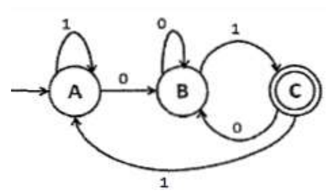
a. Construct DFA

1. $\Sigma=\{0,1\}$and strings that have an odd number of 1's and any number of 0's.
2. for $\Sigma=\{a, b, c\}$that accepts any string with aab as a substring
3. $\Sigma=\{x, y\}$, where if a substring yy is present, then it has to be followed by an x.
4. {0,1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not.
5. Over {a, b}, all strings with atleast one a.
6. Over {a, b}, strings except those ends with abb
7. Over {a, b}, all strings with b as a second letter.
8. Over {0,1} all strings ending with 00
9. Over {0,1} detects even number of 0's
10. Over {a, b}, $L = \{w| n_a(w) > 1\}$, where $n_a(w)$ is the number of a's in w
11. Over {0,1} strings with atleast 2 0's and ending with atleast 2 1's.
12. L = {w denotes an odd binary number}
13. Over {a, b}, L = {awa}
14. $L=\{w_1aw_2 \,|w_1,w_2\in\{a, b\}*,|w_1| = 2, |w_2| \geq 2\}$
15. $L = \{w\in (0, 1)*|w$ contains at least two 0s, or exactly two1s$\}$

b. What is the language of below DFA?



c. Show that the string ababa is accepted for DFA of a.13

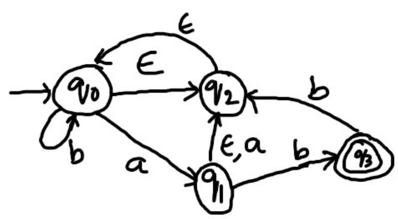
d. Check whether the language $L = \{a^{2n}b^{3m}c|n>=1, m>=0\}$ is regular.

e. Define NFA

1. Over {a, b}, L = (a + b)* b (a + b)
2. Over {a, b}, L = {w | w belongs to abab^n or aba^n
3. Over {a, b}, all strings ending with aba
4. Over {a, b}, all strings ending with ab or ba

f. Show that

1. strings abab is accepted for NFA of e.2
2. $\delta\hat{}(q2, aba)$ for

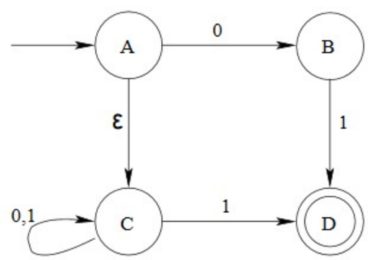


g. Convert to DFA

1. All NFA's of question e.

2.

Qqaq1aqq



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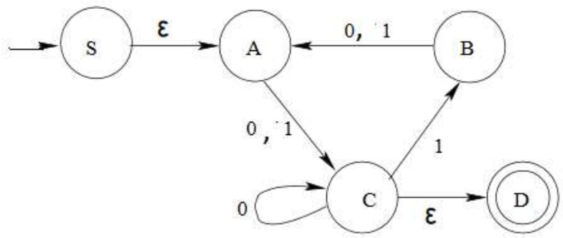

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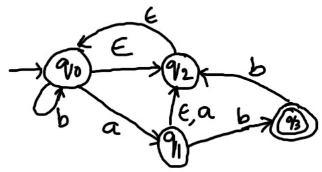


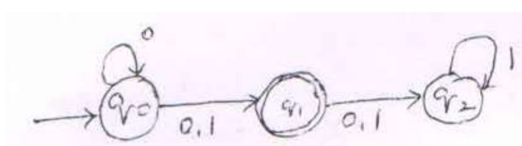
5.

δ_N	ϵ	a	b	c
$\rightarrow p$	$\{q, r\}$	$\{\phi\}$	$\{q\}$	$\{r\}$
q	ϕ	$\{p\}$	$\{r\}$	$\{p, q\}$
$* r$	ϕ	ϕ	ϕ	ϕ

6.

δ_N	0	1
$\rightarrow p$	$\{q, s\}$	$\{q\}$
$* q$	$\{r\}$	$\{q, r\}$
r	$\{s\}$	$\{p\}$
$* s$	ϕ	$\{p\}$

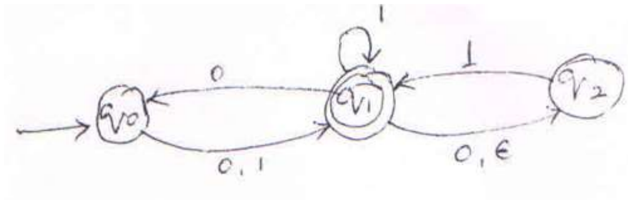
7.



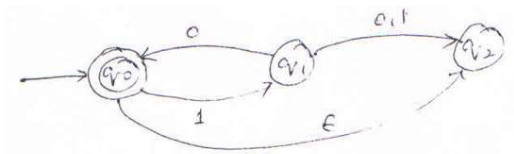
8.

δ_N	ϵ	a	b	c
$\rightarrow p$	$\{q, r\}$	ϕ	$\{q\}$	$\{r\}$
q	ϕ	$\{p\}$	$\{r\}$	$\{p, q\}$
$* r$	ϕ	ϕ	ϕ	ϕ

9.

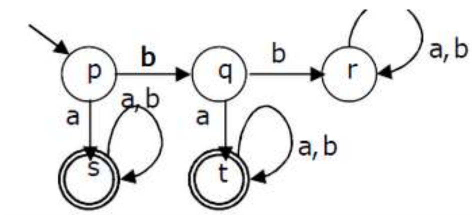


10.

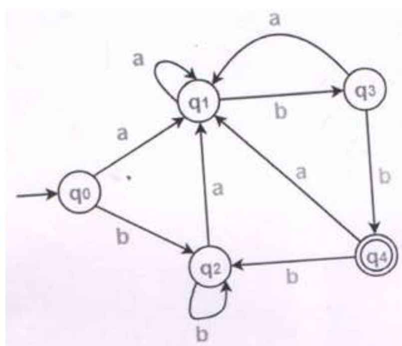


h. Minimize DFA

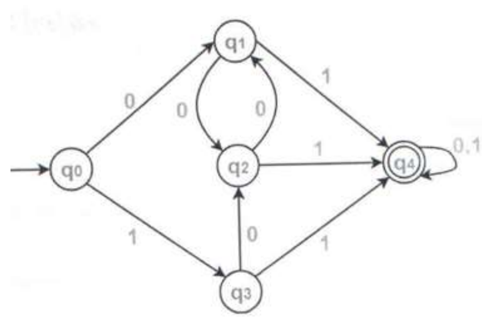
1.



2.

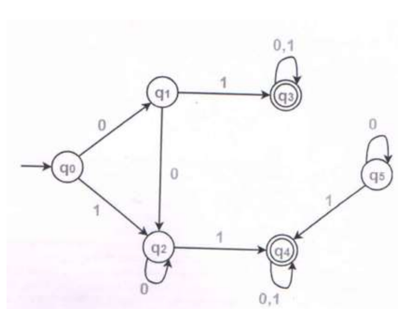


3.



