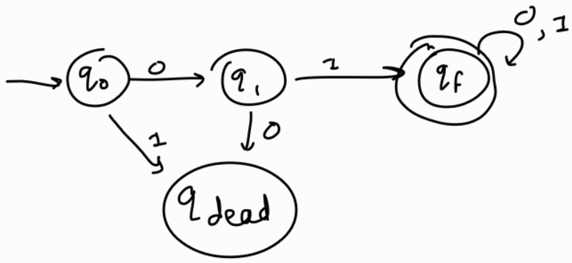
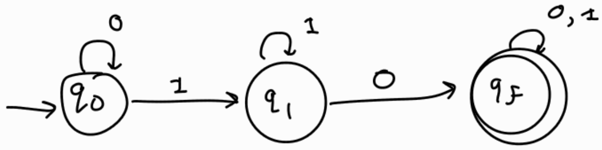
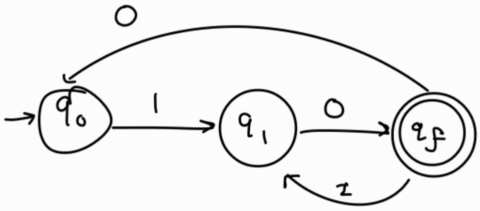
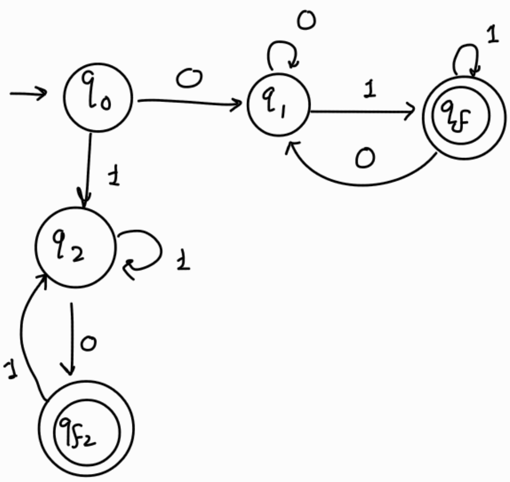
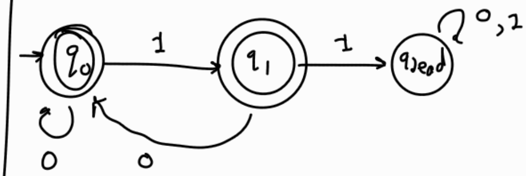








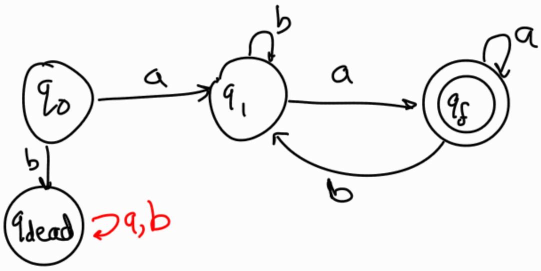
No consequtive 1's

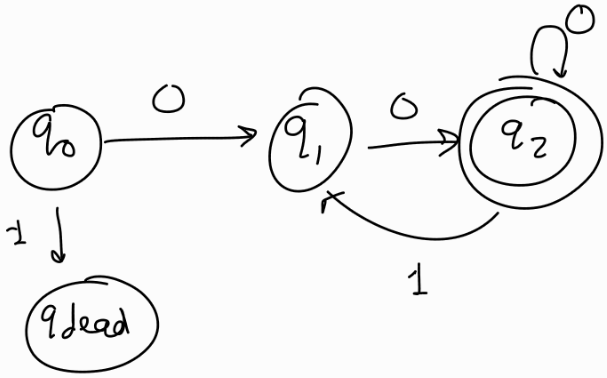


$$\hat{\delta}(q_0, 101) = \delta(\delta(\delta(\hat{\delta}(q_0, \epsilon), 1), 0), 1)$$
$$= \delta(\delta(\delta(q_0, 1), 0), 1)$$
$$= \delta(\delta(q_1, 0), 1)$$
$$= \delta(q_0, 1)$$
$$= \delta(q_1)$$
$$= \text{Accept}$$

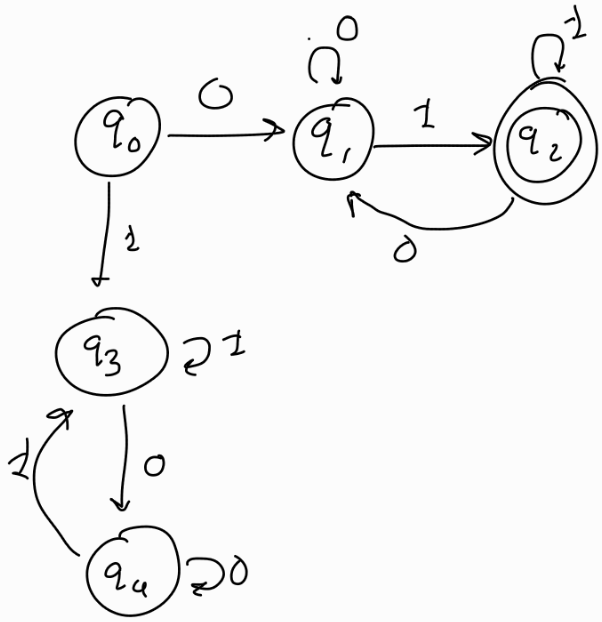
Regular if accepted by some DFA.

$$awa \mid \varepsilon \in \{a, b\}^*$$

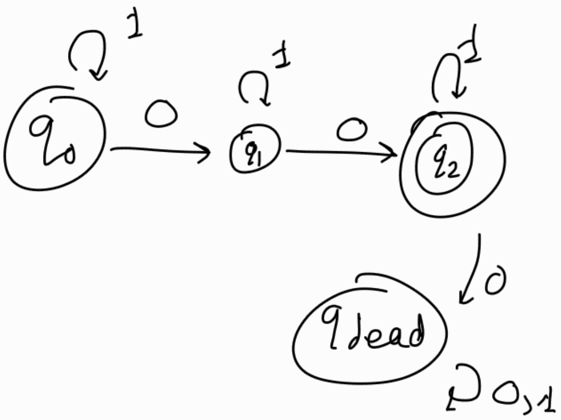




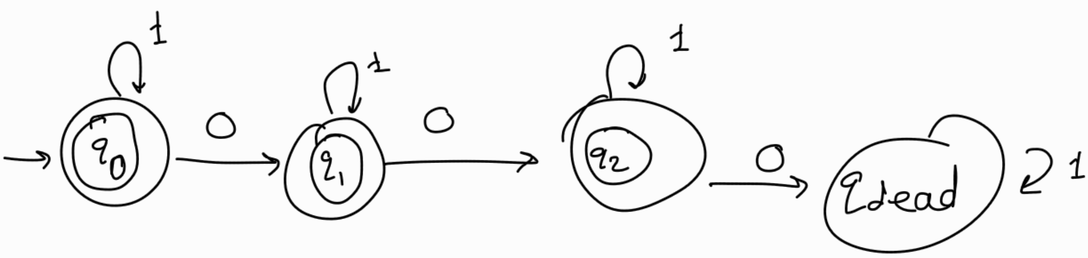
q0
q1
q2
qdead
0
0
0
1
1



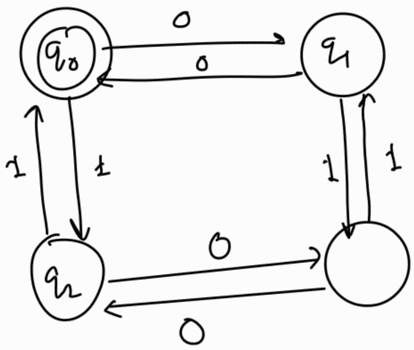
q0
q1
q2
q3
q4
0
0
1
1
0
1
1
0
1
0



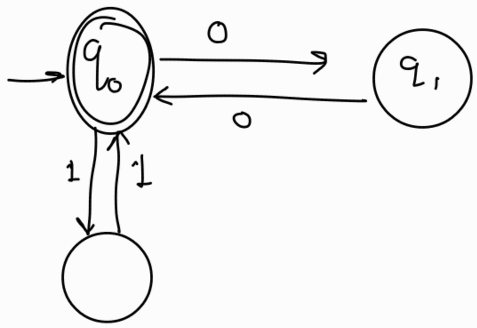
q0
q1
q2
qdead
1
1
1
0
0
0
0,1



q0
q1
q2
qdead
1
1
1
0
0
0
1



even 0's AND 1's



$a^n b a^n b$

let p be the pumping length.

