**COEP Technogical University**
Department of Mathematics
(MA- 21001) Probability and Statistics for Engineers
T.Y. B. Tech. Semester V (Computer, E and TC, Electrical, Mechanical and Instrumentation Engineering )
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# Tutorial 2 on Unit 2

1. Define random variable, discrete and continuous random variables giving five examples of each.

2. Determine, with reasons, which of the following functions are probability distribution functions and if it is then say what type of distribution it represents:

   (i) $f(x) = 2(1+x)/27$ Hint/Ans: not a pdf since no interval is specified and the area under the curve is not one.

   (ii) $f(x) = \begin{cases} x, & 0 < x < 1, \\ 2-x, & 1 \le x < 2, \\ 0, & \text{elsewhere} \end{cases}$

   (iii)

   | $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|---|
   | $f(x)$ | 0.41 | 0.35 | 0.15 | 0.10 | 0.04 | 0.01 | 0 |

   (iv)

   | $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|
   | $f(x)$ | 0.35 | 0.33 | 0.18 | 0.10 | 0.03 | 0.01 |

3. By making suitable changes in the required functions above, make them into probability distribution functions. Is it the only way? Hint/Ans:(i) can be made into a pdf in infinitely many ways since we get only one eqn involving two unknown limits. For example 2 < x < 5 is one interval which makes it into a prob distn of a cts r.v. Hint/Ans:(iii) if we have to change only one of the probabilities then then there is a unique way but then we can not do this by using the last three probabilities.

4. Find the cumulative distribution functions for the above distribution functions.

5. Consider the following distribution function of a random variable $X$ :

$$F(x) = \begin{cases} 0 & \text{if } x < 2 \\ \dfrac{-x^2}{4} + 2x - 3 & \text{if } -2 \le x \le 4 \\ 1 & \text{if } x > 4. \end{cases}$$

   (i)What is the PDF of $X$?

   (ii) Calculate $P(X < 3)$ and $P(X = 4)$.

   (iii) Determine $E(X)$ and $\text{Var}(X)$.

6. An innovative winemaker experiments with new grapes and adds a new wine to his stock. The percentage sold by the end of the season depends upon weather and various other factors. It can be modelled using the random variable $X$ with the CDF as

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 3x^2 - 2x^3 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1. \end{cases}$$

   Plot the cumulative distribution function with R. Determine $f(x)$. What is the probability of selling at least one-third of his wine, but not more than two-thirds? Define CDF in R and calculate the probability of above question again. What is the variance of X?

7. A continuous random variable $X$ defined in the interval $[1, 10]$ has a constant density function. Find it. Hence find

   (i) $P(X \leq 4)$

   (ii) $P(2 < X \leq 7)$

   (iii) $P(X = 5)$

   (iv) $k$ if $P(k \leq X < 9) = 0.5$

8. Find the mean and variance of each random variable in the above examples.

9. If $X$ represents the number of imperfections per 10 m of a fabric in example 2 (iv) then how many imperfections do you expect in a 100m roll of fabric?

10. Prove that expectation is linear. What can you say about variance?

11. How will you explain for a layman that expectation of a constant is itself while its variance is zero?

12. Find the mean and standard deviation of $X$ if $E(2X - 1) = 9$ and $E(2X - 1)^2 = 89$.

13. Let $X_1, X_2, \ldots, X_n$ be $n$ random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2,\ i = 1, 2, \ldots, n$. Find $E(\overline{X})$ and Var($\overline{X}$). $\overline{X}$ : arithmetic mean of these $n$ random variables.

14. A fair coin is tossed 10 times. What is the probability of getting more heads than tails?

15. Prove that the binomial distribution tends to Poisson as $n \to \infty$ and $p \to 0$.

16. A medical store owner knows that, on an average, 40 people visit the shop per hour.

    (i) Find the probability that no one visits the store when he wants a rest room break of 3 minutes at noon.

    (ii) Find the probability that more than 3 people visit during this break.

17. The average number of defects in a 5 meter handloom yarn is estimated to be 12. Find the probability that fewer than 7 defects are found

    (i) in a given meter.

    (ii) on 7 of the next 10 meters inspected.

    (iii) on 60 of the next 75 meters inspected.

18. Define Normal r.v and find it's mean and variance.

19. Prove that if X is normal with mean $\mu$ and variance $\sigma^2$ then $\frac{X-\mu}{\sigma}$ is standard normal.

20. If $X$ is normal with mean 100 and standard deviation 5, find

    (i) $P(95 < X < 110)$

    (ii) $P(X < 50)$

    (iii) $k$ if $P(X > k) = 0.3192$

    (iv) $X_1, X_2$ if $P(X_1 < X < X_2) = 0.4176$ and $P(Z < z) = 0.7088$

21. A pair of dice is rolled 420 times. What is the probability that a total of 8 occurs at least 50 times? Between 70 and 90 times inclusive? Exactly 100 times?

22. The life in years of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. If 1000 of these switches are installed in different systems, what is the probability that atmost 300 fail during the first year?

23. If the standard deviation of the mean for the sampling distribution of a random sample of size 36 is 2 then should the sample size be increased or decreased for reducing the standard deviation by 0.8? By how much?