\

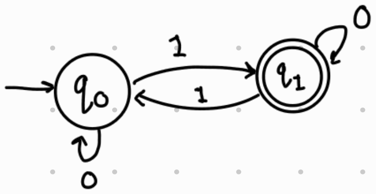
4.



TOC Question Bank 1.

a) Construct DFA

1) odd 1's, any 0's.

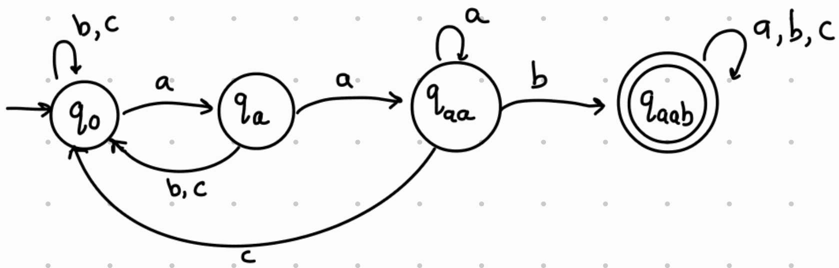


$\Sigma = \{0, 1\}$

$L = \{01, 001, 01110, 1, 111 \ldots\}$

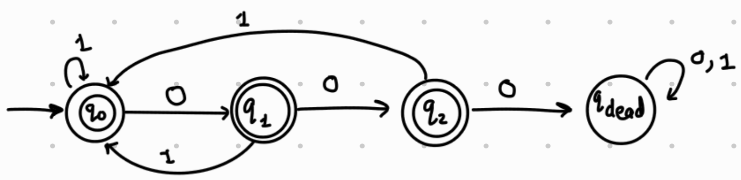
2) $\Sigma = \{a, b, c\}$

$L = \{(a+b+c)^* \, aab \, (a+b+c)^*\}$



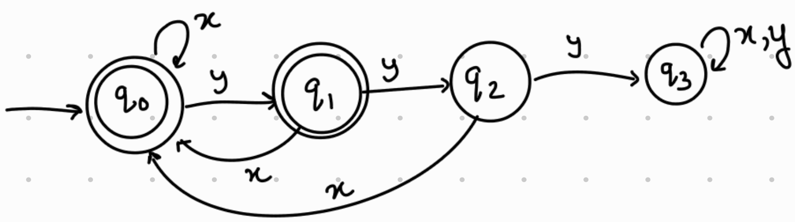
4) $\Sigma = \{0, 1\}$

$L = \{001, 010, 100, 011, 111 \ldots\}$



3) $\Sigma = \{x, y\}$

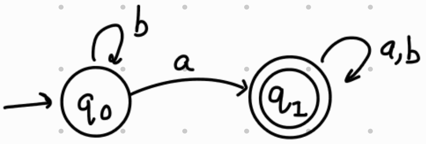
$L = \{\epsilon, x^*, y, xy, yx, yyx, xyyx, (x^*) yyx (x+y^*)\}$



	x	y
$\rightarrow^* q_0$	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

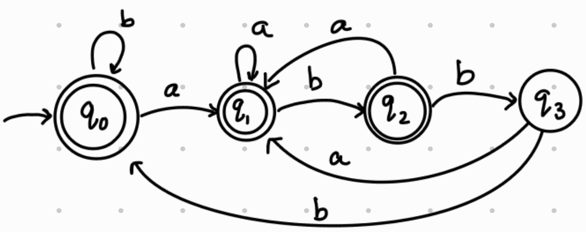
5) $\Sigma = \{a, b\}$

$L = \{a, a(a+b)^*, (a+b)^* a\}$



6) $\Sigma = \{a, b\}$

$L \neq \{(a+b)^* abb\}$

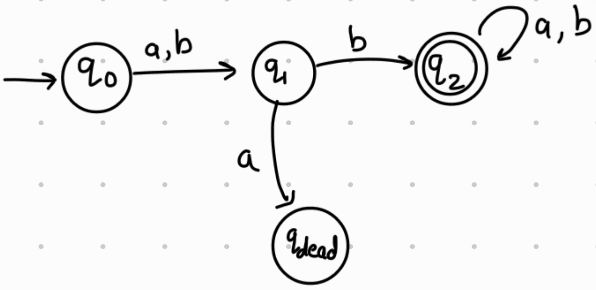


	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_0	q_0

7) $\Sigma = \{a, b\}$

$L = \{ab, bb, abab, abbb, bbab, bbbb \dots\}$

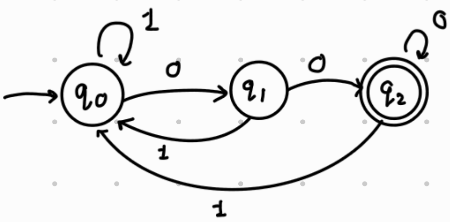
$L \neq \{a, b, aa, ba \dots\}$



8) $\Sigma = \{0, 1\}$

$L = \{(0+1)^* 00\}$

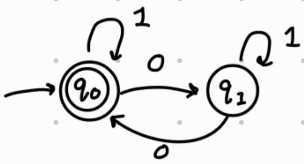
$L \neq \{0, 1, 10, 11 \dots\}$



9) $\Sigma = \{0,1\}$

$L = \{\epsilon, 100, 001, 0000 \ldots\}$

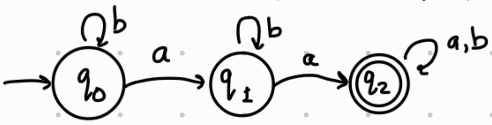
$L \neq \{101, 110, 0, 000 \ldots\}$



10) $\Sigma = \{a,b\}$

$L = \{w \mid n_a(w) > 1\}$ wher $n_a(w)$ is number of a's in w

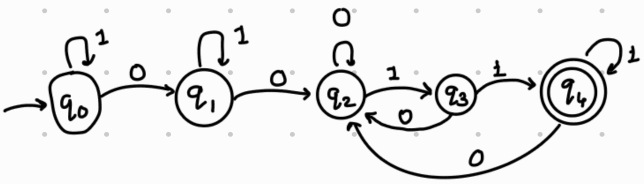
$L = \{aa, baa, aaa, aab, aba \ldots\}$



11) $\Sigma = \{0,1\}$

- at least 2 0's and ending with at least 2 1's

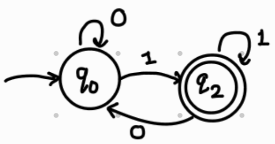
$L = \{0011\}$



12) $L = \{$an odd binary number$\}$

$\Sigma = \{0,1\}$

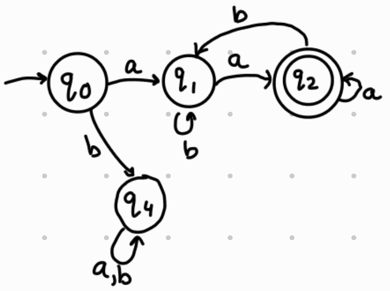
$L = \{1, 11, 101, 111, \ldots\} = \{(0+1)^* 1\}$



