

# Probability and Random Processes

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## Question:

Let  $(X, Y)$  have joint probability density function

$$f(x, y) = \begin{cases} 8xy & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

if  $E(X|Y = y_0) = \frac{1}{2}$ , then  $y_0$  equals

and

$$E(X|Y = y_0) = \int_0^{y_0} x \cdot \frac{2x}{y_0^2} dx \quad (10)$$

$$= \frac{2}{y_0^2} \left[ \frac{x^3}{3} \right]_0^{y_0} \quad (11)$$

$$= \frac{2y_0}{3} \quad (12)$$

$$\Rightarrow \frac{2y_0}{3} = \frac{1}{2} \quad (13)$$

$$y_0 = \frac{3}{4} \quad (14)$$

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- 1)  $\frac{3}{4}$
- 2)  $\frac{1}{2}$
- 3)  $\frac{1}{3}$
- 4)  $\frac{2}{3}$

## Solution:

$$E(X|Y) = \int_{-\infty}^{\infty} x p_{X|Y} dx \quad (2)$$

where

$$p_{X|Y} = \frac{p_{XY}(x, y)}{p_Y(y)} \quad (3)$$

$$p_Y(y) = \int_0^y p_{X|Y}(x, y) dx \quad (4)$$

for  $0 < y < 1$

$$= \int_0^y 8xy dx \quad (5)$$

$$= 8y \left[ \frac{x^2}{2} \right]_0^y \quad (6)$$

$$= 4y^3 \quad (7)$$

Substituting  $p_Y(y)$  we get

$$p_{X|Y} = \frac{8xy}{4y^3} \quad (8)$$

$$= \frac{2x}{y^2} \quad (9)$$