Question 12.13.3.24
Probability and Random Processes

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Question:12/13/3/24

Consider the probability distribution of a random variable X:

X	0	1	2	3	4
P(X)	0.1	0.25	0.3	0.2	0.15

Calculate

(i) $\text{var}(X)$

(ii) $\text{var}\left(\frac{X}{2}\right)$

Solution:

(i) $\text{var}(X)$

$$= E\left[X - E(X)\right]^2 \tag{1}$$

$$= E\left[X^2 + [E(X)]^2 - 2XE(X)\right] \tag{2}$$

$$= E\left(X^2\right) + [E(X)]^2 - 2[E(X)]^2 \tag{3}$$

$$= E\left(X^2\right) - [E(X)]^2 \tag{4}$$

where

$$E(X) = \sum_{k=0}^{4} k p_X(k) \tag{5}$$

$$= 2.05 \tag{6}$$

and

$$E\left(X^2\right) = \sum_{k=0}^{4} k^2 p_X(k) \tag{7}$$

$$= 5.65 \tag{8}$$

Then

$$\text{var}(X) = 5.65 - (2.05)^2 \tag{9}$$

$$= 1.4475 \tag{10}$$

(ii) $\text{var}\left(\frac{X}{2}\right)$

$$= E\left(\frac{X^2}{4}\right) - \left[E\left(\frac{X}{2}\right)\right]^2 \tag{11}$$

$$= \frac{1}{4}\left[E\left(X^2\right) - [E(X)]^2\right] \tag{12}$$

$$= \frac{1}{4}E(X) \tag{13}$$

$$= 1 - \left(\frac{2}{5} + \frac{1}{2} - \frac{1}{5}\right) \tag{14}$$

$$= 1 - \frac{7}{10} \tag{15}$$

$$= \frac{3}{10} \tag{16}$$