Tutorial - 1.

1. Verify following functions are solutions of corresponding D.E.

1. $y = \sin^{-1} xy$. $\quad xy' + y = y'\sqrt{1-x^2y^2}$

$\Rightarrow \quad y = \sin^{-1} x.y$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2y^2}} \cdot \frac{d}{dx}(x.y).$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2y^2}} \cdot (x.y' + y)$$

$$y' = \frac{1}{\sqrt{1-x^2y^2}} \cdot x.y' + y.$$

$$\boxed{x.y' + y = y'\sqrt{1-x^2y^2}}$$

2. $x^2 + y^2 = 1$. $\quad x + yy' = 0$.

$$x^2 + y^2 = 1.$$

Taking derivative w.r.t. x.

$$2x + 2y\frac{dy}{dx} = 0.$$

$$2x + 2y.y' = 0.$$

$$2(x + yy') = 0.$$

$$\boxed{x + yy' = 0.}$$

3. $y = c.e^{-x} + x^2 - 2x$, $\quad y' + y = x^2 - 2$.

$$y = c.e^{-x} + x^2 - 2x.$$

Taking derivative.

$$\frac{dy}{dx} = \frac{d}{dx}ce^{-x} + \frac{d}{dx}x^2 - \frac{d}{dx}-2x.$$

$$y' = -c.e^{-x} + 2x - 2$$

* Add and subtract x^2 in r.h.s.

$y' = -c.e^{-x} + x^2 - x^2 + 2x - 2$

$y' = -(c.e^{-x} + x^2 - 2x) + x^2 - 2$

$y' = -(y) + x^2 - 2$

$$\boxed{y' + y = x^2 - 2}$$

4. $y^2 - 2x^2 = c$ $\qquad y.y' = 2x$

Taking derivative w.r.t. x

$$\frac{d}{dx} y^2 - \frac{d}{dx} 2x^2 = \frac{d}{dx} c$$

$$2y . \frac{dy}{dx} - 4x = 0.$$

$$2y . y' - 4x = 0.$$

$$2(y.y' - 2x) = 0.$$

$$y.y' - 2x = 0.$$

$$yy' = 2x.$$

2. Obtain the general solution of each of following D.E.

1. $y' = \dfrac{1}{(x+1)(x^2+1)}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{(x+1)(x^2+1)}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+c}{(x^2+1)}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{A(x^2+1) + (Bx+c)(x+1)}{(x+1)(x^2+1)}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{Ax^2 + A + Bx^2 + Bx + cx + c}{(x+1)(x^2+1)}$$

$$\frac{1}{(x+1)(x^2+1)} = \frac{Ax^2 + Bx^2 + Bx + cx + A + c}{(x+1)(x^2+1)}$$

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$$\frac{1}{(x+1)(x^2+1)} = \frac{x^2(A+B) + x(B+C) + A+C}{(x+1)\ (x^2+1)}.$$

comparing

$x^2(A+B) = 0.$

$x\ (B+C) = 0.$

$A+C = 1$

$\Rightarrow$ $A+B=0$ — ①

$B+C=0$ — ②

$A+C=1$ — ③.

from ① and ②.

$A=-B$ $\quad B=-C.$

put above value in eqn ①.

$A-C=0$

$\boxed{A=C}$

$C+C=1$

$2C=1$

$\boxed{C=1/2}$ $\quad \boxed{A=1/2}$

$B+C=0.$

$B+\frac{1}{2}=0.$

$\boxed{B=-1/2}$

put all values in original equation.

$$\frac{dy}{dx} = \frac{1/2}{(x+1)} + \frac{-1/2x+1/2}{(x^2+1)}.$$

Integrating both side.

$$\int\frac{dy}{dx} = \int\frac{1/2}{(x+1)} + \int\frac{-1/2x+1/2}{(x^2+1)}.$$

$$y = \frac{1}{2}\log(x+1) + \left[-\int\frac{1/2x}{(x^2+1)} + \int\frac{1/2}{(x^2+1)}\right.$$

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* Adjustement

$$y = \frac{1}{2}\log(x+1) + \left(-\frac{1}{2\times 2}\cdot\frac{2x}{(x^2+1)}\right) + \frac{1}{2}(\tan^{-1}(x))$$

$$= \frac{1}{2}\log(x+1) + \left(\frac{-1}{4}\cdot\log(x^2+1)\right) + \frac{1}{2}\tan^{-1}(x)$$

$$\boxed{y = \frac{1}{2}\log(x+1) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\tan^{-1}(x)}$$

2. $$y' = \frac{y^2 - xy}{x^2 + xy}$$

This is homogeneous equation with degree 1.

Divide by x^2

$$y' = \frac{y^2/x^2 - xy/x^2}{x^2/x^2 + xy/x^2}$$

$$\frac{dy}{dx} = \frac{y^2/x^2 - y/x}{1 + y/x} \quad \text{——— (1)}$$

Now substitute $\frac{y}{x} = v$ ——— (2)

$$y = x.v$$

$$\frac{dy}{dx} = x\cdot v' + v \quad \text{——— (3)}$$

From (1), (2) and (3).

$$v + x.\frac{dv}{dx} = \frac{v^2 - v}{1+v}$$

$$x.\frac{dv}{dx} = \frac{v^2 - v}{1+v} - v$$

$$x.\frac{dv}{dx} = \frac{v^2 - v - v(1+v)}{(1+v)}$$

$$x.\frac{dv}{dx} = \frac{\cancel{v^2} - v - v - \cancel{v^2}}{(1+v)}$$

$$x\frac{dv}{dx} = \frac{-2v}{(1+v)}$$

$$\frac{(1+v)}{-2v} = -\frac{dx}{x}$$

$$\frac{1}{2v} + \frac{v}{2v} = -\frac{dx}{x}$$

Taking integration

$$\int \frac{1}{2v} + v\frac{1}{2}\,dv = -\int \frac{1}{x}\,dx$$

$$\frac{1}{2}(\log v + v) = -\log(x)$$

$$\frac{1}{2}(\log v + v) = \log\left(\frac{1}{x}\right)$$

Resubstitute $v = y/x$.

$$\boxed{\frac{1}{2}\left(\log\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)\right) = \log\left(\frac{1}{x}\right)}$$

3. $x \cdot y' = y + x\cos^2(y/x)$.

put $\frac{y}{x} = v$. —— ①

$y = x \cdot v$ —— ②

$\frac{dy}{dx} = x \cdot v' + v$ —— ③.

Now divide given equation by x.

$y' = y/x + \cos^2(y/x)$.

$\frac{dy}{dx} = y/x + \cos^2(y/x)$ —— ④

put values of ①, ②, ③ in ④.

$$x \cdot \frac{dv}{dx} + \cancel{v} = \cancel{v} + \cos^2(v).$$

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$x \cdot \frac{dv}{dx} = \cos^2 v$

$\frac{1}{\cos^2 v} dv = \frac{1}{x} dx$

Integrating.

$\int \frac{1}{\cos^2 v} dv = \int \frac{1}{x} dx$

$\int \sec^2 v \, dv = \int 1/x \, dx$

$\tan v = \log x + c$

Resubstitute.

$$\boxed{\tan\left(\frac{y}{x}\right) = \log x + c}$$

3. Find the perticular solution of each of following

1. $x^3 (\sin y) y' = 2$ $\qquad y(x) \longrightarrow \pi/2 \quad x \longrightarrow +\infty$

$x^3 (\sin y) \frac{dy}{dx} = 2$

Variable separable method.

$\sin y \, dy = \frac{2 \, dx}{x^3}$

Integrating both side.

$\int \sin y \, dy = \int 2/x^3 \, dx$

$-\cos y = 2 \int x^{-3} dx$

$-\cos y = \cancel{2} \cdot \frac{x^{-2}}{-\cancel{2}} + c$

$\cancel{-}\cos y = \cancel{+}x^{-2} + c$

$\cos y = x^{-2} + c$

$$\boxed{\cos y = \frac{1}{x^2} + c}$$

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Now.

$$\cos(\pi/2) = \frac{1}{\infty} + C.$$

$$0 = \frac{1}{\infty} + C.$$

$$\boxed{C = 0.}$$

2. $y' = y(y^2-1)$ $\quad y(0) = 2.$

$$\frac{dy}{dx} = y(y^2-1).$$

$$\frac{1}{y(y^2-1)} dy = dx. \quad \text{—— ①.}$$

Now,

$$\frac{1}{y(y^2-1)} = \frac{A}{y} + \frac{Bxy + c}{(y^2-1)}$$

$$\frac{1}{y(y^2-1)} = \frac{A(y^2-1) + Bxy + c\,(y)}{y(y^2-1)}$$

$$\frac{1}{y(y^2-1)} = \frac{Ay^2 - A + Bxy^2 + cy}{y(y^2-1)}$$

$$\frac{1}{y(y^2-1)} = \frac{(A+B)y^2 + cy - A}{y(y^2-1)}.$$

$$(A+B)y^2 + cy - A = 1.$$

$\Rightarrow$ $A + B = 0$

$c = 0$

$-A = 1.$ $\quad \therefore A = -1.$

$\therefore B = +1.$

$$\frac{1}{y(y^2-1)} = \frac{-1}{y} + \frac{1y + 0}{(y^2-1)}$$

put in equation ①.

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$$\left[\frac{-1}{y}+\frac{y}{(y^2-1)}\right]dy = dx$$

Integrating both side.

$$\int \frac{-1}{y}+\frac{y}{(y^2-1)}\,dy = \int dx.$$

$$-\log y + \int \frac{1}{2}\,\frac{2y}{y^2-1}\,dy = x$$

$$\boxed{-\log y + \frac{1}{2}\log(y^2-1) = x + c.}$$

$$\log\frac{1}{y} + \frac{1}{2}\log(y^2-1) = x + c.$$

Now put $y(0)=2$

$$\log\left(\frac{1}{y}\right) + \log(y^2-1)^{1/2} = x + c.$$

$$\log\left(\frac{1}{y}\right) + \log\sqrt{y^2-1} = x + c.$$

$\log A + \log B = \log A \cdot B.$

$$\log\frac{\sqrt{y^2-1}}{y} = x + \log c.$$

$$\log\frac{\sqrt{y^2-1}}{y} = x + \log c.$$

$$\log\frac{\sqrt{2^2-1}}{2} = 0 + \log c.$$

$$\log\frac{\sqrt{3}}{2} = \log c.$$

$$\boxed{c = \frac{\sqrt{3}}{2}}$$

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*▽ 3. $(x+2)y' - xy = 0 \qquad y(0) = 1.$

$$(x+2)\frac{dy}{dx} - xy = 0.$$

$$(x+2)\frac{dy}{dx} = xy.$$

$$\frac{1}{y}dy = \frac{x}{(x+2)}dx$$

Integrating on both side.

$$\int \frac{1}{y}dy = \int \frac{x}{(x+2)}dx.$$

$$\log y = \int \frac{x+2-2}{x+2}dx \quad *$$

$$\log y = \int \frac{x+2}{x+2} - \frac{2}{x+2}dx.$$

$$\log y = \int 1\,dx - 2\log(x+2) + \cancel{\log}\ \log c.$$

$$\boxed{\log y = x - 2\log(x+2) + \cancel{\log}\ \log c}$$

$$\log y = x - \log(x+2)^2 + \log c.$$

put $y(0) = 1$.

