Page No. ______
Date: / /

**Q.1**

1) $f(x) = \dfrac{-1}{(x+3)}$ $\quad -2 \le x \le 3$

$f'(x) = +(x+3)^{-1-1}$

$= (x+3)^{-2}$

$= \dfrac{1}{(x+3)^2}$ $\quad -2 \le x \le 3$

$f'(x) \ne 0$

$f'(x)$ is defined $\forall\, x \in [-2, 3]$.

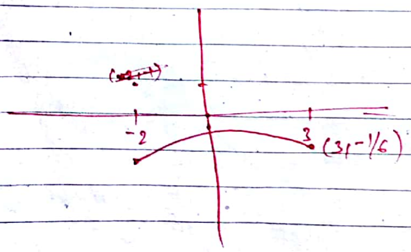
$\therefore$ Extream Value occure at $(-2, 3)$

$\therefore f(-2) = \dfrac{-1}{(-2+3)} = -1$

$f(3) = \dfrac{-1}{(3+3)} = -1/6$

$\therefore$ Absolute max value occure at $x = \cancel{-1/6}\ 3$ and Value is $-1/6$

Absolute min. value is $-1$ at $x = -2$




$\frac{1}{3} - 1$

$\frac{1-3}{3}$ $\quad -2/3$

Page No. ______
Date: / /

2) $f(x) = x^{1/3}$ $\quad -1 \le x \le 8$

$f'(x) = \frac{1}{3} x^{1/3 - 1}$ $\quad (-1 \le x \le 8$

$f'(x) = \dfrac{1}{3x^{2/3}}$ $\quad [-1, 8]$

$\therefore$ It is not defined at $x = 0$

$\therefore$ Critical point is $0$

$\therefore$ Extream Value occure at $0, -1, 8$

$\therefore f(0) = 0$

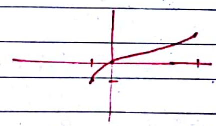
$f(-1) = (-1)^{1/3}$

$= -1$

$f(8) = (8)^{1/3} = 2$

abs. max value is 2 at $x = 8$

abs. min value is $-1$ at $x = -1$



$\sqrt{x} = \frac{1}{2}x^{1/2-1}$ $\quad \frac{1}{2}x^{-1/2}$ $\quad \frac{1}{2\sqrt{2}}$ $\quad \frac{2}{3}-1$ $\quad \frac{2-3}{3} = -\frac{1}{3}$



(e) $f(x) = -3x^{2/3} \quad -1 \le x \le 1$

$f'(x) = -\cancel{3} \times \frac{2}{\cancel{3}} x^{2/3-1} \quad -1 \le x \le 1$

$= -2x^{-1/3}$

$f'(x) = \frac{-2}{x^{1/3}} \quad -1 \le x \le 1$

$\therefore$ It is nto defined at $x = 0$

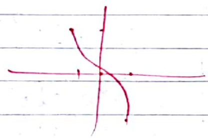
$\therefore$ 0 is C.P.

$\therefore$ Extream Value occure at $x = 0, -1, 1$

$\therefore f(0) = 0$

$f(-1) = -3(-1)^{2/3}$

$= -3$



$f(1) = -3 \times (1)^{2/3}$

$= -3$

Abs. Max 3 at $x = 0$

Abs. min $= -3$ $\quad x = -3$

e) $f(x) = \sqrt{4-x^2} \quad -2 \le x \le 1$

$\therefore f'(x) = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$

$= \frac{-\cancel{2}x}{\cancel{2}\sqrt{4-x^2}}$

$f'(x) = \frac{-x}{\sqrt{4-x^2}}$



$\therefore f'(x) = 0$

$\therefore 0 = \frac{-x}{\sqrt{4-x^2}}$

$\boxed{x = 0}$

$\therefore f^n$ is not defined at $x = -2, 2$ by it is not in the range

$\therefore$ C.P. is 0

$\therefore$ Abs. value occure at $x = 0, -2, 1$

$\therefore f(0) = \sqrt{4-0} = 2$

$f(-2) = \sqrt{4-(-2)^2} = 0$

$f(1) = \sqrt{4-1^2} = \sqrt{3}$

Page No. ____ Date: / /

(e) $f(\theta) = \sin\theta \quad -\pi/2 \le \theta \le 5\pi/6$

$\therefore f'(\theta) = -\cos\theta$

$\therefore f'(\theta)$ is 0 at $\pi/2$
But $\pi/2$ is not in Range

$\therefore$ No. cp.

