142103012

FCS: Assignment - 5

Q1) $P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$

R Using the method

s^5	1	5	1
s^4	3	4	3
s^3	11/3	0	0
s^2	4	3	
s^1	-11/4	0	
s^0	3		

1) As there are 2 sign changes 2 poles are on right half plane.
2) Remaining 3 poles are on left half plane
3) No poles on the J-ω axis

Q2) $T(s) = \dfrac{s+8}{s^5 - s^4 + 4s^3 - 4s^2 + 3s - 2}$

$$1 + T(s) = \frac{1 + s + 8}{s^5 - s^4 + 4s^3 - 4s^2 + 3s - 2} = 0$$

$$= s^5 - s^4 + 4s^3 - 4s^2 - 4s + 6 = 0$$

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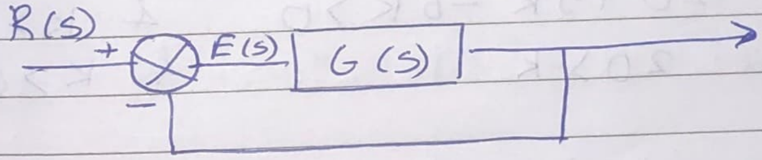
Routh's array

s^5	1	4	4
s^4	-1	-2	6
s^3	2	10	0
s^2	3	6	
s^1	6	0	
s^0	6		

1) As there are 2 sign changes, 2 poles are on the right half plane
2) Remaining 3 poles are on the left half plane
3) No poles on the $j\omega$ axis

Q3)



$$G(s) = \frac{K(s+6)}{s(s+1)(s+4)}$$

It is a unity feedback system

$$\therefore \quad GH = \frac{K(s+6)}{s(s+1)(s+4)}$$

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Characteristics equation $= 1 + GH$

$$= 1 + \frac{K(S+6)}{S(S+1)(S+4)}$$

$$= S(S+1)(S+4) + K(S+6)$$
$$= (S^2+S)(S+4) + KS + 6K$$
$$= S^3 + S^2 4 + S^2 + 4S + SK + 6K$$
$$= S^3 + 5S^2 + 4S + SK + 6K$$
$$= S^3 + 5S^2 + S(4+K) + 6K$$

Routh Array

S^3	1	$4+K$
S^2	5	$6K$
S^1	$5(4+K)-6K$	0
S^0	$6K$	

For system to be stable:

$$5(4+K) - 6K > 0 \quad \& \quad 6K > 0$$
$$20 + 5K - 6K > 0 \quad \& \quad K > 0$$
$$20 > K \quad \& \quad K > 0$$

Range of K: $0 < K < 20$

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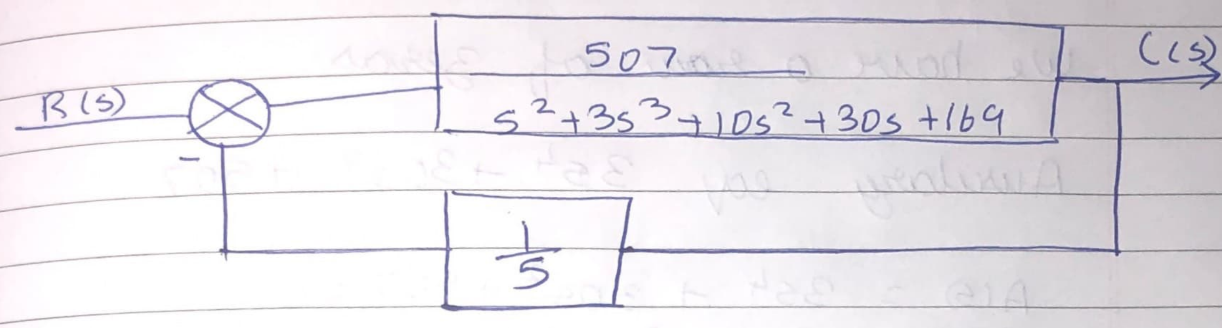
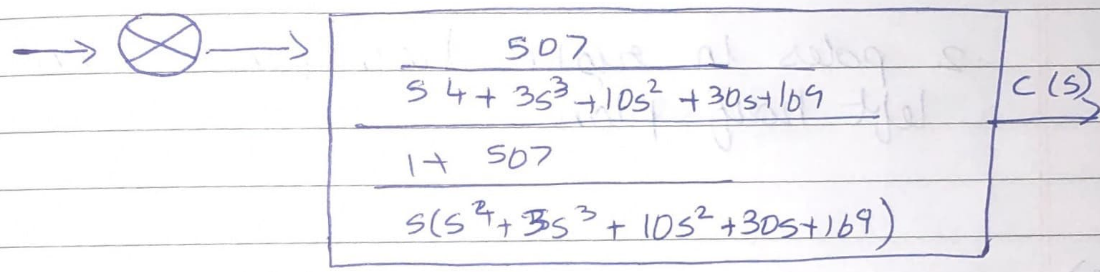
Q4