$$\log(1) = 0 - \log(4) + \log c.$$

$$\log(1) = \log\frac{1}{4} + \log c.$$

$$\log(1) = \log\left(\frac{c}{4}\right)$$

$$\boxed{c = 4}$$

4. $y' + \frac{y-x}{y+x} = 0 \qquad y(1) = 1.$

$$\frac{dy}{dx} + - \frac{y-x}{y+x}$$

Divide by x.

$$\frac{dy}{dx} = -\left(\frac{y/x - 1}{y/x + 1}\right) \quad \text{——— ①}$$

Substitute $\frac{y}{x} = v$. ---- (2)

$$y = v \cdot x.$$

$$\frac{dy}{dx} = v + x \cdot v' \quad \text{---- (3)}$$

from (1) and (2) and (3).

$$v + x \cdot \frac{dv}{dx} = \frac{-(v-1)}{(v+1)}$$

$$x \cdot \frac{dv}{dx} = \frac{-(v+1)}{v+1} - v$$

$$x \cdot \frac{dv}{dx} = \frac{-v+1 - v(v+1)}{v+1}$$

$$x \cdot \frac{dv}{dx} = \frac{-v+1 - v^2 - v}{v+1}$$

$$x \cdot \frac{dv}{dx} = \frac{-v^2 - 2v + 1}{v+1}$$

$$\frac{v+1}{-(v^2+2v-1)} dv = \frac{1}{x} dx.$$

Integrating both side

$$-\int \frac{v+1}{(v^2+2v-1)} dv = \int \frac{1}{x} dx.$$

$$-\int \frac{1}{2} \cdot \frac{2(v+1)}{(v^2+2v-1)} dv = \log x. \cancel{+\log c}$$

$$-\frac{1}{2} \int \frac{2v+2}{(v^2+2v-1)} dv = -\log x.$$

$$\frac{1}{2} \log (v^2+2v-1) = -\log (x).$$

$$\log (v^2+2v-1)^{1/2} = \log \left(\frac{1}{x}\right).$$

$$\log \sqrt{v^2+2v-1} = \log \left(\frac{1}{x}\right) + c. \quad \text{---- (4)}$$

$$\sqrt{v^2+2v-1} = \frac{c}{x}$$

Resubstituting $v = y/x$.

$$\sqrt{v^2+2v-1} = c/x.$$

$$\sqrt{\left(\frac{y}{x}\right)^2+2\left(\frac{y}{x}\right)-1} = \frac{c}{1}$$

$$\sqrt{1^2+2(1)-1} = c$$

$$\sqrt{1+2-1} = c$$

$$\therefore c = \sqrt{2} \quad \text{---} \quad (5)$$

from (4) and (5).

$$\boxed{\frac{\sqrt{2}}{x} = \sqrt{\left(\frac{y}{x}\right)^2-2\left(\frac{y}{x}\right)-1}}$$

* 5. $y' = (y-x)^2$ $\qquad y(0) = 2$

wrong

$$\frac{dy}{dx} = (y-x)^2$$

$$dy = (y-x)^2\,dx.$$

put $y-x = u$.

$$\frac{du}{dx} = -1$$

$$du = -dx.$$

$$dx = -du.$$

$$dy = -u^2\,du.$$

Integrating both side

$$\int dy = \int -u^2\,du.$$

$$y = -\frac{u^3}{3} + c.$$

$$y = -\frac{(y-x)^3}{3} + c$$

$$y = \frac{(-y+x)^3}{3} + c$$

$$y + \frac{(y-x)^3}{3} = c$$

$$\boxed{\frac{3y + (y-x)^3}{3} = c}$$

$\cancel{3y + (y-x)(y^2+yx+x^2) = c}$

$\cancel{3y + (y^3 - 3y}$

put $y(0) = 2$

$$\frac{3(2) + (2-0)^3}{3} = c$$

$$\frac{6+8}{3} = c$$

$$\frac{14}{3} = c$$

$$\boxed{c = 14/3}$$

4. Show that following diff. equation are exact and hence obtain their general solution.

1. $3x(xy-2)dx + (x^3+2y)dy = 0$

$\Rightarrow M = 3x(xy-2)$ $\qquad N = (x^3+2y)$

$\frac{dm}{dy} = 3x^2 \cdot (1) - 0$ $\qquad \frac{dN}{dx} = 3x^2$

$\frac{dm}{dy} = 3x^2$ $\qquad \frac{dN}{dx} = 3x^2$

Equation is exact

$\int M\,dx + \int N\,dy = 0$

$\int 3x^2y - 6x\,dx + \int 2y = 0$

2

$\cancel{8} \cdot \frac{x^3}{\cancel{8}} \cdot y - \cancel{6}^{3} \frac{x^2}{\cancel{2}} + \cancel{7} \cdot \frac{y^2}{\cancel{2}} = 0$

$\boxed{x^3 y - 3x^2 + y^2 = 0}$

2. $(\cos x \cdot \cos y - \cot x) dx - (\sin x \cdot \sin y) dy = 0$

$m = (\cos x \cdot \cos y - \cot x)$

$N = (\sin x \cdot \sin y)$

$\frac{dm}{dy} = -\sin y \cdot \cos x - 0$

$\frac{dN}{dx} = -\cos x \cdot \sin y$

$\frac{dm}{dy} = \frac{dN}{\partial x}$

Equation is exact

$\int m\,dx + \int N\,dy = 0$

$\int (\cos x \cdot \cos y - \cot x)\, dx + \int 0 = 0$

$\boxed{\sin x \cdot \cos y - \log|\sin x| = 0}$

3. $(y^2 \cdot e^{xy^2} + 4x^3) dx + (2xye^{xy^2} - 3y^2) dy = 0$

$m = y^2 \cdot e^{xy^2} + 4x^3$

$\frac{\partial m}{\partial y} = 2y \cdot e^{xy^2} + y^2 \cdot e^{xy^2} \cdot x \cdot 2y$

$N = 2xye^{xy^2} - 3y^2$

$\frac{\partial N}{\partial x} = 2y(e^{xy^2} + e^{xy^2} \cdot x \cdot y^2)$

$\frac{\partial m}{\partial y} = \frac{\partial N}{\partial x}$

$\int m\,dx + \int N\,dy = 0$

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$$\int y^2 . e^{xy^2} + 4x^3 + \int -3y^2 \, dy = 0.$$

$$\int \frac{\cancel{y^2} . e^{xy^2}}{\cancel{y^2}} + \frac{4x^4}{\cancel{4}} - \frac{\cancel{3} y^3}{\cancel{3}} = 0.$$

$$\boxed{e^{xy^2} + x^4 - y^3 = 0.}$$

5. Verify that $\frac{1}{x^2}$, $\frac{1}{y^2}$, $\frac{1}{xy}$, $\frac{1}{x^2+y^2}$ are I.F of differential equation $-y\,dx + x\,dy = 0$.

$\Rightarrow$ $-y\,dx + x\,dy = 0$ —— ①.

$\Rightarrow$ 1. $1/x^2$.

multiple by $1/x^2$ to eqn ①.

$$\frac{-y}{x^2} + \frac{1}{x} \, dxy = 0$$

Now $M = -y/x^2$ $\qquad$ $N = 1/x$.

$$\frac{\partial M}{\partial y} = \frac{-1}{x^2}. \qquad \frac{\partial N}{\partial x} = \frac{1}{x}$$

$$= x^{-1}$$

$$= -x^{-2}$$

$$= \frac{-1}{x^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$\therefore$ $\boxed{\frac{1}{x^2} \text{ is an integrating factor}}$

$\Rightarrow$ 2. $\frac{1}{y^2}$

multiply by $1/y^2$ to eqn ①.

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$$-\frac{1}{y}dx + \frac{x}{y^2}dy = 0$$

$M = -1/y$

$\frac{\partial M}{\partial y} = -y^{-1}$

$= -(-y^{-2})$

$= +y^{-2}$

$\frac{\partial M}{\partial y} = \frac{1}{y^2}$

$N = x/y^2$

$\frac{\partial N}{\partial x} = \frac{1}{y^2} \cdot \frac{d}{dx} x$

$\frac{\partial N}{\partial x} = \frac{1}{y^2} \cdot 1$

$\frac{\partial N}{\partial x} = \frac{1}{y^2}$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

$\boxed{\therefore \frac{1}{y^2} \text{ is an integrating factor}}$

⇒ 3. $\frac{1}{xy}$

multiply by $1/xy$ to eqn ①.

$$-\frac{1}{x}dx + \frac{1}{y}dy = 0$$

$M = -1/x$

$\frac{\partial M}{\partial y} = -1/x$

$\frac{\partial M}{\partial y} = 0$

$N = 1/y$.

$\frac{\partial N}{\partial x} = 1/y$

$\frac{\partial N}{\partial x} = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\boxed{\therefore \frac{1}{xy} \text{ is an integrating factor}}$

⇒ 4. $\frac{1}{x^2+y^2}$

multiply by $\frac{1}{x^2+y^2}$

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$$= -\left(\frac{1}{x^2+y^2}\right) \cdot y\,dx + \left(\frac{1}{x^2+y^2}\right) x \cdot dx = 0$$

$$M = \frac{-y}{x^2+y^2} \qquad N = \frac{x}{x^2+y^2}$$

$$\frac{\partial M}{\partial y} = -\frac{[(x^2+y^2) - y \cdot 2y]}{(x^2+y^2)^2}$$

$$= -\frac{[x^2+y^2 - 2y^2]}{(x^2+y^2)^2}$$

$$= \frac{-[x^2 - y^2]}{(x^2+y^2)^2}$$

$$= \frac{-x^2 + y^2}{(x^2+y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{(y^2 - x^2)}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = \frac{x^2+y^2 \cdot (1) - x \cdot 2x}{(x^2+y^2)^2}$$

$$= \frac{x^2+y^2 - 2x^2}{(x^2+y^2)^2}$$

$$= \frac{-x^2 + y^2}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = \frac{y^2 - x^2}{(x^2+y^2)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore \frac{1}{x^2+y^2}$ is an Integrating factor

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6. Verify that $\frac{y+1}{x^4}$ is an I.F of D.E $3(y+1)dx = 2x\,dy$. solve it using I.F and otherwise.

$\Rightarrow$ $3(y+1)\,dx = 2x\,dy$

$3(y+1)\,dx - 2x\,dy = 0$ —— ①

To prove $\frac{y+1}{x^4}$ is an I.F multiply $\frac{y+1}{x^4}$ to eqn ①

$$\frac{y+1}{x^4}[3(y+1)]dx - \frac{y+1}{x^4}(2x)\,dy = 0$$

$$\frac{3(y+1)^2}{x^4}\,dx - \frac{2xy+2x}{x^4}\,dy = 0$$

$$M = \frac{3(y+1)^2}{x^4} \qquad N = -\left(\frac{2xy+2x}{x^4}\right)$$

$$\frac{\partial M}{\partial y} = \frac{3}{x^4}(y^2+2y+1)$$

$$= \frac{3}{x^4}(2y+2)$$

$$\boxed{\frac{\partial M}{\partial y} = \frac{6y+6}{x^4}} \quad —— ②$$

Now.

$$\frac{\partial N}{\partial x} = -\left(\frac{2xy}{x^4} + \frac{2x}{x^4}\right)$$

$$\frac{\partial N}{\partial x} = -\left(\frac{2y}{x^3} + \frac{2}{x^3}\right)$$

$$\frac{\partial N}{\partial x} = -(2y(x^{-3}) + 2(x^{-3}))$$

$$\frac{\partial N}{\partial x} = -(2y(-3x^{-4}) + 2(-3x^{-4}))$$

$$\frac{\partial N}{\partial x} = -(-6yx^{-4} - 6x^{-4})$$

$$= -\left(\frac{-6y}{x^4} - \frac{6}{x^4}\right)$$

$$= +\frac{(6y+6)}{x^4}$$

$$\boxed{\frac{\partial N}{\partial x} = \frac{6y+6}{x^4}}$$

$\therefore$ Hence $\frac{y+1}{x^4}$ is an I.F to given equation

7. Solve following diff. equation.

1. $(2\cos y + 4x^2)dx = x \cdot \sin y \, dy$

$2\cos y + 4x^2 dx - x\sin y \, dy = 0$ —— ①

$M = 2\cos y + 4x^2$
$N = -x\sin y$

$\frac{\partial M}{\partial y} = -2\sin y$. $\quad \frac{\partial N}{\partial x} = -\sin y$.