$\therefore$ Extream Values are

$\therefore f(-\pi/2) = \sin(-\pi/2) = -1$

$f(5\pi/6) = \sin(5\pi/6) = 1/2$

ab max $= 1/2$ at $x\,\theta$
abs min $= -1$

(f) $f(x) = 3x^4 - 16x^3 + 18x^2 \quad -1 \le x \le 4$

$f'(x) = 12x^3 - 48x^2 + 36x$

$0 = 12x^3 - 48x^2 + 36x$
$0 = 12x(x^2 - 4x + 3)$

$12x = 0 \quad x = 0$
$x^2 - 4x + 3 = 0$
$\therefore x = 3 \quad x = 1$
$\therefore$ cp $= 0, 3, 1$

$+3$
$-3 \; -1$

$\therefore f(x) = 0$
$f(3) = 3\times3^4 - 16\times3^3 + 18\times3^2$
$= -27$
$f(1) = 12 - 48 + 36 = \cancel{8}\;5$
$f(-1) = -12 + 48 - 36 = \cancel{8}\;37$
$f(4) = \cancel{68}20 \;\; 32$

Page No. ____ Date: / /

2) a) $f(x) = 2x^2 - 8x + 9$
Domain is $\mathbb{R}$ $(-\infty, \infty)$

$\therefore f'(x) = 4x - 8$

$\therefore 0 = 4x - 8$
$4x = 8$
$\boxed{x = 2}$
$\therefore$ 2 is only c.p.

$\therefore$ Extream Value is $f(2) = 2\times2^2 - 8\times2 + 9$

$= 1$

b) $\cancel{f(x)} = \cancel{\frac{1}{\sqrt{x} - x}}$

$f(x) = x^3 - 2x + 4$
Domain is $\mathbb{R}$.
$f'(x) = 3x^2 - 2$

$\therefore 0 = 3x^2 - 2$
$3x^2 = 2$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{2/3}$

$\therefore$ Extream Value is $f(\sqrt{2/3}) = (\sqrt{2/3})^3 - 2(\sqrt{2/3}) + 4$

$= \cancel{2.91} \; \cancel{3.03} \; 2.91$

$= \left(-\sqrt{2/3}\right)^3 - 2\left(-\sqrt{2/3}\right) + 4$

$= \cancel{3.03} \; \cancel{6.29} \; 5.09$

Scanned by CamScanner

$(-2,$  $\sqrt{x} = \frac{1}{2} x^{1/2 - 1}$
$= \frac{1}{2\sqrt{x}}$

Page No.____ Date: / /

c) $= f(x) = \sqrt{x^2 - 1}$

$\therefore$ Domain $= \{ x / x^2 - 1 \geq 0 \}$

$= \{ x / x^2 \geq 1 \}$

Domain is $(-\infty, -1] \cup \cancel{[-1, 1]} \cup [1, \infty]$

Domain $= (-\infty, -1] \cup [1, \infty]$

$\therefore f'(x) = \frac{1}{2\sqrt{x^2-1}} \times 2x$

$f'(x) = \frac{x}{\sqrt{x^2-1}}$

$0 = \frac{x}{\sqrt{x^2-1}} \rightarrow x \times (\sqrt{x^2-1})^{1/2} = 0$

$x = 0 \quad (\sqrt{x^2-1})^{1/2} = 0$

$\boxed{x = 0}$

0 is CP $\quad (x^2-1)^{1/2} = 0$

$\therefore \cancel{f(0) = \sqrt{0-1}}$

$f(0) = \sqrt{0-1}$

$= \sqrt{-1}$

$\therefore$ Absolute max does not exist

$(-2, 2)$
$(-\infty, -1] \cup [1, \infty]$
$-\frac{1}{2} x^{-1/2 - 1}$
$-\frac{1}{2} x^{-3/2}$
$= \frac{-1}{2x^{3/2}}$

