

# CHAPTER

# 2

# *Electrostatics*

## 2.1 INTRODUCTION

If a glass rod is rubbed with silk, the rod acquires positive charge. If a hard rubber rod is rubbed with fur, the rod will acquire negative charge. This process is known as charging by friction. It means that the glass rod and rubber rods are *electrified*. Charged bodies attract each other if they have unlike charges or repel each other if they have like charges. It is subsequently discovered that electric charge is fundamentally associated with atomic particles, the electron and the proton. Electrons carry negative charge and protons carry a positive charge. Matter in its neutral state contains equal amounts of positive and negative charges. Accordingly, we interpret the electrification of the bodies as occurring due to transfer of charge. When two bodies are rubbed together, a redistribution of electrons takes place. The body which receives electrons becomes negatively charged while the body which loses electrons becomes positively charged. The study of electric forces between charged objects **at rest** is called **electrostatics**.

## 2.2 ELECTRIC CHARGES

The charges either positive or negative are always built up as a collection of elementary charges, carried by fundamental particles protons and electrons. They are always an integral multiple of the smallest unit of charge, that of an electron. When a body is said to be charged, it contains either an excess of electrons or a shortage of electrons. The charge residing on a charged body is

$$Q = \pm ne \quad (2.1)$$

where  $n$  is an integer taking values 1,2,3,....

The SI unit of charge is the Coulomb denoted by C. The value of  $e$  is

$$1e = 1.602 \times 10^{-19} \text{ C}$$

## 2.3 COULOMB'S LAW

If two charges are brought nearer, they exert forces on each other. We say that the charges are interacting. If the charges are at rest, their interaction is known as *electrostatic interaction*. The electrostatic interaction for two charged particles is given by Coulomb's law.

### The Law

*The electrostatic interaction between two charge particles is proportional to the square of the distance between them and its direction is along the line joining the two charges.*

If  $q_1$  and  $q_2$  are two **point charges at rest**, and are separated by a distance  $r$ , they exert a force on each other which is given by

$$F = k \frac{q_1 q_2}{r^2} \quad (2.2)$$

where  $k$  is a constant which has the following value in the SI system.

$$k = \frac{1}{4\pi\epsilon_0} \quad (2.3)$$

Here  $\epsilon_0$  is called the **permittivity of free space** and has the value

$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2 \\ \therefore k &= 9 \times 10^9 \text{ Nm}^2 / \text{C}^2 \end{aligned} \quad (2.4)$$

When the charges are located in a dielectric medium having a **relative permittivity** of  $\epsilon_r$ , the force is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{\epsilon_r r^2} \quad (2.5)$$

The Coulomb's force is maximum when the charges are in a vacuum and reduces when the charges are placed in a medium.

### Vector form of the law

The Coulomb's law (2.2) may be expressed in vector form as

$$\mathbf{F} = k \frac{q_1 q_2}{r^2} \hat{\mathbf{r}} = k \frac{q_1 q_2 \mathbf{r}}{r^3} \quad (2.6)$$

In the above equation,  $\hat{\mathbf{r}}$  is the unit vector along  $\mathbf{r}$ . It is given by  $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$ .

- In equ. (2.6),  $\mathbf{F}$  is the force produced by the charge  $q_1$  on the charge  $q_2$ . The force produced by the charge  $q_2$  on the charge  $q_1$  is therefore  $-\mathbf{F}$ .
- The charges are assumed to be at rest; otherwise we need to take into account the magnetic forces.
- The size of the charges must be very small compared to their separation. Hence, the charges are assumed to be point charges.

**Example 2.1:** Two equal and similar charges 3 cm apart in air repel each other with a force equivalent to 1.5 kg wt. Find the magnitude of the charges.

**Solution.** Now, 
$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$\therefore 1.5 = 9 \times 10^9 \times \frac{q_1^2}{(3 \times 10^{-2})^2} \quad \text{as} \quad \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^2$$

$$\therefore q_1^2 = \frac{1.5 \times 9 \times 10^{-4}}{9 \times 10^9} = 1.5 \times 10^{-13}$$

$$\therefore q_1 = 3.87 \times 10^{-7} \text{ C}$$

**Example 2.2:** Two point charges of 1 C each are separated from each other by a distance of 1 m in a vacuum.

(a) What is the force of their interaction?