characteristic polynomial

$$= \frac{1+507}{s\,(s^4+3s^2+10s^2+30s+169)}$$

$$= s^5+3s^4+10s^3+30s^2+169+507$$

s^5	1	10	169
s^4	3	30	507
s^3	0/12	0/60	0
s^2	15	507	0
s^1	-345.6	0	0
s^0	507	0	0

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We have a row of zeros

Auxilary eq $3s^4 + 30s^2 + 507$

$A(s) = 3s^4 + 30s^2 + 507 = 0$

$\frac{d(A(s))}{ds} = 12s^3 + 60s$

2 poles in right half plane and 3 in left half plane

(Q5)

$$T(s) = \frac{Ks^4 + 2Ks^2 + 4Ks + 8K}{(k+1)s^2 + 2(1-k)s^2 + 2k+4}$$

Since all coefficients must be positive for stability in second order polynomial

$-1 < k < 1$; $k < 1$; $\frac{-1}{2} < k$

$\therefore \frac{-1}{2} < k < 1$

$$\boxed{\frac{-1}{2} < k < 1}$$

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Q6) $G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$

1) $25u(t)$ → It is a unit singnal

$L.T = \frac{25}{s} = R(s)$

$\therefore e_{step} = \lim_{s \to 0} s \frac{R(s)}{1+G(s)} = \frac{s+25/s}{1+G(s)} = \frac{25}{1+\lim_{s \to 0} G(s)}$

$e_{step} = \frac{25}{1+\infty} = \frac{25}{\infty} = 0$

Zero steady state error

2) $35u(t)$ → ramp signal

$R(s) = \frac{37}{s^2}$

$e_{ram} \lim_{s \to 0} \frac{s \times 37/s^2}{1+G(s)} = \lim_{s \to 0} \frac{3}{s+sG(s)}$

$= \frac{3}{0+\lim_{s \to 0} sG(s)} = \frac{3 \times 7}{70 \times 12 \times 15} = \frac{7}{3000}$

$\therefore$ Finite Steady state error

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3) ~~47~~ $47t^2 u(t) \rightarrow$ parabolic signal

$$R(s) = \frac{47 \times 2}{s^3}$$

$$C_{para} = \lim_{s\to 0} \frac{s(47/s^3)\times 2}{1+G(s)} = \frac{47\times 2}{s^2+s^2G(s)} = \frac{2\times 47}{0} = \infty$$

∴ Infinite steady state error

$$G(s) = \frac{60(s+3)(s+4)(s+8)}{s^2(s+6)(s+14)}$$

$r(t) = 80t^2 u(t) \rightarrow$ Parabolic signal

$$R(s) = \frac{160}{s^3}$$

$$C_{para} = \lim_{s\to 0} \frac{s(160/s^3)}{1+G(s)}$$

$$= \lim_{s\to 0} \frac{160}{s^2+s^2G(s)} = \frac{160}{0+\lim_{s\to 0} s^2G(s)} = \frac{80\times 7\times 2}{60\times 8}$$

