Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \ \mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$
 (1)

1 Vectors

parameters	values	description	
m ₁	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$	AB	
\mathbf{m}_2	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	ВС	
m ₃	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	CA	
A - B	4.47	length of AB	
B-C	2.82	length of BC	
C - A	2	length of CA	
	3	non collinear	
n ₁	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	AB	
c_1	10		
n ₂	$\begin{pmatrix} -2\\2 \end{pmatrix}$	BC	
c_2	-2		
n ₃	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	CA	
<i>c</i> ₃	-4		
Area	2	Area of Triangle	
∠A	26.57°		
∠B	18.43°	Angles	
∠C	135°		

TABLE 1: Vectors.

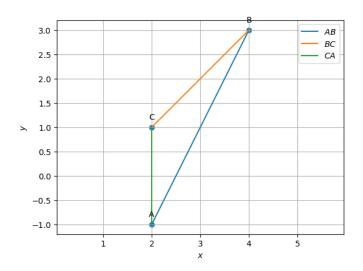


Fig. 1: triangle plotted using python

2 Median

	•	1		
parameters	value	description		
D	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	BC midpoint		
E	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	CA midpoint		
F	$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	AB midpoint		
m ₄	$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	AD		
n ₄	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$			
c_4	7			
m ₅	$\begin{pmatrix} -2 \\ -3 \end{pmatrix}$	BE		
n ₅	$\begin{pmatrix} -3\\2 \end{pmatrix}$			
c ₅	-6			
m ₆	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	C.F.		
n ₆	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	CF		
c_6	-1			
G	$\begin{pmatrix} 2.67 \\ 1 \end{pmatrix}$	Centroid		
$\begin{array}{c} \underline{BG} \\ \overline{GE} \\ \underline{CG} \\ \overline{GF} \\ \underline{AG} \\ \overline{GD} \end{array}$	2	Division ratio by G		
	2	collinear		
$rank \begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$				

TABLE 2: Median.

