{1}{(p^2 + \alpha^2)^2}$	$\frac{1}{2\alpha^3} (\sin \alpha t - \alpha t \cos \alpha t)$
13.	$\frac{1}{(p - \alpha)^2}$	$t e^{\alpha t}$
14.	$\frac{n!}{(p - \alpha)^{n+1}}$	$t^n e^{\alpha t}$
15.	$\frac{\pi}{2} - \text{arctg} \frac{p}{\alpha}$	$\frac{\sin \alpha t}{t}$

16.	$\alpha F_1(p) + \beta F_2(p)$	$\alpha f_1(t) + \beta f_2(t)$
17.	$\frac{1}{\alpha} F\left(\frac{p}{\alpha}\right)$	$f(\alpha t)$
18.	$F(p - \alpha)$	$e^{\alpha t} f(t)$
19.	$e^{-p\tau} F(p)$	$f(t - \tau)$ (bu yerda $\tau > 0$)
20.	$pF(p) - f(0)$	$f'(t)$
21.	$-F'(p)$	$tf(t)$
22.	$(-1)^n \frac{d^n}{dp^n} F(p)$	$t^n f(t)$
23.	$\frac{F(p)}{p}$	$\int_0^t f(\tau) d\tau$
24.	$\int_p^{+\infty} F(q) dq$	$\frac{f(t)}{t}$
25.	$F_1(p) \cdot F_2(p)$	$\int_0^t f_1(\tau) f_2(t - \tau) d\tau$

3. Найти оригинала по его изображению

3. Quyidagi formulalarning aslini toping:.

1. $F(p) = \frac{1}{p^2 + 4p + 5}$

3. $F(p) = \frac{1}{7 - p + p^2}$

2. $F(p) = \frac{1}{p^2 + 4p + 3}$

4. $F(p) = \frac{1}{p(p^2 - 5p + 4)}$

$$5. \quad F(p) = \frac{1}{(p-1)(p^2-4)}$$

$$6. \quad F(p) = \frac{1}{p+2p^2+p^3}$$

$$7. \quad F(p) = \frac{p+3}{p(p^2-4p+3)}$$

$$8. \quad F(p) = \frac{1}{(p^2+1)^2}$$

$$9. \quad F(p) = \frac{p}{(p^2+1)^2}$$

$$10. \quad F(p) = \frac{e^{-2p}}{p^2}$$

$$11. \quad F(p) = \frac{2e^{-p}}{p^3}$$

$$12. \quad F(p) = \frac{1}{p^2(p^2+1)}$$

$$13. \quad F(p) = \frac{p}{p^3+1}$$

$$22. \quad F(p) = \frac{p+3}{p^2+2p+10}$$

$$23. \quad F(p) = \frac{p}{p^2-2p+10}$$

$$24. \quad F(p) = \frac{1}{p^3-8}$$

$$25. \quad F(p) = \frac{p}{(p-1)^3(p+2)^2}$$

$$29. \quad F(p) = \frac{1}{p(p^2+1)(p^2+4)}$$

$$14. \quad F(p) = \frac{2p+3}{p^3+4p^2+5p}$$

$$15. \quad F(p) = \frac{p^2+2p-1}{p^3+3p^2+3p+1}$$

$$16. \quad F(p) = \frac{p+2}{(p+1)(p-2)(p^2+4)}$$

$$17. \quad F(p) = \frac{2p^3+p^2+2p+2}{p^5+2p^4+2p^3}$$

$$18. \quad F(p) = \frac{e^{-p}}{p(p-1)}$$

$$19. \quad F(p) = \frac{e^{-3p}}{(p+1)^2}$$

$$20. \quad F(p) = \frac{e^{-p}}{p^2-1} + \frac{pe^{-2p}}{p^2-4}$$

$$21. \quad F(p) = \frac{1}{p^2+1} (e^{-2p} + 2e^{-3p} + 3e^{-4p})$$

$$26. \quad F(p) = \frac{p+1}{p(p-1)(p-2)(p-3)}$$

$$27. \quad F(p) = \frac{p+3}{p(p^2-4p+3)}$$

$$28. \quad F(p) = \frac{1}{(p-1)(p^2-4)}$$

$$30. \quad F(p) = \frac{1}{p(p^4-5p^2+4)}$$

Differensial tenglama va differensial tenglamalar sistemalarini

yehichning operatsion xisob usuli.

