LAUC

Q.1 Show nullity, basis, dimension

1. $2x+y-z=0$
   $2x+y+z=0$

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}_{2\times 3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times 1}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$2z = 0$$

$$\boxed{z = 0}$$

$$2x + y - z = 0$$
$$2x + y = 0$$
$$2x = -y$$
$$\boxed{x = \frac{-y}{2}}$$

$\boxed{\begin{matrix}\text{Nullity} = 1 \\ \text{Rank} = 2\end{matrix}}$ Dimension - 1

No. of column = 3

Basis element : $\left\{ \begin{pmatrix} -1/2 \\ 1 \\ 0 \end{pmatrix} \right\}$

2. ~~2x~~ $x+y+z=0$
   $x-y=0$
   $y+z=0$

$1 - \frac{1}{2}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

~~$R_3 \to R_3 + 2R_2$~~

$$\cancel{\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & & \end{bmatrix}}$$

$R_2 \to -\frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1/2 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$\boxed{\begin{matrix} z = 0 \\ y = 0 \\ x = 0 \end{matrix}}$$

Rank = 3

Nullity = 0

Dimension = 0

Basis = (0, 0, 0)

3. $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}_{4\times 3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & -1 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 1 \\ 3 & 1 & 0 \\ 5 & 1 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 5R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 3 \\ 0 & -2 & 3 \\ 0 & -\cancel{5}4 & 6 \end{bmatrix}$$

$R_2 \rightarrow -5R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -3/5 \\ 0 & -2 & 3 \\ 0 & -4 & 6 \end{bmatrix}$$

~~Nullity = 0~~

~~Rank = 4~~

$3+2\times\left(-\frac{3}{5}\right)$

$R_3 \rightarrow R_3 + 2R_2$

$\therefore 3-\frac{6}{5}$

$R_4 \rightarrow R_4 + 4R_2$

$6+4\left(-\frac{3}{5}\right)$

$6-\frac{12}{5}$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -3/5 \\ 0 & 0 & 9/5 \\ 0 & 0 & 18/5 \end{bmatrix} \quad \cancel{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}$$

$z = 0 \quad y = 0 \quad x = 0$

$R_4 \rightarrow R_4 - 2R_3$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -3/5 \\ 0 & 0 & 9/5 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank = 3

Nullity = 0

Basis = (0, 0, 0)

Dimension = 0

4. $4x + 7y - 5z = 0$

$2x - y + z = 0$

$$\begin{bmatrix} 4 & 7 & -5 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 7 & -5 \\ 2 & -1 & 1 \end{bmatrix}$$

~~$R_1 \rightarrow R_2 \rightarrow 2R_2 - R_1$~~

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 2 & -1 & 1 \\ 4 & 7 & -5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 9 & -7 \end{bmatrix}$$

$9y - 7z = 0$

$2x - y + z = 0$

$9y = 7z$

$\boxed{y = \frac{7}{9} z}$

$\boxed{x = \frac{y - z}{2}}$

Rank = 2

Nullity = 1

Basis = $\begin{pmatrix} -1/9 \\ 7/9 \end{pmatrix}$

2

$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$ find nullity and rank

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$

$R_2 \to R_2 - 3R_1$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$

$-y - 2z = 0$

$-y = 2z$

$\boxed{y = -2z}$ — ①

$x = -2y \mp 3z$ — ②

Nullity = 1

Rank = 2

Basis = $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

No. of column = 3

According to theorm

No. of column = Nullity + Rank

L.H.S = 3

R.H.S = 1 + 2

= 3

L.H.S = R.H.S

Proved ...

Q.3 $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ Compute eigen value and eigen vecter of $A - 7I$. How are they related to those of A

$A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} = 0$

$(1-\lambda)(4-\lambda) + 2 = 0$

$4 - \lambda - 4\lambda + \lambda^2 + 2 = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda = +2$ $\boxed{\lambda = 3}$

$\boxed{\text{For } \lambda = 2}$

$(A - 2I)X = 0$

$\left(\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x_1 - x_2 = 0$

$2x_1 + 2x_2 = 0$

$\boxed{x_1 = -x_2}$

Eigen vector $= \{(x_1, x_2) : x_1 = -x_2 ;\ x_2 \in R\}$

$= \{(-x_2, x_2),\ x_2 \in R\}$

$= \{x_2(-1, 1),\ x_2 \in R\}$

~~for ei~~ Eigen vector $(-1, 1)$

for $\lambda = 3$

$$|A - 3I| x = 0$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - x_2 = 0$$
$$2x_1 + x_2 = 0$$

$$\boxed{x_1 = \frac{-x_2}{2}}$$

Eigen vector = $\left(-\frac{1}{2}, 1\right)$

~~AM = 1~~ $\qquad$ GM = 1

AM = 1 $\qquad$ GM = 1

Vector for $A - 7I$

$$\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 - x_2 = 0$$
$$2x_1 - 3x_2 = 0$$
$$x_1 = \frac{-x_2}{6}$$

