

Introduction:

Automata, Computability and Complexity:

→ What are the fundamental capabilities and limitations of computers?

"By starting from the end you can better understand the reason for the beginning."

Complexity Theory:

The Central Question of Complexity Theory! "What makes some problems computationally hard and others easy?"

Set, Sequences,
Tuples

Set → Group of objects represented as a unit
↳ in a set → elements/members

\in ← set membership

\notin ← non - ∈ -

- Multiple occurrences of same object allowed in multiset but not set.
- e.g. $\{5, 7, 7\}$ and $\{5, 7\}$ both are different multisets but are identical sets.

- Members

0 → Empty set \emptyset

1 → Singleton set

2 → Unordered pair

Sequences :

List of objects in some ORDER.

→ Order matters in sequence.

→ Sequences may be finite or infinite.

(1, 2, 4, 9, 16, ...)

→ Finite sequences → tuples

A sequence with k-elements → k-tuple.

Functions &
Relations

Function

→ An object that sets up input/output relation.

→ takes i/p gives o/p

→ $f(a) = b$
 ↑
 i/p ↳ o/p

set of inputs → Domains

set of outputs → Range

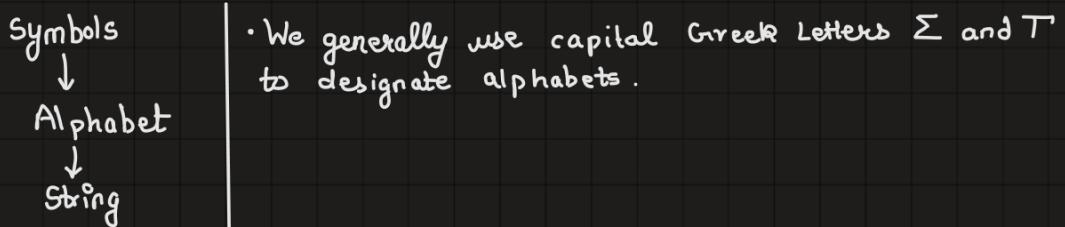
- A function that uses all the elements of the range is said to be onto the range.

Strings & Languages

Strings of chars are building blocks of CS.

alphabet → non-empty finite set of **symbols**.

T over which strings are defined may vary with application.



String over an alphabet:

- length of ω
- empty string (ϵ)
- Reverse of ω (ω^R)
- substring
- concatenation
 xy, x^n
- Lexicographic Order

↳ Shortlex order or string order

- Finite sequence of symbols from the alphabet, usually written next to one-another, not comma separated.
e.g. if $\Sigma_1 = \{0, 1\} \rightarrow 01001$ is a string over Σ_1 .
- If ω is a string over Σ , the length of ω , $|\omega|$ is number of symbol it contains.
- * String of length zero is called empty string and is written $\epsilon \rightarrow$ (epsilon)
↳ plays role of 0 in number system.
- Reverse of ω , ω^R is a string obtained by writing ' ω ' in opposite order.
- String z is a substring of ω if it appears consecutively within ω .
- Concatenation
 - x with $y \rightarrow xy : x_1, \dots, x_n, y_1, \dots, y_m \leftarrow$ resultant string
 - x with itself, k times $\underbrace{x \dots x}_{k} \rightarrow$ written as x^k
- Lexicographic order → same as dictionary order.
↓
Modification → Shortlex order / string order
difference: shorter string precedes longer string
 $\Sigma_1 = \{0, 1\} \rightarrow$ string ordering $\rightarrow (\epsilon, 0, 1, 00, 01, 10, 11, \dots)$

Let,

$y = abcde \rightarrow y$'s prefix's $\rightarrow \epsilon, a, ab, \underline{abc}, abcd, abcde$.

$x = abc$

$z = de$

* x is a prefix of y if there exist a' z' s.t. $xz = y$.

* Proper Prefix → (Non-prime prefix)

↳ x is proper prefix of y if $x \neq \epsilon$ and $x \neq y$

e.g. $y = abcd$

proper prefix 'x' $\rightarrow a, ab, abc$

Non -11- -11- 'x' $\rightarrow \epsilon, abcd$

Boolean Logic → Mathematical System built around two values, TRUE and FALSE.
↳ Boolean Values

- Boolean Values	→ Boolean Operations	→ Not (Negation)	¬
- Boolean Operations		AND (Conjunction)	∧
		OR (Disjunction)	∨
		XOR	⊕
		equality	↔
		Implication	→

Definition,
Theorem &
Proofs.

- Definition describe objects and notions we use.
- Mathematical statements are made of the previously defined objects and notions.
- Proof → Convincing, logical argument that a statement is true.
- Theorem → Mathematical statement proved true.
- Lemmas → Statements proved only because they assist in proof of another, more significant proof.
- Corollary → Theorems or proof that allow related statements to be proved easily.

Finding Proof:

- A well written proof is a set of stats, wherein each one follows with simple reasoning from previous stats in sequence.

Types of
Proof:

① Proof by Construction:

- For theorems stating a particular type of object exists.
- We can prove by demonstrating construction of object. This is proof by construction.

② Proof by Contradiction:

- Assume stat- is false, → this assumption leads to a false consequence, called contradiction.

③ Proof by Induction:

- Method to show all elements of a set have same property.
- Two parts
 - ↳ Basis
 - ↳ Induction step.