Question 1.5.2

Find the intersection $\mathbf{I}$ of the angle bisectors of $\mathbf{B}$ and $\mathbf{C}$

Solution
From **??** the bisectors of $\mathbf{B}$ and $\mathbf{C}$ are obtained as

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \quad (1)$$

and

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \quad (2)$$

respectively.
The pair of linear equations can be solved using the Augmented matrix $\begin{pmatrix} \mathbf{P} | \mathbf{Q} \end{pmatrix}$
Here,

$$\mathbf{P} = \begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix} \quad (3)$$

$$\mathbf{Q} = \begin{pmatrix} \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} \mathbf{P} | \mathbf{Q} \end{pmatrix} = \left(\begin{array}{cc|c} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} & \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{array} \right) \quad (5)$$

The augmented matrix is converted into decimal notations for easier calculations and then can be solved using row reduction as follows

$$\left(\begin{array}{cc|c} 1.81 & 0.67 & -3.21 \\ 1.7 & -0.62 & -2.03 \end{array} \right) \xleftrightarrow{R_2 \leftarrow 1.7R_1 - 1.81R_2} \left(\begin{array}{cc|c} 1.81 & 0.67 & -3.21 \\ 0 & 1.33 & -1.05 \end{array} \right) \quad (6)$$

$$\xleftrightarrow{R_1 \leftarrow 1.33R_1 - 0.67R_2} \left(\begin{array}{cc|c} 1.81 & 0 & -2.68 \\ 0 & 1.33 & -1.05 \end{array} \right) \quad (7)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{1.81}} \left(\begin{array}{cc|c} 1 & 0 & -1.48 \\ 0 & 1.33 & -1.05 \end{array} \right) \quad (8)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{1.33}} \left(\begin{array}{cc|c} 1 & 0 & -1.48 \\ 0 & 1 & -0.79 \end{array} \right) \quad (9)$$

We obtain

$$\mathbf{I} = \begin{pmatrix} -1.48 \\ -0.79 \end{pmatrix} \quad (10)$$

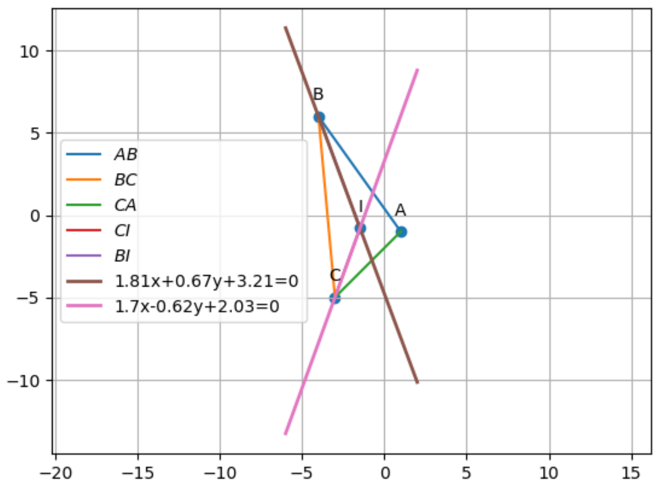

Fig. 1. img plotted using python