COLLEGE OF ENGINEERING, PUNE

(An autonomous Institute of Government of Maharashtra)

END SEM EXAM

(CT201) DISCRETE STRUCTURE AND GRAPH THEORY

Class: - S. Y. B. Tech (Computer Engineering/Information Technology)

Year: - 2013-14 | **Semester: - III**

Duration: - 3 Hrs | **Max. Marks: - 60**

Instructions:

1. Figures to right indicate full marks.
2. Draw neat diagrams wherever required.
3. Justify your answers. Answers without proper justification will not get any mark(s).
4. Write all bits of a question together as far as possible.

Q. 1 Solve all of the following (each carries TWO marks): **[12]**

(a) Show that the set of all integers {... , -2, -1, 0, 1, 2, ...} is a countably infinite set?

(b) Write the converse, inverse, contrapositive and negation of the following statement:

The home team wins whenever playing on home ground

(c) Show that: $(p \wedge q)$ *and* $(\sim (q \rightarrow \sim p)$ are logically equivalent ? (by formulas)

(d) Show that the premises :
"If I join COEP then i will get good placement". " If I get good placement then i will get good package ". "If I get good package then I will become financially stable". I joined COEP. Does this imply that "I will be financially stable." Show that the conclusion follows from the premises ?

(e) Let A and B be sets. Show that :

$$\overline{A \cap B} = \bar{A} \cup \bar{B}$$

(f) Write the following using quantifiers :

i) All humming birds are richly colored
ii) No large birds live on honey
iii) Birds that do not live on honey are dull in color
iv) Humming birds are small

Assume that the universe consists of all birds and we have P(x), Q(x), R(x) and S(x) where

P(x) → x is a Humming bird
Q(x) → x is large
R(x) → x lives on honey
S(x) → x is richly colored

Q. 2 **(a)** Use mathematical induction to show that : **[3]**

$n! >= 2^{n-1}$ for $n \geq 1$

(b) Show that the relation $\geq$ is a partial ordering on the set of integers? **[2]**

(c) Consider R to be the set of real numbers . **[2]**
Let A = {x ∈ R such that 1< x <2 } with ≤ as the partial ordered. Find :
i) All upper and lower bounds of A
ii) Greatest lower bound and least upper bound of A

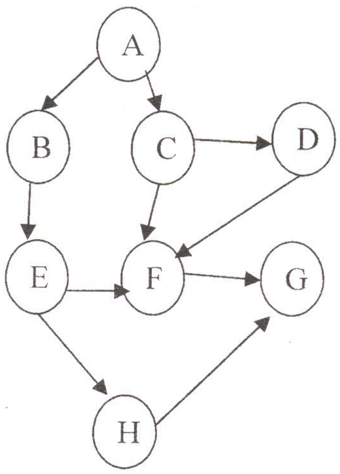
(d) Which of the following functions from N to N are surjection, which of them are injection, and which of them are bijection ? **[2]**

$f(n) = n^3$
$g(n) = n + 3$
$h(n) = n + 1$ if n is odd
$= n - 1$ if n is even

(e) Use Warshalls algorithm to find the transitive closure of the following relation on A={a,b,c,d,e} : **[3]**

R = { (a,b), (a,c), (a,e), (b,a), (b,c), (c,a), (c,b), (d,a), (e,d) }

Q. 3 **(a)** Apply BFS algorithm to the following graph with the starting vertex as A: **[4]**



(b) Using graph coloring, schedule the examinations in a university. Schedule the final examination for eight courses (MI, MII, MIII, MIV, DS,DE,PPL,DSGT) using fewest number of different time slots, if there are no students taking both MI and DSGT, both MII and DSGT, both MIV and DS, both MIV and DE, both MI and MII, both MI and MIII, both MIII and MIV but there are students in every other combination of courses ? **[4]**

(c) Prove that , for any connected planar graph : **[3]**

$$v - e + r = 2$$

where v, e, and r are the number of vertices, edges, and regions of the graph respectively ?

(d) What is the chromatic number of a cycle graph with odd number of vertices(for n > 1) and even number of vertices ? **[1]**

Q. 4 **(a)** Give a solution of a puzzle commonly known as instant insanity. Given four cubes , each of their faces being colored with one of the four colors blue, green, red and yellow, as shown in figure (4a) below, where the cubes are numbered 1,2,3, 4 and the colors of the faces are abbreviated as b, g, r and y. You are asked to stack the four cubes in a column so that the four colors will show in each of the four sides of the column. **[4]**