$\therefore$ Assuming L_2 is regular,

let $\omega = a^p b a^p b$

let $x = a^{p-k}$

let $y = a^k$

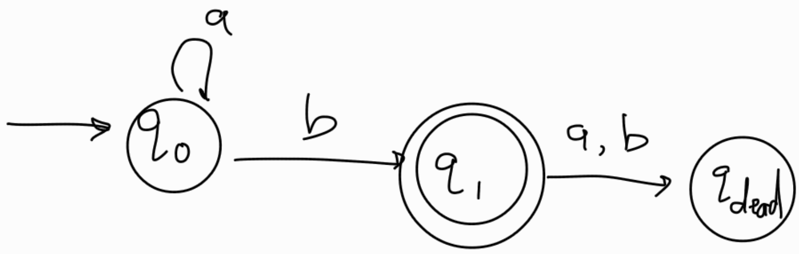
let $z = b a^p b$

$\therefore |xy| \leq p$

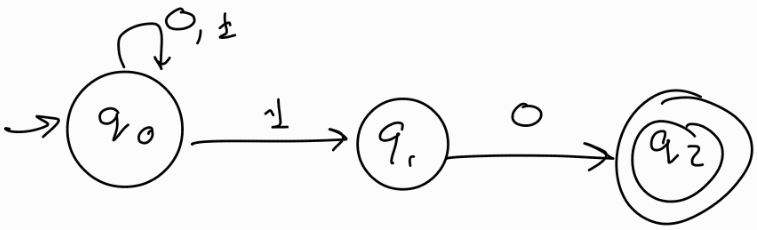
$|y| > 0$

Now, when we pump y, 0 times,

$a^{p-k} a^0 b a^p b = a^{p-k} b a^p b$ which does not belong to the language .

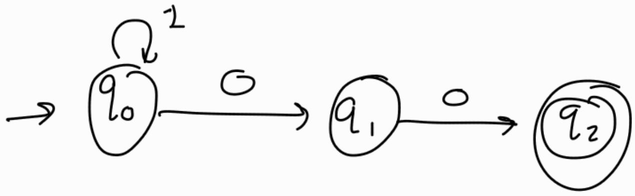


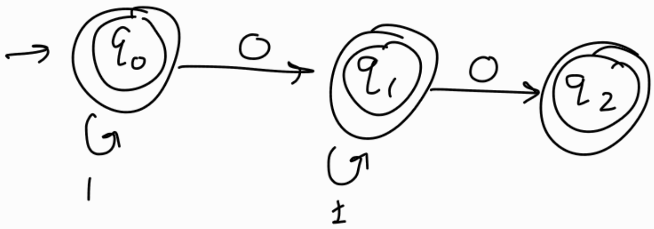
NFA:

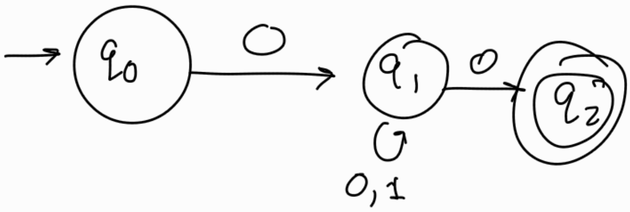


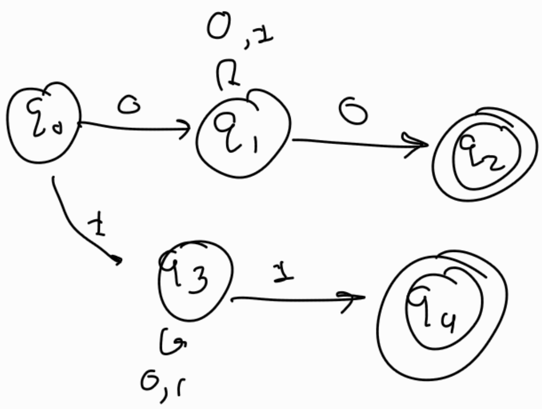
* NFA

$$\delta(q, w) = \Sigma^* \times Q \rightarrow 2^Q$$



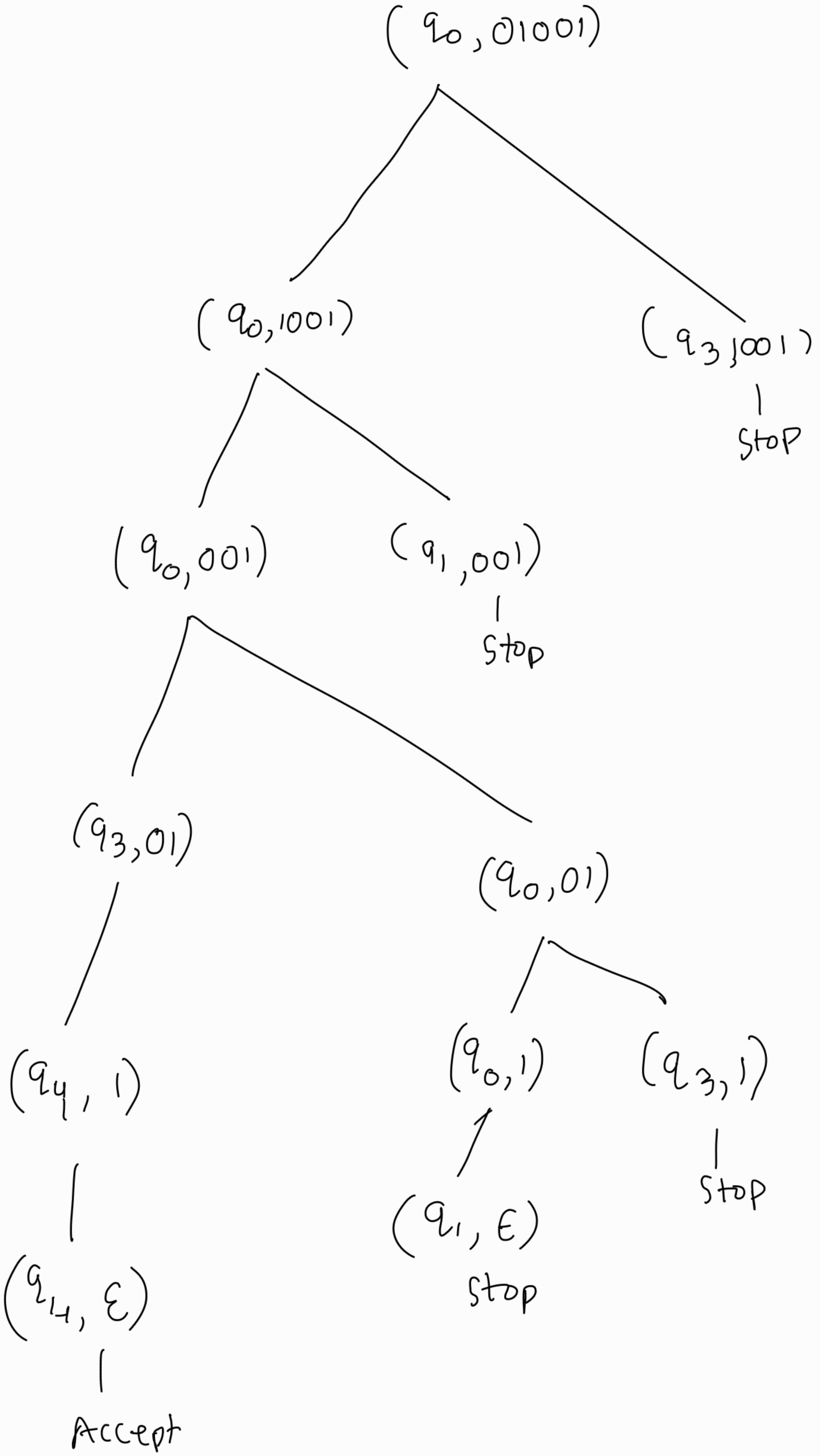




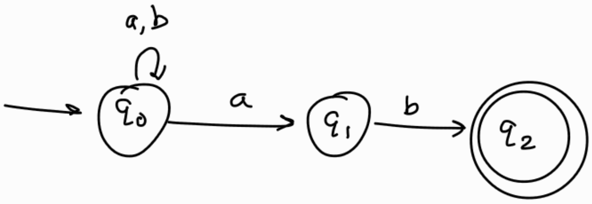


$\hat{\delta}(q_0, 01001)$

5.19

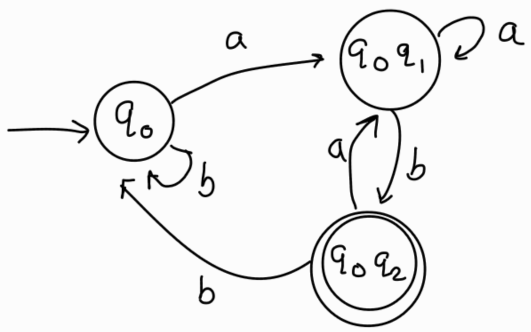


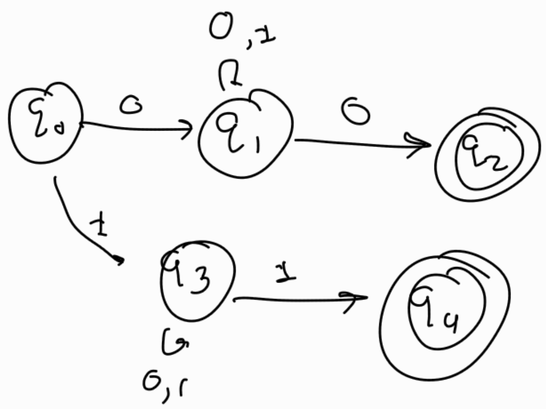
$\hat{\delta}(q_0, 01001) = \{q_0, q_3, q_4\}$ $\quad q_4 \in \{F\}$ $\quad \therefore$ $q_0, 01001$ is ✓✓



$\underline{Q'}$

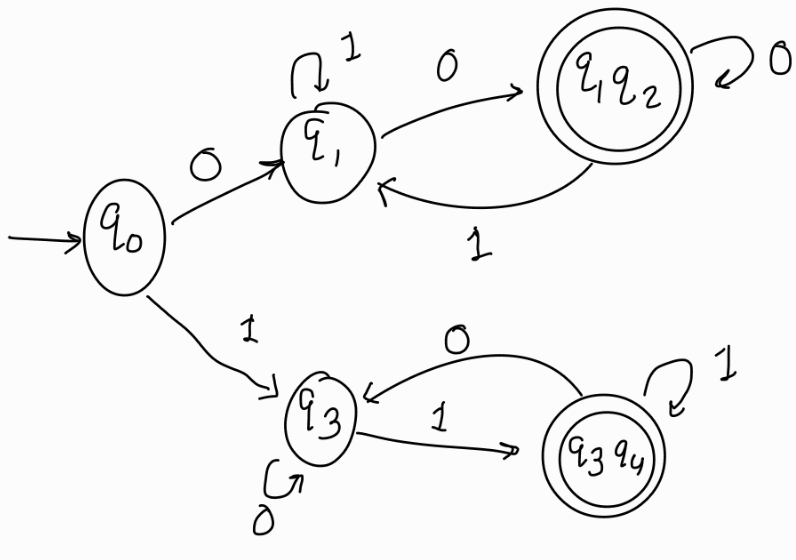
	a	b
$\rightarrow [q_0]$	$[q_0 q_1]$	$[q_0]$
not rechable x $[q_1]$	ϕ	$[q_2]$
not rechable x $*[q_2]$	ϕ	ϕ
$[q_0 q_1]$	$[q_0 q_1]$	$[q_0 q_2]$
$*[q_0 q_2]$	$[q_0 q_1]$	$[q_0]$





Φ'

	0	1	
$[q_0]$	$[q_1]$	$[q_3]$	
$[q_1]$	$[q_1 q_2]$	$[q_1]$	
$[q_2]$	$\emptyset$	$\emptyset$	unreachable
$[q_3]$	$[q_3]$	$[q_3 q_4]$	
$[q_4]$	$\emptyset$	$\emptyset$	
$[q_1 q_2]$	$[q_1 q_2]$	$[q_1]$	
$[q_3 q_4]$	$[q_3]$	$[q_3 q_4]$	

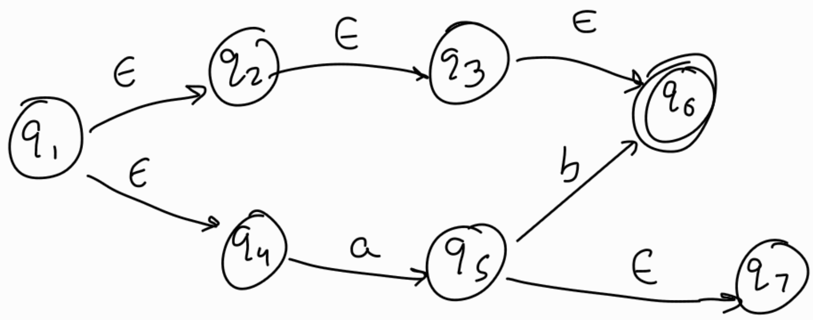


ϵ - NFA

$$\delta = \Sigma \times Q \cup \{\epsilon\} \longrightarrow 2^Q$$

ϵ - closure

set of All states visitable from current by ϵ transitions.



ϵ-closure		a	b
q_1	$\{q_1, q_2, q_3, q_4, q_6\}$	ϕ	ϕ
q_2	$\{q_2, q_3, q_6\}$	ϕ	ϕ
q_3	$\{q_3, q_6\}$	ϕ	ϕ
q_4	$\{q_4\}$	q_5	ϕ
q_5	$\{q_5, q_7\}$	ϕ	q_6
q_6	$\{q_6\}$	ϕ	ϕ
q_7	$\{q_7\}$	ϕ	ϕ

for ϵ-NFA, $\hat{\delta}$ works differently

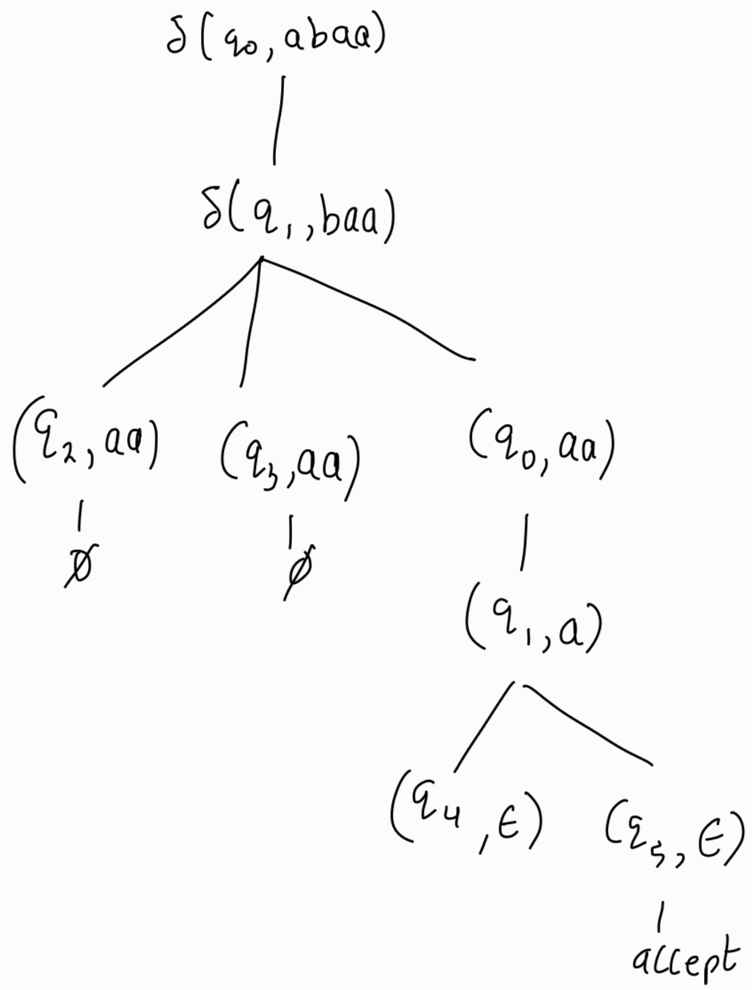
$$\hat{\delta}(q, ab) = \text{e-closure}(\delta(\delta(\hat{\delta}(q_0, \epsilon), a), b))$$

