COEP TECH. UNIVERSITY
Department of Mathematics
(MA- 21001) Probability and Statistics for Engineers
T.Y. B. Tech. Semester VI
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Tutorial 4 on Unit 4

1. What is meant by the word 'Statistic'?.

2. What is meant by an unbiased estimator of a population parameter θ? What is biased estimator of a population parameter?

3. Let $X_1, X_2, \ldots, X_n$ be n random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2,\ i = 1, 2, \ldots, n$. What is an unbiased estimator of mean μ and that of σ^2?

4. Define suitable populations from which the following samples are selected:
(a) Persons in 200 homes are called by telephone in the city of Richmond and asked to name the candidate that they favor for election to the school board
(b) A coin is tossed 100 times and 34 tails are recorded.

5. The numbers of incorrect answers on a true-false compentency test for a random sample of 15 students were recorded as follows:
$2, 1, 3, 0, 1, 3, 6, 0, 3, 3, 5, 2, 1, 4, 2$. Find the mean, the median, the mode.

6. Find the probability that a random sample of 25 observations, from a normal population with variance $\sigma^2 = 6$, will have a variance (a) S^2 greater than 9.1. (b) S^2 lying in between 3.462 and 10.745.
Ans: (a) 0.05 (b) 0.94

7. Prove that the sample mean $\overline{X} = \frac{1}{n}\sum_{1}^{n} X_i$ is an unbiased estimator for the population mean μ.

8. Find an unbiased estimator for the population variance σ^2.

9. Define type-I error and type-II error in testing of hypothesis one can do.

10. What is meant by significance level and p-value?

11. Find the $100(1-\alpha)\%$ confidence interval for the population mean μ, if variance σ^2 is (i) known (ii) unknown. Also interpret the result in terms of error. Explain what will be different if we want a one sided confidence interval.

12. Increase in sample size n will ....(increase/reduce) α and β simultaneously.

13. Explain the use of z- test, t- test, Chi-square test and F-test in testing of hypothesis.

14. Using statistical tables find the following:

    (i) $\chi^2_{0.025}$ when degree of freedom is 15 ANS: 27.488

    (ii) $\chi^2_{0.01}$ when degree of freedom is 18 ANS:34.805

    (iii) $\chi^2_{0.05}$ when degree of freedom is 25 ANS:37.652

    (iv) $t_{0.025}$ when degree of freedom is 15 ANS:2.131

    (v) $t_{0.995}$ when degree of freedom is 17 ANS: 2.898

    (vi) $f_{0.05}$ when degree of freedom are 7 and 15

    (vi) $f_{0.05}$ when degree of freedom are 7 and 15

    (vii) $f_{0.99}$ when degree of freedom are 28 and 12

    (viii) $f_{0.01}$ when degree of freedom are 24 and 19

    (ix) χ^2_α if $P(X^2 < \chi^2_\alpha) = 0.95$ when degree of freedom is 6

    (x) χ^2_α if $P(X^2 > \chi^2_\alpha) = 0.05$ when degree of freedom is 16

    (xi) χ^2_α if $P(\chi^2_\alpha < X^2 < 23.209) = 0.015$ when degree of freedom is 10

    (xii) $P(T < 2.365)$ when degree of freedom is 7 ANS: 0.975

    (xii) $P(T > -2.567)$ when degree of freedom is 17 ANS:0.99

    (xiii) $P(-t_{0.005} < T < t_{0.01})$ when degree of freedom is 20 ANS:0.985

    (xiv) k such that $P(k < T < 2.807) = 0.095$ for a random sample of size 24 from a normal population.

15. If S_1^2 and S_2^2 represent the variances of independent random samples of size 8 and 12 respectively, taken from a normal populations with equal varinces, find $P(\frac{S_1^2}{S_2^2} < 4.89)$.

16. Find the probability that a random sample of size 28 from a normal population with variance 4 will have a variance

    (i) greater than 6.1

    (ii) between 2.168 and 5.749. Assume that measurements are continuous.

17. To test the hypothesis that a coin is fair, the following decision rule is adopted. (i) Accept the hypothesis if the number of heads in a single sample of 100 tosses is between 40 and 60. (ii) Reject the hypothesis otherwise.
    (a) Find the probability of rejecting hypothesis when it is actually correct.
    (b) Interprete geometrically the decision rule.
    (c) What conclusions would you draw if a sample of 100 tosses yielded 53 heads? 60 heads?
    (d) Could you be wrong in your conclusions to (c)?

18. Design decision rule to test the hypothesis that a coin is fair if a sample of 64 tosses of the coin is taken and if the level of significance is (a) 0.05 (b) 0.01.

