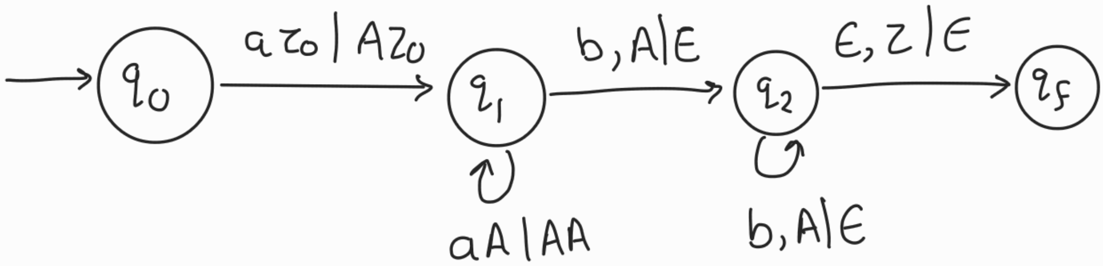
QB3

what is $L(G)$ and $\underline{N(P)}$?

1) Construct DPDA for:

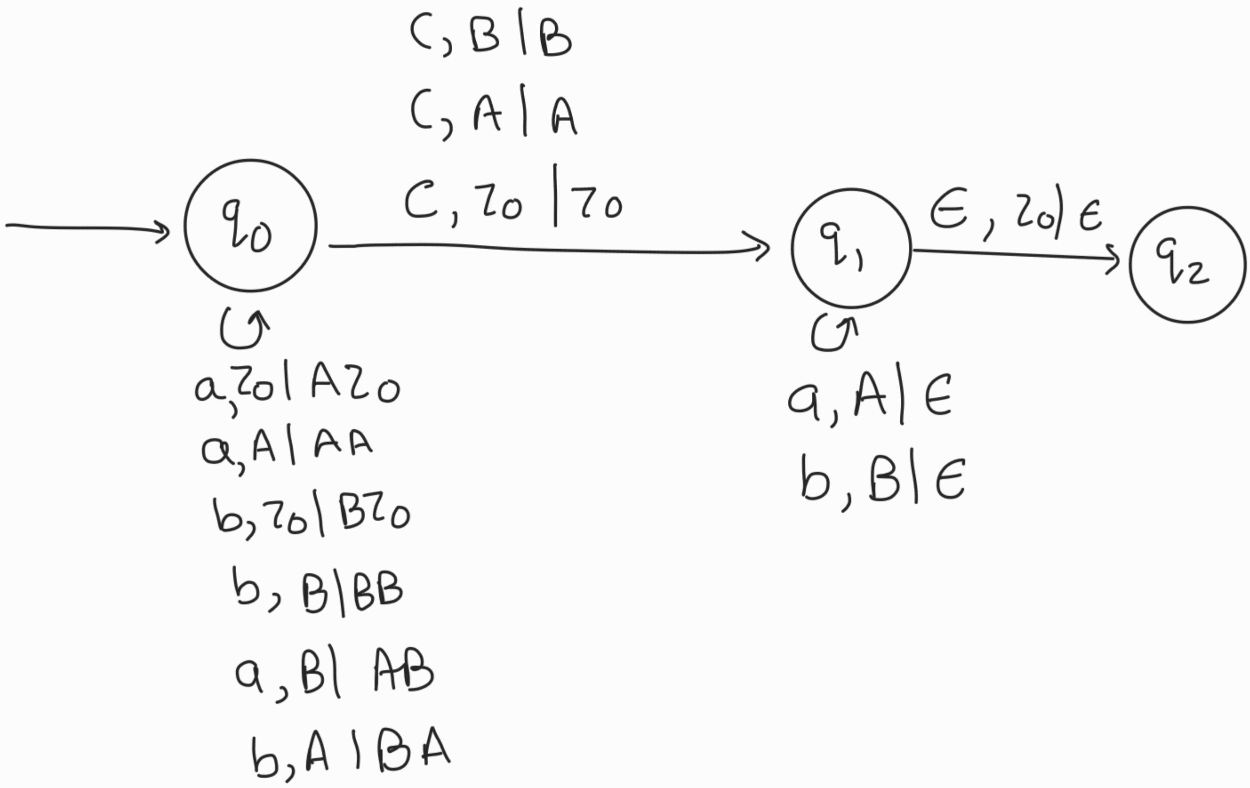
a) $L = \{ a^n b^n \mid n > 0 \}$



$$G = \{ \underbrace{\{q_0, q_1, q_2, q_3\}}_{Q}, \underbrace{\{a, b\}}_{\Sigma}, \underbrace{\{A, B, z_0\}}_{\Gamma}, \delta, q_0, z_0, \underset{?}{\underline{\underline{\{\emptyset\}}}} \}$$