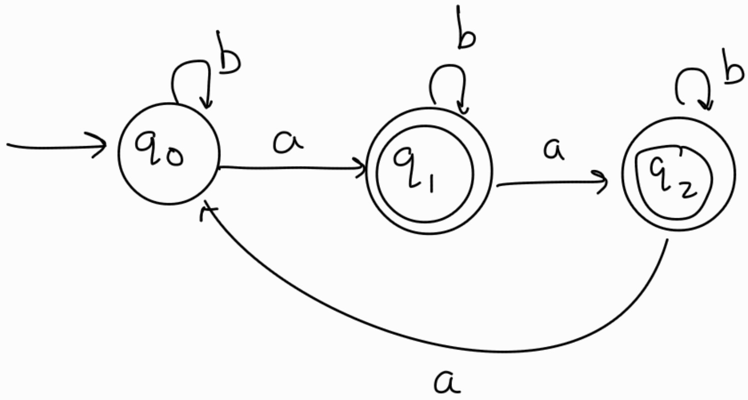
$$= \text{e-closure}(\delta(\hat{\delta}(q_0, a), b)$$

$$= \text{e-closure}(\hat{\delta}(q_1, b))$$

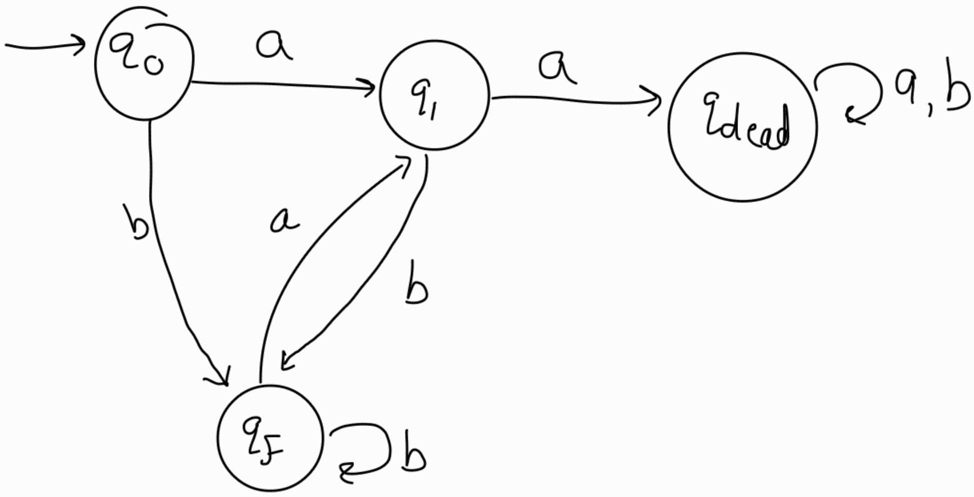
$$= \text{e-closure}(q_2)$$

$$= \{q_2, q_3, q_0\}$$

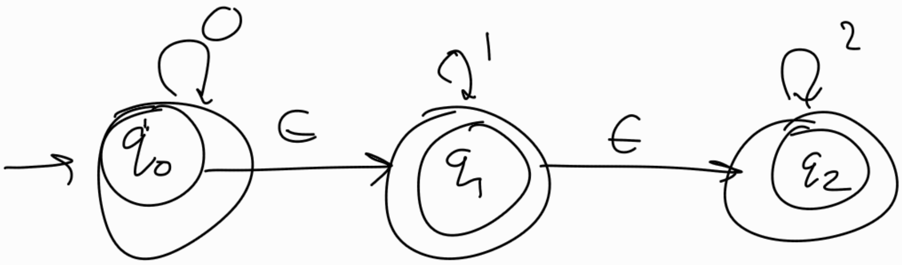




$n_a(w) \bmod 3 > 1$



without substr aa, end with b.

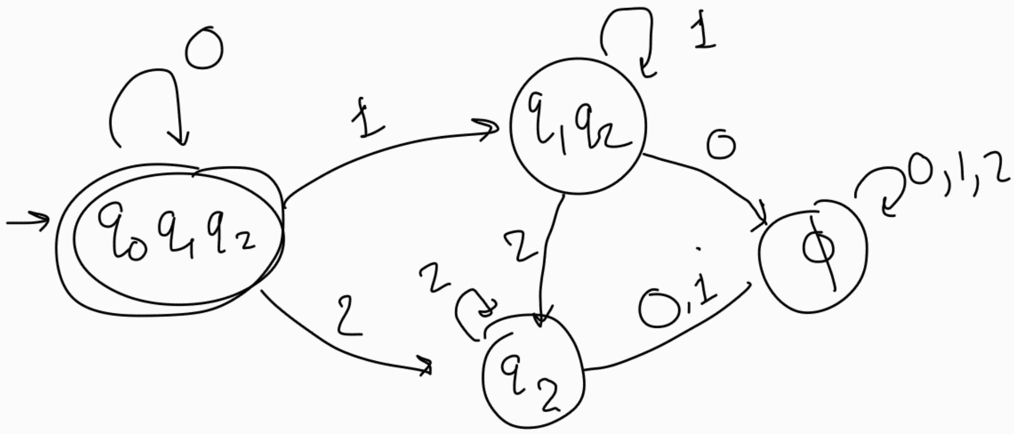


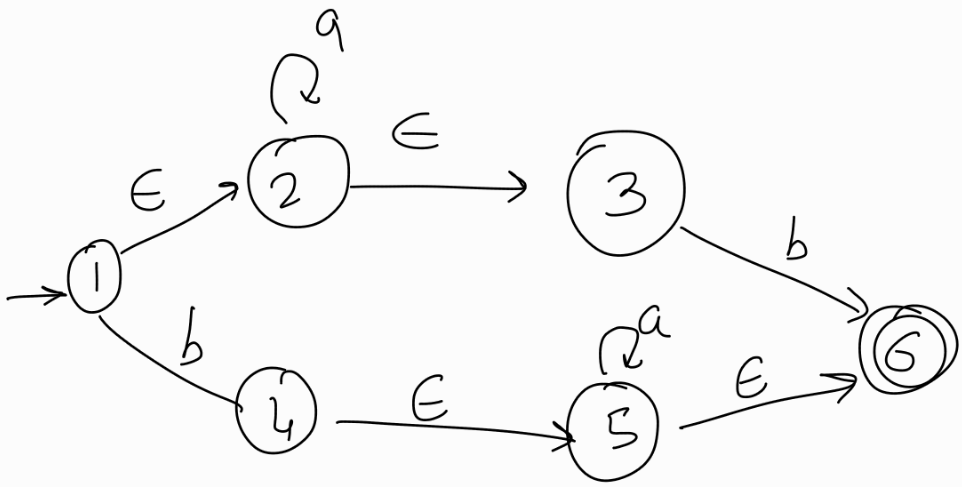
$\epsilon^*(q_0) = q_0 q_1 q_2$

$\epsilon^*(q_1) = q_1 q_2$

$\epsilon^*(q_2) = q_2$

	0	1	2
→ $[q_0 q_1 q_2]$	$q_0 q_1 q_2$	$q_1 q_2$	q_2
$q_1 q_2$	$\emptyset$	$q_1 q_2$	q_2
q_2	$\emptyset$	$\emptyset$	q_2

