Assignment

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Question:12/13/3/40

An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on k.

Solution:

parameters	values	decription
X	0	first ball
	1	second ball
Y	0	black ball
	1	white ball

TABLE 1: Random variable description

The probabilities are

$$\begin{cases} p_{XY}(00) = \frac{n}{m+n} \\ p_{XY}(01) = \frac{m}{m+n} \\ \Pr(X=1, Y=1 | X=0, Y=0) = \frac{m}{m+n+k} \\ \Pr(X=1, Y=1 | X=0, Y=1) = \frac{m+k}{m+n+k} \\ \Pr(X=1, Y=0 | X=0, Y=0) = \frac{n+k}{m+n+k} \\ \Pr(X=1, Y=0 | X=0, Y=1) = \frac{n}{m+n+k} \end{cases} \tag{1}$$

Using total probability theorem,the desired probability is

$$= p_{XY}(00) \Pr(X=1, Y=1 | X=0, Y=0) + p_{XY}(01) \Pr(X=1, Y=1 | X=0, Y=1) \tag{2}$$

$$= \left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+k}\right) + \left(\frac{m}{m+n}\right)\left(\frac{m+k}{m+n+k}\right) \tag{3}$$

$$= \frac{mn + m(m+k)}{(m+n)(m+n+k)} \tag{4}$$

$$= \frac{m(m+n+k)}{(m+n)(m+n+k)} \tag{5}$$

$$= \frac{m}{m+n} \tag{6}$$

which is independent of k.