Q 1 A

i. What mathematical model permits easy interconnection of physical systems? — 6

Transfer Function

ii. Define the transfer function.

It the ratio of Laplace transform of output to the Laplace transform of input assuming all initial conditions zero.

iii. What do we call the mechanical equations written in order to evaluate the transfer function?

Equation of motion

iv. If we understand the form the mechanical equations take, what step do we avoid in evaluating the transfer function?

Free body diagram

v. Why do transfer functions for mechanical networks look identical to transfer functions for electrical networks

There are direct analogies between the electrical variables and components and the mechanical variables and components.

vi. Instability is attributable to what part of the total response?

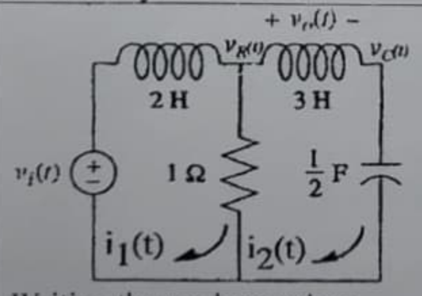
Natural response

B Compare the open loop control system and closed loop control system on the basis of following points. — 6

Accuracy	Inaccurate and unreliable	Accurate and reliable
Power consumption	consume less power	consume more power
Complexity	Simple	Complex
Response to external disturbances	The changes in output due to external disturbances are not corrected automatically	The changes in output due to external disturbances are corrected are automatically
Stability	they are generally stable	efforts are needed to design a stable system
Cost	Economical	Costlier

Q 2 A

Find the transfer function, $G(s) = \frac{V_o(s)}{V_i(s)}$, for following network. Solve the problem using mesh analysis. — 6



Writing the mesh equations,

$$(2s+1)I_1(s) - I_2(s) = V_i(s)$$
$$-I_1(s) + (3s + 1 + 2/s)I_2(s) = 0$$

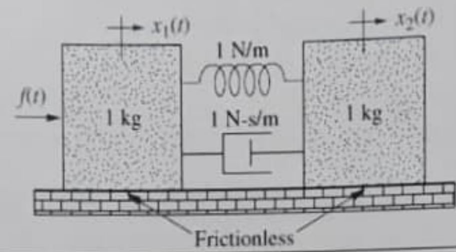
Solving for $I_2(s)$,

$$I_2(s) = \frac{\begin{vmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2s+1 & -1 \\ -1 & \frac{3s^2+s+2}{s} \end{vmatrix}}$$

But $V_o(s) = 3s\, I_2(s)$

$$\frac{I_2(s)}{V_i(s)} = \frac{s}{6s^3 + 5s^2 + 4s + 2}$$
$$\therefore I_2(s) = V_o(s)/3s$$
$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{3s^2}{6s^3 + 5s^2 + 4s + 2}$$

Q 2 B Find the transfer function, $G(s) = \frac{X_2(s)}{F(s)}$, for the translational mechanical network shown in Figure. **6**



Writing the equation of motion

$$(s^2 + s + 1)X_1(s) - (s + 1)X_2(s) = F(s)$$
$$-(s + 1)X_1(s) + (s^2 + s + 1)X_2(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} (s^2 + s + 1) & F(s) \\ -(s+1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + s + 1) & -(s+1) \\ -(s+1) & (s^2 + s + 1) \end{vmatrix}}$$
$$= \frac{(s + 1)F(s)}{s^2((s^2 + s + 1)}$$
$$\frac{X_2(s)}{F(s)} = \frac{(s + 1)}{s^2((s^2 + 1s + 2)}$$

Q 3 A Consider a unity feedback system with closed transfer function **3**

$$\frac{C(s)}{R(s)} = \frac{Ks + b}{s^2 + as + b}$$

Determine the open loop transfer function $G(s)$. Find the steady state error with unit ramp input

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{Ks + b}{s^2 + as + b}$$

Cross multiplying the above equation

$$(s^2 + as + b)G(s) = (1 + G(s))(Ks + b)$$
$$(s^2 + as + b)G(s) = (Ks + b) + (Ks + b)G(s)$$
$$[(s^2 + as + b) - (Ks + b)]G(s) = (Ks + b)$$
$$G(s) = \frac{(Ks + b)}{[(s^2 + as + b) - (Ks + b)]} = \frac{(Ks + b)}{s^2 + (a - K)s} = \frac{(Ks + b)}{s[s + (a - K)]}$$