19. A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation equal to 0.9 year. If a random sample of 10 of these batteries has standard deviation of 1.2 years, do you think that $\sigma > 0.9$ years? Use a 0.05 level of significance.

20. A study is conducted to compare the lengths of time required by men and women to assemble a certain product. Past experience indicates that the distribution of times for both men and women is approximately normal but the variance of the times for women is less than that for the men. A random sample of times for 11 men and 14 women gives respective standard deviations 6.1 and 5.3. Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 > \sigma_2^2$.

21. A manufacturer of a certain brand of energy bar claims that the average saturated fat content in the bar is 0.5 gms. Will you support his claim if the 8 bars that you examined for fat content were found to contain 0.6, 0.7, 0.7, 0.3, 0.4, 0.5, 0.4 and 0.2 gms of saturated fat? Assume normality. Take $\alpha = 0.05$.

22. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39350 kms. with the population standard deviation of 3260 kms. Test the hypothesis, at 1 percent level of significance, that the mean life of tyre is 40000 kms.

23. Weight in kg of 10 lambs are 38, 40, 45, 53, 47, 43, 55, 48, 52, 49. Can we say that the variance of the population from which the above sample is drawn has a variance of 20 sq.kg? Assume $\alpha = 0.05$

24. Two random samples were drawn from two normal populations and their values are:

$$\begin{array}{llllllllllll} A: & 66 & 67 & 75 & 76 & 82 & 84 & 88 & 90 & 92 & & \\ B: & 64 & 66 & 74 & 78 & 82 & 85 & 87 & 92 & 93 & 95 & 97 \end{array}$$

Test whether the two populations have the same variance at 10% level of significance.

25. The following data represents the number of Statistics lectures attended by 18 students : 9, 12, 18, 14, 12, 14, 12, 10, 16, 14, 13, 15, 13, 11, 13, 11, 9, 11. Perform a sign test to test the instructor's claim that the median of number of lectures attended is 12. Use a 2% level of significance.

26. The runs scored in a cricket match by 11 players is as follows:
7, 16, 121, 51, 101, 81, 1, 16, 9, 11, 16.
Find the mean, median and mode of this data.
(Ans: Mean=39.09, Median=16, Mode= 16.)

27. Find the variance and standard deviation of the following scores in an exam:
92, 95, 85, 80, 75, 50
(Ans: Variance=263.5, standard deviation=16.2)

28. Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. (i)What is the probability of an individual scoring above 500 in the GMAT ? (ii)How

high must an individual score on the GMAT in order to score in the highest 5% ? (Ans:(i)0.5948, (ii)711.24)

29. The table shows the standard Deviation and Sample Standard Deviation for both men and women. Find the f statistic considering the Men population in numerator.

Population	Population s.d	Sample s.d.
Men	30	35
Women	50	45

(Ans: 1.68)

30. A company manufactures light bulbs. The CEO of the company claims that an average light bulb lasts 300 days. A researcher randomly selects 15 bulbs for testing. The sampled bulbs last an average of 290 days, with a standard deviation of 50 days. If the CEO's claim were true, what is the probability that 15 randomly selected bulbs would have an average life of no more than 290 days? (Ans: t=- 0.7745966, cumulative probability=0.226. So if the true bulb life were 300 days, there is a 22.6% chance that the average bulb life for 15 randomly selected bulbs would be less than or equal to 290 days.)

31. The Head of Quality Control in a foundry claims that the castings produced in the foundry are 'better than average.' In support of this claim he points out that of a random sample of 60 castings inspected, 59 passed. It is known that the industry average percentage of castings passing quality control inspections is 90%. Do these results support the Head's claim? Take $S = 10$.

32. Dishwasher powder is poured into the cartons in which it is sold by an automatic dispensing machine which is set to dispense 3 kg of powder into each carton. In order to check that the dispensing machine is working to an acceptable standard (i.e. does not need adjustment), a production engineer takes a random samples of 40 cartons and weighs them. It is found that the mean weight of the sample is 3.005 kg. It is known that the dispensing machine operates with a variance of $0.015^2\ kg^2$ and that the manufacturer of the powder is willing to rely on a 5% level of significance. Does the sample provide the engineer with sufficient evidence that the true mean is not 3.00 kg and so the machine requires adjustment? (Ans: machine is not operating acceptably and needs adjustment)

33. A coin is tossed 10 times. We wish to test the hypothesis that the coin is fair. Let p be the probability that the coin shows a head. Which of the following represents the alternative hypothesis?

    a) $H1 : p > 0.5$
    b) $H1 : p < 0.5$
    c) $H1 : p = 0.5$
    d) $H1 : p \neq 0.5$

34. A large health study has found that 7% of the population suffers from a blood condition. A group of 15 people from an area near a mobile phone transmitter are tested for the condition and 2 people are found to have the condition. The locals believe that the transmitter increases the likelihood of having the condition. We wish to perform a test

of significance on whether the mobile phone transmitter increases the incidence of the condition.