Page No.____ Date: / /

Q. 2-d) $f(x) = \frac{1}{\sqrt{x^2-1}}$

Domain $= \{ x / x^2 - 1 \geq 0 \}$

$= \{ x / x^2 > 1 \}$

$= \{ x / x <> \pm 1 \}$

$\therefore$ Domain $(-\infty, -1) \cup (1, \infty)$

$\therefore f'(x) = \frac{d}{dx} \frac{1}{\sqrt{x^2-1}}$

$= \frac{-1}{\cancel{2}(x^2-1)^{3/2}} \times (\cancel{2}x)$

$f'(x) = \frac{-x}{(x^2-1)^{3/2}}$

$0 = \frac{-x}{(x^2-1)^{3/2}}$

$-x = 0 \quad \therefore x = 0$

$\therefore$ But 0 is not in domain

$\therefore$ ~~0 is the Critical point~~

no. CP

at ~~x = 1 and x = -1~~ fun is ~~not defined~~

$\therefore x = 0, -1, 1$

$f(0) = \frac{1}{\sqrt{0-1}} = \frac{1}{\sqrt{-1}}$

$\therefore$ abs. max is not possible

Scanned by CamScanner

$\therefore$ Abso

e) $f(x) = \dfrac{x}{(x^2+1)}$

$\therefore$ ~~Domain = $\{x / x^2+1 \neq 0\}$~~

$\therefore$ ~~Domain $\{x / x^2 \neq -1$~~

Domain = $\mathbb{R}$

$\therefore f'(x) = \dfrac{u}{v}$

$= \cancel{x x(x^2+1)} -$

$= \dfrac{(x^2+1)1 - (x)(2x)}{(x^2+1)^2}$

$f'(x) = \dfrac{x^2+1-2x^2}{(x^2+1)^2}$

$0 = \dfrac{1-x^2}{(x^2+1)^2}$

$1-x^2 = 0$

$\therefore x^2 = 1$

$\boxed{x = \pm 1}$

$\therefore$ Critical points are $x = +1, -1$

$\therefore \cancel{f'(+)} =$

$f(1) = \dfrac{1}{(1+1)} = \dfrac{1}{2}$

$f(-1) = \dfrac{-1}{((-1)^2+1)} = -1/2$

$2/3 - 1 \quad \dfrac{2-3}{3} \quad -1/3 \quad \dfrac{2}{3} + 1/3$

$\therefore$ abs. max = $1/2$ at $x = 1$

abs. min = $-1/2$ at $x = \cancel{+1/2}(-1)$

6) $f(x) = e^x$

$\therefore$ Domain $\mathbb{R}$

$f'(x) = e^x$.

$e^x = 0$

$\therefore$ no Extream Value occurs

Q.3

a) $f(x) = x^{2/3}(x+2)$

$\therefore$ Domain $\mathbb{R}$

$\therefore$

$f'(x) = \cancel{uv} \quad (u.v)' = uv' + vu'$

$= x^{2/3}(1) + (x+2)(\frac{2}{3}x^{-1/3})$

$\therefore f'(x) = x^{2/3} + \dfrac{2}{3}(x+2)\dfrac{1}{x^{1/3}}$

$\therefore f'(x) = 0$

$\therefore x^{2/3} + \dfrac{2(x+2)}{3x^{1/3}}$

$0 = \dfrac{3x + \cancel{2x} + 4}{3x^{1/3}}$

$0 = \cancel{5}x + 4$

$\cancel{5}x = -4$

$x = -4/\cancel{5}$

$\frac{1}{x^3 \sqrt[3]{x}}$ $\frac{1}{x^{3/2}}$

Page No.____ Date: / /

$f'(x) = \frac{1}{0}$ at $x=0$.