4. Решить Задачу Коши

4. Koshi masalasini yeching

- 1) $x'' + 3x' = e^t, \quad x(0) = 0, \quad x'(0) = -1.$
- 2) $x'' - 2x' = e^{2t}, \quad x(0) = x'(0) = 0.$
- 3) $x'' + 2x' - 3x = e^{-t}, \quad x(0) = 0, \quad x'(0) = 1.$
- 4) $x''' + x' = 1, \quad x(0) = x'(0) = x''(0) = 0.$
- 5) $x'' + 2x' = t \sin t, \quad x(0) = x'(0) = 0.$
- 6) $x'' + 2x' + x = \sin t, \quad x(0) = 0, \quad x'(0) = -1.$
- 7) $x''' - x'' = \sin t, \quad x(0) = x'(0) = x''(0) = 0.$
- 8) $x'' - 2x' + x = e^t, \quad x(0) = 0, \quad x'(0) = 1.$
- 9) $x'' + 2x' + 5x = 0, \quad x(0) = -1, \quad x'(0) = 2.$
- 10) $x'' - 2x' + 2x = 1, \quad x(0) = x'(0) = 0.$
- 11) $x'' + x' = \cos t, \quad x(0) = 2, \quad x'(0) = 0.$
- 12) $x'' + 2x' + x = t^2, \quad x(0) = 1, \quad x'(0) = 0.$
- 13) $x''' + x'' = \sin t, \quad x(0) = x'(0) = 1, \quad x''(0) = 0.$
- 14) $x'' + x = \cos t, \quad x(0) = -1, \quad x'(0) = 1.$
- 15) $x''' + x'' = t, \quad x(0) = -3, \quad x'(0) = 1, \quad x''(0) = 0.$
- 16) $x'' + 2x' + 5x = 3, \quad x(0) = 1, \quad x'(0) = 0.$
- 17) $x''' + x' = e^t, \quad x(0) = 0, \quad x'(0) = 2, \quad x''(0) = 0.$
- 18) $x''' + x'' = \cos t, \quad x(0) = -2, \quad x'(0) = x''(0) = 0.$
- 19) $x^{IV} - x'' = 1, \quad x(0) = x'(0) = x''(0) = x'''(0) = 0.$
- 20) $x^{IV} - x'' = \cos t, \quad x(0) = 0, \quad x'(0) = -1, \quad x''(0) = x'''(0) = 0.$
- 21) $x'' - x' = te^t, \quad x(0) = x'(0) = 0.$
- 22) $x'' + 2x' + x = 2\cos^2 t, \quad x(0) = x'(0) = 0.$
- 23) $x'' - 2x' + x = t - \sin t, \quad x(0) = x'(0) = 0.$
- 24) $x'' + 4x = 2 \cos t \cdot \cos 3t, \quad x(0) = x'(0) = 0.$
- 25) $x'' - x' = t^2, \quad x(0) = 0, \quad x'(0) = 1.$
- 26) $x''' + x = \frac{1}{2}t^2e^t, \quad x(0) = x'(0) = x''(0) = 0.$

$$27) \quad x'' + x = t \cos 2t, \quad x(0) = x'(0) = 0.$$

$$28) \quad x'' + x' + x = te^t, \quad x(0) = x'(0) = 0.$$

$$29) \quad x''' + 3x'' + 3x' + x = 1, \quad x(0) = x'(0) = x''(0) = 0.$$

$$30) \quad x^{IV} + 2x'' + x = t \sin t, \quad x(0) = x'(0) = x''(0) = x'''(0) = 0.$$

5. Quyidagi differensial tenglamalarni operatsion xisob usulida yeching:

$$1) \quad x'' + x' = t, \quad x(0) = x'(0) = 0.$$

$$2) \quad x''' + x' = e^t, \quad x(0) = x'(0) = x''(0) = 0.$$

$$3) \quad x'' - 2x' = t^2 e^t, \quad x(0) = x'(0) = 0.$$

$$4) \quad x'' + 2x' + 2x = \sin t, \quad x(0) = x'(0) = 0.$$

$$5) \quad x'' = \operatorname{arctg} t, \quad x(0) = x'(0) = 0.$$

$$6) \quad x'' + x = \frac{1}{2 + \cos t}, \quad x(0) = x'(0) = 0.$$

$$\text{Javob: } x = t \sin t - \frac{4}{\sqrt{3}} \sin t \operatorname{arctg} \frac{\operatorname{tg} \frac{t}{2}}{\sqrt{3}} + \cos t \ln(2 + \cos t) - \ln 3 \cos t.$$

$$7) \quad x'' + x' = \frac{1}{4 + \operatorname{tg}^2 t}, \quad x(0) = x'(0) = 0.$$

