College of Engineering Pune
Department of Mathematics
MA-16003 : LA and UC
Tutorial on Unt IV.

(1) Suppose that $\int_0^x f(t)dt = x^2 - 2x + 1$. Find $f(x)$. (x-1)x+c

(2) Find $f(4)$ if $\int_0^x f(t)dt = x cos \pi x$. -16

(3) Find the volumes of the solids generated by revolving the regions bounded by the lines and curves about the x- axis:

Formula :integration(pi (y)^2)

(a) $y = x^2, y = 0, x = 2$. 32/5 pi
(b) $y = x^3, y = 0, x = 2$. 128/7 pi
(c) $y = \sqrt{(9 - x^2)}, y = 0$. 36 pi
(d) $y = x - x^2, y = 0$. pi/30 Solve that to get the values for the upper and lower bound
(e) $y = \sqrt{cos\ x}, 0 \leq x \leq \pi/2, y = 0, x = 0$. pi
(f) $y = sec\ x, y = 0, x = -\pi/4, x = \pi/4$. 2pi

(4) Find the volumes of the solids generated by revolving the regions bounded by the lines and curves about the y- axis.

Formula :same as above

(a) The region enclosed by $x = \sqrt{5}y^2, x = 0, y = 1, y = -1$. 2pi
(b) The region enclosed by $x = y^{3/2}, x = 0, y = 2$. 4pi
(c) The region enclosed by $x = \sqrt{2sin\ 2y}, 0 \leq y \leq \pi/2, x = 0$. 2pi
(d) The region enclosed by $x = \sqrt{cos(\pi y/4)}, -2 \leq y \leq 0, x = 0$ 4
(e) $x = 2/(y + 1), x = 0, y = 0, y = 3$. 3pi

(5) The region in the first quadrant bounded above by the line y = 2, below by the curve $y = 2sin\ x, 0 \leq x \leq \pi/2$ and on the left by the y-axis, about the line $y = 2$.

(6) Find the volumes of the solids generated by revolving the regions bounded by the lines and curves about the x- axis:

Formula :integration(pi(R(y)^2 - r(x)^2))

(a) $y = x, y = 1, x = 0$. 2pi/3
(b) $y = 2\sqrt{x}, y = 2, x = 0$. 2pi
(c) $y = x^2 + 1, y = x + 3$.
(d) $y = 4 - x^2, y = 2 - x$.
(e) $y = sec\ x, y = \sqrt{2}, -\pi/4 \leq x \leq \pi/4$.
(f) $y = sec\ x, y = tan\ x, x = 0, x = 1$.

(7) The disk $x^2 + y^2 \leq a^2$ is revolved about the line $x = b, (b > a)$ to generate a solid shaped like a doughnut and called a torus. Find its volume.

(8) A bowl has a shape that can be generated by revolving the graph of $y = x^2/2$ between $y = 0$ and $y = 5$ about the y- axis. Find the volume of the bowl.

(9) Find the lengths of the following curves.

Enter directly in the arc calculator

(a) $y = (1/3)(x^2 + 2)$ from $x = 0$ to $x = 3$. 4.43...
(b) $y = x^{3/2}$ from $x = 0$ to $x = 4$. 9.07..
(c) $x = (y^3/3) + (1/(4y))$ from $y = 1$ tp $y = 3$. 8.833....
(d) $x = (y^{3/2}/3) - y^{1/2}$ from $y = 1$ to $y = 9$.
(e) $y = \int_0^x tan\ t\ dt, 0 \leq x \leq \pi/6$.

(10) The graph of the equation $x^{2/3} + y^{2/3} = 1$ is one of a family of curves called astroids (not "asteroids"!) because of their starlike appearance (Use Grapher to plot). Find the length of this particular astroids.

(11) Find the area of the surface generated by revolving the given curve about the indicated axis.
(a) $y = x^2, 0 \leq x \leq 2$, x axis. 53.22
(b) $xy = 1, 1 \leq y \leq 2, y-$ axis. 2pi
(c) $x = 2\sqrt{(4 - y)}, 0 \leq y \leq 15/4$, y- axis. 81.95

(12) Find the area of the surface geoerated by revolving about the x-axis the portion of the astroid $x^{2/3} + y^{2/3} = 1$ lying in upper half plane.

(13) Evaluate the following improper integrals:

Put directly into the calculator

(a) $\displaystyle\int_0^\infty \frac{dx}{x^2 + 1}$
(b) $\displaystyle\int_0^4 \frac{dx}{\sqrt{(4 - x)}}$
(c) $\displaystyle\int_{-1}^1 \frac{dx}{x^{2/3}}$
(d) $\displaystyle\int_0^1 \frac{dx}{\sqrt{(1 - x^2)}}$
(e) $\displaystyle\int_{-\infty}^\infty \frac{2x\ dx}{(x^2 + 1)^2}$
(f) $\displaystyle\int_{-1}^4 \frac{dx}{|x|}$
(g) $\displaystyle\int_{-1}^\infty \frac{dx}{x^2 + 5x + 6}$
(h) $\displaystyle\int_0^\infty \frac{dx}{(x + 1)(x^2 + 1)}$

(14) Test the convergence of follwoing integrals:

convergent (a) $\displaystyle\int_0^\pi \frac{sin\ \theta\ d\theta}{\sqrt{(\pi - \theta)}}$
convergent (b) $\displaystyle\int_0^\pi \frac{dt}{\sqrt{t} + sin\ t}$
Divergent (c) $\displaystyle\int_0^1 \frac{dt}{t - sin\ t}$
convergent (d) $\displaystyle\int_0^2 \frac{dt}{1 - t^2}$
(e) $\displaystyle\int_0^2 \frac{dt}{1 - t}$ convergent
(f) $\displaystyle\int_1^\infty \frac{dt}{t^3 + 1}$ Divergent
(g) $\displaystyle\int_4^\infty \frac{dt}{\sqrt{t} - 1}$ Divergent
(h) $\displaystyle\int_2^\infty \frac{dt}{\sqrt{(t - 1)}}$ Divergent