Data Communications and Networking Fourth Edition

Forouzan

Chapter 10

Error Detection and Correction

10-4 CYCLIC CODES

***Cyclic codes** are special linear block codes with one extra property. In a cyclic code, if a codeword is cyclically shifted (rotated), the result is another codeword.*

Topics discussed in this section:

Cyclic Redundancy Check
Hardware Implementation
Polynomials
Cyclic Code Analysis
Advantages of Cyclic Codes
Other Cyclic Codes

Table 10.6 *A CRC code with C(7, 4)*

Dataword	*Codeword*	*Dataword*	*Codeword*
0000	0000000	1000	1000101
0001	0001011	1001	1001110
0010	0010110	1010	1010011
0011	0011101	1011	1011000
0100	0100111	1100	1100010
0101	0101100	1101	1101001
0110	0110001	1110	1110100
0111	0111010	1111	1111111

Figure 10.14 *CRC encoder and decoder*

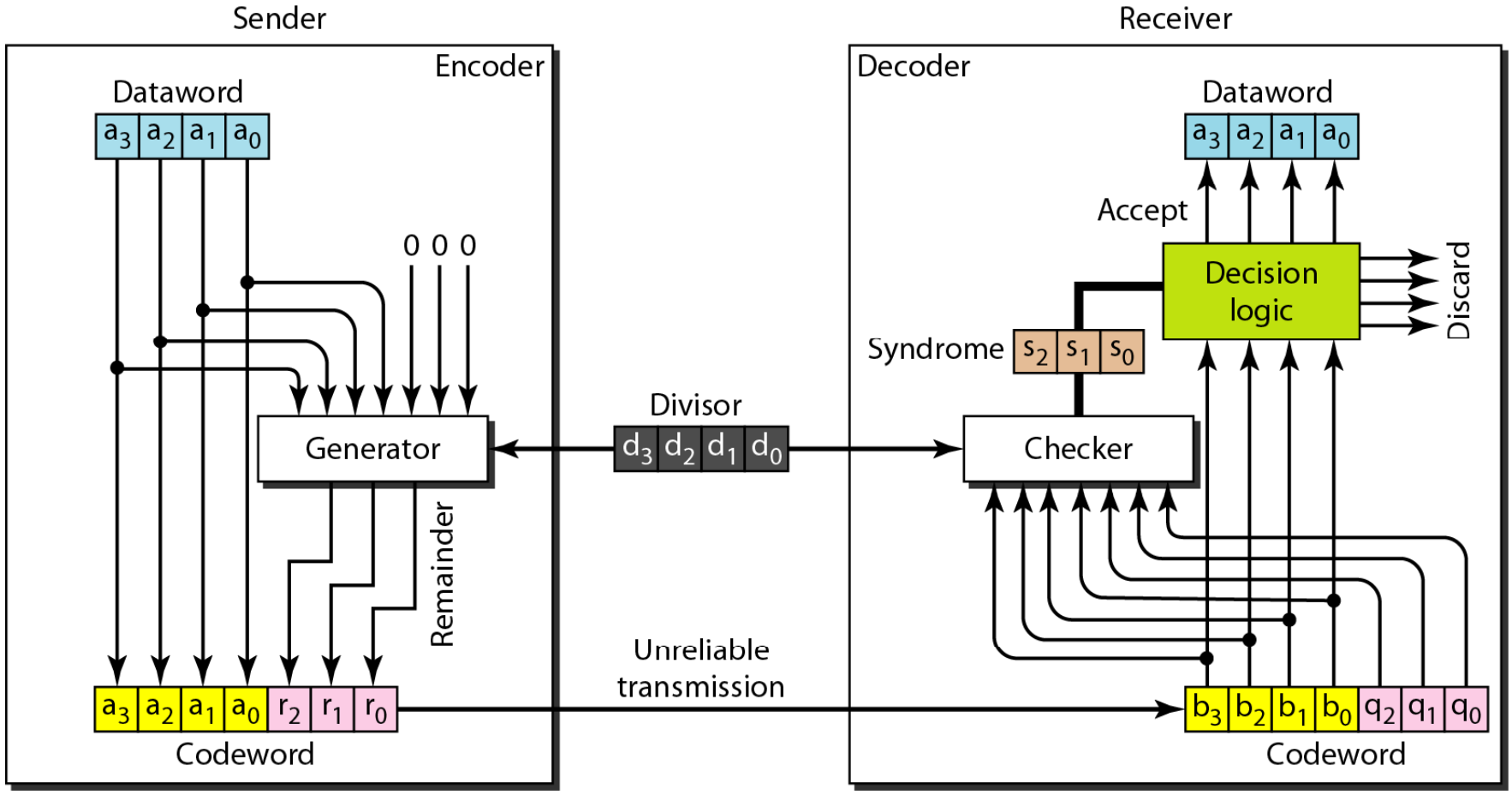


Figure 10.15 *Division in CRC encoder*

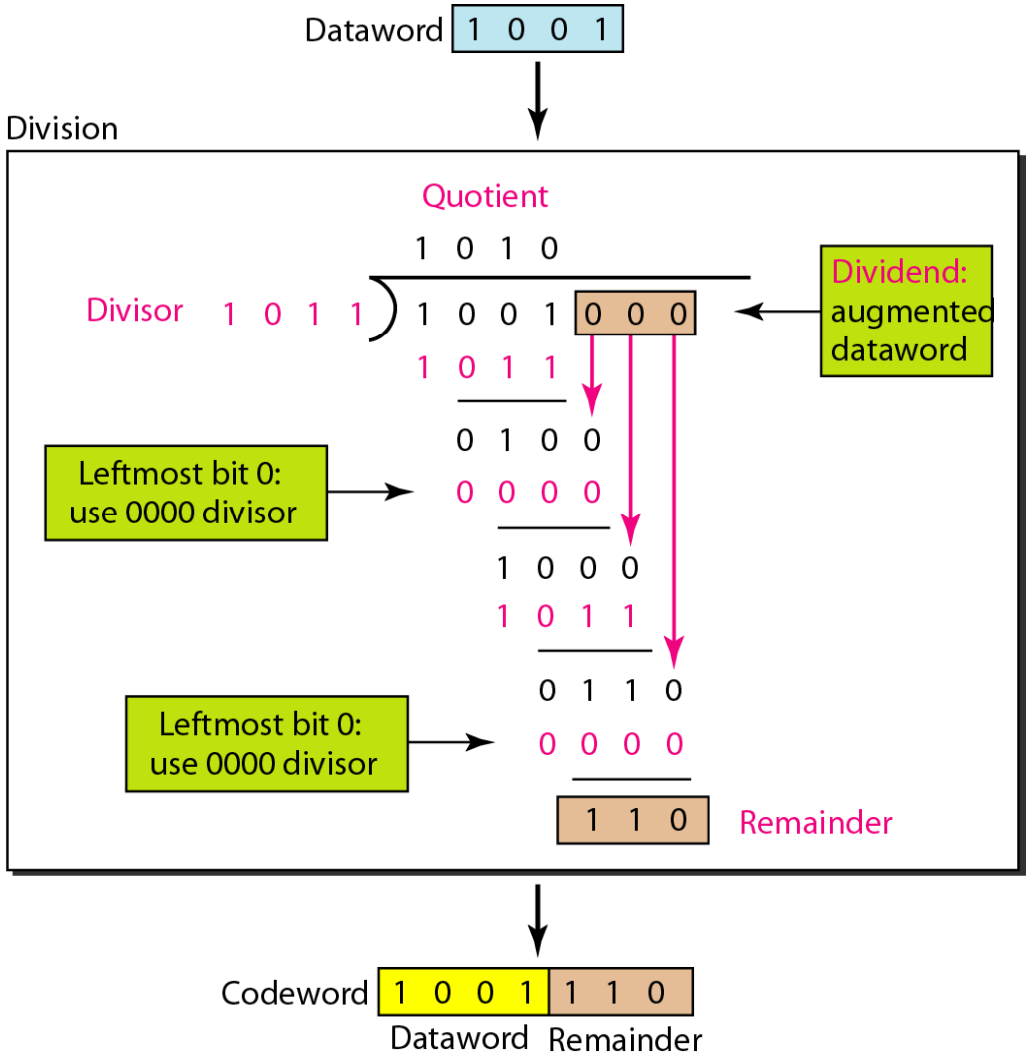


Figure 10.16 *Division in the CRC decoder for two cases*

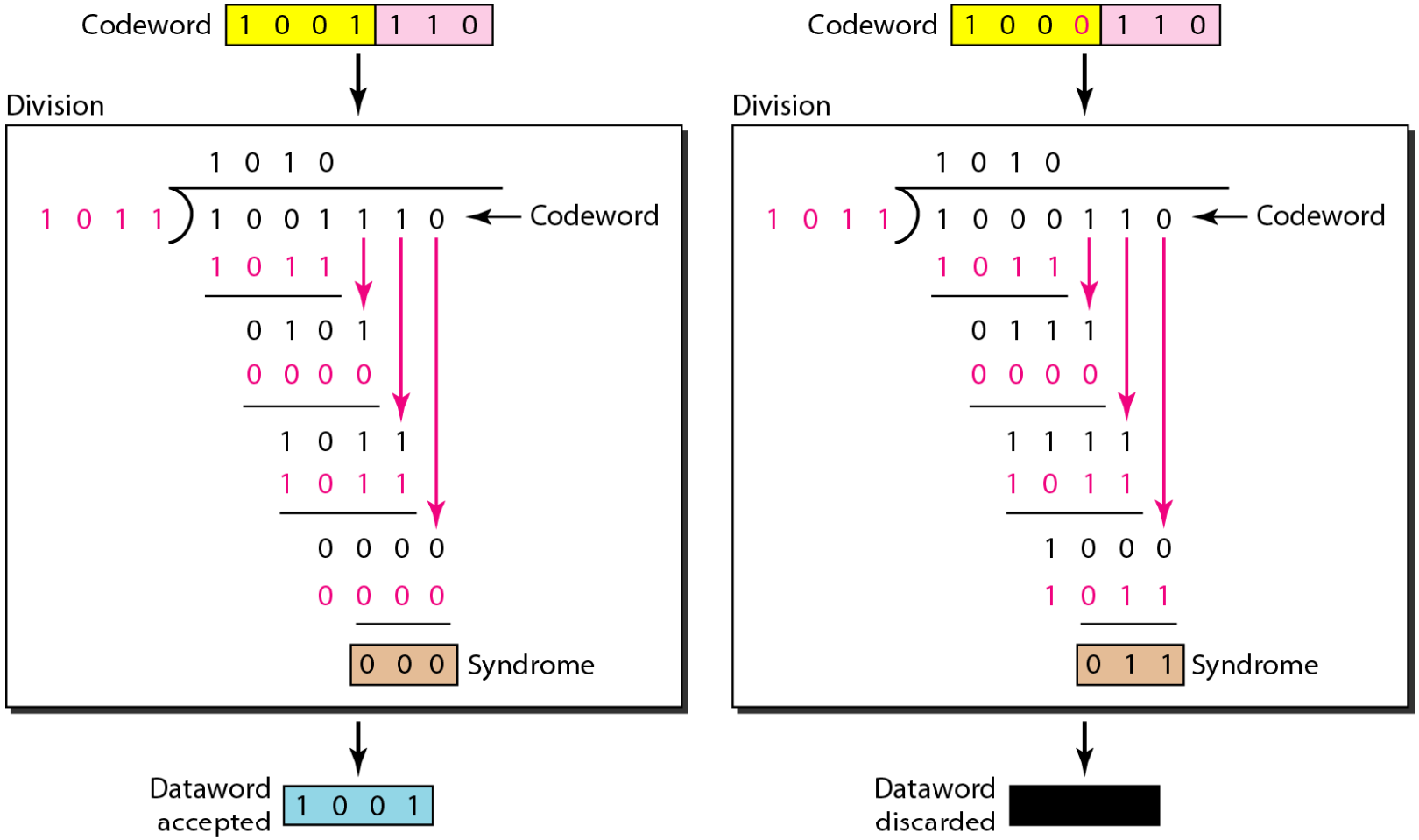


Figure 10.17 *Hardwired design of the divisor in CRC*

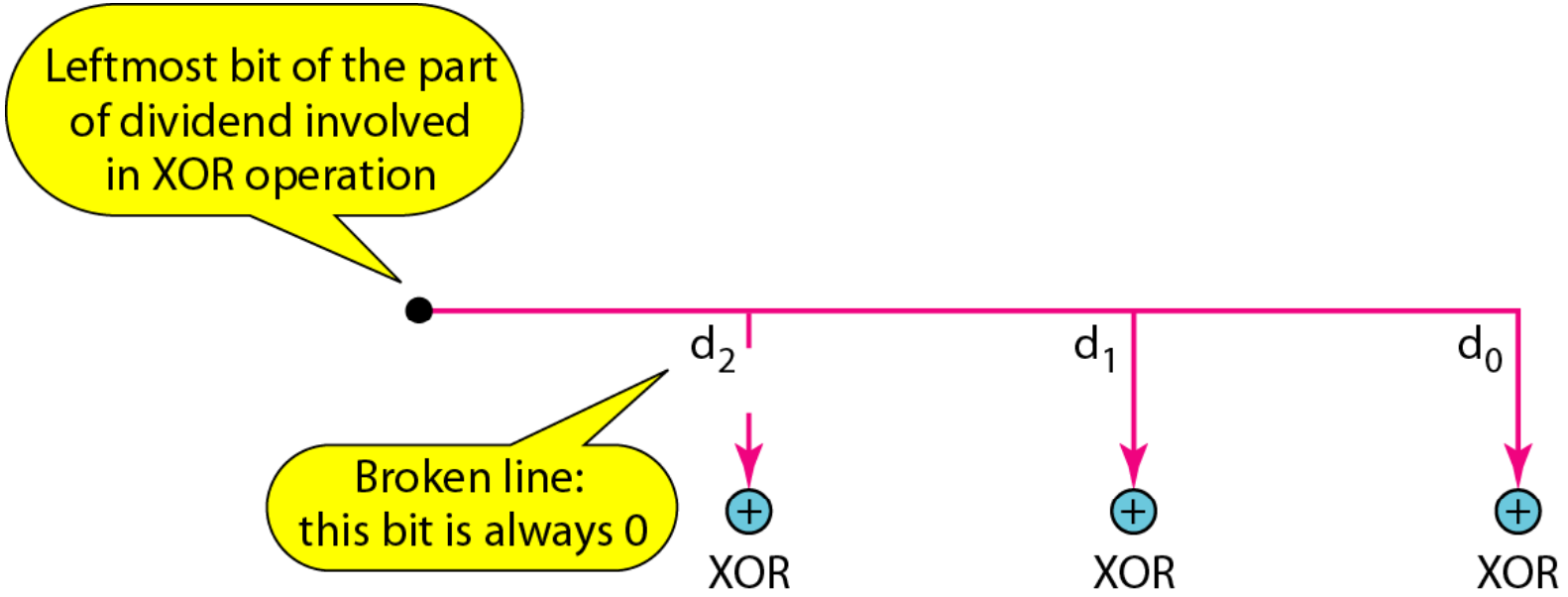