Javob: x

$$\begin{aligned} &= \frac{1}{3} - \frac{9 - \pi\sqrt{3}}{27} \cos t \\ &+ \frac{\sqrt{3}}{36} \sin t \ln \left(\frac{\sqrt{3} \sin t - 2}{\sqrt{3} \sin t + 2} \right) - \frac{\sqrt{3}}{9} \cos t \operatorname{arctg}(\sqrt{3} \cos t). \end{aligned}$$

$$8) \quad x'' + x = \cos t, \quad x(0) = x'(0) = 0.$$

$$9) \quad x'' + x' = \frac{1}{1 + \cos^2 t}, \quad x(0) = x'(0) = 0.$$

$$\text{Javob: } x = \cos t \operatorname{arcth}(\cos t) - \frac{\pi}{4} \cos t - \frac{1}{2\sqrt{2}} \sin t \ln \left| \frac{\sin t - \sqrt{2}}{\sin t + \sqrt{2}} \right|.$$

$$10) \quad x'' - x = \operatorname{sh} t, \quad x(0) = x'(0) = 0.$$

$$11) \quad x'' - 2x' + x = \operatorname{ch} t, \quad x(0) = x'(0) = 0.$$

$$12) \quad x''' + x' = \frac{1}{2 + \sin t}, \quad x(0) = x'(0) = x''(0) = 0.$$

$$\text{Javob: } x = \ln 2 \cos t -$$

$$-\cos t \ln(2 + \sin t) - t \sin t + \frac{2}{\sqrt{3}}(2 \sin t + 1) \left(\operatorname{arctg} \frac{2 \operatorname{tg} \frac{t}{2} + 1}{\sqrt{3}} - \frac{\pi}{6} \right).$$

$$13) \quad x'' - x = \operatorname{th} t, \quad x(0) = x'(0) = 0.$$

$$14) \quad x'' + x' = t, \quad x(0) = x'(0) = 0.$$

$$15) \quad x''' + x' = e^t, \quad x(0) = x'(0) = x''(0) = 0.$$

$$16) \quad x'' - 2x' = t^2 e^t, \quad x(0) = x'(0) = 0.$$

$$17) \quad x'' + 2x' + 2x = \sin t, \quad x(0) = x'(0) = 0.$$

$$18) \quad x'' = \operatorname{arctg} t, \quad x(0) = x'(0) = 0.$$

$$19) \quad x'' + x = \frac{1}{2 + \cos t}, \quad x(0) = x'(0) = 0.$$

$$\text{Javob: } x = t \sin t - \frac{4}{\sqrt{3}} \sin t \operatorname{arctg} \frac{\operatorname{tg} \frac{t}{2}}{\sqrt{3}} + \cos t \ln(2 + \cos t) - \ln 3 \cos t.$$

$$20) \quad x'' + x' = \frac{1}{4 + \operatorname{tg}^2 t}, \quad x(0) = x'(0) = 0.$$

Javob: x

$$\begin{aligned} &= \frac{1}{3} - \frac{9 - \pi\sqrt{3}}{27} \cos t \\ &+ \frac{\sqrt{3}}{36} \sin t \ln \left(\frac{\sqrt{3} \sin t - 2}{\sqrt{3} \sin t + 2} \right) - \frac{\sqrt{3}}{9} \cos t \operatorname{arctg}(\sqrt{3} \cos t). \end{aligned}$$

$$21) \quad x'' + x = \cos t, \quad x(0) = x'(0) = 0.$$

$$22) \quad x'' + x' = \frac{1}{1 + \cos^2 t}, \quad x(0) = x'(0) = 0.$$

$$\text{Javob: } x = \cos t \operatorname{arch}(\cos t) - \frac{\pi}{4} \cos t - \frac{1}{2\sqrt{2}} \sin t \ln \left| \frac{\sin t - \sqrt{2}}{\sin t + \sqrt{2}} \right|.$$

$$23) \quad x'' - x = \operatorname{sh} t, \quad x(0) = x'(0) = 0.$$

$$24) \quad x'' - 2x' + x = \operatorname{ch} t, \quad x(0) = x'(0) = 0.$$

$$25) \quad x''' + x' = \frac{1}{2 + \sin t}, \quad x(0) = x'(0) = x''(0) = 0.$$

$$\text{Javob: } x = \ln 2 \cos t -$$

$$-\cos t \ln(2 + \sin t) - t \sin t + \frac{2}{\sqrt{3}}(2 \sin t + 1) \left(\operatorname{arctg} \frac{2 \operatorname{tg} \frac{t}{2} + 1}{\sqrt{3}} - \frac{\pi}{6} \right).$$

26) $x'' - x = \operatorname{th} t, \quad x(0) = x'(0) = 0.$

27) $x'' + x' = t, \quad x(0) = x'(0) = 0.$

28) $x''' + x' = e^t, \quad x(0) = x'(0) = x''(0) = 0.$

29) $x'' - 2x' = t^2 e^t, \quad x(0) = x'(0) = 0.$

30) $x'' + 2x' + 2x = \sin t, \quad x(0) = x'(0) = 0.$

6. Berilgan differensial tenglamalar sistemasini yeching

1) $\begin{cases} x' = y - 1, & x(0) = 1; \\ y' = -x + 2, & y(0) = 0; \end{cases}$

