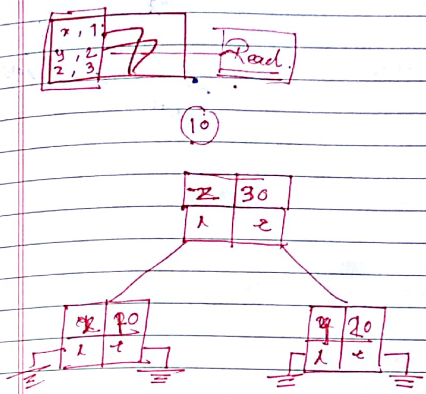
```
struct data {
        chae name [16] ;
        int maeks ;
            * left ;
    }       * eight;
        }
                atoi → conveet steing to
                       integee.

int main ( int aegc , chae* aegv [0] ) {
    File * fp1, fp2 ;
    fp1= open (aegv [1] , O_RDONLY ) ;
      int i = 1;
   if( fp1 ==0 )
   {

      }
   while ( aee[i-1] ! = '\m' ) {
        read (fp , &aee[i] , sizeof (aee[0])) ;
        i++ );
      }
   chae * token = stetok (aee, ",");
      steepy ( name, token) ;
      token = stetok (NULL, ",")
      maeks = atoi (token) ;
      return y ;

 peintf ("%s %d ", name , maeks) ;
```






$$x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = \frac{-1}{x}$$

1) $\int \sqrt[2]{x}\,dx = \int x^{\frac{1}{2}}\,dx$

$\frac{1}{2}+1 = \frac{3}{2} = x^{\frac{3}{2}} \times \frac{2}{3}\,dx = \frac{2x^{\frac{3}{2}}}{3}$

$= \frac{2\sqrt[2]{x^3}}{3}$

2) $\int \sqrt[3]{x^4}\,dx = \int x^{4/3}\,dx$

$\frac{4}{3}+1 = \frac{4+3}{3} = \frac{7}{3} = x^{\frac{7}{3}} \times \frac{3}{7}\,dx = \frac{3\sqrt[3]{x^7}}{7}$

3) $\int (3x-1)^2\,dx$ $\quad (3x)(3x-1)-1(3x-1)$

$9x^2-3x-3x+1$

→ $\int 9x^2-6x+1\,dx$ $\quad 9x^2-6x+1$

$\frac{9x^3}{3} - \frac{6x^2}{2} + 1 \quad 3x^3-3x^2+1x$

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4) $\int \frac{x^4+6x^3}{x} dx$

$\Rightarrow$ seperate into two fraction.

$\int x^3 + 6x^2$

$\frac{x^4}{4} + 2x^3$

5) $\int \frac{1}{x} dx = \ln x + C$

6) $\int \frac{1}{x-3} dx = \ln(x-3)$

6) $\int \frac{5}{x-2} dx = 5 \int \frac{1}{x-2} = 5\ln(x-2)$

7) $\int e^{4x} dx = \frac{e^{4x}}{4} + C$

8) $\int e^{5x} dx = \frac{e^{5x}}{5} + C$

9) $\int e^x = \frac{e^x}{\cancel{x}} = e^x$

10) $\int 8 e^{2x} dx = 4\cancel{8}\frac{e^{2x}}{\cancel{2}} = 4e^{2x} + C$

11) $\int \sec(x)\tan(x) dx = \sec(x)$

$\int \sec^2(x) dx = \tan(x)$

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1) Plug in substitution :

$\lim_{x\to 4} \frac{x^2-2x-3}{x^2-9} = \frac{(4)^2-2(4)-3}{(4)^2-9} = \frac{5}{7}$

2) Factor

$\lim_{x\to 3} \frac{x^2-2x-3}{x^2-9} = \frac{(x+1)(x-3)}{(x+3)(x-3)} = \frac{x+1}{x+3} = \frac{4}{6} = \frac{2}{3}$

$\Rightarrow \int x.e^x dx$

$\left[\frac{x^2}{2} \times e^x\right]$

$\rightarrow u\,dv = uv - \int v\,du$

$u = x \qquad v = e^x$

$du = dx \qquad dv = e^x dx$

$xe^x - \int e^x dx$

$xe^x - e^x + C$

4) open up parenthesis

$\lim_{x\to 0} \frac{(x+2)^2-4}{x} = \lim_{x\to 0} \frac{x^2+4x+4-4}{x} = \frac{x^2+4x}{x}$

$\lim_{x\to 0} \frac{x^2+4x}{x} = \lim_{x\to 0} \frac{\cancel{x}(x+4)}{\cancel{x}} = \lim_{x\to 0}(x+4)$

$= 0+4 = 4$

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1) $\int_0^2 \frac{dx}{x^2+1} dx$

using $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \times \tan^{-1}\left(\frac{x}{a}\right)$

$= \frac{1}{1} \times \tan^{-1}\left(\frac{x}{1}\right)$

$= \lim_{t\to\infty} \left[\tan^{-1}(x)\right]$

$(\tan^{-1}(x))\Big|_0^2$

$= \tan^{-1}(2) - \tan^{-1}(0)$

$= \tan^{-1}(2) - 0$

$= \tan^{-1}(2)$

2) $\int_0^\infty \frac{dx}{x^2+1}$

$\rightarrow \lim_{t\to\infty} \int_0^t \frac{dx}{x^2+t} = \lim_{t\to\infty} \left[\tan^{-1}(t) - \tan^{-1}(0)\right]$

$= \lim_{t\to\infty} \left(\tan^{-1}(t)\right)$

$= \left(\tan^{-1}(\infty)\right)$

$= \frac{\pi}{2}$

3) $\int_{-\infty}^0 \frac{dx}{1+x^2} = \lim_{t\to-\infty} \int_t^0 \frac{dx}{1+x^2} dx$

ln → natural log

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$= \lim_{t\to-\infty} \left[\tan^{-1}(0) - \tan^{-1}(t)\right]$

* $\int x^{-1} dx = \ln|x| + c$

$\Rightarrow \int \frac{1}{4x} dx = \frac{1}{4} \int x^{-1} dx = \frac{1}{4} \ln|x| + c$

$\Rightarrow \int \frac{x+6}{\sqrt{x}} dx = \int \left(\frac{x}{\sqrt{x}} + \frac{6}{\sqrt{x}}\right) dx = \int \frac{x}{x^{1/2}} + \frac{6}{x^{1/2}} dx$

$= \int \left(x^{\frac{1}{2}} + 6x^{-\frac{1}{2}}\right) dx = \frac{2x^{\frac{3}{2}}}{3} + 12x^{\frac{1}{2}} + c$

$\Rightarrow \int \frac{x^2}{\sqrt{36-x^2}} dx \rightarrow \sqrt{a^2-x^2} = x = a\sin\theta$
$\sqrt{6^2-x^2} = x = 6\sin\theta$

$\sqrt{36-(6\sin\theta)^2} = \sqrt{36-(36\sin^2\theta)} = \sqrt{36(1-\sin^2\theta)}$

$= 6\sqrt{(1-\sin^2\theta)} = 6\sqrt{\cos^2\theta} = 6|\cos\theta|$

$dx = 6\cos\theta \, d\theta$

$\int \frac{(6\sin\theta)^2}{6\cos\theta} (6\cos\theta \, d\theta) = \int 36\sin^2\theta \, d\theta$

$= 36 \int \sin^2\theta \, d\theta = 36 \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta$

$= 18\left(\theta - \frac{\sin 2\theta}{2}\right) = 18(\theta - \sin\theta\cos\theta) + c$

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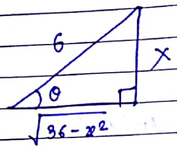
$$\boxed{\int \sin^2\theta \, d\theta = \int \left(\frac{1-\cos 2\theta}{2}\right) d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + c}$$




$\cos\theta = \frac{adj}{hypo} = \frac{\sqrt{36-x^2}}{6}$

$\sin\theta = \frac{opp}{hypo} = \frac{x}{6}$

$= 18(\theta - \sin\theta\cos\theta) + c \qquad \theta = \sin^{-1}\left(\frac{x}{6}\right)$

$= 18\sin^{-1}\left(\frac{x}{6}\right) - \frac{x}{6}\,\frac{\sqrt{36-x^2}}{6} + c$

