$$1+\frac{N'}{N} = 1+\frac{22.134}{206}=\frac{228.134}{206} = 1.107$$

$$t = \frac{1}{\lambda} ln(1.107) = \frac{4.5\times10^9\ yrs.}{0.693} ln(1.107)$$

$$= 6.6\times10^8\ years = \textbf{660 \textit{Million Years}}$$

Example 21.10: *A wooden piece of antiquity weighs 50 gm and shows ^{14}C activity of 320 disintegrations per minute. Estimate its age, assuming that the living tree, of which the wooden piece was a part, shows ^{14}C activity of 12 disintegrations per minute per gm. The half-life of ^{14}C is 5730 years.*

Solution: Let thre be N_o radioactive C-14 atoms in the tree just before it died. Its activity is

$$A_0 = \lambda N_0$$

After its death, the activity decreases exponentially

Thus
$$A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t}$$

$\therefore$
$$\frac{A}{A_0} = e^{-\lambda t}$$

$$A_0 = 12 \text{ disintegrations/min/gm}$$

$$A = \frac{320}{50} \text{ disintegrations/min/gm}$$

$$\lambda = \frac{0.693}{5730}(year)^{-1}$$

$$ln\left(\frac{A}{A_0}\right) = -\lambda t$$

$$t = \frac{1}{\lambda} ln\left(\frac{A_0}{A}\right) = \left(\frac{5730\,Yrs.}{0.693}\right) ln\left(\frac{50\times12}{320}\right) = \textbf{5197.5 yrs.}$$

21.19 NUCLEAR REACTIONS

Natural radioactivity is a nuclear reaction in which a radioactive nucleus of one element *spontaneously* changes into the nucleus of another element. It is also possible to induce a nuclear reaction artificially, by bombarding a nucleus with high-energy particles. *The reaction, in which an external bombarding particle successfully changes the identity of a target nucleus, is known as an* **artificial nuclear reaction** *or simply* **nuclear reaction**.

The first artificial nuclear reaction was produced by Rutherford in 1919 by bombarding nitrogen gas with α-particles emitted by a natural bismuth-214 source.

$$^{14}_{7}N \quad + \quad ^{4}_{2}He \quad \rightarrow \quad ^{17}_{8}O \quad + \quad ^{1}_{1}H$$

The bombardment resulted in conversion of nitrogen nucleus into oxygen nucleus. Such transformation of one nucleus into another nucleus is called **transmutation**. The important implication of this reaction is that one element can be changed into another by use of bombardment process. Subsequent to Rutherford's experiment, many isotopes were subjected to beams of high energy particles and numerous reactions were found. Not all nuclear reactions produce stable isotopes. If ${}^{25}_{12}Mg$ is bombarded with an alpha source, a radioactive isotope of aluminium ${}^{28}_{13}Al$ is produced which does not exist in nature.

$$ {}^{25}_{12}Mg \quad + \quad {}^{4}_{2}He \quad \rightarrow \quad {}^{28}_{13}Al^{*} \quad + \quad {}^{1}_{1}H $$

The asterisk (*) is used to denote a **radioactive isotope**. Artificial radioactive isotopes have characteristic half-life as do the naturally occurring ones.

In a typical nuclear reaction experiment, a target material is bombarded by high-energy particles such as protons, neutrons, deuterons, electrons or α-particles. The interaction between the target material and the incident particle depends on the nature and the energy of the particles and the target material.

Most of the alpha particles emitted from natural radioactive decay do not have enough energy to penetrate a heavy positively charged nucleus to induce a nuclear reaction. If the artificial nuclear reactions are to be studied for heavier elements, the kinetic energy of projectile particles must be increased.

A nuclear reaction is represented in a form similar to that of a chemical reaction. Thus, the reaction equation is shown as

$$ {}^{A}_{Z}X \quad + \quad a \quad \rightarrow \quad {}^{A_1}_{Z_1}Y \quad + \quad b $$

where a is the projectile particle that bombards the target nucleus X resulting in the product nucleus Y and an outgoing particle b. The reaction is represented in an abbreviated form as

$$ {}^{A}_{Z}X \ (a, b) \ {}^{A_1}_{Z_1}Y $$

Thus, the first artificial nuclear reaction is written as ${}^{14}_{7}N \ (\alpha, p) \ {}^{17}_{8}O$. The type of (a,b) nuclear reaction is determined by the nature of the projectile and the emitted particles.

In any nuclear reaction, mass and charge are conserved. Conservation of mass requires that the sums of the superscript mass numbers on each side of the equation are equal. Conservation of charge requires that the sums of the subscript protons on each side of the equation are equal.

