Rules of inference

- Addition
  - If P is a premise, we can use Addition rule to derive $P \vee Q$.

$$P$$

$$\therefore P \vee Q$$

- Example
  - Let P be the proposition, “He studies very hard” is true
  - Therefore − "Either he studies very hard Or he is a very bad student." Here Q is the proposition “he is a very bad student”.

- Conjunction
  - If P and Q are two premises, we can use Conjunction rule to derive $P \wedge Q$

$$
\begin{array}{l}
\quad P \\
\quad Q \\
\therefore P \wedge Q
\end{array}
$$

- Example
  - Let P – “He studies very hard”
  - Let Q – “He is the best boy in the class”
  - Therefore – "He studies very hard and he is the best boy in the class"

- Simplification
- If $P \wedge Q$ is a premise, we can use Simplification rule to derive P.

  $P \wedge Q$

  $\therefore P$

- Example
  - "He studies very hard and he is the best boy in the class", $P \wedge Q$
  - Therefore – "He studies very hard"

- Modus Ponens
  - "If you have a current password, then you can log on to the network“
  - "You have a current password“
    - Therefore:
  - "You can log on to the network“
    - This has the form:

$$p \rightarrow q$$

$$p$$

$$\therefore q$$

Modus Tollens

- You can't log into the network
- If you have a current password, then you can log into the network

Therefore

- You don't have a current password.
- This is an argument of the form:

$\neg q$

$p \rightarrow q$

$\therefore \neg p$

- Hypothetical syllogism
  - If it rains, we will not have a picnic.
  - If we don't have a picnic, we won't need a picnic basket.
  - Therefore, if it rains, we won't need a picnic basket.

$$P \rightarrow Q$$

$$Q \rightarrow R$$

$$\therefore P \rightarrow R$$

- Disjunctive syllogism
  - The cake has either chocolate or vanilla frosting.
  - The cake does not have vanilla frosting
  - Therefore, the cake has chocolate frosting

$P \lor Q, \neg P$ infers Q

$$
\begin{array}{r}
P \lor Q \\
\neg P \\
\text{infers } Q
\end{array}
$$

Rules of inference

	Rule of Inference	Tautology	Name
1)	P $\underline{P \to Q}$ $\therefore Q$	$(P \wedge (P \to Q)) \to Q$	Modus ponens
2)	$\neg Q$ $\underline{P \to Q}$ $\therefore \neg P$	$[\neg Q \wedge (P \to Q)] \to \neg P$	Modus tollens
3)	$P \to Q$ $\underline{Q \to R}$ $\therefore P \to R$	$[(P \to Q) \wedge (Q \to R)] \to (P \to$	Hypothetical syllogism
4)	$P \vee Q$ $\underline{\neg P}$ $\therefore Q$	$[(P \vee Q) \wedge \neg P] \to Q$	Disjunctive syllogism

Rules of Inference

5)	P $\therefore PVQ$	$P \rightarrow (P \vee Q)$	Addition
6)	$P \wedge Q$ $\therefore P$	$(P \wedge Q) \rightarrow P$	Simplification
7)	P Q $\therefore P \wedge Q$	$((P) \wedge (Q)) \rightarrow P \wedge Q$	Conjunction
8)	$P \vee Q$ $\neg P \vee R$ $\therefore Q \vee R$	$[(P \vee Q) \wedge (\neg P \vee R)] \rightarrow (Q \vee R$	Resolution

Proof of Rules of Inference

- Prove $p \wedge q, r \vdash q \wedge r$
  - 1. $p \wedge q$ Premise
  - 2. r Premise
  - 3. q simplification, from 1
  - 4. $q \wedge r$ conjunction, from 2 and 3

Prove p, $\neg\neg(q \wedge r) \vdash \neg\neg p \wedge r$

- 1. p premise
- 2. $\neg\neg(q \wedge r)$ premise
- 3. $\neg\neg p$ insertion of double negation,in 1
- 4. $q \wedge r$ elimination of double negation,
- 5. r simplification, from 4
- 6. $\neg\neg p \wedge r$ conjunction

Prove

- “It is not sunny this afternoon and it is colder than yesterday,”
- “We will go swimming only if it is sunny,” “If we do not go swimming, then we will take a canoe trip,”
- and “If we take a canoe trip, then we will be home by sunset”
- lead to the conclusion “We will be home by sunset.”

solution

- p :- “It is sunny this afternoon,”
- q the proposition “It is colder than yesterday,”
- r :- “We will go swimming,”
- s :- “We will take a canoe trip,”
- t :- “We will be home by sunset.”

  Then the premises become

- $\neg p \wedge q$, $r \rightarrow p$, $\neg r \rightarrow s$, and $s \rightarrow t$. The conclusion is simply t.

- confusing about "Swimming **only if** Sunny" with "Swimming **if** Sunny".
- The former is meant by the problem to mean "If it is Sunny, we might or might not go Swimming, by we won't go Swimming if it is not Sunny".
- The latter, which is how you interpreted it, means "If it is Sunny, we certainly go Swimming; here it is saying nothing about what happens if it is not Sunny."
- therefore had the implicatoin in the opposite direction of what was intended.