1) $\int_0^1 \frac{dx}{x-1} = \lim_{c\to 1}\int \frac{dx}{x-1} = \lim_{c\to 1}\left[\log|x-1|\right]_0^c$

$= \lim_{c\to 1}\left[\log|\cancel{x}c-1| - \log|0-1|\right]$

$= \lim_{c\to 1}\left[\log|1-1| - \log|-1|\right]$

$= \lim_{c\to 1}\left[\log|0|\right]$

$= -\infty$

2) $\int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{c\to 4}\int \frac{dx}{\sqrt{4-x}}$

$= \lim_{c\to 4}\left[-2\sqrt{4-x}\right]_0^c$

$= \left[-2\sqrt{4-c} - (-2\sqrt{4-0})\right]$

$= \left[-2\sqrt{4-4} - (-2(2))\right] = 4$

3) $\int_0^1 \frac{dx}{\sqrt{(1-x^2)}}$

$\rightarrow \lim_{c\to 1}\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{c\to 1}\left[\sin^{-1}(x)\right]_0^c$

$= \lim_{c\to 1}\left[\sin^{-1}(c) - \sin^{-1}(0)\right]$

$= \lim_{c\to 1}\left[\sin^{-1}(1) - \sin^{-1}(0)\right]$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

4) $\int_{-1}^1 \frac{dx}{x^{2/3}}$ Here p.p = 0.

$\rightarrow \lim_{c\to 0}\int_{-1}^c \frac{dx}{x^{2/3}} + \lim_{d\to 0}\int_d^1 \frac{dx}{x^{2/3}}$

$= \lim_{c\to 0}\int_{-1}^c \frac{dx}{x^{2/3}} + \lim_{d\to 0}\int_d^1 \frac{dx}{x^{2/3}}$

$= \lim_{c\to 0}\left[\frac{x^{\frac{1}{3}}}{1/3}\right]_{-1}^c + \lim_{d\to 0}\left[\frac{x^{\frac{1}{3}}}{1/3}\right]_1^d$

$$= \lim_{c\to 0}\left[3(c)^{\frac{1}{3}} - 3(-1)^{\frac{1}{3}}\right] + \lim_{d\to 0}\left[3(4)^{\frac{1}{3}} - 3(d)^{\frac{1}{3}}\right]$$

$$= \left[3(0)^{\frac{1}{3}} - (-3)\right] + \left[3(0)^{\frac{1}{3}} - 0\right]$$

$$= 3 + 3 = 6$$

5) $$\int_{-1}^{4} \frac{dx}{|x|} = \int_{-1}^{0} \frac{dx}{|x|} + \int_{0}^{4} \frac{dx}{|x|}$$

$$\rightarrow \lim_{c\to 0}\int_{-1}^{c} \frac{dx}{-x} + \lim_{d\to 0}\int_{d}^{4} \frac{dx}{x}$$

$$= \lim_{c\to 0}\left[-\ln(|x|)\right]_{-1}^{c} + \lim_{d\to 0}\left[\ln|x|\right]_{d}^{4}$$

$$= \lim_{c\to 0}\left|-\log(x)\right|_{-1}^{c} + \lim_{d\to 0}\left[\log|x|\right]_{d}^{4}$$

$$= \left[-\log|c| - (-\log|-1|)\right] + \left[\log|4| - \log|d|\right]$$

$$= \left[-\log|0| + \log|-1|\right] + \left[\log|4| - \log|0|\right]$$

$$= \infty + 0 + 0.60 + \infty$$

$$= \underline{\infty}$$

6) $$\int_{-1}^{0} \frac{dx}{x^2 + 5x + 6} = \int_{-1}^{0} \frac{dx}{(x+2)(x+3)}$$

$$= \lim_{c\to 0}\int_{-1}^{c} \frac{1}{(x+2)(x+3)}\, dx$$

$$= \cancel{\lim_{c\to 0}\left[\log|x+2| + \log\right.}$$

$$= \text{using } \int_a^b f(x)\,dx = \int_b^a -f(x)\,dx$$

$$\int_{0}^{-1} -\frac{1}{(x+2)(x+3)}\, dx$$

$$\lim_{c\to 0}\int_{0}^{-1} -\frac{1}{(x+2)(x+3)}\, dx$$

$$= \lim_{c\to 0}\left[-\ln(|x+2|) + \ln(|x+3|)\right]_{0}^{-1}$$

$$= \lim_{c\to 0}\left(-\ln(|x+2|) + \ln(|x+3|)\right)\Big|_{0}^{-1}$$

$$= \text{using } f(x)\Big|_a^b = f(b) - f(a)$$

$$-\ln(|-1+2|) + \ln(|-1+3|) - \left(-\ln(|0+2|) + \ln(|0+3|)\right)$$

$$-\ln(|1|) + \ln|2| + \ln|2| + \ln|3|$$

$$0 + \ln(2) + \ln(6)$$

$$\cancel{\ln|2|} + \cancel{\ln(6)}$$

$$= \ln(12)$$

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1. $\int_0^x f(t)\,dt = x^2 - 2x + 1$. Find $f(x)$;

$F(x) = \int_0^x f(t)\,dt$

$\rightarrow$ $\cancel{f(x)\,dx} = \int_0^x \cancel{x^2 - 2x + 1}$

$\cancel{f'(x) = x^2 - 2x + 1}$

by FTC 1

$\therefore f(x) = x^2 - 2x + 1$

2. $f(4)$ if $\int_0^x f(t)\,dt = x \cos \pi x$.

$\Rightarrow$ $F(x) = \int_0^x f(t)\,dt$

$F(x) = x \cos \pi x$

$f(4) = 4 \cos \pi 4$

$= 4 \times (-1) \times 4 = -16$

Q.3 $\longrightarrow$ ? Volume of solid : (about $x$-axis)

a) $y = x^2$, $y = 0$; $x = 2$.

For limit :-

$\longrightarrow$ $V = \int_a^b \pi\, f(x)^2\,dx$

put $y=0$ in eqn

$\therefore 0 = x^2 \therefore$

$\therefore x = 0$

$\therefore$ limit $x=0$ to $x=2$

$V = \int_0^2 \pi\,(x^2)^2\,dx$

$= \int_0^2 \pi x^4\,dx = \pi \int_0^2 x^4\,dx = \pi \left[\frac{x^5}{5}\right]_0^2$

$= \pi \left[\frac{2^5}{5} - \frac{0^5}{5}\right] = \frac{32\pi}{5}$

b) $y = x^3$, $y = 0$, $x = 2$

$\rightarrow$ $V = \int_a^b \pi\,(f(x))^2\,dx$ limit $x=0$ and $x=2$

$V = \int_0^2 \pi\,(x^3)^2\,dx = \int_0^2 \pi x^6\,dx = \pi \int_0^2 x^6\,dx$

$= \pi \left[\frac{x^7}{7}\right]_0^2 = \pi \left[\frac{2^7}{7} - \frac{0^7}{7}\right] = \frac{128\pi}{7}$

c) $y = \sqrt{(9 - x^2)}$, $y = 0$

$\rightarrow$ put ~~y = 0~~ ~~0 = $\sqrt{(9-x^2)}$~~

$y^2 = 9 - x^2$ $\because y^2 + x^2 = r^2 \ldots$ (circle formula)

$y^2 + x^2 = 9.$ $\therefore r^2 = 9$ $\therefore r = \pm 3$

$V = \int_a^b \pi\,(f(x))^2\,dx = \int_{-3}^{3} \pi \left(\sqrt{9 - x^2}\right)^2\,dx$

$= \int_{-3}^{3} \pi\,(9 - x^2)\,dx = \pi \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$

$= \pi \left[9(+3) - \frac{(+3)^3}{3} - 9(-3) - \frac{(-3)^3}{3}\right]$

$= \pi \left[\left(27 - \frac{27}{3}\right) - \left(-27 + \frac{27}{3}\right)\right]$