b. $L = \{ wcw^r \mid w \in \{a,b\}^* \}$

aabacabaa



M_2 for L_2

$G = \{$

$\{q_0, q_1, q_2\},$

$\{a, b\},$

$\{Z_0, A, B\},$

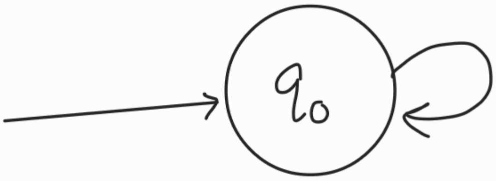
$\delta \quad ,$

$q_0,$

$Z_0,$

$\} \quad \{\emptyset\}$

(c) $L = \{ n_a(\omega) = n_b(\omega) \mid \omega \in \{a,b\}^* \}$



$a, Z_0 \mid A$

$b, Z_0 \mid B$

$a, A \mid AA$

$b, B \mid BB$

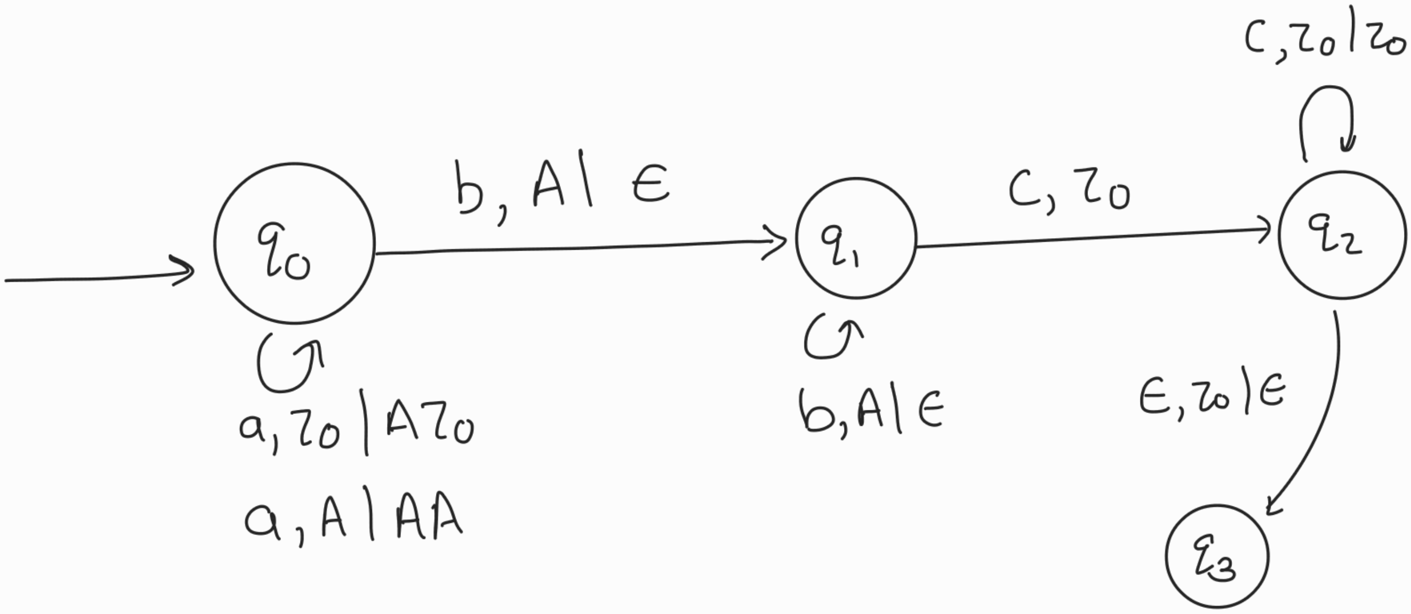
$a, B \mid \epsilon$

$b, A \mid \epsilon$

$\epsilon, Z_0 \mid \epsilon$

$G = \{ \{q_0\}, \{a,b\}, \{A, B, Z_0\}, \delta,$
$\{q_0\}, \{Z_0\}, \varnothing \}$

(d) $a^n b^n c^m \mid n > 0, m > 0$

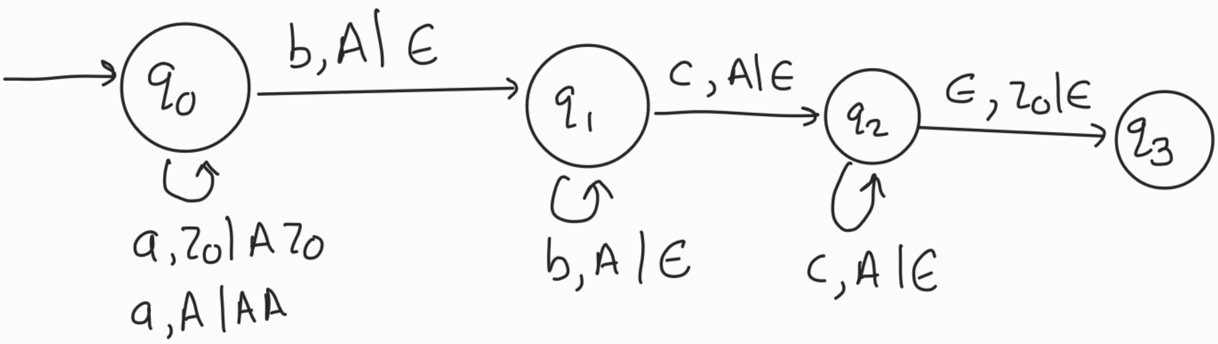


$G = \{ \{q_0, q_1, q_2, q_3\}, \{a, b\}, \{A, B, Z_0\}, \delta, \{q_0\}, \{Z_0\}, \{\emptyset\} \}$

