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MIS 142103012   FCS

Assignment - 3

1) A system a transfer function, $G(s) = \frac{100}{s+50}$
Find T_c settling time $= T_s$ &
rise time T_r.

=> $G(s) = \frac{100}{s+50}$ => $\boxed{a = 50}$

$T_c = \frac{1}{a} = \cancel{0.025}\ 0.02\ s$

$T_s = \frac{4}{a} = 0.08\ s$

$T_r = \frac{2.2}{a} = 0.044\ s$

Q2)

=> $G(s) = \frac{81}{s^2 + 6s + 81} = \frac{w_n^2}{s^2 + 2 w_n \delta s + w_n^2}$

Comparing => $[w_n = 9\ rad/s]$

$= 2 w_n \delta = 6$

$= \delta = \frac{1}{3} = 0.333$

as $\delta < 1$, the motion is underdamped

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3)

$\Rightarrow$ $G(s) = \dfrac{225}{s^2 + 25s + 225}$

$\omega_n = 15$ $\quad\quad \zeta = \dfrac{25}{30} = \boxed{\dfrac{5}{6}}$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{15\sqrt{1-\left(\frac{5}{6}\right)^2}} = \dfrac{\pi}{15\sqrt{\frac{36-25}{25}}} = \dfrac{\pi \times 5}{15\sqrt{11}} = \dfrac{\pi}{3\sqrt{11}}$

$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$

$= e^{-\left(\frac{5}{6} \times \frac{\pi}{\sqrt{11}} \times 5\right)} \times 100$

$= \boxed{e^{\frac{-25\pi}{6\sqrt{11}}} \times 100}$

$T_s = \dfrac{4}{\zeta \times \omega_n} = \dfrac{4 \times 6}{5 \times 15} = \dfrac{24}{75}$

$T_r = \dfrac{\pi - \phi}{\omega_n\sqrt{1-\zeta^2}}$ $\quad\quad \phi = \tan^{-1}\dfrac{\sqrt{1-\zeta^2}}{\zeta}$

$= \phi = \tan^{-1}\left(\dfrac{\sqrt{11}\,6}{5 \times 5}\right)$

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Q4) $G(s) = \dfrac{361}{s^2+16s+361}$

$\omega_n = 19$

$\zeta = \dfrac{16}{38} = \dfrac{8}{19}$

$T_s = \dfrac{4}{\zeta \times \omega_n} = \dfrac{4}{16} \times \dfrac{38}{19} = \dfrac{1}{2}$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{19\sqrt{1-\frac{64}{361}}} = \dfrac{\pi \times 19}{19\sqrt{297}} = \dfrac{\pi}{\sqrt{297}}$

$T_r = \dfrac{\pi - \left(\tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)\right)}{\omega_n\sqrt{1-\zeta^2}}$

$= \dfrac{\pi - \tan^{-1}\left(\dfrac{\sqrt{297} \times 19}{19 \times 8}\right)}{19\sqrt{297}} \times 19$

$= \dfrac{\pi - \tan^{-1}\left(\frac{\sqrt{297}}{8}\right)}{\sqrt{297}}$

$\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$

$= e^{-\left(\frac{8}{19}\frac{\pi}{\sqrt{297}}19\right)} \times 100$

$= 100\, e^{-\frac{8\pi}{\sqrt{297}}}$

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Q5) Find ξ, w_n, T_P, T_S, $\%OS$ & T_r for a system whose transfer function is

$$G(s) = \frac{441}{s^2 + 25s + 441}$$

$$w_n = \sqrt{c} \Rightarrow w_n \sqrt{441}$$

$$\therefore w_n = 21$$

$$\xi = \frac{a/2}{w_n} = \frac{25/2}{21} = 0.595$$

$$T_s = \frac{4}{\xi w_n} = \frac{4}{0.59 \times 21} = \boxed{0.325}$$

$$T_p = \frac{\pi}{w_n\sqrt{1-\xi^2}} = \frac{\pi}{21\sqrt{1-(0.595)^2}} = \boxed{0.1869}$$

