Question 1.5.2

Find the intersection **I** of the angle bisectors of B and C

Solution

From (1.5.1) the bisectors of B and C are obtained as

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \quad (1)$$

and

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \quad (2)$$

respectively.

The pair of linear equations can be written in the form

$$AX = B \quad (3)$$

$$X = A^{-1}B \quad (4)$$

Here,

$$A = \begin{bmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{bmatrix} \quad (5)$$

$$B = \begin{bmatrix} \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{bmatrix} \quad (6)$$

We obtain A^{-1} as

$$\frac{1}{(\frac{6}{\sqrt{61}} + \frac{24}{\sqrt{37}\sqrt{61}} + \frac{6}{\sqrt{37}})} \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{122}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{-11}{\sqrt{122}} - \frac{7}{\sqrt{74}} \end{bmatrix}$$

and $A^{-1}B$ as

$$\frac{1}{(\frac{6}{\sqrt{61}} + \frac{24}{\sqrt{37}\sqrt{61}} + \frac{6}{\sqrt{37}})} \begin{bmatrix} \frac{6}{\sqrt{37}} - \frac{96}{\sqrt{37}\sqrt{61}} - \frac{18}{\sqrt{61}} \\ -\frac{30}{\sqrt{61}} + \frac{144}{\sqrt{37}\sqrt{61}} - \frac{6}{\sqrt{37}} \end{bmatrix}$$

on simplification we otain

$$\mathbf{I} = \frac{1}{\sqrt{37} + 4 + \sqrt{61}} \begin{pmatrix} \sqrt{61} - 16 - 3\sqrt{37} \\ -\sqrt{61} + 24 - 5\sqrt{37} \end{pmatrix} \quad (7)$$