Question 1.5.2

Find the intersection **I** of the angle bisectors of **B** and **C**

Solution:

From **??** the bisectors of **B** and **C** are obtained as

$$\left(\frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} \quad \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}}\right)\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \quad (1)$$

and

$$\left(\frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}}\right)\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \quad (2)$$

respectively.

The pair of linear equations can be solved using the Augmented matrix $\left(\mathbf{P}|\mathbf{Q}\right)$ Here,

$$\mathbf{P} = \begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix} \quad (3)$$

$$\mathbf{Q} = \begin{pmatrix} \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{pmatrix} \quad (4)$$

$$\left(\mathbf{P} \mid \mathbf{Q}\right) = \left(\begin{array}{cc|cc} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} & \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{array}\right) \quad (5)$$

The augmented matrix is converted into decimal notations for easier calculations and then can be solved using row reduction as follows

$$\begin{pmatrix} 1.81 & 0.67 & -3.21 \\ 1.7 & -0.62 & -2.03 \end{pmatrix} \xrightarrow{R_2 \leftarrow 1.7R_1 - 1.81R_2} \begin{pmatrix} 1.81 & 0.67 & -3.21 \\ 0 & 1.33 & -1.05 \end{pmatrix}$$
$$(6)$$

$$\xrightarrow{R_1 \leftarrow 1.33R_1 - 0.67R_2} \begin{pmatrix} 1.81 & 0 & -2.68 \\ 0 & 1.33 & -1.05 \end{pmatrix}$$
$$(7)$$

$$\xrightarrow{R_1 \leftarrow \frac{R_1}{1.81}} \begin{pmatrix} 1 & 0 & -1.48 \\ 0 & 1.33 & -1.05 \end{pmatrix}$$
$$(8)$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{1.33}} \begin{pmatrix} 1 & 0 & -1.48 \\ 0 & 1 & -0.79 \end{pmatrix}$$
$$(9)$$

We obtain

$$\mathbf{I} = \begin{pmatrix} -1.48 \\ -0.79 \end{pmatrix} \quad (10)$$
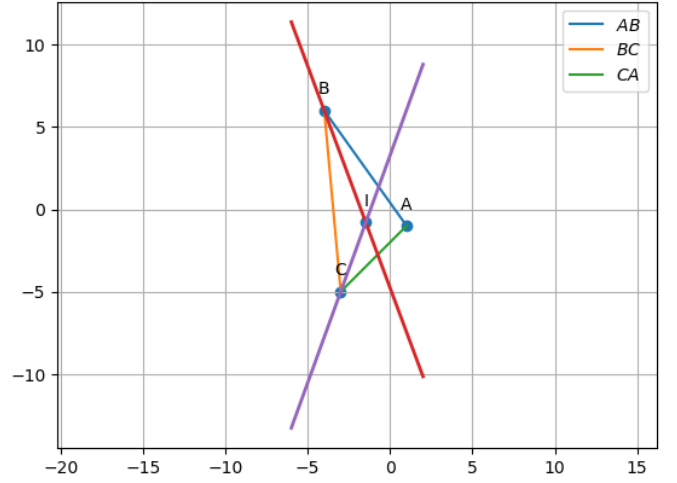

Fig. 1. Intersection point **I** of angle bisectors of **B** and **C** plotted using python