CHAPTER

21 Atomic Nucleus and Nuclear Energy

21.1 INTRODUCTION

The nucleus of an atom occupies an astonishingly small volume at the center of the atom and is most densely packed with protons and neutrons. Almost all the entire mass of the atom is accounted by the nucleus alone. The particles in the nucleus are held together by nuclear forces which are a fundamental force like gravitational force. The heavier nuclei exhibit instability and transform into stable nuclei through radioactive disintegration. The natural radioactivity of nuclei is effectively used in the determination of ages of organic as well as inorganic substances. Radioactivity can be induced in nuclei through nuclear reactions, which assist mankind in different fields such as medicine, industry and agriculture. The neutron induced nuclear reactions occupy special place. The neutrons have the ability of causing fission of uranium and plutonium nuclei which liberates tremendous energy. The nucleus is thus a storehouse of enormous power, which can be utilized in wars as well as in production of electrical power. For the past century fossil fuels namely, coal, oil and natural gas have supplied the major portion of our energy requirements. These sources will be nearly exhausted in the near future and it is imminent that alternative sources of power are to be searched for. Nuclear power is one of the alternative sources. The energy reserve in the form of uranium is many times greater than that of fossil fuels. The generation of power through nuclear fission, however, poses its own threats and problems. Nuclear fusion power is believed to be an inexhaustible source of energy and is without any attendant hazards. Efforts are in progress for producing electrical power through nuclear fusion on commercial scale.

21.2 THE ATOMIC NUCLEUS

The British physicist Ernest Rutherford proposed the nuclear model of atom in 1911 in an attempt to explain the large angle scattering of α-particles which were directed at a thin gold foil. He postulated that all the positive charges of an atom were concentrated in a central massive core and named it nucleus. A **nucleus** is the centre of the atom where most of the mass of the atom and all of the positive charge are concentrated.

There is a vast difference between the size of an atom and its nucleus. The nucleus is a very small part of an atom. The radius of nucleus is of the order of 10^{-14} m while that of an atom is of the order of a few angstroms (10^{-10} m). Therefore, an atom occupies about a million times more space than does a nucleus. In between the atomic electrons and the nucleus, there is a lot of void of space.

An atomic nucleus is not a single indivisible point mass. It is composed of smaller particles. It is obvious that nucleus contains protons. However, all of the nuclear mass could not be accounted for by protons. In 1920, Rutherford suggested the existence of a neutral particle in the nucleus which he called a **neutron**. Neutron was discovered in 1932 by James Chadwick. All nuclei, with the exception of hydrogen nucleus, contain neutrons. Protons and neutrons are about 2000 times more massive than electrons. They are collectively known as **nucleons**. Since a stable atom is electrically neutral, the number of positively charged protons in the nucleus is always equal to the number of negatively charged electrons around the nucleus. The number of protons or electrons in an atom is known as the **atomic number** and is denoted by Z. Total number of protons and neutrons in an atomic nucleus is called the **mass number** and is denoted by A. A special notation is used to designate a particular nucleus. The nucleus of an atom is denoted by the chemical symbol of the atom subscripted and superscripted respectively by atomic number Z and mass number A. Thus, if the chemical of an atom is X, its nucleus is denoted by $^{A}_{Z}X$. For example, $^{12}_{6}C$ (read as carbon-six-twelve) denotes the nucleus of carbon atom, containing 6 protons and 12 – 6 = 6 neutrons.
There are 92 stable species of atom available in nature.

21.3 ISOTOPES

A particular kind of atom of any element is called a **nuclide**. A nuclide is distinguished from other nuclides by the number of protons and neutrons it contains. The atomic number Z determines the chemical nature of an element. Although for a particular element the number of electrons and protons is fixed, the number of neutrons in the nucleus may vary. It implies that the mass number A may differ though the atomic number Z remains the same. Such atoms will be chemically identical but their nuclei show marked differences in stability. Nuclei of the same element having different numbers of neutrons are called isotopes. Thus, **isotopes** are atoms of a given element that have different masses. To cite an example, hydrogen has three isotopes. $^{1}_{1}H$ is the most common isotope. It is just a proton. The other isotopes $^{2}_{1}H$ and $^{3}_{1}H$ are called **deuteron** and **triton** respectively. In atomic form they are known as deuterium and tritium respectively. The deuteron denoted by D is made of one proton and one neutron. For about every 6500 atoms of ordinary hydrogen in water, there is one atom of deuterium. Triton denoted by T is made of one proton and two neutrons. It is radioactive with a half-life period of 12.26 years.

