

Assignment

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Question 9.3.18

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Solution:

Parameter	Value	Description
X	$\{0,1,2,3,4\}$	Number of defective bulbs taken
n	4	Number of bulbs taken
p	0.2	Taking a defective bulb
q	0.8	Taking a non defective bulb
$\mu = np$	0.8	Mean of Binomial distribution
$\sigma^2 = npq$	0.64	Variance of Binomial distribution

TABLE 1: Parameter description

let Y be a gaussian Random variable

$$Y \sim N(np, npq) \quad (1)$$

$$\sim N(0.8, 0.64) \quad (2)$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$p_Y(x) \approx \Pr(x - 0.5 < Y < x + 0.5) \quad (3)$$

$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \quad (4)$$

$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \quad (5)$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y < x) \quad (6)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \quad (7)$$

$$\Rightarrow \frac{Y - \mu}{\sigma} \sim N(0, 1) \quad (8)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (9)$$

$$= \begin{cases} 1 - Q\left(\frac{x - \mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu - x}{\sigma}\right) & x < \mu \end{cases} \quad (10)$$

Then probability in terms of Q function is

$$\Rightarrow p_Y(x) \approx Q\left(\frac{(x - 0.5) - \mu}{\sigma}\right) - Q\left(\frac{(x + 0.5) - \mu}{\sigma}\right) \quad (11)$$

1)

$$X = 0 \quad (12)$$

The Gaussian approximation for $\Pr(X = 0)$ is

$$\approx Q(-8.125) - Q(-1.875) \quad (13)$$

$$\approx 0.3017 \quad (14)$$

2)

$$X = 1 \quad (15)$$

The Gaussian approximation for $\Pr(X = 1)$ is

$$\approx Q(-1.875) - Q(4.375) \quad (16)$$

$$\approx 0.4555 \quad (17)$$

3)

$$X = 2 \quad (18)$$

The Gaussian approximation for $\Pr(X = 2)$ is

$$\approx Q(4.375) - Q(10.625) \quad (19)$$

$$\approx 0.1739 \quad (20)$$

4)

$$X = 3 \quad (21)$$

The Gaussian approximation for $\Pr(X = 3)$ is

$$\approx Q(10.625) - Q(16.875) \quad (22)$$

$$\approx 0.0164 \quad (23)$$

5)

$$X = 4 \quad (24)$$

The Gaussian approximation for $\Pr(X = 4)$ is

$$\approx Q(16.875) - Q(23.125) \quad (25)$$

$$\approx 0.00036 \quad (26)$$

Number of defective bulbs	Binomial distribution	Gaussian approximation	Error
0	0.4096	0.3017	26.342773437
1	0.4096	0.4555	11.206054688
2	0.1536	0.1739	13.216145833
3	0.0256	0.0164	35.9375
4	0.0016	0.00036	77.5

TABLE 2: Comparing the gaussian approximation with binomial

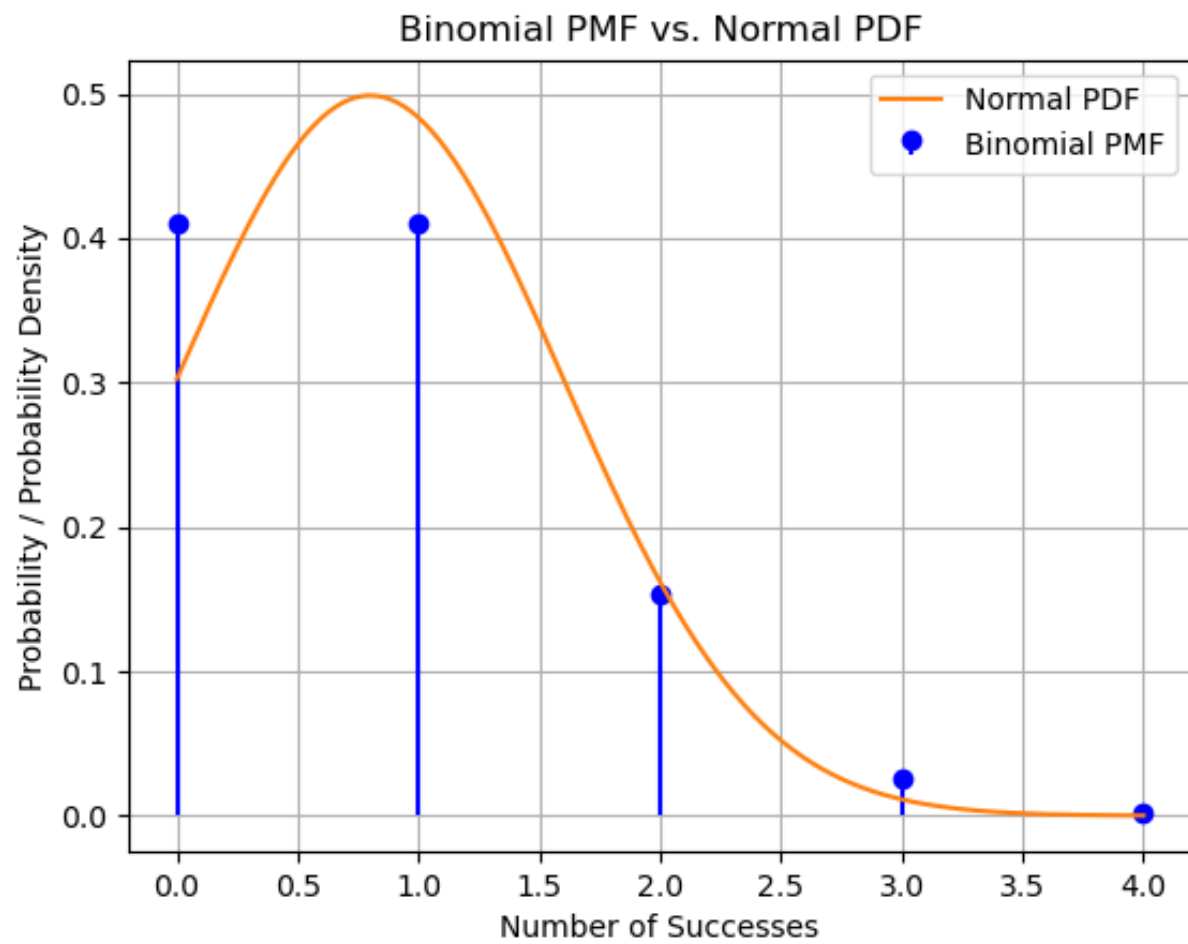


Fig. 1: Binomial and gaussian distribution