1) Prove L is not regular.

(i) $L = \{a^i b^i \mid i \geq 0\}$

→ Assume L is regular.

∴ there exists pumping length 'p' such that, P is a positive real integer.

Let $w \in L$ and $|w| \geq P$

e.x ⇒ w = aaabbb,

let x = aa

y = ab

z = bb

∴ $|y| > 0$ and $|xy| \leq P$

but, when we pump y, 2 times

$w = xy^2z$ = aaababbb

$xy^2z \notin L$, ∴ We arrive at a contradiction.

∴ L is not regular.

2) $L = \{a^i b^j \mid i > j\}$

→ Assume L is reg.

$P = |Q| > 0$

Let w be a string such that,

1) $w \in L$

2) $|w| \geq p$

ex, w = aaabbbbb

x = aaa

y = bb

z = bbb

here, $|y| > 0$ and $|xy| < P$,

but when we pump y 0 times,

∴ P is pumping length

$a^{P+K} b^P$

$x = a$

$y = a^{P-1}$

$z = a^K b^P$

Assume $K < P$,

$xy^0z = aa^K b^P$

$= a^{K+1} \leq b^P$

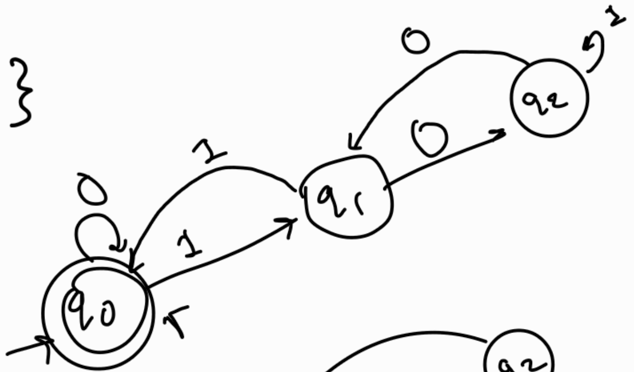
∴ $K < P$

Contradiction

xy^0z = aaabbb, which $\notin L$.

$\therefore$ Contradiction, and L non regular.

iii) $L = \{ ww \mid w \text{ in } \{a,b\}^* \}$



Assume, $L \rightarrow$ reg.

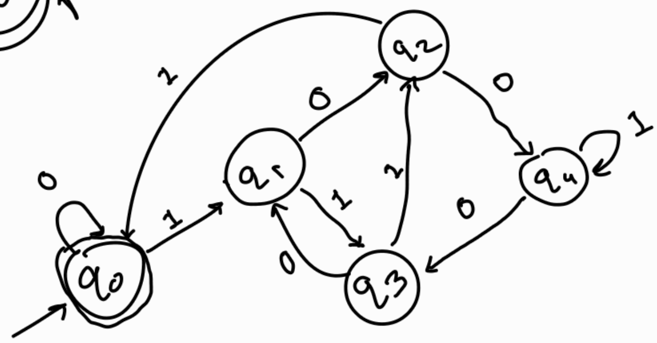
$P = |Q| > 0$

$w =$ abbabb

$x =$ ab

$y =$ ba

$z =$ bb



here $|y| > 0$, $|xy| \leq P$,

but, if we pump y 0 times,

xy^0z = abbb, but abbb $\notin L$

$\therefore$ Contradiction.

L is non regular.

(iv) $L = \{ w_1 w_2 : w_1, w_2 \in \{a,b\}^*, |w_1| = |w_2| \}$

L is regular.

$\therefore$ $P = |Q| > 0$

$w =$ aba bba

let,

$x = ab$

$y = abb$

$z = a$

$\therefore |xy| \leq P$ and $|y| > 0$

but when we try pumping y 2 times,

xy^2z = ababbabba

here $|xy^2z| = 9$, we cannot split xy^2z in two strings w_1, w_2

S.T. $|w_1| = |w_2|$.

$\therefore$ Contradiction. L is non regular.

(v) $L = \{a^n b^n c^n \mid n \geq 0\}$

$L \to$ regular

$P = |Q| > 0$

$w \in L$, $|w| \geq P$

let, $w =$ aabbcc

$\therefore$ $x = aa$

$y = bb$

$z = cc$

$\therefore |y| > 0$, $|xy| \leq P$.

but, when we pump y 2 times,

$xy^2z =$ aabbbbcc.

$xy^2z \notin L$

$\therefore$ L is not regular.

(vi) $L = \{a^n b a^m b a^{n+m} \mid n, m > 1\}$

Let $L =$ reg.

$\therefore P = |Q| > 0$

Let $w \in L$, $|w| \geq P$, $\quad \underline{w = aababaaa} \quad \{a^2 b a^1 b a^{2+1}$

$\therefore w = xyz$ S.T.

$x = aab$

$y = ab$

$z = aaa$

$\therefore |y| > 0$, $|xy| < P$

but when we try to pump y, 0 times,

$xy^0z = aabaaa$

$xy^0z \notin L$

$\therefore$ contradiction. $\therefore$ L is not regular.

(vii) $L = \{w \mid w$ has equal number of 0's and 1's $\}$

$L \rightarrow$ reg $\}$ assumption

$\therefore p = |\varphi| > 0$

Let $w \in L$, $|w| \geq p$,

$w = 01100011$

$w \in L$,

$\therefore$ let $w = xyz$, S.T,

$x = 01$

$y = 100$

$z = 011$

$\therefore |xy| \leq p$, $|y| > 0$,

if we pump y twice,

$xy^2z = 01100100011$,

here 0's occur 6 times an 1's occur 5 times.

$\therefore xy^2z \notin L$

$\therefore$ L is not regular.

(viii) $L = \{1^{n^2}\}$

$L \rightarrow$ reg

$\therefore p = |\varphi| > 0$

$w \in L$, $|w| \geq p$

$w = 1111$ $\quad \therefore n = 2$

let $w = xyz$, S.T.

$x = 1$
$y = 11$
$z = 1$

$\therefore |y| > 0$
$|xy| \leq P$

$\therefore$ when we pump y 2 times,

$xy^2z = 111111$

but $xy^2z \in L$ as $1^6 \neq 1^{n^2}$ for any number.

$\therefore L$ is not regular.

(ix) $L = \{a^i b a^j \mid i > j \geq 0\}$

let L be regular.

$\therefore P = |\varphi| > 0$

Let $\omega \in L$, S.T. $|\omega| > P$

let $\omega =$ aaabaa ... $\{a^3ba^2$ ... $i = 3, j = 2.\}$

$\therefore \omega = xyz$

$x =$ aaa
$y =$ ba
$z =$ a

$\therefore |y| > 0$, $|xy| \leq P$

when we pumb y'2', we get,

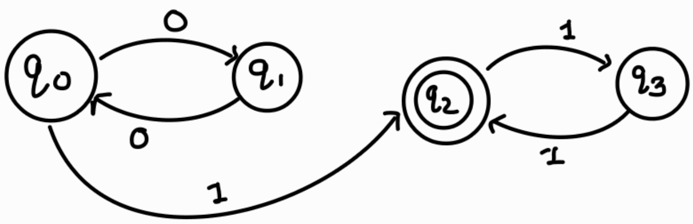
$xy^2z =$ aaababaa $\notin L$

$\therefore$ contradiction.

L is not regular.

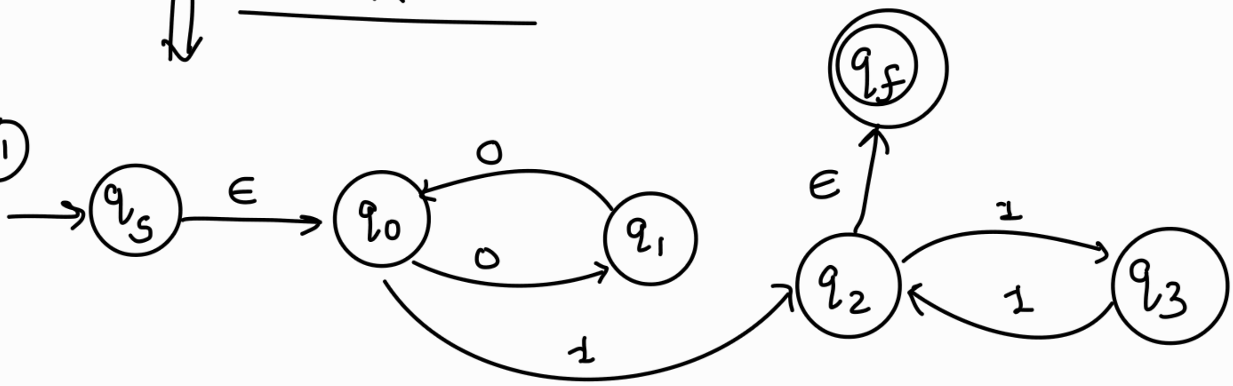
2.

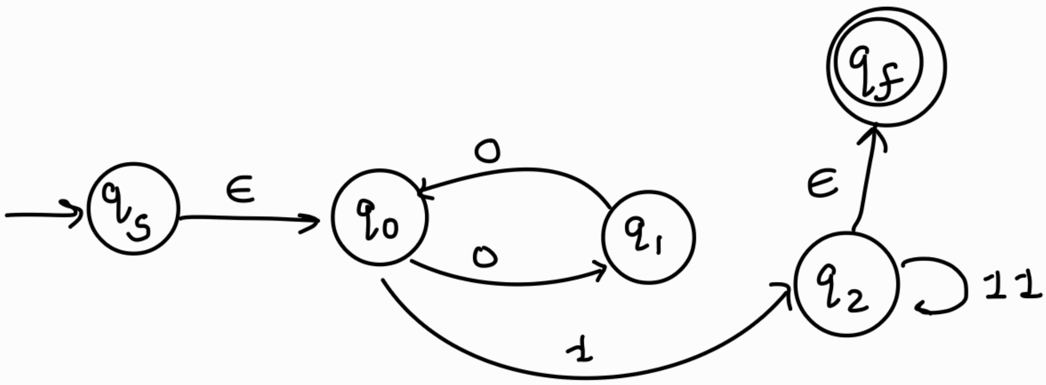
i) Even 0's followed by Odd 1's

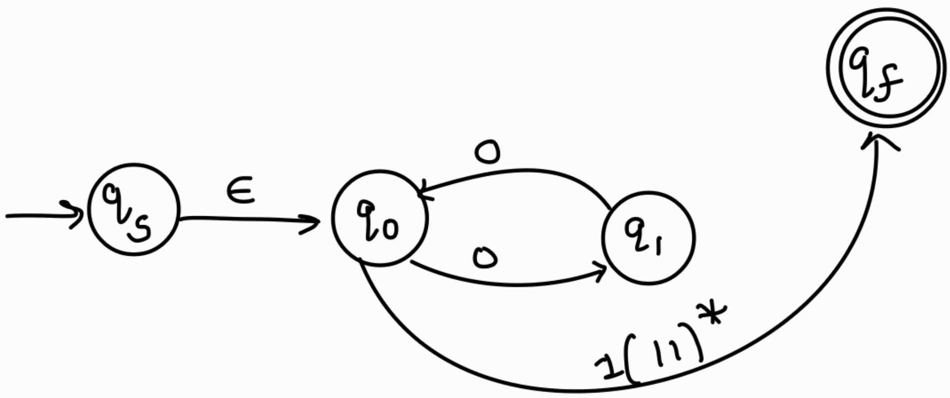


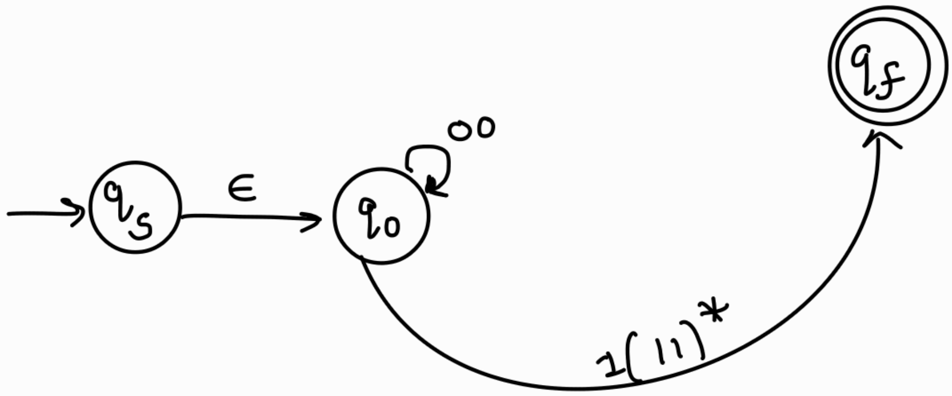
$L = (00)^*(11)^*1$

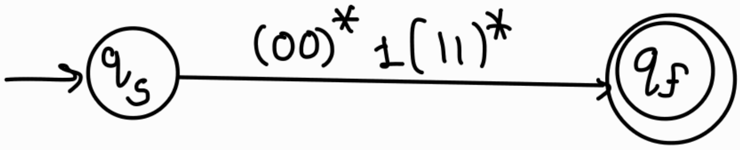
1









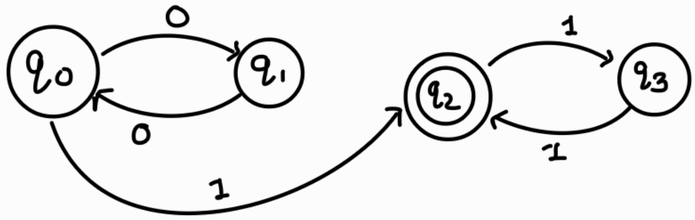


Arden's theorem:

$R = Q + RP$

and $\epsilon \notin P$,

$R = QP^*$



$q_0 = \epsilon + q_1 0$

$q_1 = q_0 0$

$q_2 = q_0 1 + q_3 1$

$q_3 = q_2 1$

$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\underbrace{00}_{P}$$

$\therefore R = QP^* \Rightarrow q_0 = \epsilon(00)^* \quad \ldots (1)$

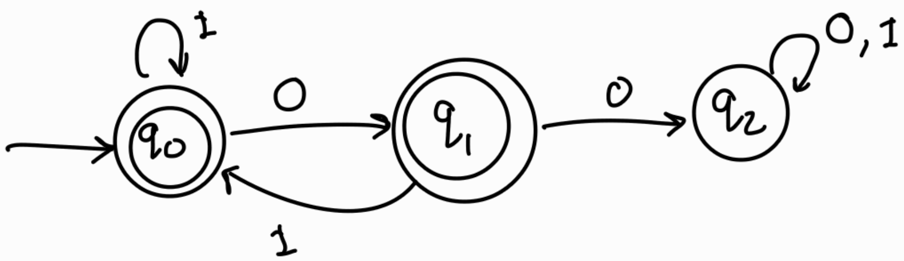
$$q_2 = q_0 1 + q_3 1$$

$$\underbrace{q_2}_{R} = \underbrace{(00)^* 1}_{Q} + \underbrace{q_2}_{R}\underbrace{11}_{P}$$

$$q_2 = (00)^* 1 (11)^*$$

$\therefore$ Regular expression $= (00)^* 1 (11)^*$

(ii) Two 0's do not come together:



$$q_0 = \epsilon + q_0 1 + q_1 1$$

$$q_1 = q_0 0$$

$$q_2 = q_1 0 + q_2 0 + q_2 1$$

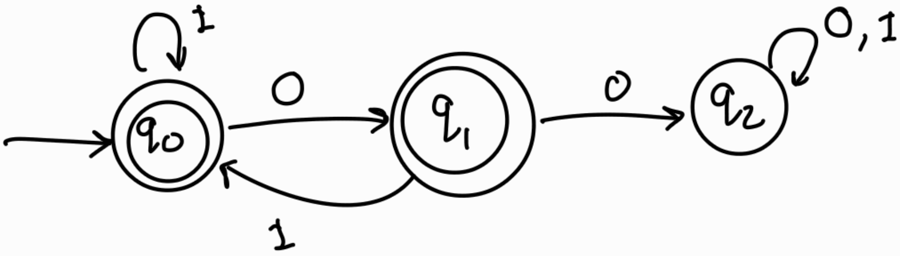
$$q_0 = \epsilon + q_0 1 + q_0 0 1$$

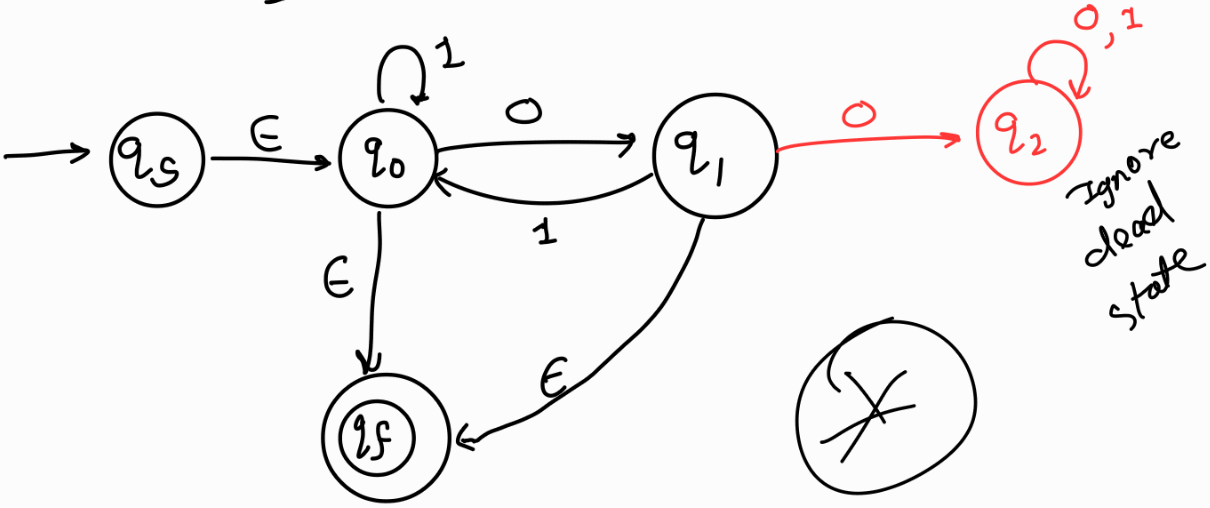
$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\underbrace{(1+01)}_{P}$$

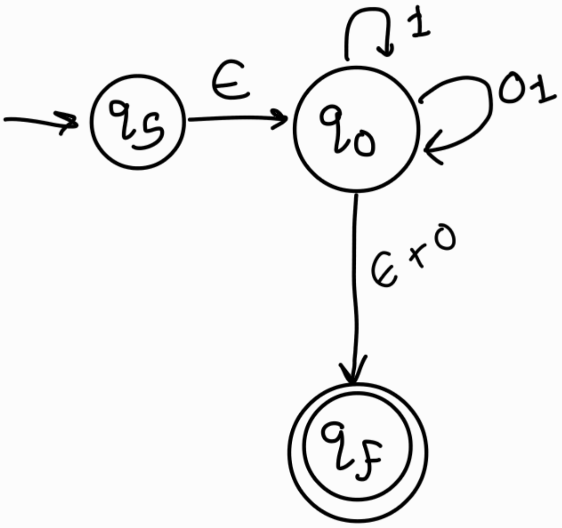
$$\therefore q_0 = \epsilon (1+01)^*$$

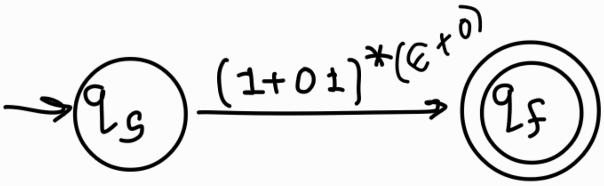
Regex $= q_0 + q_1$

$= (1+01)^* + (1+01)^* 0 = (1+01)^* (\epsilon + 0)$

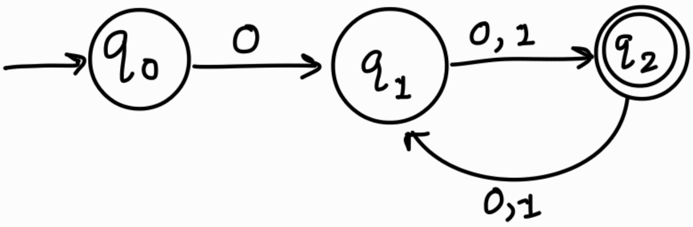

q0
q1
q2
0
1
0, 1


qs
q0
q1
q2
qf
ε
0
1
0, 1
Ignore dead state


qs
q0
qf
ε
1
01
ε+0


qs
qf
(1+01)*(ε+0)

(iii)



$$q_0 = \epsilon$$

$$q_1 = q_0 0 + q_2 0 + q_2 1$$

$$q_2 = q_1 0 + q_1 1$$

$$q_1 = \epsilon 0 + (q_1 0 + q_1 1) 0 + (q_1 0 + q_1 1) 1$$

$$= 0 + q_1 (00 + 10) + q_1 (01 + 11)$$

$$q_1 = 0 + q_1 (00 + 10 + 01 + 11)$$

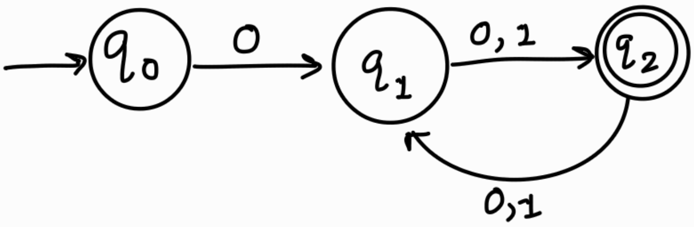
$$q_1 = 0 (00 + 10 + 01 + 11)^*$$

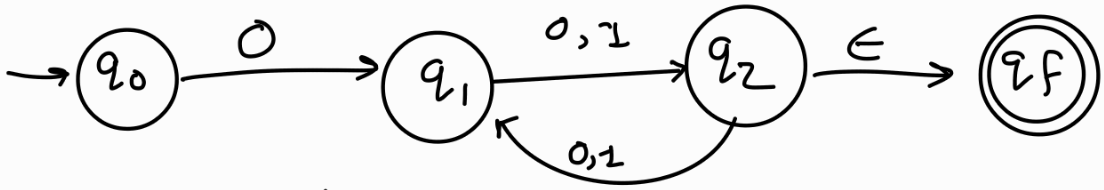
$$q_2 = q_1 0 + q_1 1$$

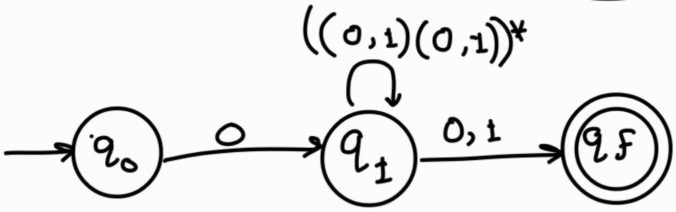
$$= q_1 (0 + 1)$$

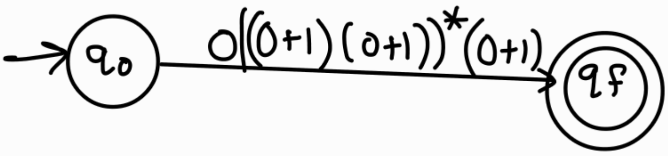
$$q_2 = 0 (00 + 10 + 01 + 11)^* (0 + 1)$$

$\therefore$ Regular expression is $0(00+10+01+11)^*(0+1)$

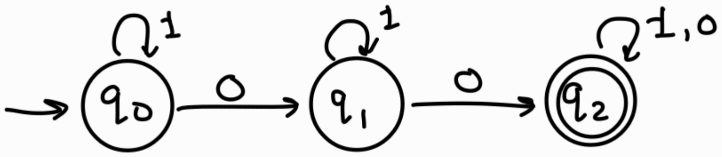








(iv) String containing at least 2 zeroes:



$$q_0 = \epsilon + q_0 1$$

$$q_1 = q_0 0 + q_1 1$$

$$q_2 = q_1 0 + q_2 1 + q_2 0$$

$$\underbrace{q_0}_{R} = \underbrace{\epsilon}_{Q} + \underbrace{q_0}_{R}\underbrace{1}_{P}$$

$$q_0 = \epsilon 1^*$$

$$q_0 = 1^*$$

$q_1 = q_0 0 + q_1 1$

$q_1 = 1^* 0 + q_1 1$

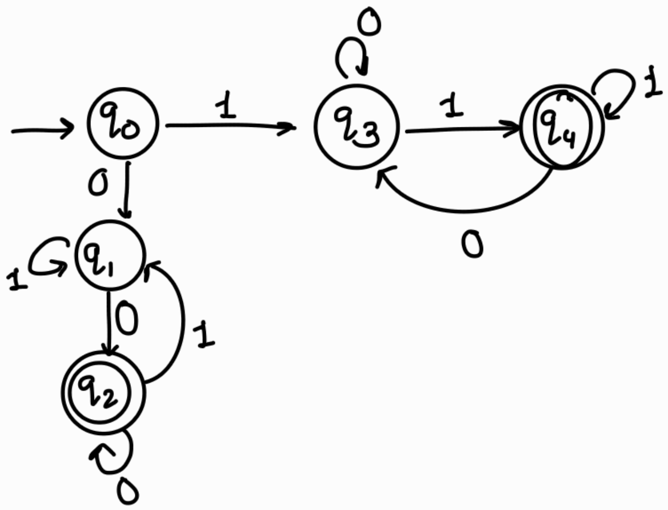
$q_1 = 1^* 0 1^*$

$q_2 = q_1 0 + q_2 1 + q_2 0$

$q_2 = 1^* 0 1^* 0 + q_2 (1+0)$

$q_2 = 1^* 0 1^* 0 (1+0)^*$

(v) String being & end with 0 or 1.



