Question 12.13.3.104
Probability and Random Processes

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Question:12/13/3/104
Let (X, Y) have joint probability density function

$$f(x, y) = \begin{cases} 8xy & if 0 < x < y < 1 \\ 0 & otherwise \end{cases} \tag{1}$$

if $E(X|Y = y_0) = \frac{1}{2}$,then y_0 equals
Solution:

$$E(X|Y = y_0) = \int_{-\infty}^{\infty} xf(x|y)\, dx \tag{2}$$

where

$$f(x|y) = \frac{f(x, y)}{f(y)} \tag{3}$$

$$f(y) = \int_0^y f(x, y)\, dx \tag{4}$$

$$= \int_0^y 8xydx \tag{5}$$

$$= 8y\left[\frac{x^2}{2}\right]_0^y \tag{6}$$

$$= 4y^3 \tag{7}$$

Substituting f(y) we get

$$f(x|y) = \frac{8xy}{4y^3} \tag{8}$$

$$= \frac{2x}{y^2} \tag{9}$$

and

$$E(X|Y = y_0) = \int_0^{y_0} x.\frac{2x}{y_0^2} dx \tag{10}$$

$$= \frac{2}{y_0^2}\left[\frac{x^3}{3}\right]_0^{y_0} \tag{11}$$

$$= \frac{2y_0}{3} \tag{12}$$

$$\implies \frac{2y_0}{3} = \frac{1}{2} \tag{13}$$

$$y_0 = \frac{3}{4} \tag{14}$$