Assignment

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Question 9.3.18
From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.
Solution:

Parameter	Value	Description
X	{0,1,2,3,4}	Number of defective bulbs taken
n	4	Number of bulbs taken
p	0.2	Taking a defective bulb
q	0.8	Taking a non defective bulb
$\mu = np$	0.8	Mean of Binomial distribution
$\sigma^2 = npq$	0.64	Varience of Binomial distribution

TABLE 1: Parameter description

Binomial
The PMF of the distribution is,

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{1}$$

1)

$$k = 0 \tag{2}$$
$$\implies p_X(0) = {}^4C_0 (0.2)^0 (0.8)^4 \tag{3}$$
$$= 0.4096 \tag{4}$$

2)

$$k = 1 \tag{5}$$
$$\implies p_X(1) = {}^4C_1 (0.2)^1 (0.8)^3 \tag{6}$$
$$= 0.4096 \tag{7}$$

3)

$$k = 2 \tag{8}$$
$$\implies p_X(2) = {}^4C_2 (0.2)^2 (0.8)^2 \tag{9}$$
$$= 0.1536 \tag{10}$$

4)

$$k = 3 \tag{11}$$
$$\implies p_X(3) = {}^4C_3 (0.2)^3 (0.8)^1 \tag{12}$$
$$= 0.0256 \tag{13}$$

5)

$$k = 4 \tag{14}$$
$$\implies p_X(2) = {}^4C_4 (0.2)^4 (0.8)^0 \tag{15}$$
$$= 0.0016 \tag{16}$$

Gaussian
let Y be a gaussian Random variable

$$Y \sim N(np, npq) \tag{17}$$
$$\sim N(0.8, 0.64) \tag{18}$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$\Pr(X = x) \approx \Pr(x - 0.5 < Y < x + 0.5) \tag{19}$$
$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \tag{20}$$
$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \tag{21}$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y < x) \tag{22}$$
$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \tag{23}$$
$$\implies \frac{Y - \mu}{\sigma} \sim N(0, 1) \tag{24}$$
$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \tag{25}$$
$$= 1 - Q\left(\frac{x - \mu}{\sigma}\right) \tag{26}$$

Then PDF in terms of Q funtion is

$$\implies \Pr(X = x) \approx Q\left(\frac{(x - 0.5) - \mu}{\sigma}\right) - Q\left(\frac{(x + 0.5) - \mu}{\sigma}\right) \tag{27}$$

1)

$$X = 0 \tag{28}$$

The Gaussian approximation for $\Pr(X = 0)$ is

$$\approx Q(-8.125) - Q(-1.875) \tag{29}$$
$$\approx 0.3017 \tag{30}$$

2)

$$X = 1 \tag{31}$$

The Gaussian approximation for $\Pr(X = 1)$ is

$$\approx Q(-1.875) - Q(4.375) \tag{32}$$
$$\approx 0.4555 \tag{33}$$

3)

$$X = 2 \tag{34}$$

The Gaussian approximation for $\Pr(X = 2)$ is

$$\approx Q(4.375) - Q(10.625) \tag{35}$$
$$\approx 0.1739 \tag{36}$$

4)

$$X = 3 \tag{37}$$

The Gaussian approximation for $\Pr(X = 3)$ is

$$\approx Q(10.625) - Q(16.875) \tag{38}$$
$$\approx 0.0164 \tag{39}$$

5)

$$X = 4 \tag{40}$$

The Gaussian approximation for $\Pr(X = 4)$ is

$$\approx Q(16.875) - Q(23.125) \tag{41}$$
$$\approx 0.00036 \tag{42}$$

Number of defective bulbs	Binomial distribution	Gaussian approximation	Error
0	0.4096	0.3017	26.342773437
1	0.4096	0.4555	11.206054688
2	0.1536	0.1739	13.216145833
3	0.0256	0.0164	35.9375
4	0.0016	0.00036	77.5

TABLE 2: Comparing the gaussian approximation with binomial

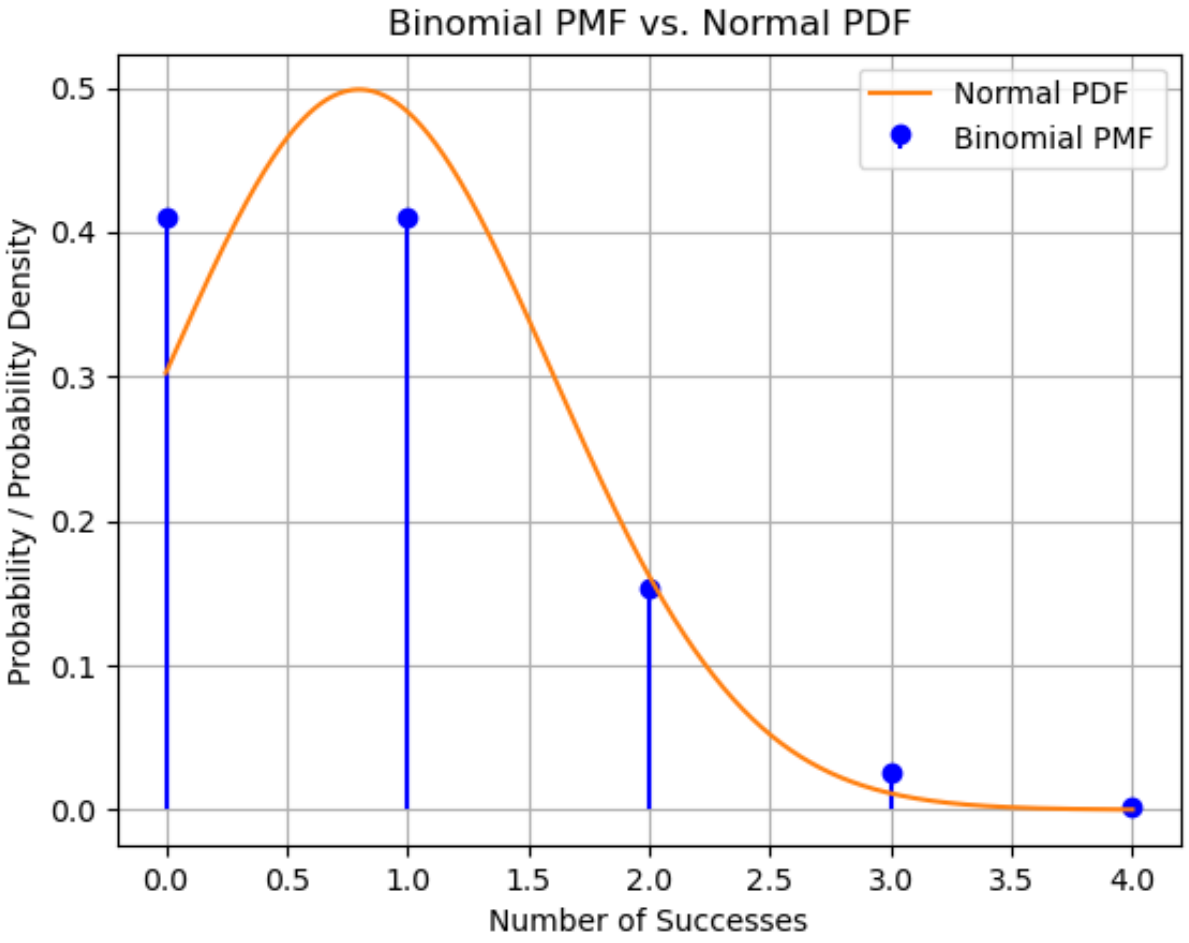

Fig. 1: Binomial and gaussian distribution