$\therefore$ Equation is not exact.

From type 1.

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$$

$$= \frac{1}{-x\sin y}\left(-2\sin y - (-\sin y)\right)$$

$$= \frac{1}{-x\sin y}(-2\sin y + \sin y)$$

$$= \frac{1}{-x\cancel{\sin y}} - \cancel{\sin y}$$

$= \frac{1}{x}$.

Integrating factor $= e^{\int f(x)\,dx}$

$= e^{\int 1/x\,dx}$

$= e^{\log x}$

I.F $= x$.

multiply x by x to equation 1.

$x(\cancel{e_0}\,2\cos y + 4x^2)dx - x(x\sin y)dy = 0$

$\underline{2e_0}$

$(2x\cos y + 4x^3)dx - x^2\sin y\,dy = 0$.

Solution.

$\int m\,dx + \int N\,dy = c$.

$\int 2x.\cos y + 4x^3 + \int 0 = c$.

$\int 2x.\cos y + 4x^3 = c$.

$\cancel{2}\left(\frac{x^2}{\cancel{2}}\right).\cos y + \cancel{4}\left(\frac{x^4}{\cancel{4}}\right) = c$.

$\boxed{x^2\cos y + x^4 = c}$

Wrong

2. $y e^{x/y}\,dx + (y - x e^{x/y})dy = 0$.

$m = y.e^{x/y}$

$N = (y - x e^{x/y})$.

$\frac{\delta m}{\partial y} = y.e^{x/y}.x.\left(\frac{-1}{y^2}\right) + e^{x/y}$.

$= e^{x/y}\left(\frac{-x}{y}\right) + e^{x/y}$.

$= e^{x/y}\left(\frac{-x}{y} + 1\right)$

$\frac{\partial m}{\partial y} = e^{x/y}\left(1 - \frac{x}{y}\right)$

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3.

$N = y - x \cdot e^{x/y}$

$\frac{\partial N}{\partial x} = 1 - \left[x \cdot e^{x/y} \cdot \frac{1}{y} + e^{x/y}\right]$

$= 1 - \left[e^{x/y} \cdot \frac{x}{y} + e^{x/y}\right]$

$= -(e^{x/y})(-x/y + 1)$

$= -e^{x/y} \cdot x/y - 1$

$= -e^{x/y}\frac{x}{y} - e^{x/y}$

$= -e^{x/y}\left(\frac{x}{y} + 1\right)$

$= e^{x/y}\left(-1 - \frac{x}{y}\right)$

$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

Equation is not exact.

—X—

2. $y \cdot e^{x/y}\, dx + y - x e^{x/y}\, dy = 0$

$-y + x \cdot e^{x/y}\, dy = y \cdot e^{x/y}\, dx$

$\frac{-y + x \cdot e^{x/y}}{y \cdot e^{x/y}} = \frac{dx}{dy}$

put $x/y = v$ — ①

$x = v \cdot y$

$\frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$ — ②

$\frac{x \cdot e^{x/y} - y}{y \cdot e^{x/y}} = \frac{dx}{dy}$

$\frac{v \cdot y e^{v} - y}{y \cdot e^{v}} = v + y\frac{dv}{dy}$ — (①, ②)

$\frac{y(v \cdot e^{v} - 1)}{y \cdot e^{v}} = y\frac{dv}{dy} + v$

$\frac{v \cdot e^{v} - 1}{e^{v}} = y\frac{dv}{dy} + v$

Wrong

$\frac{v \cdot e^{v} - 1}{e^{v}} - v = y\frac{dv}{dy}$

$\frac{v/e^{v} - 1 - v/e^{v}}{e^{v}} = y \cdot \frac{dv}{dy}$

$\frac{-1}{e^{v}} = y \cdot \frac{dv}{dy}$

$\frac{-1}{y}\, dy = e^{v} dv$

$-\log y + \log c = e^{v}$

$\log \frac{1}{y} + \log c = e^{v}$

$\log \frac{c}{y} = e^{v}$

Resubstitute $v = x/y$

$\boxed{\log \frac{c}{y} = e^{x/y}}$

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3. $(2x+e^y)\,dx + x.e^y\,dy = 0$

$M = (2x+e^y)$

$N = x\,e^y$

$\frac{\partial M}{\partial y} = e^y$ $\qquad$ $\frac{\partial N}{\partial x} = e^y$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ Equation is exact.

Solution.

$$\int M\,dx + \int N\,dy = 0$$

$$\int (2x+e^y) + \int 0\,dy = 0$$

$$\int 2x+e^y = 0$$

$$\frac{\cancel{2}x^2}{\cancel{2}} + e^y x = 0$$

$$\therefore x^2 + e^y x = 0$$

$$\boxed{\therefore\ x^2+e^y x = 0}$$

Wrong 4. $(x+y^2)\,dy - dx = 0$

$(x+y^2)\,dy - dx = 0$

$M = (x+y^2)$ $\qquad$ $N = -1$

$\frac{\partial M}{\partial y} = (x+y^2)$ $\qquad$ $\frac{\partial N}{\partial x} = -1$

$\frac{\partial M}{\partial y} = 2y$ $\qquad$ $\frac{\partial N}{\partial x} = 0$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

we have to find I. F.

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{-1}\left(2y-0\right)$$

$$= \frac{2y}{-1}$$

$$= -2y$$

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$M\,dx + N\,dy = 0$

4. $(x+y^2)\,dy - dx = 0.$

$dx - (x+y^2)\,dy = 0.$

$m = 1$ $\qquad\qquad$ $N = -(x+y^2)$

$\frac{\partial m}{\partial y} = 0.$ $\qquad\qquad$ $\frac{\partial N}{\partial x} = -x - y^2$

$\frac{\partial N}{\partial x} = -1.$

$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$

$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{1}\,(-1-0)$

$= \frac{-1}{1} = -1.$

$I.F = e^{\int f(x)\,dx}$

$= e^{\int -1 x\,dx}$

$= e^{\int 1}\,dx$ $\qquad$ Type-2. so $I.F = e^{-y}$

$\boxed{I.F = e^{-x}}$

multiply by I.F

$e^{-x}\,dx - e^{-x}(x+y^2)\,dy = 0.$

$m = e^{-x}.$ $\qquad\qquad$ $N = -e^{-x}(x+y^2)$

Solution $\int m\,dx + \int N\,dy = 0.$

$\int e^{-x} + \int 0 = c$

$\frac{e^{-x}}{-1} = c$

$\boxed{c = -e^{-x}}$

Check * $\frac{-x}{e^y} - \frac{y^2}{e^y} - \frac{2y}{e^y} - 2e^y = c.$

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6. $y' - x^{-1}y = x^{-1}y^2$

$\frac{dy}{dx} - x^{-1}y = x^{-1}y^2$

$\frac{dy}{dx} = x^{-1}y^2 + x^{-1}y$

$\frac{dy}{dx} = x^{-1}.(y^2+y)$

$\frac{1}{y^2+y}dy = x^{-1}dx$

Now integrating both sides

$\int \frac{1}{y+y^2} dy = \int \frac{1}{x} dx$

—x—

Solve using Bernoullis Equation.

Divide by y^2

$\frac{1}{y^2}\frac{dy}{dx} - \frac{x^{-1}\cancel{y}}{y^{\cancel{2}}} = \frac{x^{-1}\cancel{y^2}}{\cancel{y^2}}$

$y^{-2}\frac{dy}{dx} - x^{-1}y^{-1} = x^{-1}$ — ①

put $y^{-1} = v$

$-y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$

$y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$ — ②

~~-dv~~ put above value in equation ①.

$-\frac{dv}{dx} - x^{-1}v = x^{-1}$

multiply by -1

$\frac{dv}{dx} + x^{-1}v = -x^{-1}$

$p(x) = x^{-1}$

$q(x) = -x^{-1}$

$$I.F = e^{\int p\,dx} = e^{\int x^{-1}} = e^{\int 1/x} = e^{\log x} = x$$

Solution:

$$v \cdot e^{\int p\,dx} = \int e^{\int p\,dx} \cdot Q\,dx + c$$

$$v \cdot x = \int x \cdot -x^{-1} + c$$

$$v \cdot x = \int x \cdot \frac{-1}{x} + c$$

$$v \cdot x = -x + c$$

Resubstitute $v = y^{-1}$

$$\boxed{y^{-1} \cdot x = -x + c}$$

5. $(x+y)^2 y' = 1$

$$(x+y)^2 \frac{dy}{dx} = 1 \quad \text{—} ①$$

put $x+y=t$ — ③

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1 \quad \text{—} ②$$

from ①, ②, ③

$$(t)^2\left(\frac{dt}{dx} - 1\right) = 1$$

$$t^2 \frac{dt}{dx} - t^2 = 1$$

$$\frac{dt}{dx} = \frac{1+t^2}{t^2}$$

$$\frac{t^2}{(1+t^2)} dt = dx$$

$$\int \frac{t^2}{t^2+1} dt = \int dx$$

$$\int \frac{t^2+1-1}{t^2+1} dt = \int dx$$

$$\int \frac{t^2+1}{t^2+1} dt - \int \frac{1}{1+t^2} dt = \int dx$$

$$\int 1\,dt - \int \frac{1}{1+t^2} dt = x$$

$$t - \tan^{-1}|t| = x + c$$

$$c + x = t - \tan^{-1}(t)$$

put $t = x+y$.

$$\boxed{x + c = (x+y) - \tan^{-1}(x+y)}$$

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* सर solved it.