$q_0 = \epsilon$

$q_1 = q_0 0 + q_1 1 + q_2 1$

$q_2 = q_1 0 + q_2 0$

$q_3 = q_0 1 + q_3 0 + q_4 0$

$q_4 = q_3 1 + q_4 1$

- - - - .

$q_1 = 0 + q_1 1 + q_2 1$

$\underbrace{q_2}_{R} = \underbrace{q_1 0}_{Q} + \underbrace{q_2}_{R} \underbrace{0}_{P}$

$q_2 = q_1 0 0^*$

$$q_1 = 0 + q_1 1 + q_1 00^*$$

$$q_1 = 0 + q_1 (1 + 00^*)$$

$$q_1 = 0 (1 + 00^*)^*$$

$$q_2 = 0 (1 + 00^*)^* 00^*$$

$$q_2 = 0 (1 + 0^*)^* 0^*$$

$$q_3 = q_0 1 + q_3 0 + q_4 0$$

$$q_4 = q_3 1 + q_4 1$$

$$q_4 = q_3 1 1^*$$

$$q_4 = q_3 1^*$$

$$q_3 = 1 + q_3 0 + q_3 1^*$$

$$= 1 + q_3 (0 + 1^*)$$

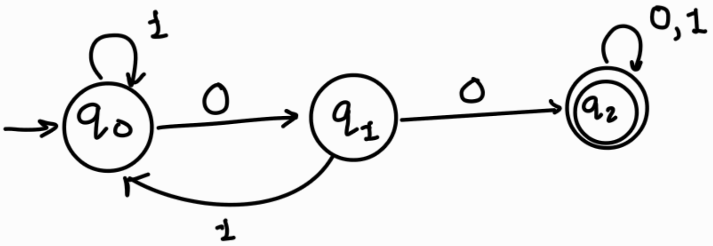
$$q_3 = 1 (0 + 1)^*$$

$$q_4 = 1 (0 + 1^*)^* 1^*$$

$\therefore$ Regular expression $= q_2 + q_4$

$$= 0 (0 + 1^*)^* 0 \quad + 1 (0 + 1^*)^* 1$$

(vi) Strings contain substring 00.



$$q_0 = \epsilon + q_0 1 + q_1 1$$

$$q_1 = q_0 0$$

$$q_2 = q_1 0 + q_2 0 + q_2 1 \longrightarrow q_2 = q_1 0 + q_2(0+1)$$

$$q_2 = q_1 0 (0+1)^*$$

$$q_0 = \epsilon + q_0 1 + q_0 0 1$$

$$q_0 = \epsilon + q_0 (1 + 01)$$

$$q_0 = \epsilon (1+01)^*$$

$$q_1 = (1+01)^* 0$$

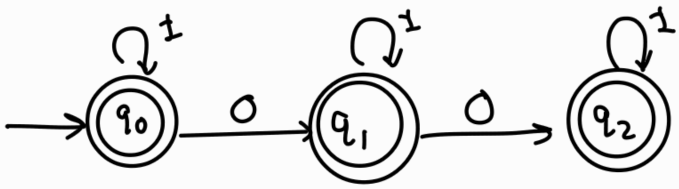
$$q_2 = (1+01)^* 0 0 (0+1)^*$$

$$q_1 = (1+01)^* 0$$

$$q_2 = (1+01)^* 00 + q_2 (0+1)$$

$$q_2 = (1+01)^* 00 (0+1)^*$$

(vii) Strings containing at most 2 zeroes



$$q_0 = \epsilon + q_0 1$$

$$q_1 = q_0 0 + q_1 1$$

$$q_2 = q_1 0 + q_2 1$$

$q_0 = \epsilon 1^*$

$q_0 = 1^*$

$q_1 = 1^*0 + q_1 1$

$q_1 = 1^*01^*$

$q_2 = 1^*01^*0 + q_2 1$

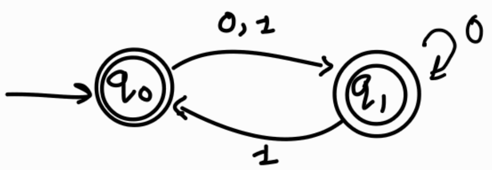
$q_2 = 1^*01^*01^*$

$1^*(\epsilon + 01^* + 01^*01^*)$

$1^*(\epsilon + 01^*(\epsilon + 01^*))$

P. 1,8

(ix) string has 1 at every even position



Resolve

$q_0 = \epsilon + q_1 0$

$q_1 = q_0 0 + q_0 1 + q_1 0$

$q_1 = q_0(0+1) + q_1 0$

$= q_0(0+1)0^*$

$q_0 = \epsilon + q_0(0+1)0^*0$

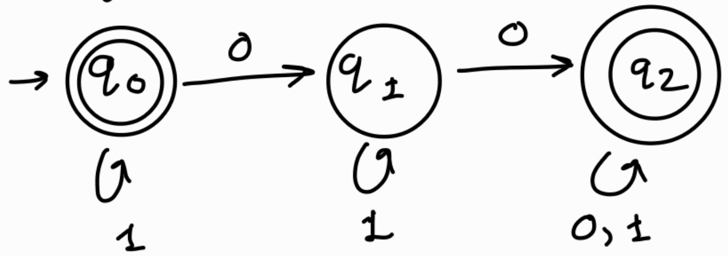
$q_0 = \epsilon((0+1)0^*0)^*$

$q_1 = ((0+1)0^*0)^*(0+1)0^*$

Regex $= ((0+1)0^*0)^* + ((0+1)0^*0)^*(0+1)0^*$

$= ((0+1)0^*0)^*(\epsilon + (0+1)0^*)$

(x) Strings that do not contain a single 0.



$q_0 = \epsilon + q_0 1$

$q_1 = q_0 0 + q_1 1$

$q_2 = q_1 0 + q_2 0 + q_2 1$

$q_0 = \epsilon + q_0 1$

$q_0 = \epsilon 1^*$

$q_1 = 1^* 0 + q_1 1$

$q_1 = 1^* 0 1^*$

$q_2 = 1^* 0 1^* 0 + q_2 (0+1)$

$q_2 = 1^* 0 1^* 0 (0+1)^*$

$\therefore$ Regex $= q_0 + q_2 = 1^* + 1^* 0 1^* 0 (0+1)^*$

(3) Simplify CFG

(i) $aa(b^* + a)a(ab^* + aa)$

$\Rightarrow aa(b^* + a)aa(b^* + a)$

$\Rightarrow (aa(b^* + a))^2 \rightarrow (aab^* + a^3)^2$

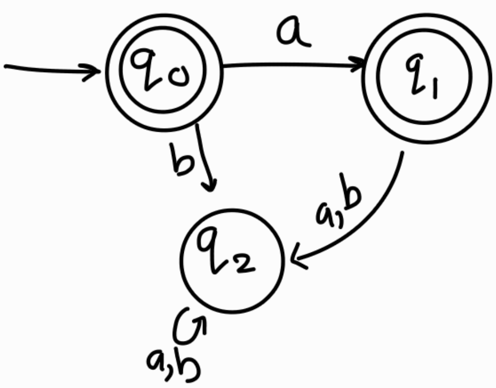
(ii) $(a^* b^*)^* + a^*$

$\Rightarrow a^* b^* + a^*$

$a^*(b^* + \epsilon)$

$a^*(b^*) \longrightarrow a^* b^*$

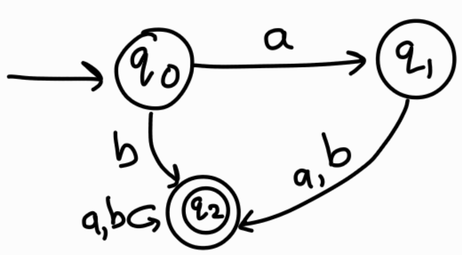
(4) $L = \{\epsilon, a\}$ over $\Sigma = \{a, b\}$



M_1 for L

$\bar{L}$ will be,

$(b + a(a+b))(a+b)^*$



$L = \{(a+b)^* - \{a, \epsilon\}\}$

5) $\Sigma = \{0, 1\}$

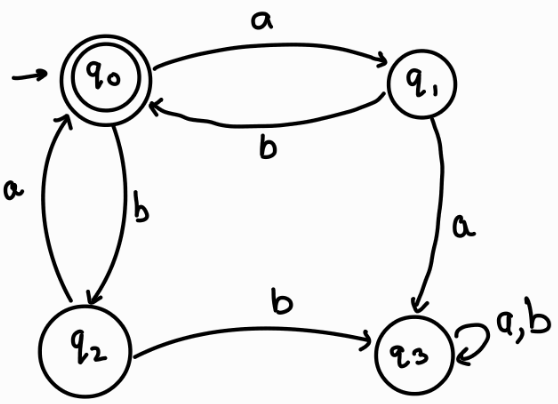
$\Delta = \{a, b\}$

$h(0) = aa$

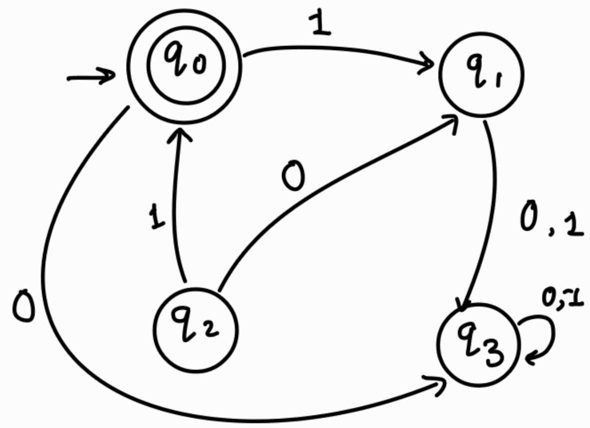
$h(1) = aba$

$L = \{ab + ba\}^*$

$h^{-1}(L) = ?$



M_1 for L



(6) Design CFG for

(i) $\{a^n b^n \mid n >= 1\}$

$G = \{V, T, P, S\}$

$V = \{S, A \quad \}$

$T = \{a, b\}$

$S = S$

$S \rightarrow aSb \mid aAb$

$A \rightarrow ab \mid \epsilon$

Kulkarni's way:

$S \longrightarrow aSb \mid ab$

(ii) $\{a^n b^{2n} \mid n >= 0\}$

$G = \{V, T, P, S\}$

$V = \{S \quad \}$

$T = \{a, b\}$

$P = \{S \quad \}$

$S = S$

$S \Rightarrow \epsilon \mid aSbb$

(iii) $\{a^m b^n c^{n+m} \mid n >= 0, m >= 0\}$

$G = \{V, T, P, S\}$

$V = \{ \quad \}$

$T = \{a, b, c\}$

$P = \{S, \quad \}$

$S = S$

$a^m b^n c^n c^m$

$a^m b^n c^n c^m$

$S \to aSc \mid B$

$B \to bBC \mid \varepsilon$

$a^5 b^4 c^9$

$a^5 b^4 c^4 c^5$

$S \to aSc \mid B \mid \epsilon$

unnecessary

$B \to bBc \mid \epsilon$

Intuition:

① the $a^m b^n c^{m+n}$ can be written as $a^m b^n c^n c^m$ as $c^{m+n} = c^n c^m$

② Now, we need to focus on productions: we know there are 'm' a's in start and 'm' c's in the end.

$\therefore$ we use aSc ... this would give us $a^m S c^m$

③ IMP we can't have bSC here in the

Same production as aSC as we don't want to regenerate aSC within the bSC.
∴ we create a new production B.
B will give us $b^n c^n$.