$$= \frac{7\times 2}{6} = \frac{14}{6} = \frac{7}{3}$$

Finite steady state error

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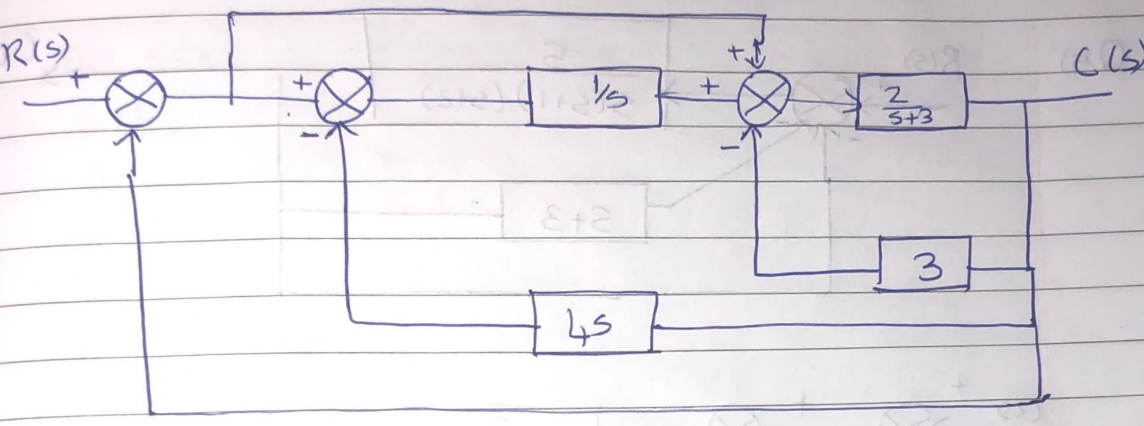
Q3)



$$G_e(s) = \left(\frac{(s+1)}{s}\right)\left(\frac{2/(s+3)}{1+14(s+3)}\right) = \frac{2(s+1)}{s(s+5)}$$

1) $r(s)$ $15u(t)$

$R(t) = \frac{15}{s}$

$$e_{step} = \frac{\frac{15 \times s}{s}}{1+\lim_{s\to 0} G(s)} = \frac{15}{1+\infty} = 0$$

2) $r(s) = 15t\,u(t)$

$R(t) = 15/s^2$

$$e_{ramp} = \frac{15/s^2 + s}{0+\lim_{s\to 0} G(s)} = \frac{15}{\frac{2}{17}} = 127.5$$

3) $r(s) = 15t^2\,u(t)$

$R(t) = \frac{15}{s^3}$

$$e_{para} = \frac{\frac{15}{s^5}+s}{0+\lim_{s\to 0} s^2 G(s)} = \infty$$

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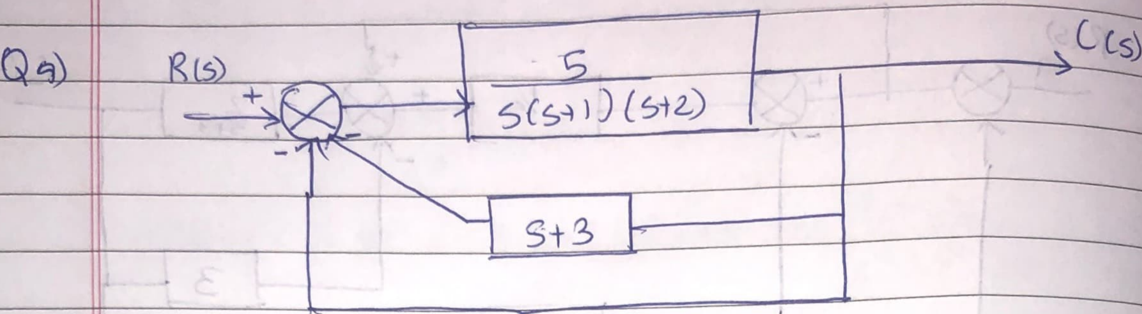
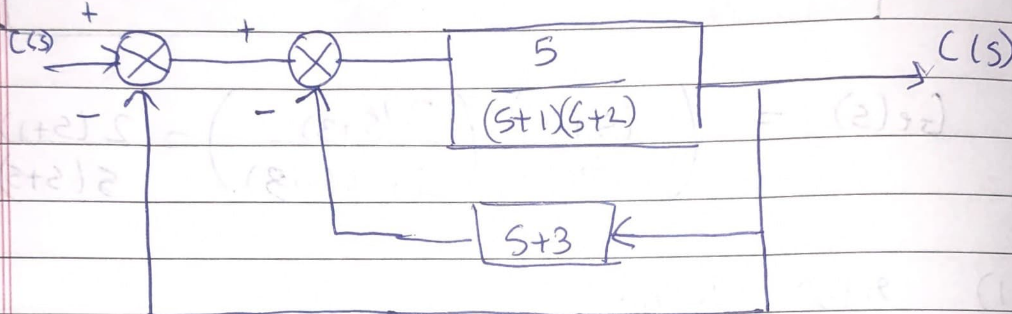
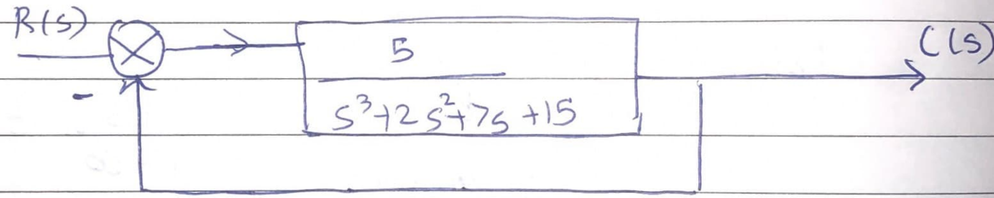
Q9)







