# Assignment 3

1. Generate 100 real number for the variable X from the uniform distribution U [0,1].

2. Construct the training set T = { $(x_1,y_1),(x_2,y_2),\ldots\ldots,(x_{100},y_{100})$} using the relation

   $$Yi = \sin(2\pi x_i) + \epsilon_i \text{ where } \epsilon_i \sim N(0,0.25).$$

   In the similar way construct a testing set of size 50

   $$\text{I,e. Test} = \{ (x'_1,y'_1),(x'_2,y'_2),\ldots\ldots,(x'_{50},y'_{50})\}.$$

3. Estimate the regularized Least Squares Polynomial Regression model of order M= 9, using the training set T using direct method. You also need to tune the regularization parameter λ which corresponds to minimum RMSE. After tuning the parameter λ, evaluate your estimated function using NMSE, RMSE, MAE and $R^2$ on test set.

4. Estimate the regularized Least Squares Polynomial Regression model of order M= 9, on the training set T using gradient descent method with the tuned value of λ obtained in question 3 by selecting an appropriate step length η. Compare the solution obtained by direct method and gradient descent method. Evaluate your estimated function using NMSE, RMSE, MAE and $R^2$ on test set.

5. Estimate the regularized Least Squares Polynomial Regression model of order M= 9 on the training set T using stochastic gradient descent method with the tuned value of λ obtained in question 3 by selecting an appropriate step lengths $\eta_i$. Compare the solution with the solution obtained by the direct method and gradient descent method. Evaluate your estimated function using NMSE, RMSE, MAE and $R^2$ on test set. Also obtain the norm of the obtained gradient at each iteration (epoch) of stochastic gradient descent method and plot it.

6. Estimate the regularized Least Squares Kernel Regression model by using the Gaussian (RBF) kernel which is given by

   $$K\ (x_i, x_j) = e^{\frac{-||x_i - x_j||^2}{\sigma^2}}.$$

   Also, tune the value of the regularization parameter λ and kernel parameter σ and obtain the best possible estimate using NMSE, RMSE, MAE and $R^2$ on test set.

7. Estimate the regularized Least Squares Kernel Regression model by using the Gaussian (RBF) kernel with stochastic gradient descent method. Make use of the tuned value of regularization parameter λ and kernel parameter σ of question 6. Compare the obtained solution with the solution obtained at question 6.

8. Modify the training set T by picking up randomly 5 data points from the training set T and scale their $y_i$ values by 20. Now, find the estimate of Least Squares Polynomial Regression model of order M= 9 with modified training set. Plot the estimated function along with data points of modified training set. Also, write down your observations.