Integrals & antiderivatives

Problem solutions by sarvarbek

Integrals

1. $\int(3x^2 + 2x + 1)\,dx$

   Integrating each term separately:

   $\int(3x^2)\,dx = x^3$

   $\int(2x)\,dx = x^2$

   $\int(1)\,dx = x$

   Adding the constant of integration, we get:

   Final result: $(x^3) + (x^2) + x + C$

2 $\int(\sin(x) + \cos(x))\,dx$

$\int\sin(x)\,dx = -\cos(x)$

$\int\cos(x)\,dx = \sin(x)$

Final result: $-\cos(x) + \sin(x) + C$

Explanation:

We used the fact that the antiderivative of sin(x) is -cos(x) and the antiderivative of cos(x) is sin(x). We then added the constant of integration.

Using ln (x)

1. ∫(sqrt(x) / x) dx

Rewrite sqrt(x) as x^(1/2). Then, the integral becomes ∫(x^(1/2) / x) dx.

Simplifying, we have ∫(x^(-1/2)) dx.

Using the power rule for integration, we add 1 to the exponent and divide by the new exponent:

∫(x^(-1/2)) dx = 2x^(1/2) + C

Final result: 2sqrt(x) - 2ln(|x|) + C

Explanation:

We applied the power rule for integration to integrate x^(-1/2). Then, we used the property of logarithms to rewrite x^(-1/2) as ln(|x|^(1/2)). Finally, we added the constant of integration.

2. ∫(ln(x)) dx

This is an example of integration by parts. Let u = ln(x) and dv = dx. Then,

du = (1/x) dx and v = x.

Applying the integration by parts formula:

∫(ln(x)) dx = x * ln(x) - ∫(x * (1/x)) dx

Simplifying, we get: ∫(ln(x)) dx = x * ln(x) - ∫(1) dx

The integral of 1 with respect to x is simply x.

Final result: x * ln(x) - x + C

Explanation: We applied the integration by parts formula, then integrated the resulting expression to obtain the final result.

Using multiplication

1 . ∫(tan(x) sec(x)) dx

For this integral, we recognize that the derivative of sec(x) is sec(x) * tan(x). Therefore, ∫tan(x) sec(x) dx = sec(x) + C.

Final result: sec(x) + C

Explanation:

We used the known antiderivative of tan(x) sec(x) to obtain the final result.

Integration

1 . $\int(2x + 3)\, dx$

Integrating each term separately:

$\int(2x)\, dx = x^2$

$\int(3)\, dx = 3x$

Adding the constant of integration, we get:

Final result: $x^2 + 3x + C$

Antiderivatives

1. $\int(x^2 + 2x + 1)\,dx$

The antiderivative of x^2 with respect to x is $(1/3)x^3$, the antiderivative of 2x with respect to x is x^2, and the antiderivative of 1 with respect to x is x.

Final result: $\int(x^2 + 2x + 1)\,dx = (1/3)x^3 + x^2 + x + C$, where C is the constant of integration.

Antiderivatives

∫(2cos(x) - 3sin(x)) dx

Therefore, the antiderivative of

$2\cos(x)-3\sin(x)$

$2\cos(x)-3\sin(x)$ with respect to x x is: $\int(2\cos(x)-3\sin(x))\,dx=2\sin(x)-3\cos(x)+C$ $\int(2\cos(x)-3\sin(x))dx= 2\sin(x)-3\cos(x)+C$

Where C C is the constant of integration.

So, the solution is $2\sin(x)-3\cos(x)+C$