

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (1)$$

1 VECTORS

parameters	values	description
\mathbf{m}_1	$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$	AB
\mathbf{m}_2	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	BC
\mathbf{m}_3	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	CA
$\ A - B\ $	4.47	length of AB
$\ B - C\ $	2.82	length of BC
$\ C - A\ $	2	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
\mathbf{n}_1	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	AB
c_1	10	
\mathbf{n}_2	$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$	BC
c_2	-2	
\mathbf{n}_3	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	CA
c_3	-4	
Area	2	Area of Triangle
$\angle A$	26.57°	Angles
$\angle B$	18.43°	
$\angle C$	135°	

TABLE 1: Vectors.

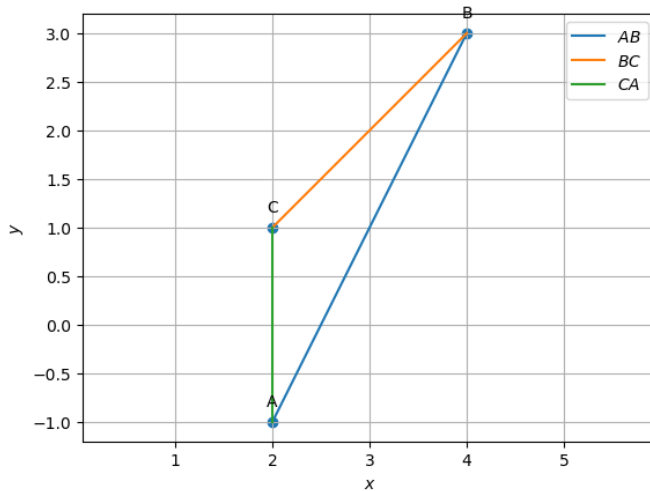


Fig. 1: triangle plotted using python

2 MEDIAN

parameters	value	description
\mathbf{D}	$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$	BC midpoint
\mathbf{E}	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	CA midpoint
\mathbf{F}	$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$	AB midpoint
\mathbf{m}_4	$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$	AD
\mathbf{n}_4	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$	
c_4	7	
\mathbf{m}_5	$\begin{pmatrix} -2 \\ -3 \end{pmatrix}$	BE
\mathbf{n}_5	$\begin{pmatrix} -3 \\ 2 \end{pmatrix}$	
c_5	-6	
\mathbf{m}_6	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	CF
\mathbf{n}_6	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	
c_6	-1	
\mathbf{G}	$\begin{pmatrix} 2.67 \\ 1 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by \mathbf{G}
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

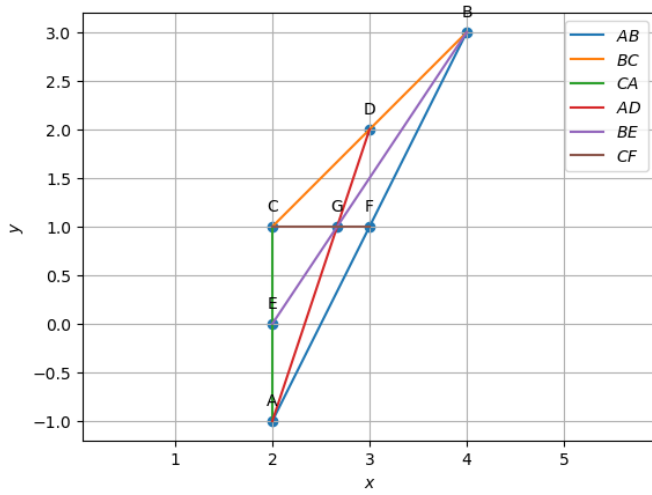


Fig. 2: medians plotted using python

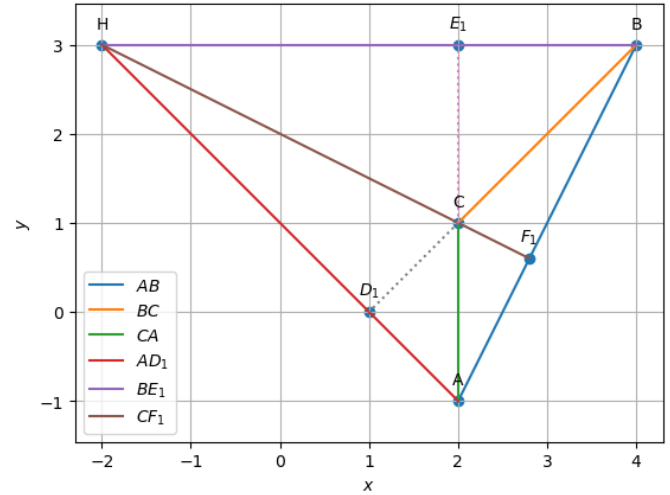


Fig. 3: altitudes plotted using python

3 ALTITUDE

parameters	value	description
D_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	Foot of altitude from A
E_1	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$	Foot of altitude from B
F_1	$\begin{pmatrix} 2.8 \\ 0.6 \end{pmatrix}$	Foot of altitude from C
m_7	$\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	AD_1
n_7	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	
c_7	1	
m_8	$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$	BE_1
n_8	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$	
c_8	6	
m_9	$\begin{pmatrix} 0.8 \\ -0.4 \end{pmatrix}$	CF_1
n_9	$\begin{pmatrix} -0.4 \\ -0.8 \end{pmatrix}$	
c_9	-1.6	
H	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 PERPENDICULAR BISECTOR