$$T_r = \frac{\pi - \phi}{w_n\sqrt{1-\xi^2}}$$

$$\therefore \phi = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi^2}\right) = \tan^{-1}\left(\frac{\sqrt{1-(0.595)^2}}{0.595}\right)$$

$$= \boxed{53.48} \text{ — } 1$$

$$T_r = \frac{\pi - 53.482}{w_n\sqrt{1-\xi^2}} = \frac{\pi - 53.487}{21 \times \sqrt{1-(0.595)^2}}$$

$$T_r = \boxed{-2.983}$$ ✓

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% OS = 9.77 %

Q6) Determine the validity of second order approximation for each of these two transfer

a) $G(s) = \dfrac{700}{(s+15)(s^2+4s+100)}$ → $s+15$, $s = -15$

$s^2+4s+100$

$s = \dfrac{-4 \pm \sqrt{4^2 - 4\times100}}{2}$

$s = -2 \pm j\sqrt{384}$

Dominant poles have real part of -2 & the higher order pole is at -15 i.e More than 5 times be further

⇒ second order approximation is valid

b) $G(s) = \dfrac{360}{(s+4)(s^2+2s+90)}$ → $s+4=0$ = $s=-4$

$s^2+2s+90$, $s = \dfrac{-2 \pm \sqrt{2^2 - 4\times90}}{2}$

$s = -1 \pm \sqrt{356}$

Dominant poles have a real part of -1 & the higher order pole is at -4 i.e not more than 5 times

⇒ second order is Not valid

Q7) Determine the validity of a second order step-response approximation for each transfer function shown below

a) $G(s) = \cancel{185.7} \frac{185.71(s+7)}{(s+6.5)(s+10)(s+20)}$

Expanding $G(s)$

$$G(s) = \frac{1}{s} + \frac{0.8942}{s+20} - \frac{1.5918}{s+10} - \frac{0.3023}{s+6.5}$$

But -0.3023 is not an order of magnitude less than residues of second order term i

$\Rightarrow$ Second orde approximation is not valid.

b) $G(s) = \frac{197.14(3+7)}{(s+6.9)(s+10)(s+20)}$

Expanding

$$G(s) = \frac{1}{s} + \frac{0.9782}{s+20} - \frac{1.9078}{s+10} - \frac{0.0704}{s+6.5}$$

0.0704 is an order of magnitude less than residue of second order terms i.e term 2 & 3

$\Rightarrow$ Second order approximation is valid

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Q8)

=> For (a)

$G(s) = \frac{a}{s+a} = \frac{5}{s+5} = \frac{1}{s} - \frac{1}{s+5}$

$c(t) = 1-e^{-at}$ output response

$c(t) = 1-e^{-5t}$

and time constant $T_0 = \frac{1}{a}$

$\therefore T_e = \frac{1}{5} = 0.2$

$T_e = 0.2$

Rise time $T_r = \frac{2.2}{a} = \frac{2.2}{5} = \boxed{0.44}$

$T_r = 0.44$

Setteling time $T_s = \frac{4}{a} = \frac{4}{5} = \boxed{0.8}$

For (b)

$G(s) = \frac{a}{s+a} = \frac{20}{s+20} = \frac{1}{s} = \frac{1}{s+20}$

$c(t) = 1-e^{-at}$

$= 1-e^{-20t}$

$T_c = \frac{1}{a} = \frac{1}{20} = 0.05$

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Rise time $(T_r) = \frac{2.2}{a} = \frac{2.2}{20} = 0.11$

Settling time $(T_s) = \frac{4}{a} = \frac{4}{20} = \frac{1}{5} = 0.2$

Q3)

⇒ using voltage division rule.

