Question 1.5.2

Find the intersection I of the angle bisectors of B and C

Solution:

From ?? the bisectors of **B** and **C** are obtained as

$$\left(\frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} \quad \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}}\right) \mathbf{x} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}}$$
 (1)

and

$$\left(\frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}}\right) \mathbf{x} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}}$$
 (2)

respectively. The pair of linear equations can be solved using the Augmented matrix (P|Q) Here,

Fig. 1. Intersection point I of angle bisectors of B and C plotted using python

$$\mathbf{P} = \begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix}$$
(3)
$$\mathbf{Q} = \begin{pmatrix} \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{pmatrix}$$
(4)

$$(\mathbf{P} \mid \mathbf{Q}) = \begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} & \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} & \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{pmatrix}$$

$$(5)$$

The augmented matrix is converted into decimal notations for easier calculations and then can be solved using row reduction as follows

$$\begin{pmatrix}
1.81 & 0.67 & | & -3.21 \\
1.7 & -0.62 & | & -2.03
\end{pmatrix}
\xrightarrow{R_2 \leftarrow 1.7R_1 - 1.81R_2}
\begin{pmatrix}
1.81 & 0.67 & | & -3.21 \\
0 & 1.33 & | & -1.05
\end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow 1.33R_1 - 0.67R_2}
\begin{pmatrix}
1.81 & 0 & | & -2.68 \\
0 & 1.33 & | & -1.05
\end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow \frac{R_1}{1.81}}
\begin{pmatrix}
1 & 0 & | & -1.48 \\
0 & 1.33 & | & -1.05
\end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{1.33}}
\begin{pmatrix}
1 & 0 & | & -1.48 \\
0 & 1 & | & -0.79
\end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{1.33}}
\begin{pmatrix}
1 & 0 & | & -1.48 \\
0 & 1 & | & -0.79
\end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{1.33}}
\begin{pmatrix}
1 & 0 & | & -1.48 \\
0 & 1 & | & -0.79
\end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{1.33}}$$

We obtain

$$\mathbf{I} = \begin{pmatrix} -1.48 \\ -0.79 \end{pmatrix} \tag{10}$$