$RL \subseteq CFL$

• every regular language is a CFL, but not vice versa

To prove: Every regular is context free Grammar.

① Context free languages:

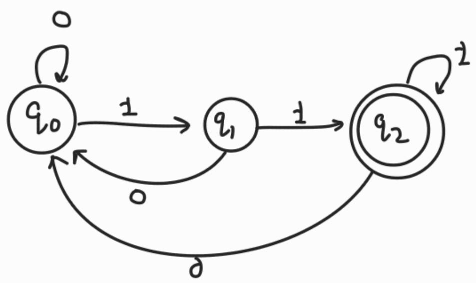
V - finite set of variables

R - finite set of rules $\longrightarrow$ $R_i \in (\Sigma \cup V)^*$

T - finite set of terminals

S - Start Variable. $\longrightarrow$ $S \in V$

DFA:



$V_0 \longrightarrow q_0$

$V_1 \longrightarrow q_1$

$V_2 \longrightarrow q_2$

$$V_0 \longrightarrow 0V_0 \mid 1V_1$$

$$V_1 \longrightarrow 0V_0 \mid 1V_2$$

$$V_2 \longrightarrow 0V_0 \mid 1V_2 \mid \epsilon$$

str = "10011"

$V_0 \Rightarrow 1V_1$

$\Rightarrow 10V_0$

$\Rightarrow 100V_0$

$\Rightarrow 1001V_1$

$\Rightarrow 10011V_2 \Rightarrow 10011\epsilon \Rightarrow 10011$

* Grammars are more powerful than DFA.

Push down automata:
- Same as DFA but has access to infinite stack.

$$\boxed{PDA = DFA + stack}$$

CFL:

$0^n 1^n$ ✓

$0^n 1^n 0^n$ ✗

$0^n 1^n 0^n 1^n$ ✗ $\because$ it will accept $0^5 1^5 0^4 1^4$ which is not part of $0^n 1^n$

$0^n 1^n 0^m 1^m$ ✓

---

1. L_0 is a regular language (which is infinite)
2. M_0 is a DFA such that $L(M_0) = L_0$
3. M_0 has finite number of states p.
4. List strings of L_0 in increasing order of length

$\omega \in L_0$	$\lvert\omega\rvert$
ω_0	0
ω_1	1
ω_2	2
⋮	⋮
ω_n	n

Pumping Lemma for regular languages.

If A is a Context Free Language, then there exists a number P such that for all strings S in A of length at least P, S can be divided into 5 pieces: u, v, x, y, z satisfying the following 3 conditions.

1. $uv^i xy^i z \in A$ } → Pump splitted string.
2. $|vy| > 0$
3. $|vxy| \leq P$

} → to decide where to split the string

We talk about height of parse tree

for DFA value of $P = |Q|$

for CFG value of P is calculated by fixing a parse tree.

Steps:

① CFL is infinite
② It has a CFG
③ CFG is finite
④ list strings in increasing height of parse tree

- A point will occur when height of parse tree will be greater than number of variables. This indicates a loop in the parse tree.

Q. $0^n 1^n 2^n$

1. Assume $L_5 = 0^n 1^n 2^n$ is context free language.

2. $\therefore$ L_5 has a CFG.

$\therefore$ There exists a P (Pumping Constant)

string $S \Rightarrow$ $S = 0^P 1^P 2^P$

$= uv^i x y^i z$

Case 1: v and y contain all 0's

$0^{P+K} 1^P 2^P \notin L_5$

Case 2: v and y contains all 1's

$0^P 1^{P+K} 2^P \notin L_5$

Case 3: v and y contain all 2's

$0^P 1^P 2^{P+K} \notin {}_5$

$\therefore$ Contradiction! L_5 is not CFL.