21.20 Q-VALUE

Nuclear reactions resemble ordinary chemical reactions and are accompanied by energy changes. The energy liberated or absorbed during a nuclear reaction is called **nuclear reaction energy**. This energy is denoted by Q in the reaction equation and is called the *energy balance* of the reaction or more commonly, its **Q-value**. The Q-value of a reaction can be positive or negative depending on the nature of the reaction. According to Einstein's mass-energy equivalence principle, Q-value must be balanced by the changes in mass associated with the nuclear reaction.

Let us consider a general nuclear reaction

$$ X + a \rightarrow Y + b. $$

Let M_0 be the mass of the stationary target nucleus X,

M_1, E_1 be the mass and energy of the projectile a,

M_2, E_2 be the mass and energy of the product nucleus Y, and

M_3, E_3 be the mass and energy of the emitted particle b.

The nuclear reaction may be written as

$$ M_0 + (M_1 + E_1) = (M_2 + E_2) + (M_3 + E_3) $$

or

$$ (M_0 + M_1) - (M_2 + M_3) = (E_2 + E_3) - E_1 $$

As Q-value is the energy balance, it may be expressed as

$$Q = (E_2 + E_3) - E_1 \qquad (21.16\text{ a})$$

or

$$Q = (M_0 + M_1) - (M_2 + M_3) \qquad (21.16\text{ b})$$

The above equations show that the Q-value of a nuclear reaction may be determined either from the known kinetic energies of the particles involved or from the known masses of the reactants and the product nuclei. However, we define *Q-value as the difference in masses of the reactants and the products.*

(*i*) If $Q > 0$, the reaction is said to be **exothermic** or **exoergic**. That is, energy is released in the reaction. In this case, $(M_0 + M_1) > (M_2 + M_3)$. Hence, the total mass of the products is less than that of the reactants. The difference in masses (mass defect) is converted into energy.

(*ii*) If $Q < 0$, the reaction is said to be **endothermic** or **endoergic**. In this case $(M_0 + M_1) < (M_2 + M_3)$. That is, the total mass of the reactants is less than that of the products. It means that there is a gain of mass in the reaction, which could happen when there is absorption of energy. Therefore, energy is to be supplied in the form of kinetic energy of the projectile *a*. The kinetic energy of the projectile particle must have some minimum value, below which the reaction cannot occur. The minimum energy necessary for an endothermic reaction to occur is called the **threshold energy**.

1. Example of Exothermic Nuclear Reaction:

${}^7_3Li\ (p, \alpha)\ {}^4_2He$ is an example of exothermic reaction. Let us calculate the Q-value of this reaction.

Reactants total mass	Products total mass
$M_0 = 7.01822$ amu	$M_2 = 4.00387$ amu
$M_1 = 1.00814$ amu	$M_3 = 4.00387$ amu
$M_0 + M_1 = 8.02636$ amu	$M_2 + M_3 = 8.00744$ amu

$$\Delta M = (M_0 + M_1) - (M_2 + M_3) = 0.01862 \text{ amu}$$

$$Q = c^2\Delta M = 931.4\ (\Delta M)\ MeV = 931.4\ (0.01862) \text{ MeV}$$

or

$$\boldsymbol{Q = 17.34 \text{ MeV.}}$$

2. Example of Endothermic Nuclear Reaction:

${}^{14}_7N\ (\alpha, p)\ {}^{17}_8O$ is an example of endothermic reaction. Let us calculate the Q-value of this reaction.

Reactants total mass	Products total mass
$M_0 = 14.00753$ amu	$M_2 = 17.00450$ amu
$M_1 = 4.00387$ amu	$M_3 = 1.00814$ amu
$M_0 + M_1 = 18.01140$ amu	$M_2 + M_3 = 18.01264$ amu

$$\Delta M = (M_0 + M_1) - (M_2 + M_3) = -\ 0.00124 \text{ amu}$$

$$\text{Q} = c^2\Delta M = -931.4\ (\Delta M) \text{ MeV} = -931.4\ (0.00124) \text{ MeV}$$

or

$$\mathbf{Q = -1.15 \text{ MeV.}}$$

Example 21.11. *A nuclear reaction is given by* ${}^{10}_5B + {}^4_2He \rightarrow {}^{13}_6C + {}^1_1H$