$\therefore$ Critical points are ~~-5/~~ $-4/5$, $0$

$\therefore$ ~~f~~ Extream Value occures at $x = -4/5$

$\therefore f(-4/5) = (-4/5)^{2/3}((-4/5)+2)$

$f(-4/5) = 1.03$

$\therefore f(0) = 0$

$\therefore$ Abs. max $= 1.03$ at $-4/5$
and min $= 0$

(b) $f(x) = x^2\sqrt{3-x}$

$\therefore$ Domain $= \{x \mid 3-x \geq 0\}$

$\therefore = \{x \mid x \leq 3\}$

$\therefore$ Domain $= (-\infty, 3]$

$\therefore f'(x) = u \cdot v$

$= x^2\left(\frac{1}{2\sqrt{3-x}}\right) \times (-1) + \sqrt{3-x}\,(2x)$

$= -\frac{x^2}{2(\sqrt{3-x})} + 2x(\sqrt{3-x})$

$= \frac{-x^2 + \cancel{2}\,4x(3-x)}{2\sqrt{3-x}}$

$= \left(\frac{-x^2+12x-\cancel{8}\,4x^2}{2}\right) \times (3-x)^{-1/2} = 0$

3/ $x^{1/2} \times x^{3/2}$

Page No.____ Date: / /

$(3-x)^{1/2} = 0$

$f'(x) = 0$

$(3-x)^{-1/2} = 0$

$0 = -x^2 + 12x - 4x^2$

$= 12x - 5x^2$

$0 = x(12-5x)$

$\therefore x = 0$ $\quad 12-5x=0$

$5x = 12$

$x = \frac{12}{5}$

$\therefore$ ~~at~~ Critical points are

$0, \frac{12}{5}$

$\therefore f(0) = 0$

$f(12/5) = (12/5)^2 \times \sqrt{3-(12/5)}$

$= 4.46$

$f(3) = 3^2 \times \sqrt{3-3}$

$= 0$

$\therefore$ Abs. max $= 4.46$

min $= 0$

Scanned by CamScanner

$f'(x) = \frac{u}{v} \quad \frac{vu' - uv'}{v^2}$

c) $= f(x) = x|x| - x$

$\frac{1}{\sqrt{x^2-1}} \quad \frac{1}{(x^2-1)^{\frac{1}{2}}} \rightarrow = -\frac{1}{2}(x^2-1)^{-\frac{1}{2}-1} \times 2x$

$\frac{x}{\sqrt{x^2-1}} \quad \frac{\sqrt{x^2-1}}{x}$

$= -\frac{1}{2}(x^2-1)^{-3/2}(2x)$

$= -\frac{(\cancel{2}x)}{\cancel{2}(x^2-1)^{3/2}}$

$= \frac{-x}{(x^2-1)^{3/2}}$

$$\begin{cases} x^2 - x & x \geq 0 \\ -x^2 - x & x < 0 \end{cases}$$

$\therefore f'(x) \begin{cases} 2x - 1 & x \geq 0 \\ -2x - 1 & x < 0 \end{cases}$

$\therefore f'(x) = 0$

$$\begin{cases} 2x = 1, \quad x = 1/2 & x \geq 0 \\ -2x = -1, \quad x = -1/2 & x < 0 \end{cases}$$

Critical points are $-1/2$, $1/2$

$\therefore f(-1/2) = -1/2|-1/2| - (-1/2)$

$-1/2 \times (1/2) - 1/2$

$= -0.25 + 1/2$

$= 0.25$

$\therefore 1/2(1/2) - 1/2 \quad = -0.25$

$0.25 - 1/2$

$\therefore$ Abs max is 0.25 at $-1/2$

min is $-0.25$ at $1/2$

f) $f(x) = \begin{cases} 3 - x & x > 0 \\ 3 + 2x - x^2 & x \leq 0 \end{cases}$

$f'(x) = \begin{cases} -1 & x > 0 \\ 2 - 2x & x \leq 0 \end{cases}$

$\therefore 0 = 2 - 2x$

$2x = 2$

$\boxed{x = 1}$

$\boxed{C.P = 1}$

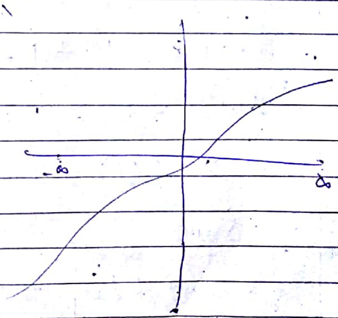
$\therefore 3 + 2\times1 - (1)^2 = 4$

Abs max $= 4$.