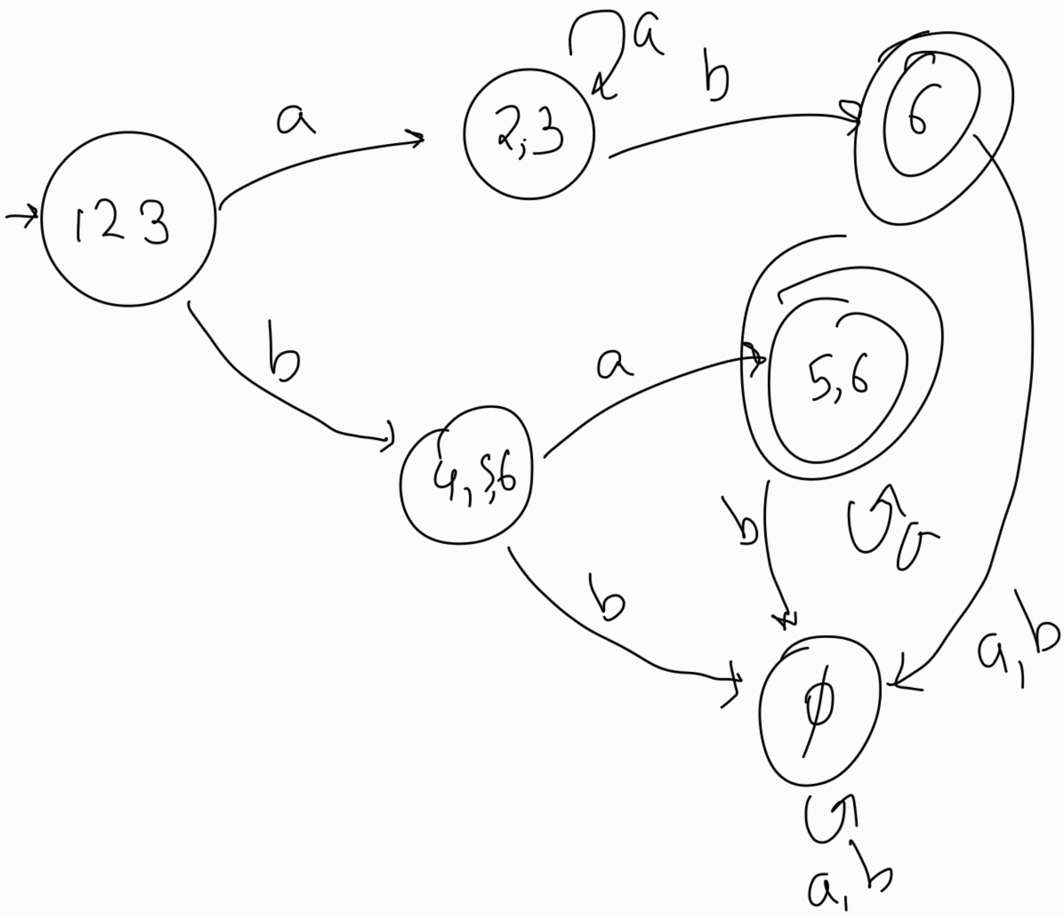


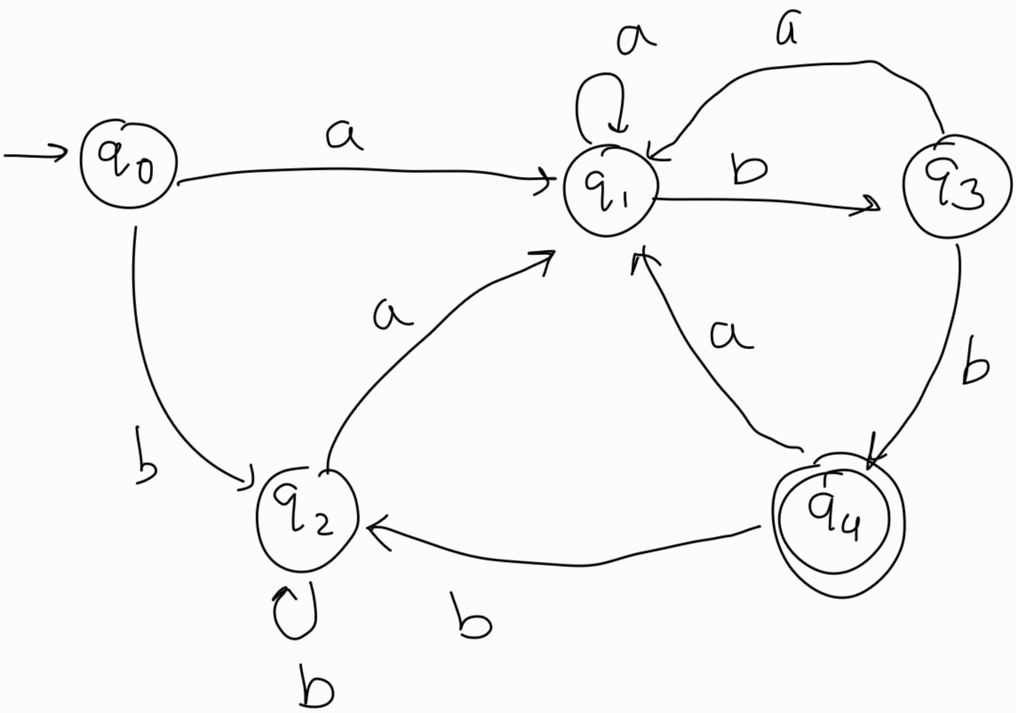


State	ε-closure
1	1,2.3
2	2,3
3	3
4	4,5,6
5	5,6
6	6

	a	b
→ [1,2,3]	[2,3]	[4,5,6]
[4,5,6]	[5,6]	∅
[2,3]	[2,3]	[6]
[6]	∅	∅
[5,6]	[5,6]	∅

