Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	CA
$\|\|A-B\|\|$	4.47	length of AB
$\|\|B-C\|\|$	2.82	length of BC
$\|\|C-A\|\|$	2	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	AB
c_1	10	
$\mathbf{n_2}$	$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$	BC
c_2	-2	
$\mathbf{n_3}$	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	CA
c_3	-4	
Area	2	Area of Triangle
$\angle A$	26.57°	Angles
$\angle B$	18.43°	
$\angle C$	135°	

TABLE 1: Vectors.

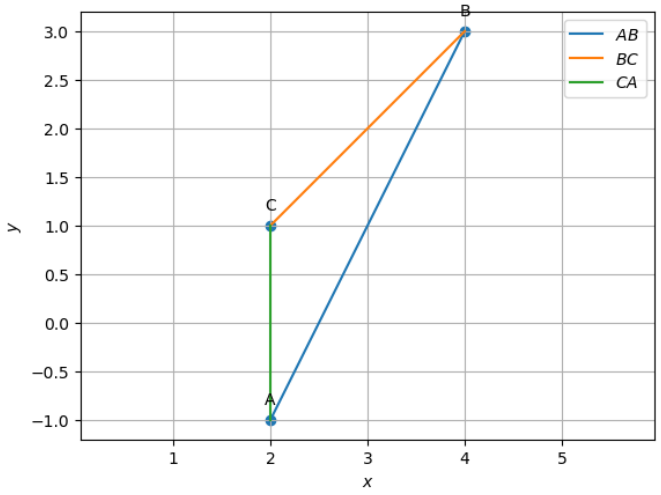

Fig. 1: triangle plotted using python

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	
c_4	7	
$\mathbf{m_5}$	$\begin{pmatrix} -2 \\ -3 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	
c_5	-6	
$\mathbf{m_6}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	
c_6	-1	
$\mathbf{G}$	$\begin{pmatrix} 2.67 \\ 1 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

