College of Engineering Pune
Multiple Integration
Tutorial 4

(1) Sketch the region of integration and evaluate the integrals.
(a) $\int_0^3 \int_0^2 (4-y^2)dy\ dx$ (b) $\int_0^3 \int_{-2}^0 (x^2y-2xy)dy\ dx$
(c) $\int_\pi^{2\pi} \int_0^\pi (sinx+cosy)dy\ dx$ (d) $\int_0^\pi \int_0^{sinx} ydy\ dx$
(e) $\int_1^{ln8} \int_0^{lny} e^{x+y}dy\ dx$ (f) $\int_1^2 \int_y^{y^2} dydx$
(g) $\int_0^1 \int_2^{4-2x} dy\ dx$ (h) $\int_0^2 \int_{y-2}^0 dy\ dx$
(i) $\int_0^1 \int_1^{e^x} dx\ dy$ (j) $\int_0^{ln2} \int_{e^x}^2 dy\ dx$
(k) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3ydx\ dy$ (l) $\int_0^2 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 6xdy\ dx$

(2) Integrate f over the region.
(a) $f(x,y)=x^2+y^2$ over the triangular region with vertices (0,0), (0,1), (1,0).
(b) $f(x,y)=y\ cosxy$ over the rectangle $0 \le x \le \pi$, $0 \le y \le 1$.

(3) Find the volume of the region bounded by the paraboloid $z=x^2+y^2$ and below by the triangle enclosed by the lines $y=x, x=0$, and $x+y=2$ in the $xy-$ plane.

(4) Find the volume of the solid that is bounded by the cylinder $z=x^2$ and below by the region enclosed by the parabola $y=2-x^2$ and the line $y=x$ in the $xy-$ plane.

(5) Find the volume of the solid in the first octant bounded by the coordinate planes, the plane $x=3$, and the parabolic cylinder $z=4-y^2$.

(7) In following exercise change the Cartesian integral into polar integral and then evaluate the polar integral.
(a) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dy\ dx$ (b) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\ dx$
(c) $\int_{-a}^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\ dx$ (d) $\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2)dx\ dy$
(e) $\int_0^6 \int_0^y xdx\ dy$ (f) $\int_0^2 \int_0^x ydy\ dx$
(g) $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} ln(x^2+y^2+1)dx\ dy$ (h) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dx\ dy.$

(8) Find the area of the region that lies inside the cardioid $r=1+cos\ \theta$ and outside the circle $r=1$.

(9) Find the area of the region cut from the first quadrant by the cardioid $r=1+sin\ \theta$.

(10) Find the area of the region common to the interiors of the cardioid $r=1+cos\ \theta$ and cardioid $r=1-cos\ \theta$.
(11) Write six different iterated triple integrals for the volume of the rectangular solid in the first octant bounded by the coordinate planes and the planes $x=1, y=2$ and $z=3$. Evaluate one of the intgrals.

(12) Write six different iterated triple integrals for the volume of the tetrahedron cut from the first octant by the plane $6x+3y+2z=6$. Evaluate one of the intgrals.

(13) Write six different iterated triple integrals for the volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 4$ and the plane $y = 3$. Evaluate one of the intgrals.

(14) Let D be the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Write six different iterated triple integrals for the volume D. Evaluate one of the intgrals.

(15) Evaluate the triple integrals.
(a) $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz\ dy\ dx$ (b) $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx\ dy\ dz$
(c) $\int_0^1 \int_0^\pi \int_0^\pi ysinz\ dx\ dy\ dz$ (d) $\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz\ dy\ dx$.

(16) Evaluate the cylindrical coordinate integrals.
(a) $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} dz\ r\ dr\ d\theta$ (b) $\int_0^{2\pi} \int_0^3 \int_{r^2/3}^{\sqrt{18-r^2}} dz\ r\ dr\ d\theta$
(c) $\int_0^{2\pi} \int_0^{\theta/2\pi} \int_0^{\sqrt{3+24r^2}} dz\ r\ dr\ d\theta$ (d) $\int_0^\pi \int_0^{\theta/\pi} \int_{-\sqrt{4-y^2}}^{3\sqrt{4-y^2}} z\ dz\ r\ dr\ d\theta$.

(17) D is the solid right cylinder whose base is the region between the circles $r = cos\theta$ and $r = 2cos\theta$ and whose top lies in the plane $z = 3 - y$.

(18) Evaluate the spherical coordinate integrals.
(a) $\int_0^{\pi/2} \int_0^\pi \int_0^{2sin\phi} \rho^2 sin^2\phi\ d\rho\ d\phi\ d\theta$
(b) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^2 (\rho\ cos\phi)\ d\rho^2 sin\phi\ d\rho\ d\phi\ d\theta$
(c) $\int_0^{2\pi} \int_0^\pi \int_0^{(1-cos\phi)/2} \rho^2 sin^2\phi\ d\rho\ d\phi\ d\theta$
(d) $\int_0^{3\pi/2} \int_0^\pi \int_0^1 5\rho^3 sin^3\phi\ d\rho\ d\phi\ d\theta$

(19) Evaluate the integral
(a) $\int_0^4 \int_{y/2}^{y/2+1} \dfrac{2x - y}{2} dx\ dy$

(20) Use the transformation, $u = x - y$, $v = 2x + y$ to evaluate the integral
$\int \int_R (2x^2 - xy - y^2) dx\ dy$
for the region R in the first quadrant bounded by the lines $y = -2x + 4$, $y = -2x + 7, y = x - 2$ and $y = x + 1$.

(21) Use the transformation, $u = 3x + 2y$, $v = x + 4y$ to evaluate the integral
$\int \int_R (3x^2 + 14xy + 8y^2) dx\ dy$
for the region R in the first quadrant bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3, y = -(1/4)x$ and $y = -(1/4)x + 1$.

(22) Use the transformation, $u = 2x - 3y$, $v = -x + y$ of the parallelogram R to evaluate the integral $\int \int_R 2(x - y) dx\ dy$
for the region R in the xy-plane with the boundaries $x = -3, x = 0, y = x$ and $y = x + 1$.

(23) Sphere and cones : Find the volume of the portion of the solid sphere $\rho \leq a$ that lies between the cones $\phi = \pi/3$ and $\phi = 2\pi/3$

(24) Sphere and half-planes : Find the volume of the region cut from the solid sphere $\rho \leq a$ by the half-planes $\phi = 0$ and $\phi = \pi/6$ in the first octant.

(25) Sphere and plane : Find the volume of the smaller region cut from the solid sphere $\rho \leq 2$ by the plane $z = 1$.

(26) Cone and planes : Find the volume of the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ between the planes z = I and $z = 2$.