$= 36\pi$

d) $y = x - x^2$ , $y = 0$

→ put $y = 0$

∴ $0 = x - x^2$

∴ $x(1 - x^2) = 0$

$x = 0$ & $x = +1$

$$V = \int_0^{+1} \pi (x - x^2)^2 \, dx$$

$$= \pi \int_0^{+1} x^2 - 2x(x)^2 + x^4 \, dx$$

$$= \pi \int_0^{+1} x^2 - 2x^3 + x^4 \, dx$$

$$= \pi \left[ \frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right]_0^{+1}$$

$$= \pi \left[ \frac{(+1)^2}{3} - \frac{2(+1)^4}{4} + \frac{(+1)^5}{5} \right]$$

$$= \pi \left[ \frac{1}{3} - \frac{2}{4} + \frac{1}{5} \right]$$

$$= \frac{\pi}{30}$$



e) $y = \sqrt{\cos x}$ , $0 \leq x \leq \pi/2$ , $y = 0$ , $x = 0$

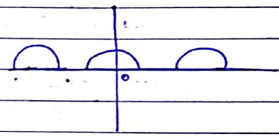
→ $V = \int_a^b \pi (f(x))^2 \cdot dx$

$$= \int_0^{\pi/2} \pi (\sqrt{\cos x})^2 \cdot dx$$

$$= \int_0^{\pi/2} \pi \cos x \cdot dx = \pi \int_0^{\pi/2} \cos x \cdot dx$$

$$= \pi [\sin x]_0^{\pi/2}$$

$$= \pi \left[ \sin \frac{\pi}{2} - \sin 0 \right]$$



$$= \pi$$

f) $y = \sec x$ , $y = 0$, $x = -\pi/4$ , $x = \pi/4$

→ $V = \int_{-\pi/4}^{\pi/4} \pi (\sec x)^2 \cdot dx$

$$= \pi \int_{-\pi/4}^{\pi/4} (\sec x)^2 \cdot dx$$ ∵ $\int \sec^2 x = \tan x$

$$= \pi [\tan x]_{-\pi/4}^{\pi/4}$$

$$= \pi \left[ \tan\left[\frac{\pi}{4}\right] + \tan \frac{\pi}{4} \right]$$ $\tan 45^\circ = 1$ $(\pi/4)$

$$= 2\pi$$

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Q.4 : on revolve around y-axis.

1) $x = \sqrt{5}\, y^2$ , $x = 0$ , $y = 1$ , $y = -1$

$\rightarrow$ $V = \int_{y=-1}^{1} \pi \left(\sqrt{5}\, y^2\right)^2 \cdot dy$

$= \pi \int_{-1}^{1} 5y^4 \cdot dy$

$= \pi \left[\frac{5y^5}{5}\right]_{-1}^{1}$

$= \pi \left[\frac{5(+1)^5}{5} - \frac{5(-1)^5}{5}\right]$

$= \pi \left[\frac{5}{5} + \frac{5}{5}\right] = 2\pi$

2) $x = y^{\frac{3}{2}}$ , $x = 0$ , $y = 2$

put $x = 0$ $\quad 0 = y^{\frac{3}{3}}$ $\therefore y = 0$

$V = \int_{0}^{2} \pi \left(y^{\frac{3}{2}}\right)^2 \cdot dy$

$= \pi \int_{0}^{2} y^3 \cdot dy = \pi \left[\frac{y^4}{4}\right]_0^2$

$\frac{\cancel{2}\times 2\times 2\times 2}{4\times}$

$= \pi \left[\frac{2^4}{4} - \frac{0^4}{4}\right] = 4\pi$

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3) $x = \sqrt{2\sin 2y}$ , $0 \le y \le \pi/2$ , $x = 0$

$\rightarrow$ $V = \int_{y=0}^{\pi/2} \pi \left(\sqrt{2\sin 2y}\right)^2 \cdot dy$

$= \pi \int_{0}^{\pi/2} 2 \sin 2y \cdot dy$

$= 2\pi \int_{0}^{\pi/2} \sin 2y \cdot dy = 2\pi \left[\frac{-\cos 2y}{2}\right]_0^{\pi/2}$

$= 2\pi \left[\frac{-\cos 2(\pi/2)}{2} + \frac{\cos 2(0)}{2}\right]$

$= 2\pi \left[\frac{1}{2} + \frac{1}{2}\right]$

$= 2\pi (1) = 2\pi$

* 4) $x = \sqrt{\cos(\pi y/4)}$ , $-2 \le y \le 0$ , $x = 0$

$\rightarrow$ $V = \int_{y=-2}^{0} \pi \left(\sqrt{\cos(\pi y/4)}\right)^2 \cdot dy$

$= \int_{-2}^{0} \pi \left(\cos(\pi y/4)\right) \cdot dy$

$= \pi \int_{-2}^{0} \cos\left(\frac{\pi y}{4}\right) \cdot dy$

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put $t = \frac{\pi y}{4}$ $\quad dy = \frac{1}{t} \times dt$

$\frac{dt}{dy} = \frac{\pi}{4}$

$\boxed{dy = \frac{4}{\pi} dt}$

$= \pi \int_{-2}^{0} \frac{4\cos(t)}{\pi} dt$

$= \frac{4\pi}{\pi} \int_{-2}^{0} \cos(t)\, dt$

$= 4 \int_{-2}^{0} \cos(t)\, dt$

$= 4 \left[\sin(t)\right]_{-2}^{0}$

$= \cancel{4 [\sin(0) - \sin(-2)]}$

$= 4 \left[\sin \frac{\pi y}{4}\right]_{-2}^{0}$

$= 4 \left[\sin \frac{\pi(0)}{4} - \sin \frac{\pi(-2)}{4}\right]$

$= 4 [0 - 0]$

$= 0$

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e) $x = 2/(y+1)$ $\quad x = 0 \quad y = 0, \; y = 3$

$\rightarrow V = \int_0^3 \pi \left(\frac{2}{(y+1)}\right)^2 dy$

$= \pi \int_0^3 \frac{4}{(y+1)^2} dy$

$= 4\pi \int_0^3 (y+1)^{-2} \cdot dy$

$= 4\pi \left[\frac{(y+1)^{-1}}{-1}\right]_0^3$

$= 4\pi \left[\frac{-1}{(1+y)}\right]_0^3$

$= 4\pi \left[\frac{-1}{(1+3)} + \frac{1}{(1+0)}\right]$

$= 4\pi \left[\frac{-1}{4} + \frac{1}{1}\right]$

$= 4\pi \left[-\frac{1}{4} + 1\right] = 4\pi \left[\frac{-1+4}{4}\right] = 4\pi \left[\frac{3}{4}\right]$

$= \frac{12\pi}{4} = 3\pi$

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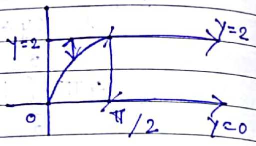
5] The region in the first quadrant bounded above by the line $y = 2$, below by the curve $y = 2\sin x$, $0 \leq x \leq \pi/2$ and the left by the axis, about the line $y = 2$.

→ $y = 2\sin x \quad 0 \leq x \leq \pi/2 \quad y = 2$

X $V = \int_0^{\pi/2} \pi (2\sin x)^2 \, dx$




$= \pi \int_0^{\pi/2} 4 \sin^2 x \, dx$

$= 4\pi \int \sin^2 x \, dx$

$\sin(x)^2 = \frac{1 - \cos(2x)}{2}$

$\therefore = 4\pi \int_0^{\pi/2} \frac{1 - \cos(2x)}{2} \cdot dx$

$= \frac{4\pi}{2} \int_0^{\pi/2} (1 - \cos(2x)) \cdot dx$

$= 2\pi \int_0^{\pi/2} 1 - \cos(2x) \cdot dx$

$= 2\pi \int_0^{\pi/2} 1 \cdot dx - \int \cos 2x \cdot dx$

$= 2\pi \left[ x - \frac{\sin 2x}{2} \right]_0^{\pi/2}$

$= 2\pi \left[ \frac{2x - \sin 2x}{2} \right]_0^{\pi/2}$

$\sin \pi = 0$ on $x$
$\sin 0 = 0$
$\cos \pi$
$\cos$



$= \pi \left[ 2x - \sin 2x \right]_0^{\pi/2}$

$= \pi \left[ 2\left(\frac{\pi}{2}\right) - \sin 2\left(\frac{\pi}{2}\right) - \left(2(0) - \sin 2(0)\right) \right]$