8. For a diffn Equation $m dx + N dy = 0$ if $\dfrac{\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is a function that $e^{\int f(x)}$ is an I.F. of given equation.

wrong

⇒ $m dx + N dy = 0.$

We have to prove $e^{\int f(x)}$ is an I.F.

let $e^{\int f(x)} = \mu(x).$

$\mu(x) m dx + \mu(x) N dy = 0$ is an exact fn.

$$\frac{\partial(\mu m)}{\partial y} = \frac{\partial(\mu \cdot N)}{\partial(x)}.$$

$$\mu(x)\frac{dm}{\partial y} = \frac{\partial \mu}{\partial x} N + \mu \cdot \frac{\partial N}{\partial x}.$$

$$N \frac{\partial \mu}{\partial x} = \mu \cdot \frac{\partial m}{\partial y} - \mu \frac{\partial N}{\partial x}.$$

$$\frac{\partial \mu}{\partial x} = \frac{1}{N}\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right)\mu.$$

$$\boxed{\frac{\partial \mu}{\partial x} - \frac{1}{N}\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right)\mu = 0.}$$

9. Obtain the solutions of following D.E.

Wrong 1. $x dy = (y + x^2 + 9y^2) dx$

$(y + x^2 + 9y^2) dx - x dy = 0.$

$m = y + x^2 + 9y^2$ $\quad N = -x$

$\frac{\partial m}{\partial y} = 1 + 9(2y).$ $\quad \frac{\partial N}{\partial x} = -1.$

$\frac{\partial m}{\partial y} = 1 + 18y.$ $\quad \frac{\partial N}{\partial x} = -1.$

$$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$$

∴ Equation is not Exact.

$$\frac{1}{N}\left(\frac{\partial m}{\partial y}-\frac{\partial N}{\partial x}\right)=\frac{1}{-x}\left(1+18y-(-1)\right)$$

— x —

1. $x\,dy=(y+x^2+9y^2)\,dx$

$$\frac{dy}{dx}=\frac{(y+x^2+9y^2)}{x}$$

$$\frac{dy}{dx}=\frac{y}{x}+\frac{x^2}{x}+\frac{9y^2}{x}$$

$$\frac{dy}{dx}=\frac{y}{x}+x+9y\left(\frac{y}{x}\right) \quad \text{--- (1)}$$

put $y/x=v$.

$y=v.x$ --- (2)

$$\frac{dy}{dx}=v+\frac{dv}{dx}.x \quad \text{--- (3)}$$

$$v+\frac{dv}{dx}.x=v+x+9(vx).v$$

$$\cancel{v}+\frac{dv}{dx}.x=\cancel{v}+x+9v^2x$$

$$\frac{dv}{dx}.x=x+9v^2x$$

$$\frac{dv}{dx}=\frac{x+9v^2x}{x}$$

$$=\frac{\cancel{x}}{x}+\frac{9v^2\cancel{x}}{\cancel{x}}$$

$$\frac{dv}{dx}=1+9v^2$$

$$\frac{1}{1+9v^2}dv=dx$$

$$\int\frac{1}{1+9v^2}dv=\int dx$$

$$\int\frac{1}{(3v)^2+1}dv=\int dx$$

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$$\frac{dy}{dv} = 3. \qquad dv = \frac{1}{3} dy.$$

put $3v = y.$

$$\frac{1}{3}\int \frac{1}{y^2+1}\, dy = \int dx.$$

$$\frac{1}{3}\tan^{-1}(y) = x.$$

$$\frac{1}{3}\tan^{-1}(3v) = x$$

$$\frac{1}{3}\tan^{-1}\left(\frac{3y}{x}\right) = x.$$

$$\frac{1}{3}\tan^{-1} 3\left(\frac{y}{x}\right) + c = x$$

2. $(4xy + 3y^2 - x)\,dx + (x(x+2y))\,dy = 0$ —— ①.

$m = 4xy + 3y^2 - x.$

$N = x^2 + 2xy$

$$\frac{\partial m}{\partial y} = 4x + 6y \qquad \frac{\partial N}{\partial x} = 2x + 2y.$$

$$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$$

finding I.F.

$$\frac{1}{N}\left(\frac{\partial m}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{x^2+2xy}\left(4x+6y-(2x+2y)\right)$$

$$= \frac{1}{x^2+2xy}(2x + 4y).$$

$$= \frac{2x+4y}{x^2+2xy}$$

$$= \frac{2(x+2y)}{x(x+2y)}$$

$$f(x) = \frac{2}{x}.$$

$$I.F = e^{\int f(x)} = e^{\int 2/x\, dx} = e^{2\log x} = \underline{\underline{x^2}}$$

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Now multiply equation ① by I.F

$x^2(4xy+3y^2-x)dx+x^3(x+2y)dy=0.$

$(4x^3y+3x^2y^2-x^3)dx+(x^4+2x^3y)dy=0.$

$M=(4x^3y+3x^2y^2-x^3).$

$N=(x^4+2x^3y)$

$\int M dx + \int N dy = C.$

$\int 4x^3y+3x^2y^2-x^3 + \int 0 = C.$

$\cancel{4}y\frac{x^4}{\cancel{4}} + \cancel{3}y^2\cdot\frac{x^3}{\cancel{3}} - \frac{x^4}{4} = c$

$$\boxed{\therefore x^4y+x^3y^2-\frac{x^4}{4}=c}$$

4. $y' = \operatorname{cosec}x - y\cot x.$

$\frac{dy}{dx} = \operatorname{cosec}x - y\cot x$

$\frac{dy}{dx} + y\cot x = \operatorname{cosec}x$

$y' + p(x)y = q(x).$

$p(x) = \cot x$

$q(x) = \operatorname{cosec}x$

$e^{\int p dx} = e^{\int \cot x} = e^{\log|\sin x|}.$

$= \sin x$

Now, general solution.

$y.e^{\int p dx} = \int e^{\int p dx}.q(x)\, dx$

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$$y \cdot \sin x = \int \sin x \cdot \operatorname{cosec} x \, dx$$

$$= \int \sin x \cdot \frac{1}{\sin x} \, dx$$

$$y \cdot \sin x = x + c$$

$$\boxed{y = \frac{x + c}{\sin x}}$$

10. Verify that given functions are linearly independent soln of given diff. eqn hence write general soln and solve given initial value problem.

1. $y'' + 9y = 0$.

$y(0) = 4 \quad y'(0) = 6$

$y_1 = \cos 3x \quad y_2 = \sin 3x$.

→ $y_1 = \cos 3x \qquad y_2 = \sin 3x$

To check L.I.

$\frac{y_1}{y_2} = \frac{\cos 3x}{\sin 3x} = \cot 3x$ ____ (not scalar).

Hence y_1 and y_2 are L.I.

general solution.

$y = c_1 \cos 3x + c_2 \sin 3x$.

$y(0) = 4$

$4 = c_1 \cos 3(0) + c_2 \sin 3(0)$

$4 = c_1 + 0$

$\boxed{c_1 = 4}$

Now $y'(0) = 6$.

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$\frac{dy}{dx} = -c_1 \sin 3x \cdot 3 + c_2 \cos 3x \cdot 3.$

$\frac{dy}{dx} = -c_1 \sin 3x \cdot 3 + c_2 \cos 3x \cdot 3$

$6 = -3c_1 \sin 3x(6) + 3c_2 \cos 3(0).$

$6 = 0 + 3c_2(1)$

$6/3 = c_2$

$\boxed{c_2 = 2}$

$\boxed{y = 4\cos 3x + 2\sin 3x.}$

2. $4x^2y - 3y = 0 \quad y(1) = 3 \quad y'(1) = 2.5 \quad y_1 = x^{-1/2}$
$y_2 = x^{3/2}$

→ check

$\frac{y_1}{y_2} = \frac{x^{-1/2}}{x^{3/2}} = x^{-1/2} \cdot x^{-3/2}$

$= x^{-4/2}$

$= x^{-2}$

$= \frac{1}{x^2}$ — (not scalar)

General solution.

$y = c_1 x^{-1/2} + c_2 x^{3/2}$

$y' = c_1 \cdot \frac{-1}{2} x^{-3/2} + c_2 \frac{3}{2} x^{1/2}$

$y' = \frac{-1}{2} c_1 x^{-3/2} + \frac{3c_2}{2} x^{1/2}$

$2.50 = \frac{-1}{2} c_1 (1)^{-3/2} + \frac{3}{2} c_2 (x)^{1/2}$

$2.5 = -\frac{c_1}{2} + \frac{3}{2} c_2$

11

$$2.5 = \frac{-c_1 + 3c_2}{2}$$

$$\boxed{5 = -c_1 + 3c_2} \quad \text{—} ①$$

Now put $y(1) = 3$.

$$3 = c_1 (1)^{-1/2} + c_2 (1)^{3/2}$$

$$\boxed{3 = c_1 + c_2} \quad \text{—} ②$$

Adding ① + ②.

$$8 = 4c_2$$

$$\boxed{c_2 = 2}$$

Now $3 = c_1 + 2$

$$\boxed{c_1 = 1}$$

General solution.

$$\boxed{y = 1\, x^{-1/2} + 2\, x^{3/2}}$$

11 Using method of reduction of order obtain second L.I soln of given diff. equation.

1. $xy'' + 2y' + xy = 0.$ $\quad y_1 = \frac{\sin x}{x}$

→ To find y_2.

$$x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$$

Divide by x.

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y = 0.$$

Here $p(x) = 2/x$ $\quad q(x) = 1.$

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$$u = \int \frac{1}{y_1^2} \cdot e^{-\int p\,dx}$$

$$= \int \frac{1}{\left(\frac{\sin x}{x}\right)^2} \cdot e^{-\int 2/x\,dx}$$

$$= \int \frac{1 \cdot \cancel{x^2}}{\sin^2 x} \cdot 1/\cancel{x^2}$$

$$= \int \frac{1}{\sin^2 x}$$

$$= \int \operatorname{cosec}^2 x$$

$$u = -\cot x$$

$$y_2 = u \cdot y_1$$

$$y_2 = -\cot x \cdot \frac{\sin x}{x}$$

$$= -\frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{x}$$

$$\boxed{y_2 = \frac{-\cos x}{x}}$$

b 2. $(1-x^2)y'' - 2xy' + 2y = 0$. $y_1 = x$

$\rightarrow$ Divide by $(1-x^2)$.

$$y'' - \frac{2xy'}{(1-x^2)} + \frac{2}{(1-x^2)} y = 0.$$

$$p(x) = \frac{-2x}{(1-x^2)} \qquad q(x) = \frac{2}{(1-x^2)}.$$

$$e^{-\int p\,dx} = e^{-\int -\frac{2x}{(1-x^2)}} = e^{\int \frac{2x}{1-x^2}} = e^{\frac{-1}{-1}\int \frac{-2x}{(1-x^2)}}$$

$$= e^{-\log(1-x^2)}.$$

$$\boxed{e^{-\int p\,dx} = \frac{1}{(1-x^2)}}.$$

$x^2 \quad \left(\frac{1}{x^2}+\frac{1}{x^2}\right) \quad x - x^2$

$x^2(1-x^2)$

$x^2(1-x)(1+x)$

$$u = \int \frac{1}{y_1^2} \cdot e^{-\int p\,dx}$$

$$= \int \frac{1}{x^2} \cdot \frac{1}{(1-x^2)}.$$

$$= \int \frac{1}{x^2(1-x^2)}$$

$$= \int \frac{1}{x^2 - x^4}$$

$$= \int \frac{1}{x^{-2}}$$

$$= \int x^2$$

$$\boxed{u = \frac{x^3}{3}}$$

$$y_2 = u \cdot y_1$$

$$= \frac{x^3}{x^3} \cdot x$$

$$\boxed{y_2 = \frac{x^4}{3}}$$

$$\to \frac{1}{x^2(1-x)(1+x)}$$

$$\to \frac{\ln|x+1| - \ln|x-1|}{2} - \frac{1}{x}$$

$$\boxed{u = \frac{\log|x+1| - \log|x-1|}{2} - \frac{1}{x}}$$

$$y_2 = u \cdot y_1$$

$$= \frac{\log|x+1| - \log|x-1|}{2} - \frac{1}{x} \cdot x$$

$$\boxed{y_2 = x\left(\frac{\log|x+1| - \log|x-1|}{2}\right) - 1}$$

12. Obtain general solution of following diffn equation.

1. $25y'' + 40y' + 16y = 0$

$$25\frac{d^2y}{dx^2} + 40\frac{dy}{dx} + 16y = 0.$$

$$(25D^2 + 40D + 16)y = 0.$$

$$25D^2 + 40D + 16 = 0.$$

$$25D^2 + 20D + 20D + 16 = 0.$$

$$5D(5D+4) + 4(5D+4) = 0$$

$$(5D+4) = 0 \quad (5D+4) = 0.$$

$$5D = -4 \quad 5D = -4$$

$$D = -4/5 \quad D = -4/5.$$

400

+20 +20

Roots are real and repeated

$y = e^{mx}(c_1 + c_2 x).$

$$\boxed{y = e^{-4/5x}(c_1 + c_2 x).}$$

2. $y'' + 4y' + (4+\omega^2)y = 0.$

$D^2 y + 4D + (4+\omega^2)y = 0.$

$(D^2 + 4D + (4+\omega^2) = 0$

By formula method.