Given that: ${}^{10}_5B = 10.016125$ amu, ${}^4_2He = 4.003874$ amu

${}^{13}_6C = 13.007440$ amu and ${}^1_1H = 1.008146$ amu

Example 21.12. *A 0.01 mm thick* ${}^{7}_{3}Li$ *target is bombarded with 10^{13} protons/s. As a result 10^{8} neutrons/s are produced. What would be the cross-section for this reaction? The density of lithium is 500 kg/m^{3}.*

Solution. $$\sigma = \frac{N}{N_0 N_t}$$

The number of target nuclei per unit volume $= \dfrac{\rho N_A}{M} = \dfrac{\left(500\ \text{kg/m}^3\right)\left(6.02\times10^{26}\right)}{7\ \text{kg}} = 4.3 \times 10^{28}/\text{m}^3.$

The number of target nuclei/unit area $N_t = \left(\dfrac{\rho N_A}{M}\right)t = (4.3 \times 10^{28}/\text{m}^3)\ (0.01 \times 10^{-3}\ \text{m})$

$= 4.3 \times 10^{23}/\text{m}^2.$

Number of nuclei undergoing interaction per second = Number of neutrons produced /s

$= 10^{8}.$

$$\therefore \qquad \sigma = \frac{10^8}{10^{13}\times 4.3\times 10^{23}/\text{m}^2} = \mathbf{2.3 \times 10^{-29} m^2}.$$

21.22 NEUTRONS AND NEUTRON INDUCED REACTIONS

The neutron was discovered by the English physicist James Chadwick in 1932. He found that when beryllium is bombarded with α-particles, it emits neutral particles with a mass close to that of the proton.

$$ {}^{9}_{4}Be \quad + \quad {}^{4}_{2}He \quad \rightarrow \quad {}^{12}_{6}C \quad + \quad {}^{1}_{o}n $$

The neutron produced in this reaction carries sufficient energy to cause additional nuclear reactions in nuclei with which the neutron collides. Chadwick received Nobel Prize in physics in 1935 for his discovery of neutron.

Neutrons are very effective in initiating nuclear reactions because they do not possess electric charge. As such they penetrate nuclei more deeply than any other particle. Neutrons interact with nucleus differently, according to whether they are fast or slow. **Fast neutrons** are those having energies in the range of 100 keV to 50 MeV. **Slow neutrons** are those which have energies not exceeding 100 keV.

The heaviest naturally available element is uranium having an atomic number 92. Enrico Fermi, an Italian physicist, who undertook a systematic study of neutron-induced nuclear reactions, speculated that neutron bombardment of uranium would yield new elements that would be more massive than uranium. In 1934 Fermi found that uranium bombarded with neutrons yielded radioactive products which were assumed to be trans-uranium elements. However, in 1938 Otto Hahn and Fritz Strassmann , the German chemists, using precise radiochemical identification established beyond doubt that the neutron bombardment of uranium produced an isotope of barium $\left({}^{139}_{56}Ba\right)$. More similar identifications followed. In 1939, two Austrian physicists Lise Meitner and Otto R.Frisch suggested that the neutron caused a division of the uranium nucleus into 'two nuclei of roughly equal size". They called the process **nuclear fission**, by analogy to the biological fission of a living cell into two parts. Shortly afterwards, it was found that transuranium elements may also form when uranium is bombarded with neutrons. Thus, neutron bombardment of uranium leads sometimes to fission and sometimes to formation of transuranium elements neptunium $\left({}^{239}_{93}Np\right)$ and plutonium $\left({}^{239}_{94}Pu\right)$.

21.23 NUCLEAR FISSION

Nuclear fission is a neutron-induced nuclear reaction in which a heavy nucleus such as uranium splits into two intermediate lighter nuclei. In a nucleus there is a competition between the nuclear force, which holds the nucleus together, and the electrostatic repulsion of the protons which tires to tear the nucleus apart. In case of heavy nuclei there is a delicate balance between the nuclear and electric forces. This balance can be easily upset. The energy necessary to cause fission is about 6 MeV.