$= \pi \left[ \pi - 0 \right]$

$= \pi^2$

Q. 6 Find volume of the solid generated by revolving the regions bounded by the lines and curve about $x$-axis.

X a) $y = x$, $y = 1$, $x = 0$

→ $V = \int_a^b \pi (f(x))^2 \cdot dx$

$= \int_a^b \pi (x)^2 \cdot dx = \int_0^1 \pi x^2 \cdot dx$

$= \pi \int_0^1 x^2 \cdot dx = \pi \left[ \frac{x^3}{3} \right]_0^1 = \pi \left[ \frac{1}{3} - \frac{0}{3} \right]$

$= \frac{\pi}{3}$

$\int \sin x = x \cos x \, dx$

5] $V = \int_0^{\pi/2} \pi (2 - 2\sin x)^2 \, dx$ $(a-b)^2$

$4\int 1 - \frac{1-\cos(x)^2}{2}$

$= \int_0^{\pi/2} \pi (4 - 8\sin x + 4\sin(x)^2 \, dx$

$= \pi \int 4 \cdot dx - \int 8\sin x \, dx + \int 4 \sin(x)^2 dx$

$= \pi (4x + 8\cos x + 2x - \sin(2x))$

$\int 4\sin(x)^2 dx = 4 \int \sin(x)^2 dx$

$= 4 \int \frac{1 - \cos(2x)}{2} dx$

$= \frac{4}{2} \int 1 - \cos(2x) \cdot dx$

$= 2 \int 1 \cdot dx - \int \cos(2x) \, dx$

$= 2x - \sin(2x) + c$

$= \pi (6x + 8\cos x - \sin(2x)) \Big|_0^{\pi/2}$

$= (6\pi x + 8\pi \cos x - \sin \pi \, 2x) \Big|_0^{\pi/2}$

$= 6\pi \times \frac{\pi}{2} + 8\pi \cos\frac{\pi}{2} - \sin \pi \cancel{2} \times \frac{\pi}{\cancel{2}} - ($

$6\pi \times 0 + 8\pi \cos(0) - \pi \sin(2 \times 0))$

$= 3\pi^2 - 8\pi$

Q.6.2) $y = 2\sqrt{x}$, $y = 2$, $x = 0$ (solve it by washer)

→ For limit put $y = 2$

$\cancel{2} = \cancel{2}\sqrt{x}$ $x = 1$ $\therefore x = 1$

$V = \int_{x=0}^{1} \pi (y)^2 \, dx$

$= \int_0^{1} \pi (2\sqrt{x})^2 \cdot dx = \int_0^{2} \pi \, 4x \cdot dx$

$= \pi \int_0^2 4x \cdot dx = \pi \left[ \frac{4x^2}{2} \right]_0^2$

$= \pi \left[ \frac{4 \times 1}{2} - \frac{4(0)}{2} \right] = 2\pi$

3) $y = x^2 + 1$, $y = x + 3$

$x^2 + 1 = x + 3$ $x = 2$

$x^2 - x - 2 = 0$ $x = -1$.

$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) =$

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$$V = \int_{x=-1}^{2} \pi (x^2+1)^2 \cdot dx$$

$$= \pi \int_{-1}^{2} (x^2+1)^2 \, dx$$

$\frac{8+6}{3}$

$\frac{14}{3}$

$$= \cancel{\pi \left[ \frac{x^3}{3} + x \right]_{-1}^{2}}$$

$$= \pi \left[ \cancel{\frac{2^3}{3} + 2 - \left( \frac{(-1)^3}{3} + (-1) \right)} \right]$$

$$\cancel{= \pi \left[ \frac{8\,14}{3} - \right.}$$

$$= \pi \int_{-1}^{2} 2x^4 + 2x^2 + 1 \cdot dx$$

$$= \pi \int_{-1}^{2} (x^4) + 2x^2 + 1 \cdot dx$$

$$= \pi \int x^4 + 2 \int x^2 + x$$

$$= \pi \left[ \frac{x^5}{5} + \frac{2x^3}{3} + x \right]_{-1}^{2}$$

$$= \pi \left[ \frac{(2)^5}{5} + \frac{2(2)^3}{3} + (2) - \left( \frac{(-1)^5}{5} + \frac{2(-1)^3}{3} - 1 \right) \right]$$

$$\int_{-1}^{2} \pi (x+3)^2 \, dx$$

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$$= \pi \left[ \frac{78}{5} \right] = \frac{78\pi}{5}$$

d) $y = 4 - x^2$, $y = 2 - x$

$$4 - x^2 = 2 - x$$
$$4 - x^2 - 2 + x = 0$$
$$-x^2 + x + 2 = 0$$
$$x^2 - x - 2 = 0$$
$$x^2 - 2x + x - 2 = 0$$
$$x(x-2) + 1(x-2) = 0$$
$$\therefore x = 2 \quad x = -1$$

$$V = \int_{-1}^{2} \pi (4 - x^2)^2 \cdot dx$$

$$= \pi \int_{-1}^{2} (4)^2 - 8x^2 + x^4 \cdot dx$$

$$= \pi \int_{-1}^{2} 16 - 8x^2 + x^4 \cdot dx$$

$$= \pi \left[ 16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_{-1}^{2}$$

$\frac{2\times2\times2\times2\times2}{4 \quad 4}$ 32

$$= \pi \left[ 16 \times 2 - \frac{8(2)^3}{3} + \frac{(2)^5}{5} - \left( 16(-1) - \frac{8(-1)^3}{3} + \frac{(-1)^5}{5} \right) \right]$$

$$= \pi \left[ 32 - \frac{64}{3} + \frac{32}{5} - \left( -16 + \frac{8}{3} - \frac{1}{5} \right) \right]$$

$$\frac{64}{15} - \left( -\frac{203}{15} \right) = \frac{256}{15} + \frac{203}{15} = \frac{153}{5}\pi$$

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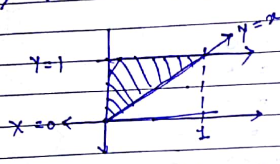
e) $y = \sec x$ , $y = \sqrt{2}$ , $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$

*

→ By washer method.

Q.6 1) $y = x$ $y = 1$ , $x = 0$




put $y = 1$
$x = 1$

$R = 1$ $r = x$

$$v = \int_{x=a}^{b} \pi \ (R^2 - r^2) \ dx$$

$$= \int_{\cancel{x=0}}^{\cancel{1}} \cancel{\pi \ (1^2 - (0)^2) \ dx}$$

$$= \pi \int_{x=0}^{1} (1^2 - x^2) \ dx.$$

$$= \pi \int_0^1 1 - x^2 \cdot dx$$

$$= \pi \left[ x - \frac{x^3}{3} \right]_0^1$$

$$= \pi \cdot \left[ 1 - \frac{(1)^3}{3} - \left( 0 - \frac{0}{3} \right) \right] = \pi \left[ \frac{3-1}{3} \right] = \frac{2\pi}{3}$$

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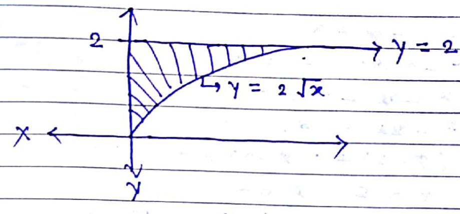
2) $y = 2\sqrt{x}$ , $y = 2$ , $x = 0$

→




→ $y = 2\sqrt{x}$ $\frac{y}{2} = \sqrt{x}$ $= \frac{y^2}{4}$ ∴ $x = \frac{y^2}{4}$

$2\sqrt{x}$ 2

put $x = 0.5$ as intermediate

$2\sqrt{0.5} < 2$
↓ ↓ ↓
r R

$$v = \int_0^1 \pi \left( (2)^2 - (2\sqrt{x})^2 \right) \cdot dx$$

$$= \pi \int_0^1 4 - 4x \cdot dx$$

$$= \pi \int 4 \cdot dx - \int 4x \ dx$$

$$= \pi \left[ 4x - \frac{4x^2}{2} \right]_0^1 = \pi \left[ \frac{8x - 4x^2}{2} \right]_0^1$$

$$= \pi \left[ \frac{8(1) - 4(1)^2}{2} - \frac{8(0) - 4(0)^2}{2} \right]$$

$$= \pi \left[ \frac{8-4}{2} \right] = \pi \left[ \frac{4}{2} \right] = 2\pi$$

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c) $y = x^2 + 1$ , $y = x + 3$