Q.4

$x^3 + x - 1$

$3x^2 + 1$



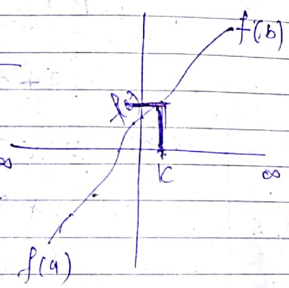
Page No. Date: / /

1) $= (x^3 + x - 1)$ has exactly one real root.
$f^n$ is continuous
$\therefore$ When $x \to (-\infty)$, $f(x) \to (-\infty)$
$x \to (\infty)$, $f(x) \to \infty$

So $\because$ $f^n$ is continuous
$f^n$ will cut the $x$ axis.
$\therefore$ $\exists$ a root

Intermediate Value theorem

$f^n$ is continuous on $[a, b]$
& $k$ is a number s.t
$\exists$ $f(c) = k$




$\therefore$ $x^3 + x - 1 = 0$
$[-1, 1]$
$f(-1) = (-1)^3 + (-1) - 1$
$= -1 - 1 - 1$
$= -3$

$f(1) = 1^3 + 1 - 1 = 1$
$f(-1) < 0 < f(1)$
$\therefore$ $f(c) = 0$ $c \in [a, b]$
$\therefore$ Suppo
Rolle
c and d are $f(c)$ the two roots
$f(d) = 0$
$f(c) = 0$
By LMVT $\frac{f(d) - f(c)}{d - c} = f'(x)$

f'(

$\frac{0 - 0}{x} = .$
$0 = f'(x)$

$f'(x) = 3x^2 + 1$

$\therefore f'(x) \neq 0$
our supposition was wrong
$\therefore$ there is only one real root.

Q.5 $f(x) = \tan x$ $[0, \pi]$
$f'(x) = \sec^2 x$
$\therefore 0 = \sec^2 x$
$0 = \frac{1}{\cos^2 x}$
$1 = 0$
But $1 \neq 0$
$\therefore$ 0 is not critical point
$\therefore$ $f'(x) = \frac{1}{\cos^2 x}$
at when $\cos^2 x = 0$ $f^n$ is N.D
$\therefore$ at $x = \pi/2$ $f^n$ is not defined.
$\therefore$ Extream Value occur at
$\pi/2$, 0, $\pi$
$\tan \pi/2 = \infty$
$\tan(0) = 0$ $\tan \pi = 0$

Q.6

$f(x) = 2 + (x-5)^3$

$f'(x) = 3(x-5)^2 \times$

$\therefore 0 = 3(x-5)^2$

$3 \neq 0 \quad \therefore x^2 - 10x + 25 = 0$

25

-5 -5

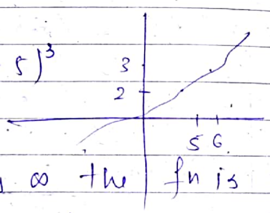
$\boxed{x = 5}$

$\therefore$ 5 is a C.P

$f(5) = 2 + (5-5)^3$

$f(5) = 2$

$f(6) = 2 + (6-5)^3$





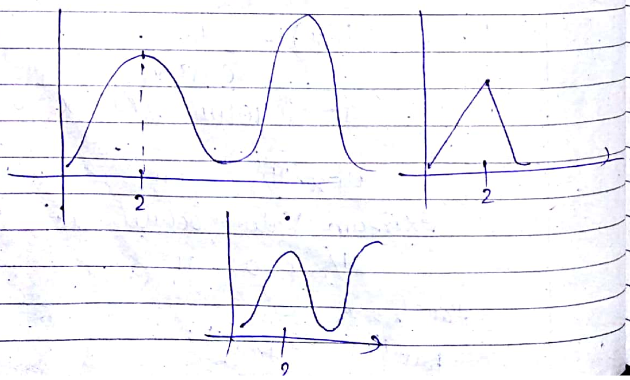
$= 2+1$

$= 3$

$\therefore f'(x) = 3(x-5)^2$ is increasing

$\therefore f'(x)$ As we go towords $\infty$ the fn is increasing $\therefore$ it does not have local Extream Value at 5

Q.7





Q.8

$f'(0) = 1$

$f'(1) = 1$

Suppose fn Satisfy all the condition of LMVT

Applying LMVT $f'$ at $[0, 1]$

$\frac{f(b) - f(a)}{b - a} = f'(c)$

$\therefore \frac{f(1) - f(0)}{1 - 0} = f'(c)$

$\frac{f'(1) - f'(0)}{1 - 0} = f''(c)$

$\frac{1 - 1}{1 - 0} = f''(c)$

$f''(c) = 0$

$\therefore$ But it is given that

$f''(x) > 0 \;\; \forall x$

$\therefore$ Our Supposition is wrong

Such fn never Exist

Scanned by CamScanner

$3x+1$

$[-a, +a]$

Page No.
Date: / /

Q.9

As $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$

$\therefore$ $\exists$ a $f^n$ must cut $x$ axis

$\therefore$ $\exists$ a root of $f^n$.