$\emptyset$ $\emptyset$ $\emptyset$





	q_0	q_1	q_2	q_3
q_1	X			
q_2		X		
q_3	X	X	X	
q_4	X	X	X	X

q_4 is final else aren't hence mark q_4 row

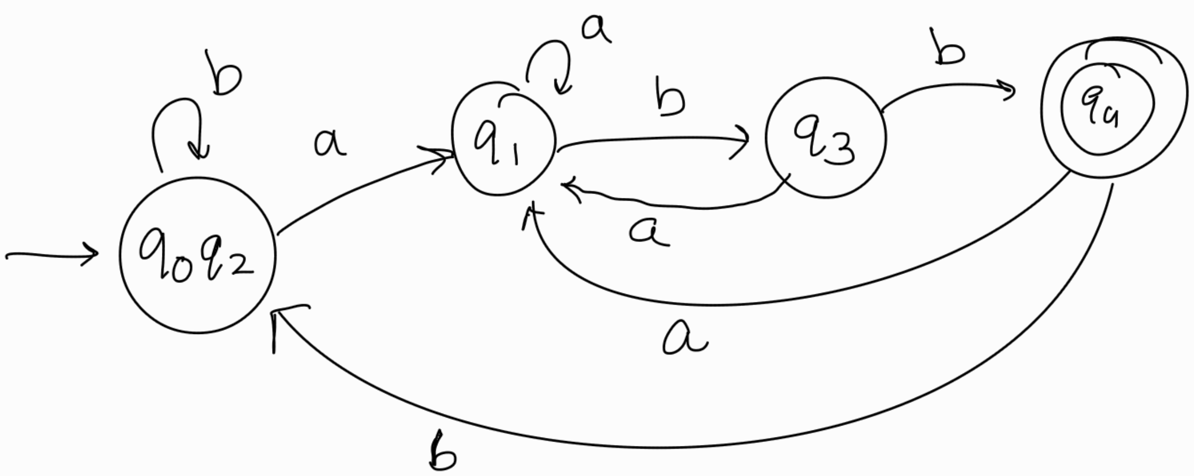
Now

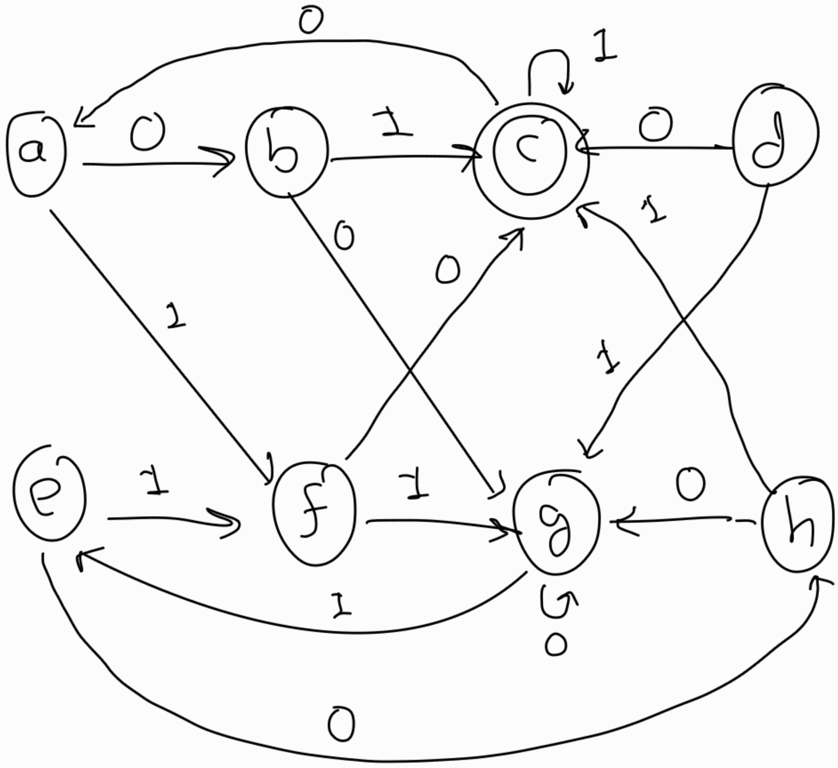
loop 1:

	a	b
q_0	q_1	q_2
q_1	q_1	q_3
q_2	q_1	q_2

	a	b
q_3	q_1	q_4
q_4	q_1	q_2

	a	b	
$q_0 q_1$	$q_1 q_1$	$q_2 q_3$	
$q_0 q_2$	$q_1 q_1$	$q_2 q_2$	
$q_0 q_3$	$q_1 q_1$	$q_2 q_4$	✓ final state
$q_1 q_2$	$q_1 q_1$	$q_3 q_2$	
$q_1 q_3$	$q_1 q_1$	$q_3 q_4$	✓✓
$q_2 q_3$	$q_1 q_1$	$q_2 q_4$	✓✓





	0	1
a	b	f
b	g	c
c	a	c
d	c	g
e	h	f
f	c	g
g	g	e
h	g	c

D is unreachable
so remove first

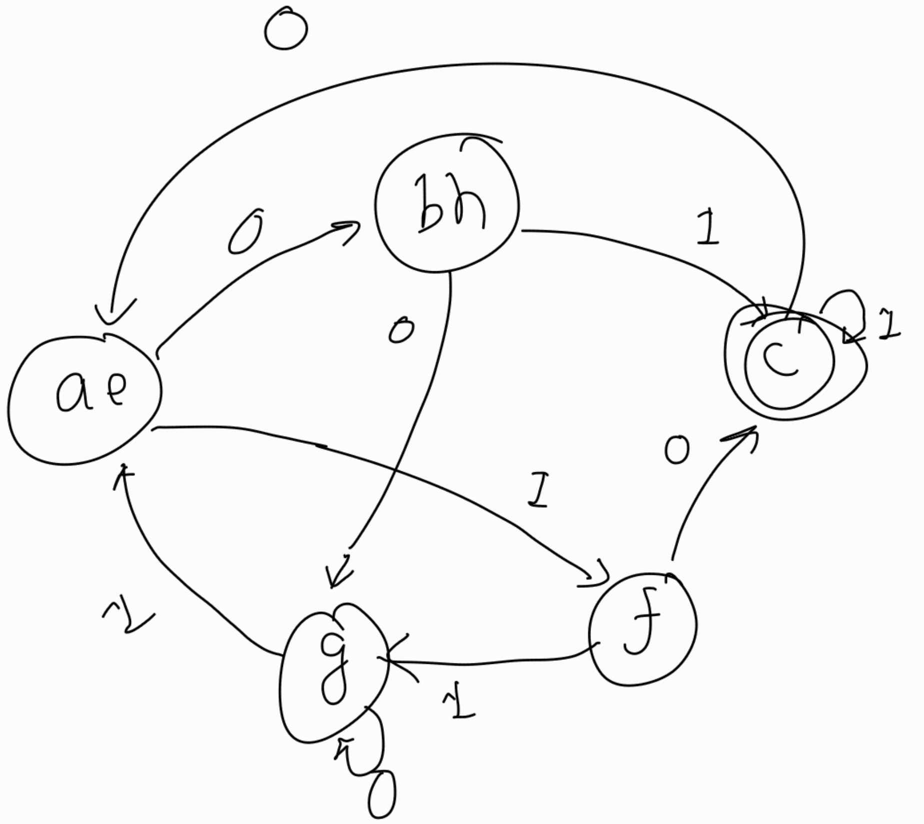
	a	b	c	d	e	f	g
b	X						
c	X	X					
~~d~~	~~X~~	~~X~~	~~X~~				
e		X	X				
f	x	x	X		X		
g	X	X	X		X	X	
h	X		X		X	x	XX

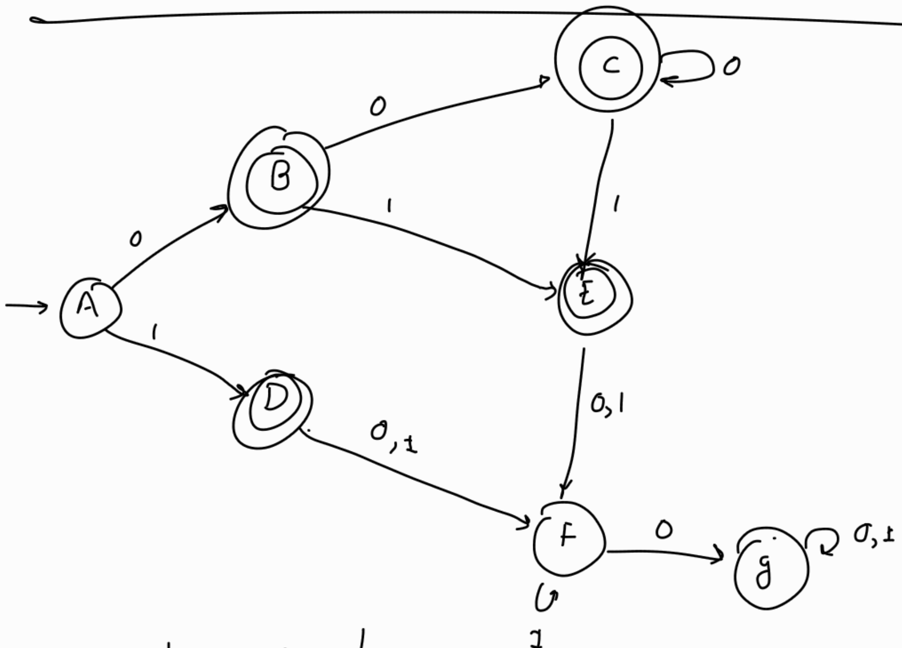
states:

{ ae, bh, c, f, g }

	a	b
ab	bg	fc √√
ad	bc √√	fg
ae	bh	ff
af	bc √√	fg
ag	bg √√	fe
ah	bg	fc √√
bd	gc	cg √√
be	gh	cf √√
bf	gc	cg √√
bg	gg	ce √√
bh	gg	cc
de	ch √√	gf
df	cc	gg
dg	cg	ge
dh	cg	gc
ef	hc √√	fg
eg	hg	fe √√
eh	hg	fc √√
fg	cg √√	ge
fh	cg √√	gc

gh | 99 | ec(r)



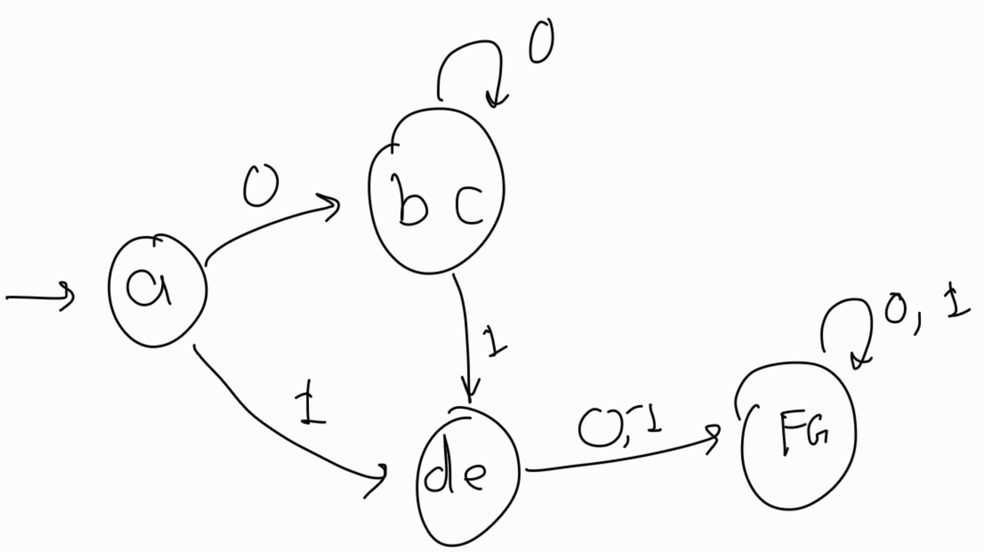


	0	1
a	B	D
b	C	E
c	C	E
d	F	F
e	F	F
f	g	F
g	g	g

	0	1
af	Bg ✓✓	DF
ag	Bg ✓✓	DG
→ bc	C C	E E
bd	C F ✓	E F
be	C F ✓✓	E F
cd	CF ✓✓	E F
ce	CF ✓✓	EF
→ de	FF	FF
fg	gg	Fg

	a	b	c	d	e	f
b	X					
c	X					
d	X	X	X			
e	X	X	X			
f	X	X	X	X	X	
g	X	X	X	X	X	

∴ States = { a, bc, de, fg }



Regular expressions:

Another way for describing regular languages.

$$(1+01)^*(\epsilon+0)$$

Union :

q_0, q_1

r_0, r_1 $\Rightarrow$ $q_0r_0 \xrightarrow{0} q_0r_1 \; \ldots$

$\ldots$

if one final state, then final.

Intersection :

q_0, q_1 $\Rightarrow$ $q_0r_0 \rightarrow q_0r_1 \; \ldots$

r_0, r_1

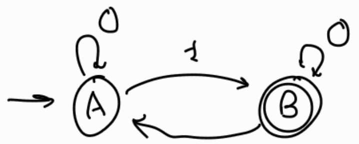
$\ldots$

if all final present, then final.

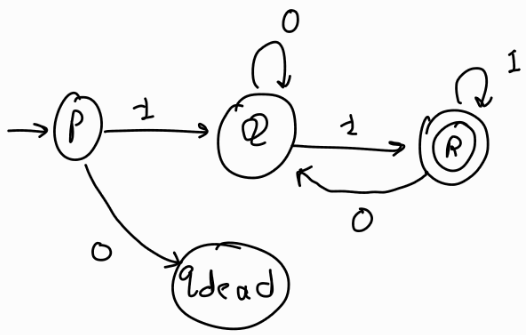
---

e.g. Odd 1's and start & end with 1

$L_1 \cap L_2$

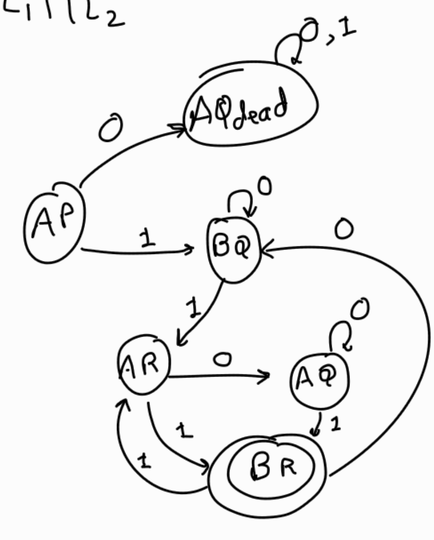


L_1

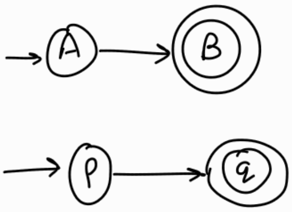


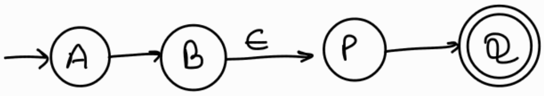
l_2

$L_1 \cap L_2$

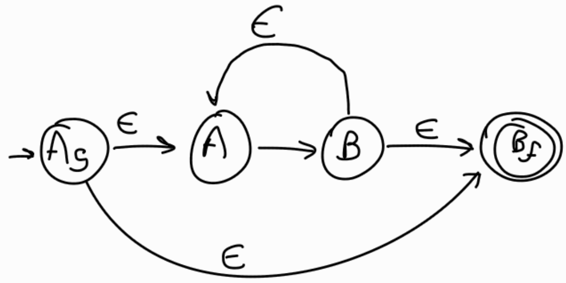


Concatenation :