(b) What will be the force if the medium between the charges is water?

**Solution:** 
$$F = \frac{q_1 q_2}{4\pi\epsilon_0 \epsilon_r r^2}; \quad \text{For air } \epsilon_r = 1 \text{ and for water } \epsilon_r = 80.$$

$$(a) \therefore F_{\text{air}} = 9 \times 10^9 \times \frac{1 \times 1}{1} = 9 \times 10^9 \text{ N}$$

$$(b) F_{\text{water}} = 9 \times 10^9 \times \frac{1 \times 1}{80 \times 1} = 1.1 \times 10^8 \text{ N}$$

## 2.4 PRINCIPLE OF SUPERPOSITION

If a number of point charges  $q_1, q_2, q_3, \dots$  are present in a region, then the total force on any particular charge is the vector sum of forces it experiences due to all other charges. This is called the **principle of superposition**. For the sake of simplicity, let there be only three charges. The force on  $q_3$  is

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_{13} + \mathbf{F}_{23} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{|r_{13}|^3} \mathbf{r}_{13} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{|r_{23}|^3} \mathbf{r}_{23} \end{aligned} \quad (2.7)$$

Generalizing, one finds the force acting on a charge  $q_j$  due to a number of other charges present in the region is

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{|r_{ij}|^3} \mathbf{r}_{ij} \quad (2.8)$$

## 2.5 ELECTRIC FIELD

Any region where an electric charge experiences a force is called an **electric field**. The force is due to the presence of other charged body in that region. For example, a charge  $q$  is placed in a region where a charged body  $Q$  is present. According to the standard convention, we take always  $Q$  to be positive. The charge  $q$  experiences a force  $\mathbf{F}$  and we say that it is in an electric field produced by the charge  $Q$ . The force that the charge  $Q$  produces is proportional to  $q$ . Thus, the force on a charged particle placed in an electric field is proportional to the charge of the particle.

$$F \propto q$$

or

$$\mathbf{F} = q\mathbf{E} \quad (2.9)$$

where  $E$  is the proportionality constant and is known as **electric field strength** or **intensity**. Note that electric field  $\mathbf{E}$  is a vector quantity since force  $\mathbf{F}$  is a vector quantity. The direction of  $\mathbf{E}$  is along the direction of force, that is along the line joining  $Q$  to  $q$ .

### Electric Field Intensity

The intensity of electric field at a point in the electric field is equal to the force per unit test charge placed at that point. Thus,

$$\mathbf{E} = \frac{\mathbf{F}}{q} \quad (2.10)$$

If  $q$  is positive, the force  $\mathbf{F}$  acting on the charge has the same direction as that of the electric field  $\mathbf{E}$ . If  $q$  is negative, the force  $\mathbf{F}$  acting on the charge has the direction opposite to the electric field  $\mathbf{E}$ .

Ideally,  $q$  must be as small as possible in order to avoid possible disturbance of the original field  $\mathbf{E}$ . For this reason, the following definition of  $\mathbf{E}$  is more commonly used.

$$E = \lim_{q \rightarrow 0} \frac{F}{q} \quad (2.11)$$

In SI system of units, the unit of electric field is Newton/Coulomb or N/C.

In the expression (2.5), let  $q_1 = Q$ ,  $q_2 = q$  and the medium be air,  $\epsilon_r = 1$ . Then the force produced by the charge  $Q$  on the charge  $q$  placed at a distance  $r$  from  $Q$  is given by

$$\mathbf{F} = q \left[ \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} \right] \quad (2.12)$$

Comparing the above expression (2.12) with (2.9), we may say that the electric field  $E$  at the point where  $q$  is placed is

$$\mathbf{E} = \left[ \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} \right]$$

Therefore, the field produced by the charge  $Q$  is

$$\mathbf{E} = \left[ \frac{Q}{4\pi\epsilon_0 r^2} \hat{\mathbf{r}} \right] \quad (2.13)$$

From the above equation, we see that the electric field decreases as one moves away from the charged body  $Q$ . It reduces to zero at infinity.