The uranium element which plays a very important role in the production of nuclear energy exists in three isotopic forms.

Isotope	Relative abundance	Half-life
$^{238}_{92}U$	99.28%	4.5×10^9 years
$^{235}_{92}U$	0.714%	7.1×10^8 years
$^{234}_{92}U$	0.006%	2.5×10^5 years

21.4 THE NUCLEAR FORCE

The nucleons are clustered together within the small volume of the nucleus. Large repulsive electrical forces operate between the positively charged protons, which tend to tear the nucleus apart. Yet the nuclei of most atoms are stable. It means that there must be some strong attractive force which more than balances the electrostatic repulsion and holds the nucleus together. This strong attractive force is called the **nuclear force**. The nuclear interaction between nucleons is, therefore, a **strong interaction**. The nuclear force is a fundamental force like the gravitational and electrical forces, but is more complicated and not completely understood. The nuclear forces have the following properties.

Salient Features of the Nuclear Forces:

1. The nuclear force is strongly attractive and much larger in magnitude than either the electrostatic force or gravitational force. They act between any pair of nucleons – proton-proton, proton-neutron, and neutron-neutron.
2. Nuclear forces are very **short-ranged** forces. A nucleon interacts only with its nearest neighbours, over distances of the order of 10^{-15} m.
3. Nuclear forces are **charge-independent**. They do not depend on the charge of nucleons. The nuclear forces acting between two protons, between a proton and a neutron or between two neutrons have the same magnitude. That is *n*-*n* force is no different from the *p*-*p* and *n*-*p* force.
4. Nuclear forces are **spin-dependent** and depend on the mutual orientation of the spins of the nucleons. For example, a neutron and a proton are kept together forming a nucleus of deuteron, only if their spins are parallel to each other.
5. Nuclear forces are **not central** forces. They cannot be represented as directed along the straight line connecting the centers of the interacting nucleons.
6. Nuclear forces have the property of **saturation**. It means that each nucleon in a nucleus interacts with a limited number of neighbours. Saturation manifests itself in that the binding energy per nucleon does not grow with an increase in the number of nucleons, but remains approximately constant. Further, the saturation of the nuclear forces is indicated by the volume of a nucleus being proportional to the number of nucleons forming it.
7. The magnitude of nuclear force is so high that the work required to divide a nucleus into its constituents is about 8 MeV in contrast to a few eV required to separate the extra-nuclear electrons from its atom.

21.4.1 Proton-neutron Theory

For elements of low mass numbers, the atomic number Z is nearly half the mass number A. Thus the number of protons is nearly equal to the number of neutrons. With increasing mass number, the value of Z becomes less than half of A which means that the number of neutrons exceeds that of protons. In case of ${}^{238}_{92}U$, $Z = 92$ and $A = 238$. Therefore, $(A–Z) = 146$. Thus, uranium nucleus consists of 92 protons and 146 neutrons. For elements of low mass numbers, the neutron-to-proton ratio is close to unity. But as the number of protons in a nucleus exceeds 20, the neutron to proton ratio is greater than unity in stable nuclides. It appears that in order to maintain nuclear stability, the neutrons must exceed protons in number (Fig.21.1). Further, the neutron excess increases with the increasing atomic number. For the heaviest stable nuclides such as ${}^{208}_{82}Pb$ the neutron-to-proton ratio

