Question 1.5.2

Find the intersection **I** of the angle bisectors of B and C

Solution

From **??** the bisectors of B and C are obtained as

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \quad (1)$$

and

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \quad (2)$$

respectively.

The pair of linear equations can be solved using the Augmented matrix $\left[A|B\right]$

Here,

$$A = \begin{bmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{bmatrix} \quad (3)$$

$$B = \begin{bmatrix} \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{bmatrix} \quad (4)$$

$$\left[A|B\right] = \left[\begin{array}{cc|c} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} & \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{array}\right] \quad (5)$$

The augmented matrix is converted into decimal notations for easier calculations and then can be solved using row reduction as follows

$$\left[\begin{array}{cc|c} 1.81 & 0.67 & -3.21 \\ 1.7 & -0.62 & -2.03 \end{array}\right] \xleftrightarrow{R_2 \leftarrow R_1 - (\frac{1.81}{1.7})R_2} \left[\begin{array}{cc|c} 1.81 & 0.67 & -3.21 \\ 0 & 1.33 & -1.05 \end{array}\right] \quad (6)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{0.67}{1.33}R_2} \left[\begin{array}{cc|c} 1.81 & 0 & -2.68 \\ 0 & 1.33 & -1.05 \end{array}\right] \quad (7)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{1.81}} \left[\begin{array}{cc|c} 1 & 0 & -1.48 \\ 0 & 1.33 & -1.05 \end{array}\right] \quad (8)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{1.81}} \left[\begin{array}{cc|c} 1 & 0 & -1.48 \\ 0 & 1 & -0.75 \end{array}\right] \quad (9)$$

We obtain

$$\mathbf{I} = \begin{pmatrix} -1.48 \\ -0.75 \end{pmatrix} \quad (10)$$