---

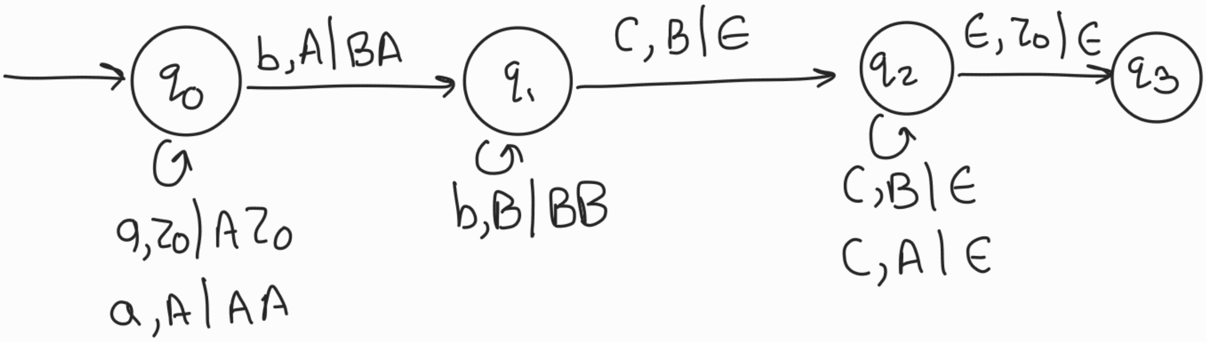
(5) $L = \{ a^{n+m} b^n c^m \mid n > 0, m > 0 \}$

$L = \{ a^m a^n b^n c^m \mid n > 0, m > 0 \}$



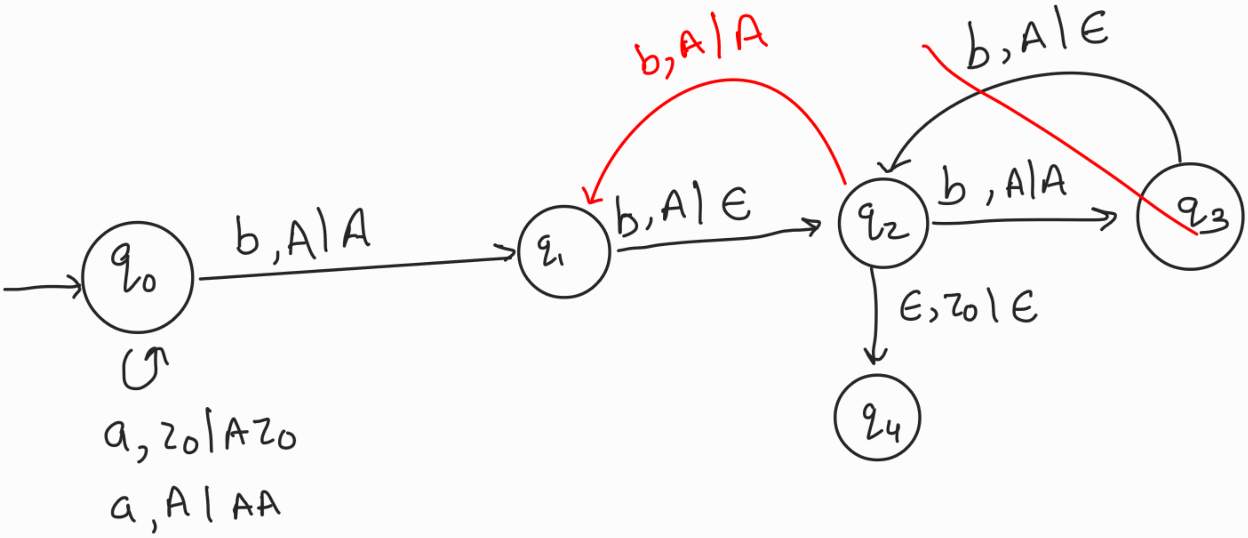
$G = \{ \{q_0, q_1, q_2, q_3\}, \{a, b\}, \{A, B, Z_0\}, \delta, \{q_0\}, \{Z_0\}, \emptyset \}$

(f) $L = \{a^n b^m c^{m+n} \mid n>0 \;, m>0\}$



$G = \{ \{q_0, q_1, q_2, q_3\}, \{a, b\}, \{A, B, Z_0\}, \delta, q_0, Z_0, \{\emptyset\}\}$