$$V_C(S) = \left(\frac{1/CS}{R + 1/CS}\right) \cdot V(S)$$

$u(t) = 5$

$V_i(S) = \frac{5}{9}$

$$= \frac{1/RC}{S + 1/RC} \cdot V(S)$$

$$= \frac{0.703}{S + 0.703}$$

$$V_C(S) = \frac{5}{S}\left(\frac{0.703}{S+0.703}\right) = \frac{5}{S} - \frac{5}{S+0.703}$$

$$V_C(t) = 5 - 5e^{-0.703t}$$

$$T_C = \frac{1}{a} = \frac{1}{0.703} = 1.422$$

$$T_r = \frac{2.2}{a} = \frac{2.2}{0.703} = 3.129$$

$$T_s = \frac{4}{a} = \frac{4}{0.703} = 5.69$$

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$\therefore$ Time constant $= 1.422$ Rise time 3.129 and
Settling time $= 5.695$

Q10)

$\Rightarrow$ Writing the equation of motion

$$(ms^2 + bs)\, X(s) = F(s)$$

Then the transfer function $= \dfrac{X(s)}{F(s)} = \dfrac{1}{ms^2+bs}$

differentiating

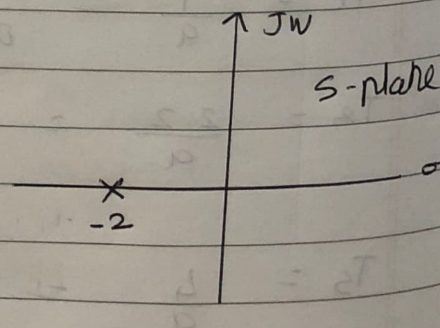
$$\frac{s X(s)}{F(s)} = \frac{1}{ms+b} = \frac{1/m}{s+\frac{b}{m}}$$

thus

Settling time $T(s) = \dfrac{4}{a} = \dfrac{4}{b/m} = \dfrac{2}{3} m = 0.66667\, m$

Q11)

a) $T(s) = \dfrac{2}{s+2}$



For the given transfer function the pole is at -2

Pole : -2

Output response $c(t) = (A + Be^{-2t})\, u(t)$

$\therefore$ It is a first order response

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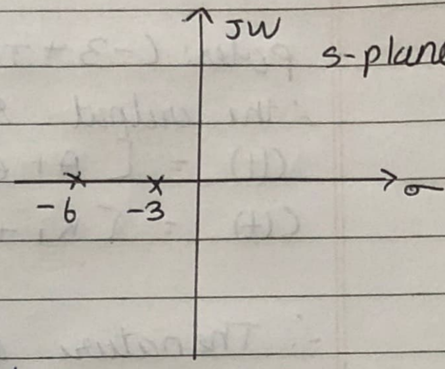
b) $T(s) = \dfrac{5}{(s+3)(s+6)}$



For the given transfer function there are 2 real poles at -3 & -6.

Poles : -3, -6

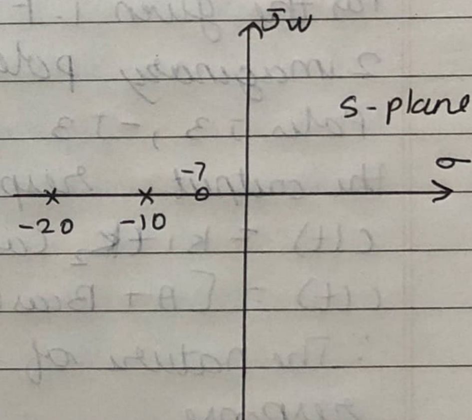
∴ output response $c(t) = (K_1 + K_2 e^{-2t} + K_3 e^{-6t})\, u(t)$

or $c(t) = (A + Be^{-3t} + Ce^{-6t})\, u(t)$

i.e

The nature of response of the system is <u>overdamped response</u>

c) $T(s) = \dfrac{10\,(s+7)}{(s+10)(s+20)}$



For the given transfer function there are 2 real poles at -10 and -20 & zero at -7

