# Assignment

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**Question 9.3.18**

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

**Solution:**

**Binomial**

Let X be a binomial random variable representing the number of defective bulbs in a sample of 4 bublbs with the parameters n and p as,

$$n = 4 \tag{1}$$

$$p = \frac{6}{30} \tag{2}$$

$$= 0.2 \tag{3}$$

PMF of the distribution is,

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{4}$$

1)

$$k = 0 \tag{5}$$

$$\implies p_X(0) = {}^4C_0 (0.2)^0 (0.8)^4 \tag{6}$$

$$= 0.4096 \tag{7}$$

2)

$$k = 1 \tag{8}$$

$$\implies p_X(1) = {}^4C_1 (0.2)^1 (0.8)^3 \tag{9}$$

$$= 0.4096 \tag{10}$$

3)

$$k = 2 \tag{11}$$

$$\implies p_X(2) = {}^4C_2 (0.2)^2 (0.8)^2 \tag{12}$$

$$= 0.1536 \tag{13}$$

4)

$$k = 3 \tag{14}$$

$$\implies p_X(3) = {}^4C_3 (0.2)^3 (0.8)^1 \tag{15}$$

$$= 0.0256 \tag{16}$$

5)

$$k = 4 \tag{17}$$

$$\implies p_X(2) = {}^4C_4 (0.2)^4 (0.8)^0 \tag{18}$$

$$= 0.0016 \tag{19}$$

**Gaussian**

Mean and varience of X are

$$\mu_X = np \tag{20}$$
$$= 0.8 \tag{21}$$
$$\sigma_X^2 = np\,(1-p) \tag{22}$$
$$= 0.16 \tag{23}$$

Let,Z be a rondom variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$, such that,

$$Z = \frac{X - \mu_X}{\sigma_X} \tag{24}$$

Z converges to normal distribution for large value of n

$$f\,(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{25}$$

And the Q funtion is

$$Q\,(x) = \Pr\,(Z > x) \tag{26}$$

Then

$$\Pr\left(Z = \frac{X - \mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X + 0.5 - \mu_X}{\sigma_X} < Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{27}$$
$$\approx \Pr\left(Z < \frac{X + 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{28}$$
$$\approx \Pr\left(Z > \frac{X - 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{29}$$
$$\approx Q\left(\frac{X - 0.5 - \mu_X}{\sigma_X}\right) - Q\left(\frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{30}$$

1)

$$X = 0 \tag{31}$$

$$\approx Q\,(-8.125) - Q\,(-1.875) \tag{32}$$
$$\approx 0.3017 \tag{33}$$

2)

$$X = 1 \tag{34}$$

$$\approx Q\,(-1.875) - Q\,(4.375) \tag{35}$$
$$\approx 0.4555 \tag{36}$$

3)

$$X = 2 \tag{37}$$

$$\approx Q\,(4.375) - Q\,(10.625) \tag{38}$$
$$\approx 0.1739 \tag{39}$$

4)

$$X = 3 \tag{40}$$

$$\approx Q(10.625) - Q(16.875) \tag{41}$$
$$\approx 0.0164 \tag{42}$$

5)

$$X = 4 \tag{43}$$

$$\approx Q(16.875) - Q(23.125) \tag{44}$$
$$\approx 0.00036 \tag{45}$$

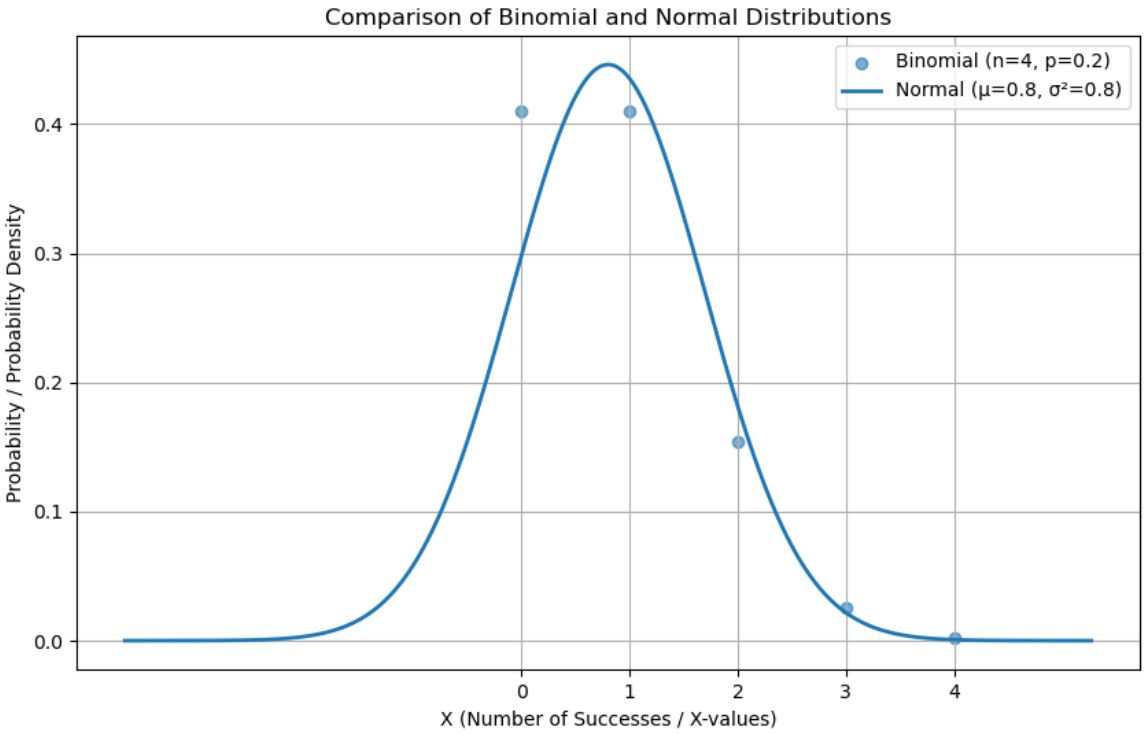


Fig. 1: Binomial and gaussian distribution