$$\left(-\frac{1}{6}, 1\right)$$

2. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(-\lambda) = 0$$

$$-\lambda + \lambda^2 = 0$$

$$\cancel{\lambda^2 \rightarrow}$$

$$\lambda^2 - \lambda = 0$$

$$\lambda(\lambda - 1) = 0$$

$$\lambda = 0 \quad \text{or} \quad \lambda = 1$$

$$\boxed{\begin{matrix} AM = 1 \\ GM = 1 \end{matrix}}$$

For $\lambda = 1$

$(A - I)x = 0$

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~$x_1 = 0$~~ $\quad x_2 = 0$

~~x_2 = any arbitary~~ x_1 - any arbitary

~~(0, 1)~~ — Eigen vector $(1, 0)$

For $\lambda = 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

~~$x_1 = 0$~~ any arbitar

~~$x_2 = 0$~~ — ~~Eigen vector~~

~~(1, 0)~~

$x_1 = 0$

x_2 = any arbitary

$(0, 1)$

$$\cancel{P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$$

$$V_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$P^{-1} = \left[\begin{array}{cc|cc} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$P = P^{-1} A P$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

It is diagonazable.

3. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 0-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda^2 = 1$$

$\lambda = 1$ $\underline{\text{or}}$ $\lambda = -1$ $\underline{AM = GM = 1}$

For $\lambda = 1$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 = 0$$

$$x_1 - x_2 = 0$$

$$\boxed{x_1 = x_2}$$

$(1, 1)$ — Eigen vector

For $\lambda = -1$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 = 0$$

$$x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$(-1, 1)$ Eigen vector

$$\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & -2 \\ 2 & -2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)(-2-\lambda) + 4 = 0$$

$$-4 - 2\lambda + 2\lambda + \lambda^2 + 4 = 0$$

$$\cancel{\lambda^2 + 8 = 0}$$

$$\cancel{\lambda^2 - 2 = 0}$$

$$\cancel{\lambda^2 = 2}$$

$$\cancel{\lambda = \sqrt{2} \quad \lambda = -\sqrt{2}}$$

AM = 2
GM = 1

$$\lambda^2 = 0$$

$$\boxed{\lambda = 0}$$

$$\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 - 2x_2 = 0$$

$$\boxed{x_1 = x_2}$$

$(1, 1) \longleftarrow$ Eigen vector

It is not diagonazable

5. $\begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix}$

$$\begin{vmatrix} 2-\lambda & 0 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(-2-\lambda) = 0$$

$$-4 - 2\lambda + 2\lambda + \lambda^2 = 0$$

$$\lambda^2 - 4 = 0$$

$$\lambda^2 = 4$$

$\lambda = -2$

$\underline{AM = 1.}$

For $\lambda = 2$

$(A - 2I)\,X = 0$

$$\left[\begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 - 4x_2 = 0$$

$$2x_1 = 4x_2$$

$$\boxed{x_1 = 2x_2} \quad \text{—— (1)}$$

Eigen vector $= (2, 1)$

for $\lambda = -2$

$$\left[\begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$4x_1 = 0$$

$$\boxed{x_1 = 0}$$

x_2 = any arbitary

$\underline{(0, 1)}$ ← Eigen vector —— (2)

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = P$$

$$P^{-1} = \left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$$\cancel{\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{array}\right]}$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to 2R_2 - R_1$

$$\cancel{\left[\begin{array}{cc|cc} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{array}\right]}$$

$$\cancel{\left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 \end{array}\right]}$$

$1/2 \cdot \quad \frac{1}{2} \times 2$

$$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$$

Not diagonazable

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -2 \end{array}\right]$$

~~$R_1 \to 2R_1 + R_2$~~

$-2R_2$

$$\left[\begin{array}{cc|cc} 1 & 1 & 0 & 1 \\ 0 & 1 & -1/2 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$\left[\begin{array}{cc|cc} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1/2 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ +1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

It is diagonazable

LAUC

1. $\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\rightarrow \quad |A - \lambda I| = 0$

$$\begin{vmatrix} 3-\lambda & 4 & 2 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & -\lambda \end{vmatrix} = 0$$

$$3-\lambda \begin{vmatrix} 1-\lambda & 2 \\ 0 & -\lambda \end{vmatrix} - 4 \begin{vmatrix} 0 & 2 \\ 0 & -\lambda \end{vmatrix} + 2 \begin{vmatrix} 0 & 1-\lambda \\ 0 & 0 \end{vmatrix}$$

$3-\lambda \ (1-\lambda)(-\lambda)$

$(3-\lambda)\ \cancel{(1+\lambda^2)}\ (-\lambda+\lambda^2)$

$\underline{\lambda = 3}$ or $\cancel{\lambda^2 +}$ $\lambda^2 - \lambda = 0$

$\lambda(\lambda - 1) = 0$

For $\lambda = 3$

$(A - 3I)x = 0$

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 2 \\ 0 & -2 & 2 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 \rightarrow$ arbitary

