Question 1.5.2

Find the intersection **I** of the angle bisectors of B and C

Solution
From (1.5.1) the bisectors of B and C are obtained as

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}}$$

and

$$\begin{pmatrix} \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}}$$

respectively.
The pair of linear equations can be written in the form

$$AX = B$$

then

$$X = A^{-1}B$$

Here,

$$A = \begin{bmatrix} \frac{11}{\sqrt{122}} + \frac{7}{\sqrt{74}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{122}} - \frac{1}{\sqrt{2}} \end{bmatrix}$$

and

$$B = \begin{bmatrix} \frac{2}{\sqrt{74}} - \frac{38}{\sqrt{122}} \\ \frac{2}{\sqrt{2}} - \frac{38}{\sqrt{122}} \end{bmatrix}$$

We obtain A^{-1} as

$$\frac{1}{(\frac{6}{\sqrt{61}} + \frac{24}{\sqrt{37}\sqrt{61}} + \frac{6}{\sqrt{37}})} \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{122}} & \frac{1}{\sqrt{122}} + \frac{5}{\sqrt{74}} \\ \frac{11}{\sqrt{122}} + \frac{1}{\sqrt{2}} & \frac{-11}{\sqrt{122}} - \frac{7}{\sqrt{74}} \end{bmatrix}$$

and $A^{-1}B$ as

$$\frac{1}{(\frac{6}{\sqrt{61}} + \frac{24}{\sqrt{37}\sqrt{61}} + \frac{6}{\sqrt{37}})} \begin{bmatrix} \frac{6}{\sqrt{37}} - \frac{96}{\sqrt{37}\sqrt{61}} - \frac{18}{\sqrt{61}} \\ -\frac{30}{\sqrt{61}} + \frac{144}{\sqrt{37}\sqrt{61}} - \frac{6}{\sqrt{37}} \end{bmatrix}$$

on simplification we otain

$$I = \frac{1}{\sqrt{37}+4+\sqrt{61}} \begin{pmatrix} \sqrt{61}-16-3\sqrt{37} \\ -\sqrt{61}+24-5\sqrt{37} \end{pmatrix}$$