Figure 10.18 *Simulation of division in CRC encoder*

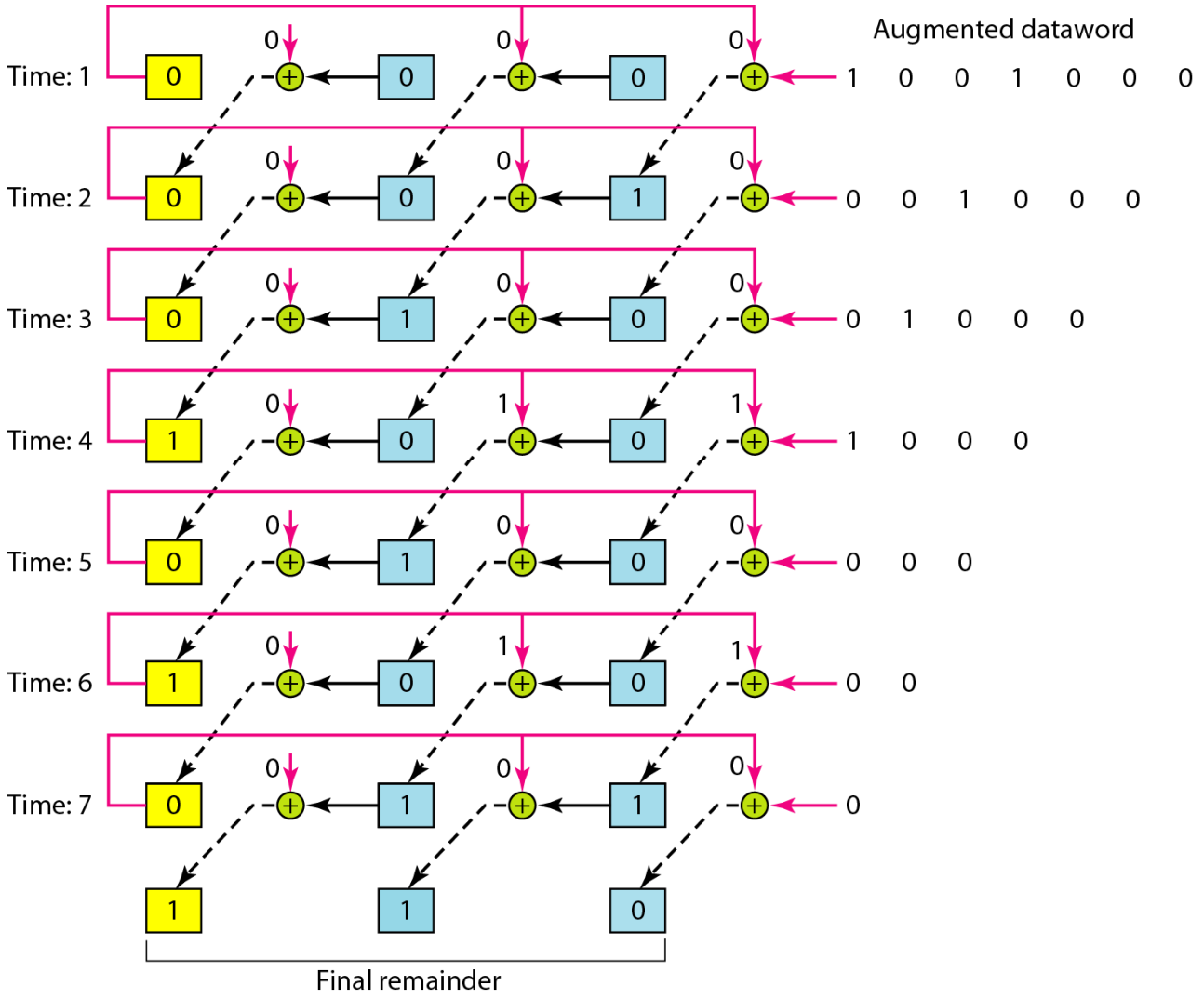


Figure 10.19 *The CRC encoder design using shift registers*

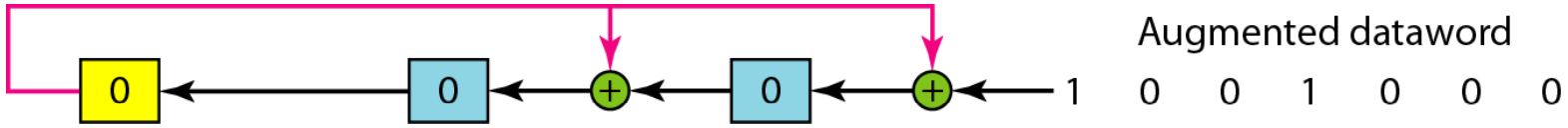


Figure 10.20 *General design of encoder and decoder of a CRC code*

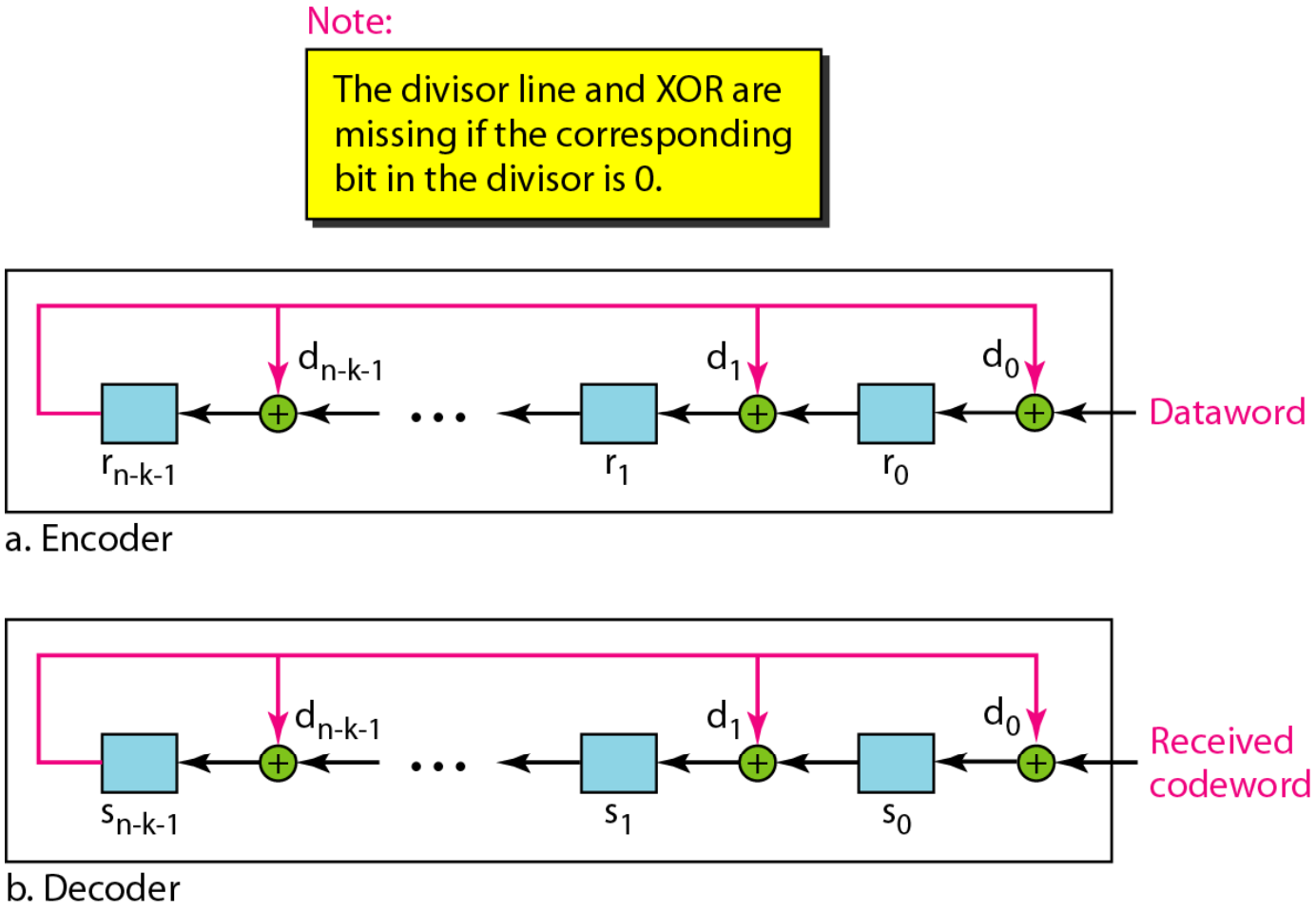
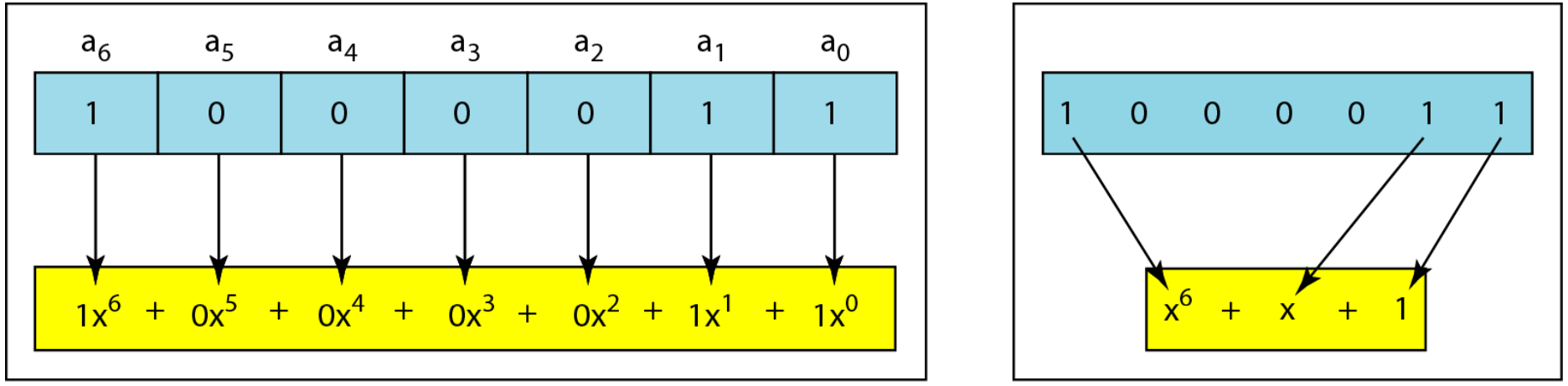