$\therefore$ Let $[-a, b]$

$\therefore$ $f(a) = -a$

$f(b) = b$

$\therefore$ $\exists$ c s.t. $f(c) = 0$, $\in c\,[a, b]$

Suppose c and d are the two real root.

$\therefore$ $f(c) = 0$

$f(D) = 0$

By LMVT on $[c, d]$

$$\frac{f(d) - f(c)}{d - c} = f'(x)$$

$$\frac{0 - 0}{d - c} = f'(x)$$

$f(x) = 0$

$0 = f'(x)$

But it is given that $f' \neq 0$

$\therefore$ Our Supposition was wrong.

Page No.
Date: / /

$\therefore$ $f^n$ has exactly one real root.

Q.10.

$|\sin a - \sin b| \leq |a - b| \;\forall a, b \in \mathbb{R}$

$\therefore$ Let $f(x) = \sin x$

$\therefore$ $f(a) = \sin a$

$f(b) = \sin b$

Applying LMVT on $[a, b]$

$$\therefore \frac{f(b) - f(a)}{b - a} = f'(c) \;\exists\, c \in (a, b)$$

$$\frac{\sin b - \sin a}{b - a} = \cos x$$

$$\frac{\sin b - \sin a}{b - a} = \cos x \leq 1.$$

$$\frac{\sin b - \sin a}{b - a} \leq 1.$$

$\sin b - \sin a \leq b - a$

$\therefore$ $\sin a - \sin b \leq a - b$

Taking Mod on b.s

$|\sin a - \sin b| \leq |b - b|$

hence proved.

Page No. ____ Date: / /

Q.11 $f(0) = -3$

$f'(x) = -5$

find $f(2)$

Applying LMVT on $[0,2]$

$\therefore \frac{f(b)-f(a)}{b-a} = f'(x)$

$\frac{f(2)-f(0)}{2-0} = f'(x)$

$\frac{f(2)-(-3)}{2} = -5$

$\frac{f(2)+3}{2} = -5$

$f(2)+3 = -10$

$\therefore f(2) = -10-3$

$f(2) = -13$

$\therefore$ Max value of $f'(2)$ is $-13$

Q.12 Let f & g are the two runners

Let $f(t)$ be the distance covered by runner A at time t

& $g(t)$ be the distance covered by the runner B at time t

$\therefore [0, t_0]$

let 0 be the starting point

$t_0$ be the finishing point

Page No. ____ Date: / /

$\therefore f(t_0) = g(t_0)$

$f(t) = g(t) = h(t)$

Let $h(t) = f(t) - g(t)$ —— (1)

$\therefore h(0) = 0$

$h(t_0) = \cancel{t_0}\ 0$

By Rolles Theorem on $[0, t_0]$

$c \in (0, t_0)$ s.t

$h'(c) = 0$

$f'(c) - g'(c) = 0$ from (1)

$\therefore f'(c) = g'(c)$

$\therefore$ c is the point where runner A and B have same velocity

Q.13 $a>0$ f $[-a,a]$

$f'(x) \leq 1 \ \forall x \in (-a,a)$ $f(a) = a$

$f(-a) = -a$

Show that $f(0) = 0$

applying LMVT on $(-a,a)$

$\therefore$ ~~f(~~ $f(-a) < 0 < f(a)$

$\therefore \exists$ ~~f(c)~~ 0 in bet $[-a,a]$

~~applying~~

$\frac{f(a)-f(-a)}{a-(-a)} = f'(x)$

$\frac{a-(-a)}{a+a}$ ~~f'(x)~~ $\leq 1$

Page No. Date: / /

$a + a \leq a$

Q.14 $\frac{b-a}{\sqrt{1-a^2}} \leq \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$