2) $\begin{cases} x' = y - 1, & x(0) = 1; \\ y' = -x - 2y, & y(0) = -1; \end{cases}$

3) $\begin{cases} 3x' + 2x + y' = 1, & x(0) = 0; \\ x' + 4y' + 3y = 0, & y(0) = 0; \end{cases}$

4) $\begin{cases} x' = 3x + 4y, & x(0) = 1; \\ y' = 4x - 3y, & y(0) = 1; \end{cases}$

5) $\begin{cases} x' = -y, & x(0) = 1; \\ y' = 2x + 2y, & y(0) = 1; \end{cases}$

6) $\begin{cases} x' = -x + 3y + 1, & x(0) = 1; \\ y' = x + y, & y(0) = 2; \end{cases}$

7) $\begin{cases} x' = x + 4y, & x(0) = 1; \\ y' = 2x - y + 9, & y(0) = 0; \end{cases}$

8) $\begin{cases} x' = 2y + 1, & x(0) = -1; \\ y' = 2x + 3, & y(0) = 0; \end{cases}$

9) $\begin{cases} x' = 3y, & x(0) = 2; \\ y' = 3x + 1, & y(0) = 0; \end{cases}$

10) $\begin{cases} x' = -2x + y, & x(0) = 0; \\ y' = 3x, & y(0) = 1; \end{cases}$

11) $\begin{cases} x' = -2x + y + 2, & x(0) = 1; \\ y' = 3x, & y(0) = 0; \end{cases}$

12) $\begin{cases} x' = -3x - 4y + 1, & x(0) = -1; \\ y' = 4x - 2y, & y(0) = 0; \end{cases}$

- 13) $\begin{cases} x' = 2y, & x(0) = 2; \\ y' = 2x, & y(0) = 2; \end{cases}$
- 14) $\begin{cases} x' = 2x - 2y, & x(0) = 3; \\ y' = -4x, & y(0) = 1; \end{cases}$
- 15) $\begin{cases} x' = 3x + 5y + 2, & x(0) = 0; \\ y' = 3x + y + 1, & y(0) = 2; \end{cases}$
- 16) $\begin{cases} x' = 2x + 2y + 2, & x(0) = 0; \\ y' = 4y + 1, & y(0) = 1; \end{cases}$
- 17) $\begin{cases} x' = x + 2y, & x(0) = 0; \\ y' = 2x + y + 1, & y(0) = 5; \end{cases}$
- 18) $\begin{cases} x' = y + 3, & x(0) = 1; \\ y' = x + 2, & y(0) = 0; \end{cases}$
- 19) $\begin{cases} x' = -y - z, & x(0) = -1; \\ y' = -x - z, & y(0) = 0; \\ z' = -x - y, & z(0) = 1; \end{cases}$
- 20) $\begin{cases} x' = y + z, & x(0) = 0; \\ y' = 3x + z, & y(0) = 1; \\ z' = 3x + y, & z(0) = 1. \end{cases}$
- 21) $\begin{cases} x' = 3x + 4y, & x(0) = 1; \\ y' = 4x - 3y, & y(0) = 1; \end{cases}$
- 22) $\begin{cases} x' = -y, & x(0) = 1; \\ y' = 2x + 2y, & y(0) = 1; \end{cases}$
- 23) $\begin{cases} x' = -x + 3y + 1, & x(0) = 1; \\ y' = x + y, & y(0) = 2; \end{cases}$
- 24) $\begin{cases} x' = x + 4y, & x(0) = 1; \\ y' = 2x - y + 9, & y(0) = 0; \end{cases}$
- 25) $\begin{cases} x' = 2y + 1, & x(0) = -1; \\ y' = 2x + 3, & y(0) = 0; \end{cases}$
- 26) $\begin{cases} x' = 3y, & x(0) = 2; \\ y' = 3x + 1, & y(0) = 0; \end{cases}$
- 27) $\begin{cases} x' = -2x + y, & x(0) = 0; \\ y' = 3x, & y(0) = 1; \end{cases}$
- 28) $\begin{cases} x' = -2x + y + 2, & x(0) = 1; \\ y' = 3x, & y(0) = 0; \end{cases}$

$$29) \begin{cases} x' = -3x - 4y + 1, & x(0) = -1; \\ y' = 4x - 2y & y(0) = 0; \end{cases}$$

$$30) \begin{cases} x' = 2y, & x(0) = 2; \\ y' = 2x, & y(0) = 2; \end{cases}$$