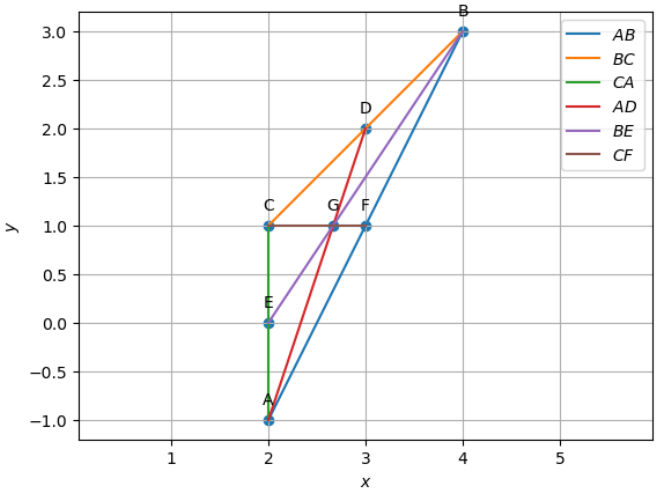

Fig. 2: medians plotted using python

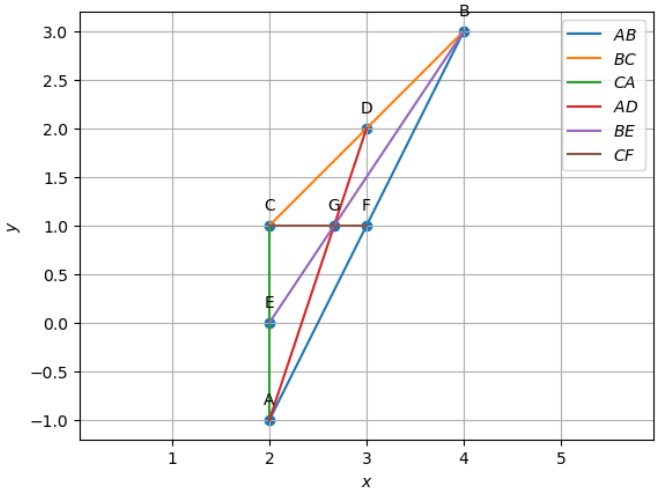

Fig. 3: altitudes plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}1\\0\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}2\\3\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}2.8\\0.6\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}-1\\1\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}1\\1\end{pmatrix}$	
c_7	1	
$\mathbf{m_8}$	$\begin{pmatrix}-2\\0\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}0\\2\end{pmatrix}$	
c_8	6	
$\mathbf{m_9}$	$\begin{pmatrix}0.8\\-0.4\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}-0.4\\-0.8\end{pmatrix}$	
c_9	-1.6	
$\mathbf{H}$	$\begin{pmatrix}-2\\3\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}-2\\2\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}-2\\-2\end{pmatrix}$	
c_{10}	-10	
$\mathbf{m_{11}}$	$\begin{pmatrix}2\\0\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}0\\2\end{pmatrix}$	
c_{11}	0	
$\mathbf{m_{12}}$	$\begin{pmatrix}-4\\2\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}-2\\-4\end{pmatrix}$	
c_{12}	-10	
$\mathbf{O}$	$\begin{pmatrix}5\\0\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	3.16	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	53.13°	$\angle BOC = 2\angle BAC$
$\angle BAC$	26.565°	
$\angle AOC$	36.8°	$\angle AOC = 2\angle ABC$
$\angle ABC$	18.4°	
$\angle AOB$	270°	$\angle AOB = 2\angle BCA$
$\angle BCA$	135°	

TABLE 4: Perpendicular Bisector.

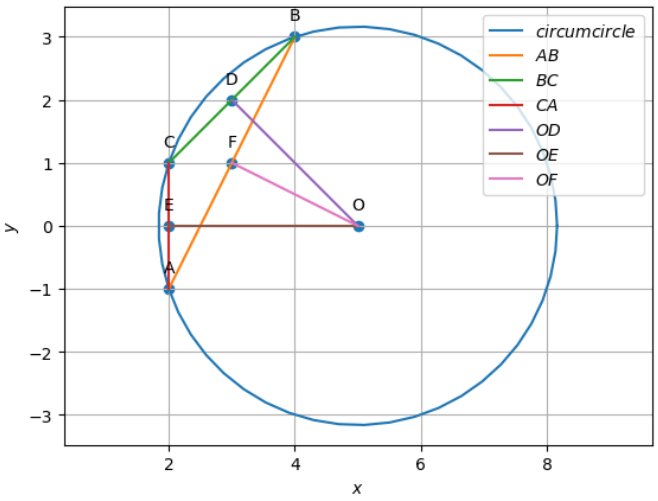

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}-0.45\\-1.89\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}-1.89\\0.45\end{pmatrix}$	
c_{13}	-4.23	
$\mathbf{m_{14}}$	$\begin{pmatrix}-1.15\\-1.6\end{pmatrix}$	BI
$\mathbf{m_{14}}$	$\begin{pmatrix}1.6\\-1.15\end{pmatrix}$	
c_{14}	2.943	
$\mathbf{m_{15}}$	$\begin{pmatrix}-0.71\\0.29\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}-0.29\\-0.70\end{pmatrix}$	
c_{15}	-1.29	
$\mathbf{I}$	$\begin{pmatrix}2.43\\0.82\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}2.13\\1.13\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}2\\0.82\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}2.81\\0.62\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	0.43	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	13.28°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	9.21°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	67.5°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

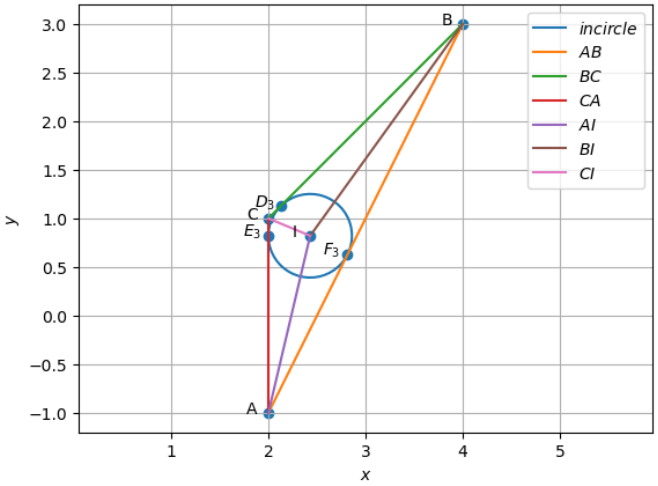

Fig. 5: Angle bisectors plotted using python