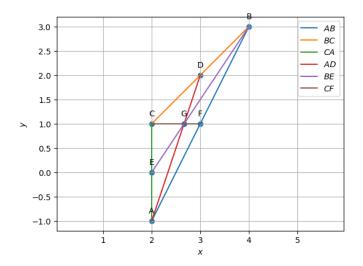


Fig. 2: medians plotted using python

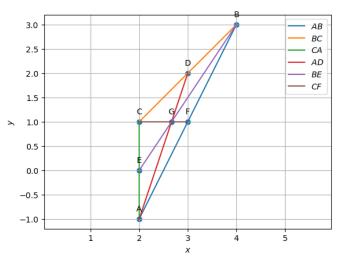


Fig. 3: altitudes plotted using python

4 Perpendicular Bisector

description

value

parameters

3 ALTITUDE

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				parameters	varac	description
parameters value description D_1 $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Foot of altitude from A E_1 $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ Foot of altitude from B E_1 $\begin{pmatrix} 2.8 \\ 0.6 \end{pmatrix}$ Foot of altitude from C m_1 $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ m_7 $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ AD_1 m_7 $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ AD_1 m_1 $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$ CF_1 m_8 $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ BE_1 m_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ BE_1 m_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ BE_1 m_9 $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$ $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$ m_9 $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$ $\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$ $\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$ m_9 $\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$				m ₁₀	1 1	AD
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	naramatara	voluo	description	n ₁₀	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	AD_1
D_1 $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ Foot of altitude from A m_{11} $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ BE_1 E_1 $\begin{bmatrix} 2.8 \\ 0.6 \end{bmatrix}$ Foot of altitude from B m_{11} $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ BE_1 m_7 $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ AD_1 m_{12} $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ CF_1 m_7 $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ AD_1 a_{12} $\begin{bmatrix} -2 \\ -4 \end{bmatrix}$ CF_1 m_8 $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ BE_1 AD_1 AD	parameters	(.)	description			
E1 $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ Foot of altitude from B R	\mathbf{D}_1	1 1	Foot of altitude from A			
F1 $\begin{pmatrix} 2.8 \\ 0.6 \end{pmatrix}$ Foot of altitude from C n_{11} $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ m_7 $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ AD_1 m_{12} $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$ CF_1 m_7 $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ AD_1 m_{12} $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ CF_1 m_8 $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ BE_1 O $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ O O m_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ BE_1 $ O - A $ $ O - A $ $ O - B $ $OA = OB = OC = R$ m_9 $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$ CF_1 ABC AB	E.	(2)	Foot of altitude from B	m ₁₁	(0)	BE_1
\mathbf{F}_1 $\begin{pmatrix} 0.6 \\ 0.6 \end{pmatrix}$ Foot of altitude from \mathbf{C} \mathbf{C}_{11} \mathbf{O} \mathbf{C}_{11} \mathbf{C}_{11} \mathbf{C}_{11} \mathbf{C}_{11} \mathbf{C}_{11} \mathbf{C}_{11} \mathbf{C}_{12} \mathbf{C}_{12} \mathbf{C}_{11} \mathbf{C}_{12} \mathbf{C}_{11} \mathbf{C}_{1	L 1	(3)	1 oot of annual from B	n ₁₁		BL ₁
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\mathbf{F_1}$		Foot of altitude from C	c_{11}		
$\mathbf{n_7}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ c_7 \end{pmatrix}$ $\mathbf{n_{12}}$ $\begin{pmatrix} -2 \\ -4 \end{pmatrix} \\ c_{12} \end{pmatrix}$ $\mathbf{m_8}$ $\begin{pmatrix} -2 \\ 0 \end{pmatrix} \\ c_8 \end{pmatrix}$ $\mathbf{BE_1}$ \mathbf{O} $\begin{pmatrix} 5 \\ 0 \end{pmatrix} \end{pmatrix}$ Circumcentre $\mathbf{m_8}$ $\begin{pmatrix} 0 \\ -2 \end{pmatrix} \\ c_8 \end{pmatrix}$ $\mathbf{BE_1}$ $\ \mathbf{O} - \mathbf{A}\ $ $\ \mathbf{O} - \mathbf{A}\ $ $\ \mathbf{O} - \mathbf{B}\ $ $\ \mathbf{O} - \mathbf{B}\ $ $\ \mathbf{O} - \mathbf{C}\ $ 3.16 $OA = OB = OC = R$ $\mathbf{m_9}$ $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix} \\ c_{0.4} \end{pmatrix}$ $\mathcal{C}F_1$ $\mathcal{C}F_2$ $$	m ₇	(-1)			1 1	CE
m_8 $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ BE_1 O $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ Circumcentre m_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ $ O - A $ $ O - B $ $ O - C - C $ $ O - C - C $ $ O - C - C $ $ O - C - C - C $ $ O - C - C - C - C $ $ O - C - C - C - C - C $ $ O - C - C - C - C - C $ $ O - C - C - C - C - C - C $ $ O - C - C - C - C - C - C $ $ O -$	\mathbf{n}_7	1 1 1	AD_1	n ₁₂		CF ₁
m_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ BE_1 O O O O m_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ $ O - A $ $ O - B $	c_7	1		$ c_{12} $	-10	
n_8 $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ $ O - A $ c_8 6 $ O - B $ $ O - C $ m_9 $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$ $ O - C $ $ O - C $ n_9 $\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$ $ C - C $ $ C - C - C $ $ C - C - C - C $ $ C - C - C - C - C - C - C - C - C - C $	m ₈		D.C.	О		Circumcentre
c_8 6 m_9 $\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$ R m_9 $\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$ $\angle BOC$ 53.13° $\angle BAC$ 26.565° $\angle BOC = 2\angle BAC$ $\angle AOC$ 36.8° $\angle AOC = 2\angle ABC$ $AOC = 2\angle ABC$	n ₈	1 1	BE_1		2.16	OA OB OG B
m9 CF_1 CF_2 CF_1 CF_2 <	c_8	6		O - C	3.16	OA = OB = OC = R
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		(0.8)		R		
n_9 $\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$ $\angle AOC$ 26.505° c_9 -1.6 $\angle ABC$ 18.4° $\angle AOC = 2\angle ABC$ $AOC = 2\angle ABC$ $AOC = 2\angle ABC$ $AOC = 2\angle ABC$ $AOC = 2\angle ABC$	1119	(-0.4)	CF_1			/ROC = 2/RAC
$ \begin{array}{c ccccc} \hline & (-0.8) & & \angle AOC & 36.8^{\circ} \\ \hline & c_9 & -1.6 & & \angle ABC & 18.4^{\circ} \\ \hline & H & (-2) & Orthocentre & & \angle AOB & 270^{\circ} \\ \hline & & & & & & & & & & & & & & & & & & $	no	$\left(-0.4\right)$		∠BAC	26.565°	ZDOC - ZZDNC
C_9 -1.6 $\angle ABC$ 18.4 $\angle AOB$ 270° $\angle AOB = 2 \angle BCA$	119	(-0.8)		∠AOC_	36.8°	440G 244BG
H Orthocentre $/AOR - 2/RCA$	<i>C</i> 9	-1.6		∠ABC	18.4°	$\angle AOC = 2\angle ABC$
	н	(-2)	Orthocentre	∠AOB	270°	$\angle AOB = 2\angle BCA$
	11	(3)		∠BCA	135°	