(iv) $\{a^n b^m \mid n \neq m\}$

$G = \{V, T, P, S\}$

$S = S$

$V = \{S, \quad \}$

$T = \{a, b\}$

P:

$S \rightarrow aSb \mid A \mid B$

$A \rightarrow Aa \mid a$

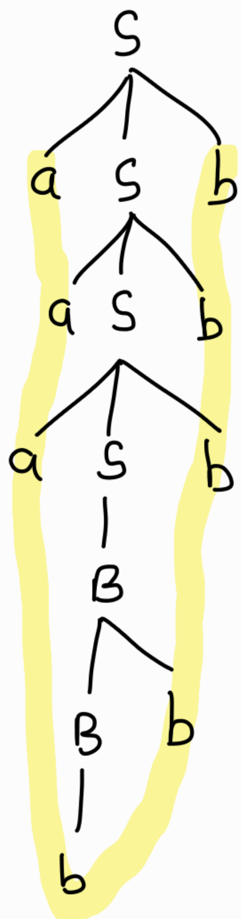
$B \rightarrow Bb \mid b$

$S \rightarrow aSb \mid A \mid B$

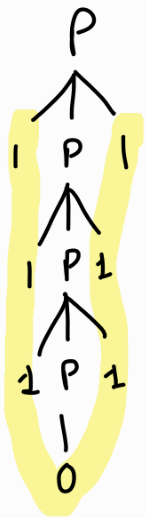
$A \rightarrow aA \mid a$

$B \rightarrow bB \mid b$

$a^3 b^5$

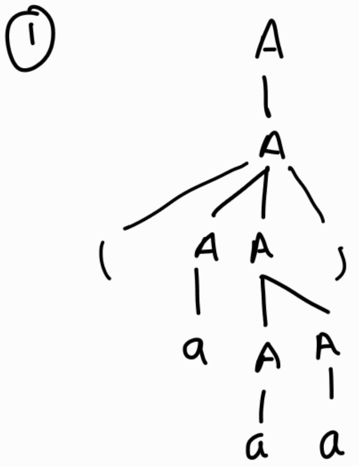


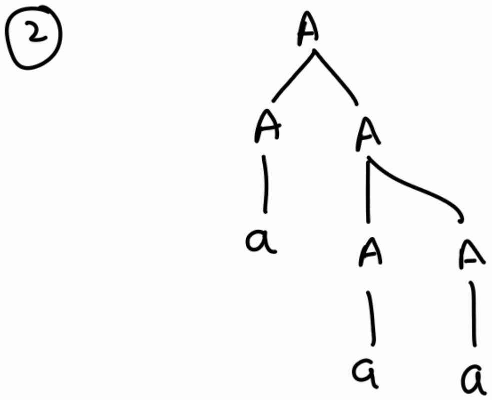
7. find parse tree for 1110111 for CFG $P \rightarrow 0P0 \mid 1P1 \mid 0 \mid 1 \mid \epsilon$



8. $A \rightarrow AA \mid (A) \mid a$

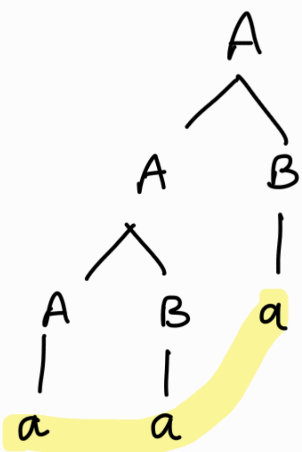
string = aaa





$A \rightarrow AB \mid a$

$B \rightarrow a$



Intuition:

- Here the problem is AA, where the recursion may occur either in the left subtree or right subtree.
- To avoid this we change AA to AB thus limiting recursion to left of tree. Stopping multiple parse tree formation.

(ii) $S \to AB \mid C$

$A \to aAB \mid ab$

$B \to cBd \mid cd$

$C \to aCd \mid aDd$

$D \to bDc \mid bc$

3 Step:

Remove ambiguity

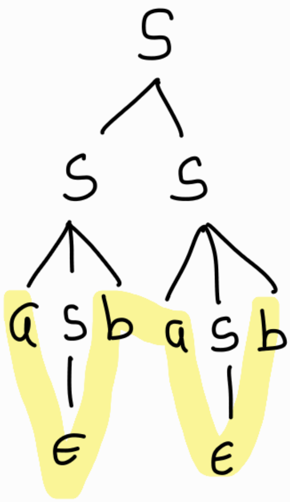
From notes

1) Remove useless symbols, ϵ-productions, unit production.

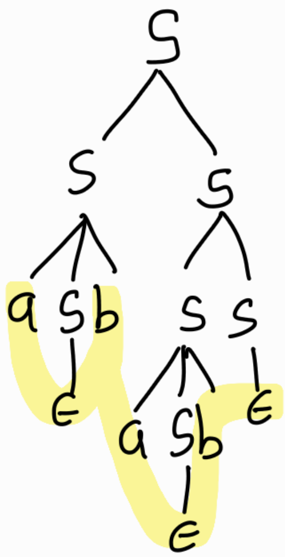
(iii) $S \longrightarrow aSb \mid SS \mid \epsilon$

String = abab

1

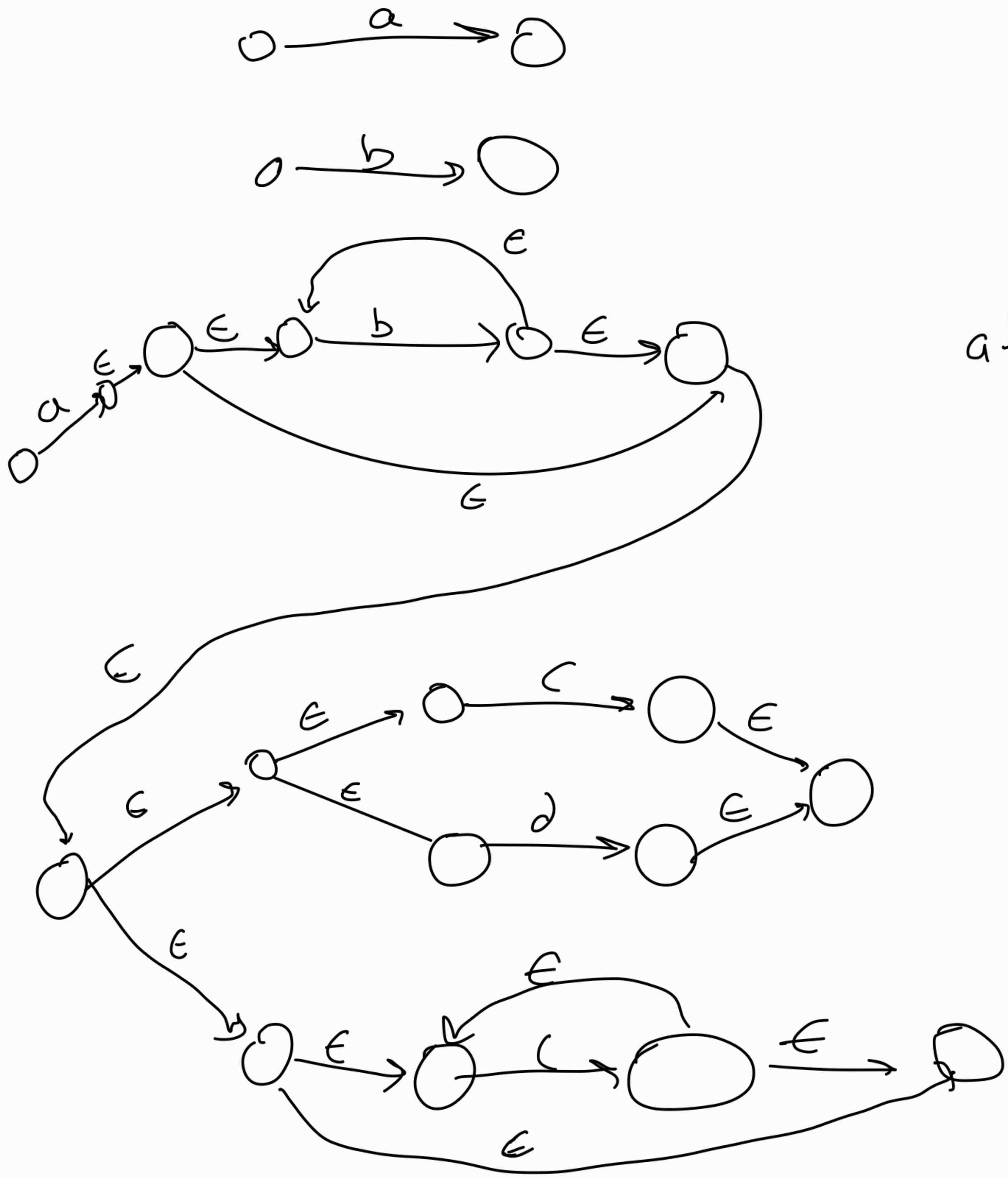


2



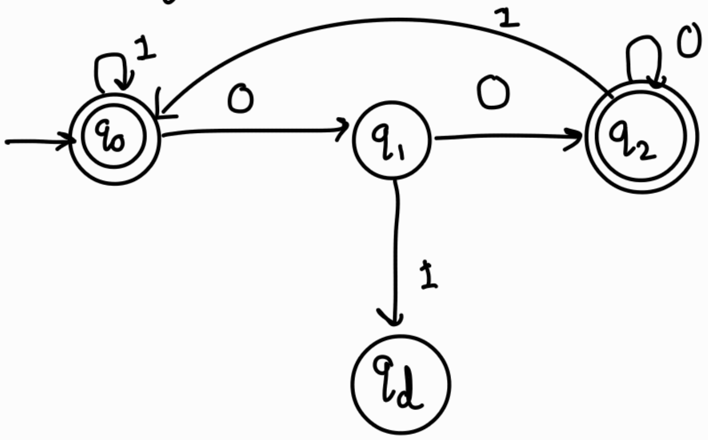
Q.9) Find NFA for given regex :

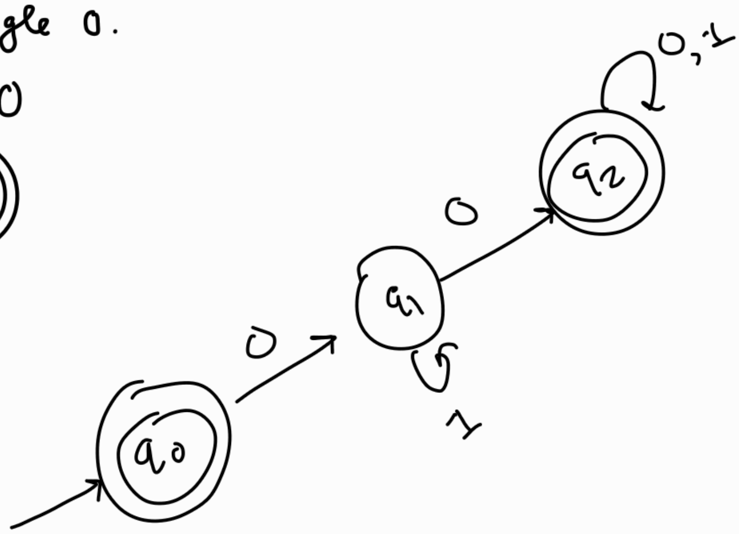
(i) $ab^*((c+d)+c^*)$



$ab^*((c+d)+c^*)$

2] (x) Strings that do not contain a single 0.