13) $\Sigma = \{a,b\}$

$L = \{awa \mid w = (a+b)^*\}$

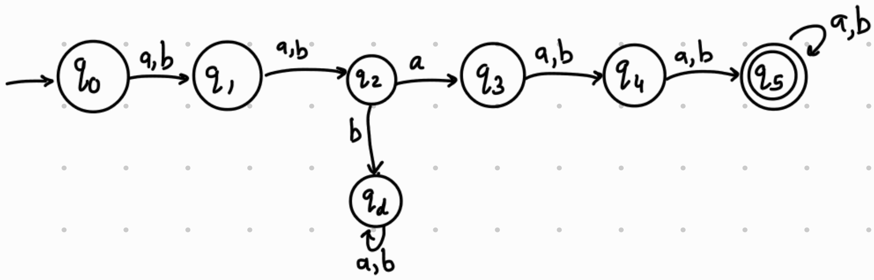
$L = \{aa, aba, abba \dots\}$



aaba

14) $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a+b\}^*, |w_1| = 2, |w_2| \geq 2$

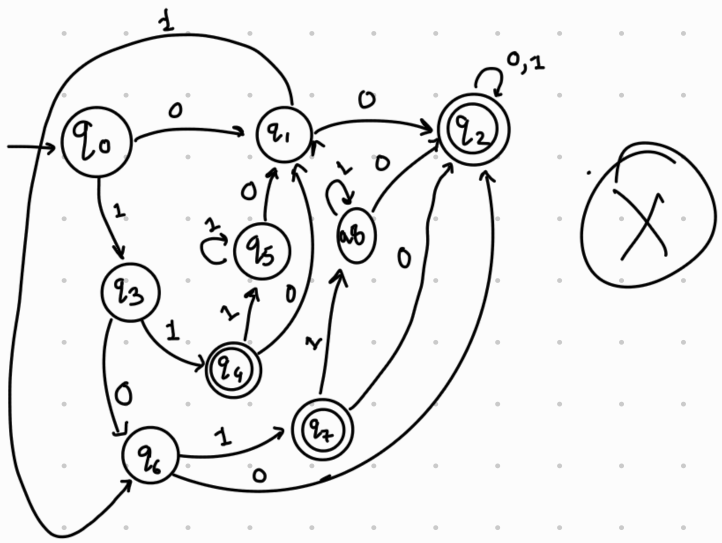
$\Sigma = \{a,b\}$



15) $\Sigma = \{0,1\}$

$L = \{w \mid w$ contains at least two zeroes or exactly to ones$\}$

$L = \{00, 001, 0011, 11 \dots\}$



/ /

b) Language:

$\Sigma = \{1, 0\}$

$L = \{01, 101, 1101, 1001101, \ldots\}$

$L = \{(0+1)^* 01\}$

c) $\delta(q_0, ababa) = \delta(\delta(\delta(\delta(\delta(\hat{\delta}(q_0, \epsilon), a), b), a), b), a)$

$= \delta(\delta(\delta(\delta(\delta(q_0, a), b), a), b), a)$

$= \delta(\delta(\delta(\delta(q_1, b), a), b), a)$

$= \delta(\delta(\delta(q_1, a), b), a)$

$= \delta(\delta(q_2, b), a)$

$= \delta(q_1, a)$

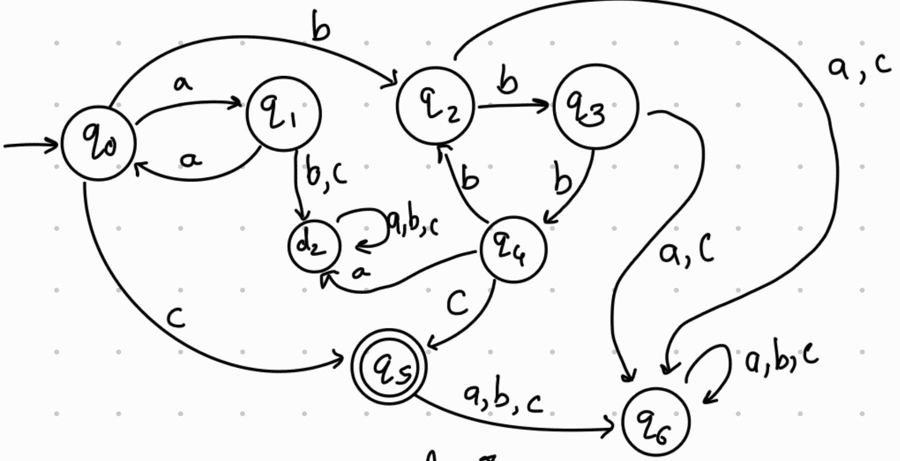
$= q_2$

$q_2 \subseteq F$ i.e. $q_2 \in F$

$\therefore$ ababa is accepted.

d) $L = \{a^{2n} b^{3m} c \mid n >= 1, m >= 0\}$ is regular.

$L = \{aac, aabbbc, aaaabbbc, aaaabbbbbbc \ldots\}$



M

Assume, L is regular,

$\therefore$ there exist some DFA, 'M', S.T. $M = \{Q, \Sigma, \delta, q_0, F\}$,

By pumping Lemma, $P = |Q|$ ... pigeon hole principle,

$S = aabbbc$, $x = aa$, $y = bbb$, $z = c$, $\therefore P = 6$,

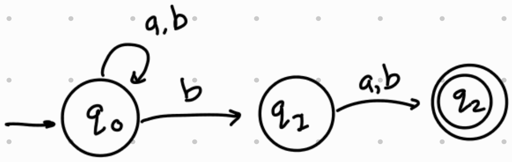
here,

① $|y| > 0$ ② $|xy| \leq P$ i.e. $5 \leq P$ ③ $xy^i z \in L$, → if we take any value of i, it will be a subset of L. $\therefore$ Regular language.

$i \geq 0$

e) 1) $\Sigma = \{a, b\}$

$L = (a+b)^* \, b \, (a+b)$



	a	b
$\rightarrow q_0$	q_0	q_0q_1
q_1	q_2	q_2
$*q_2$	–	–

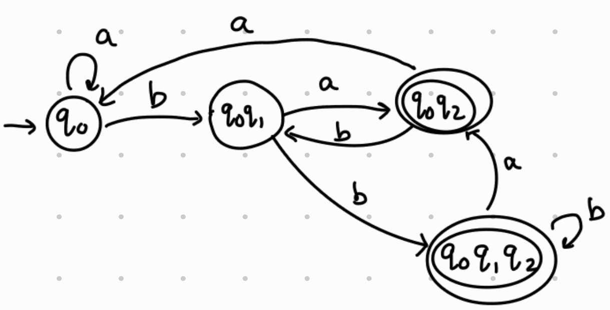
① Copy first row

	a	b
$\rightarrow q_0$	q_0	q_0q_1
q_0q_1	q_0q_2	$q_0q_1q_2$
q_0q_2	q_0	q_0q_1
$q_0q_1q_2$	q_0q_2	$q_0q_1q_2$