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{-4^2 - 4 \times 1 \times (4+\omega^2)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{\cancel{16} - \cancel{16} + 4\omega^2}}{2}$$

$$= \frac{-4 \pm \sqrt{4\omega^2}}{2}$$

$$= \frac{-4 \pm 2\omega i}{2}$$

$D_1 = \frac{-4 + 2\omega i}{2}$ $\qquad D_2 = \frac{-4 - 2\omega i}{2}$

Roots are complex.

$D_1 = \frac{-4}{2} + \frac{2\omega i}{2}$ $\qquad D_2 = \frac{-4}{2} - \frac{2\omega i}{2}$

$D_1 = -2 + \omega i$ $\qquad D_2 = -2 - \omega i$

$\alpha = -2 \qquad \beta = \omega$

$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x).$

$$\boxed{y = e^{-2x}(c_1 \cos \omega x + c_2 \sin \omega x)}$$

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3. $y'' - k^2 y = 0.$

$(D^2 - k^2)y = 0.$

$-k^2$

$D^2 - k^2 = 0.$

$(D+k)(D-k) = 0.$ $+k \quad -k$

$D = -k \qquad D = +k$

roots are non repeated

$y = c_1 e^{mx} + c_2 e^{mx}$

$$\boxed{y = c_1 e^{-kx} + c_2 e^{kx}}$$

4. $2y'' - 9y' = 0.$

$2\frac{d^2y}{dx^2} - 9\frac{dy}{dx} = 0.$

$2D^2y - 9Dy = 0$

$(2D^2 - 9D)y = 0.$

$2D^2 - 9D = 0$

By formula method.

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times 0}}{2 \times 2}$$

$$= \frac{9 \pm \sqrt{81}}{4}$$

$$= \frac{9 \pm 9}{4}$$

$$= \frac{9+9}{4} \text{ or } \frac{9-9}{4}$$

$$= \frac{18}{4} \text{ or } \frac{0}{4}$$

$$D = \frac{9}{2} \text{ or } 0$$

Roots are real and non-repeated

$y = c_1 e^{mx} + c_2 e^{mx}$

$= c_1 e^0 + c_2 e^{9/2x}$

$\boxed{y = c_1 + c_2 e^{9/2x}}$

5. $y'' - 2\sqrt{2}y' + 2.5y = 0$

$D^2y - 2\sqrt{2}\,Dy + 2.5y = 0$

$(D^2 - 2\sqrt{2}D + 2.5)y = 0$

$(D^2 - 2\sqrt{2}D + 2.5) = 0$

By formula method

$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{+2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4 \times 1 \times 2.5}}{2 \times 1}$

$= \frac{2\sqrt{2} \pm \sqrt{4 \times 2 - 10}}{2}$

$= \frac{2\sqrt{2} \pm \sqrt{-2}}{2}$

$= \frac{2\sqrt{2} \pm \sqrt{2}i}{2}$

$= \frac{\sqrt{2}(2 \pm i)}{2}$

$= \frac{\cancel{\sqrt{2}}(2 \pm i)}{\sqrt{2}\cdot\cancel{\sqrt{2}}}$

$D = \frac{2 \pm i}{\sqrt{2}}$

$D = \sqrt{2} \pm \frac{1}{\sqrt{2}}i$

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$\alpha = \sqrt{2}\sqrt{\cancel{2}}/\sqrt{\cancel{2}}$

$\alpha = 2/\sqrt{2}$ $\beta = 1/\sqrt{2}$

$= \cancel{\sqrt{2}/\sqrt{2}}$

$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \cancel{\cos} \sin \beta x)$

$\boxed{y = e^{\sqrt{2}x}(c_1 \cos 1/\sqrt{2}x + c_2 \sin 1/\sqrt{2}x).}$

6. $4y'' + 16y' + 17y = 0.$

$4D^2 + 16D + 17 = 0.$

By formula method.

$17 \times 4 = \cancel{28}\ 68$

? ?

$D = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-16 \pm \sqrt{16^2 - 4 \times 4 \times 17}}{2 \times 4}$

$= \dfrac{-16 \pm \sqrt{256 - 272}}{8}$

$= \dfrac{-16 \pm \sqrt{-16}}{8}$

$= \dfrac{-16 \pm \sqrt{16}\, i}{8}$

$= \dfrac{-16 \pm 4i}{8}$

$= \dfrac{-\cancel{4}(4 \pm i)}{\cancel{8}2}$

$= \dfrac{-4 \pm i}{2}.$

$D. = \dfrac{-4 + i}{2}$ or $\dfrac{-4 - i}{2}$

$\alpha = -4/2$ $\beta = \dfrac{1}{2}$

$\alpha = -2$ $\beta = 1/2$

$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

$\boxed{y = e^{-2x}(c_1 \cos \tfrac{1}{2}x + c_2 \sin \tfrac{1}{2}x)}$

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14. Solve following initial value problem:

1. $x^2y - 2xy' + 2y = 0.$ $y(1) = 1.5$ $y'(1) = 1.$

→ we have,

$y = x^m.$
$y' = mx^{m-1}$
$y'' = (m-1)\,mx^{m-2}.$

$x^2 m(m-1)x^{m-2} - 2x \cdot mx^{m-1} + 2x^m = 0.$
$\cancel{x^2}(m^2-m)x^{m-\cancel{2}} - 2m\cancel{x} \cdot x^{m-\cancel{1}} + 2x^m = 0.$
$(m^2-m)x^m - 2m \cdot x^m + 2x^m = 0.$
$x^m(m^2-m-2m+2) = 0.$
$m^2-m-2m+2 = 0.$
$m^2 - 3m + 2 = 0$
$(m-2)(m-1) = 0.$
$m = 2$ or $m = 1$

2
-2 -1

Roots are real and non-repeated

$y = x^m = x^2$ or x^1

$y = c_1 \cdot y_1 + c_2 y_2.$
$y = c_1 \cdot x^1 + c_2 x^2$

We have. $y(1) = 1.5$
$1.5 = c_1(1) + c_2(1)^2$
$\boxed{1.5 = c_1 + c_2}$ ①.

$y'(1) = 1$
$y' = c_1 + 2c_2x.$
$1 = c_1 + 2c_2 \cdot 1$

$$\boxed{1 = c_1 + 2c_2} \quad — ②$$

Subtract eqn ① - ②

$$1.5 = c_1 + c_2$$
$$1 = c_1 + 2c_2$$
$$- \quad - \quad -$$

$$0.5 = \quad -c_2$$

$$\boxed{c_2 = -\frac{1}{2}}$$

Now. put value in eqn ②.

$$1 = c_1 + 2\left(\frac{-1}{2}\right)$$

$$1 = c_1 - 1$$

$$\boxed{c_1 = 2}$$

2. $x^2y'' + xy' + 9y = 0 \quad y(1) = 2 \quad y'(1) = 0.$

We have

$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)x^{m-2}$$

$$x^2 . m(m-1)x^{m-2} + x . mx^{m-1} + 9x^m = 0.$$
$$\cancel{x^2}(m^2-m)x^{m-2} + \cancel{x}mx^{m-1} + 9x^m = 0$$
$$x^m((m^2-m) + m\cancel{x^m} + 9) = 0.$$
$$m^2 - m + m + 9 = 0.$$
$$m^2 + 9 = 0.$$
$$m^2 = \cancel{\pm 9i} -9$$
$$m = \sqrt{-9}.$$
$$m = \pm 3i$$

$m = 3i \qquad m = -3i.$ (out of syllabus).

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3. $(x^2D^2 + 3xD + 1)y = 0$. $y(1) = 3$ $y'(1) = -4$.

Comparing with standard form.

$x^2D^2 + 3xD + 1 = 0$.

By using formula method.

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3x \pm \sqrt{(3x)^2 - 4 \times x^2 \cdot 1}}{2 \times x^2}$$

$$= \frac{-3x \pm \sqrt{9x^2 - 4x^2}}{2x^2}$$

$$= -3x \pm \sqrt{5x^2} / 2x^2$$

$$= \frac{-3x \pm x\sqrt{5}}{2x^2}$$

$$= \frac{\cancel{x}(-3 \pm \sqrt{5})}{2x^{\cancel{2}}}$$

$$= \frac{-3 \pm \sqrt{5}}{2x}$$

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17. Solve the following differential equations.

Ans 1. $(D^4+4D^3+8D^2+8D+4)y=0$.

$d^2 = D^4$
$d = D^2$

Let $D = x$

$\rightarrow 1 \quad -4 \quad 8 \quad -8 \quad +4$

$x^4+4x^3+8x^2+8x+4=0$.

$\rightarrow x^4+4x^3+4x^2+4x^2+8x+4=0$.

$(x^4+4x^3+4x^2)+(4x^2+8x+4)=0$.

$x^2(x^2+4x+4)+4(x^2+2x+1)=0$.

$x^2(x+2)^2+4(x+1)^2=0$.

$[x(x+2)]^2+4(x+1)^2=0$.

$(x^2+2x)^2+4(x+1)^2=0$.

$(\cancel{x^2+2x}$

$(\underbrace{x^2+2x+1}-1)^2+4(x+1)^2=0$.

$[(x+1)^2-1]^2+4(x+1)^2=0$.

$(x+1)^4-2(x+1)^2+1^2+4(x+1)^2=0$.

$(x+1)^4-\underline{2(x+1)^2}+1+\underline{4(x+1)^2}=0$.

$(x+1)^4+2(x+1)^2+1=0$.

$[(x+1)^2+1]^2=0$

$(x+1)^2+1=0$

$(x+1)^2=-1$

$(x+1)=\sqrt{-1}$

$x+1=\pm i$

$\boxed{x=-1\pm i}$

power 4 है इसलिए 4 रूट Roots

Roots are complex.

$D=-1\pm i \qquad \alpha=-1 \quad \beta=1$

$y=e^{-x}[c_1\cos x+c_2\sin x] + (c_3\cos x+c_4\sin x)$.

Complex Repeated $* \; y=e^{-x}\,(c_1+c_2x)\cos x+(c_3+c_4x)\sin x$.