(g) $L = \{a^n b^{2n} \mid n > 0\}$

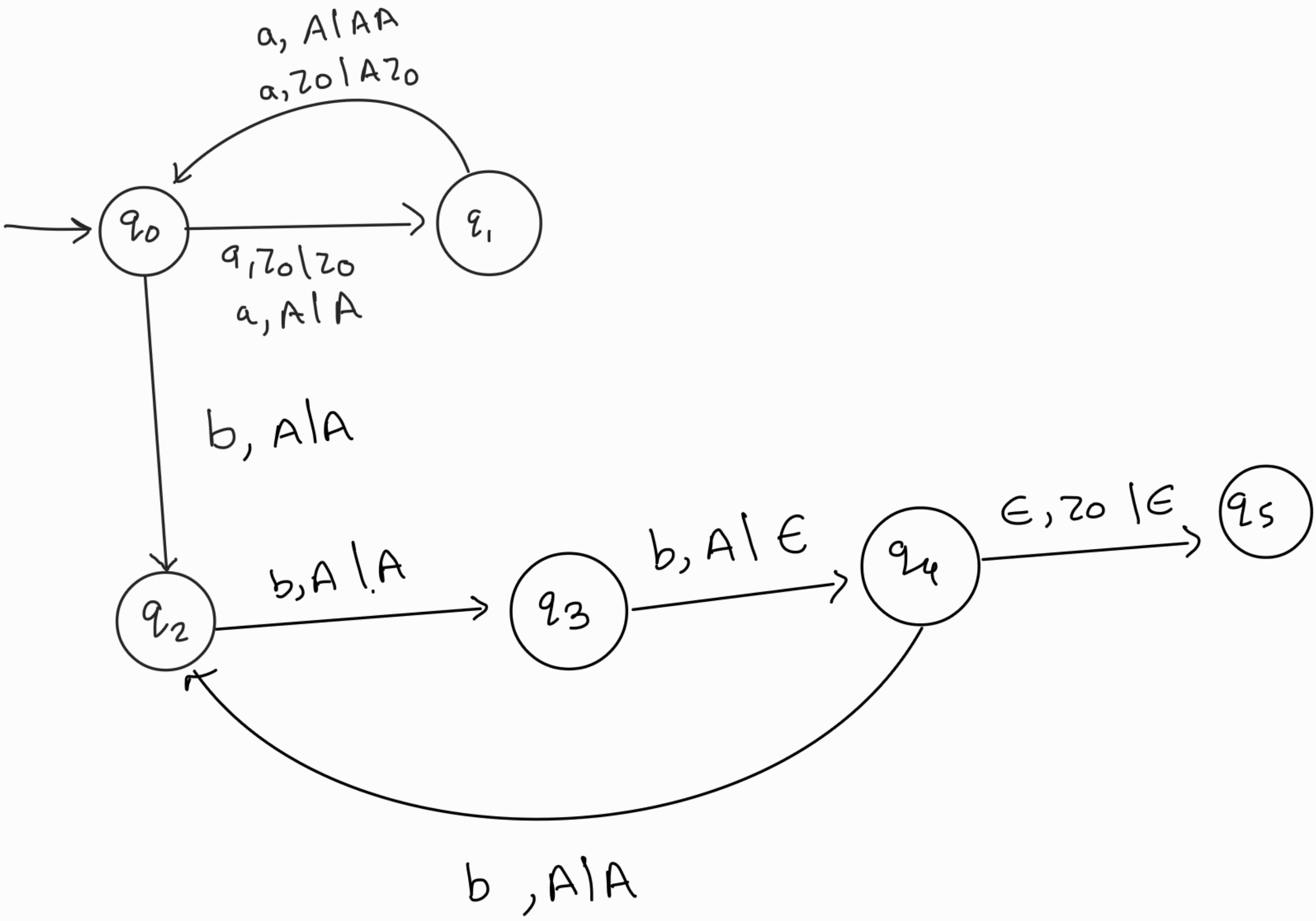


$G = \{ \; \{q_0, q_1, q_2, q_3, q_4\},$

$\{a, b\}, \{A, B, Z_0\},$

$\delta, q_0, Z_0, \{\emptyset\}\}$

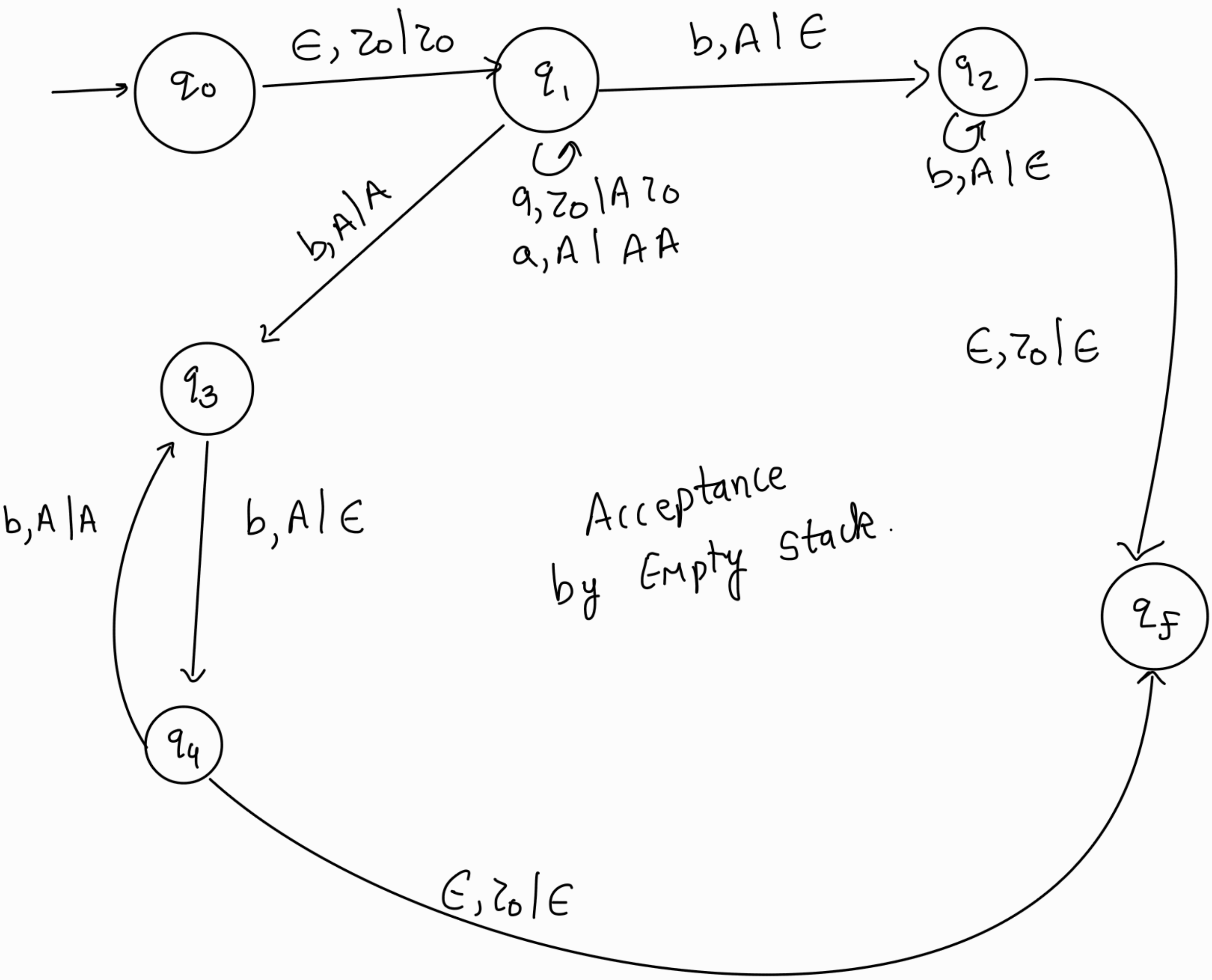
(h) $L = \{ a^{2n} b^{3n} \mid n > 0 \}$