$$q_0 = \epsilon + q_0 1 + q_2 1$$

$$q_1 = q_0 0$$

$$q_2 = q_1 0 + q_2 0$$

$$q_2 = (q_0 0)0 + q_2 0$$

$$q_2 = q_0 00 + q_2 0$$

$$q_2 = q_0 000^*$$

$$q_0 = \epsilon + q_0 1 + q_0 000^*$$

$$q_0 = \epsilon + q_0 (1 + 000^*)$$

$$q_0 = (1 + 000^*)^*$$

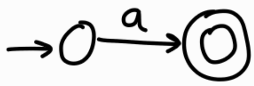
$$q_2 = (1 + 000^*)^* 000^*$$

Regular expression $= (1+000^*)^* + (1+000^*)^* 000^*$

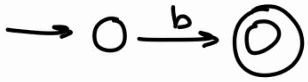
$$= (1+000^*)^* (\epsilon + 000^*)$$

Q.9) a) $ab^*((c+d)+c^*)$

a

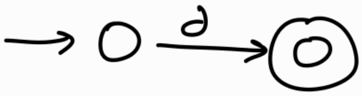


b

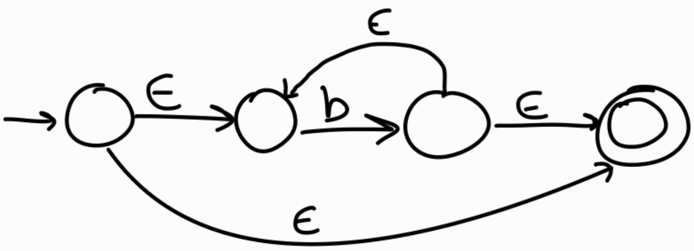


c

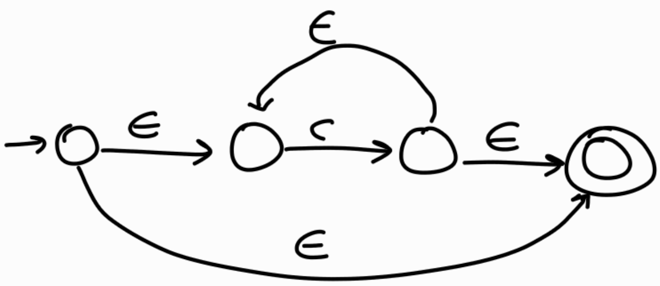
d



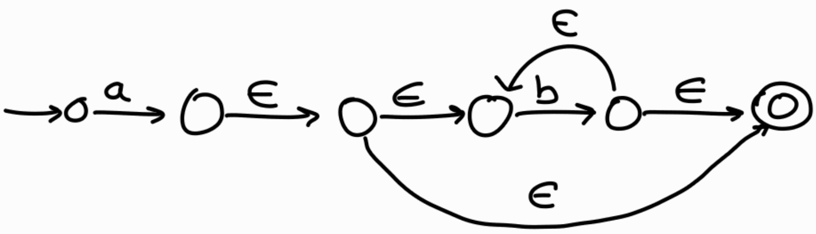
b^*



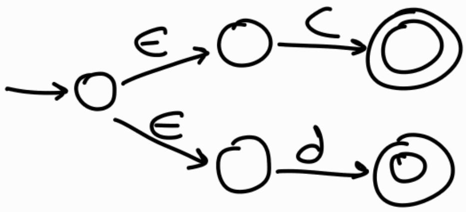
c^*



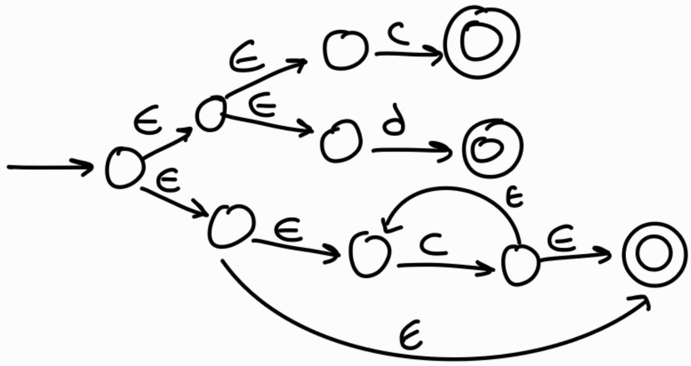
ab^*



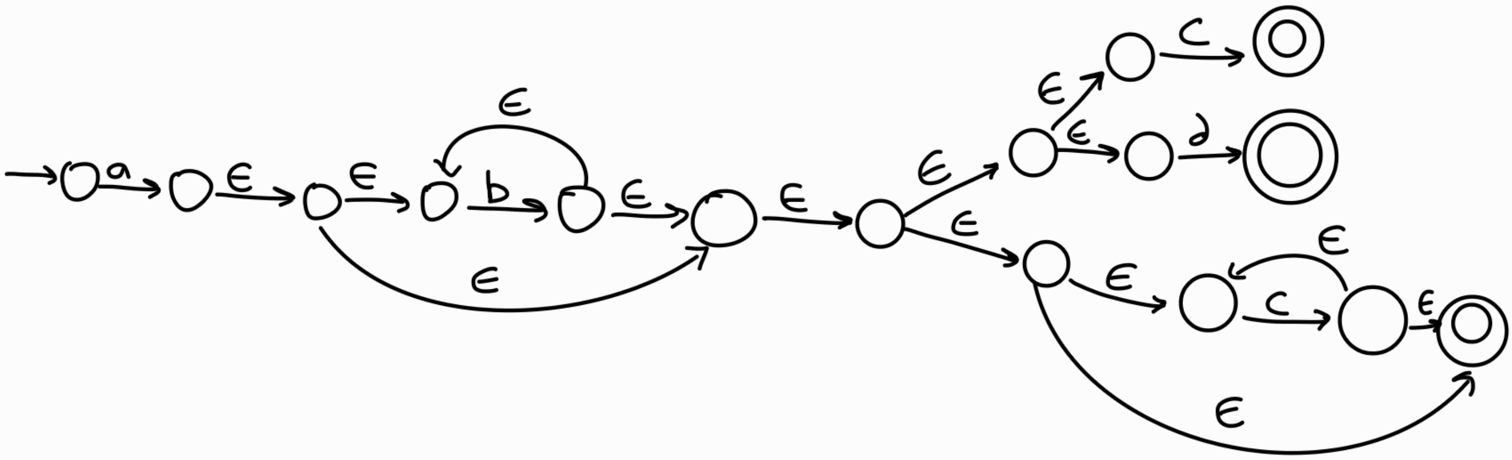
$c+d$



$((c+d)+c^*)$



$ab^*((c+d)+c^*)$



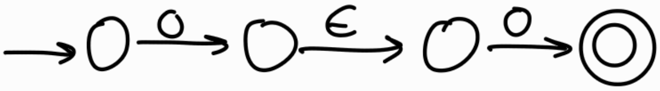
$\underline{ab^*((c+d)+c^*)}$

b) $(0+1)^*(00+11)$

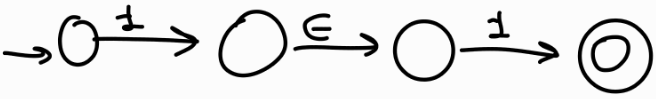
0

1

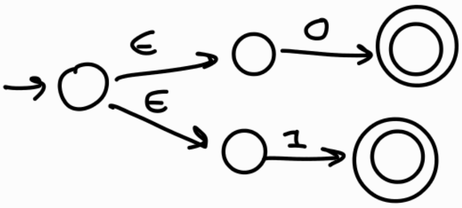
00



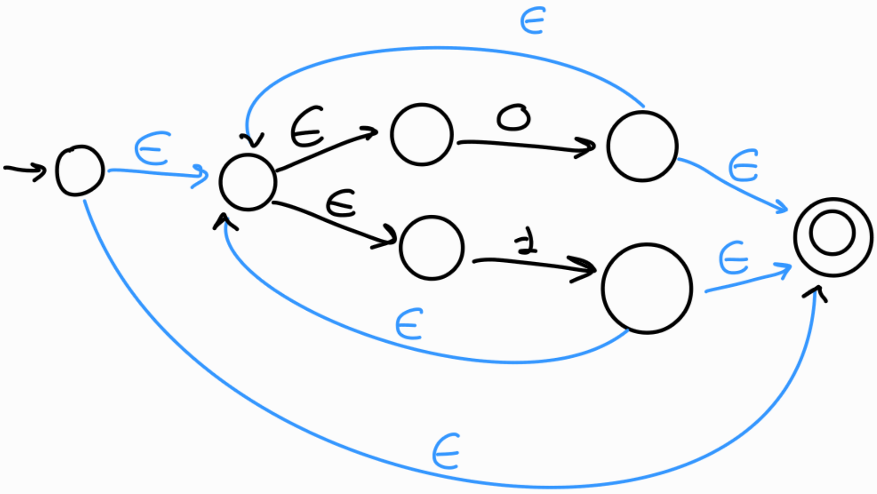
11



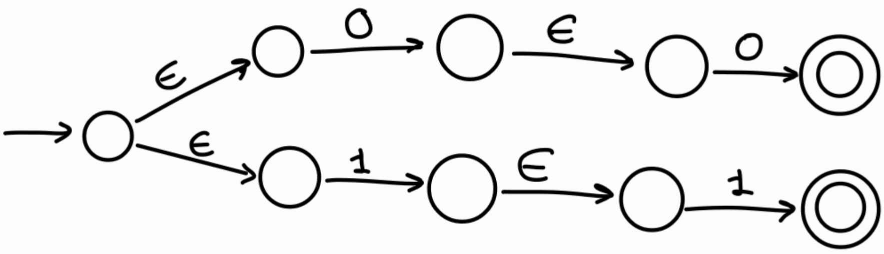
$(0+1)$



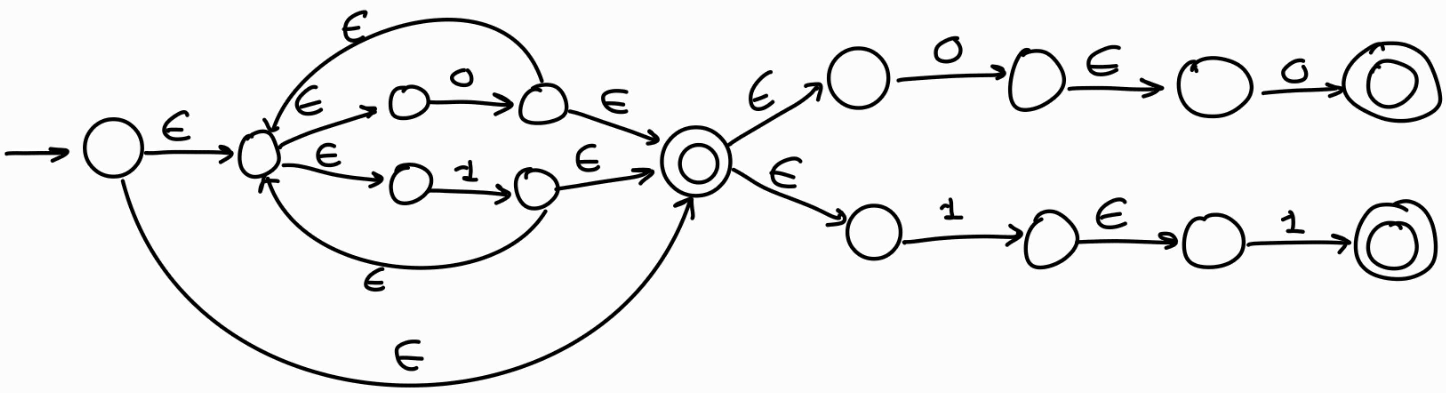
$(0+1)^*$



$(00+11)$



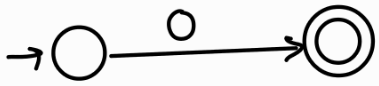
$(0+1)^*(00+11)$



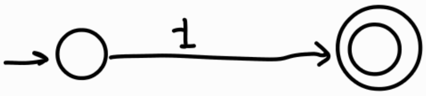
$(0+1)^*(00+11)$

(c) $L = (01 + 2^*)1$