### 2.5.1 Electric Field Due to a Group of Point Charges

The net electric field at a point due to a group of point charges can be found by applying the superposition principle. Since the Coulomb force obeys the superposition principle, the electric field intensity (force per unit charge) obeys the superposition principle. The electric field at a point  $P$  due to  $n$  point charges  $q_1, q_2, q_3, \dots, q_n$  is equal to the vector sum of electric fields due to  $q_1, q_2, q_3, \dots, q_n$  at point  $P$ . Thus, the resultant electric field is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots + \mathbf{E}_n \quad (2.14)$$

where  $\mathbf{E}_1$  is the electric field intensity at  $P$  due to  $q_1$ ,

$\mathbf{E}_2$  is the electric field intensity at  $P$  due to  $q_2$ ,

$\mathbf{E}_3$  is the electric field intensity at  $P$  due to  $q_3$ , and so on.

**Example 2.3:** A force of  $0.015 \text{ N}$  acts upon a charge of  $2 \times 10^{-7} \text{ C}$  at a point in an electric field. What is the strength of electric field at that point?

**Solution:** Electric field  $E$ , is given by  $E = F/q$

$$\therefore E = \frac{0.015 \text{ N}}{2 \times 10^{-7} \text{ C}} = 7.5 \times 10^4 \text{ N/C}$$

## 2.6 COMPUTATION OF ELECTRIC FIELD IN SOME SPECIFIC CASES

### 1. Field due to a linear charge

Let us consider an infinitely long charged wire of negligible thickness and having a constant linear charge density  $\lambda$ . Let a point  $P$  be at a distance  $y$  from the wire, as shown in Fig. 2.1.

It is required to find the electric field intensity at  $P$ . Let us assume that the wire is made up of a number of infinitely small elements of length,  $dx$ . Let one of such elements be at a distance  $x$ , as shown in Fig. 2.1. Let the small charge on element be  $dq$ .

$$dq = \lambda dx$$

The field due to this charge  $dq$  at point is

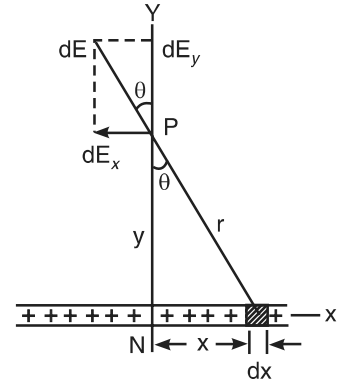
$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{(NP)^2} = \frac{dq}{4\pi\epsilon_0 r^2}$$

The  $x$  and  $y$  components of  $d\mathbf{E}$  are:

$$dE_x = -dE \sin \theta \quad \text{and} \quad dE_y = -dE \cos \theta$$

The  $x$ -components of field at P cancel out each others effect. Therefore, the net field will be due to  $y$ -components only and is directed along  $y$ -axis.

The resultant field is



**Fig. 2.1**

$$\begin{aligned} E &= \int_{x=-\infty}^{x=+\infty} dE_y = \int_{x=-\infty}^{x=+\infty} dE \cos \theta = \int_0^\infty 2 dE \cos \theta = \int_0^\infty \frac{2 dq}{4\pi\epsilon_0 r^2} \cos \theta \\ &= \int_0^\infty \frac{2 \lambda dx}{4\pi\epsilon_0 r^2} \cos \theta = \frac{\lambda}{2\pi\epsilon_0} \int_0^\infty \frac{dx}{r^2} \cos \theta \end{aligned}$$