$G = \{ \{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \{A, B, Z_0\},$
$\delta, q_0, Z_0, \{\phi\} \}$

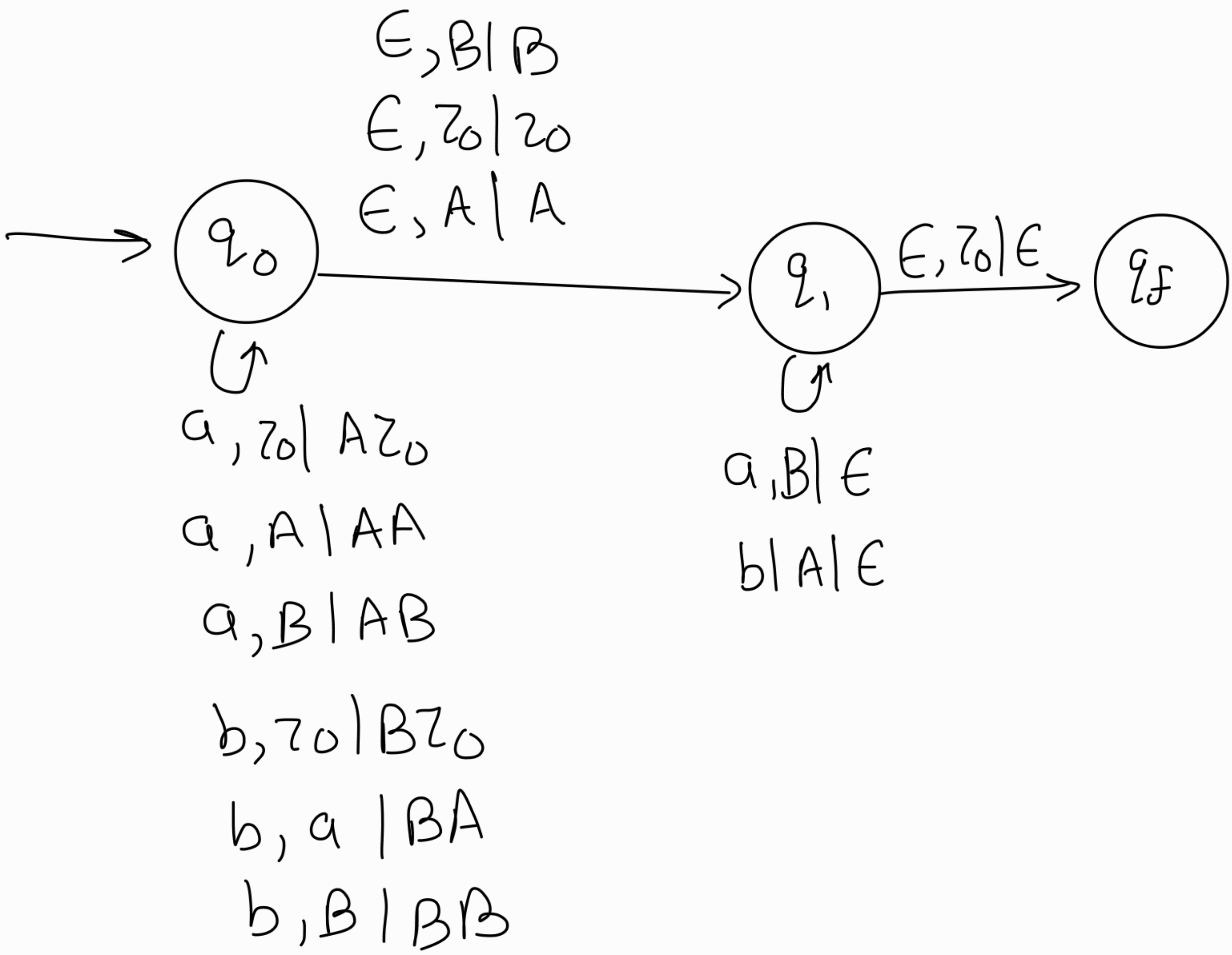
2. Construct PDA. Check if DPDA.