The velocity error constant

$$K_v = \lim_{s\to 0} sG(s)$$
$$= \lim_{s\to 0} s \times \frac{(Ks + b)}{s[s + (a - K)]} = \frac{b}{a - K}$$

Therefore, the steady state error for a unit ramp input is

$$e(\infty) = \frac{1}{K_v} = \frac{a-K}{b}$$

B A unity feedback system is characterised by an open loop transfer function **5**

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain K so that the system will have a damping ratio of 0.5. For this value of K determine the settling time, peak overshoot and time to peak overshoot for unit step input

The closed loop transfer function of the given unity feedback system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{K}{s^2+10s+K}$$

Compare with standard form of transfer function of a second order system

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+10s+K} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n^2 = K \quad \therefore \omega_n = \sqrt{K} \text{ and}$$

$$2\zeta\omega_n = 10$$

$$2 \times 0.5 \times \omega_n = 10 \qquad \therefore \omega_n = 10$$

$$\therefore K = \omega_n^2 = 100$$

The settling time for 5% criterion is

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5 \times 10} = 0.8\ seconds$$

The peak overshoot is

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = e^{-\pi\times 0.5/\sqrt{1-0.5^2}} = 0.163$$

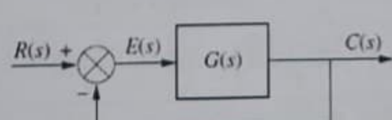
The peak time is

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.5^2}} = 0.363\ seconds$$

C For the unity feedback system shown in Figure, where **4**

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

Find the steady-state errors for the following test inputs:
$25\,u(t), 37\,t\,u(t), and\ 47t^2u(t)$.



$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{sR(s)}{1+\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}}$$

For step input $R(s) = 25/s$

$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}} = 0$$

For ramp input $R(s) = 37/s^2$

$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}} = 6.075 \times 10^{-2}$$

	For parabolic input $R(s) = 47/s^3$ $$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}} = \infty$$

Q 4 A	By means of Routh criterion, determine the stability of the system represented by the following characteristics equations. For system found to be unstable, determine the number of roots of the characteristics equation in the right half of the s plane. $s^6 + 2s^5 + s^4 + 2s^3 + 3s^2 + 4s + 5 = 0.$	4

s^6	1	1	3	5
s^5	2	2	4	
s^4	$\frac{2\times 1 - 1\times 2}{2} = 0$	$\frac{2\times 3 - 1\times 4}{2} = 1$	$\frac{2\times 5 - 1\times 0}{2} = 5$	
s^3	∞			

The first element in the s^4 row its zero, whereas there are some non-zero elements in the same row. So the system is unstable. To find the location of the roots replace the first zero element by a small positive number epsilon and proceed with formation of root table

s^6	1	1	3	5
s^5	2	2	4	
s^4	ε	$\frac{2\times 3 - 1\times 4}{2} = 1$	$\frac{2\times 5 - 1\times 0}{2} = 5$	
s^3	$\frac{\varepsilon\times 2 - 2\times 1}{\varepsilon}$	$\frac{4\varepsilon - 10}{\varepsilon}$		
s^2	$\frac{-4\varepsilon^2 + 12\varepsilon - 2}{\varepsilon}$	5		
s^1	$\frac{\left(\frac{-4\varepsilon^2+12\varepsilon-2}{\varepsilon}\right)\left(\frac{4\varepsilon-10}{\varepsilon}\right) - \left(\frac{2\varepsilon-2}{\varepsilon}\right)\times 5}{\frac{-4\varepsilon^2+12\varepsilon-2}{\varepsilon}}$	0		
s^0	5			

As $\varepsilon \to 0$, there are 2 sign changes in the elements of first column of Routh array. Therefore, there are 2 roots of the characteristic equation in the right half of the s plane. So the system is unstable.