parameters	value	description
m_{10}	$\begin{pmatrix} -2 \\ 2 \end{pmatrix}$	AD_1
n_{10}	$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$	
c_{10}	-10	
m_{11}	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	BE_1
n_{11}	$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$	
c_{11}	0	
m_{12}	$\begin{pmatrix} -4 \\ 2 \end{pmatrix}$	CF_1
n_{12}	$\begin{pmatrix} -2 \\ -4 \end{pmatrix}$	
c_{12}	-10	
O	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$	Circumcentre
$\ O - A\ $	3.16	$OA = OB = OC = R$
$\ O - B\ $		
$\ O - C\ $		
R		
$\angle BOC$	53.13°	$\angle BOC = 2\angle BAC$
$\angle BAC$	26.565°	
$\angle AOC$	36.8°	$\angle AOC = 2\angle ABC$
$\angle ABC$	18.4°	
$\angle AOB$	270°	$\angle AOB = 2\angle BCA$
$\angle BCA$	135°	

TABLE 4: Perpendicular Bisector.

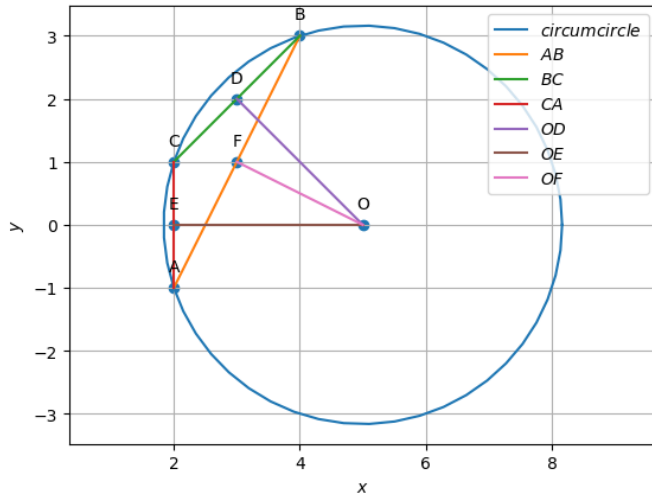


Fig. 4: perpendicular bisectors plotted using python

5 ANGLE BISECTOR

parameters	value	description
\mathbf{m}_{13}	$\begin{pmatrix} -0.45 \\ -1.89 \end{pmatrix}$	AI
\mathbf{n}_{13}	$\begin{pmatrix} -1.89 \\ 0.45 \end{pmatrix}$	
c_{13}	-4.23	
\mathbf{m}_{14}	$\begin{pmatrix} -1.15 \\ -1.6 \end{pmatrix}$	BI
\mathbf{m}_{14}	$\begin{pmatrix} 1.6 \\ -1.15 \end{pmatrix}$	
c_{14}	2.943	
\mathbf{m}_{15}	$\begin{pmatrix} -0.71 \\ 0.29 \end{pmatrix}$	CI
\mathbf{n}_{15}	$\begin{pmatrix} -0.29 \\ -0.70 \end{pmatrix}$	
c_{15}	-1.29	
\mathbf{I}	$\begin{pmatrix} 2.43 \\ 0.82 \end{pmatrix}$	Incentre
\mathbf{D}_3	$\begin{pmatrix} 2.13 \\ 1.13 \end{pmatrix}$	Point of contact with BC
\mathbf{E}_3	$\begin{pmatrix} 2 \\ 0.82 \end{pmatrix}$	Point of contact with AC
\mathbf{F}_3	$\begin{pmatrix} 2.81 \\ 0.62 \end{pmatrix}$	Point of contact with AB
$\ \mathbf{I} - \mathbf{D}_3\ $	0.43	$ID_3 = IE_3 = IF_3 = r$
$\ \mathbf{I} - \mathbf{E}_3\ $		
$\ \mathbf{I} - \mathbf{F}_3\ $		
r		
$\angle BAI$	13.28°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	9.21°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	67.5°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

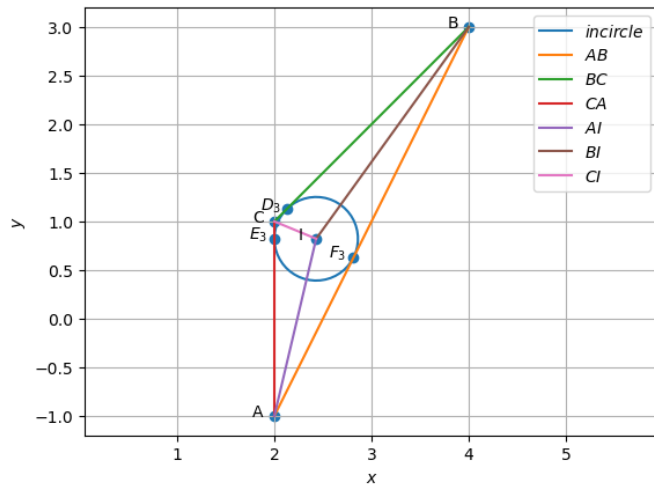


Fig. 5: Angle bisectors plotted using python