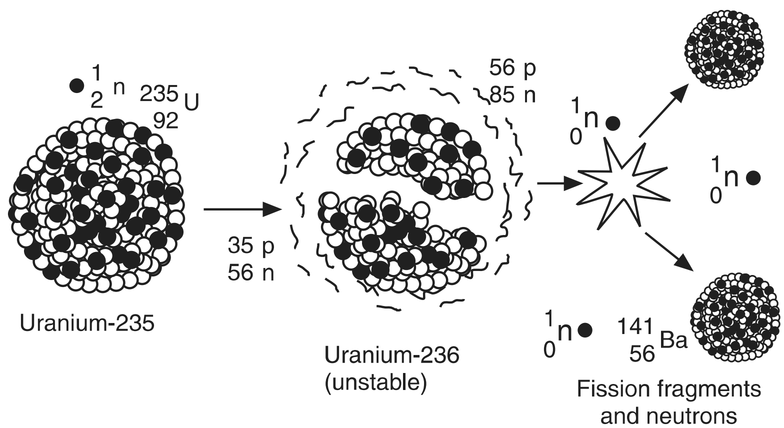

Fig. 21.11

Only certain heavy nuclei can undergo fission and the probability of fission reaction for a particular nucleus depends on the energy of the incident neutrons. Nuclei with odd number of neutrons, ${}^{233}_{92}U$, ${}^{235}_{92}U$ and ${}^{239}_{94}Pu$ undergo fission with slow neutrons. On the other hand, the nucleus ${}^{232}_{90}Th$ with an even number of neutrons requires fast neutrons with energies of 1 MeV or more.

Nuclear fission *is the phenomenon of breaking up the nucleus of a heavy atom into two more or less equal segments with the release of a large amount of energy.* Hahn discovered in 1938 that when uranium was bombarded with thermal neutrons, the uranium nucleus broke up into barium and krypton nuclei of atomic numbers 56 and 36 and liberated 3 neutrons accompanied by a tremendous amount of energy (Fig. 21.11). The nuclear reaction is represented as follows:

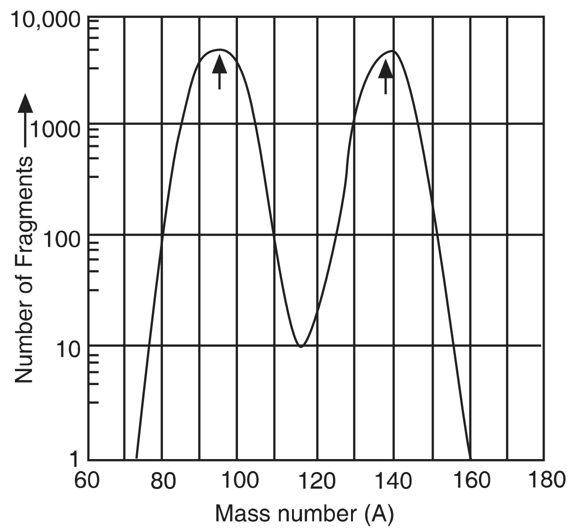

Fig. 21.12

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q \qquad (21.19)$$

The U^{235} nuclei do not all split up into those of *Ba* and *Kr*. The fission reaction can proceed in about 40 different ways, yielding different final products. The heavy nuclei may split up into nuclei of several pairs of elements lying in the central region of the periodic table with slightly unequal masses. These are known as *fission fragments*.

The fission products such as $^{141}_{56}Ba$ and $^{92}_{36}Kr$ are radioactive and undergo disintegrations until stable isotopes are formed. After several steps they form the stable isotopes $^{141}_{59}Pr$ and $^{90}_{40}Zr$ respectively. Radio-chemical analysis showed that nuclides resulting from fission have atomic numbers between 30 and 63 and mass numbers between 72 and 158. Fig. 21.12 shows the mass distribution of the fission fragments from the fission of U-235 nuclei. It is most likely that one fragment will have a mass number of about 95 and the other about 140. Most of the energy (about 80%) released in fission goes into kinetic energy of the two fission fragments, and the remaining (20%) appears as decay products (β and γ rays) and kinetic energy of neutrons emitted in the fission process. The neutrons typically have energies of one to several MeV.

The most important features of nuclear fission is that the process is accompanied by

(*i*) the release of a large amount of energy and

(*ii*) the emission of two or more energetic neutrons which under appropriate conditions cause fission in neighbouring nuclei.

21.23.1 Energy Released during Nuclear Fission

The energy released during the process of fission is known as *nuclear energy or atomic energy*. The amount of energy released during fission may be estimated using mass defect method. We illustrate the method by taking the fission reaction (21.19) as an example.