$\Downarrow$

DFA

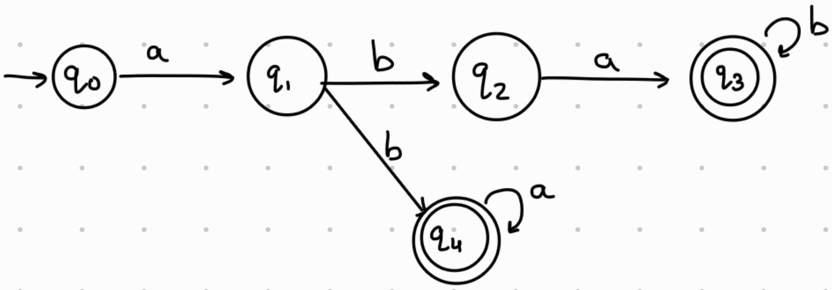
$\Downarrow$



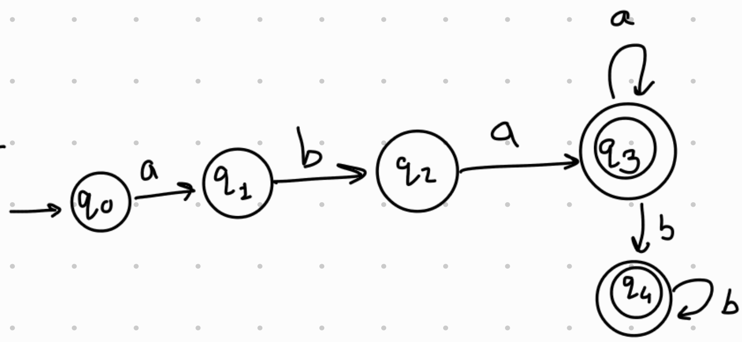
$F = \{q_0q_2, \; q_0q_1q_2\}$

2) $\Sigma = \{a, b\}$

$L = \{\omega \mid \omega \in abab^n \text{ OR } \omega \in aba^n\}$

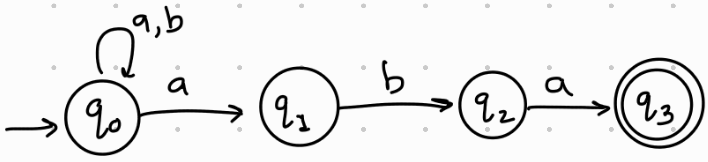


OR



③ $\Sigma = \{a, b\}$

$L = \{aba, aaba, baba \ldots (a+b)^* aba\}$

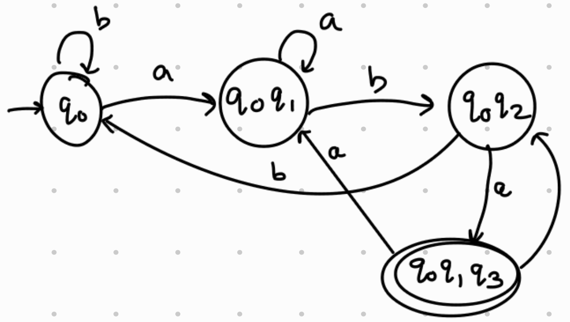


NFA

	a	b
→ q_0	q_0q_1	q_0
q_1	–	q_2
q_2	q_3	–
* q_3	–	–

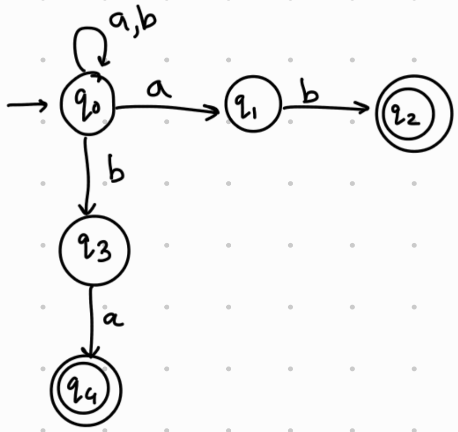
DFA

	a	b
→ q_0	q_0q_1	q_0
q_0q_1	q_0q_1	q_0q_2
q_0q_2	$q_0q_1q_3$	q_0
$q_0q_1q_3$	q_0q_1	q_0q_2



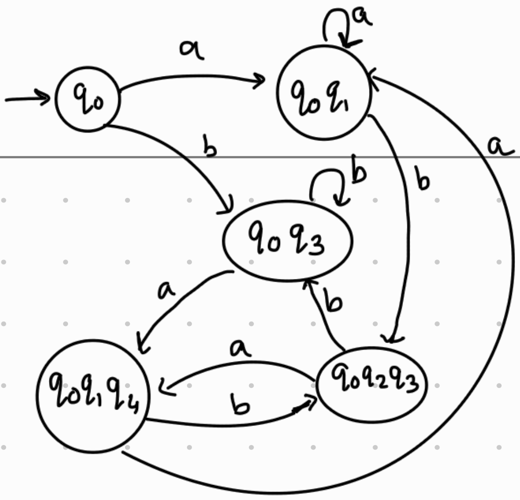
④ $\Sigma = \{a, b\}$

$L = \{ab, ba, (a+b)^* ab, (a+b)^* ba\}$



	a	b
→ q_0	q_0q_1	q_0q_3
q_1	–	q_2
* q_2	-	–
q_3	q_4	–
* q_4	–	-

DFA

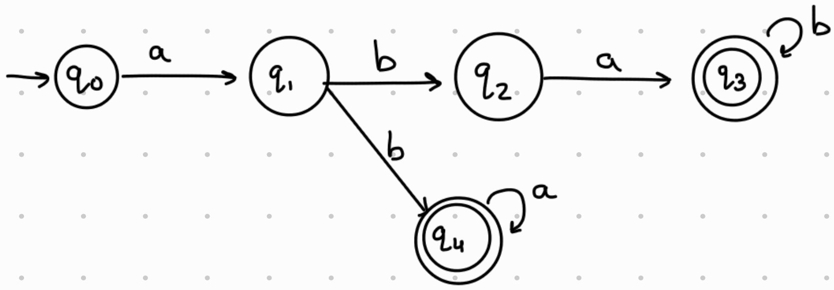


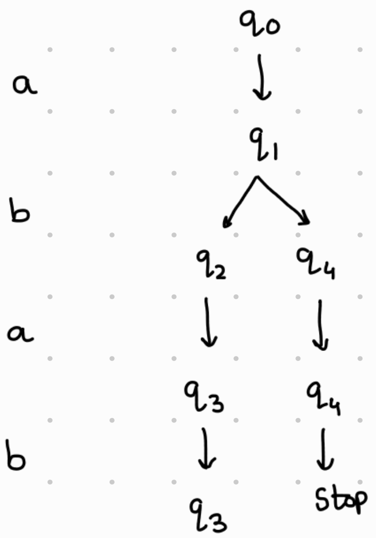
	a	b
$\to q_0$	q_0q_1	q_0q_3
q_0q_1	q_0q_1	$q_0q_2q_3$
q_0q_3	$q_0q_1q_4$	q_0q_3
$q_0q_2q_3$	$q_0q_1q_4$	q_0q_3
$q_0q_1q_4$	q_0q_1	$q_0q_2q_3$

DFA

f. Show That :

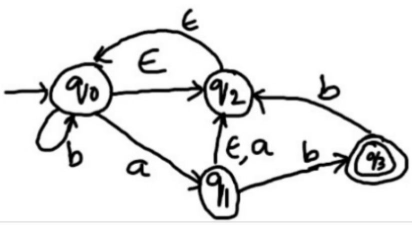
① abab is accepted in 'e.2' →





$q_3 \subseteq F$. $\therefore$ abab is accepted by NFA of e.2.

