MIS : 142103012 Div - 2

FCS

Assigment - 2

Q1) Find the laplace transformation of
$f(t) = A e^{-at} u(t)$

$\Rightarrow F(s) = \int_0^\infty f(t) e^{-st} dt$

$= \int_0^\infty A e^{-at} e^{-st} dt$

$= A \int_0^\infty e^{-(s+a)t} dt$

$= \frac{-A}{s+a} e^{-(s+a)t} \Big|_0^\infty = \frac{A}{s+a}$

Q2) Find the inverse laplace transformation of $F(s) = \frac{1}{(s+3)^2}$

$\Rightarrow$ We know the first ~~shofle~~ shifting theorem

$L[e^{-at} f(t)] = F(s+a)$

the Laplace transform of $f(t) = t u(t)$ is $\frac{1}{s^2}$

the inverse transform of $F(s+a) = \frac{1}{(s+a)^2}$

is $e^{-at} t u(t)$

$f(t) = e^{-3t} t u(t)$

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Q3) Given the following differential eq, solve for y (t) if all initial conditions are 0 ($y(0) = 0$)

Use : $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 21y = 32u(t)$

$\Rightarrow$ Substitute the corresponding L.T for each term in above eq using table & the initial condition $y(t)$ & $\frac{dy(t)}{dt}$ by $y(0-)=0$ & $\frac{dy(0-)}{dt} = 0$

So we get

$$s^2Y(s) + 10sY(s) + 21Y(s) = \frac{32}{s}$$

solving for the response $Y(s)$

$$Y(s) = \frac{32}{s(s^2+10s+21)} = \frac{32}{s(s+7)(s+3)}$$

$$Y(s) = \frac{32}{s(s+7)(s+3)} = \frac{K_1}{s} + \frac{K_2}{(s+7)} + \frac{K_3}{(s+3)}$$

$$K_1 = \left.\frac{32}{(s+7)(s+3)}\right|_{s\to 0} = \frac{32}{21}$$

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$$K_2 = \left.\frac{32}{s(s+3)}\right|_{s=-7} = \frac{32}{28} = \frac{8}{7}$$

$$K_3 = \left.\frac{32}{s(s+7)}\right|_{s\to -3} = \frac{32}{-12} = -\frac{8}{3}$$

$$Y(s) = \frac{32/21}{s} + \frac{8/7}{(s+7)} + \frac{-8/3}{(s+3)}$$

$$= \frac{32}{21}u(t) + \frac{8}{7}e^{-7t} + \left(-\frac{8}{3}e^{-3t}\right)$$

$$= \frac{32}{21}u(t) + \frac{8}{7}e^{-7t} - \frac{8}{3}e^{-3t}$$

$$= \frac{8}{7}e^{-7t} - \frac{8}{3}e^{-3t} + \frac{32}{21}$$

Q4) Find the Laplace transform of $f(t) = te^{-5t}$

=> We know laplace T of $\frac{1}{s^2}$ $t = \frac{1}{s^2}$

& we know $e^{-at} = \frac{1}{s+a}$

$$L\{te^{-5t}\} = \frac{1}{(s+5)^2}$$

$$= te^{-5t} = \frac{1}{(s+5)^2}$$

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Q5) Find the inverse Laplace transform of
$$F(s) = \frac{10}{s(s+2)(s+3)^2}$$

⇒ Expanding F(s) by partial fraction

$$F(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+3)^2} + \frac{D}{s+3}$$

$$A = \left.\frac{10}{(s+2)(s+3)^2}\right|_{s-0} = \frac{5}{9}$$

$$B = \left.\frac{10}{s(s+3)^2}\right|_{s\to 2} = -5$$

$$C = \left.\frac{10}{s+(s+2)}\right|_{s\to -3} = \frac{10}{3}$$

$$D = \left.(s+3)^2 \frac{dF(s)}{ds}\right|_{s\to -3} = \frac{40}{9}$$

Taking the inverse Laplace transform yeild

$$f(t) = \frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t}$$

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Q6) Find the transfer function of $\frac{d^2c(t)}{dt^2}$ + $\frac{5\,dc(t)}{dt} + 8\,c(t) = r(t)$

⇒ Taking Laplace transform of both sides

$s^2 C(s) + 5s\,C(s) + 8\,C(s) = R(s)$

The laplace function G(s) is

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s^2+5s+8}$$

Q7) Find the ~~to~~ transfer function of

$$\frac{d^3c(t)}{dt^3} + 6\frac{d^2c(t)}{dt^2} + 7\frac{d^2c(t)}{dt^2} + 8\,c(t) =$$