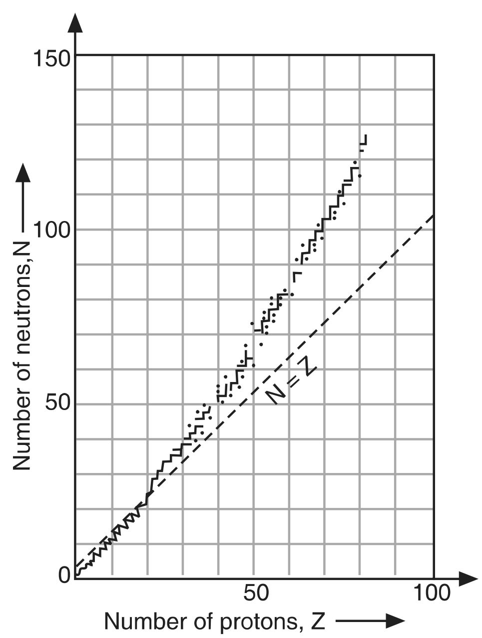

Fig. 21.1: A Plot of neutron number N versus proton number Z in nuclides. Nuclides with proton numbers greater than 20 have more neutrons than protons. The dashed line corresponding to the condition N = Z is called the line of stability.

exceeds 1.5. It is inferred from the above facts that neutrons act as a nuclear "glue" to hold the nucleus together. As the number of protons increases for larger stable nucleus, the number of neutrons also increases, so that the short range nuclear forces are greater than the long range repulsive electrical forces. Neutrons provide attractive nuclear forces between both protons and other neutrons. The proton-neutron theory was put forward by Heisenberg in 1932.

Origin of the Strong Interaction

In 1935, H.Yukawa suggested a successful theoretical model to explain the origin of nuclear force. He proposed that the nuclear force is an **exchange force** acting through virtual particles. The virtual particles responsible for nuclear binding forces are called π **mesons** (or pions). The existence of mesons was discovered experimentally in 1947. Three different types of pions, namely positive, negative and neutral pions exist. The mass of a π meson is about 273 times the mass of an electron. It is postulated that nucleons consist of some sort of common core surrounded by a pulsating cloud of pions. The pions are supposed to rapidly jump back and forth between the nucleons, thereby changing their identity equally fast and at the same time keeping them bound together. When neutral pions are exchanged, the nucleons do not undergo any change. Thus, neutral π^o mesons are associated with the forces that exist between neutrons and neutrons. When a negative pion is exchanged between a neutron and proton, the neutron emits a negative pion and turns into a proton and the proton absorbing the negative pion turns into a neutron. Thus, negative π^- mesons are associated with the forces that exist between neutrons and protons. When a positive pion is exchanged between a neutron and proton, the proton emits a positive pion and turns into a neutron and the neutron absorbing the positive pion turns into a proton. No particular proton will remain a proton in the nucleus and no particular neutron remains a neutron in the nucleus. If a proton is in the field of the negative meson, it is converted into a neutron. When a neutron is in the field of a positive meson, it is converted into a proton. Thus, the nucleus is seen to be an ever-changing complex structure. Thus, according to Yukawa's theory, the protons and neutrons in the nucleus are held together by the continual exchange of π mesons. The interactions are summarized as follows.

$$\begin{gathered}
n - \pi^- \rightarrow p \\
p + \pi^- \rightarrow n \\
p - \pi^+ \rightarrow n \\
n + \pi^+ \rightarrow p \\
n + \pi^o \rightarrow n \\
p + \pi^o \rightarrow p
\end{gathered}$$

21.5 STATIC PROPERTIES OF NUCLEUS

21.5.1 Nuclear Mass

Mass of an atom, a nucleus or an elementary particle is extremely small. Expressing their masses in conventional units involves cumbersome negative powers of ten. Therefore, a separate unit called **atomic mass unit** has been devised for expressing the masses of atomic particles. Nuclear mass means the mass or weight of the nucleus alone. Atomic mass unit (a.m.u.) is defined as $1/12^{th}$ the mass of the ${}^{12}_{6}C$ atom.

$$1 \text{ a.m.u.} = \frac{1}{12} \times \text{Mass of one carbon atom} = \frac{1}{12} \times \frac{12 \text{ kg}}{6.02 \times 10^{26}} = 1.66 \times 10^{-27} \text{ kg.}$$

The uranium nucleus U-238 has a mass of 238 a.m.u. As the mass of the electron is negligibly small in comparison to that of protons and neutrons, the mass of the nucleus is taken as the mass of the corresponding atom also. Thus, the mass of the uranium atom is 238 a.m.u.