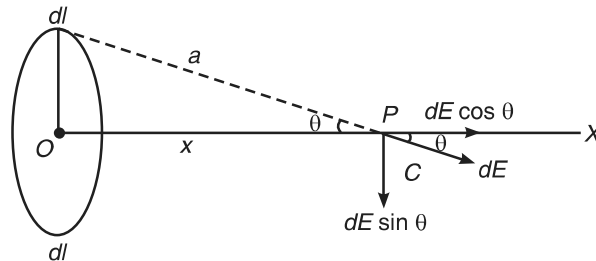
From Fig. 2.1, we have  $\frac{x}{y} = \tan \theta$

$$\therefore \frac{dx}{y} = \sec^2 \theta d\theta \quad \text{or} \quad dx = y \sec^2 \theta d\theta. \quad \text{Also, } x^2 + y^2 = r^2.$$

$$\begin{aligned} \therefore E &= \frac{\lambda}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{y \sec^2 \theta d\theta}{x^2 + y^2} \cos \theta = \frac{\lambda}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{y \sec^2 \theta d\theta}{y^2 \tan^2 \theta + y^2} \cos \theta \\ &= \frac{\lambda}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{y \sec^2 \theta d\theta}{y^2 \sec^2 \theta} \cos \theta = \frac{\lambda}{2\pi\epsilon_0} \int_0^{\pi/2} \frac{\cos \theta}{y} d\theta \\ &= \frac{\lambda}{2\pi\epsilon_0 y} [\sin \theta]_0^{\pi/2} \end{aligned}$$

$$\text{or} \quad E = \frac{\lambda}{2\pi\epsilon_0 y} \quad (2.15)$$

## 2. Field due to a uniformly charged ring at an axial point



**Fig. 2.2**

Let us consider a uniformly charged ring of charge  $q$  and radius  $a$  is shown in Fig. 2.2. Let P be a point on the axis of the ring at a distance  $x$  from its centre. If a positive charge  $q$  is on the

ring, then the charge per unit length of the ring will be  $\lambda = q / 2\pi a$ . Now let us consider a small element of the ring of length  $dl$ . The charge on the element  $dl$  is  $\frac{q dl}{2\pi a}$ .

$$\text{Electric field due to this element is given by } dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q dl}{2\pi a}$$

This field due to small element can be resolved into two components along  $x$ -axis (axis of the ring) and  $y$ -axis (perpendicular to the axis). Owing to symmetry, the perpendicular components cancel out. Hence, the resultant field will be due to the components along the axis of the ring. Therefore, the resultant field is given by

$$E = \int dE = \int dE \cos \theta$$

$$\therefore E = \int \frac{1}{4\pi\epsilon_0} \frac{q dl}{2\pi a} \frac{1}{r^2} \cos \theta$$

$$\text{From Fig. 2.2, we have } a^2 + x^2 = r^2 \quad \text{and} \quad \cos \theta = \frac{x}{\sqrt{a^2 + x^2}}.$$

$$\therefore E = \int \frac{1}{4\pi\epsilon_0} \frac{q dl}{2\pi a} \frac{1}{(a^2 + x^2)} \cdot \frac{x}{\sqrt{a^2 + x^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q x}{2\pi a} \frac{1}{(a^2 + x^2)^{3/2}} \int dl$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q x}{2\pi a} \frac{1}{(a^2 + x^2)^{3/2}} \times 2\pi a$$

$$\text{or} \quad E = \frac{1}{4\pi\epsilon_0} \frac{q x}{(a^2 + x^2)^{3/2}} \quad (2.16)$$

### 3. Field due to a uniformly charged disc

A disc of radius ' $a$ ' units is charged uniformly with a charge density  $\sigma$  C/m<sup>2</sup>. Let P be at a distance  $r$  from the centre of the disc as shown in Fig. 2.3. The disc may be regarded as formed by several annular rings of increasing radius. Let us consider a ring of radius  $x$  and the area of the hatched ring be  $ds$ .