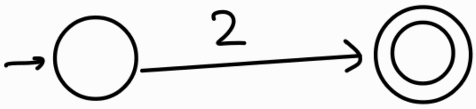
0



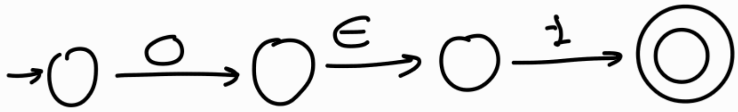
1



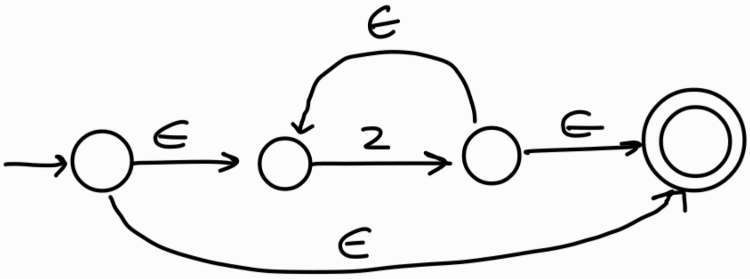
2



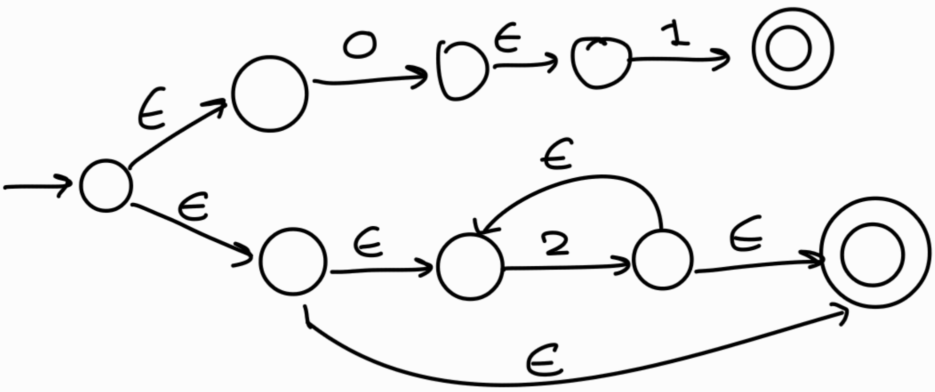
01



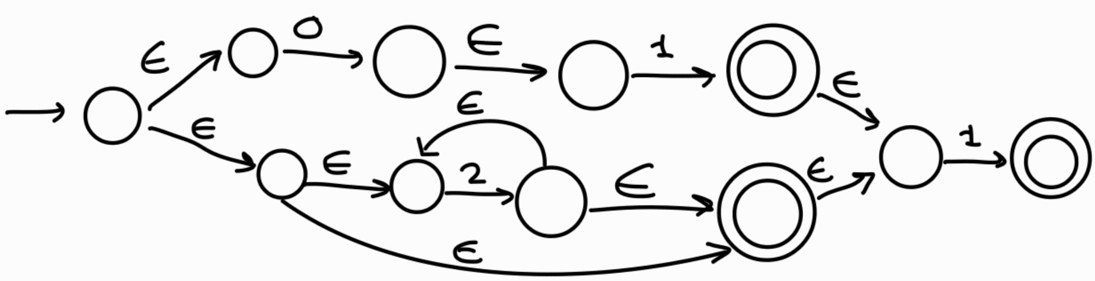
2^*



$(01 + 2^*)$

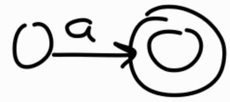


$(01 + 2^*)1$



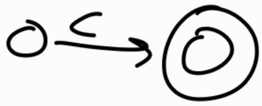
(iv) bc (ab+c)* a

a

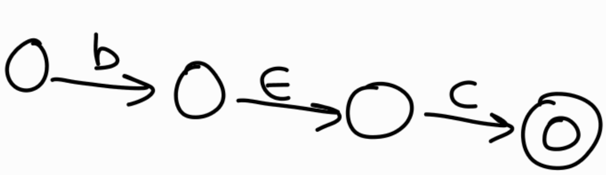


b

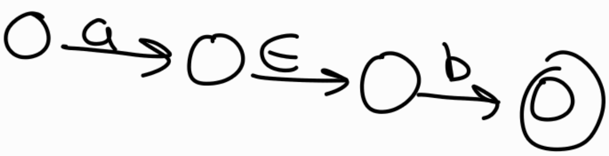
c



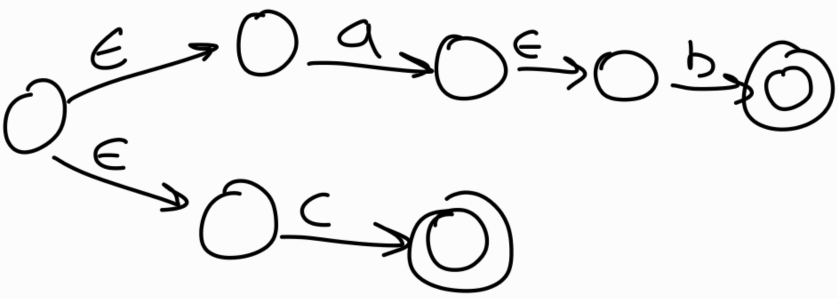
bc



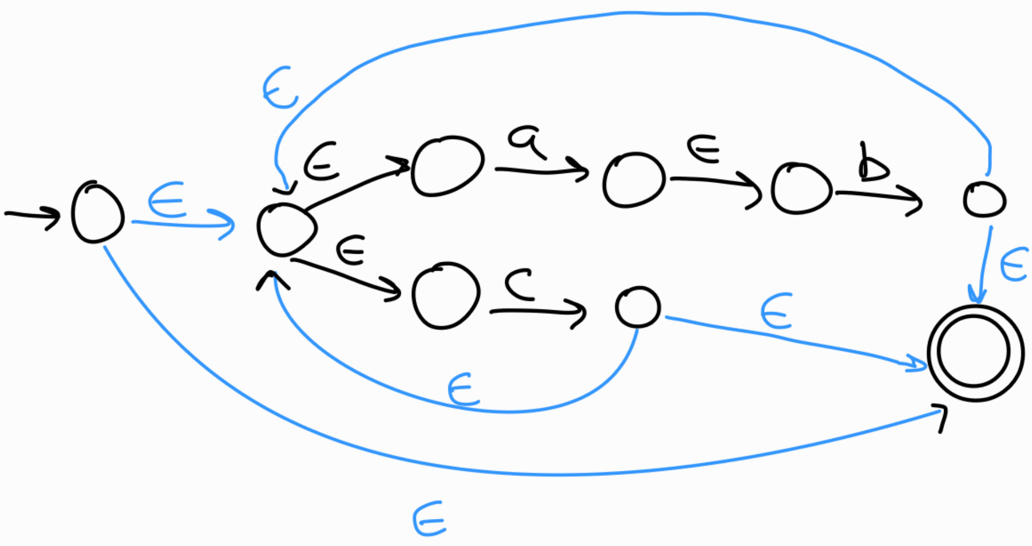
ab



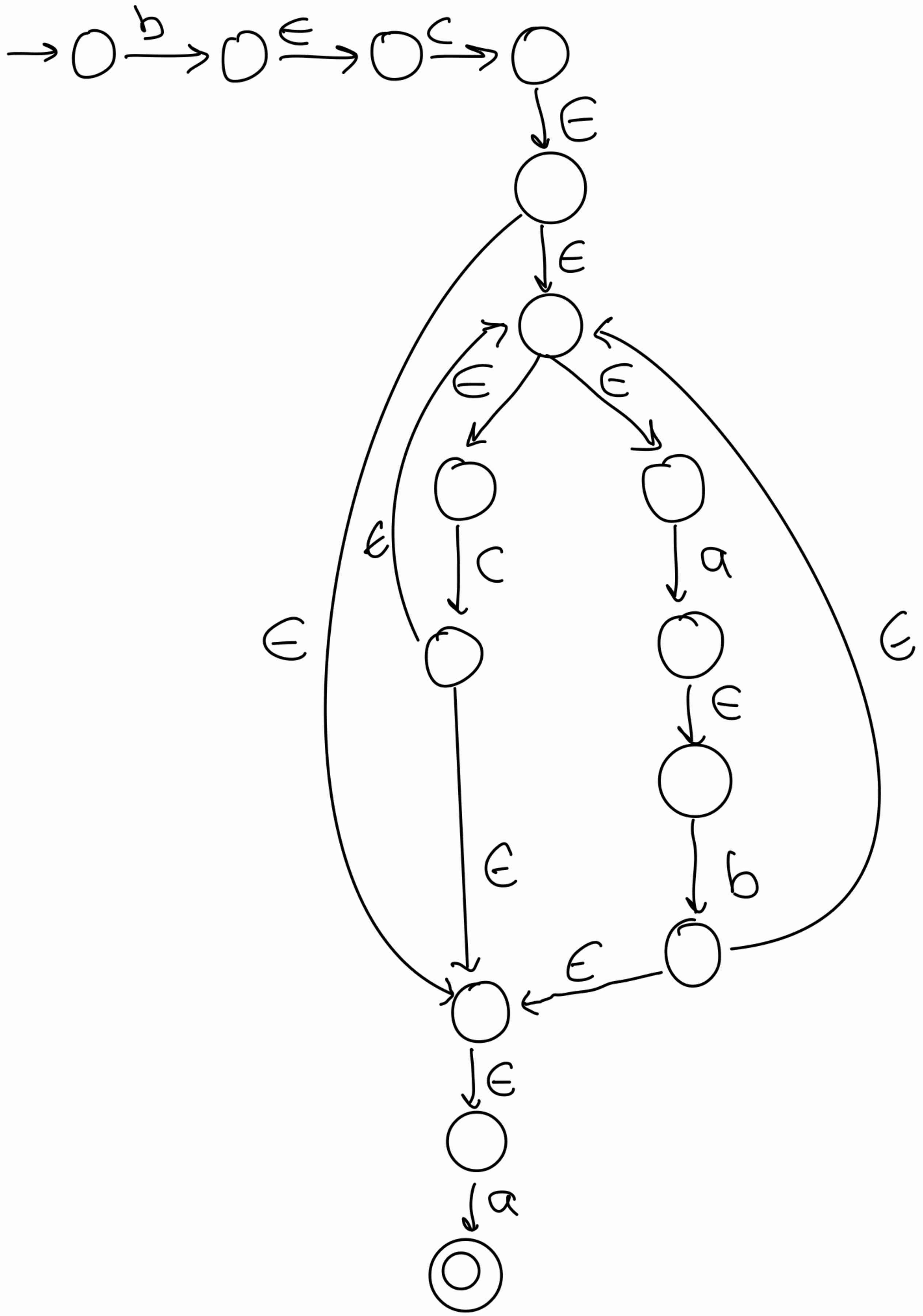
ab+c



(ab+c)*

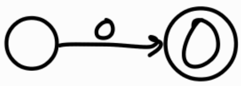


$bc(ab+c)^*a$

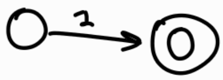


(v) $00(01+10)^*11$

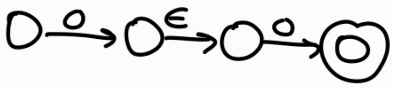
0



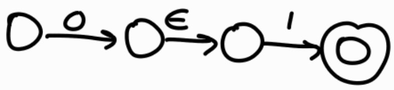
1



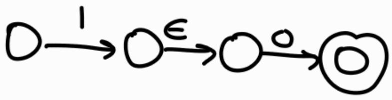
00



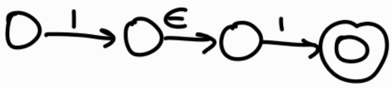
01



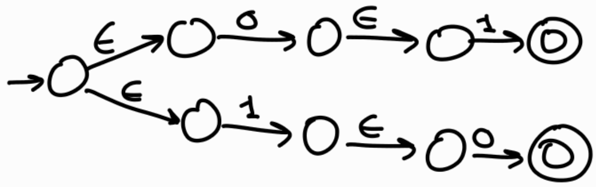
10



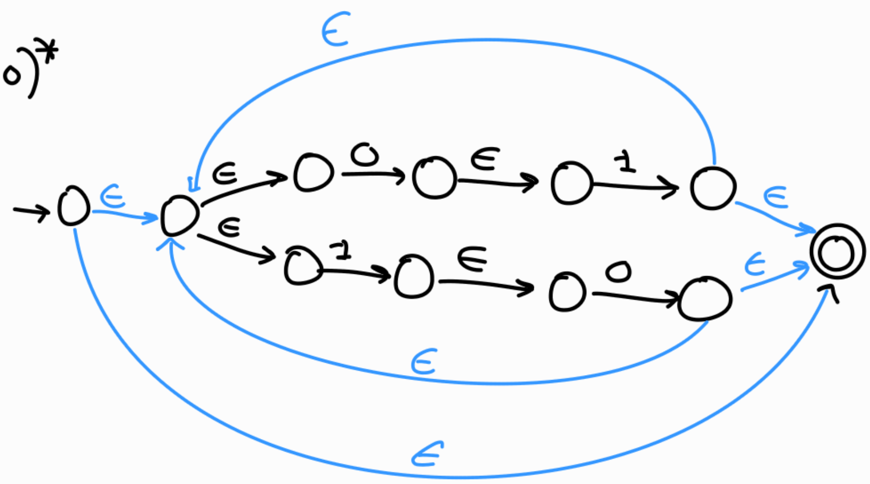
11



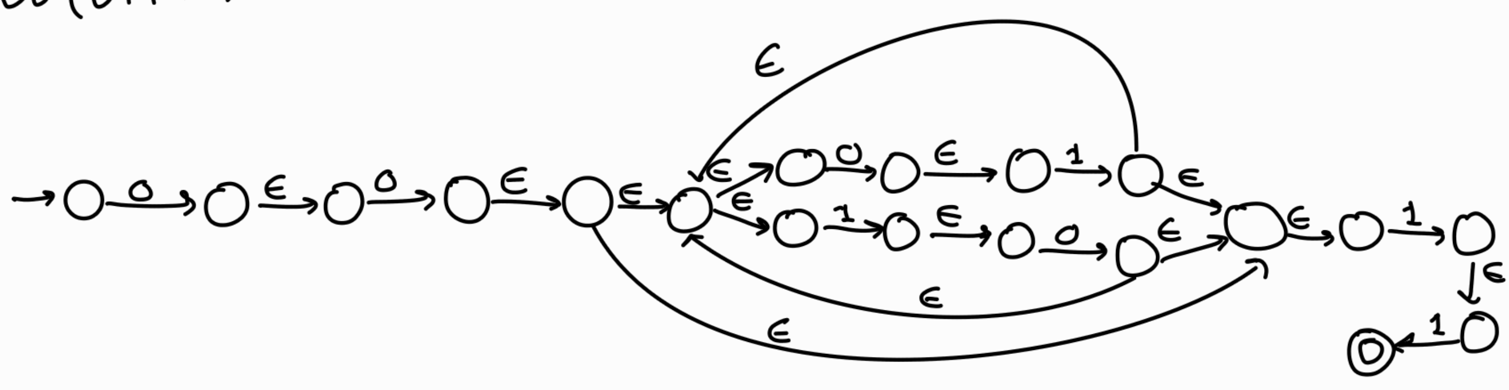
$(01+10)$



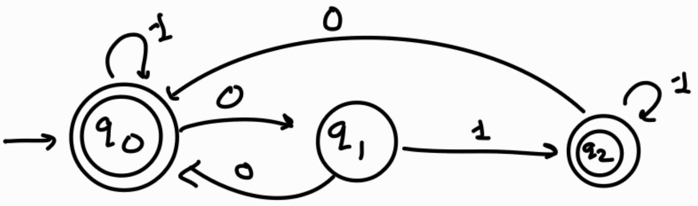
$(01+10)^*$



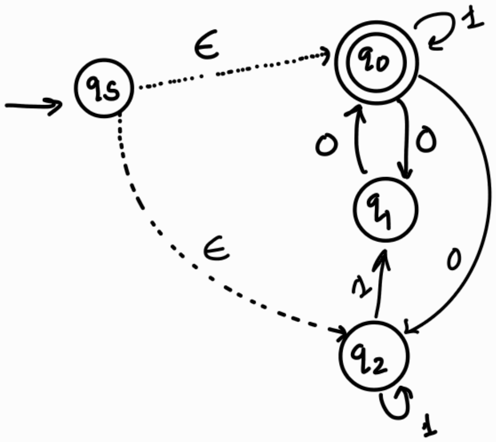
$00(01+10)^*11$



(10) Consider DFA for L. Find FA for L^R



M for L



M for L^R

$q_0 = \epsilon + q_0 1 + q_1 0 + q_2 0$

$q_1 = q_0 0$

$q_2 = q_1 1 + q_2 1$

$q_2 = q_1 1 1^*$ ...(1)