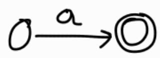


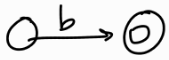
$L_1 \cdot L_2 =$ 

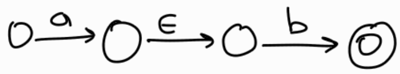
R

$\Longrightarrow R^* \Longrightarrow$ 

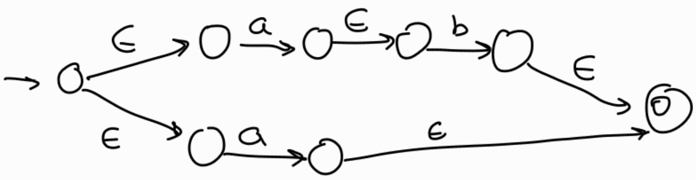
$(ab+a)^*$

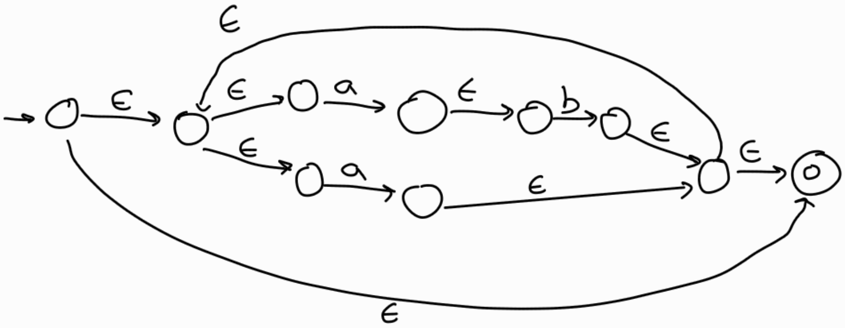
a 

b 

ab 

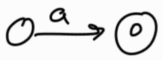
ab +a



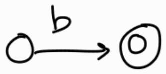
$(ab+a)^*$ 

$a+(bc)^*$

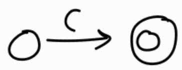
a



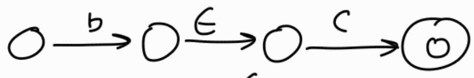
b



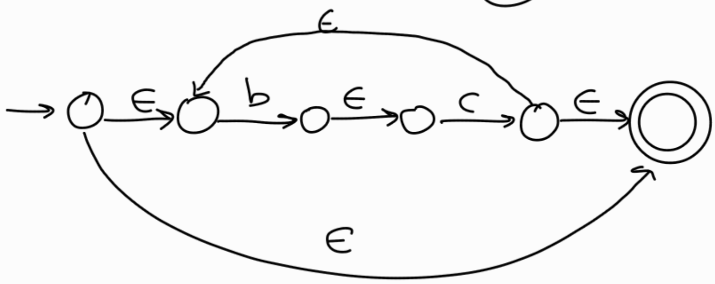
c



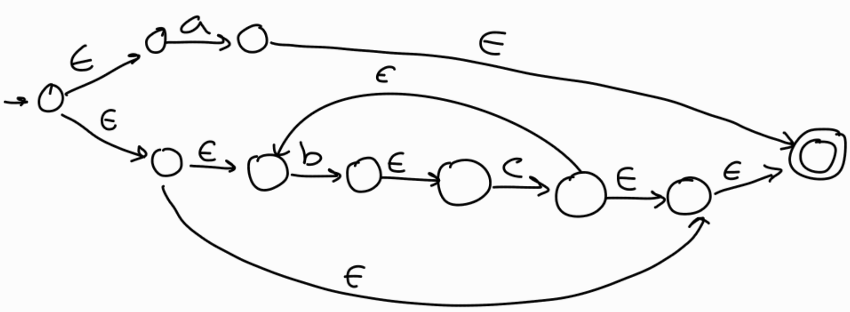
bc



$(bc)^*$



$a+(bc)^*$

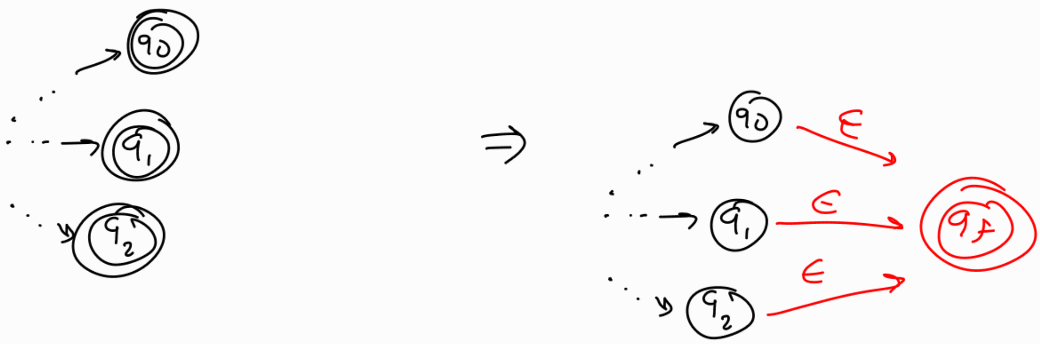


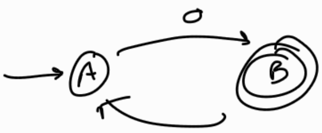
DFA to Regex:

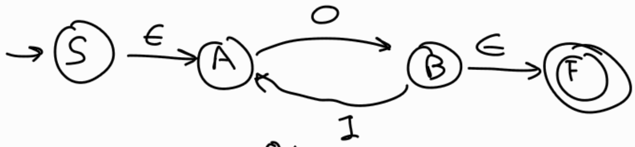
(i) State elim:

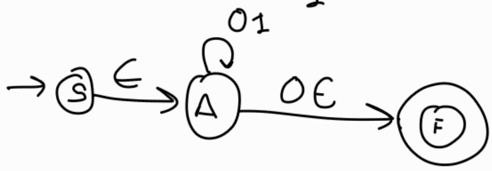
(a) if incoming edge to start, create new start

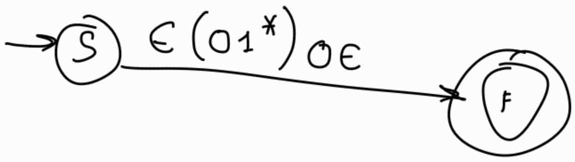
(b) Multiple final states:

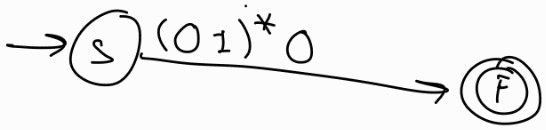


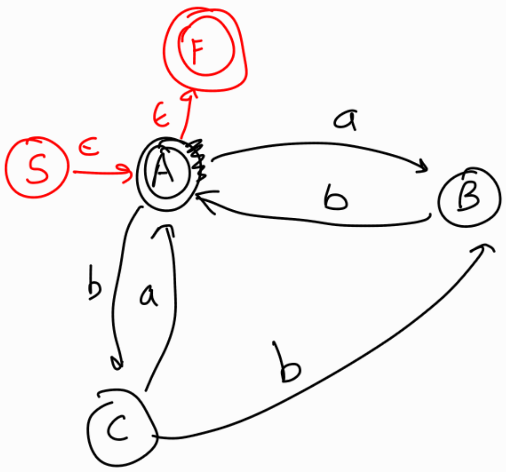




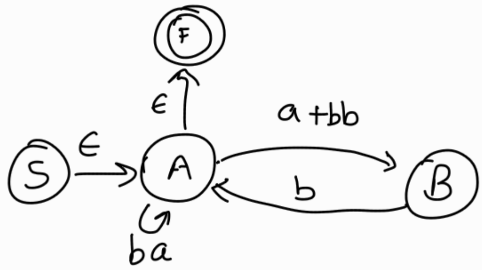


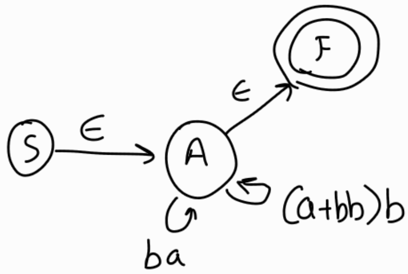


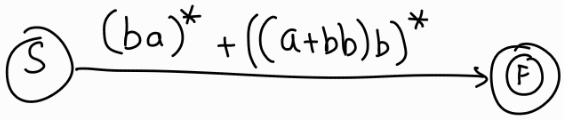




$\Rightarrow$







$(ba)^* + ((ab)^* + (bbb)^*)$

Arden's theorem:

$R = Q + RP$

$R = QP^*$

* ϵ-transitions not allowed.
  must have only one ϵ state.

e.g. 13.12

$q_1 = \epsilon + q_2 a + q_3 a$

$q_2 = q_1 b + q_2 b + q_3 b$

$q_3 = q_2 a \quad \ldots (1)$

$q_2 = q_1 b + q_2 b + q_2 ab$

$$\underbrace{q_2}_{R} = \underbrace{q_1 b}_{Q} + \underbrace{q_2}_{R} \underbrace{(b + ab)}_{P}$$

$q_2 = q_1 b (b + ab)^* \quad \ldots (11)$

$q_3 = q_2 a$

$= q_1 b (b + ab)^* a$

$q_1 = \epsilon + q_1 a + q_3 a$

$= \epsilon + q_1 a + q_1 b (b + ab)^* a a$

$q_1 = \epsilon + q_1 (a + b (b + ab)^* aa)$

$q_1 = (a + b (b + ab)^* aa)^* \quad \ldots (3)$