Electric field due to the hatched ring is

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\sigma ds}{r^2} \cos \theta$$

$$\text{The area of the annular ring } ds = \pi \left[ (x + dx)^2 - x^2 \right] = \pi(2x dx + dx^2) \cong 2\pi x dx.$$

The term  $dx^2$  is negligible compared to  $x$ .

$$\therefore dE = \frac{1}{4\pi\epsilon_0} \frac{\sigma 2\pi x dx}{r^2} \cos \theta = \frac{\sigma x dx}{2\epsilon_0 r^2} \cos \theta$$

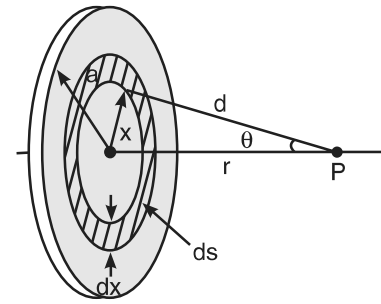


Fig. 2.3

From Fig. 2.3, it is seen that  $\cos \theta = d/r \quad \therefore \quad r = d \sec \theta$

$$\tan \theta = x/d \quad \therefore \quad x = d \tan \theta \quad \text{and} \quad dx = d \sec^2 \theta d\theta$$

Using these expressions

$$dE = \frac{\sigma(d \tan \theta) \cdot (d \sec^2 \theta d\theta)}{2\epsilon_0 d^2 \sec^2 \theta} \cos \theta = \frac{\sigma \sin \theta d\theta}{2\epsilon_0}$$

Electric field due to the entire disc is obtained by integrating the above expression between the limits of  $\theta$  from 0 to  $\alpha$ .

$$\therefore \quad E = \frac{\sigma}{2\epsilon_0} \int_0^\alpha \sin \theta d\theta = \frac{\sigma}{2\epsilon_0} [-\cos \theta]_0^\alpha = \frac{\sigma}{2\epsilon_0} [1 - \cos \alpha]$$

$$\text{From Fig. 2.3, we have } \cos \alpha = \frac{d}{\sqrt{a^2 + d^2}}.$$

$$\therefore \quad E = \frac{\sigma}{2\epsilon_0} \left[ 1 - \frac{d}{\sqrt{a^2 + d^2}} \right] \quad (2.17)$$

$$\text{If } d \gg a, \quad E = \frac{\sigma}{2\epsilon_0} \quad (2.18)$$

#### 4. Field due to an electric dipole

A system of two equal and opposite charges separated by a small distance is called an **electric dipole**.

Fig. 2.4 shows two charges  $q$  separated by a distance  $2a$ .

##### (i) Field at an axial point of the dipole

Let P be a point on the axis of the dipole at a distance  $x$  from the centre of the dipole. The field due to the negative charge  $-q$  at P is given by

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x-a)^2}$$

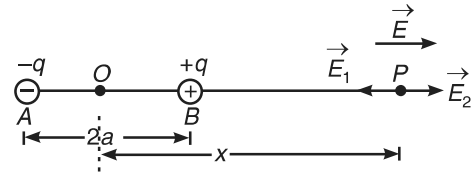


Fig. 2.4

The field due to the positive charge  $q$  at P is given by

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x+a)^2}$$

The resultant intensity  $E = E_1 - E_2$

$$\begin{aligned} &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x-a)^2} - \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(x+a)^2} \\ &= \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{(x-a)^2} - \frac{1}{(x+a)^2} \right] = \frac{q}{4\pi\epsilon_0} \left[ \frac{(x+a)^2 - (x-a)^2}{(x-a)^2 (x+a)^2} \right] \\ &= \frac{q}{4\pi\epsilon_0} \left[ \frac{4ax}{(x-a)^2 (x+a)^2} \right] \end{aligned}$$

or 
$$E = \frac{qax}{\pi\epsilon_0(x^2 - a^2)^2} \quad (2.19)$$

If  $x \gg a$ , 
$$E = \frac{qa}{\pi\epsilon_0 x^3} = \frac{2aq}{2\pi\epsilon_0 x^3}$$

or 
$$E = \frac{\mu}{2\pi\epsilon_0 x^3} \quad (2.20)$$

In the above  $\mu = 2aq$  is known as the electric **dipole moment**.

- The resultant field **E** is along the axis of the dipole and is directed from  $-q$  to  $q$ .
- The electric field due to a dipole is inversely proportional to cube of the distance.