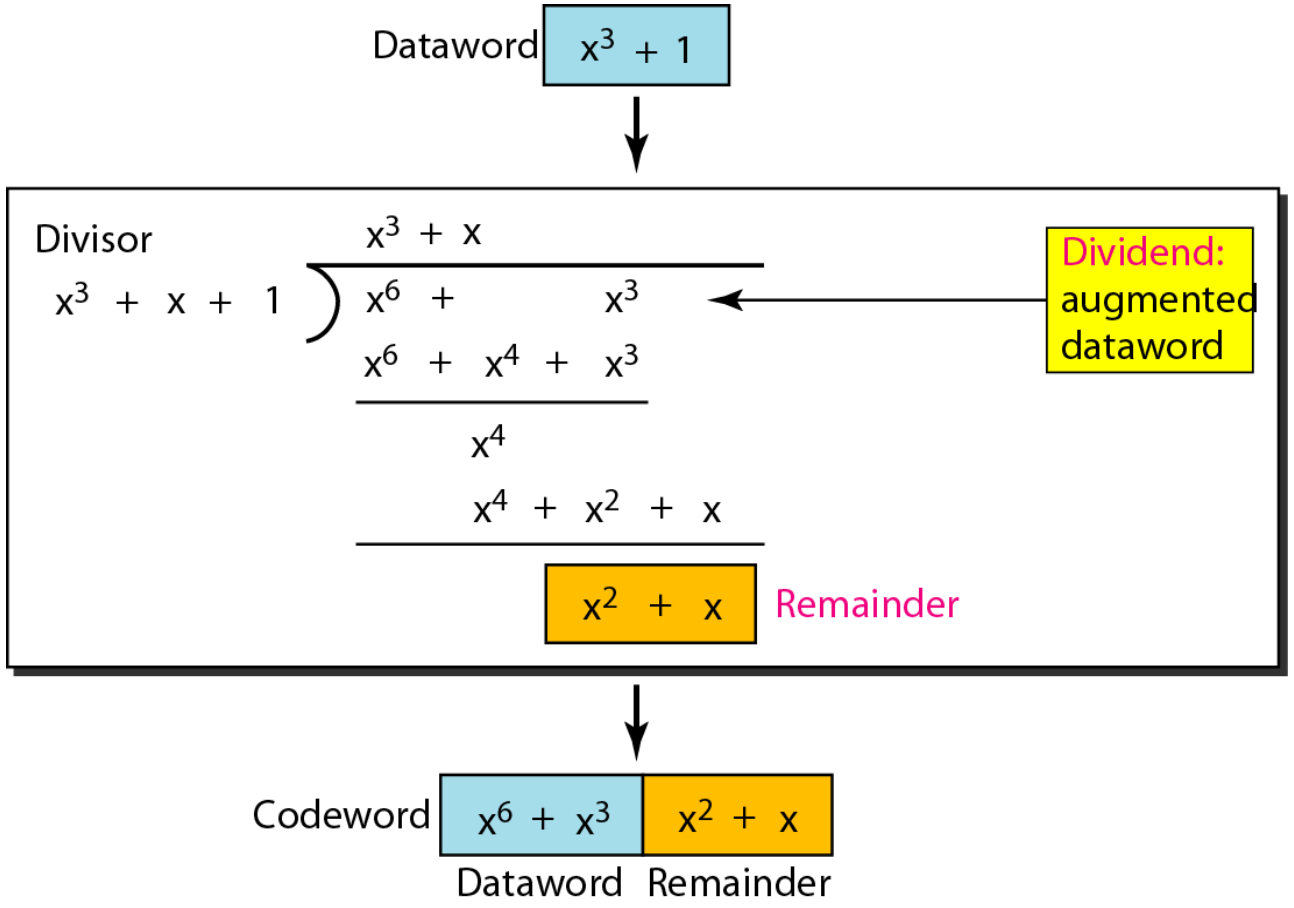
Figure 10.21 *A polynomial to represent a binary word*



a. Binary pattern and polynomial

b. Short form

Figure 10.22 *CRC division using polynomials*



Note

The divisor in a cyclic code is normally called the generator polynomial or simply the generator.

Note

In a cyclic code,

If s(x) ≠ 0, one or more bits is corrupted.

If s(x) = 0, either

a. No bit is corrupted. or

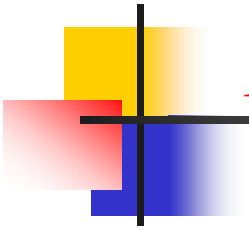
b. Some bits are corrupted, but the decoder failed to detect them.

Note

In a cyclic code, those *e*(x) errors that are divisible by *g*(x) are not caught.

Note

If the generator has more than one term and the coefficient of x^0 is 1, all single errors can be caught.