→ $x^2 + 1 = x + 3$

$x^2 - x + 1 - 3 = 0$

$x^2 - x - 2 = 0$

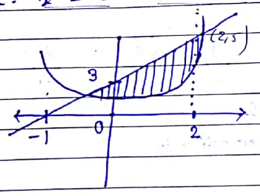
$x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$\therefore x = 2 \quad x = -1$

if $x = 0$

$\therefore y = 3$




$y = \quad x^2 + 1 \qquad x + 3$

put $x = 0$

$0 + 1 < 0 + 3$

$1 < 3$

$\therefore R = x + 3 \qquad r = x^2 + 1$

$$V = \int_{-1}^{2} \pi \left(R^2 - r^2\right) dx$$

$$= \int_{-1}^{2} \pi \left((x+3)^2 - (x^2+1)^2\right)$$

$$= \cancel{\pi \int_{-1}^{2} \left(x^2 + 6x + 9 - x^4 + 2x^2 + 1\right)}$$

$$= \cancel{\pi \int_{-1}^{2} 3x^2 + 2x + 10 - x^4}$$

$$= \cancel{\pi \left[\frac{3x^3}{3} + \frac{6x^2}{2} + 10x - \frac{x^5}{5}\right]_{-1}^{2}}$$

$$= \cancel{\pi \left[\frac{3(2)^3}{3} + \frac{6(2)^2}{2} + 10(2) - \frac{(2)^5}{5} - \left(\frac{3(-1)^3}{3} + \frac{2(-1)^2}{2} + 10(-1) - \frac{(-1)^5}{5}\right)\right]}$$

$$= \cancel{\pi \left[\frac{24}{3} + \frac{24}{2} + 20 - \frac{32}{5} - \left(\frac{-3}{3} + \frac{-2}{2} + -10 + \frac{1}{5}\right)\right]}$$

$$= \cancel{\pi \left[\frac{168}{5} - \left(\frac{-59}{5}\right)\right]}$$

$$= \pi \int_{-1}^{2} (x^2 + 6x + 9) - \int_{-1}^{2} x^4 + 2x^2 + 1$$

$$= \pi \left[\frac{x^3}{3} + \frac{6x^2}{2} + 9x\right]_{-1}^{2} - \left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_{-1}^{2}$$

$$= \pi \left[\frac{(2)^3}{3} + \frac{6(2)^2}{2} + 9(2) - \left(\frac{(-1)^3}{3} + \frac{6(-1)^2}{2} + 9(-1)\right)\right] -$$

$$\left[\frac{(2)^5}{5} + \frac{2(2)^3}{3} + (2) - \left(\frac{(-1)^5}{5} + \frac{2(-1)^3}{3} + (-1)\right)\right]$$

$$= \pi \left[\frac{8}{3} + \frac{24}{2} + 18 - \left(\frac{-1}{3} + \frac{6}{2} - 9\right)\right] - \left[\frac{32}{5} + \frac{16}{3} + 2 - \left(-\frac{1}{5} - \frac{2}{3} - 1\right)\right.$$

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$= \pi[45 - 15.6]$

$= 29.4\pi$

d) $y = 4 - x^2$ ; $y = 2 - x$

$4 - x^2 = 2 - x$

$-x^2 + x - 2 + 4 = 0$

$-x^2 + x + 2 = 0$

$x^2 - x - 2 = 0$

$\therefore x = -1 \quad x = 2$

put $x = -1$

$y = 4 - (-1)^2$

$y = 3$

put $x = 2$ $(4-(2)^2)$

$\therefore y = 0$

put $x = -1$

$= 2 - (-1)$

$y = 3$

$y = 2 - (-2)$

$y = 4$

put $x = 0$ $\therefore y = 2 - x$

$= 2 - 0$

$y = 2.$

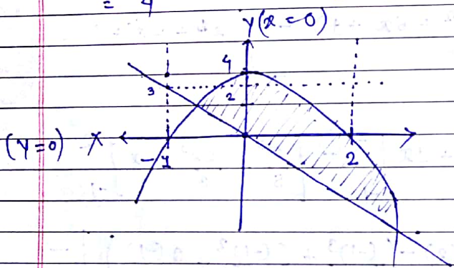
$y = 4 - x^2$

$= 4 - (0)^2$

$= 4$

$\therefore R = 4 - x^2$

$r = 2 - x.$




$$V = \int_{x=-1}^{2} \pi (R^2 - r^2) \cdot dx$$

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$$V = \pi \int_{-1}^{2} (4 - x^2)^2 - (2 - x)^2 \cdot dx$$

$$= \pi \left[ \int_{-1}^{2} (4 - x^2)^2 dx - \int_{-1}^{2} (2 - x)^2 dx \right]$$

$$= \pi \left[ \int_{-1}^{2} 16 - 8x^2 + x^4 \, dx - \int_{-1}^{2} 4 - 4x + x^2 \cdot dx \right.$$

$$= \pi \left[ 16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_{-1}^{2} - \left[ 4x - \frac{4x^2}{2} + \frac{x^3}{3} \right]_{-1}^{2}$$

$$= \pi \left[ 16(2) - \frac{8(2)^3}{3} + \frac{(2)^5}{5} - \left( 16(-1) - \frac{8(-1)^3}{3} + \frac{(-1)^5}{5} \right) \right] -$$

$$\left[ 4(2) - \frac{4(2)^2}{2} + \frac{(2)^3}{3} - \left( 4(-1) - \frac{4(-1)^2}{2} + \frac{(-1)^3}{3} \right) \right]$$

$$= \pi \left[ 32 - \frac{64}{3} + \frac{32}{5} - \left( -16 + \frac{8}{3} - \frac{51}{5} \right) \right] -$$

$$\left[ 8 - \frac{16}{2} + \frac{8}{3} - \left( -4 + \frac{4}{2} - \frac{1}{3} \right) \right]$$

$$= \pi \left[ \frac{256}{15} + \frac{203}{15} - \frac{8}{3} + \frac{7}{3} \right]$$

$$= \pi \left[ \frac{153}{5} - 5 \right] = 25.6\pi$$

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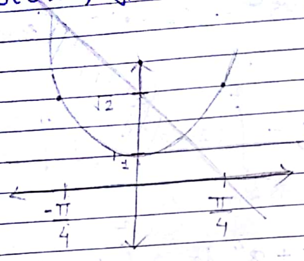
e) $y = \sec x$ , $y = \sqrt{2}$ , $-\pi/4 \leq x \leq \pi/4$




put $x = 0$ in ~~both~~ eqn

$y = \sec 0$ $\qquad y = \sqrt{2}$

$y = 1$ $\qquad y = \sqrt{2}$

$\therefore 1 < \sqrt{2}$

$\therefore R = \sqrt{2}$ $\qquad r = \sec x$

$$\therefore V = \int_a^b \pi (R^2 - r^2)\, dx$$

$$= \int_{-\pi/4}^{\pi/4} \pi \left((\sqrt{2})^2 - (\sec x)^2\right) \cdot dx$$

$$= \pi \int_{-\pi/4}^{\pi/4} 2 - \sec(x)^2 \, dx$$

$$= \pi \int_{-\pi/4}^{\pi/4} 2 \cdot dx - \int_{-\pi/4}^{\pi/4} \sec^2 x \, dx$$

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$$= \pi \left[ 2x - \tan(x) \right]_{-\pi/4}^{\pi/4}$$

$$= \pi \left[ 2 \times \frac{\pi}{4} - \tan\left(\frac{\pi}{4}\right) - \left(2 \times -\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right)\right]$$

$$= \pi \left[ \frac{\pi}{2} - \tan\frac{\pi}{4} - \left(-\frac{\pi}{2} + \tan\frac{\pi}{4}\right)\right]$$