Poles -10, -20

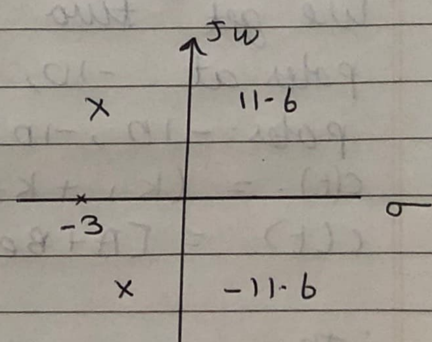
Zero : - 7

Output response $c(t) = (K_1 + K_2 e^{-10t} + K_3 e^{-20t})\, u(t)$

or $c(t) = (A + Be^{-10t} + Ce^{-20t})\, u(t)$

∴ the respons is <u>overdamped response</u>

d) $T(s) = \dfrac{20}{s^2+6s+144}$



For the given transfer function we get 2 complex ~~ab~~ poles of the system at $(-3 + j3\sqrt{15})$ & $(-3 - j3\sqrt{15})$

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Poles: $(-3+J3\sqrt{15}), (-3-J3\sqrt{15})$

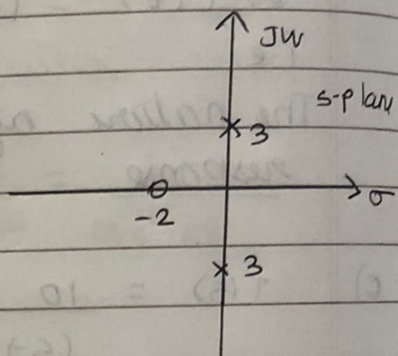
∴ the output response is

$c(t) = [A + e^{-3t}(B\cos 3\sqrt{15}t) + C\sin(3\sqrt{15}t)]u(t)$

$c(t) = [K_1 + e^{-3t}(K_2\cos 3\sqrt{15}t) + K_3\sin 3\sqrt{15}t]\, ut$

∴ The nature of respone of the system is underdamped

e) $T(s) = \frac{s+2}{s^2+9}$



For the given T.F there are 2 imaginary poles at $\pm J3$

Poles $J3, -J3$ and zero: -2

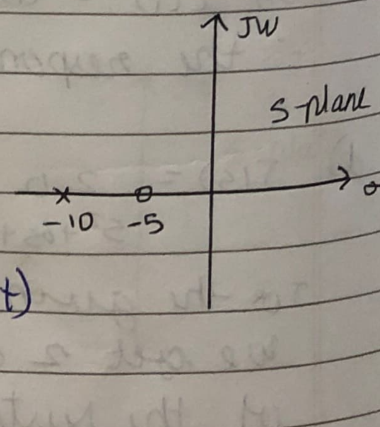
the output response $c(t) =$

$c(t) = k_1 + k_2\cos(3t) + k_3\sin(3(t))\, u(t)$

$c(t) = [A + B\cos(3t) + C\sin(3t)]u(t)$

∴ The nature of response of the system is undamped response

b) $T(s) = \frac{s+5}{(s+10)^2}$



For the given transfer function we get two multiple real poles at $-10, -10$ and 0 at -5

poles: $-10, -10$ & zero : -5

$c(t) = (K_1 + K_2e^{-3t} + k_3e^{-10t})\, u(t)$

$c(t) = [A + Be^{-10t} + Ce^{-10t}]\, u(t)$

∴ The respose of the system is critically damped

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Q12)

=> writing the nod eq at capacitas V

$$V_C(s)\left(\frac{1}{R_2} + \frac{1}{Ls} + Cs\right) + \frac{V_C(s) - V_E(s)}{R_1} = 0$$

Hence

$$\frac{V_C(s)}{V(s)} = \frac{1/R_1}{1/R_1 + 1/R_2 + 1/Ls + Cs} = \frac{10s}{s^2 + 20s + 500}$$

The Step response is

$$\frac{10}{s^2 + 20s + 500}$$

The poles are at $-10 \pm j20$

$$\therefore V_C(t) = Ae^{-10t} \cos(20t + \phi)$$

Q13)

→ The eq of motion $(Ms^2 + f_y s + K_s) X(s) = F(s)$

Therefore $$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + F_{ys} + K_s} = \frac{1}{s^2 + s + 5}$$