Example 10.15

Which of the following g(x) values guarantees that a single-bit error is caught? For each case, what is the error that cannot be caught?

a. $x + 1$ *b.* x^3 *c.* 1

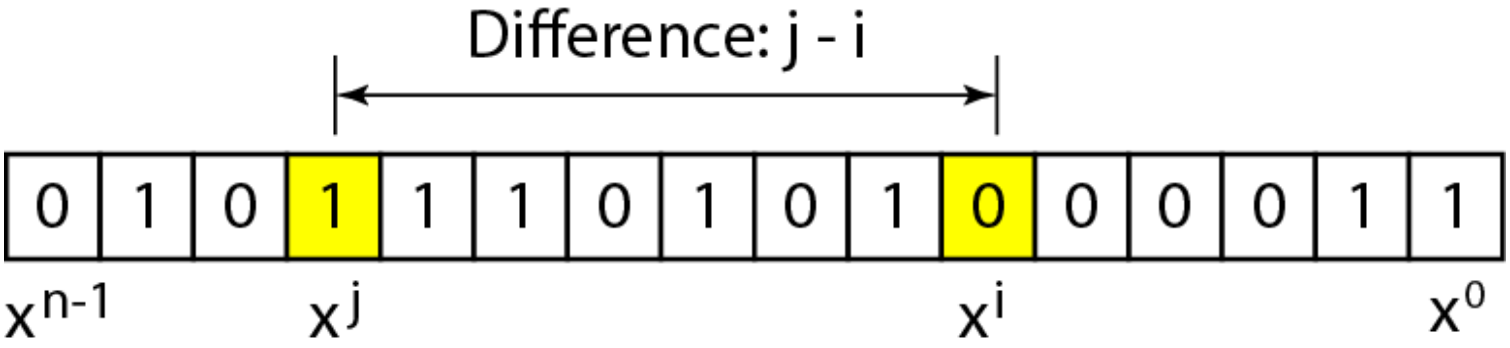
Solution

a. No x^i can be divisible by $x + 1$. Any single-bit error can be caught.

b. If i is equal to or greater than 3, x^i is divisible by g(x). All single-bit errors in positions 1 to 3 are caught.

c. All values of i make x^i divisible by g(x). No single-bit error can be caught. This g(x) is useless.

Figure 10.23 *Representation of two isolated single-bit errors using polynomials*



Note

If a generator cannot divide $x^t + 1$ (t between 0 and n – 1), then all isolated double errors can be detected.

Example 10.16

Find the status of the following generators related to two isolated, single-bit errors.

a. $x + 1$ *b.* $x^4 + 1$ *c.* $x^7 + x^6 + 1$ *d.* $x^{15} + x^{14} + 1$

Solution

a. This is a very poor choice for a generator. Any two errors next to each other cannot be detected.

b. This generator cannot detect two errors that are four positions apart.

c. This is a good choice for this purpose.

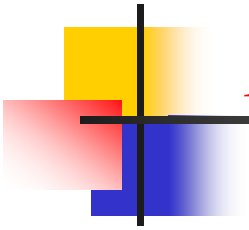
d. This polynomial cannot divide $x^t + 1$ *if t is less than 32,768. A codeword with two isolated errors up to 32,768 bits apart can be detected by this generator.*

Note

A generator that contains a factor of *x* + 1 can detect all odd-numbered errors.

Note

- All burst errors with $L \leq r$ will be detected.
- All burst errors with $L = r + 1$ will be detected with probability $1 - (1/2)^{r-1}$.
- All burst errors with $L > r + 1$ will be detected with probability $1 - (1/2)^{r}$.



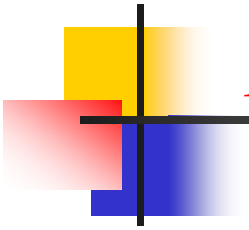
Example 10.17

Find the suitability of the following generators in relation to burst errors of different lengths.

a. $x^6 + 1$ *b.* $x^{18} + x^7 + x + 1$ *c.* $x^{32} + x^{23} + x^7 + 1$

Solution

a. This generator can detect all burst errors with a length less than or equal to 6 bits; 3 out of 100 burst errors with length 7 will slip by; 16 out of 1000 burst errors of length 8 or more will slip by.



Example 10.17 (continued)

b. ***This generator can detect all burst errors with a length less than or equal to 18 bits; 8 out of 1 million burst errors with length 19 will slip by; 4 out of 1 million burst errors of length 20 or more will slip by.***

c. ***This generator can detect all burst errors with a length less than or equal to 32 bits; 5 out of 10 billion burst errors with length 33 will slip by; 3 out of 10 billion burst errors of length 34 or more will slip by.***

Note

A good polynomial generator needs to have the following characteristics:

1. **It should have at least two terms.**
2. **The coefficient of the term x^0 should be 1.**
3. **It should not divide $x^t + 1$, for t between 2 and $n - 1$.**
4. **It should have the factor $x + 1$.**

Table 10.7 *Standard polynomials*

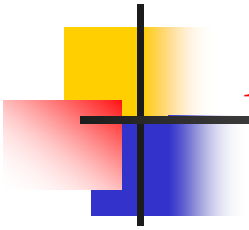
Name	*Polynomial*	*Application*
CRC-8	$x^8 + x^2 + x + 1$	ATM header
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^2 + 1$	ATM AAL
CRC-16	$x^{16} + x^{12} + x^5 + 1$	HDLC
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$	LANs

10-5 CHECKSUM

The last error detection method we discuss here is called the checksum. The checksum is used in the Internet by several protocols although not at the data link layer. However, we briefly discuss it here to complete our discussion on error checking

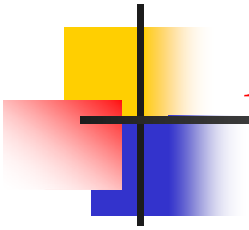
Topics discussed in this section:

Idea
One's Complement
Internet Checksum



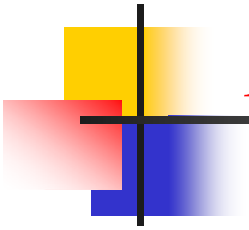
Example 10.18

Suppose our data is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers. For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, 36), where 36 is the sum of the original numbers. The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum. Otherwise, there is an error somewhere and the data are not accepted.



Example 10.19

We can make the job of the receiver easier if we send the negative (complement) of the sum, called the checksum. In this case, we send (7, 11, 12, 0, 6, −36). The receiver can add all the numbers received (including the checksum). If the result is 0, it assumes no error; otherwise, there is an error.