$1 - (-\pi/2 + 1)$

$$= \pi \left[ \frac{\pi}{2} - 1 + \frac{\pi}{2} - 1 \right]$$

$$= \pi \left[ \pi - 2 \right] = \pi^2 - 2\pi$$

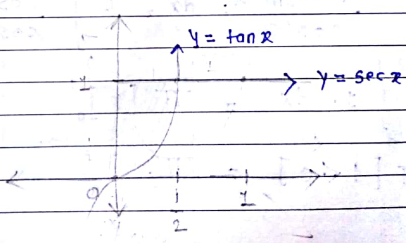
f) $y = \sec x$ , $y = \tan x$ , $x = 0$ , $x = 1$ ;

put $x = 0$ in $y = \sec x$ $\qquad$ put

$\therefore y = \sec 0 = 1$

put $x = 1$ in $y = \sec x$

$y = \sec(1) = 1.00015$



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put $x = 0.5$ in both

$y = \sec(0.5)$ $\quad y = \tan(0.5)$

$1.0.. > 8.7\times10^{-3}$

$\therefore R = \sec x \quad r = \tan x$

$$V = \int_0^1 \pi\left[(\sec x)^2 - (\tan x)^2\right]\cdot dx$$

$$= \pi\int_0^1 \sec(x)^2\cdot dx - \int_0^1 \tan^2 x\; dx$$

$$= \pi\left[\tan x\right]_0^1 - \left[\tan x\right]_0^1$$

$$= \pi\left[\tan(1) - \tan(0)\right] - \left[\tan(1) - \tan(0)\right]$$

$$= \pi\int_0^1 \frac{1}{\cos(x)^2} - \frac{\sin^2 x}{\cos^2 x}\cdot dx$$

$$= \pi\int_0^1 \frac{1-\sin^2 x}{\cos^2 x}\,dx = \pi\int_0^1 \frac{\cos^2 x}{\cos^2 x}\cdot dx$$

$$\pi\int_0^1 1\cdot dx = \pi\left[x\right]_0^1$$

$$= \pi\left[1-0\right] = \pi.$$

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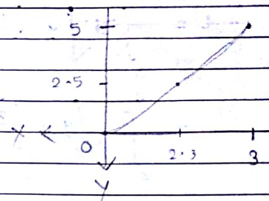
8) $y = \frac{x^2}{2}$ $\quad y = 0$, $y = 5$ around y-axis

$x = \sqrt{2y}$ $\quad y = 0$, $y = 5$ ~~about~~

$$\rightarrow V = \int_{y=0}^{5} \pi\left(\sqrt{2y}\right)^2\cdot dy$$




$$= \pi\int_{y=0}^{5} 2y\cdot dy$$

$$= \pi\int_0^5 \left[\frac{2y^2}{2}\right]_0^5$$

$$= \pi\left[y^2\right]_0^5 = \pi\left[(5)^2 - (0)^2\right]$$

$$= 25\pi$$

Q.9 Find the length of cure.

b) Length of cure $= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)}\cdot dx$

1) $y = \frac{1}{3}(x^2+2)$ from $x=0$ to $x=3$

$\frac{dy}{dx} = \frac{1}{3}(x^2+2)$

$$L = \int_0^3 \sqrt{1+\left(\frac{2x}{3}\right)^2}\,dx \qquad \frac{dy}{dx} = \frac{2x}{3}$$

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$\frac{3}{2}-1 = \frac{3-2}{2} = \frac{1}{2}$

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b) $y = x^{\frac{3}{2}}$ from $x=0$ to $x=4$

$\rightarrow \frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$

$$L = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1+\left(\frac{3\sqrt{x}}{2}\right)^2}\, dx$$

$$= \int_0^4 \sqrt{1+\frac{9x}{4}}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \int_0^4 \frac{\sqrt{4+9x}}{2}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\int_0^4 (4+9x)^{\frac{1}{2}}\, dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{2}{54}\left[(4+9x)^{3/2}\right]_0^4$$

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$$\frac{2}{54}\left[(4+9\times 4)^{3/2} - (4+0)^{3/2}\right]$$

$$= \frac{1}{27}\left((4+36)^{3/2} - (4)^{3/2}\right]$$

$$= 9.07$$

9\. $L = \int_0^3 \sqrt{1+\left(\frac{2x}{3}\right)^2}\, dx = \int_0^3 \sqrt{\frac{\sqrt{9+4x^2}}{3}}\, dx$

$$= \frac{1}{3}\int_0^3 \sqrt{9+4x^2}\, dx$$

$$= \frac{1}{3}\left[\frac{2(9+4x^2)^{\frac{3}{2}}}{3} \times \frac{1}{8}\right]_0^3$$

$$= \frac{2}{81}\left[(9+4x^2)^{\frac{3}{2}}\right]_0^3$$

$$= \frac{2}{72}\left[(9+4(3^2))^{\frac{3}{2}} - (9+4(0^2))^{\frac{3}{2}}\right]$$

$$\cancel{= \frac{2}{81}\left[(21)^{\frac{3}{2}} - (9)^{\frac{3}{2}}\right]}$$

=

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c) $x = \left(\frac{y^3}{3}\right) + \left(\frac{1}{4y}\right)$ from $y = 1$ to $y = 3$

$\rightarrow x = \frac{4y^4 + 3}{12y}$

$\frac{dx}{dy} = \frac{1}{3} 3y^2 + \frac{1}{4} \frac{1}{y^{-2}}$

$= y^2 + \frac{1}{4y^{+2}} = y^2 + \frac{1}{4y^2}$

Q.103 —— ? improper integrals

a) $\int_0^\infty \frac{dx}{x^2+1}$

$\rightarrow \lim_{c\to\infty} \int_0^c \frac{dx}{x^2+1}$ ..... Type I improper I.

$\int \frac{1}{x^2+1} dx . \int \frac{+}{1x^2+x^2} = \tan^{-1}(x)$

$\therefore \lim_{c\to\infty} \int_0^c \frac{dx}{x^2+1} = \lim_{c\to\infty} \left[\tan^{-1}(x)\right]_0^c$

$= \lim_{c\to\infty} \left[\tan^{-1}(c) - \tan^{-1}(0)\right]$

$= \left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$

$= \frac{\pi}{2}$

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b) $\int_0^4 \frac{dx}{\sqrt{(4-x)}}$

$\rightarrow \lim_{c\to 4} \int_0^4 \frac{dx}{(4-x)^{1/2}}$ .... Type I

$= \lim_{c\to 4} \left[\frac{(4-x)^{\frac{1}{2}}}{1/2} \times \frac{1}{-1}\right]_0^c$

$= \lim_{c\to 4} \left[\frac{(4-x)^{\frac{1}{2}}}{1/2} \times \frac{1}{-1}\right]_0^c$

$= \lim_{c\to 4} \left[-2(4-x)^{\frac{1}{2}}\right]_0^c$

$= \lim_{c\to 4} \left[-2(4-c)^{\frac{1}{2}} + 2(4-0)^{\frac{1}{2}}\right]$

$= \left[-2(4-4)^{\frac{1}{2}} + 2(4)^{\frac{1}{2}}\right]$

$= \left[0 + 2(2)\right] = 4$

c) $\int_{-1}^1 \frac{dx}{x^{2/3}}$

$\rightarrow \lim_{c\to 0} \int_{-1}^0 \frac{dx}{x^{2/3}} + \lim_{d\to 0} \int_0^1 \frac{dx}{x^{2/3}}$

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$$= \lim_{c\to 0}\int_{-1}^{c}\frac{dx}{x^{2/3}} + \lim_{d\to 0}\int_{d}^{1}\frac{dx}{x^{2/3}}$$

$$= \lim_{c\to 0}\int_{-1}^{c} x^{-\frac{2}{3}}\,dx + \lim_{d\to 0}\int_{d}^{1} x^{-\frac{2}{3}}\,dx$$

$$= \lim_{c\to 0}\left[3x^{\frac{1}{3}}\right]_{-1}^{c} + \lim_{d\to 0}\left[3x^{\frac{1}{3}}\right]_{d}^{1}$$

$$= \lim_{c\to 0}\left[3(c)^{\frac{1}{3}} - 3(-1)^{\frac{1}{3}}\right] + \lim_{d\to 0}\left[3(1)^{\frac{1}{3}} - 3(d)^{\frac{1}{3}}\right]$$

$$= \lim_{c\to 0}\left[3(0)^{\frac{1}{3}} + 3\right] + \lim\left[3 - 0\right]$$

$$= 3 + 3 = 6$$

d) $\displaystyle\int_{0}^{1}\frac{dx}{\sqrt{(1-x^2)}}$ p.p = 1

$$\rightarrow \lim_{c\to 1}\int_{0}^{1}\frac{dx}{\sqrt{(1-x^2)}} = \lim_{c\to 1}\int_{0}^{c}\frac{dx}{\sqrt{(1-x^2)}}$$

$$= \lim_{c\to 1}\left[\sin^{-1}(x)\right]_{0}^{c}$$

$$= \lim_{c\to 1}\left[\sin^{-1}(c) - \sin^{-1}(0)\right]$$

$$= \left[\sin^{-1}(1) - \sin(0)\right]$$

$$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

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e) $\displaystyle\int_{-\infty}^{\infty}\frac{2x}{(x^2+1)^2}\,dx$

$$\rightarrow \int_{-\infty}^{0}\frac{2x}{(x^2+1)^2}\,dx + \int_{0}^{\infty}\frac{2x}{(x^2+1)^2}\,dx$$

$$\lim_{c\to -\infty}\int_{c}^{0}\frac{2x}{(x^2+1)^2}\,dx + \lim_{d\to\infty}\int_{0}^{d}\frac{2x}{(x^2+1)^2}\,dx$$

No need to interchang limits. becausof ($\infty$)

use 'u' subtitution method.