Let p be the probability that an individual has the condition. Which of the following would be the appropriate null and alternative hypotheses?

a) $H0 : p = 2/15$
   $H1 : p \neq 2/15$

b) $H0 : p = 0.07$
   $H1 : p \neq 0.07$

c) $H0 : p = 2/15$
   $H1 : p > 2/15$

d) $H0 : p = 0.07$
   $H1 : p > 0.07$

35. A random sample of 30 households was selected as part of a study on electricity usage, and the number of kilowatt-hours (kWh) was recorded for each household in the sample for the March quarter of 2006. The average usage was found to be 375kWh. In a very large study in the March quarter of the previous year it was found that the standard deviation of the usage was 81kWh. Assuming the standard deviation is unchanged and that the usage is normally distributed provide an expression for calculating a 99% confidence interval for the mean usage in the March quarter of 2006.

a) $375 \pm 2.756 \times \dfrac{81}{\sqrt{30}}$.

b) $375 \pm 2.575 \times \dfrac{9}{\sqrt{30}}$.

c) $375 \pm 2.33 \times \dfrac{81}{\sqrt{30}}$.

d) $375 \pm 2.575 \times \dfrac{81}{\sqrt{30}}$.

36. Conduct an F-Test on the following samples:
Sample-1 having variance = 109.63, sample size = 41.
Sample-2 having Variance = 65.99, sample size = 21.

37. Test the null hypothesis that the following sample is from a population with median 100 against the alternative the median is greater than 100 i.e., Use normal approximation to Wilcoxon Signed Rank Test (without continuity correction) Assume that the distribution of differences is symmetric. 98.38, 115.33, 98.62, 114.38, 87.79, 84.06, 96.18, 98.74, 91, 107.82, 108.28, 112.62, 124.18, 101.99, 112.51, 75.65, 83.77, 84.91, 109.73, 109.41, 100.4, 95.37, 115.46, 111.78, 86.13, 82.14, 78.47, 98.18

a) Null hypothesis H0 is rejected against the alternative H1 at 5% level of significance

b) Null hypothesis H0 is rejected against the alternative H1 at 10% level of significance

c) Null hypothesis H0 is not rejected against the alternative H1 at 5% level of significance

d) Null hypothesis H0 is not rejected against the alternative H1 at 10% level of significance

38. Suppose that an allergist wishes to test the hypothesis that at least 30 percent of the public is allergic to some cheese products. Explain how the allergist could commit (a) a type I error; (b) a type II error.

39. A large manufacturing firm is being charged with discrimination in its hiring practices.
(a) What hypothesis is being tested if a jury commits a type I error by finding the firm guilty?
(b) What hypothesis is being tested if a jury commits a type II error by finding the firm guilty?

40. The proportion of adults living in a small town who are college graduates is estimated to be p=0.6. To test this hypothesis, a random sample of 15 adults is selected. If number of college graduates in our sample is anywhere from 6 to 12 , we shall not reject the null hypothesis that $p = 0.6$; otherwise we shall conclude that $p \neq 0.6$.
(a) Evaluate α assuming that $p = 0.6$. Use binomial distribution.
(b) Evaluate β for the alternatives $p = 0.5$ and $p = 0.7$.
(c) Is this a good test procedure?

41. Repeat example above for if 200 adults are selected and the fail to reject region is defined to be $110 \leq x \leq 130$, where x is the number of college graduates in our sample. Use the normal approximation. Ans : $\alpha = 0.1498$; $\beta = 0.0793 \, for \, p = 0.5$; $\beta = 0.0618 \, for \, p = 0.7$.

42. Lots of 40 components each are deemed unacceptable if they contain 3 or more defectives. The procedure for sampling a lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defective is found in the sample if there are 3 defectives in the entire lot?

43. The PQR company claims that the lifetime of a type of battery that it manufactures is more than 250hrs. A consumer advocate wishing to determine whether the claim is justified measures the lifetimes of 24 of the company's batteries; the results are : $271, 230, 198, 275, 282, 225, 284, 219, 253,$
$216, 262, 288, 236, 291, 253, 224, 264, 295, 211, 252, 294, 242, 272, 268$. Assuming the sample to be random, Using **Sign test**, determine whether the company's claim is justified at the 0.05 significance level.

———THE END———-