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2. $(D^4+10D^2+9)y=0.$

$\cancel{((D^2)^2+10D.D+9)y=0}$

$\Rightarrow (D^2+9)(D^2+1)=0.$

$\Rightarrow D^4+D^2+9D^2+9=0$

$(D^4+10D^2+9)=0.$

So equation is $(D^2+9)(D^2+1)=0.$

$D^2+9=0. \qquad D^2+1=0.$

$D^2=-9 \qquad D^2=-1$

$D=\sqrt{-9} \qquad D=\sqrt{-1}$

$D=\pm 3i \qquad D=\pm 1 i$

$D=+3i \quad D=-3i \quad D=i \quad D=-i$

Roots are complex

$\cancel{y=e^{\alpha x}(c_1+c_2x)\cos 2x+e^{\alpha x}(c_3+c_4x)}$

$y=e^{0x}(c_1\cos bx+c_2\sin bx)+e^{0x}(c_3\cos bx+c_4\sin$

$\boxed{y=(c_1\cos 3x+c_2\sin 3x)+(c_3\cos x+c_4\sin x)}$

3. $(D^5-3D^4+3D^3-D^2)y=0.$

put $D=x.$

$(x^5-3x^4+3x^3-x^2)y=0.$

$(x^5-3x^4+3x^3-x^2)=0.$

$\cancel{x^4(x-3)+x^2(3x-1)}$

$x^2(x^3-3x^2+3x-1)=0.$

$x^2=0. \qquad x^3-3x^2+3x-1=0.$

$\boxed{x=0}$

One Root is $\boxed{1}$

1	1	-3	3	-1
		1	-2	1
	1	-2	1	$\boxed{0}$

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$x^2 - 2x + 1 = 0$.
$(x-1)(x-1) = 0$
$x = 1, x = 1$.

$\begin{array}{c} 1 \\ \wedge \\ -1 \quad -1 \end{array}$

So all roots are
$x = 0, 0, 1, 1, 1$.

$y = e^{mx}(c_1 + c_2x) + e^{mx}(c_3 + c_4x + c_5x^2)$
$= e^{0x}(c_1 + c_2x) + e^{1x}(c_3 + c_4x + c_5x^2)$.

$\boxed{y = (c_1 + c_2x) + e^x(c_3 + c_4x + c_5x^2)}$

~~4. y~~

5. $(D^3 + 3D^2 + 3D + 1)y = e^{-x} \cdot \sin x$.

Corresponding eqn will be

$x^3 + 3x^2 + 3x + 1 = 0$.
$\because (a+b)^3 = a^3 + b^3 + 3ab \cdot (a+b)$.
$= x^3 + 1^3 + 3x(1)(x+1)$
$= x^3 + 1 + 3x(x+1)$
$= x^3 + 1 + 3x^2 + 3x$
$(x+1)^3 = x^3 + 3x^2 + 3x + 1$

$(x+1)^3 = 0$
$x = -1, -1, -1$ are roots.
$m = -1, -1, -1$.
$y_h = e^{mx}(c_1 + c_2x + c_3x^2)$.
$\boxed{y_h = e^{-x}(c_1 + c_2x + c_3x^2)}$.

4. $y''' - y' = 2x^2 \cdot e^x$

$$\frac{d^3y}{dx^2} - \frac{dy}{dx} = 2x^2 \cdot ex$$

put $\frac{dy}{dx} = m.$

$(m^3 - m) = 2x^2 \cdot ex.$

Comparing with auxiliary equation.

$(m^3 - m) = 0.$

$m(m^2 - 1) = 0.$

$m = 0 \quad , \quad m^2 - 1 = 0$

$m^2 = 1$

$m = 0. \quad m = \pm 1$

$m = -1, 0, 1.$

$y_h = c_1 e^{mx} + c_2 e^{mx} + c_3 e^{mx}$

$y_h = c_1 e^{-x} + c_2 + c_3 e^x$

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16. using method of variation of parameters Solve following equations

(wrong) 1. $y'' - 4y' + 4y = e^{2x}/x$ (wrong).

$(m^2 - 4m + 4)y = 0$

$m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m = 2, 2$

4 → -2 -2

$y_h = e^{2x}(c_1 + c_2 x)$.

By variation of ~~variati~~ parameter

we have.

$u = \int \frac{-r(x) \cdot y_2}{w}$ $v = \int \frac{r(x)\, y_1}{w}$

$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_1' \end{vmatrix}$

$= \begin{vmatrix} 2 & 2 \\ 0 & 0 \end{vmatrix}$

$= 0$.

$u = 0$. $v = u$.

$y_p = 0 + 0 = 0$.

$y = y_h + y_p$

$\boxed{y = e^{2x}(c_1 + c_2 x)}$

2. ~~$3y'' + 10y' + 3y = 9x + 5\cos x$~~

~~$3D^2y + 10Dy + 3y = 9x + 5\cos x$~~

~~$(3D^2 + 10D + 3)y = 0$~~

~~$3D^2 + 10D + 3 = 0$~~

~~$3 \times 3 = 9$~~ → 9 1

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2. $y'' + 9y = \sec 3x$

$D^2y + 9y = \sec 3x$

$(D^2+9)y = 0$

$D^2+9=0.$

$D = \pm 3i$

$y_h = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$

$= e^0 (c_1 \cos 3x + c_2 \sin 3x).$

$\boxed{y_h = (c_1 \cos 3x + c_2 \sin 3x)}$

$y_1 = \cos 3x.$ $\qquad y_2 = \sin 3x.$

$y_p = u \cdot y_1 + v \cdot y_2.$

$u = -\int \frac{r(x) y_2}{w} \quad + \quad v = \int \frac{r(x) y_1}{w}$

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & \cos 3x \cdot 3 \end{vmatrix}$$

$= 3\cos 3x \times \cos 3x - \sin 3x \cdot (-3\sin 3x)$

$= 3\cos^2 3x + 3\sin^2 3x$

$= 3(\sin^2 3x + \cos^3 3x).$

$= 3(1)$

$\boxed{W. = 3}$

$u = -\int \frac{\sec 3x \cdot \sin 3x}{3}.$

$= \frac{-1}{3}\int \frac{1}{\cos 3x} \cdot \sin 3x.$

$= \frac{-1}{3}\int \tan 3x$

$= \frac{-1}{9} \cdot \log \frac{|\sec 3x|}{3.} = \frac{-1}{9} \log |\sec 3x| + c$

$\boxed{u. = \frac{1}{9} \log |\cos 3x| + c.}$

$$V = \int \frac{r(x) \cdot y_1}{W}$$

$$= \int \frac{\sec 3x \cdot \cos 3x}{3}$$

$$= \frac{1}{3} \int \sec 3x \cdot \cos 3x$$

$$= \frac{1}{3} \int \frac{1}{\cos 3x} \cdot \cos 3x$$

$$= \frac{1}{3} \int 1 \, dx$$

$$\boxed{V = \frac{1}{3} x}$$

$$y_p = u \cdot y_1 + v \cdot y_2$$

$$= \frac{1}{9} \log|\cos 3x| \cdot \cos 3x + \frac{1}{3} x \sin 3x$$

$$\boxed{y_p = \frac{\log|\cos 3x| \cdot \cos 3x}{9} + \frac{x \sin 3x}{3}}$$

$$y = y_h + y_p$$

$$\boxed{y = (c_1 \cos 3x + c_2 \sin 3x) + \frac{\log|\cos 3x| \cdot \cos 3x}{9} + \frac{x \cdot \sin 3x}{3}}$$

3. $y'' - 4y' + 5y = e^{2x} \operatorname{cosec} x$

$\cancel{D^2y - 4D + 5}$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = e^{2x} \operatorname{cosec} x$$

$D^2y - 4Dy + 5 = 0.$

$(D^2 - 4D + 5)y = 0.$

$D^2 - 4D + 5 = 0.$

$D = 2 \pm i$

5

$\alpha = 2 \qquad \beta = 1.$

$y_h = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x).$

$y_h = e^{2x}(c_1 \cos x + c_2 \sin x).$

$y_1 = \cos x \qquad y_2 = \sin x.$

$$w = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos^2 x + \sin^2 x$

$w = 1$

$$u = -\int \frac{r(x)\, y_2}{w} = -\int \frac{e^{2x} . \operatorname{cosec} x . \sin x}{1}$$

$$= -\int e^{2x} . \frac{1}{\cancel{\sin x}} . \cancel{\sin x}$$

$$= -\int e^{2x}\, dx$$

$$u = -\frac{e^{2x}}{2}$$

$$V = \int \frac{r(x)\, y_1}{w}$$

$$= \int \frac{e^{2x} \operatorname{cosec} x \cdot \sin \cos x}{1}$$

$$= \int e^{2x} \cdot \frac{1}{\sin x} \cdot \cos x$$

$$= \int \underset{v}{e^{2x}} \cdot \underset{u}{\cot x} \, dx$$

$$= \cot x \cdot \frac{e^{2x}}{2} - \int (-\operatorname{cosec}^2 x) \cdot e^{2x}$$

$$= \cot x \cdot \frac{e^{2x}}{2} + \int \operatorname{cose}^2 x \cdot e^{2x} \, dx$$

$$= \cot x \cdot \frac{e^{2x}}{2} + \operatorname{cosec}^2 x \cdot \frac{e^{2x}}{2}$$

Solve problem from youtube link. (Academics Videos).

4. $(D^2+6D+9)y = 16e^{-3x}/(x^2+1)$.

$(D^2+6D+9)y = 0$.

$D^2+6D+9 = 0$.

$(D+3)(D+3) = 0$

$\boxed{D = -3, -3}$

$\begin{array}{c} 9 \\ \wedge \\ 3 \quad 3 \end{array}$

$\boxed{y_h = e^{-3x}(c_1 + c_2 x)}$

$y_1 = e^{-3x}$. $\qquad y_2 = x \cdot e^{-3x}$.

$$W = \begin{vmatrix} e^{-3x} & x \cdot e^{-3x} \\ -3e^{-3x} & e^{-3x} - x \cdot e^{-3x} \cdot 3 \end{vmatrix}$$

$$= \cancel{e^{-3x}(e^{-3x} + x e^{-3x} \cdot 3)} -$$

$$\begin{vmatrix} e^{-3x} & x \cdot e^{-3x} \\ -3 \cdot e^{-3x} & e^{-3x}(1 - 3x) \end{vmatrix}$$

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$$= e^{-3x} \times e^{-3x} \begin{vmatrix} 1 & x \\ -3 & 1-3x \end{vmatrix}$$

$$= e^{-6x} (1 - \cancel{3x} + \cancel{3x})$$

$$\boxed{W = e^{-6x}}$$

$$u = \int \frac{-r(x) \cdot y_2}{w}$$

$$= \int \frac{-16e^{-3x}}{(x^2+1)} \cdot \frac{x \cdot e^{-3x}}{e^{-6x}}$$

$$= \int \frac{-16x\, \cancel{e^{-6x}}}{(x^2+1)\, \cancel{e^{-6x}}}$$

$$= \int \frac{-16x}{(x^2+1)}$$

$$= -16 \int \frac{x}{(x^2+1)}$$

$$= \frac{-16}{2} \int \frac{2x}{(x^2+1)}$$

$$u = -8 \log |x^2+1)$$

$$V = \int \frac{r(x)\, y_1}{w}$$

$$= \int \frac{16\, e^{-3x} \cdot e^{-3x}}{(x^2+1) \cdot e^{-6x}}$$

$$= \int \frac{16\, \cancel{e^{-6x}}}{(x^2+1) \cdot (\cancel{e^{-6x}})}$$

$$= \int \frac{16}{(x^2+1)}$$

$$= 16 \int \frac{1}{1+x^2}$$

$$V = 16 \tan^{-1} x$$

$$y_p = u \cdot y_1 + v \cdot y_2$$

$$y(p) = -8 \log|x^2+1| \cdot e^{-3x} + 16\tan^{-1}x \cdot x \cdot e^{-3x}$$
$$= -8e^{-3x} \log|x^2+1| + 16xe^{-3x} \cdot \tan^{-1}x$$

$$y = y_h + y_p$$
$$= e^{-3x}(c_1 + c_2 x) - 8e^{-3x}\log|x^2+1| + 16xe^{-3x} \cdot \tan^{-1}x.$$

5. $y'' + 4y' + 4y = e^{-2x}/x^2.$

$(D^2+4D+4)y = 0$

$D^2+4D+4 = 0.$

$(D+2)(D+2) = 0$

$\boxed{D = -2, -2}$

4 → +2, +2.

