

Assignment

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Question:12/13/3/40

An urn contains m white and n black balls. A ball is drawn at random and is put back into the urn along with k additional balls of the same colour as that of the ball drawn. A ball is again drawn at random. Show that the probability of drawing a white ball now does not depend on k .

Solution:

parameters	values	decription
X	0	first ball
	1	second ball
Y	0	black ball
	1	white ball

TABLE 1: Random variable description

The probabilities are

$$\left\{ \begin{array}{l} p_{XY}(00) = \frac{n}{m+n} \\ p_{XY}(01) = \frac{m}{m+n} \\ \Pr(X=1, Y=1|X=0, Y=0) = \frac{m}{m+n+k} \\ \Pr(X=1, Y=1|X=0, Y=1) = \frac{m+k}{m+n+k} \\ \Pr(X=1, Y=0|X=0, Y=0) = \frac{n+k}{m+n+k} \\ \Pr(X=1, Y=0|X=0, Y=1) = \frac{n}{m+n+k} \end{array} \right. \quad (1)$$

The desired probability is

$$= p_{XY}(11) \quad (2)$$

Using total probability theorem,

$$= p_{XY}(00) \Pr(X=1, Y=1|X=0, Y=0) + p_{XY}(01) \Pr(X=1, Y=1|X=0, Y=1) \quad (3)$$

$$= \left(\frac{n}{m+n} \right) \left(\frac{m}{m+n+k} \right) + \left(\frac{m}{m+n} \right) \left(\frac{m+k}{m+n+k} \right) \quad (4)$$

$$= \frac{mn + m(m+k)}{(m+n)(m+n+k)} \quad (5)$$

$$= \frac{m(m+n+k)}{(m+n)(m+n+k)} \quad (6)$$

$$= \frac{m}{m+n} \quad (7)$$

which is independent of k .