Step response now evaluated : $$X(s) = \frac{1}{s(s^2+s+5)}$$

$$= \frac{1/5}{s} - \frac{1/5 + 1/5}{(s + 1/2) + 19/4}$$

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$$= \frac{\frac{1}{5}(s+1/2) + \frac{1}{5\sqrt{19}}\frac{-\sqrt{19}}{2}}{(s+\frac{1}{2})^2 + 19/4}$$

Taking the inverse Laplace Transform

$$x(t) = \frac{1}{5} - \frac{1}{5}e^{-0.5t}\left(\cos\frac{\sqrt{19}}{2}t + \frac{1}{\sqrt{19}}\sin\frac{\sqrt{19}}{2}t\right)$$

$$= \frac{1}{5}\left[1 - 2\sqrt{\frac{5}{19}}\, e^{-0.5t}\cos\left(\frac{\sqrt{19}}{2}t - 12.92°\right)\right]$$

Q14)

$\Rightarrow$ $G(s) / T(s) = \frac{b}{s^2 + as + b}$

$b = \omega_n^2$

$\omega_n^2 = 16$

$\omega_n = \sqrt{16} = 4$

$a = 2\zeta\omega_n$

$3 = 2\zeta\omega_n$

$3 = 2\zeta 4$

$\zeta = \frac{3}{2\times 4} = \frac{3}{8} = 0.375$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{4\sqrt{1-0.375^2}} = 0.8472\ s$$

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$$T_s = \frac{4}{\zeta \omega_n} = \frac{4}{0.375(4)} = \underline{2.6675}$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100 = e^{-(}$$

$$= e^{-(0.375\pi/\sqrt{1-0.375^2})} \times 100 = 28.0596\%$$

$$= \underline{28.06\%}$$

$$T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$$

$$T_r = \frac{1.4238}{\omega_n} = \frac{1.4238}{4} = 0.356\,s$$

b) $$T(s) = \frac{0.04}{s^2 + 0.02s + 0.04}$$

$$G(s)/T(s) = \frac{b}{s^2 + as + b}$$

$\omega_n^2 = b$

$\omega_n^2 = 0.04$

$\omega_n = \sqrt{0.04} = 0.2$

$a = 2\zeta\omega_n$

$0.02 = 2\zeta\, 0.2$

$$\zeta = \frac{0.02}{2 \times 0.2}$$

$\zeta = 0.05$

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$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{0.2\sqrt{1-0.05^2}} = 15.727\,s = \underline{15.73\,s}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.05(0.2)} = \underline{400\ s}$$

$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100 = e^{-(0.05\pi/\sqrt{1-(0.05)^2})} \times 100$$

$$= \underline{85.45\ \%}$$

$$\omega_n T_r = (1.76\zeta^3 + 0.417\zeta^2 + 1.039\zeta + 1)$$
$$= 1.05112$$

$$T_r = \frac{1.05112}{\omega_n} = \underline{5.26}\ s$$

c) $$T(s) = \frac{1.05 \times 10^7}{s^2 + 1.6 + 10^3 s + 1.05 \times 10^7}$$

Comparing with

$$G(s) = \frac{b}{s^2 + as + b}$$

$\omega_n^2 = b$

$\omega_n^2 = 1.05 \times 10^7$

$\omega_n = \sqrt{1.05 \times 10^7}$

$\omega_n = 3240.370$

$\underline{\omega_n = 3240}$

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$a = 2\zeta\omega_n$

$1.6\times10^3 = 2\zeta\, 3240$

$\zeta = \dfrac{1.6\times10^3}{2\times3240}$

$\zeta = 0.2469$

$\zeta = 0.247$

$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} \quad \dfrac{\pi}{3240\sqrt{1-(0.247)^2}} = 1.00\times10^{-3} = \underline{0.001\,s}$

$\cancel{T(s)} =$

$T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{0.247(3240)} = 4.99\times10^{-3} = \underline{0.005\,s}$

$\%\,OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\times100$

$= e^{-(0.247\pi/\sqrt{1-(0.247)^2}}\times100 = 44.89\,\%$

$\omega_n t_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.257$

$T_r = \dfrac{1.257}{3240} = 3.88\times10^{-4}\,s$