$\boxed{y_h = c_1 e^{-2x} + c_2 x e^{-2x}}$

$$W = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ e^{-2x} \cdot -2 & e^{-2x} \cdot + e^{-2x} \cdot (-2) \cdot x \end{vmatrix}$$

$$= \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x} - e^{-2x} \cdot 2x \end{vmatrix} \rightarrow e^{-2x}(1-2x)$$

$$= e^{-2x} \cdot e^{-2x} \begin{vmatrix} 1 & x \\ -2 & 1-2x \end{vmatrix}$$

$$= e^{-4x} \left| 1 - \cancel{2x} + \cancel{2x} \right|$$

$\boxed{W = e^{-4x}}$

$$u = \int \frac{-r(x) \cdot y_2}{W}$$

$$= -\int \frac{e^{-2x}}{x^2} \cdot \frac{x \cdot e^{-2x}}{e^{-4x}}$$

$$= -\int \frac{x \cdot \cancel{e^{-4x}}}{x^2 \cdot \cancel{e^{-4x}}}$$

$$= -\int \frac{1}{x}\, dx$$

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$= -\log |x|$

$$\boxed{u = \log \left|\frac{1}{x}\right|}$$

$$V = \int \frac{r(x) \cdot y_1}{w}$$

$$= \int \frac{e^{-2x}}{x^2} \cdot \frac{e^{-2x}}{e^{-4x}}$$

$$= \int \frac{\cancel{e^{-4x}}}{x^2 \cdot \cancel{e^{-4x}}}$$

$$= \int \frac{1}{x^2}$$

$$= \int x^{-2}$$

$$= \frac{x^{-1}}{-1}$$

$$\boxed{v = -\frac{1}{x}}$$

$y_p = u \cdot y_1 + v \cdot y_2$

$y_p = \log \left|\frac{1}{x}\right| \cdot e^{-2x} + \frac{-1}{x} \cdot x \cdot e^{-2x}$

$y = y_h + y_p$

$$\boxed{y = c_1 e^{-2x} + c_2\, e x e^{-2x} + \log \left|\frac{1}{x}\right| e^{-2x} - \frac{1}{x} x e}$$

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1. $y'' - 4y' + 4y = e^{2x}/x$

$(m^2 - 4m + 4)\,y = 0$

$m^2 - 4m + 4 = 0$

$(m-2)\,(m-2) = 0$

$\boxed{m = 2, 2}$

$$\begin{array}{c} 4 \\ \wedge \\ -2 \quad -2 \end{array}$$

$y_h = c_1 e^{2x} + c_2 x . e^{2x}$

we have

$$W = \begin{vmatrix} e^{2x} & x . e^{2x} \\ e^{2x}.2 & e^{2x}.(1) + e^{2x}.2.x \end{vmatrix}$$

$$= \begin{vmatrix} e^{2x} & x . e^{2x} \\ 2e^{2x} & e^{2x} + e^{2x}.2x \end{vmatrix}$$

$$= \begin{vmatrix} e^{2x} & x . e^{2x} \\ 2e^{2x} & e^{2x}(1+2x) \end{vmatrix}$$

$$= e^{2x}.e^{2x} \begin{vmatrix} 1 & x \\ 2 & (1+2x) \end{vmatrix}$$

$$= e^{2x}.e^{2x}\,|1 + \cancel{2x} - \cancel{2x}|$$

$\boxed{W = e^{4x}}$

$$u = -\int \frac{r(x) . y_2}{W}\,dx$$

$$= -\int \frac{e^{2x}}{x} . \frac{x . e^{2x}}{e^{4x}}\,dx$$

$$= -\int \frac{x . \cancel{e^{4x}}}{x . \cancel{e^{4x}}}\,dx$$

$$= -\int 1\,dx$$

$\boxed{u = -x}$

$$V = \int \frac{r(x) . y_1}{W} = \int \frac{e^{2x}}{x} . \frac{e^{2x}}{e^{4x}}$$

$$= \int \frac{\cancel{e^{4x}}}{x . \cancel{e^{4x}}}$$

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$$= \int \frac{1}{x}\,dx$$

$$\boxed{v = \log x}$$

$y_p = u.y_1 + v.y_2$

$y_p = -x.e^{2x} + \log x.\,x.e^{2x}$

$y = y_h + y_p$

$$\boxed{y = c_1 e^{2x} + c_2 x.e^{2x} - x e^{2x} + x \log x\, e^{2x}}$$

Q.3.

5. $y' = (y-x)^2 \qquad y(0) = 2$

$\frac{dy}{dx} = (y-x)^2$

put $y - x = u$.

$\frac{du}{dx} = \frac{dy}{dx} - 1$

$\frac{du}{dx} + 1 = \frac{dy}{dx}$

$\cancel{\frac{dy}{dx}} = u^2 \; \cancel{\frac{dy}{dx} + 1}$

$\frac{du}{dx} + 1 = (u)^2$

$\frac{du}{dx} = u^2 - 1$

$\frac{du}{u^2-1} = dx$

$\int \frac{du}{u^2-1} = \int dx$

$\int \frac{1}{(u+1)(u-1)} = \int dx$

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partial fraction.

$$\frac{1}{(u+1)(u-1)} = \frac{A}{(u+1)} + \frac{B}{(u-1)}$$

$$\frac{1}{(u+1)(u-1)} = \frac{A(u-1) + B(u+1)}{(u+1)(u-1)}$$

$$\frac{1}{(u+1)(u-1)} = \frac{Au - A + Bu + B}{(u+1)(u-1)}$$

$$\frac{1}{(u+1)(u-1)} = \frac{u(A+B) - A + B}{(u+1)(u-1)}$$

$A+B=0$.

$-A+B=1$. —— (A)

$A=-B$ —— (1).

Put in eqn (A).

$+B+B=1$

$2B=1$

$\boxed{B=1/2}$

$\boxed{A=-1/2}$

$$\int \frac{-1/2}{(u+1)} + \frac{1/2}{(u-1)} = x + c$$

$$-\frac{1}{2}\int \frac{1}{u+1} + \frac{1}{2}\int \frac{1}{u-1} = x + c$$

$$-\frac{1}{2}\log|u+1| + \frac{1}{2}\log|u-1| = x + c$$

$$\frac{1}{2}(-\log|u+1| + \log|u-1|) = x + c$$

$$\frac{1}{2}\log\left|\frac{1}{u+1}\right| + \log|u-1| = x + c$$

$$\frac{1}{2}\log\left|\frac{u-1}{u+1}\right| = x + c$$

$$\boxed{\frac{1}{2}\log\left|\frac{y-x-1}{y-x+1}\right| = x} + c$$

$$c + 0 = \frac{1}{2} \log \left| \frac{2-0-1}{2+0+1} \right|$$

$$= \frac{1}{2} \log \left| \frac{1}{3} \right|$$

$$c = \frac{1}{2} \log \left(\frac{1}{3}\right)$$

$$\boxed{c = \log \sqrt{\frac{1}{3}}}$$

6. $e^x y' = 2(x+1) y^2$

$$e^x \frac{dy}{dx} = (2x+2) y^2$$

$$e^x \frac{dy}{dx} = 2xy^2 + 2y^2$$

$$\frac{dy}{dx} = \frac{y^2(2x+2)}{e^x}$$

$$\frac{dy}{y^2} = (2x+2) \cdot e^{-x} \, dx$$

$$\int \frac{dy}{y^2} = \int (2x+2) \cdot e^{-x} \, dx$$

$$\frac{-1}{y} = \int \underset{u}{2(x+1)} \cdot \underset{v}{e^{-x}} \, dx$$

$$= 2 \int (x+1) \cdot e^{-x} \, dx$$

$$\frac{-1}{y} = 2 \cdot (x+1) \cdot \frac{e^{-x}}{-1} - \int 1 \cdot \frac{e^{-x}}{-1} \, dx$$

$$\frac{-1}{y} = 2 \, (x+1) \cdot \frac{e^{-x}}{-1} + \int e^{-x} \, dx + c$$

$$\frac{-1}{y} = -2 \left((x+1) e^{-x} - e^{-x} \right) + c$$

$$\frac{1}{y} = 2 \left[(x+1) e^{-x} - e^{-x} \right] + c$$

putting values $\quad \frac{6}{\cdot} = 2 \left[(1) e^{-0} - e^{0} \right] + c$

$6 = 2(1-1)+c$

$6 = 2(0)+c$

$\boxed{c = 6}$

Q.4. 6. $\left(\frac{\cos y}{x+3}\right)dx - \left(\sin y \log|5x+15| - \frac{1}{y}\right)dy = 0$

$M = \left(\frac{\cos y}{x+3}\right) \cancel{dx}$ $\quad N = -\left(\sin y \log|5x+15| + \frac{1}{y}\right)$

$\frac{\partial M}{\partial y} = \frac{1}{x+3} \cdot -\sin y \quad \frac{\partial N}{\partial x} = -\sin y \cdot \frac{1}{5x+15} \cdot 5 + 0$

$\frac{\partial M}{\partial y} = \frac{-\sin y}{x+3} \quad \frac{\partial N}{\partial x} = -\frac{\sin y \cancel{5}}{\cancel{5}(x+3)}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Equation is exact.

$\int M dx + \int N dy = 0$

$\int\left(\frac{\cos y}{x+3}\right)dx + \int \frac{1}{y} = 0$

$\boxed{\cos y \cdot \log|x+3| + \log|y| = c}$

Q.7. 7. $xy' = y(\ln y - \ln x)$

$xy' = y \cdot \left(\log \frac{y}{x}\right)$

$y' = \frac{y}{x} \log\left(\frac{y}{x}\right)$

$\frac{dy}{dx} = \frac{y}{x} \cdot \log\left(\frac{y}{x}\right)$

~~put y/x = v~~

$y = x \cdot v$

$\frac{dy}{dx} = x \cdot \frac{dv}{dx} + v(1)$

$$v + x\frac{dv}{dx} = v \cdot \log v$$

$$x\frac{dv}{dx} = v \cdot \log v - v$$

$$\int \frac{dv}{v \cdot \log v - v} = \int \frac{dx}{x}$$

$$\int \frac{dv}{v(\log v - 1)} = \log x + c$$

$$\int \frac{1/v}{\log v - 1}\, dv = \log x + c$$

$$\log|\log v - 1| = \log x + \log c$$

$$\log(\log v - 1) = \log(x + c)$$

$$|\log v - 1| = x + c$$

put $v = \frac{y}{x}$

$$\boxed{\left(\log\frac{y}{x} - 1\right) = x + c}$$

12. $(\cos y \cdot \sin 2x)\, dx + (\cos^2 y - \cos^2 x)\, dy = 0$

$m = \cos y \cdot \sin 2x$ $\qquad$ $N = \cos^2 y - \cos^2 x$

$\frac{\partial m}{\partial y} = \sin 2x \cdot -\sin y$ $\qquad$ $\frac{\partial N}{\partial x} = 0 - (-\sin 2x)$

$$= \sin 2x$$

$$= 2\sin x \cdot \cos x$$

$$\frac{1}{m}\left(\frac{\partial N}{\partial x} - \frac{\partial m}{\partial y}\right)$$

$$\frac{1}{\cos y \cdot \sin 2x}\left(2\sin x \cdot \cos x - (\sin 2x \cdot -\sin y)\right)$$

$$= \frac{1}{\cos y \sin 2x}(\sin 2x + \sin 2x \cdot \sin y)$$

$$= \frac{\cancel{\sin 2x}}{\cos y \cdot \cancel{\sin 2x}} + \frac{\cancel{\sin 2x} \cdot \sin y}{\cos y \cdot \cancel{\sin 2x}}$$

* $\cos^2 y \cdot \sec y \cdot \tan y$
$\cos^2 y \frac{1}{\cos y} \cdot \frac{\sin y}{\cos y}$
$= \sin y$.