$$\frac{d^2r(t)}{dr^2} - 6\frac{dr(t)}{dt} = 3\,r(t)$$

⇒ Taking Laplace

$s^3 C(s) + 6s^2 C(s) + 7s^2 C(s) + 8\,C(s) =$

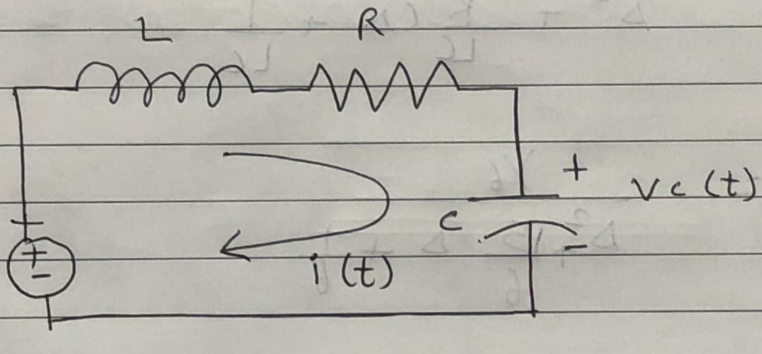
$s^2 R(s) + 6s\,R(s) + 3R(s)$

$= \frac{C(s)}{R(s)} = G(s)$

$$= \cancel{\frac{s^2 x(s) + 6s\, x(s) + 3x(s)}{s^3 x(s) + 13 s^2 x(s)}}$$

$$= \frac{s^2 + 6s + 3}{s^3 + 13s^2 + 8}$$

Q3 Find the transfer function relating the capacitor voltage (V_C) to input voltage $V(s)$ If $L = 2$ units $C = 3$ units & $R = 5$ units



Using mesh analysis

$V(s) = I(s) \cdot Z$

$V(s) = I(s) \left(R + Ls + \frac{1}{Cs} \right)$

$\therefore I(s) = \frac{V(s)}{R + Ls + \frac{1}{Cs}}$

But the voltage across the capacitor $V_C(s)$ is the product of current of impedance of capacitor

$$V_c(s) = I(s) \times \cancel{\frac{1}{C(s)}} \; \frac{1}{Cs}$$

Putting the value of $I(s)$,

$$V_c(s) = \frac{1}{Cs} \cdot \frac{V(s)}{Ls + R + \frac{1}{Cs}} = \frac{V(s)}{LCs^2 + RCs + 1}$$

$$V_c(s) = \frac{\left(\frac{1}{LC}\right) V(s)}{s^2 + \frac{R}{LC} Cs + \frac{1}{LC}}$$

$$\frac{V_c(s)}{V(s)} = \frac{1/6}{s^2 + \frac{15}{6} s + \frac{1}{6}}$$

$$= \frac{1/6}{s^2 + \frac{5}{2} s + \frac{1}{6}}$$

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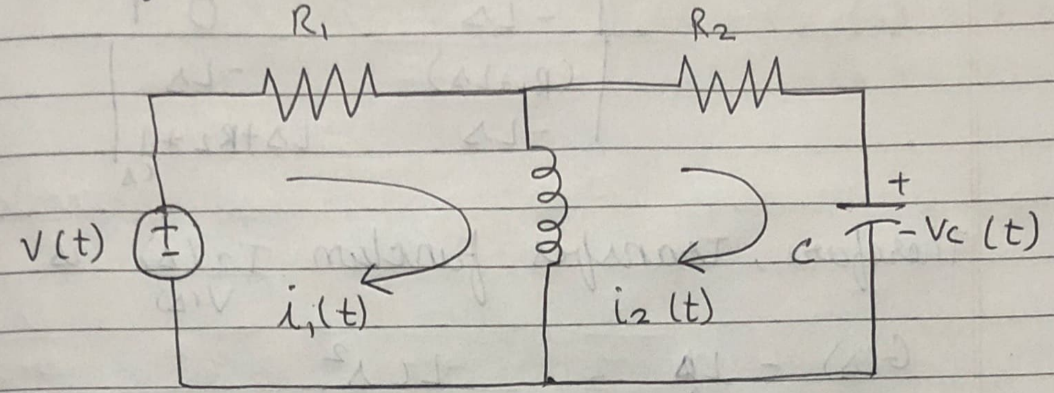
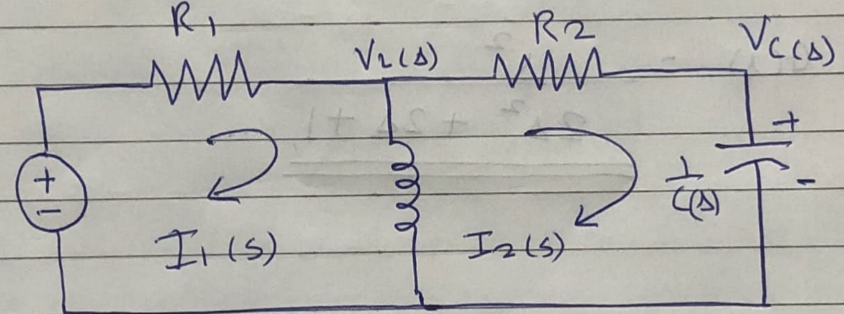
9) Find the transfer function, $I_2(s)/V(s)$. Assume value of each component is 1 unit.