B	By means of Routh criterion, ddetermine the stability of system using Routh-Hurwitz criteria having characteristic equation $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0.$ Also find the closed loop poles.	5

s^5	1	8	7
s^4	4	8	4
s^4	1	2	1

s^3	$\frac{1\times 8-1\times 2}{1}=6$	$\frac{1\times 7-1\times 1}{1}=6$	
s^2	$\frac{6\times 2-1\times 6}{6}=1$	$\frac{6\times 1-1\times 0}{6}=1$	
s^1	$\frac{1\times 6-6\times 1}{1}=0$	0	
s^0			

All the elements in the s^1 are zeros. So, the Routh's test fails. The system is unstable. To complete the Routh table, form an auxiliary equation using the coefficients of the row s^2 (the row just above the row of zeroes)

$$A(s) = s^2 + 1 = 0$$

Taking the first derivative of auxiliary equation

$$\frac{d}{ds}A(s) = \frac{d}{ds}(s^2 + 1) = 0$$

$$\therefore 2s + 0 = 0$$

Replace the row of zeros by the elements of first derivative of the auxiliary equation and process with the formation of Routh table

s^5	1	8	7
s^4	1	2	1
s^3	6	6	
s^2	1	1	
s^1	2	0	
s^0	1		

Since all the elements in the first column of the Routh array positive, there are no roots of characteristics equation in the right half of the s plane. Still the system is unstable due to existence of the row of zeros, which means that there must be roots on imaginary axis of the s plane. To determine the roots on imaginary axis, solve the auxiliary equation. To determine the other roots divide characteristics equation by auxiliary equation.

$$s^2 + 1 = 0, \qquad s^2 = -1 \qquad i.e.\ \ s = \pm j1$$

To find the other poles factorize the characteristic equation

$$\begin{aligned} s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 &= (s^2+1)(s^3+4s^2+7s+4) \\ &= (s^2+1)(s+1)(s^2+3s+4) \\ &= (s^2+1)(s+1)(s+1.5+j1.3229)(s+1.5-j1.3229) \end{aligned}$$

Therefore, the poles are at

$$s = \pm j1, \quad s = -1, \quad s = -1.5 \pm j1.3229$$

C	Determine the range of K for the system to be stable by using Routh array. $s^3 + 3Ks^2 + (K+2)s + 4 = 0.$	3

$$\infty > K > 0.526 \quad or \quad 0.526 < K < \infty$$

Q 5 A	Sketch the root locus for $$G(s)H(s) = \frac{K}{(s+2)(s+4)(s+8)}$$ Find K for stability	8
	Number of poles 3 number of zeros 0 Number of root locus branches = 3 Centroid = (-2-4-8)/3 = -14/3 = -4.6667 Angle $\theta = \frac{(2q+1)180^0}{P-Z}; \quad q = 0, 1, 2, \dots, (P-Z)-1.$ $\theta_1 = \frac{(2\times 0+1)180^0}{3-0} = 60^0; \theta_2 = \frac{(2\times 1+1)180^0}{3-0} = 180^0;$ $\theta_3 = \frac{(2\times 2+1)180^0}{3-0} = 300^0;$ Break away point = -2.9 $j\omega$ axis crossing point = $\pm 7.48j$ K < 718	
B	Find K for the above system (described in Q 5 A) when it is operating with damping factor 0.6.	4
	K = 65	
Q 6 A	Sketch the root locus for	8

	$G(s)H(s) = \frac{K(s+3)}{s^2(s+9)}$ Find K for stability	
	Number of poles 3 number of zeros 1 Number of root locus branches = 2 Centroid = -4.5 Angle $\theta = \frac{(2q+1)180^0}{P-Z}; \qquad q = 0, 1, 2, \ldots, (P-Z)-1.$ $\theta_1 = \frac{(2\times 0+1)180^0}{3-1} = 90^0; \theta_2 = \frac{(2\times 1+1)180^0}{3-1} = 270^0;$ Break away point = NIL $j\omega$ axis crossing point = Nil K > 0	
	Find dominant pole for the above system (described in Q 6 A) when the system is operating with 0.5 damping factor.	4
	No dominant pole	