# Assignment

FWC22244 - Sarvesh K

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## 1 Vectors

1. Prove that the points $\left(3,0\right), \left(6,4\right)$ and $\left(-1,3\right)$ are the vertices of a right angled isosceles triangle.

2. If the point $P\left(x,y\right)$ is equidistant from the points $A\left(a+b, b-a\right)$ and $B\left(a-b, a+b\right)$. Prove that $bx = ay$.

3. In fig 3, the vertices of $\triangle ABC$ are $A\left(4,6\right), B\left(1,5\right)$and $C\left(7,2\right)$. A line-segment $DE$ is drawn to intersect the sides $AB$ and $AC$ at $D$ and $E$ respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$ . Calculate the area of $\triangle ADE$ and compare it with area of $\triangle ABC$.

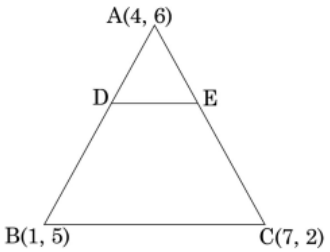


Figure 3

4. Let $P$ and $Q$ be the points of trisection of the line segment joining the points $A\left(2,-2\right)$ and $B\left(-7,4\right)$ such that $P$ is nearer to $A$. Find the coordinates of $P$ and $Q$.

## 2 Circles

5. In Fig 5, $PQ$ is a tangent at point $C$ to a circle with centre $O$. If $AB$ is a diameter and $\angle CAB = 30°$, Find $\angle PCA$.

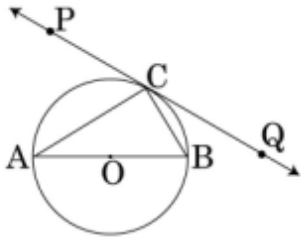


Figure 5

6. In Fig 6, a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre $O$, in such a way that the sides $AB, BC, CD$ and $DA$ touch the circle at the points $P, Q, R$ and $S$ respectively. Prove that $AB + CD = BC + DA$.

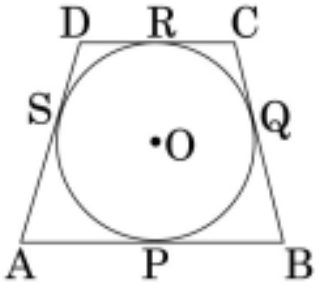


Figure 6

7. In Fig 7, from an external point $P$, two tangents $PT$ and $PS$ are drawn to a circle with centre $O$ and radius $r$. If $OP = 2r$, show that $\angle OTS = \angle OST = 30°$.

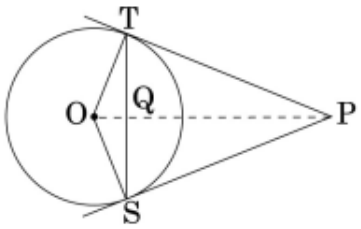


Figure 7

8. In fig 8, $O$ is the centre of a circle such that diameter $AB = 13$ cm and $AC = 12$ cm. $BC$ is joined. Find the area of the shaded region. (Take $\pi = 3.14$)

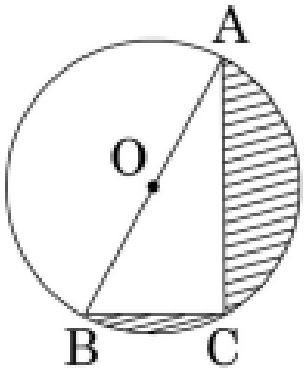


Figure 8

9. In Fig 9 , two equal circles, with centres $O$ and $O'$, touch each other at $X$.$OO\prime$ produced meets the circle with centre $O'$ at $A$. $AC$ is tangent to the circle with centre $O$, at the point $C$. $O'D$ is perpendicular to $AC$. Find the value of $\frac{DO'}{CO'}$.

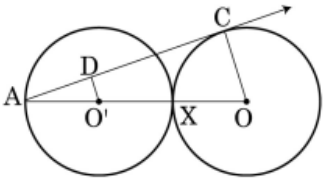


Figure 9

10. Prove that the lengths of the tangents drawn from an external point to a circle are equal.

# 3 Probability

11. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen.

12. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card

    (i) is divisible by 9 and is a perfect square

    (ii) is a prime number greater than 80

13. A number $x$ is selected at random from the numbers $1, 2, 3$ and 4. Another number $y$ is selected at random from the numbers $1, 4, 9$ and 16. Find the probability that product of $x$ and $y$ is less than 16.

14. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is :

    (i) a black King

    (ii) a card of red colour

    (iii) a card of black colour

15. Three different coins are tossed together. Find the probability of getting

    (i) exactly two heads

    (ii) at least two heads

    (iii) at least two tails.