$q_2 = q_0 0 1 1^*$ ...(2)

$q_0 = \epsilon + q_0 1 + \underbrace{q_0 0}_{q_1} 0 + \underbrace{q_0 0 1 1^*}_{q_2} 0$

$q_0 = \epsilon + q_0 (1 + 00 + 011^* 0)$

$q_0 = \epsilon (1 + 00 + 011^* 0)^*$ ....(3)

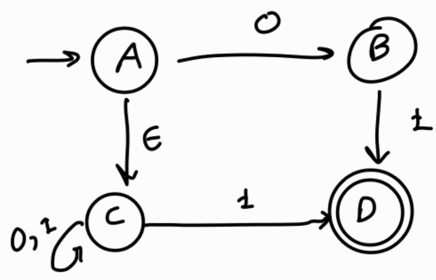
$q_2 = (1 + 00 + 011^* 0)^* 011^*$

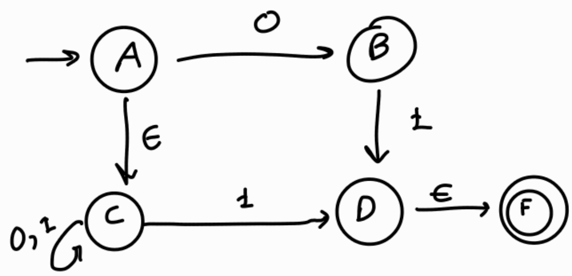
$\therefore$ Regex $= ((1+00+011^*0)^*$
$+ (1+00+011^*0)^* 011^*)$

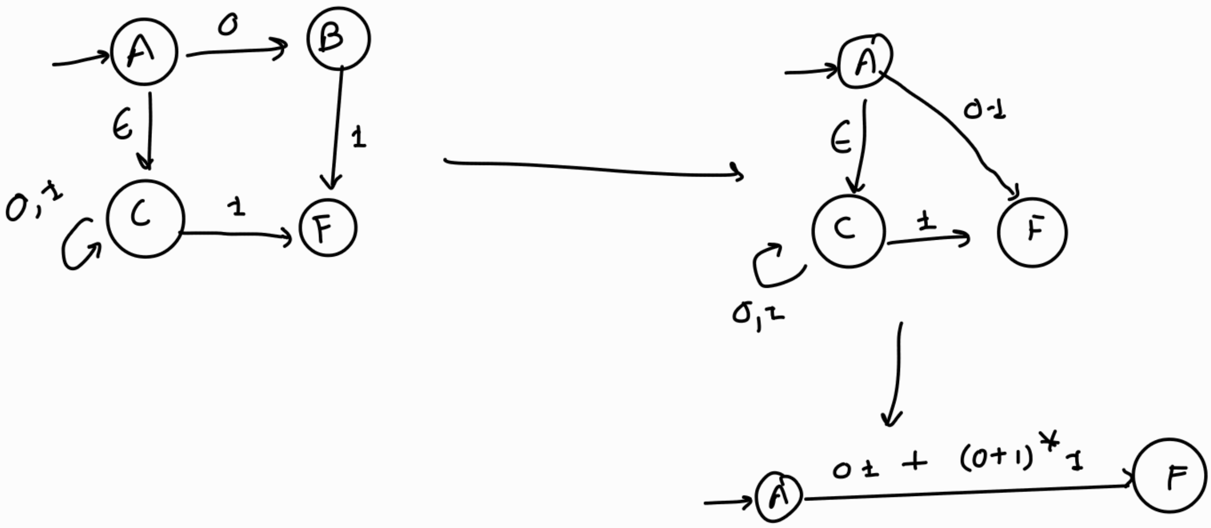
Regex $= \underline{(1+00+011^*0)^* (\epsilon + 011^*)}$

8 Check if Ambiguous ! Remove Ambiguity.

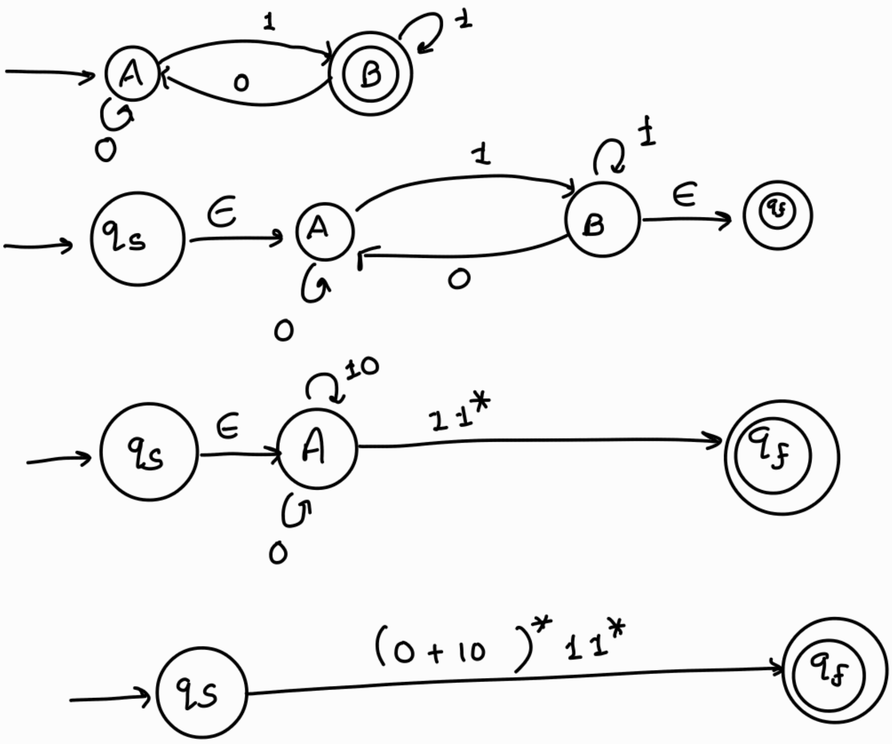
Q.12] a] Find Regex :