2. δ^(q2, aba) for



$\Rightarrow \hat{\delta}(q_2, aba)$

$= \delta(\hat{\delta}(q_2, ab), a)$

$= \delta(\delta(\hat{\delta}(q_2, a), b), a)$

$= \delta(\delta(\delta(\hat{\delta}(q_2, \epsilon), a), b), a)$

$= \delta(\delta(\delta(q_0), a), b), a)$

$= \delta(\delta(\delta(\hat{\delta}(q_0, \epsilon), a), b), a)$

$= \delta(\delta(\delta((q_0q_2), a), b) a)$

$= \delta(\delta(\hat{\delta}(q_1, \epsilon), b), a)$

$= \delta(\delta((q_1 q_2 q_0), b), a)$

$= \delta(\hat{\delta}(q_3 q_0), \epsilon), a)$

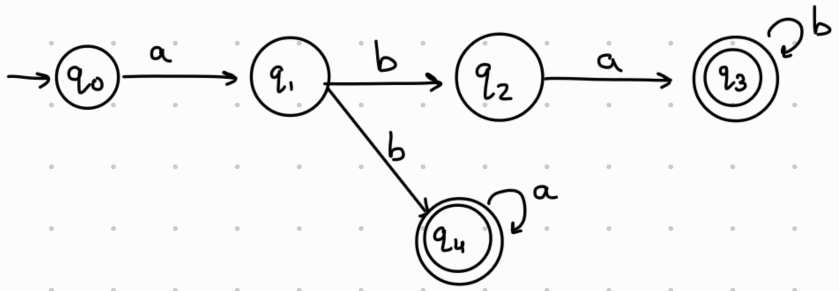
$= \delta(\{q_0 q_2\}, a)$

$= q_1$

$q_1 \notin F$

$\hat{\delta}(q_2, aba)$ is not accepted.

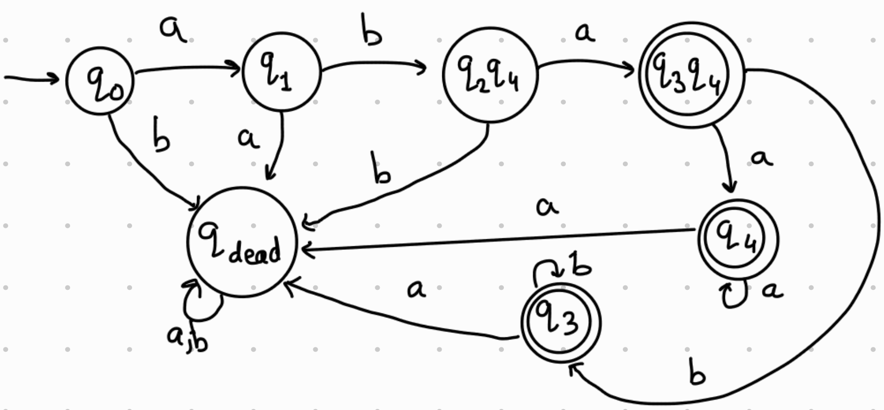
(g) Convert NFA to DFA



δ	a	b
$\rightarrow q_0$	q_1	–
q_1	–	$q_2 q_4$
q_2	q_3	–
$* q_3$	–	q_3
$* q_4$	q_4	–

$\Rightarrow$

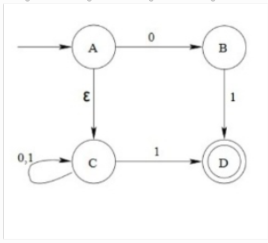
	a	b
$\rightarrow q_0$	q_1	–
q_1	–	$q_2 q_4$
$q_2 q_4$	$q_3 q_4$	–
$* q_3 q_4$	q_4	q_3
$* q_3$	–	q_3
$* q_4$	q_4	–



$abab^n$

aba^n

2.



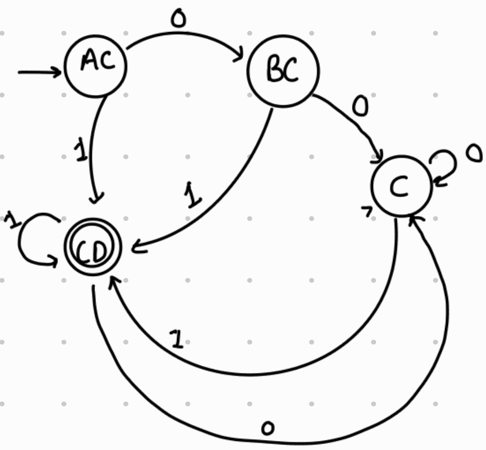
$\epsilon\text{-clo}(A) = \{A, C\}$

$\epsilon\text{-clo}(B) = \{B\}$

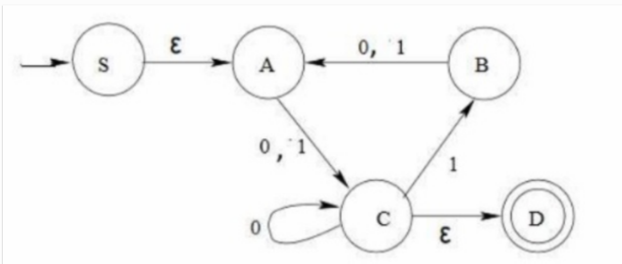
$\epsilon\text{-clo}(C) = \{C\}$

$\epsilon\text{-clo}(D) = \{D\}$

		0	1
→	[A,C]	[B,C]	[C,D]
	[B,C]	[C]	[C,D]
*	[C,D]	[C]	[C,D]
	[C]	[C]	[C,D]



3.



$\epsilon\text{-clo}(S) = \{S, A\}$

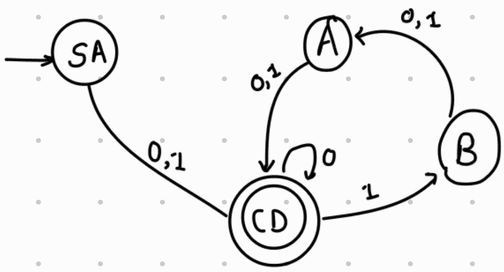
$\epsilon\text{-clo}(A) = \{A\}$

$\epsilon\text{-clo}(B) = \{B\}$

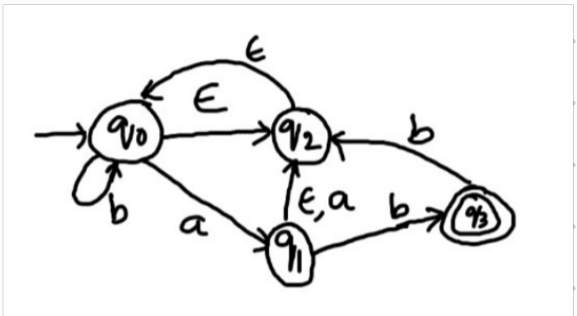
$\epsilon\text{-clo}(C) = \{C, D\}$

$\epsilon\text{-clo}(D) = \{D\}$

	δ	0	1
→	[S,A]	[C D]	[C D]
*	[C D]	[C D]	[B]
	B	A	A
	A	[C D]	[C D]

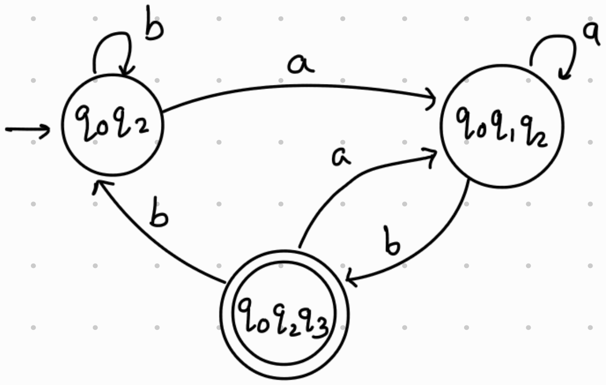


4.