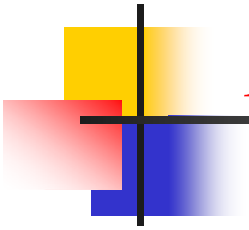


Example 10.20

How can we represent the number 21 in one's complement arithmetic using only four bits?

Solution

The number 21 in binary is 10101 (it needs five bits). We can wrap the leftmost bit and add it to the four rightmost bits. We have (0101 + 1) = 0110 or 6.

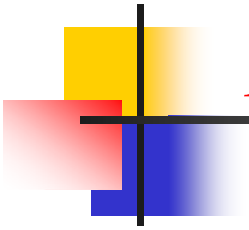


Example 10.21

How can we represent the number −6 in one's complement arithmetic using only four bits?

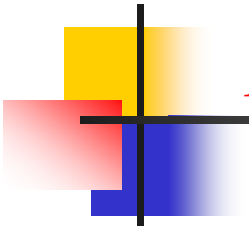
Solution

In one's complement arithmetic, the negative or complement of a number is found by inverting all bits. Positive 6 is 0110; negative 6 is 1001. If we consider only unsigned numbers, this is 9. In other words, the complement of 6 is 9. Another way to find the complement of a number in one's complement arithmetic is to subtract the number from $2^n - 1$ (16 − 1 in this case).



Example 10.22

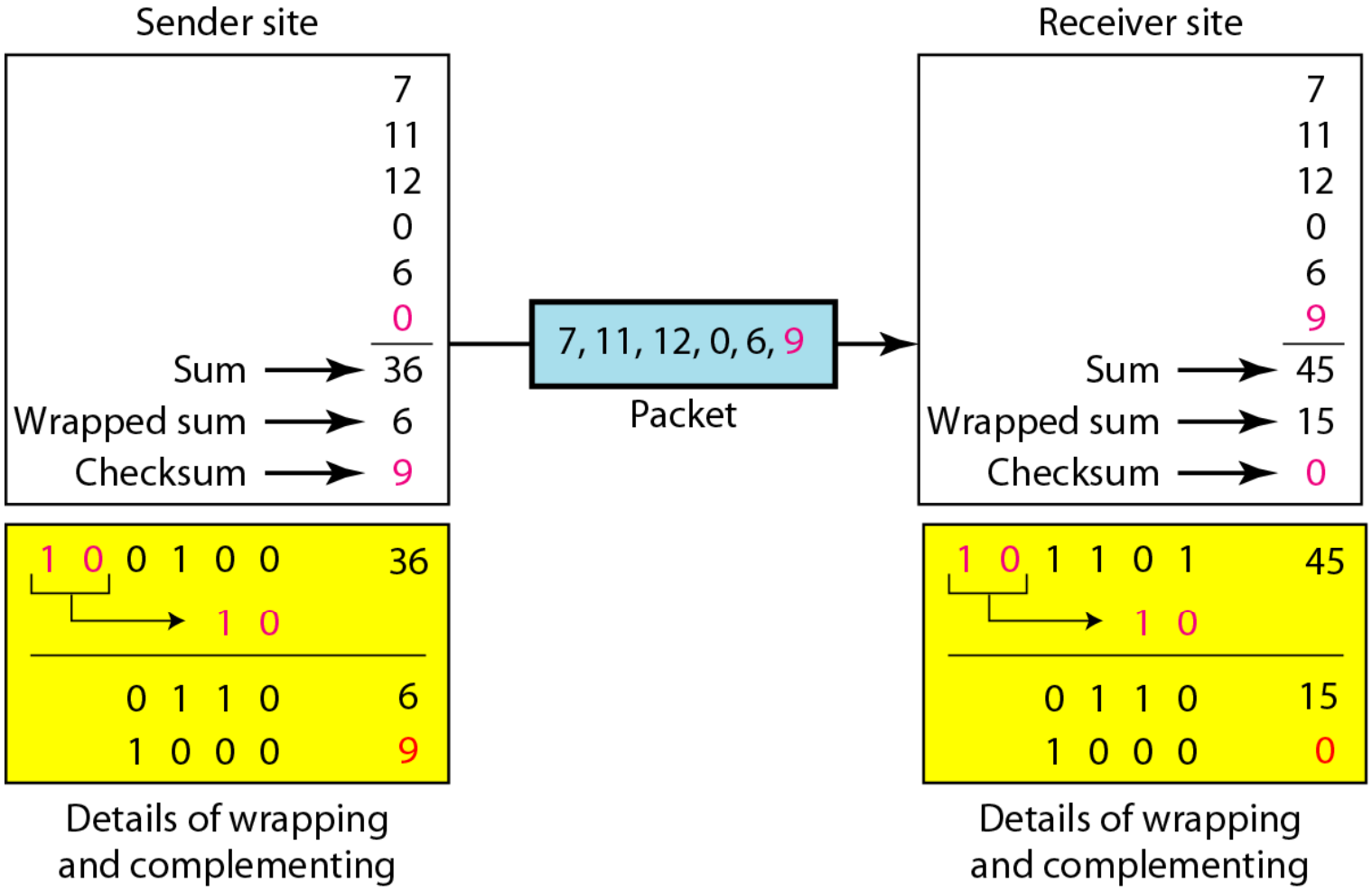
Let us redo Exercise 10.19 using one's complement arithmetic. Figure 10.24 shows the process at the sender and at the receiver. The sender initializes the checksum to 0 and adds all data items and the checksum (the checksum is considered as one data item and is shown in color). The result is 36. However, 36 cannot be expressed in 4 bits. The extra two bits are wrapped and added with the sum to create the wrapped sum value 6. In the figure, we have shown the details in binary. The sum is then complemented, resulting in the checksum value 9 (15 − 6 = 9). The sender now sends six data items to the receiver including the checksum 9.



Example 10.22 (continued)

The receiver follows the same procedure as the sender. It adds all data items (including the checksum); the result is 45. The sum is wrapped and becomes 15. The wrapped sum is complemented and becomes 0. Since the value of the checksum is 0, this means that the data is not corrupted. The receiver drops the checksum and keeps the other data items. If the checksum is not zero, the entire packet is dropped.