The a.m.u. is now called **unified atomic mass** and is denoted by the letter "u".

21.5.2 Nuclear Radius

Nuclear radii are estimated from the measurements on the maximum scattering angles of α–particles when they approached the target nuclei. The nucleus is assumed to be spherical and the following empirical relation gives its radius.

$$R = R_o A^{1/3} \tag{21.1}$$

where R_o has an average value of 1.4×10^{-15} m.

∴ The radius of uranium nucleus = 1.4×1^{-15} m × $238^{1/3}$ = 8.68×10^{-15} m.

21.5.3 Nuclear Density

The density of nuclear matter is tremendous. All nuclei have nearly the same density. The density is calculated as follows.

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of the nucleus}} = \frac{M_N}{\frac{4}{3}\pi R^3} = \frac{m_p A}{\frac{4}{3}\pi R^3} \tag{21.2}$$

$$= \frac{1.673 \times 10^{-27}\text{kg}}{11.5 \times 10^{-45}\text{m}^3} = 1.45 \times 10^{17}\ \text{kg/m}^3.$$

21.5.4 Nuclear Charge

Nucleus is electrically positive and the magnitude of its charge is equal to the number of protons in the nucleus. Thus, the charge on a nucleus is equal to its atomic number, Z. Thus, a uranium nucleus carries a charge of 92 units.

21.5.5 Nuclear Quantum States

The studies of α- and γ-ray spectra show that every nucleus possesses a set of quantum states. Transitions between different nuclear states cause emission of γ-rays.

21.5.6 Spin and Magnetic Moment

The hyperfine structures observed in atomic spectra indicated that the nucleus has spin motion. It is concluded on the basis of experimental evidence that the protons and neutrons are in continuous motion in discrete quantized orbits. Because of this motion, the nucleus possesses angular momentum and magnetic moment.

The magnetic moment of nuclei is given by

$$\mu_1 = g \frac{h}{2\pi} \cdot \frac{e}{2M} \tag{21.3}$$

The product $\frac{h}{2\pi} \cdot \frac{e}{2M}$, often written as $\frac{e\hbar}{2M}$ is known as **nuclear magneton**.

21.6 MASS DEFECT

The nucleus is formed by bringing protons and neutrons together. The mass of the resulting nucleus is less than the sum of the masses of the constituent protons and neutrons. This mass difference is called **mass defect** and is denoted by Δm.

If Z is the number of protons in the nucleus, then the number of neutrons in the nucleus is $(A–Z)$. If m_p is the mass of the proton and m_n is that of the neutron, then the sum of the masses of the protons and neutrons

$$= Zm_p + (A - Z)m_n$$

If M is the actual mass of the nucleus, then mass defect is

$$\Delta m = Zm_p + (A - Z)m_n - M \quad (21.4)$$

Because the nucleus is more stable than the separated neutrons and protons, the nucleus is in a lower energy state. It implies that the mass, which disappears, is released in the form of energy when nucleons are bound together in a nucleus.

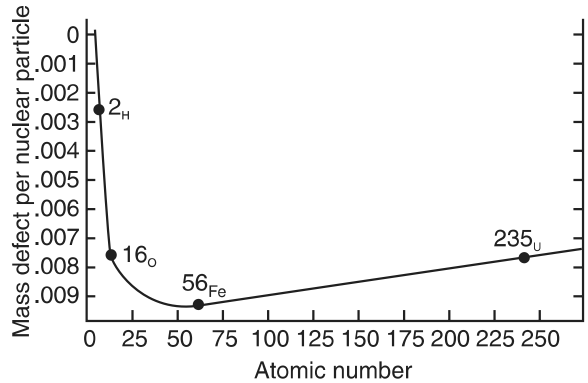

Fig. 21.2: Mass defect for different nuclides. The most stable nuclei center around Fe-56 which has the largest mass defect per nucleon.

For example, a $_2He^4$ nucleus is formed from two neutrons and two protons.

