---

1. **Fun with Big-O notation.** (3 points; 0.5 point each) Mark the following as `True` or `False`. Briefly but convincingly justify all of your answers, using the definitions of $O(\cdot)$, $\Theta(\cdot)$ and $\Omega(\cdot)$. [**To see the required detail, 1^{st} question has been solved for you.**]

   (z) $n = \Omega(n^2)$. This statement is `False`. To see this, we will use a proof by contradiction. Suppose that, as per the definition of $\Omega(\cdot)$, there is some n_0 and some $c > 0$ so that for all $n \geq n_0$, $n \geq c \cdot n^2$. Choose $n = \max\{1/c, n_0\} + 1$. Then $n \geq n_0$, but we have $n > 1/c$, which implies that $c \cdot n^2 > n$. This is a contradiction.

   (a) $n = O(n \log(n))$.

   (b) $n^{1/\log(n)} = \Theta(1)$.

   (c) If
   $$f(n) = \begin{cases} 5^n & \text{if } n < 2^{1000} \\ 2^{1000}n^2 & \text{if } n \geq 2^{1000} \end{cases}$$
   and $g(n) = \frac{n^2}{2^{1000}}$, then $f(n) = O(g(n))$.

   (d) For all possible functions $f(n), g(n) \geq 0$, if $f(n) = O(g(n))$, then $2^{f(n)} = O(2^{g(n)})$.

   (e) $5^{\log\log(n)} = O(\log(n)^2)$

   (f) $n = \Theta\left(100^{\log(n)}\right)$

2. **Fun with recurrences.** (3 points; 0.5 point each)

   Solve the following recurrence relations; i.e. express each one as $T(n) = O(f(n))$ for the tightest possible function $f(n)$, and give a short justification. Be aware that some parts might be slightly more involved than others. Unless otherwise stated, assume $T(1) = 1$. [**To see the required detail, 1^{st} question has been solved for you.**]

   (z) $T(n) = 6T(n/6) + 1$. We apply the master theorem with $a = b = 6$ and with $d = 0$. We have $a > b^d$, and so the running time is $O(n^{\log_6(6)}) = O(n)$.

   (a) $T(n) = 2T(n/2) + 3n$

   (b) $T(n) = 3T(n/4) + \sqrt{n}$

   (c) $T(n) = 7T(n/2) + \Theta(n^3)$

   (d) $T(n) = 4T(n/2) + n^2 \log n$

   (e) $T(n) = 2T(n/3) + n^c$, where $c \geq 1$ is a constant (that is, it doesn't depend on n).

   (f) $T(n) = 2T(\sqrt{n}) + 1$, where $T(2) = 1$

3. **Different-sized sub-problems.** (4 points) Solve the following recurrence relation.
   $$T(n) = T(n/2) + T(n/4) + T(n/8) + n,$$
   where $T(1) = 1$. [**We are expecting a detailed solution with all the steps. You need to finally state your running time with $O(\cdot)$ notation.**]