Figure 10.24 *Example 10.22*



Note

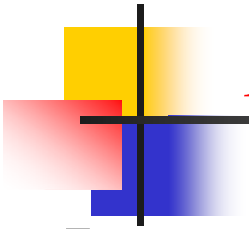
Sender site:

1. The message is divided into 16-bit words.
2. The value of the checksum word is set to 0.
3. All words including the checksum are added using one's complement addition.
4. The sum is complemented and becomes the checksum.
5. The checksum is sent with the data.

Note

Receiver site:

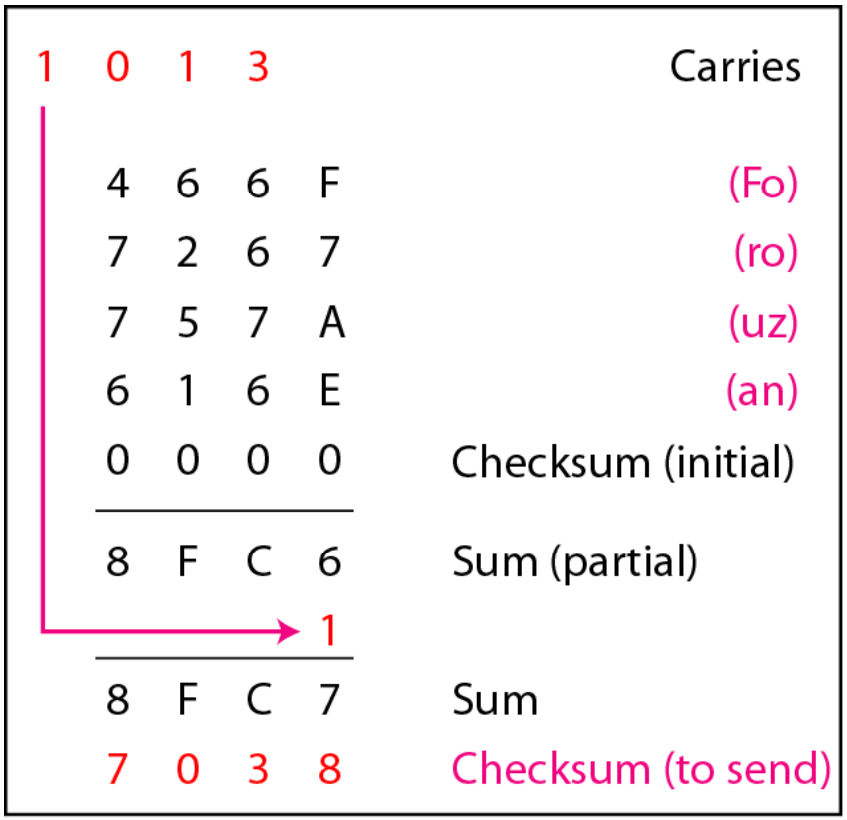
1. The message (including checksum) is divided into 16-bit words.
2. All words are added using one's complement addition.
3. The sum is complemented and becomes the new checksum.
4. If the value of checksum is 0, the message is accepted; otherwise, it is rejected.



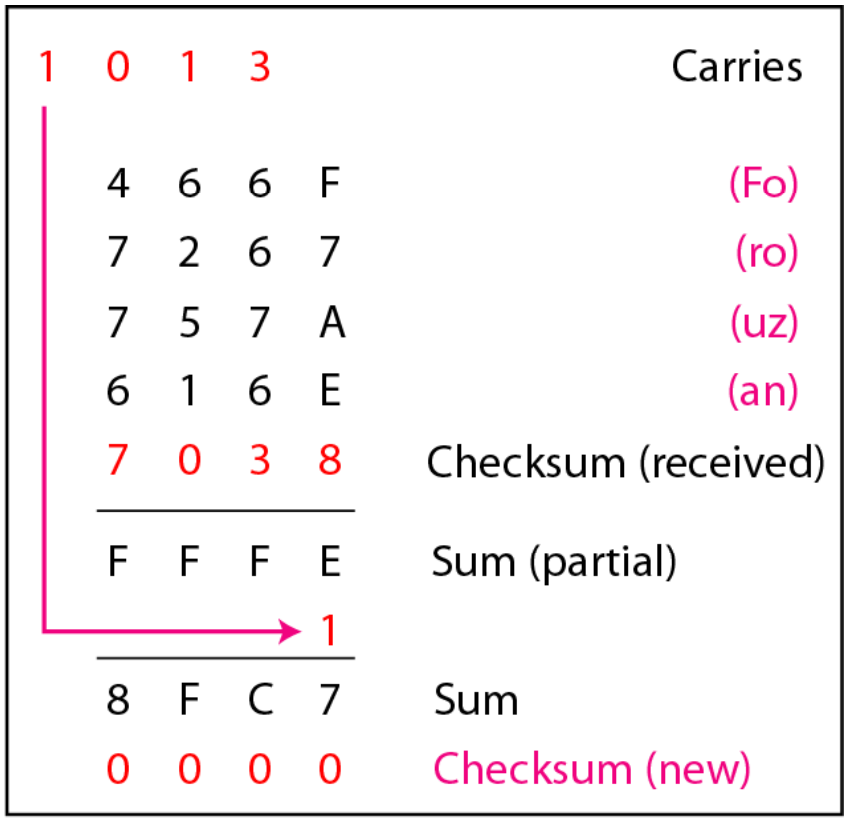
Example 10.23

Let us calculate the checksum for a text of 8 characters (“Forouzan”). The text needs to be divided into 2-byte (16-bit) words. We use ASCII (see Appendix A) to change each byte to a 2-digit hexadecimal number. For example, F is represented as 0x46 and o is represented as 0x6F. Figure 10.25 shows how the checksum is calculated at the sender and receiver sites. In part a of the figure, the value of partial sum for the first column is 0x36. We keep the rightmost digit (6) and insert the leftmost digit (3) as the carry in the second column. The process is repeated for each column. Note that if there is any corruption, the checksum recalculated by the receiver is not all 0s. We leave this an exercise.

Figure 10.25 *Example 10.23*



a. Checksum at the sender site



a. Checksum at the receiver site