COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)

(MA-16003) Linear Algebra and Univariate Calculus

End Semester Examination

Programme: Direct Second Year, Sem I
Academic Year: 2019-2020
Duration: 3 Hours
Branches: All

Max. Marks: 60
Date: 26/11/2019

Student MIS NO. :

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Mobile phones and programmable calculators are strictly prohibited.
4. Writing anything on question paper is not allowed.
5. Exchange/Sharing of stationery, calculator etc. is not allowed.
6. Write your MIS Number on Question Paper.

Attempt the following

1. (a) [CO3] Find the eigenvalues and their associated eigenspaces of the matrix A: (6)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$$

(b) [CO4] Is the following matrix diagonalizable? Justify your answer. (4)

$$A = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 1 \end{pmatrix}$$

(c) [CO3] Let $V = \{a + bx + cx^2 \mid a, b, c \in \mathbb{R}\}$. Let $D : V \longrightarrow V$ be a derivative map. Find a matrix associated to D. (2)

(d) [CO2] Give an example of a 2×2 matrix such that algebraic multiplicity of one of the eigenvalue is not equal to geometric multiplicity. (2)

OR

Let $U \subset \mathbb{R}^n$, $U^{\perp} = \{X \in \mathbb{R}^n \mid X^T Y = 0; \forall\, Y \in U\}$. Prove that U is a subspace of $\mathbb{R}^n$.

2. (a) [CO5] Find the area of the surface generated by revolving the curve $xy = 1,\ 1 \leq y \leq 2$ about the y-axis. (3)

   (b) [CO1] Define Gamma and Beta functions. Write the formula which gives the relation between them. (3)

   (c) [CO2] Can we apply the Rolle's theorem on the function $f : [-1, 1] \longrightarrow \mathbb{R}$ defined by $f(x) = |x| + 1$? Justify your answer. (2)

   (d) [CO4] Evaluate the improper integral (3)

$$\int_0^\infty \frac{dx}{\sqrt[4]{1+x}}$$

3. (a) [CO2] **Say True or False** (5)

      i. Any six vectors in $\mathbb{R}^5$ are linearly dependent.

      ii. Let U and W be any two subspaces of a vector space V then $U \cap W$ is always a subspace of V.

      iii. There exist a linear mapping $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ such that dimension of $Ker(T)$ is greater than 3.

      iv. Let $f : [0, 5] \longrightarrow \mathbb{R}$ be a continuous function defined by $f(x) = x^2 + 1$ then there exist $c \in (0, 5)$ such that $f(c) = 25$.

      v. There exist a matrix A for which dimension of column space of A may not be equal to dimension of row space of A.

   (b) [CO3] Determine all values of c which satisfy the conclusions of the Mean value theorem of the function $f(x) = x^3 + 2x^2 - x$ on $[-4, 2]$. (3)

   (c) [CO4] Test the convergence of the integral: (3)

$$\int_0^\pi \frac{\sin t}{\sqrt{\pi - t}} dt.$$

   (d) [CO4] Evaluate $I = \int_0^2 (3x + 2)dx$ using Riemann sum. (3)

OR

Show that the function $f(x) = 4x^5 + x^3 + 7x - 2$ has exactly one real root.

4. [CO3] Sketch graph of the function $f(x) = x^3 - \frac{3}{2}x^2 - 6x + 3$ using following steps:

   (a) What is domain of f and find the critical points of f. (2)

   (b) Find the intervals on which f is increasing or decreasing. (2)

   (c) Find Inflection points. (1)

   (d) Find the intervals where f is concave up or concave down. (2)

   (e) Plot key points, such as the y-intercept and the points found in above steps and sketch the curve together. (2)

   (f) Describe the end behaviour of the function. (1)

5. (a) [CO3] Let $I_n = \int (\ln x)^n dx,\ n \in \mathbb{N}$. Show that $I_n = x(\ln x)^n - nI_{n-1},\ n \geq 1$. (3)

OR

Find the arc length of $y = \frac{1}{3}(x^2+2)$ from $x = 0$ to $x = 3$.

(b) [CO5] Find the volume of the solid obtained by rotating the region bounded by the graphs of $y = \sqrt{x};\ y = 2 - x$ and $y = 0$ about the x-axis? (4)

(c) [CO3] Find the derivative of $f(x) = \int_{x}^{3x} \sin t\, dt$. (2)

(d) [CO1] Find the value of a if the limit of the following function exists at $x = \pi$ (2)

$$f(x) = \begin{cases} ax+5, & x < \pi \\ \cos x, & x \geq \pi \end{cases}$$

$***** \ END \ *****$