$4x_2 + 2x_3 = 0$

$-3x_3 = 0$

$\boxed{x_3 = 0}$

$\boxed{x_2 = 0}$

Nullity = 1

Eigen vector = $(1, 0, 0)$

For $\lambda = 1$

$(A - I)x = 0$

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_3 = 0$

$\boxed{x_3 = 0}$

$2x_1 + 4x_2 = 0$

$x_1 + 2x_2 = 0$

$x_1 = -2x_2$

Eigen vector = $(-2, 1, 0)$

for $\lambda = 0$

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x_1 + 4x_2 + 2x_3 = 0$$

$$x_2 + 2x_3 = 0$$

$$\underline{x_2 = -2x_3}.$$

$$3x_1 + 4(-2x_3) + 2x_3 = 0$$

$$3x_1 - 8x_3 + 2x_3 = 0$$

$$3x_1 - 6x_3 = 0$$

$$x_1 - 2x_3 = 0$$

$$\underline{x_1 = 2x_3}$$

Eigen vector = $(2, -2, 1)$

2. $V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ $V_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 + 2R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 2 & 4 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$P^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = P^{-1} A P$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

It is diagonazable

4. $$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

$\rightarrow$ $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 0 & 2 \\ 0 & 2-\lambda & 0 \\ 2 & 0 & -\lambda \end{vmatrix} = 0$$

$$-\lambda \begin{vmatrix} 2-\lambda & 0 \\ \cancel{0} & -\lambda \end{vmatrix} + 2$$

$$-\cancel{\lambda}(2-\lambda)(-\lambda)$$

$$-\lambda(-2\lambda + \lambda^2) \qquad \lambda(\lambda - 2) = 0$$

$$\lambda^2 - 3\lambda = 0$$

$$\cancel{\lambda^2 - 3 \quad \lambda(\lambda-3) = 0}$$

$$\lambda = 0, \lambda = 2, \lambda = 0$$

$-\lambda(2-\lambda)(-\lambda) + 2 \begin{vmatrix} 0 & 2-\lambda \\ 2 & 0 \end{vmatrix}$ $\quad 2(0-4-2\lambda)$

$+2(-4-2\lambda)$

$-\lambda(2-\lambda)(-\lambda) + 2(-4-2\lambda)$ $\quad -8-4\lambda$

$-\lambda(2-\lambda)(-\lambda) + 8 - 4\lambda$

$\cancel{\lambda^2 - 3\lambda + 8 - 4\lambda}$

$-\lambda(2-\lambda)(-\lambda) + 4(2-\lambda)$

$2-\lambda\left[(-\lambda)(-\lambda) - 4\right]$

$\lambda = 2$ or $\cancel{\lambda^2 + 4 = 0}$ $\quad \lambda^2 - 4 = 0$

$\lambda = 2$ or $\cancel{\lambda^2 = -4}$ $\quad \lambda^2 = 4$

$\lambda = 2, -2$

$\boxed{\lambda = 2, -2, 2}$

AM = 2 . for $\lambda = 2$

GM = 2 for $\lambda = 2$

For $\lambda = 2$

$(A - 2I)x = 0$

$$\left[\begin{matrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{matrix}\right] - \left[\begin{matrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{matrix}\right]\left[\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right] = \left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right]$$

$$\left[\begin{matrix} -2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -2 \end{matrix}\right]\left[\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right] = \left[\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right]$$

x_2 = arbitary value $\quad (x_1, x_2, x_3) : x_2 \in R\}$

$(x_3, 0, x_3) : x_2 \in R$

$x_3(1,0,0\}$

$x_1 - x_3 = 0$ $\quad \cancel{1,1,1}$

$\boxed{x_1 = x_3}$ $\quad \cancel{0,1,0}$

$(x_1, x_2, x_3) : x_3$

Eigen vector - $(1,0,1)\ (0,1,0)$

$\cancel{x_1, x_2, x_3}$ $\quad \{(x_1, x_2, x_3)\}\ x_2 \in R$ (

$\cancel{\{x_3, x_2, x_3\}}$

For $\lambda = -2$

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2x_1 + 2x_3 = 0$

$4x_2 = 0$

$2x_1 + 2x_3 = 0$

$\boxed{x_2 = 0}$

$x_1 = -x_3$

~~At~~ $\cancel{-2x_3 + 2x_3} = 0$

$\cancel{-2x_3 = 0}$

$\cancel{x_3 = 0}$

Eigen vectors

$(-1, 0, 1)$

$V_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad V_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$P^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to 2R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 0 & 1/2 \end{array}\right]$$

$P = P^{-1}AP$

$$= \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

It is diagonazable.