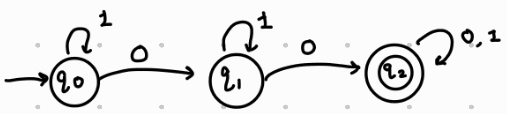
$\epsilon\text{-clo}(q_0) = \{q_0 q_2\}$
$\epsilon\text{-clo}(q_1) = \{q_1 q_2\}$
$\epsilon\text{-clo}(q_2) = \{q_0 q_2\}$
$\epsilon\text{-clo}(q_3) = \{q_3\}$

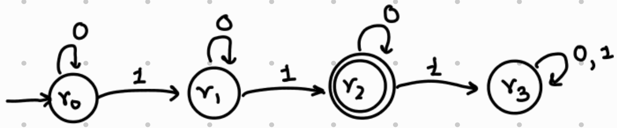
	a	b
$\rightarrow [q_0q_2]$	$[q_0q_1q_2]$	$[q_0q_2]$
$[q_0q_1q_2]$	$[q_0q_1q_2]$	$[q_0q_2q_3]$
$*[q_0q_2q_3]$	$[q_0q_1q_2]$	$[q_0q_2]$



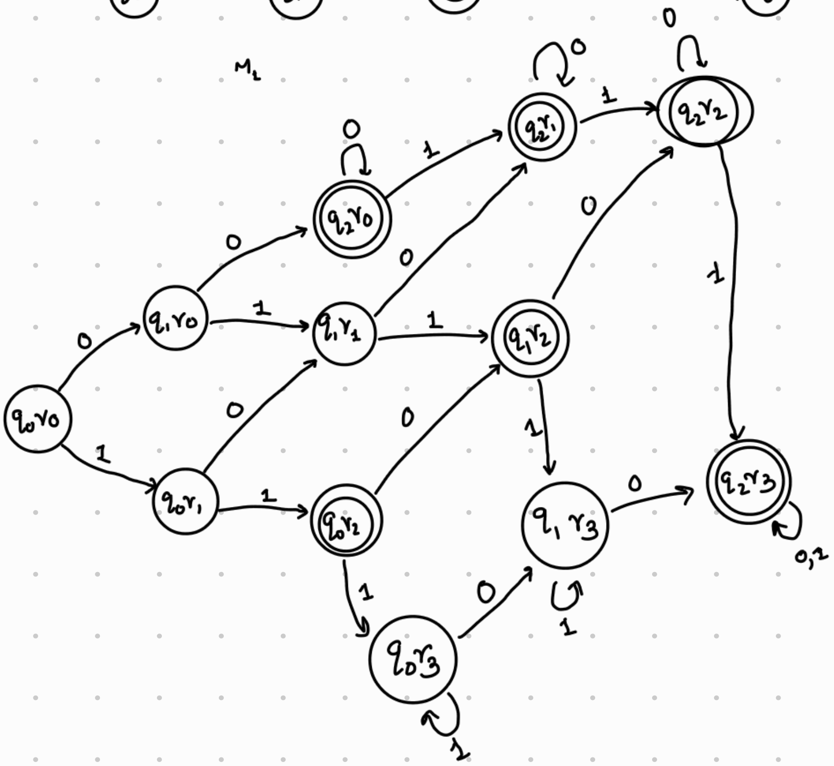
$L = \{w \mid w$ is a string ending with ab$\}$

A .15) At least 2 0's or Exactly 2 1's





M_1 M_2



	0	1
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_2	q_2

	0	1
r_0	r_0	r_1
r_1	r_1	r_2
r_2	r_2	r_3
r_3	r_3	r_3

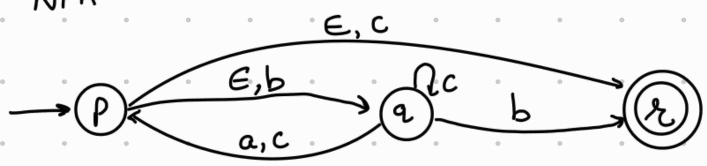
$q_0r_2 \Rightarrow 0 \Rightarrow q_1r_2$
$\Rightarrow 1 \Rightarrow q_0r_3$

$q_0r_3 \Rightarrow 0 \Rightarrow q_1r_3$
$\Rightarrow 1 \Rightarrow q_0r_3$

(9) 5)

δ_N	ϵ	a	b	c
$\rightarrow p$	{q,r}	{q}	{q}	{r}
q	ϕ	{p}	{r}	{p,q}
$*r$	ϕ	ϕ	ϕ	ϕ

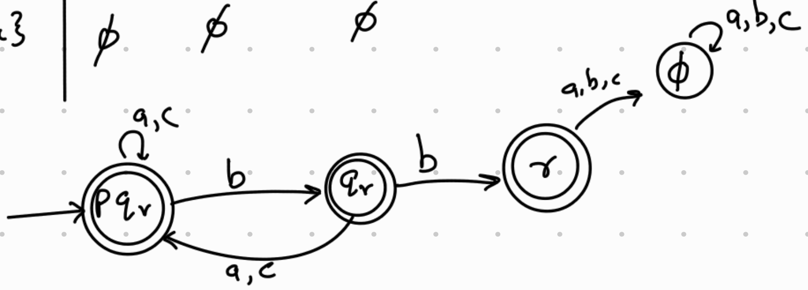
NFA :



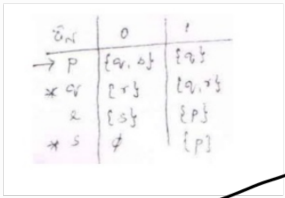
$\epsilon\text{-clo}(p) = \{p, q, r\}$
$\epsilon\text{-clo}(q) = \{q\}$
$\epsilon\text{-clo}(r) = \{r\}$