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$I.F = \sec y + \tan y$

$I.F = e^{\int \sec y + \tan y \cdot dy}$
$= e^{\log(\sec y + \tan y) + \log(\sec y)}$
$= e^{\log(\sec y + \tan y \times \sec y)}$

$I.F = \sec^2 y + \sec y \cdot \tan y$
$I.F = \sec y(\sec y + \tan y)$. —— ①

multiply eqn ① x original equation.

$[\sec y(\sec y + \tan y)] \times (\cos y \cdot \sin 2x] + [\sec y(\sec y + \tan y)]$
$\times (\cos^2 y - \cos^2 x)\, dy = 0$

$\Rightarrow \cancel{\sec^2 y + \sec y \cdot \tan y \cdot \frac{1}{\sec y} \cdot \sin 2x + \sec^2 y + \sec y \tan y}$.

$\cancel{\sin 2x}$

$\Rightarrow \cancel{\sec y}(\sec y + \tan y) \frac{1}{\cancel{\sec y}} \cdot \sin 2x + (\cos^2 y - \cos^2 x)(\sec^2 y + \sec y \cdot \tan y) = 0$

$\Rightarrow \sin 2x(\sec y + \tan y) + (1 - \sec^2 y \cdot \cos^2 x + \sin y - \cos^2 x \cdot \sec y \cdot \tan y) = 0$.

*

$M = \sin 2x(\sec y + \tan y)$.
$N = 1 + \sin y$

$\int M + \int N = c$
$\int \sin 2x(\sec y + \tan y) + \int 1 + \sin y = c$

$$\boxed{(\sec y + \tan y)\frac{-\cos 2x}{2} + y - \cos y = c.}$$

Q.9. 6. $(4xy+3y^4)dx + (2x^2+5xy^3)dy = 0$

$m = (4xy+3y^4)$ $\qquad$ $N = (2x^2+5xy^3)$

$\frac{\partial m}{\partial y} = 4x + 3.4y^3$ $\qquad$ $\frac{\partial N}{\partial x} = 4x + 5y^3$

$\frac{\partial m}{\partial y} = 4x + 12y^3$ $\qquad$ $\frac{\partial N}{\partial x} = 4x + 5y^3$

$$\frac{\partial m}{\partial y} \neq \frac{\partial N}{\partial x}$$

Let x^a, y^b are IF to given equation

$m = x^a y^b (4xy+3y^4)dx$
$= (4x^{a+1} y^{b+1} + 3x^a . y^{b+4}) dx$

$N = x^a y^b (2x^2+5xy^3) dy$
$= (2x^{a+2} y^b + 5x^{a+1} y^{b+3}) dy$

Now

$\frac{\partial m}{\partial y} = (4x^{a+1}(b+1)y^b + 3x^a (b+4) y^{b+3})$

$\frac{\partial N}{\partial x} = (2(a+2)x^{a+1} . y^b + 5(a+1)x^a . y^{b+3})$

So considering It is exact.

$4(b+1)x^{a+1}y^b + 3(b+4)x^a y^{b+3} =$
$2(a+2)x^{a+1} . y^b + 5(a+1)x^a . y^{b+3}$

Comparing.

$\Rightarrow 4(b+1)\cancel{x^{a+1}}y^b = 2(a+2)\cancel{x^{a+1}y^b}$

$4b+4 = 2a+2$

$4b+4-2 = 2a$ $\qquad$ $\therefore 4b+2 = 2a$ $\qquad$ $\therefore \cancel{2(2b+}$

$\therefore 2 = 2a-4b$ $\qquad$ $\therefore \cancel{2} = \cancel{2}(a-2b)$ $\qquad$ $\therefore a-2b=0$

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$3(b+4) = 5(a+1)$.
$3b+12 = 5a+5$
$3b+12-5 = 5a$
$\boxed{7 = 5a-3b.}$ — (2)

eqn (1) x (2) . 5.

$5a - 10b = 0$. — (3).

Solving (2) and (3).
~~$5a - 3b =$~~
~~$5a - 10b = 0$~~
$5a - 10b = 0$
$- \; 5a - 3b = 7$
$+ \quad \cdot$

$0 - 7b = -7$
$7b = 7$
$\boxed{b = 1}$
put in eqn (1).
$a - 2(1) = 0$
$a - 2 = 0$
$\boxed{a = 2}$
So integrating factor will be x^2y.

$M_1 = x^2y(4xy + 3y^4)$.
$M_1 = 4x^3y^2 + 3x^2y^5$

$N_1 = x^2y(2x^2 + 5xy^3)$.
$N_1 = 2x^4y + 5x^3y^4$

Now finding Solution.
$\int M\,dx + \int N\,dy = 0$.

$$\int 4x^3y^2 + 3x^2y^5 + \int 0 = 0$$

$$4y^2 \frac{x^4}{4} + 3y^5 \frac{x^3}{3}$$

$$y^2 \cdot x^4 + y^5 \cdot x^3 = 0$$

Solution.

$$\boxed{x^4y^2 + x^3 \cdot y^5 = 0.}$$

1. $x\,dy = (y + x^2 + y^2)\,dx.$

$$\frac{x\,dy}{x} = \frac{y}{x} + x + yy\left(\frac{y}{x}\right) dx$$

$$dy = \left(\frac{y}{x}\right) + x + y\left(\frac{y}{x}\right) dx$$

put $y = vx$

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x \cdot \frac{dv}{dx} = v + x + vx(v)$$

$$\cancel{x}\frac{dv}{\cancel{dx}} = \cancel{v + x + v^2x\,dx \cdot v}\ dx$$

$$\cancel{\frac{dv}{dx} = v + v^2x\,dx}$$

$$= \cancel{v(v + vx)}$$

$$x \cdot \frac{dv}{dx} = \cancel{v} + x + \cancel{vx}\,v^2x - \cancel{v} \cdot dx$$

$$x \cdot \frac{dv}{dx} = x + v^2x$$

$$x\frac{dv}{dx} = x(1 + v^2)$$

$$\frac{dv}{1 + v^2} = \frac{x}{x}\,dx$$

$$\int \frac{1}{1 + v^2}\,dv = \int \frac{x}{x}\,dx$$

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$\tan^{-1}(v) = x + c.$

$\tan^{-1}\left(\frac{y}{x}\right) = x + c.$

—x—

1. $x\,dy = (y + x^2 + 9y^2)\,dx.$

Divide by x on both side

$\frac{dy}{dx} = \frac{y}{x} + x + 9 \cdot y \cdot \left(\frac{y}{x}\right)$ ~~dx~~

put $y = v \cdot x$

$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}.$

$v + x \cdot \frac{dv}{dx} = v + x + 9 \cdot v \cdot x (v)$

$v + x \cdot \frac{dv}{dx} = v + x + 9v^2 x$

$x \frac{dv}{dx} = \cancel{v} + x + 9v^2 \cdot x - \cancel{v}$

$x \frac{dv}{dx} = x + 9v^2 x$

$x \frac{dv}{dx} = x(1 + 9v^2).$

$\frac{dv}{(1+9v^2)} = \frac{x}{x}\,dx$

$\int \frac{1}{1+(3v)^2}\,dv = \int 1\,dx$

$\frac{1}{3}\tan^{-1}(3v) \cdot 3 = x + c.$

$\frac{1}{3}\tan^{-1}\left(3\frac{y}{x}\right) = x + c.$

$$\boxed{\frac{1}{3}\tan^{-1} 3\left(\frac{y}{x}\right) = x + c}$$

5. $2(y+1)y' - \frac{2}{x}(y+1)^2 = x^4.$

put $y+1 = v.$

$$\frac{dy}{dx} = \frac{dv}{dx}$$

$$2(v)\frac{d\cancel{y}v}{dx} - \frac{2}{x}(v)^2 = x^4.$$

$$2v\frac{dv}{dx} = x^4 + \frac{2}{x}v^2$$

$$\frac{dv}{dx} = \frac{x^4}{2v} + \frac{\cancel{2}v^{\cancel{2}}}{x.\cancel{2}\cancel{v}}$$

$$\frac{dv}{dx} = \frac{x^4}{2v} + \frac{v}{x}$$

$$\frac{dv}{dx} = \frac{x^5+2v^2}{2vx}.$$

$$2vx\,dv = x^5+2v^2\,dx$$

$$(x^5+2v^2)dx - (2vx)\,dv = 0. \quad \text{—— (1).}$$

$m = x^5 + 2v^2$ $\qquad$ $N = -2vx$

$\frac{\partial m}{\partial \cancel{y}v} = 2.2v$ $\qquad$ $\frac{\partial N}{\partial x} = -2v.$

$\frac{\partial m}{\partial v.} = 4v$ $\qquad$ $\frac{\partial N}{\partial x} = -2v$

So it is not exact.

$$f(x) = \frac{1}{N}\left(\frac{\partial m}{\partial v} - \frac{\partial N}{\partial x}\right)$$

$$= \frac{1}{-2vx}\left(4v+2v\right)$$

$$= \frac{6v}{-2vx}$$

$$f(x). = \frac{-3}{x}$$

$$I.F = e^{\int p\,dx} = e^{\int -3/x\,dx} = e^{\int \cancel{1/3x} - 3/x\,dx}$$

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$$= e^{-3\log x}$$

$$= e^{\log x^{-3}}$$

$$I.F = \frac{1}{x^3}$$

So multiply by I.F to given eqn.

$$\frac{1}{x^3}(x^5+2v^2)\,dx - \frac{1}{x^3}(2vx) = 0$$

$$\left(\frac{x^5}{x^3}+\frac{2v^2}{x^3}\right)dx - \frac{2v\cancel{x}}{x^3}\,dv = 0$$

$$\left(x^2+\frac{2v^2}{x^3}\right)dx - \frac{2v}{x^2}\,dv = 0$$

$$\int x^2+\frac{2v^2}{x^3}\,dx + \int 0 = c$$

$$\frac{x^3}{3} + \frac{2v^2 \cdot x^{-2}}{-2} = c$$

$$\frac{x^3}{3} - v^2x^{-2} = c$$

put $v = y+1$

$$\boxed{\frac{x^3}{3} - (y+1)^2 x^{-2} = c}$$