a. $L = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$

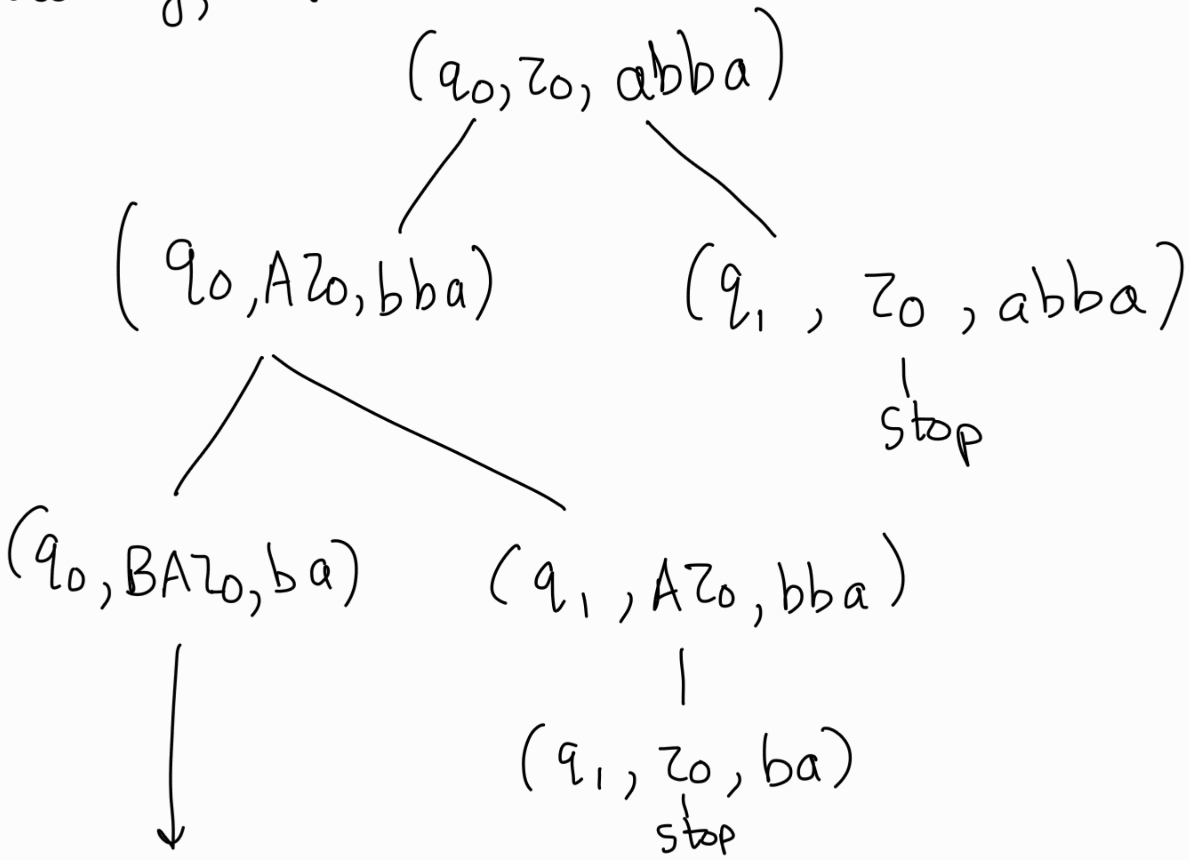


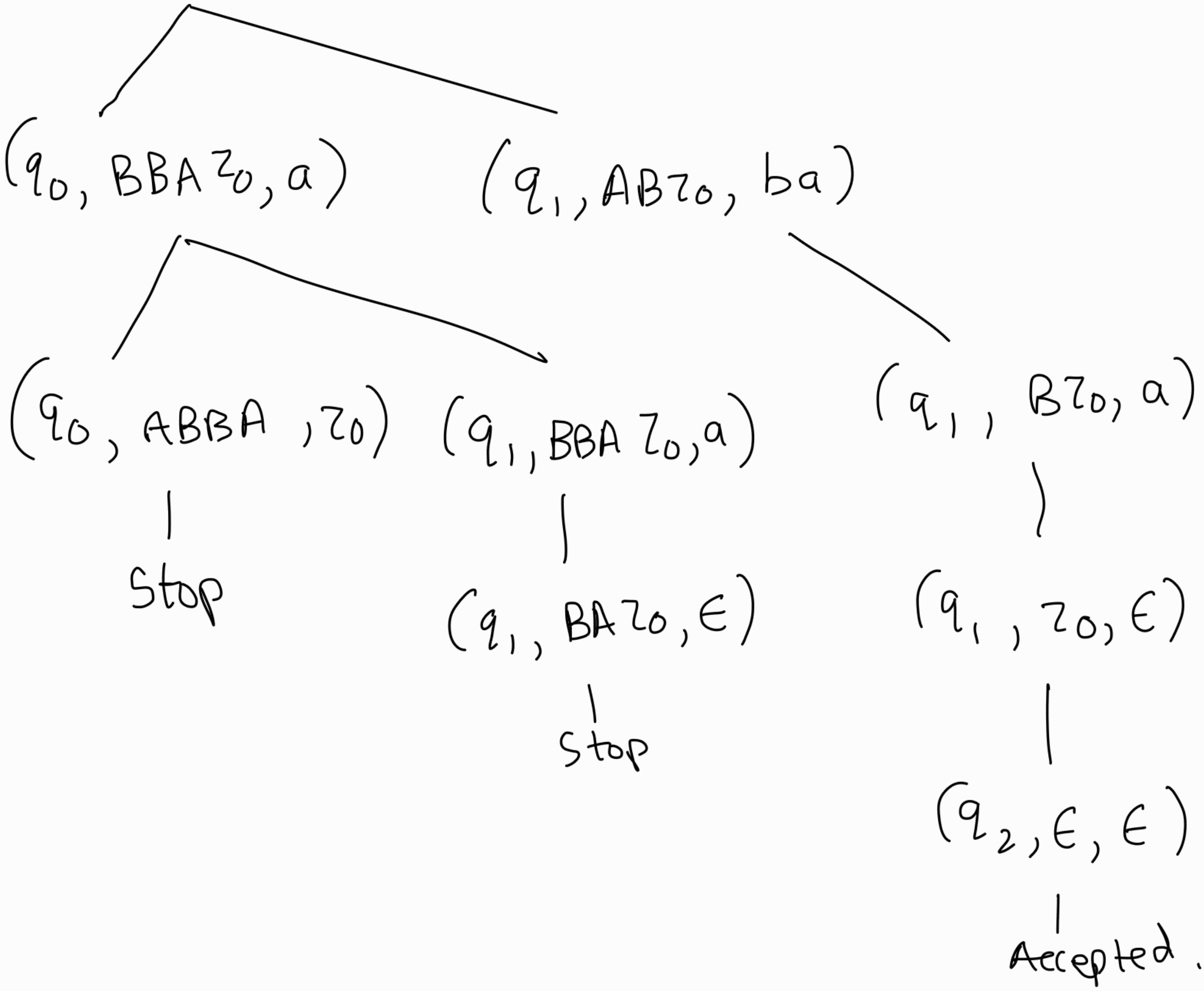
The above PDA is not a DPDA as we cannot determine the transition at 'q_1'.

b. $\{ww^R \mid w = \{a,b\}^*\}$