$u = x^2+1$

$du = 2x\cdot dx$

$$\lim_{c\to -\infty}\int_{c}^{0}\frac{du}{(u)^2} + \lim_{d\to\infty}\int_{0}^{d}\frac{du}{(u)^2}$$

$$\lim_{c\to -\infty}\left[\frac{1}{u}\right]_{c}^{0} + \lim_{d\to\infty}\left[\frac{1}{u}\right]_{0}^{d}$$

put $u = x^2+1$

$$\lim_{c\to -\infty}\left[\frac{1}{\cancel{x^2+1}}\ \frac{1}{\emptyset} - \frac{1}{-\infty}\right]\left[\frac{1}{x^2+1}\right]_{c}^{0} + \left[\frac{1}{x^2+1}\right]_{0}^{d}$$

$$\lim_{c\to -\infty}\left[\frac{1}{0+1} - \frac{1}{-\infty+1}\right] + \lim\left[\frac{1}{\infty+1} - \frac{1}{0+1}\right]$$

$$= \left[1 - \infty\right] + \left[\infty - 1\right]$$

$$= \infty$$

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f) $\int_{-1}^{4} \frac{dx}{|x|}$

$\rightarrow$ First solve $\int \frac{dx}{|x|}$

separate two integrals.

$\int \frac{dx}{x}$, $x \geq 0$ and $\int \frac{dx}{-x}$, $x \leq 0$

$= \ln(|x|) + c, x \geq 0$ and $-\ln(|x|) + c$ $x \leq 0$

$\lim_{c \to 0} \int_{-1}^{0} \frac{1}{x} dx + \lim_{d \to 0} \int_{d}^{4} \frac{-1}{x} dx$

$= \lim_{c \to 0} \int_{-1}^{c} \frac{1}{x} dx + \lim_{d \to 0} \int_{d}^{4} -\frac{1}{x} dx$

$= \lim_{c \to 0} [\log(x)]_{-1}^{c} + \lim_{d \to 0} [-\log|x|]_{d}^{4}$

$= [\log|-1| - \log|0|] + [-\log|0| + \log|0|]$

$= \infty - \infty + (-\infty) + 0.6$

$= \infty$

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g) $\int_{-1}^{\infty} \frac{dx}{x^2+5x+6}$

$\Rightarrow$ $\int \frac{dx}{x^2+5x+6} = \int \frac{1}{(x+2)(x+3)} dx$

$\frac{1}{(x+2)(x+3)} = \frac{A}{(x+2)} + \frac{B}{(x+3)}$

$\frac{1}{(x+2)(x+3)} = \frac{A(x+3) + B(x+2)}{(x+2)(x+3)}$

$1 = A(x+3) + B(x+2)$

put $x+3 = 0$ $\therefore x = -3$

$B(x+2) = 1$

$B(-3+2) = 1$ $\therefore B = -1$

put $x+2 = 0$ $x = -2.$

$1 = A(x+3)$

$1 = A(-2+3)$ $\therefore A = 1$

$\lim_{t \to \infty} \left[ \int_{-1}^{t} \frac{1}{(x+2)} + \int_{-1}^{t} \frac{-1}{(x+3)} \right]$

$= \lim_{t \to \infty} \left[ \log|(x+2)| + \log|(x+3)| \right]_{-1}^{t}$

$= \left[ \cancel{\log|(t+2)|} + \log|-t+3| \right] - \left[ \log|-1+2| + \log|-1+3| \right]$

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$$\left[\log|t+2| - \log|-1+2|\right]_{-1}^{t} + \left[\log|t+3| - \log|-1+3|\right]$$

$$= \lim_{t\to\infty}\left[\log|t+2| - \log|1|\right]_{-1}^{t} + \left[\log|t+3| - \log|2|\right]$$

$$= \lim_{t\to\infty}\left[\log|t+2| - \log|t+3|\right] + \log 2$$

$$= \log\left(\frac{t+2}{t+3}\right) + \log 2$$

$$\lim_{t\to\infty}\left[\log\frac{1+\frac{2}{t}}{1+\frac{3}{t}}\right] + \log 2$$

$$= \left[\log\frac{1+\frac{2}{\infty}}{1+\frac{3}{\infty}}\right] + \log 2$$

$$= \log\left(\frac{1}{1}\right) + \log 2$$

$$0 + \log 2$$

h) $\displaystyle\int_0^\infty \frac{dx}{(x+1)(x^2+1)}$

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$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)}$$

the factor in denominator is second order. hence $(Bx+C)$.

$$= \frac{A(x^2+1)+Bx+C\,(x+1)}{(x+1)(x^2+1)}$$

$$= A(x^2+1) + (x+1)Bx + c$$

$$= Ax^2 + A + Bx^2 + Cx + Bx + c$$

$$= Ax^2 + Bx^2 + cx + Bx + A + C$$

$$1x^0 = (A+B)x^2 + (C+B)x + (A+C)x^0$$

$A + C = 1$    $A = \frac{1}{2}$    $B = -\frac{1}{2}$    $c = \frac{1}{2}$

$A + B = 0$

$C + B = 0$

$$= \frac{\frac{1}{2}}{(x+1)} + \frac{\left(-\frac{1}{2}\right)x + \frac{1}{2}}{(x^2+1)} = \frac{\frac{1}{2}}{(x+1)} + \frac{-\frac{1}{2}x + \frac{1}{2}}{(x^2+1)}$$

$$= \frac{\frac{1}{2}}{(x+1)} + \frac{\frac{-x+1}{2}}{(x^2+1)} = \frac{\frac{1}{2}}{(x+1)} + \frac{\frac{-x+1}{2}}{(x^2+1)}$$

$$= \int \frac{1}{2(x+1)} + \int \frac{-x+1}{2(x^2+1)}\,dx$$

$$= \lim_{t\to\infty}\int_0^t \frac{1}{2(x+1)}\,dx + \lim_{t\to\infty}\int_0^t \frac{-x+1}{2(x^2+1)}\,dx$$

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$= \frac{1}{2} \lim_{t\to\infty} \frac{1}{2}\left[\log(x+1)\right]_0^t + \lim_{t\to\infty} \frac{1}{2}\left[-\frac{1}{2}\times\log(x^2+1) + \tan^{-1}(x)\right]_0^t$

$\int \frac{x-1}{2(x^2+1)}\,dx$

$= \frac{1}{2}\left(\int \frac{x}{(x^2+1)}\,dx + \int\frac{1}{(x^2+1)}\right)$

$= \frac{1}{2}\left(-\int \frac{x}{(x^2+1)}\,dx + \int \frac{1}{(x^2+1)}\right) \quad \because \int\frac{1}{x^2+1} = \tan^{-1}(x)$

$= \frac{1}{2}\Big($ put $x^2+1 = u$

$\therefore du = 2x$

$\therefore x = du/2$

$= \frac{1}{2}\int\frac{x\,dx}{(x^2+1)} = \int\frac{du}{2u} = \frac{1}{2}\int\frac{1}{u}\,du$

$= \frac{1}{2}\log(|u|) = \frac{1}{2}\log(|x^2+1|)$

$= \frac{1}{2}\left[\left(-\frac{1}{2}\times\log(x^2+1) + \tan^{-1}(x)\right)\right]_0^t$