$\phi\ q\ \phi$

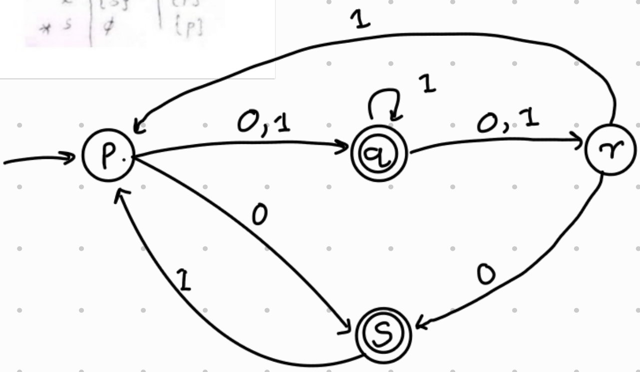
	a	b	c
$\rightarrow * \{p,q,r\}$	{pqr}	{q,r}	{p,q,r}
~~{q}~~	~~{p,q,r}~~	~~{r}~~	~~{pqr}~~
$*\{q,r\}$	{p,q,r}	{r}	{pqr}
$*\{r\}$	ϕ	ϕ	ϕ



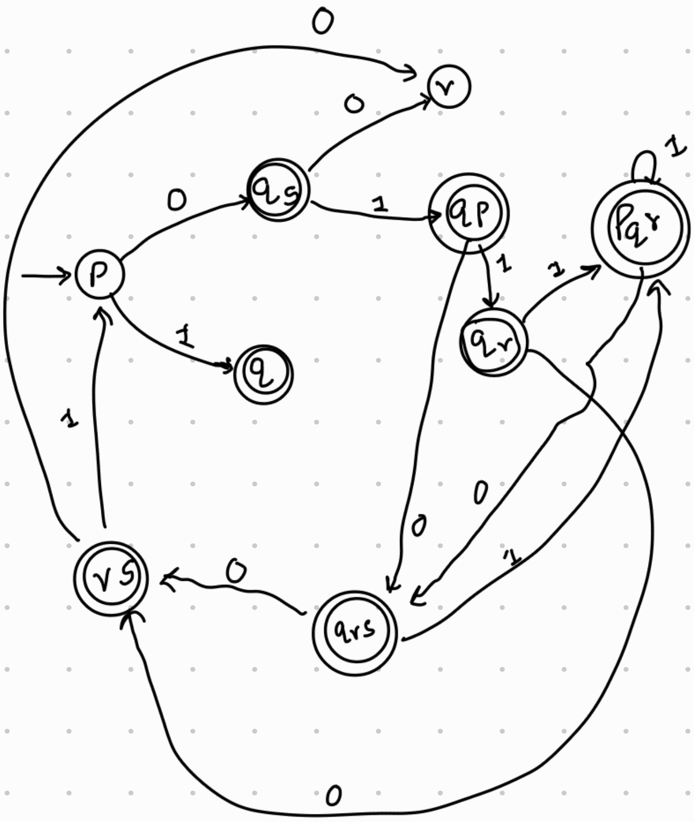
9.6)



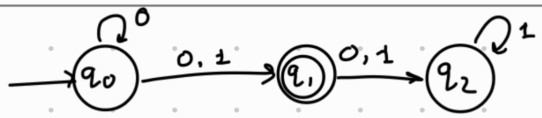
δ_N	0	1
→ p	{q, s}	{q}
* q	{r}	{q, r}
r	{s}	{p}
* s	∅	{p}



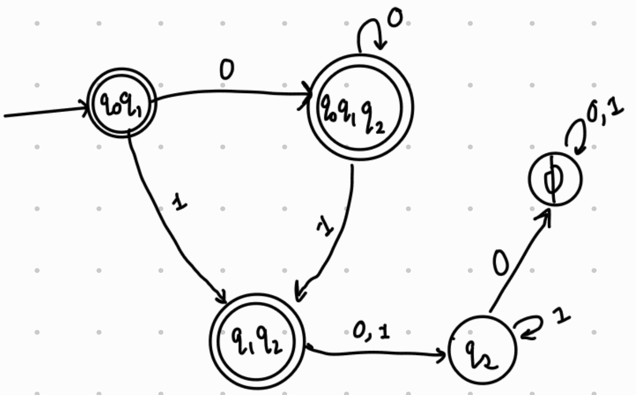
	0	1
→ p	{q,s}	{q}
* q	{r}	{q,r}
r	{s}	{p}
* s	∅	{p}
{q,s}	{r}	{q,p}
{q,r}	{r,s}	{p,q,r}
{q,p}	{q,r,s}	{q,r}
{p,q,r}	{q,r,s}	{p,q,r}
{q,r,s}	{r,s}	{p,q,r}
{r,s}	{r}	{p}



7)

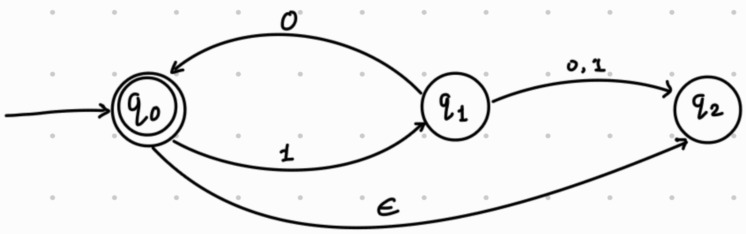


	0	1
q_0	q_0q_1	q_1
q_1	q_2	q_2
q_2	$\emptyset$	q_2
$\rightarrow * \ q_0q_1$	$q_0q_1q_2$	q_1q_2
$* \ q_0q_1q_2$	$q_0q_1q_2$	q_1q_2
$* \ q_1q_2$	q_2	q_2



$L = \{0^n 1\}$

10)

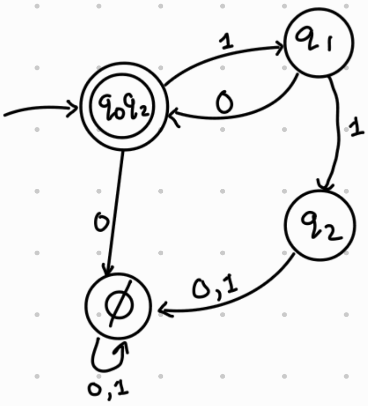


$\epsilon\text{-clo}(q_0)$ $\quad q_0 q_2$

$\epsilon\text{-clo}(q_1)$ $\quad q_1$

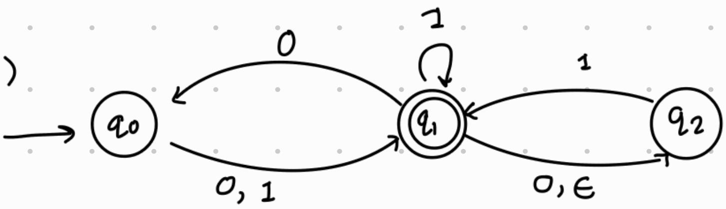
$\epsilon\text{-clo}(q_2)$ $\quad q_2$

	ϵ	0	1
q_0	q_2	$\emptyset$	q_1
q_1	$\emptyset$	q_0q_2	q_2
q_2	$\emptyset$	$\emptyset$	$\emptyset$