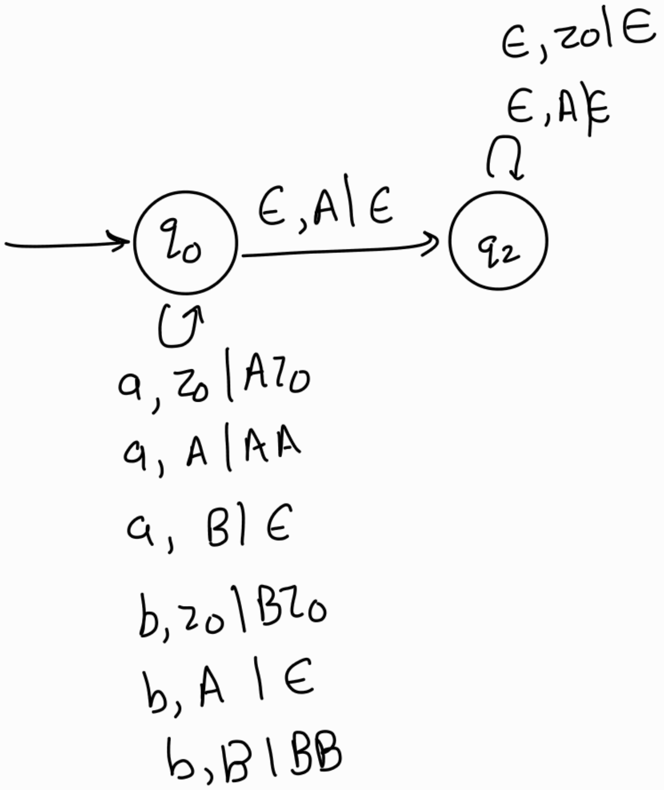
deriving, abba





$(q_0, BBAZ_0, a)$
(q_1, ABZ_0, ba)
$(q_0, ABBA, Z_0)$
$(q_1, BBAZ_0, a)$
(q_1, BZ_0, a)
Stop
(q_1, BAZ_0, ϵ)
(q_1, Z_0, ϵ)
Stop
$(q_2, \epsilon, \epsilon)$
Accepted.