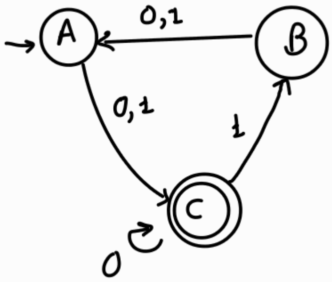


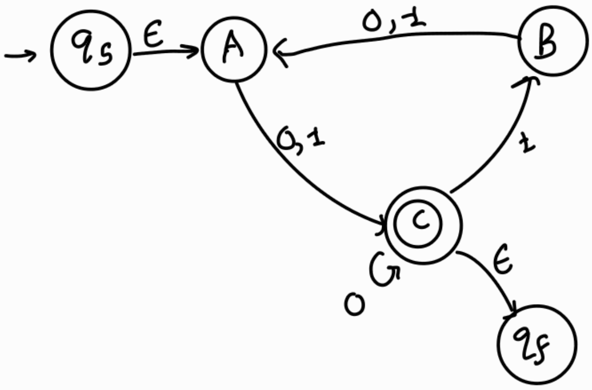


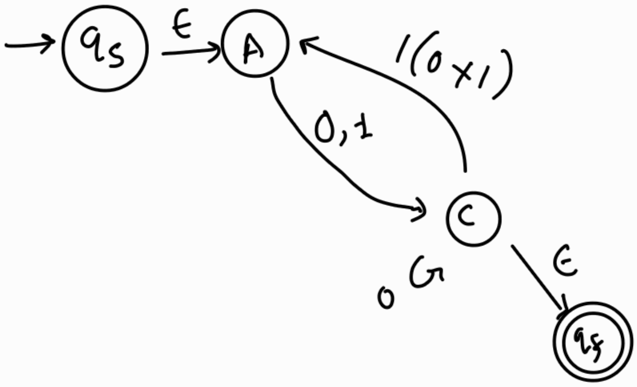
②



3

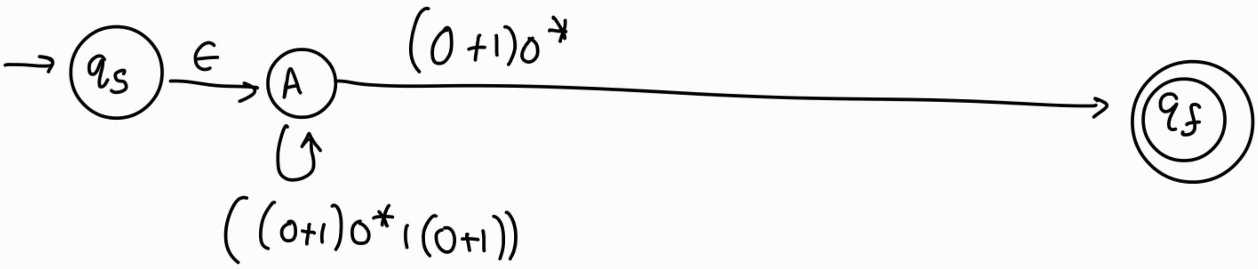


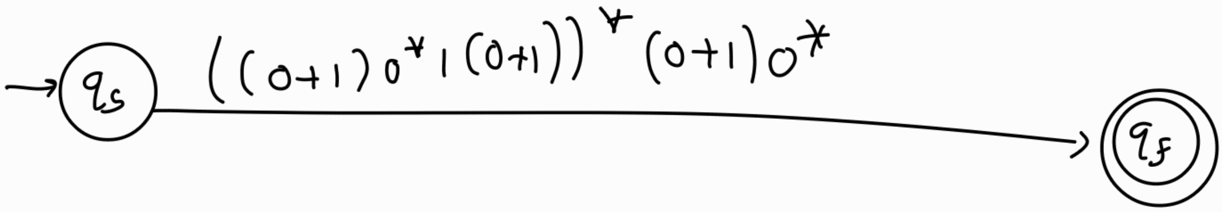




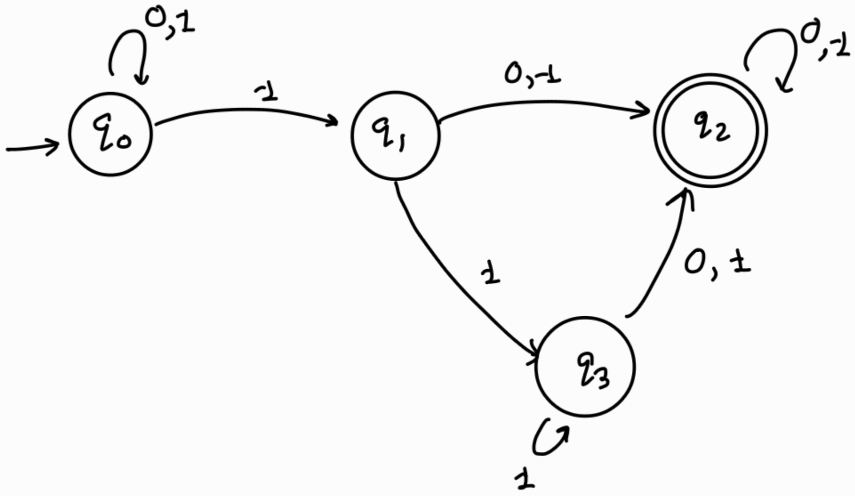
Remove B

Remove C

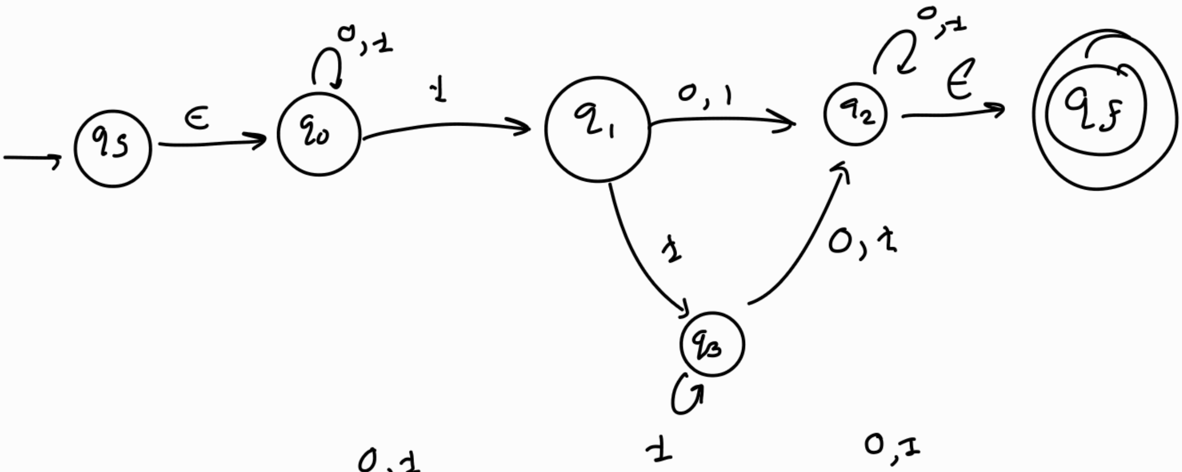




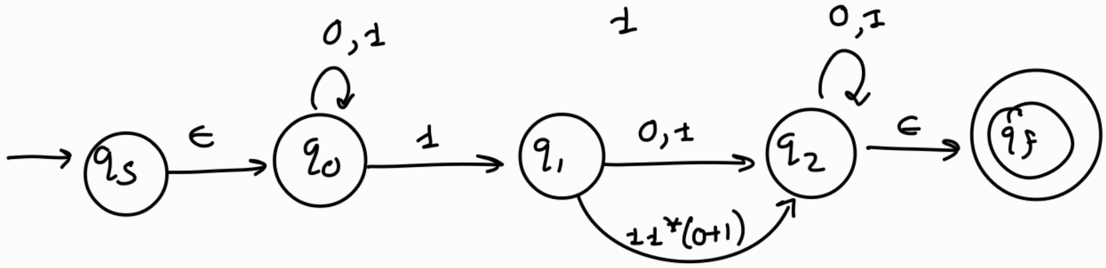
④



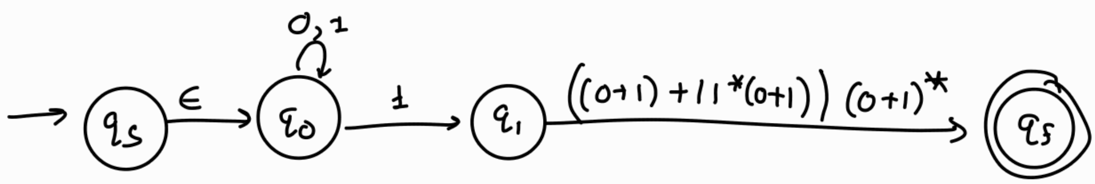
0,1
1
0,1
0,1
1
0, 1
1
q0
q1
q2
q3



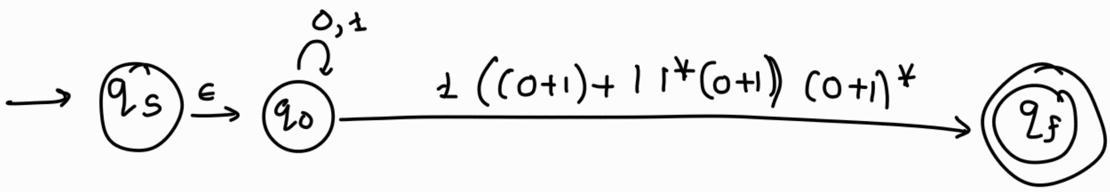
qs
∈
q0
0,1
1
q1
0, 1
q2
0,1
∈
qf
1
0, 1
q3
1



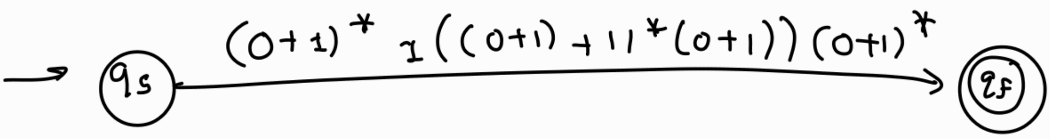
qs
∈
q0
0, 1
1
q1
0, 1
q2
0, 1
∈
qf
11*(0+1)



qs
∈
q0
0, 1
1
q1
((0+1) + 11*(0+1)) (0+1)*
qf

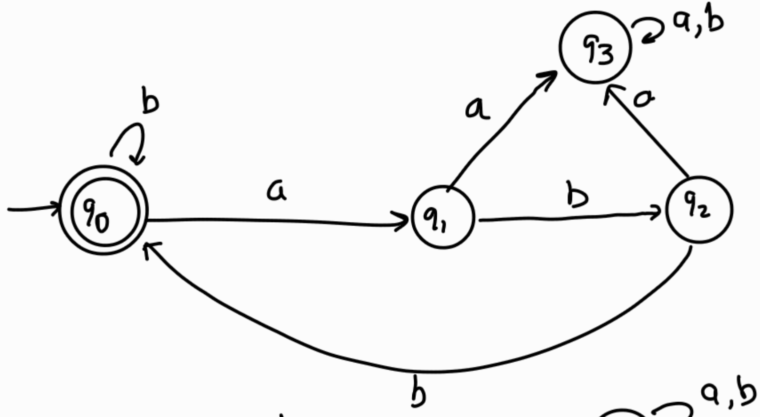


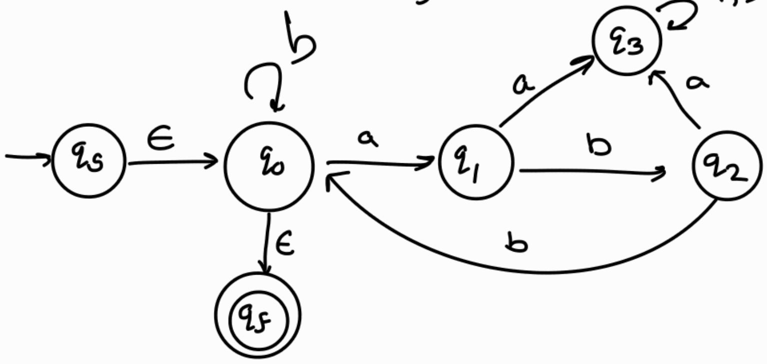
qs
∈
q0
0, 1
1 ((0+1) + 11*(0+1)) (0+1)*
qf



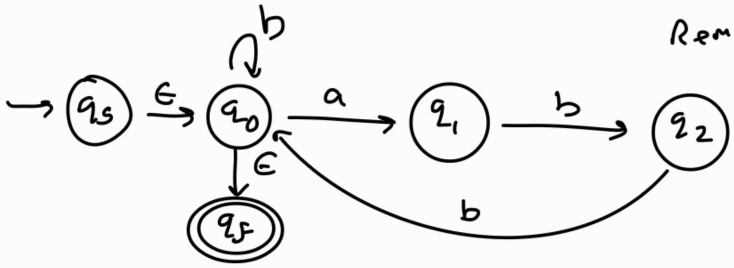
qs
(0+1)* 1 ((0+1) + 11*(0+1)) (0+1)*
qf

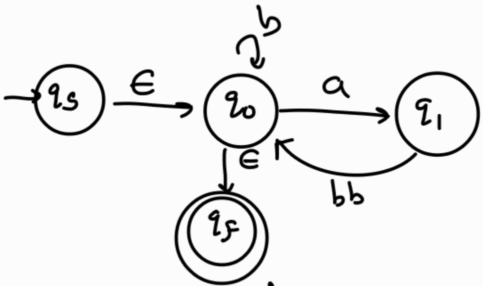
(4)

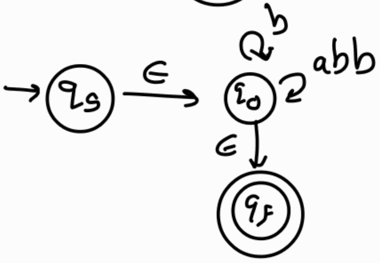


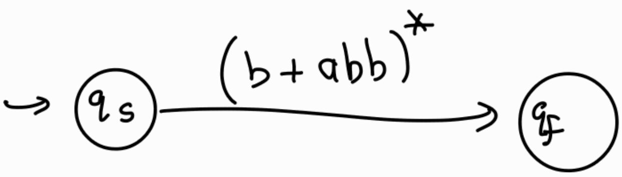


Remove q_3









$q_0 = \epsilon + q_0 b + q_2 b$

$q_1 = q_0 a$

$q_2 = q_1 b$

$q_0 = \epsilon + q_0 b + q_1 bb$

$= \epsilon + q_0 b + q_0 abb$

$q_0 = \epsilon + q_0 (b + abb)$

$q_0 = (b+abb)^*$

11.

i) $S \rightarrow aS \mid AB$

$A \rightarrow \epsilon$

$B \rightarrow \epsilon$

$D \rightarrow b$

$S \rightarrow aS \mid a$

$S_1 \rightarrow aS \mid a$

$X \rightarrow a$

P2 :

$S_1 \rightarrow XS \mid a$

$X \rightarrow a$

(ii) $S \rightarrow XY \mid YX \mid XX \mid X \mid Y$

$X \rightarrow 0X \mid 0$

$Y \rightarrow 1Y \mid 1$

$A \rightarrow 0$

$B \rightarrow 1$

$S \rightarrow XY \mid YX \mid XX \mid 0X \mid 0 \mid 1Y \mid 1$

$S \rightarrow XY \mid YX \mid XX \mid AX \mid 0 \mid BY \mid 1$

$X \rightarrow AX \mid 0$

$Y \rightarrow BY \mid 1$

$A \rightarrow 0$

$B \rightarrow 1$

(iii) $S \rightarrow a \mid Xb \mid aYa$

$X \rightarrow Y \mid \epsilon$

$Y \rightarrow b \mid X$

Remove Null productions

$S \rightarrow a \mid Yb \mid YbY$

$Y \rightarrow b$

$R \rightarrow b$

$S \rightarrow a \mid Xb \mid b \mid aYa \mid aa$

$X \rightarrow b$

$C \rightarrow a$

$S \rightarrow a \mid XX \mid b \mid RX \mid CC$

$R \rightarrow CX$

$S \rightarrow a \mid YR \mid YRY$

$Y \rightarrow b$

$R \rightarrow b$

$\Longrightarrow$

$S \rightarrow a \mid YR \mid CY$

$Y \rightarrow b$

$R \rightarrow b$

$C \rightarrow YR$

$S \rightarrow aXbX$

$X \rightarrow aY \mid bY \mid \epsilon$

$Y \rightarrow X \mid c$

① Remove ϵ productions

$S \rightarrow aXbX \mid aXb \mid abX \mid ab$

$X \rightarrow aY \mid bY \mid a \mid b$

$Y \rightarrow c$

$S \rightarrow aXbX \mid aXb \mid abX \mid ab$

$A \rightarrow a$

$B \rightarrow b$

$X \rightarrow AY \mid BY \mid a \mid b$

$Y \rightarrow c$

$S \to AXbX \mid AXB \mid ABX \mid AB$

$A \to a$

$B \to b$

$X \to AY \mid BY \mid a \mid b$

$Y \to c$

$R_1 \to AX$

$R_2 \to R_1B$

$R_3 \to R_2X$

$R_4 \to AB$

$R_5 \to R_4X$

$S \to R_2X \mid R_1B \mid R_4X \mid AB$

$A \to a$

$B \to b$

$X \to AY \mid BY \mid a \mid b$

$Y \to c$

$\{Q, \Sigma, \delta, q_0, F\}$

$\Gamma \quad z_0$

$L = \{0^n 1^m 2^m 3^n \mid n >= 1, m >= 1\}$