COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)

END Semester Examination

Programme: B.Tech **Semester: III**

Course Code: CT-16006 **Course Name: Discrete Structures and Graph Theory**

Branch: CE and IT **Academic Year: 2022-23**

Duration: 3 Hr **Max Marks: 60**

Student MIS No. /PRN

1	4	2	2	0	3	0	1	3

Instructions:

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of stationery, calculator etc. not allowed.
5. Write your PRN Number on Question Paper.

Section A

		Marks	CO	PO
Q1	**Attempt All of the following**			
a	Let the universe be a social club, and let x and y range over the members of the club. Define the predicate P(x, y) as P(x, y) := x likes y. Translate the following quantified predicates into English sentences i. $\forall x \forall y$ P(x, y) ii. $\exists x \exists y$ P(x, y) iii. $\forall x \exists y$ P(x, y) iv. $\exists x \forall y$ P(x, y)	4	1	1
b	Prove following by using Mathematical induction such that for all $n \geq 1$, the sum of the squares of the first 2n positive integers is given by the formula $1^1 + 2^2 + 3^2 + \ldots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$	3	1	2
	OR			
b	Solve the following recurrence relation $a_r = 6a_{r-1} - 8a_{r-2}$	3	2	2
c	Consider Premises: If Kiran has wide support, then he'll be asked to run for the senate. If Kiran yells "Eureka" in Iowa, he will not be asked to run for the senate. Kiran yells "Eureka" in Iowa. Conclusion: Kiran does not have wide support. Determine whether the conclusion follows logically from the premises. Explain by representing the statements symbolically and using rules of inference.	3	2	1
d	Let A ={1,2,3,4,5} and let M_R and M_S be the matrices of the relations R and S on A. Compute i. $M_R \circ M_S$ ii. $M_R \circ M_R$	4	2	1

$$M_R=\begin{bmatrix}1&1&0&0&1\\0&0&0&1&0\\1&1&0&0&1\\0&1&0&1&1\\1&0&0&0&0\end{bmatrix} \text{ and } M_S=\begin{bmatrix}0&0&0&1&1\\1&0&0&0&1\\0&1&0&1&0\\1&1&0&1&1\\1&0&1&0&0\end{bmatrix}$$

Q 2 **a** You decide to have a dinner party. Even though you are incredibly popular and have 14 different friends, you only have enough chairs to invite 6 of them. — 2,3 — 1

i. How many choices do you have for which 6 friends to invite? — 2

ii. What if you need to decide not only which friends to invite but also where to seat them along your long table? How many choices do you have then?y — 2

b If 30 dictionaries in library contain a total number of 61,327 pages then find out minimum number of pages in one of the dictionaries must have in it. — 2 — 3 — 4

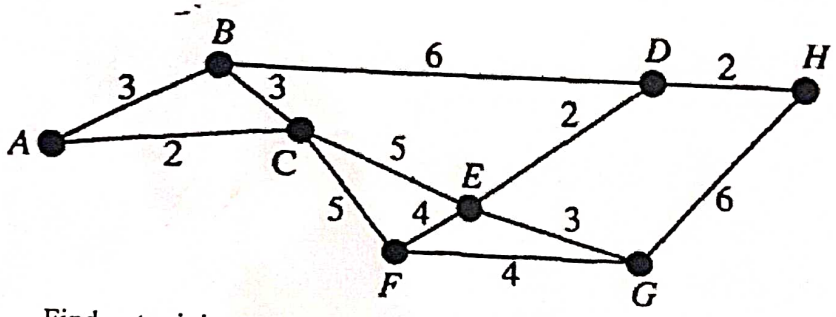
c Solve and answer the following using Binomial Coefficients for the equation as $(2x - 5y)^6$ — 3 — 1

i. Value of coefficient of the last term. — 1

ii. Find the 5^{th} term. — 2

iii. Determine the pascal's triangle for all powers of given equation. — 3

Q 3 **a** The small town of social circle maintains a system of walking between the recreational areas in town. The system is modelled by the weighted graph in following figure, where the weights represent the distances in kilometres between sites. — 4 — 1



i. Find out minimum spanning tree using Prim's Algorithm. — 3

ii. What will be the total weight of the minimum spanning tree using Breadth first search if we start with A. Draw tree. — 3

OR

a For the same graph in above question, answer, the following: — 4 — 1

i. Find out minimum spanning tree using Kruskal's Algorithm. — 3

ii. What will be the total weight of the minimum spanning tree using Depth first search if we start with A. Draw tree. — 3

b A tropical fish hobbist had six different types of fish: Alphas, Betas, Certas, Deltas, Epsalas and Fetas which are designated by A, B, C, D, E and F respectively. Because of water conditions and size only some types of fishes can survive with some other types of fishes in the same tank. The following table — 4 — 12

gives information about fishes that cannot be together

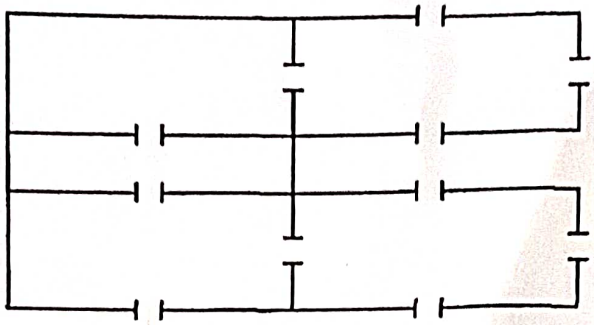
Type	A	B	C	D	E	F
Cannot be with	B,C	A,C	A,B,D,E	B,C	C,F	E

Our task is to arrange the fishes in a minimum number of tanks

i. Draw a graph for the fishes can survive together as from the above table 1

ii. Find out minimum number of tanks required to arrange the fishes based on above situation 2

c An art museum has arranged its current exhibition in the five rooms shown in below diagram. 4 12



i. Is there a way to tour the exhibit so that you pass through each door exactly once? 1

ii. Determine which type of tour it is ? 1

iii. Give a sketch of your tour in a graph , where each open door is considered as a vertex 1

Q 4 a If $6x \equiv 23 \bmod 31$, 3 1

i. Calculate multiplicative inverse for 6 mod 31 using extended Euclidean algorithm (bezout's coefficient) 2

ii. Find value of x 1

iii. Find set of different values that x can have 1

b A bag has some pens, If these pens were equally distributed to 4 3 1

- Three students, then two pens left in the bag,
- Five students, then four pens left in the bag,
- Seven students, then five pens left in the bag.

Find the minimum number of pens in the bag.

c Find integers p and q such that $1124p + 84q$ also find the GCD(1124, 84) using extended Euclidian algorithm. 4 3 4

Q 5 a Complete the given table so that the binary operation * is associative. Show the associative property is true for all elements in this table. 2 5 4

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c				
d	d	c	c	d

b If an algebraic structure is defined on a set of positive integers N with respect to binary operation a*b=lcm(a,b), ∀ a,b ∈ N. 5 1

i. Whether the algebra holds closure, associative property? If yes explain 2

ii. If exists, find out the identity element for the algebra 1

c Consider the algebra on a set S= {1,3,6,9,12} is GCD(a,b), where a,b ∈ S determine whether the set together with the binary operation is 5 1

i. Determine coposition table for the algebra on the operation GCD 1

ii. Determine whether the algebra is semigroup, or a monoid, or both with respect to its properties? 2

iii. If it is a monoid, specify the identity, If it is a semigroup or monoid, determine if its is commutative? 2

---*---All the best---*---