$= \lim_{t\to\infty}\frac{1}{2}\left[\log(t+1) - \log(0+1)\right] + \lim_{t\to\infty}\frac{1}{2}\left[\left(-\frac{1}{2}\times\log(t^2+1) + \tan^{-1}(t)\right) - \left(-\frac{1}{2}\times\log(0+1) + \tan^{-1}(0)\right)\right]$

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$= \left[\log(\infty+1) - \log(1)\right] + \left[\left(-\frac{1}{2}\times\log(\infty+1) + \tan^{-1}(\infty)\right) - \left(\frac{1}{2}\times\log(1) + \tan^{-1}(0)\right)\right]$

$= [\infty - 0] + \left[\left(-\frac{1}{2}\times\infty + \frac{\pi}{2}\right) - \left(-\frac{1}{2}\times 0 + 0\right)\right]$

$= \infty$

Q.14 Test the convergence of following integrals.

a) $\int_0^{\pi} \frac{\sin\theta}{\sqrt{(\pi-\theta)}}\,d\theta$

when we use substitute method limit also changes.

put $\theta = \pi$ in ① $\therefore u = 0$

put $\theta = 0$ in ① $\therefore u = \pi$

$\rightarrow$ put $\pi - \theta = u$ — ① $\boxed{\therefore \pi - u = \theta}$

$\therefore du = -d\theta$

$\therefore \frac{du}{d\theta} = d\theta = -du$

$\int_{\pi}^{0} \frac{\sin(\pi-u)}{\sqrt{u}}(-du)$

$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$

$\sin(\pi-u) = \sin(\pi)\cos(u) - \cos(\pi)\sin(u)$

$= 0 - (-1)\sin(u)$

$= \sin(u)$

$\sin(\pi) = 0$

$\cos(\pi) = -1$

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(~~- du removed by~~ by interchanging limits)

$$\int_0^{\pi} \frac{\sin u}{\sqrt{u}}\, du$$

By test of improper integrals.

$$f(u) = \frac{\sin u}{\sqrt{u}} \qquad g(u) = \frac{1}{\sqrt{u}}$$

$$\therefore \frac{\sin u}{\sqrt{u}} \leq \frac{1}{\sqrt{u}}$$

$$\therefore f(u) < g(u)$$

$$\int_0^{\pi} g(u)\, du = \lim_{c\to 0}\int_c^{\pi} \frac{1}{\sqrt{u}}\, du$$

$$\lim_{c\to 0}\int_c^{\pi} u^{-\frac{1}{2}}\, du$$

$$= \left[2u^{\frac{1}{2}}\right]_c^{\pi}$$

$$= \left[2\sqrt{\pi} - 2\sqrt{0}\right]$$

$$= 2\sqrt{\pi} - 0$$

$$= 2\sqrt{\pi}$$

b) $$\int_0^{\pi} \frac{dt}{\sqrt{t} + \sin t}$$

→ Here 0 is problemetic point.

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By test of improper integral

$$f(t) = \frac{1}{\sqrt{t} + \sin t} \qquad g(t) = \frac{1}{\sqrt{t}}$$

$$\sqrt{t} + \sin t \geq \sqrt{t}$$

$$\therefore \frac{1}{\sqrt{t}+\sin t} \leq \frac{1}{\sqrt{t}}$$

$$\therefore f(t) < g(t)$$

$$\therefore \int_0^{\pi} g(t)\, dt = \lim_{c\to 0}\int_c^{\pi} \frac{1}{\sqrt{t}}\, dt$$

$$\lim_{c\to 0}\int_c^{\pi} t^{-\frac{1}{2}}\, dt$$

$$= \left[2t^{\frac{1}{2}}\right]_c^{\pi}$$

$$= \left[2\pi^{\frac{1}{2}} - 2c^{\frac{1}{2}}\right] = \left[2\pi^{\frac{1}{2}} - 0\right] = 2\sqrt{\pi}$$

c) $$\int_0^{1} \frac{dt}{t - \sin t}$$

→ Here p.p is zero. (0)

By test of improper integral

$$f(t) = \frac{1}{t - \sin t} \qquad g(t) = \frac{1}{t}$$

$$t - \sin t < t$$

$$\therefore \frac{1}{t - \sin t} \geq \frac{1}{t}$$

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$f(t) \geq g(t)$

$$\lim_{c\to 0}\int_c^1 \frac{1}{t}\,dt = \lim_{c\to 0}\int_c^1 \frac{dt}{t}$$

$$= \lim_{c\to 0}\left[\log t\right]_c^1$$

$$\lim_{c\to 0}\left[\log(1) - \log(c)\right]$$

$$= \left[\log(1) - \log(0)\right]$$

$$= 0 - (-\infty)$$

$$= \infty \quad \text{— Diverges.}$$

$\because$ By D.C.T $f(t) \geq g(t)$

$\therefore$ $f(t)$ is also divergent.

d) $\int_0^2 \frac{dt}{1-t^2}$

→ Here problemetic point is 1

$$\therefore \int_0^1 \frac{dt}{1-t^2} - \int_1^2 \frac{dt}{1-t^2}$$

$$\therefore I_1 = \int_0^1 \frac{dt}{1-t^2}$$

$$f(t) = \frac{1}{1-t^2} \qquad g(t) = \frac{-1}{t^2}$$

$f(t) \leq g(t)$

$\int \frac{1}{1-t}\,dt$

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$$\lim_{c\to 0}\int_c^1 \frac{dt}{1-t^2} = \lim_{c\to 0}\int_c^1 \frac{dt}{(1-t)(1+t)}$$

$$= \lim_{c\to 0}\int_c^1 \frac{1}{(1-t)(1+t)}\,dt = \lim_{c\to 0}\int_c^1 \frac{1}{(1-t)}\,dt + \lim_{c\to 0}\int_c^1 \frac{1}{(1+t)}\,dt$$

$$= \lim_{c\to 0}\left[-\log(1-t)\right]_c^1 + \lim_{c\to 0}\left[\log(1+t)\right]_c^1$$

$$= \lim_{c\to 0}\left[-\log(1-1) + \log(1-c)\right] + \lim_{c\to 0}\left[\log(1+1) - \log(1+c)\right]$$

$$= \left[-\log(1-1) + \log(1-0)\right] + \left[\log(2) - \log(1+0)\right]$$

$$= -(-\infty) + 0 + \log(2) - 0$$

$$= \infty$$

$\therefore$ By D.C.T. $f(t)$ is divergent.

$\therefore$ $I_1$ is $\infty$, and any number or term is add in it, it remamin $\infty$,

$\therefore$ Answer is $\infty$.

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g) $\int_4^\infty \frac{dt}{\sqrt{t}-1}$

→ p.p is 1

$\lim_{c\to\infty} \int_4^c \frac{1}{\sqrt{t}-1} dt$

$f(t) = \frac{1}{\sqrt{t}-1}$ $g(t) = \frac{1}{\sqrt{t}}$

$\sqrt{t} - 1 \leq \sqrt{t}$

$\frac{1}{\sqrt{t}-1} \geq \frac{1}{\sqrt{t}}$

$\therefore f(t) \geq g(t)$

$\lim_{c\to\infty} \int_4^c g(t)\, dt = \lim_{c\to\infty} \int_4^c \frac{1}{\sqrt{t}} dt$

$= \lim_{t\to\infty} \left[ \lim_{c\to\infty} \int_4^c \frac{1}{t^{1/2}} dt. \right.$

By p - integral test.

$t^p = t^{\frac{1}{2}}$ $\therefore$ $\frac{1}{2} < p$

$\therefore$ this eqn diverges.

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h) $\int_2^\infty \frac{dt}{\sqrt{t-1}}$

→ $\lim_{c\to\infty} \int_2^c \frac{dt}{\sqrt{t-1}}$

$\therefore f(t) = \frac{1}{\sqrt{t-1}}$ $g(t) = \frac{1}{\sqrt{t}}$

$\lim_{t\to\infty} \frac{f(t)}{g(t)} = \lim_{t\to\infty} \frac{1}{\sqrt{t-1}} \times \frac{\sqrt{t}}{1}$

$= \lim_{t\to\infty} \frac{\sqrt{t}}{\sqrt{t-1}}$