$0 < a < 1$ on $0 \leq 0 \leq 1$

Let $f(x) = \sin^{-1}(x)$ $\forall$ $x \in [a, b]$

$\therefore$ $f(a) = \sin^{-1}(a)$

$f(b) = \sin^{-1}(b)$

Applying IMVT on $[a, b]$

$\therefore$ $\frac{f(b) - f(a)}{b-a} = f'(c)$ $\quad c \in (a, b)$

$\frac{\sin^{-1}(b) - \sin^{-1}(a)}{b-a} = \frac{1}{\sqrt{1-c^2}}$

$a < c < b$

$a^2 < c^2 < b^2$

$-a^2 > -c^2 - b^2$

$1 - a^2 > 1 - c^2 > 1 - b^2$

$\frac{1}{1-a^2}$

$\sqrt{1-a^2} > \sqrt{1-c^2} > \sqrt{1-b^2}$

$\frac{1}{\sqrt{1-a^2}} < \frac{1}{\sqrt{1-c^2}} < \frac{1}{\sqrt{1-b^2}}$

$\frac{1}{\sqrt{1-a^2}} < \frac{\sin^{-1}(b) - \sin^{-1}a}{b-a} < \frac{1}{\sqrt{1-b^2}}$

Multiply by $b-a$

$\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$

Page No. Date: / /

Q.13 Applying IMVT on $[0, a]$

$\therefore$ $\frac{f(a) - f(0)}{a - 0} = f'(x)$

$\frac{a - f(0)}{a - 0} = f' \leq 1$

$a - f(0) \leq a$

$\therefore$ $a - a \leq f(0)$

$0 \leq f(0)$

$\therefore$ $f(0) \geq 0$

$\boxed{f(0) = 0}$

Page No. ___ Date: / /

Q. b $\tan x > x$ for $0 < x < \pi/2$

Let $f(t) = \tan t$ $[0, x]$

Applying LMVT on $[0, x]$

$\therefore \frac{f(x) - f(0)}{x - 0} = f'(c)$ $c \in [a, x]$

$\frac{\tan(x) - \tan(0)}{x - 0} = \sec^2 x$

$\frac{\tan x - \cancel{\tan 0}}{x} = \sec^2 x$

$\sec^2 x = \frac{1}{\cos^2 x}$

$\cos x \leq 1$

$\frac{1}{\cos x} \geq 1$

$\therefore \sec^2 x \geq 1$

$\frac{\tan x}{x} \geq 1$

$\tan > x$ (hence proved.

c) $\frac{x}{1+x} < \log(1+x) < x$ ; $x > 0$

let $f(t) = \log(t)$ at $\forall x \in \cancel{[0, x]}$

Applying LMVT on $[1, 1+x]$ $[1, 1+x]$

Page No. ___ Date: / /

$\therefore \frac{f(b) - f(a)}{b - a} = \cancel{f'(c) \ \forall\ c \in (a, 1, 1+x)}$

$\frac{\log(1+x) - \log(1)}{(1+x) - 1} = \frac{1}{c}$

$\frac{\log(1+x) - 0}{x} = \frac{1}{c}$

$\log(1+x)$

$1 < c < 1+x$

$\frac{1}{1} > \frac{1}{c} > \frac{1}{1+x}$

$1 > \frac{\log(1+x)}{x} > \frac{1}{1+x}$

$x > \log(1+x) > \frac{1}{1+x}$

$\cancel{x < \log(1+x)}$

$\frac{1}{(1+x)} < \log(1+x) < x$

Proved.

Page No.

Date: / /

Q.15 $f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$

$\therefore 0 = 3x^2 - 3$

$0 = 3(x^2 - 1)$

$0 = x^2 - 1$

$x^2 = 1$

$\boxed{x = \pm 1}$

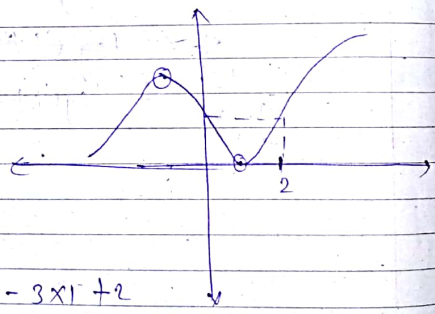
| Interval | $(-\infty, -1)$ | $(-1, 1)$ | $(1, \infty)$ |
|---|---|---|---|
| Sign of $f'(x)$ | + | − | + |
| I/D | ↑ increasing | decreasing | Increasing |

$f''(x) = 6x$

$0 = 6x$

$x = 0$

limit




$f(1) = 1 - 3\times1 + 2$

$= 0$

$f(-1) = -1 - 3(-1) + 2$

$= -1 + 3 + 2$

$= 4$

Page No.

Date: / /

$\therefore$ at poin $c = 2$ fn is Increasing.