**(ii) Field at a point on the perpendicular bisector of the dipole**

Let P be a point on the perpendicular bisector of the dipole as shown in Fig. 2.5.

Let  $E_1$  and  $E_2$  be the fields due to  $-q$  and  $q$  respectively. The fields due to the charges at P are given by

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2 + x^2}$$

The resultant intensity  $E = E_1 + E_2 = 2E_1 \cos \theta$ .

From the Fig. 2.5, we have  $\cos \theta = \frac{a}{\sqrt{a^2 + x^2}}$

$$E = 2 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{a^2 + x^2} \cdot \frac{a}{\sqrt{a^2 + x^2}} = \frac{2aq}{4\pi\epsilon_0(a^2 + x^2)^{3/2}}$$

or 
$$E = \frac{\mu}{4\pi\epsilon_0(a^2 + x^2)^{3/2}} \quad (2.21)$$

If  $x \gg a$ , 
$$E = \frac{\mu}{2\pi\epsilon_0 x^3} \quad (2.22)$$

- The resultant field **E** is parallel the axis of the dipole and is directed from  $q$  to  $-q$ .
- The electric field due to a dipole is inversely proportional to cube of the distance.

**Example 2.4:** A very large sheet of charge has density of  $5 \mu \text{ C/m}^2$ . Determine the electric field at a distance of 25 cm. Take medium as air.

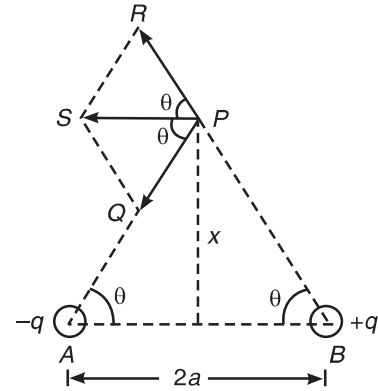
**Solution:** Now,  $\phi = \frac{q}{4\pi r^2}$

$\therefore q = 4\pi\phi r^2 = 4\pi \times 5 \times 10^{-6} \times (0.25)^2 = 1.57 \times 10^{-5} \text{ C}$

But 
$$E = \frac{q}{4\pi\epsilon_0 r^2} = \frac{1.57 \times 10^{-5}}{4\pi \times 8.854 \times 10^{-12} \times (0.25)^2} = 2.258 \times 10^6 \text{ N/C}$$

**Example 2.5:** A charged sphere of  $80 \mu \text{ C}$  is placed in air. Find the electric field intensity at a point 20 cm from the centre of the sphere.

**Solution:** Now, 
$$E = \frac{q}{4\pi\epsilon_0 \epsilon_r r^2} = 9 \times 10^9 \times \frac{80 \times 10^{-6}}{1 \times (0.2)^2} = 1.8 \times 10^5 \text{ N/C}$$



**Fig. 2.5**



**Example 2.6:** The charge per unit length on a long straight filament is  $-70 \mu\text{C/m}$ . Find the electric field at a distance 30 cm from the filament.

**Solution:** From Gauss' law, the electric field at a distance  $r$  from a long wire, is given by,

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2 \times \frac{1}{4\pi\epsilon_0} \times \frac{\lambda}{r}$$

$$\therefore E = 2 \times (9 \times 10^9) \times \frac{-70 \times 10^{-6}}{0.3} = -4.2 \times 10^6 \text{ N/C.}$$

## 2.7 ELECTROSTATIC POTENTIAL

When an electric charge is moved towards a like charge or away from an unlike charge, work is done against the electric forces by the external agency that moves the charge. As a result, the electrical charge acquires **potential energy**. If the charge is released, work is done by the field and the charge accelerates. It means that its potential energy is converted into kinetic energy. In mechanics work done on a particle is expressed in terms of changes in potential energy. In case of electric field also, the work done on a charge can be expressed in terms of the potential energy of the charge.