TABLE 3: Altitude.

TABLE 4: Perpendicular Bisector.

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

narameters	value	description	
parameters	(-0.45)	description	
m ₁₃	$\begin{pmatrix} -0.43 \\ -1.89 \end{pmatrix}$	AI	
n ₁₃	$\begin{pmatrix} -1.89 \\ 0.45 \end{pmatrix}$		
c_{13}	-4.23		
m ₁₄	$\begin{pmatrix} -1.15 \\ -1.6 \end{pmatrix}$	DI.	
m ₁₄	$\begin{pmatrix} 1.6 \\ -1.15 \end{pmatrix}$	BI	
c_{14}	2.943		
m ₁₅	$\begin{pmatrix} -0.71\\ 0.29 \end{pmatrix}$	CI	
n ₁₅	$\begin{pmatrix} -0.29 \\ -0.70 \end{pmatrix}$	CI	
C ₁₅	-1.29		
I	$\begin{pmatrix} 2.43 \\ 0.82 \end{pmatrix}$	Incentre	
\mathbf{D}_3	$\binom{2.13}{1.13}$	Point of contact with BC	
E ₃	$\begin{pmatrix} 2 \\ 0.82 \end{pmatrix}$	Point of contact with AC	
F ₃	$\begin{pmatrix} 2.81 \\ 0.62 \end{pmatrix}$	Point of contact with AB	
$ \mathbf{I} - \mathbf{D_3} $			
$ I-E_3 $	0.42	ID IE IE	
$ I-F_3 $	0.43	$ID_3 = IE_3 = IF_3 = r$	
r			
∠BAI	13.28°	$\angle BAI = \angle CAI$	
∠CAI			
∠ABI ∠CBI	9.21°	$\angle ABI = \angle CBI$	
∠ACI			
∠BCI	67.5°	$\angle ACI = \angle BCI$	

TABLE 5: Angle Bisectors.

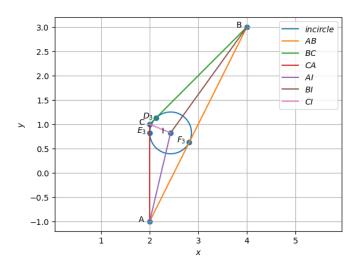


Fig. 5: Angle bisectors plotted using python