Mass of two protons	= 2 × 1.007826 = 2.015652 amu
Mass of two neutrons	= 2 × 1.008665 = 2.017330 amu
Total mass	= 4.032982 amu

Measured mass of $_2He^4$ = 4.002604 amu

∴ Mass defect Δm = (4.032982 – 4.002604) amu = 0.030378 amu

Atoms with atomic numbers between 30 and 63 have a greater mass defect per nuclear particle than very light elements or very heavy ones, as seen in Fig. 21.2. The most stable nuclei are in the atomic number range from 30 to 63.

21.7 BINDING ENERGY

The energy required to remove any nucleon from the nucleus is called the **binding energy** of that nucleon in the nucleus. It would be equal to the work that must be done in order to remove the nucleon from the nucleus without imparting it any kinetic energy. The total binding energy of a nucleus is defined as the energy required to break up the nucleus into its constituent protons and neutrons and place them at rest at infinite distances from one another. It means that the binding energy is the energy equivalent of the mass defect. Thus,

$$\Delta E_b = (\Delta m)c^2$$

$$\therefore \quad \Delta E_b = [Zm_p + (A–Z)m_n – M]c^2$$

The binding energy of the neucleus in MeV is expressed as

$$\Delta E_b = 931.4\ [Zm_p + (A–Z)m_n – M]\ MeV/u \quad (21.5)$$

The **average binding energy per nucleon** is given by

$$\Delta\xi = \Delta E_b/A \quad (21.6)$$

On the average, the binding energy per nucleon is found to be about 8 MeV. The nuclear binding energy ΔE_b depends mainly on the total number of nucleons in the nucleus. To a first approximation, it rises linearly with an increase in mass number A. It implies that each nucleon addition to a nucleus causes the liberation of about the same amount of energy. A plot of the average binding energy per nucleon as a function of mass number A is shown in Fig. 21.3.

The following important features are seen from the binding energy curve.

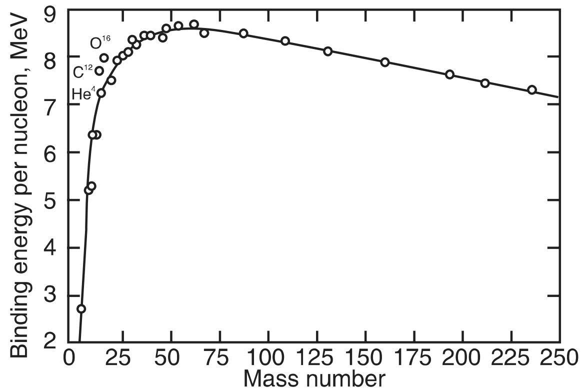

Fig. 21.3

1. The binding energies of very light nuclei such as ${}_1H^2$ are very small. In case of light nuclei, most nucleons will be in the nuclear surface and as such they will not be in a position to use all their bonds. Consequently, $\Delta E_b/A$ is lower in these cases.
2. The binding energy is high in the middle of the periodic table, for elements whose A is in the range $28 < A < 138$, i.e. from ${}_{14}Si^{28}$ to ${}_{56}Ba^{138}$. For these nuclei, the binding energy per nucleon is nearly 8.7 MeV.
3. The binding energy per nucleon decreases in case of elements having A>138. It is found to decrease to 7.6 *MeV* for uranium. The decrease may be attributed to the repulsive forces between protons, whose number increases in heavy nuclei.

Two most important conclusions may be drawn from the binding energy curve:

(*i*) If a heavy nucleus ($A \approx 240$) is divided into two intermediate nuclei ($A \approx 120$), the resulting nuclei will be more stable than the initial heavy nucleus.

(*ii*) If a single nucleus is synthesized from two light nuclei, again the resulting nucleus will be more stable than the intitial light nuclei.

The first one points out the possibility of nuclear fission while the second one points out the possibility of nuclear fusion. Both the fission and fusion processes will be accompanied by the release of great amounts of energy.