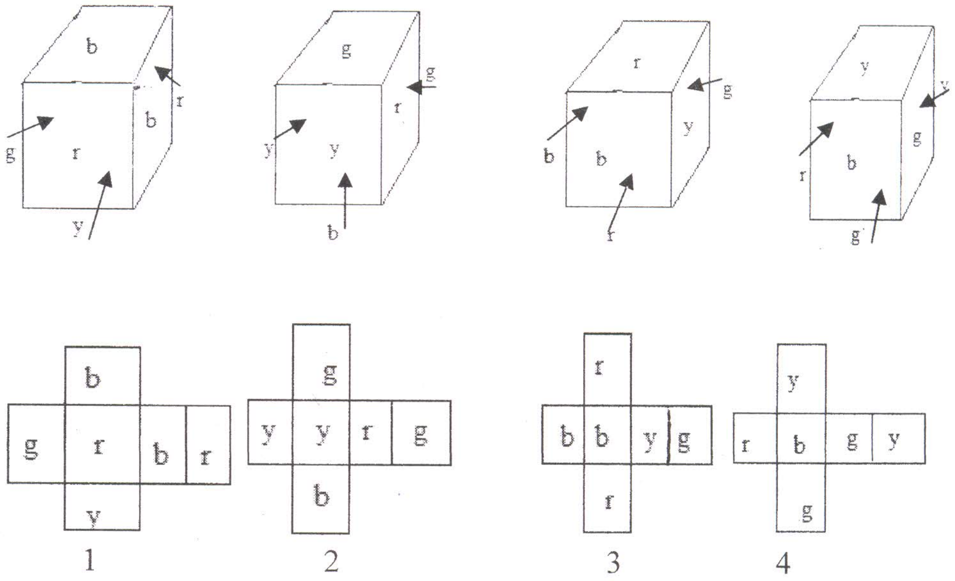
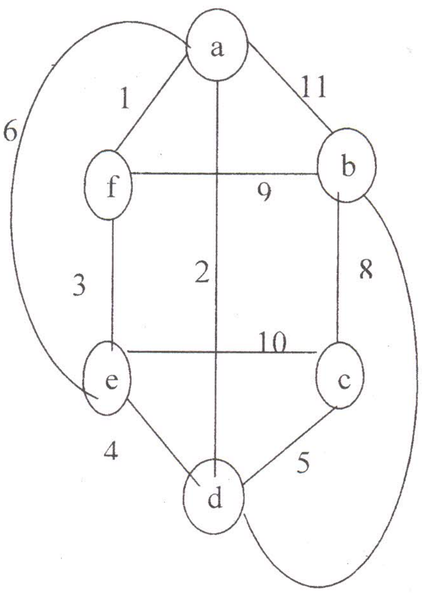
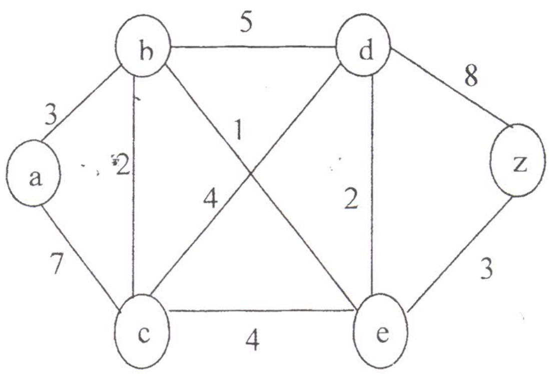


Figure : 4a

(b) Find the minimum spanning tree using Prim's algorithm for the graph shown below. Also, determine the cost of the spanning tree. **[4]**



(c) Using Dijkstra's shortest path algorithm, find the length of the shortest path between the vertices **a** and **z** in the weighted graph given below : **[4]**



Q. 5 Solve any FOUR of the following (each carries THREE marks): **[12]**

(a) For any a, b, c, d in a lattice $(A, \leq)$, if

$$a \leq b \quad \text{and} \quad c \leq d$$

then prove that :

$$a \vee c \leq b \vee d$$
$$a \wedge c \leq b \wedge d$$

(b) Let A = {1, 2, 3, 4, 5}. Show that (A , /) is a lattice by constructing a meet-join table ?

(c) For an algebraic system $(A, \vee, \wedge)$ defined by a lattice $(A, \leq)$, prove that both the join ($\vee$) and meet ($\wedge$) operations are associative , i.e., for a, b, c in A ,

$$a \vee (b \vee c) = (a \vee b) \vee c$$
$$a \wedge (b \wedge c) = (a \wedge b) \wedge c$$

(d) A chess player wants to prepare for a championship match by playing some practice games in 77 days. She wants to play at least one game a day but no more than 132 games altogether. Show that no matter how she schedules the games, there is a period of consecutive days within which she plays exactly 21 games.

(e) Let (A,*) be a semigroup. Furthermore, for every a and b in A, if $a \neq b$ then $a*b \neq b*a$. Therefore, $a*b = b*a$ implies $a = b$.

a) Show that for every a in A,
$$a * a = a$$
b) Show that for every a, b in A
$$a * b * a = a$$
c) Show that for every a, b, c in A,
$$a * b * c = a * c$$

(f) Show that $(Z^+, \oplus)$ is a group where Z^+ is the set of positive integers and $\oplus$ is mod n addition operation (addition modulo n) ?

(g) Define normal subgroup ? Discuss the properties of normal subgroups ?

(h) Define Ring as an algebraic system. Give an example of a ring and show that how it satisfies the conditions given in the definition ?