24. A random variable $X$ has the following p.m.f.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| p(X=x) | k | 3k | 5k | 7k | 9k | 11k | 13k |

Find (i) $k$ (ii) $p(X \geq 2)$ (iii) $p(0 < X < 5)$ (iv) What is the minimum value of $C$ for which $p(X \leq c) > 0.5$ ?
(Ans: (i) $k = \frac{1}{49}$ (ii) $\frac{45}{49}$ (iii) $\frac{15}{49}$ (iv) Minimum value of $C$ is $4$.)

25. Verify whether the assignment $P(X = n) = 2^{-n}$, $n = 1, 2, 3, \ldots$ is a probability mass function for random variable $X$. (Ans: It is a p.m.f.)

26. A box contains 8 items of which 2 are defective. A person draws 3 items from the box. Determine the expected number of defective items he has drawn. (Ans: 3/4)

27. A random variable has mean $2$ and standard deviation $0.5$. Find (i) $E(2X - 1)$ (ii) $Var(X + 2)$ (iii) $sd(\frac{3X-1}{-4})$. (Ans: (i) 3 (ii) $\frac{1}{4}$ (iii)$\frac{3}{8}$. )

28. The mean height of 500 students is 151 cm and the s.d. is 15cm. Assuming that heights are normally distributed, find how many students' height lie between 120 and 155. (Ans: 291)

29. Suppose that a pair of fair dice are to be tossed, and let the random variable $X$ denote the sum of the points. (a) Obtain probability distribution for $X$. (b) Find the distribution function $F(x)$ for the random variable $X$ and (c) Graph this distribution function.

30. A random variable $X$ has the density function $f(x) = \frac{c}{(x^2+1)}$, where $-\infty < x < \infty$. (a) Find the value of the constant $c$. (b) Find the probability that $X^2$ lies between $1/3$ and $1$.

31. Find the expectation of the sum of points in tossing a pair of fair dice. (Ans: 7)

32. Find the probability of getting a total of 7 at least once in three tosses of a pair of fair dice. (Ans: $\frac{91}{216}$)

33. If the probability that an individual will suffer a bad reaction from injection of a given serum is 0.001, determine the probability that out of 2000 individuals, (a) exactly 3, (b) more than 2, individuals will suffer a bad reaction. (Ans: (a) 0.180, (b) 0.323)

34. Classify the following random variables as discrete or continuous.

    (a) X: the number of automobile accidents per year in Virginia

    (b) Y: the length of time to play 18 holes of golf

    (c) M: the amount of milk produced yearly by a perticular cow

    (d) N: the number of eggs laid each month by one hen

    (e) P: the number of building permits issued each month in a certain city

    (f) Q: the weight of grain produced per acre.

35. Find the probability distribution for the number of jazz records when four records are selected at random from a collection consisting of five jazz records, two classical records and three polka records. Express your results by means of a formula. Also find the cumulative distribution of the random variable X representing the number of jazz records. Using $F(x)$, find $P(X = 1)$ and $P(0 < X \leq 2)$

36. Define: Binomial random variable, Binomial distribution.

37. The probability that a patient recovers from a certain disease is $0.4$. If 15 people are known to have contracted this disease, what is the probability that (i) at least 10 survive; (ii) from 3 to 8 surview, (iii) Exactly 5 survive.

38. The average number of oil tankers arriving each day at a certain city is 10. The facilities at a port can handle at most 15 tankers per day. What is the probability that on a given day tankers will have to be sent away?(Poisson's)

39. What is mean and Variance of Poisson's distribution?

40. A basketball player's batting average is 0.25. What is the probability that he gets exactly one hit in his next four times at bat?

41. A study examined national attitudes about antidepressants. Study says that approximately $70$ percent believe "antidepressants do not really cure anything. They just cover up the real trouble." According to this study what is the probability that at least 3 of next 5 people selected at random will be of this openion?

42. A secretary makes 2 errors per page on an average. What is the probability that on the next page he/she will make (a) 4 or more errors; (b) no errors? (Ans: (a) 0.1429, (b) 0.1353)

43. Given standard normal distribution, find the area under the curve (a) that lies to the right of $z = 1.84$ (b) between $z = -1.97$ and $z = 0.86$ (Ans:0.0329, 0.7807)

44. A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and they enriched with vitamins and proteins. Assuming that the lifetime of such mice are normally disributed with a standard deviation of 6.3 months. Find the probability that the given mice will live (a) more than 32 months (b) less than 28 months (c) more than 37 months and less than 49 months. (Ans: 0.8962; 0.0287; 0.608)

45. A lawyer commutes daily from his suburban home to his midtown office. The average time for a one way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assuming the distribution of trip times to be normally distributed.(a) What is the probability that a trip will take at least 1/2 hour?(b) If the office opens at 9 am and he leaves his house at 8.45 am daily, What percentage of time is he late for work?(c) If he leaves house at 8.35 am and coffee is served at office from 8.50 am until 9.00 am, what is the probability that he misses coffee? (d) Find the length of time above which we find the slowest 15 percent of the trips? (e) Find the probability that 2 of the next 3 trips will take at least 1/2 hour. (Ans: 0.0571, 99.11 percent, 0.3974, 27.952, 0.009222)

46. Define: Exponential distribution.

47. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probabililty that a person is served in less than 3 minutes on at least 4 of next 6 days? (Ans: 0.3437)

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