PHY F242 : QUANTUM MECHANICS-I

Assignment-1

Total Marks: 20

1. A wavefunction ψ_0 is expressed as:

$$\psi_0(x) = Axe^{-ax^2/2}, x \in (-\infty, \infty)$$

where A and a are constants.

i. Find A such that ψ_0 represents a normalized wavefunction.
ii. If a particle of mass m has state ψ_0 at a particular time instance, calculate the expected value of the kinetic energy $\langle T \rangle$ of the particle at that time instance.

[3+4]

2. Given that $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$ and V is the potential energy function of the system, prove that

$$\frac{d\langle p \rangle}{dt} = \langle -\frac{\partial V}{\partial x} \rangle$$

[4]

3. A particle at time t=0 is represented by the state:

$$\psi_0(x) = \begin{cases} A(x - x^3) & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

i. Find A such that ψ_0 is normalized
ii. Calculate the expectation values, $\langle x \rangle$ and $\langle x^2 \rangle$
iii. Calculate the expectation values, $\langle p \rangle$ and $\langle p^2 \rangle$
iv. Find σ_x and σ_p. Is their product consistent with the uncertainty principle?

[1+3+3+2]