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Q$$

Actual mass before the fission reaction

Mass of U-235 nucleus = 235.125 amu

Mass of the neutron = 1.009 amu

Total mass = 236.134 amu

Actual mass after the fission reaction

Mass of Ba-141 nucleus = 140.958 amu

Mass of Kr-92 nucleus = 91.926 amu

Mass of three neutrons = 3.027 amu

Total mass = 235.911 amu

Mass decrease during the reaction = (236.134 – 235.911) amu = 0.223 amu

= 0.223 amu × 931.4 MeV/amu = **207 MeV**

Therefore, each fission event produces about 200 MeV of energy. The energy converted per atom is roughly 10^8 times greater in nuclear reactions than in chemical reactions. The energy, 200 MeV = $200 \times 10^6 \times 1.6 \times 10^{-19}$ J $= 3.2 \times 10^{-11}$ J, is in fact very small. We spend about 5J when we do a simple job, say picking up an object from a table. However, as there are billions of nuclei even in a small sample of uranium, the total energy that could be produced will be enormous.

As an example, les us compute the energy produced when nuclei contained in 1 gram of *U*-235 undergo fission.

Atomic weight of *U*-235 = 235

There are 6.023×10^{26} atoms in 235 kg of Uranium.

$$\therefore \quad \text{Number of nuclei in one gram of } U-235 = \frac{6.023\times10^{6}}{235\times1000} = 2.56\times10^{21}$$

Energy produced by 1 gm of U–235 $= (2.56 \times 10^{21}) \times 200$ MeV

$= 2.56 \times 10^{21} \times 3.2 \times 10^{-11}$ J

$= 8.2 \times 10^{10}$ J.

1 kWh $= (1 \times 10^3 \text{ J/s})\,(3600\text{s}) = 3.6 \times 10^6$ J

$\therefore$ *Energy produced by 1 gm of U–235* $= \dfrac{8.2\times10^{10}\text{ J}}{3.6\times10^{6}\text{ J/kWh}} = 22.8\times10^{3}\text{ kWh} = 22.8\text{ MWh}.$

We can as well express the energy in calories.

1 cal = 4.187 J

Energy produced by 1gm of U-235 $= \dfrac{8.2\times10^{10}}{4.187}\text{cal} = 2 \times 10^{10}$ cal

The quantity of coal required to produce an energy equivalent to the above figure may be calculated assuming that the particular grade of coal used in thermal power plant yields 7×10^3 kcal energy per kg.

Quantity of coal required $= \dfrac{2\times10^{10}\text{cal}}{7\times10^{3}\text{kcal/kg}} = \dfrac{2\times10^{10}}{7\times10^{6}}\text{ kg} \cong 3000$ kg.

It means that about 3 tonnes of coal is required to produce as much energy as 1 gm of U-235 produces.

Enrico Fermi, the Italian physicist was honoured in 1938 with the Nobel prize in physics for his discovery of nuclear reactions brought about by slow neutrons.

21.23.2 Theory of Nuclear Fission

Niels Bohr and John A. Wheeler explained in 1939 the process of nuclear fission using the liquid drop model. According to this model the stable nucleus may be compared to a spherical liquid drop. The shape of the drop depends on the balance between the short range forces and coulomb repulsion forces. When a nucleus captures a neutron, a compound nucleus is formed. The nucleus is excited by being given mechanical energy and is in a state of higher energy. Just as an excited liquid drop oscillates, the nucleus will be in a state of oscillations. The oscillations tend to distort the spherical shape so that the nucleus assumes an ellipsoid shape. The restoring forces arising from short-range nuclear forces tend to make the nucleus return to is original spherical shape. If the excitation energy is sufficiently large, the ellipsoid deforms further into a dumb bell shape. The coulomb force of repulsion between the two parts of the deformed nucleus can overcome the short-range attractive forces causing the nucleus to split and the fragments to separate with higher speeds. Each of the fragments will then quickly take spherical form because within it the attractive nuclear forces predominate again. A possible sequence of stages of fission is shown in Fig. 21.13. Generally, the two fission fragments are not of the same size. As neutron/proton ratio for heavy nuclei is higher than for lighter nuclei, the fragments contain excess neutrons. To reduce the excess, the fragments release two or three neutrons as soon as they are formed.

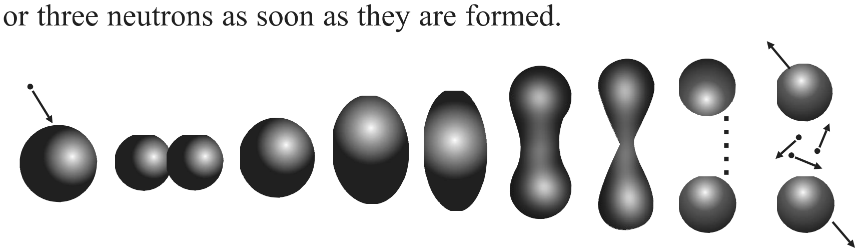

Fig. 21.13