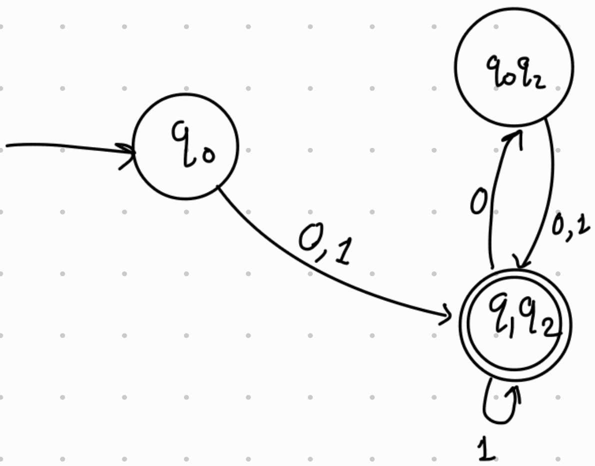
	0	1
q_0q_2	$\emptyset$	q_1
q_1	q_0q_2	q_2
q_2	$\emptyset$	$\emptyset$

9)



	0	1
$\rightarrow q_0$	q_1q_2	q_1q_2
q_1	q_0q_2	q_1q_2
q_2	ϕ	q_1q_2
$* \, q_1q_2$	q_0q_2	q_1q_2
q_0q_2	q_1q_2	q_1q_2

	ϵ-closure
q_0	$\{q_0\}$
q_1	$\{q_1, q_2\}$
q_2	$\{q_2\}$
q_1q_2	$\{q_1q_2\}$
q_0q_2	$\{q_0\, q_2\}$



h. Minimize DFA

h. 1.

q	X			
r	X	X		
s	x	X	x	
t	x	X	X	
	p	q	r	s

		a	b
	→ p	s	q
	q	t	r
	r	r	r
x	s	s	s
X	t	t	t

State	a	b	
PR	SR ✓	QR	✓
PQ	St	QR	X
QR	tr ✓	rr	✓
ST	st	st	X

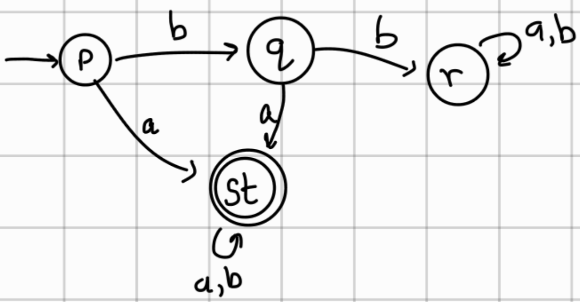
∴ States:

$$Q = \{q, r, p, st\}$$

↓

Minimized DFA

↓



Iteration 2:

State	a	b	
PQ	ST	QR ✓	✓
ST	ST	ST	X

Iteration 3:

State	a	b	
ST	ST	ST	X

2.

q_1	X			
q_2		X		
q_3	X	X	X	
q_4	X	X	X	X
	q_0	q_1	q_2	q_3

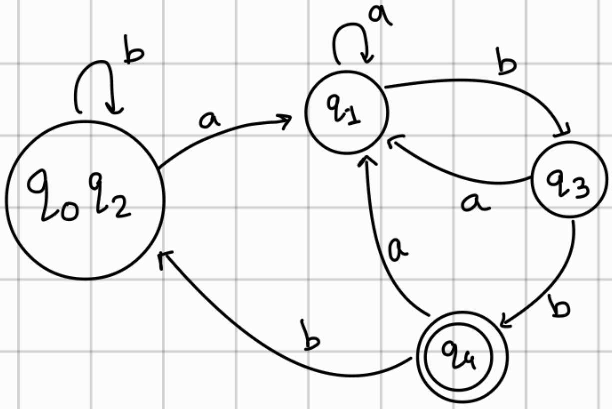
δ	a	b
$\rightarrow q_0$	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2
q_3	q_1	q_4
$* q_4$	q_1	q_2

δ	a	b	
q_0q_3	q_1q_1	q_2q_4 ✓	✓
q_0q_2	-‖-	$q_2\ q_2$	✗
q_0q_1	-‖-	q_2q_3	✗
q_1q_3	q_1q_1	$q_3\ q_4$ ✓	✓
q_1q_2	q_1q_1	$q_3\ q_2$	✗
q_2q_3	q_1q_1	$q_2\ q_4$	✓

Iteration 2:

	a	b	
q_0q_1	q_1q_1	q_2q_3 ✓	✓
q_0q_2	-‖-	q_2q_2	✗
q_1q_2	-‖-	q_3q_2 ✓	✓
q_0q_2	q_1q_1	q_2q_2	✗

$\therefore\ Q = \{q_0q_2, q_1, q_3, q_4\}$



3.

δ	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_2	q_4
q_2	q_1	q_4
q_3	q_2	q_4
$*q_4$	q_4	q_4

q_1	X			
q_2	X			
q_3	X			
q_4	X	X	X	X
	q_0	q_1	q_2	q_3

δ	0	1	
q_0q_3	q_1q_2	q_3q_4	✓
q_0q_2	q_1q_1	q_3q_4	✓
q_0q_1	q_1q_2	q_3q_4	✓
q_1q_3	q_2q_2	q_4q_4	X
q_1q_2	q_2q_1	q_4q_4	X
q_2q_3	q_1q_2	q_4q_4	X

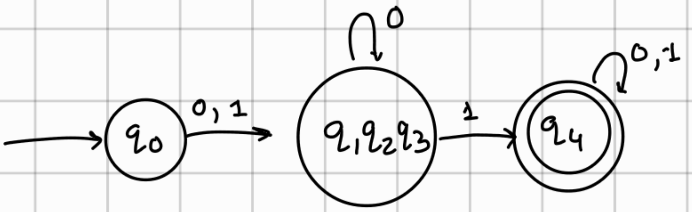
$q_1q_2 \cup q_2q_3 \cup q_1q_3$

$= q_1q_2q_3$

Iteration 2 :

q_1q_3	q_2q_2	q_4q_4	X
q_1q_2	q_2q_1	q_4q_4	X
q_2q_3	q_1q_2	q_4q_4	X

$\therefore Q = \{ q_0, q_1q_2q_3, q_4 \}$



(4)

Step 1: remove unreachable from q_0, q_5.

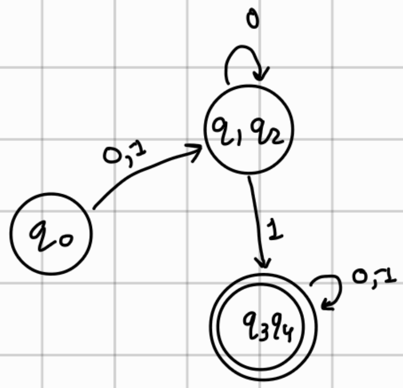
δ	0	1
$\rightarrow q_0$	q_1	q_2
q_1	q_2	q_3
q_2	q_2	q_4
$* q_3$	q_3	q_3
$* q_4$	q_4	q_4
q_5	q_5	q_4

Step 2:

q_1	X			
q_2	X			
q_3	X	X	X	
q_4	X	X	X	
	q_0	q_1	q_2	q_3

	0	1	
q_0q_2	q_1q_2	q_2q_4 ✓	✓
q_0q_1	q_1q_2	q_2q_3 ✓	✓
q_1q_2	q_2q_2 ×	q_3q_4 ×	X
q_3q_4	q_3q_4	q_3q_4	X

Iteration 2:			
q_1q_2	q_2q_2	q_3q_4	X
q_3q_4	q_3q_4	q_3q_4	X



$Q = \{ q_0, q_1q_2, q_3q_4 \}$