Example 21.1. *Calculate the binding energy of a nitrogen nucleus in MeV from the following data:*

m_H = 1.00783 u, *and* m_n = 1.00867 u *and* $m\left({}^{14}_{7}N\right) = 14.00307$ u

Solution. Mass defect ΔM = 7 × 1.00783 + 7 × 1.00867 − 14.00307 u = 0.11243 u

∴ Binding energy = 0.11243 u × 931.4 MeV/u = **104.7 MeV.**

Example 21.2. *What is the binding energy per nucleon in* ${}^{7}_{3}Li$ *nuclide?*

Proton mass, m_p *= 1.00814 amu*

Neutron mass, m_n *= 1.008665 amu*

Mass of lithium nucleus. M = 7.01822 amu

and *1 amu = 931 MeV*

Solution. Number of neutrons in lithium nucleus = 7 – 3 = 4

Total binding energy ΔE_b = [3 × 1.00814 + 4 × 1.008665 − 7.01822]931 *MeV* = **38.041 MeV**

$$\text{Binding energy per nucleon } \Delta\xi = \frac{\Delta E_b}{A} = \frac{38.041}{7}\text{MeV} = \mathbf{5.43\,MeV}$$

Example 21.3. *The atomic masses of* ${}^{15}_{7}N, {}^{16}_{8}O$ *and O are 15.0001u, 15.0030u and 15.9949u respectively. How much energy is needed to remove one proton from* ${}^{16}_{8}O$*? How much energy* *is needed to remove one neutron from* ${}^{16}_{8}O$ *? Why are these figures different from each other? (Proton rest mass = 1.0072766 u, Neutron rest mass = 1.0086654 u)*

Solution.

(*i*) ${}^{16}_{8}O - {}^{1}_{1}H \rightarrow {}^{15}_{7}N + Q$

Q = [15.9949 – 1.0072766 – 15.0001]931 MeV = [–0.0124766]931 MeV = **–11.62 MeV**

(*ii*) ${}^{16}_{8}O - {}^{1}_{0}n \rightarrow {}^{15}_{8}O + Q$

Q = [15.99.49 – 1.008654 – 15.0030]931 MeV = [–0.0167654]931 MeV = **–15.62 MeV**

Example 21.4. *Find the binding energy of an* $\propto$*-particle in MeV and Joule. Given : Mass of proton = 1.00758 amu, mass of neutron = 1.00897 amu and mass of helium nucleus = 1.0028 amu.*

Solution. ΔE_b = [2×1.00758 – 2×1.00897–4.0028]931 MeV

= 28.22 MeV = 28.22 × 10^6 × 1.602 × 10^{-19} J **= 4.5×10^{-12} J**

21.8 NUCLEAR MODELS

Models are devised to account for the properties and behaviour of a system. In case of an atom, Thomson's model, Rutherford model, Bohr model, de Broglie model, vector atom model and quantum mechanical model are devised which successively incorporated refinements one over the other. Ultimately, the quantum-mechanical model succeeded in interpreting all the properties and behaviour of an atom. In the same way efforts are made to develop a model, which can successfully explain the properties of the nucleus such as stability, spin, magnetic moment, etc. Various models have been proposed for nucleus. However, each of the models can explain only a limited number of properties of the nucleus. One of the earliest models was α- particle model proposed by Gamow.

21.8.1 Gamow Model

It is also known as α-particle model. According to this model, nucleus is assumed to have sub-groups in the form of α-particles. Each sub group has two protons and two neutrons. Hydrogen and deuterium nuclei are exceptions. The model was successful in explaining the emission of α-particles by the radioactive nuclei. The model is discarded subsequently.

Out of the other different models, two models namely *shell model* and *liquid drop model* are of importance and we study them here.

21.8.2 Nuclear Shell Model

Nuclear shell model is similar to the Bohr's model for the atom. By analogy with the closed sub-shells and shells in the case of atoms, it is assumed that nucleons also form similar closed sub-shells and shells within the nucleus. The electrons in an atom are supposed to revolve in the Coulomb electrostatic field of the nucleus in allowed orbits. In a similar manner, it is assumed in the shell model that each nucleon moves inside the nucleus in a fixed orbit under the influence of a central field of force produced by the average interaction between all the remaining nucleons.

It is found that the nuclear properties vary periodically with Z and N, in a way similar to the periodic variation of atomic properties with Z. Secondly, a nucleus is stable if it has a certain definite number of either protons or neutrons. Nuclei containing the following numbers of protons and neutrons exhibit high stability.

Z	2	8	20	50	82	
N	2	8	20	50	82	126