# 4 Algebra

16. If $-5$ is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p\left(x^2 + x\right) + k = 0$ has equal roots, find the value of $k$.

17. Solve for $x$ :

$$\sqrt{2x + 9} + x = 13$$

18. Solve for $x$ :

$$\sqrt{6x + 7} - (2x - 7) = 0$$

19. If the roots of the quadratic equation $(a - b)\, x^2 + (b - c)\, x + (c - a) = 0$ are equal,prove that $2a = b + c$.

20. Solve for $x$ :

$$\frac{1}{(x - 1)\,(x - 2)} + \frac{1}{(x - 2)\,(x - 3)} = \frac{2}{3}, x \neq 1, 2, 3$$

21. Solve for $x$ :

$$\frac{1}{x + 1} + \frac{2}{x + 2} = \frac{4}{x + 4}, x \neq -1, -2, -4$$

22. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

23. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers.

24. Two pipes running together can fill a tank in $11\frac{1}{5}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately.

# 5 Geometry

25. A ladder, leaning against a wall, makes an angle of 60° with the horizontal.If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

26. In 26, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹500/sq.metre.$\left(\text{Use } \pi = \frac{22}{7}\right)$

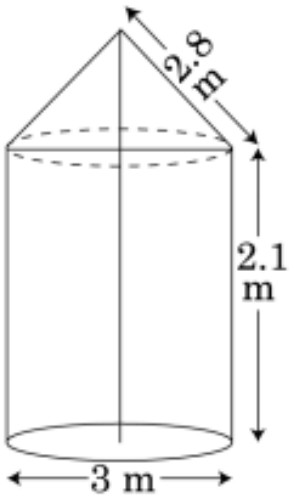


Figure 26

27. In 27, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$ $\left(\text{Use } \pi = \frac{22}{7}\right)$.

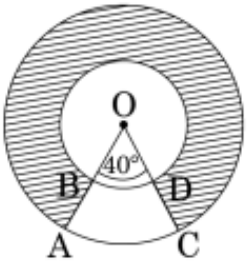


Figure 27

28. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. $\left(\text{Use } \pi = \frac{22}{7}\right)$

29. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If a sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$cm. Find the diameter of the cylindrical vessel.

30. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq.m, find the amount shared by each school to set up the tents. What value is generated by the above problem ? (Use $\pi = \frac{22}{7}$)

31. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

32. The angle of elevation of the top $Q$ of a vertical tower $PQ$ from a point $X$ on the ground is 60°. From a point $Y$, 40 m vertically above $X$, the angle of elevation of the top $Q$ of tower is 45°. Find the height of the tower $PQ$ and the distance $PX$. $\left(\text{Use } \sqrt{3} = 1.73\right)$

33. A rectangular park is to be designed whose breadth is 3 m less than its length.Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

34. In Fig 34 , is shown a sector $OAP$ of a circle with centre $O$, containing $\angle\theta$. $AB$ is perpendicular to the radius $OA$ and meets $OP$ produced at $B$. Prove

that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$

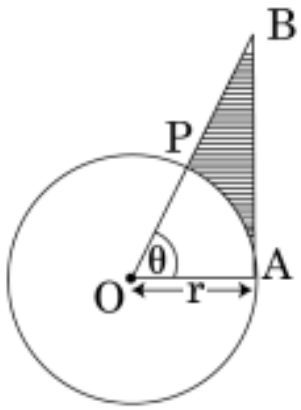


Figure 34

# 6 Discrete

35. For what value of $k$ will $k + 9, 2k - 1$ and $2k + 7$ are the consecutive terms of an A.P ?

36. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

37. If the ratio of the sum of first $n$ terms of two A.P's is $(7n + 1) : (4n + 27)$, find the ratio of their $m$ th terms.

38. The sums of first $n$ terms of three arithmetic progressions are $S_1, S_2$ and $S_3$ respectively. The first term of each A.P. is 1 and their common differences are $1, 2$ and 3 respectively. Prove that $S_1 + S_3 = S_2$.

39. The digits of a positive number of three digits are in A.P. and their sum is 15.The number obtained by reversing the digits is 594 less than the original

number. Find the number.

40. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of $X$ such that sum of numbers of houses preceeding the house numbered $X$ is equal to sum of the numbers of houses following $X$.

# 7 Constructions

41. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

42. Draw a triangle with sides 5 cm, 6 cm and 7 cm. Then draw another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of first triangle.

43. Draw an isosceles $\triangle ABC$ in which $BC = 5.5cm$ and altitude $AL = 5.3cm$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.