Qu7.

(7.1) Find out the successor of the set $A = \{\emptyset, \{\emptyset\}\}$. [1]

(7.2) prove that additive group of complex numbers(C,+) is isomorphic to multiplicative group of positive reals $(R^+, .)$ under the mapping $(a + ib) = 2^a . 3^b$ [1.5]

(7.3) Draw the binary search tree of the following sequence of entries 60, 17, 50, 20, 18, 9, 14, 35, 70, 62, 58, 1, 13, 45, 11, 0, 92, 7 and traverse the resultant binary search tree using preorder method. [2]

7.4 Count the number of n –bit (digit 0 and 1) strings with exactly k zeros, with no two consecutive zeros. [1]

Qu8. Choose the correct option from the following with appropriate justification. [3.5]

8.1 Which of the following is not valid

i) $(x)A(x) \rightarrow B \Leftrightarrow (\exists x)(A(x) \rightarrow B)$ ii) $(\exists x)A(x) \rightarrow B \Leftrightarrow (x)(A(x) \rightarrow B)$

iii) $A \rightarrow (x) B(x) \Leftrightarrow (x)(A(x) \rightarrow B)$ iii) $A \rightarrow (\exists x) B(x) \Leftrightarrow (\exists x)(A(x) \rightarrow B)$

(a) only i) (b) only ii) (c) ii) and iv) (d) i) and iii)
(e) none of the above

8.2 Which of the following is not a logical implication

(i) $(\exists y)(\forall x)P(x,y) \rightarrow (\forall x)(\exists y)P(x,y)$

(ii) $(\exists x)(\forall y) P(x,y) \rightarrow (\forall y)(\exists x)P(x,y)$

(iii) $(\forall y)(\exists x) P(x,y) \rightarrow (\exists x)(\forall y) P(x,y)$

(iv) $(\forall x)(\exists y)P(x,y) \rightarrow (\exists y)(\exists x) P(x,y)$

8.3 How many relations ate there on a set with n elements that are reflexive and symmetric?

(a) $2^{n(n-1)/2}$ (b) $2^{n(n+1)/2}$ (c) $2^{n(n-1)}$ (d) $2^{n(n+1)}$

SECTION B

Qu1. X, an engineer, puts a proposition "All engineers lie all the time". Decide the truth value, if any, of X's proposition and justify. [3]

Qu2. Calculate the number of graphs of n labeled vertices which are *surely* connected? [2]

Qu3. A chain in a partially ordered set S is a subset A of S such that every two elements of A are related by the partial order. Set $S = \{1,2,3,\ldots, p^{10}q^{20}\}$ where p and q are odd primes and $p < q$, has a partial order relation R of [3] divisibility defined on S, such that for any a, b in S aRb if and only if a divides b. Find the size of the longest chain containing $p^{10}q^{20}$.

Qu4.

4.1 Give a combinatorial argument proof for below identity, where n> 3. [3]

$$n(n-1)(n-2)\ {}^{n}C_{n} + (n-1)(n-2)(n-3)\ {}^{n}C_{n-1} + \ldots + 3!\ {}^{n}C_{3} = n(n-1)(n-2)\ 2^{n-3}$$

Note: Strictly combinatorial proof is needed. No marks for algebraic proof

4.2 Show that exactly one of any k consecutive integers is divisible by k [2]

OR

4.1 Count how many positive integral solutions exist of the following linear equation of p variables? Clearly explain the method of counting. [3]

$x_1+x_2+x_3+\ldots+x_p = q$, where $q > p$ is a positive integer and $\forall x_i$, $x_i >= 1$

4.2 Prove that in any non-empty group (G, *) where * is a binary operator over G, equations $a*x= b$ and $y*b = a$ have unique solutions $x \in G, y \in G, \forall a \in G, b \in G$. [2]

Qu5. Prove that a relation R on set S is an equivalence relation if and only if it partitions the set S into disjoint subsets such that union of these subsets is S. [2]

Qu6. Let G= (V, E) be an undirected simple graph with k components and |V|=n, |E|=m. Prove that m >= n - k . [3]

Qu7. Prove that, for any non-empty group G under binary operator *, if A={G_1, G_2, ... } is any collection of subgroups of G then $\cap G_i$, is a subgroup. [2]

Qu8. Every tree has the chromatic number 2. Prove or disprove that every undirected simple graph of n vertices having chromatic number 2 is isomorphic to a tree of n vertices. [1]

Qu9. Prove or disprove that every acyclic connected directed graph of n vertices has n-1 edges. [1]

Qu10. Prove that every first-order logic expression has an equivalent propositional logic expression. [3]