### Work done by the electric field on a charge

Let us consider a positive point test charge  $q$  placed at point A in a uniform electric field,  $\vec{E}$ . The force on the charge due to electric field is  $q\vec{E}$ . If we wish to move the charge with constant velocity *against* the electric field from point A to point B, we must exert a force of  $qE$ . The work done by the external force is positive and is equal to

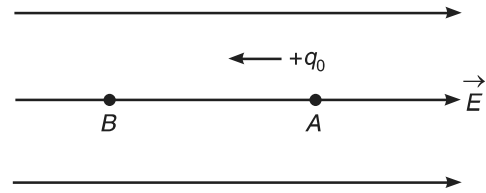


Fig. 2.6

$$W_{\text{ext}} = F_{\text{ext}}x \cos 0^\circ = +F_{\text{ext}}x = +qEx$$

At the same time, the work done by the electric field on the charge is negative since force and displacement are in opposite direction. The work done by the field is

$$W_E = F_{\text{ext}}x \cos 180^\circ = -F_{\text{ext}}x = -qEx$$

By moving the charge from point A to point B, the external force increases the **electric potential energy**,  $U$  by an amount equal to the work done on the charge.

The change in potential energy of the charge is  $\Delta U = U_B - U_A$

$$\therefore \Delta U = U_B - U_A = qEx$$

where  $U_A$  and  $U_B$  are the potential energies of the charge at location A and location B respectively.

In terms of the work done *by the electric field*, the change in potential energy of the charge may be expressed as

$$\Delta U = U_B - U_A = -(-qEx) = -W_E \quad (2.23)$$

Thus, if a charge moves from one point to another in an electric field, the difference in the electric potential energy of the charge between the points is the negative of the work done by the electric field on that charge.

### Line Integral

As the charge moves a distance  $dx$  along the path from A to B, the electric field does an element of work  $dW$  on it.

$$dW = -\mathbf{F} \cdot d\mathbf{x} = -q\mathbf{E} \cdot d\mathbf{x}$$

∴ The total work done by the field is

$$W_{AB} = -q \int_A^B \mathbf{E} \cdot d\mathbf{x} \quad (2.24)$$

The integral in the above equation is called the **line integral**.

### Electric potential difference

The electric field may be characterized by eliminating the dependence on the test charge  $q$  by defining **electric potential difference** between any two points in an electric field as the change in potential energy per unit positive test charge. We have

$$\begin{aligned} W_{AB} &= - \int_A^B \mathbf{F} \cdot d\mathbf{x} \\ \therefore W_{AB} &= - \frac{Qq}{4\pi\epsilon_0} \int_{x_1}^{x_2} \frac{1}{x^2} dx \\ &= - \frac{Qq}{4\pi\epsilon_0} \left[ \frac{1}{x_1} - \frac{1}{x_2} \right] \\ &= (U_B - U_A) \end{aligned} \quad (2.25)$$

Therefore, the work done per unit charge is

$$\frac{W_{AB}}{q} = \frac{U_B}{q} - \frac{U_A}{q} = V_B - V_A \quad (2.26)$$

where  $V_A = U_A/q$  is the potential energy per unit charge at location A and similarly  $V_B = U_B/q$  is the potential energy per unit charge at location B. Then,

$$\Delta V = \frac{\Delta U}{q} = \frac{W}{q} \quad (2.27)$$

From now onwards, we denote  $W_{AB} = W_E$  by  $W$ .

### Electric potential

A charged particle placed in an electric field has potential energy because of its interaction with the field. The potential energy  $U$  of a charge at any point is equal to the negative of the work done on the charge by the electric field as the charge moves from infinity to that point in the field. Usually the point A is chosen at infinity and hence  $V_A = 0$ . Then we denote  $V_B$  by  $V$ . Using equ. (2.26), we define the electric potential at a point as the potential energy per unit charge placed at that point. Thus,

$$V = \frac{U}{q} \quad (2.28)$$

where  $U$  is the potential energy.

The electric potential is measured in unit of volts, which is equal to Joules/Coulomb or J/C.

$$1 \text{ V} = \frac{1\text{J}}{1\text{C}}$$

Electric potential is a scalar quantity and hence it is often referred to as **electrostatic scalar potential**.