$$G(S) = \frac{5}{S^3+2S^2+7S+15}$$

1) $r(t) = 50\,u(t)$ → Step input

$$R(S) = \frac{50}{S}$$

$$e_{step} = \lim \frac{\frac{50}{S} \times S}{1+G(S)} = \frac{50}{1+\lim_{S\to 0} G(S)} = \frac{50}{1+\frac{1}{3}}$$

$$= \frac{150}{4}$$

= Finite error state

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2) 50 u(t) → ramp input

$r(t) = 50t\,u(t)$

$R(s) = \frac{50}{s}$

$e_{ramp} = \lim_{s \to 0} \frac{\frac{50}{s^2} \times s}{1+G(s)} = \frac{50}{0 + \lim_{s \to 0} sG(s)}$

$= \frac{50}{0} = \infty$

Infinite steady state error

3) $50t^2u(t)$

$R(s) = \frac{50}{s^5}$

$e_{para} = \lim_{s \to 0} \frac{\frac{50}{s^3} + s}{1+G(s)} = \frac{50}{0 + \lim_{s \to 0} (s) \cdot s^2}$

$= \frac{50}{0} = \infty$

∴ Infinite steady state error

4) $K_p = \lim_{s \to 0} G(s) = \frac{5}{15} = \frac{1}{5}$

$K_v = \lim_{s \to 0} s(G(s)) = 0$

$K_a = \lim_{s \to 0} s^2(G(s)) = 0$

∴ It is a type 0 system

Q10) $G(s) = \dfrac{K(s^2+3s+30)}{s^n(s+5)}$

a) $r(t) = 10tu(t) \rightarrow$ ramp input

$R(s) = \dfrac{10}{s^2}$

$e_{ram} = \dfrac{1}{600} = \lim \dfrac{s \times \frac{10}{s^2}}{1+G(s)}$

$= \dfrac{10}{\lim\limits_{s\to0} sG(s)}$

$= \dfrac{10}{\lim\limits_{s\to0} \dfrac{Ks(s^2+3s+30)}{s^n(s+5)}}$

For finite error for a ramp i/p $n=1$

$\dfrac{1}{6000} = \dfrac{10}{\lim\limits_{s\to0} \dfrac{K(s^2+3s+50)}{(s+5)}}$

$\dfrac{1}{6000} = \dfrac{10}{\dfrac{30K}{5}}$

$\therefore K = 10{,}000$

b) $K_p = \lim\limits_{s\to0} G(s) = \infty$

$K_v = \lim\limits_{s\to0} sG(s) = 60000$

$K_a = \lim\limits_{s\to0} s^2 G(s) = 0$