=> Transformed Network :-



For mesh 1

$R_1 I_1(s) + Ls I_1(s) - Ls I_2(s) = V(s)$

For mesh 2

$Ls I_2(s) + R_2 I_2(s) + \frac{1}{Cs} I_2(s) - Ls I_1(s) = 0$

simulateous eq in $I_1(s)$ & $I_2(s)$

$(R_1 + Ls) I_1(s) - Ls I_2(s) = V(s)$

$-Ls I_1(s) + \left(Ls + R_2 + \frac{1}{Cs}\right) I_2(s) = 0$

Solving for $I_2(s)$:-

$$I_2(s) = \frac{\begin{vmatrix} (R_1 + Ls) & V(s) \\ -Ls & 0 \end{vmatrix}}{\Delta}$$

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$$\therefore I_2(s) = \frac{\begin{vmatrix} (R_1+Ls) & V(s) \\ -Ls & 0 \end{vmatrix}}{\begin{vmatrix} (R_1+Ls) & -Ls \\ -Ls & Ls+R_2+\frac{1}{Cs} \end{vmatrix}}$$

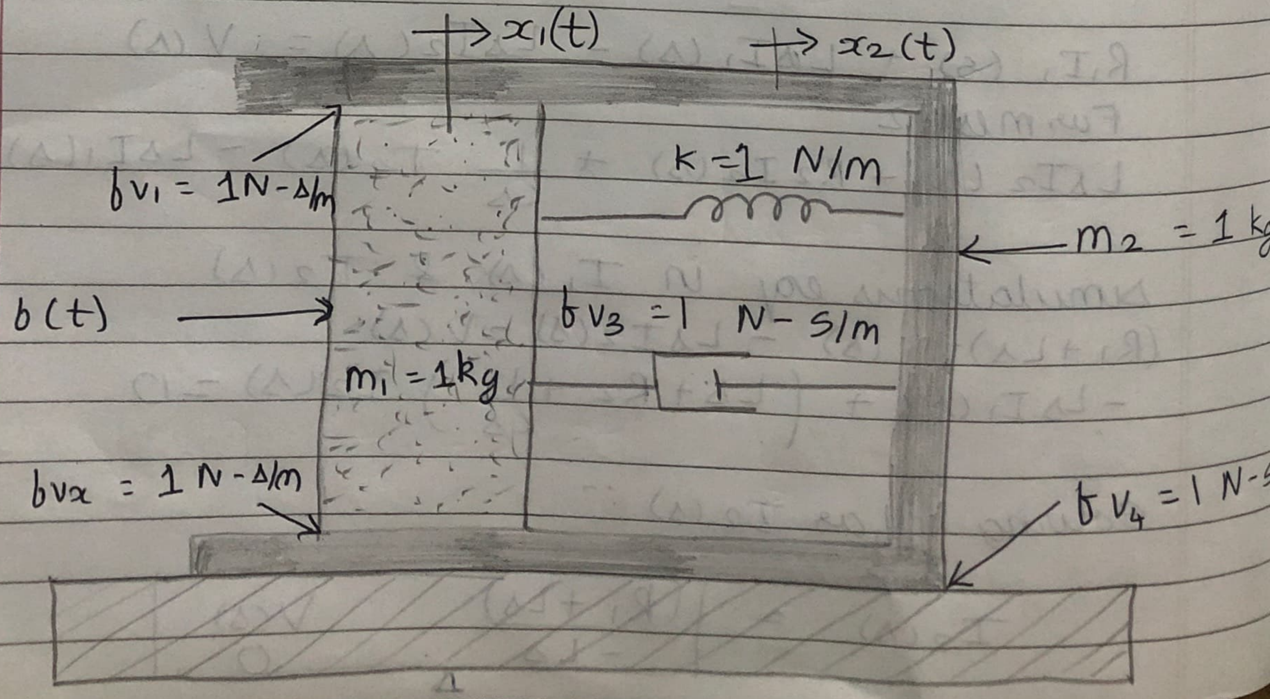
Therefore, Transfer function $\frac{I_2(s)}{V(s)}$ is

$$G(s) = \frac{L s}{\Delta} = \frac{LCs^2}{(R_1+R_2)LCs^2+(R_1R_2+C+L)s+R}$$

Given value for each component is 1 unit

$$\therefore G(s) = \frac{s^2}{2s^2+2s+1}$$

10) Find the transfer function, $G(s) = X_2(s)/F(s)$ for the translational mechanical system



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$\Rightarrow$ Writing the eq of motion

$$(s^2 + 3s + 1)\,X_1(s) - (3s+1)\,X_2(s) = F(s)$$
$$-(3s+1)\,X_1(s) + (s^2 + 4s + 1)\,X_2(s) = 0$$

solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} (s^2+3s+1) & (Fs) \\ -(3s+1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2+3s+1) & -(3s+1) \\ -(3s+1) & (s^2+4s+1) \end{vmatrix}}$$

$$= \frac{(3s+1)\,F(s)}{s(s^3+7s^2+5s+1)}$$

Hence, $\dfrac{X_2(s)}{F(s)} = G(s) = \dfrac{(3s+1)}{s(s^3+7s^2+5s+1)}$