Introduction:

Automata, Computability and Complexity:

→ What are the fundamental capabilities and limitations of computers?

"By starting from the end you can better understand the reason for the beginning."

Complexity Theory:

The Central Question of Complexity Theory! "What makes some problems computationally hard and others easy?"

Set, Sequences, Tuples

Set → Group of objects represented as a unit
↳ objects in a set → elements/members

$\in$ ← set membership
$\notin$ ← -11- non -11-

- Multiple occurances of same object allowed in multiset but not set.
→ e.g $\{5, 7, 7\}$ and $\{5, 7\}$ both are different multisets but are identical sets.

- Members
  - 0 → Empty set $\emptyset$
  - 1 → singleton set
  - 2 → unordered pair

Sequences:

List of objects in some ORDER.
→ Order matters in sequence.
→ Sequences may be finite or infinite.
$(1, 2, 4, 9, 16 \ldots)$
→ Finite sequences → tuples
A sequence with k-elements → k-tuple.

Functions & Relation

Function
→ An object that sets up input/output relation.
→ takes i/p gives o/p
→ $f(a) = b$ ← o/p
↑ i/p (at a)

set of inputs → Domains
set of outputs → Range

- A function that uses all the elements of the range is said to be onto the range.

Strings & Languages

Strings of chars are building blocks of CS.

alphabet → Non-empty finite set of symbols.
↳ over which strings are defined may vary with application.

Symbols
↓
Alphabet
↓
String

- We generally use capital Greek Letters Σ and Γ to designate alphabets.

String over an alphabet:
- length of ω
- empty string (ε)
- Reverse of ω (ω^R)
- substring
- concatenation xy, x^k
- Lexicographic Order
  ↳ Shortlex order or String order

- Finite sequence of symbols from the alphabet, usually written next to one-another, not comma separated.
  e.g. if $\Sigma_1 = \{0,1\} \rightarrow 01001$ is a string over Σ_1
- If ω is a string over Σ, the length of ω, $|\omega|$ is number of symbol it contains.

* String of length zero is called empty string and is written ε. → (epsilon)
  ↳ plays role of 0 in number system.

- Reverse of ω, ω^R is a string obtained by writing 'ω' in opposite order.
- String z is a substring of ω if it appears consecutively within ω.
- Concatenation
  - x with $y \rightarrow xy$ ⋮ $x_1 \ldots x_n y_1 \ldots y_n$ ← resultant string
  - x with itself, k times → $\underbrace{xxx\ldots x}_{k}$ → written as x^k

- Lexicographic order → same as dictionary order.
  ↓
  Modification → shortlex order / string order
  difference: shorter string precedes longer string
  $\Sigma_1 = \{0,1\}$ → string ordering → (ε, 0, 1, 00, 01, 10, 11 . . .)

Language is a set of strings

Let,
y = abcde → y's prefix's → ε, a, ab, abc, abcd, abcde.
x : abc
z = de

* x is a prefix of y if there exist a 'z' s.t. $xz = y$.

* Proper Prefix → (Non-prime prefix)
↳ x is proper prefix of y if $x \neq \varepsilon$ and $x \neq y$

e.g. y = abcd

Proper prefix 'x' → a, ab, abc
Non -"- -"- 'x' → ε, abcd

Boolean logic

- Boolean Values
- Boolean Operations

→ Mathematical System built around two values, TRUE and FALSE.
↳ Boolean Values

→ Boolean Operations → Not (Negation) $\neg$
AND (Conjunction) $\wedge$
OR (Disjunction) $\vee$
XOR $\oplus$
equality $\leftrightarrow$
Implication $\rightarrow$

Definition, Theorem & Proofs.

- Definition describe objects and Notions we use.
- Mathematical statements are made of the previously defined objects and Notions.
- Proof → Convincing, logical argument that a statement is true.
- Theorem → Mathematical statement proved true.
- Lemmas → Statements proved only because they assist in proof of another, more Significant proof.
- Corollary → Theorems or proof that allow related statements to be proved easily.

Finding Proof:

- A well written proof is a set of stmts, wherein each one follows with simple reasoning from previous stmts in sequence.

Types of Proof:

① Proof by Construction:

→ For theorems stating a particular type of object exists.
→ We can prove by demonstrating construction of object. This is proof by construction.

② Proof by Contradiction:

→ Assume stmt is false, → this assumption leads to a false consequence, called contradiction.

③ Proof by Induction:

→ Method to show all elements of a set have same property.
→ Two parts
↳ Basis
↳ Induction step.