- "We will go swimming only if it's sunny" can be rephrased in a few ways:
- *it's not the case that [we will go swimming and it won't be sunny]*, that won't happen; equivalently,
- *we won't go swimming if it isn't sunny*; in other words,
- *if it isn't sunny then we won't go swimming*. This is equivalent to:
- *if we go swimming then it is (or, will be) sunny*.
- Another way to see that this is the correct rendering is to apply De Morgan's law and then the equivalence $\neg r \vee p \equiv r \Rightarrow p$ to arrive at the same "if-then' form:
- By De Morgan's law, 1. above is equivalent to *either we won't go swimming or it will be sunny*. This is of the form $\neg r \vee p$, equivalent to $r \Rightarrow p$.
- *if we (will) go swimming then it will be sunny*.
-

$\neg p \wedge q, r \rightarrow p, \neg r \rightarrow s, s \rightarrow t \vdash t$

- 1. $\neg p \wedge q$ Premise
- 2. $\neg p$ simplification
- 3. $r \rightarrow p$ premise
- 4. $\neg r$ Modus tollens using (2) and (3)
- 5. $\neg r \rightarrow s$ premise
- 6. s Modus ponens using (4) and (5)
- 7. $s \rightarrow t$ premise
- 8. t Modus ponens using (6) and (7)

Prove

- “If you send me an e-mail message, then I will finish writing the program,”
- “If you do not send me an e-mail message, then I will go to sleep early,”
- and “If I go to sleep early, then I will wake up feeling refreshed
- lead to the conclusion “If I do not finish writing the program, then I will wake up feeling refreshed.”

- p :- “You send me an e-mail message,”
- q :- “I will finish writing the program,”
- r : “I will go to sleep early,” and
- s :- “I will wake up feeling refreshed.”
- The premises are $p \rightarrow q$, $\neg p \rightarrow r$, & $r \rightarrow s$
- The desired conclusion is $\neg q \rightarrow s$

Proof

- 1. $p \rightarrow q$ Premise
- 2. $\neg q \rightarrow \neg p$ Contrapositive of (1)
- 3. $\neg p \rightarrow r$ Premise
- 4. $\neg q \rightarrow r$ Hypothetical syllogism using (2) and (3)
- 5. $r \rightarrow s$ Premise
- 6. $\neg q \rightarrow s$ Hypothetical syllogism using (4) and (5)

Derive the following

- (~p v q, ~q v r, r->s) leads to p->s

1. ~pvq	Premise
2. p->q	From 1
3. p	Assumed Premise
4. q	Modus ponens, From 2 ,3
5. ~qvr	Premise
6. r	Disjunctive syllogism, From 4, 5
7. r->s	Premise
8. s	Modus ponens, From 6, 7
9. p->s	(From 1, 5, 7)

Fallacy of affirming the conclusion (consequence

$$p \rightarrow q$$

$$q$$

$$\therefore p$$ Can't conclude this

p If you do every problem in this book $\qquad p \rightarrow q$

q then you will learn discrete mathematics

You learned discrete mathematics $\qquad q$

Therefore,

you did every problem in this book $\qquad p$

Fallacy of denying the hypothesis (antecedent)

$$p \rightarrow q$$

$$\neg p$$

$$\therefore \neg q$$ Can't conclude this

p If you do every problem in this book $\quad p \rightarrow q$

q then you will learn discrete mathematics

You didn't do every problem in this book $\quad \neg p$

Therefore,
you didn't learn discrete mathematics $\quad \neg q$

Rules of inference for quantifiers

Universal Instantiation

$$\frac{\forall x P(x)}{\therefore P(c) \text{ for any } c}$$

Universal Generalization

$$\frac{P(c) \text{ for arbitrary } c}{\therefore \forall x P(x)}$$

Existential Instantiation

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some } c}$$

Existential Generalization

$$\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

Premises:

$C(x)$ A student in this class

$\neg B(x)$ has not read the book

$$\exists x(C(x) \wedge \neg B(x))$$

$C(x)$ Everyone in this class

$P(x)$ passed the first exam

$$\forall x(C(x) \rightarrow P(x))$$

Conclusion:

$P(x)$ Someone who passed the first exam

$\neg B(x)$ has not read the book

$$\exists x(P(x) \wedge \neg B(x))$$

1. $\exists x(C(x) \land \neg B(x))$ Premise
2. $C(a) \land \neg B(a)$ Existential instantiation from 1
3. $C(a)$ Simplification from 2
4. $\forall x(C(x) \to P(x))$ Premise
5. $C(a) \to P(a)$ Universal instantiation from 1
6. $P(a)$ Modus Ponens from 3,5
7. $\neg B(a)$ Simplification from 2
8. $P(a) \land \neg B(a)$ Conjunction from 6,7
9. $\exists x(P(x) \land \neg B(x))$ Existential generalization from 8