ⓒ $\{n_a(\omega) > n_b(\omega) \mid \omega \in (a,b)^*\}$



abbaa

$$(q_0, z_0, abbaa) \vdash (q_0, Az_0, bbaa) \vdash (q_0, z_0, baa)$$

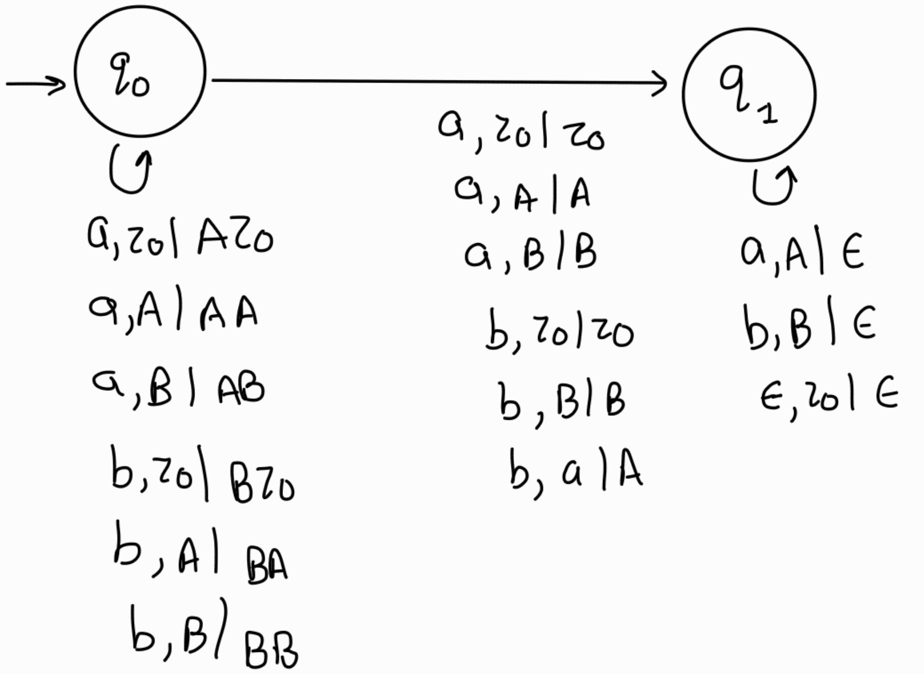
$$\vdash (q_0, Bz_0, aa) \vdash (q_0, z_0, a)$$

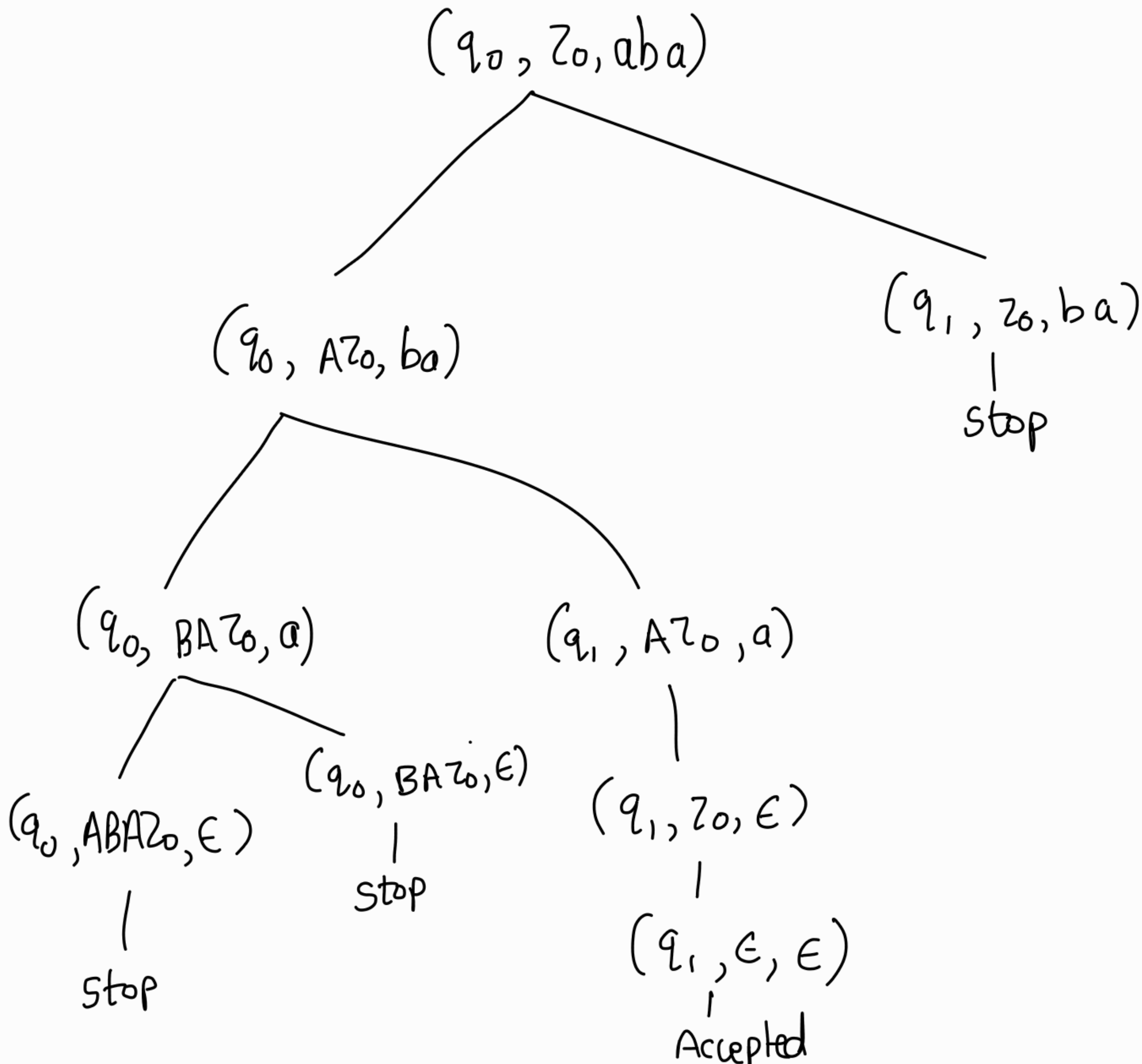
$$\vdash (q_0, Az_0, \epsilon) \vdash (q_1, z_0, \epsilon)$$

$$\vdash (q_1, \epsilon, \epsilon) \text{ (Accepted)}$$

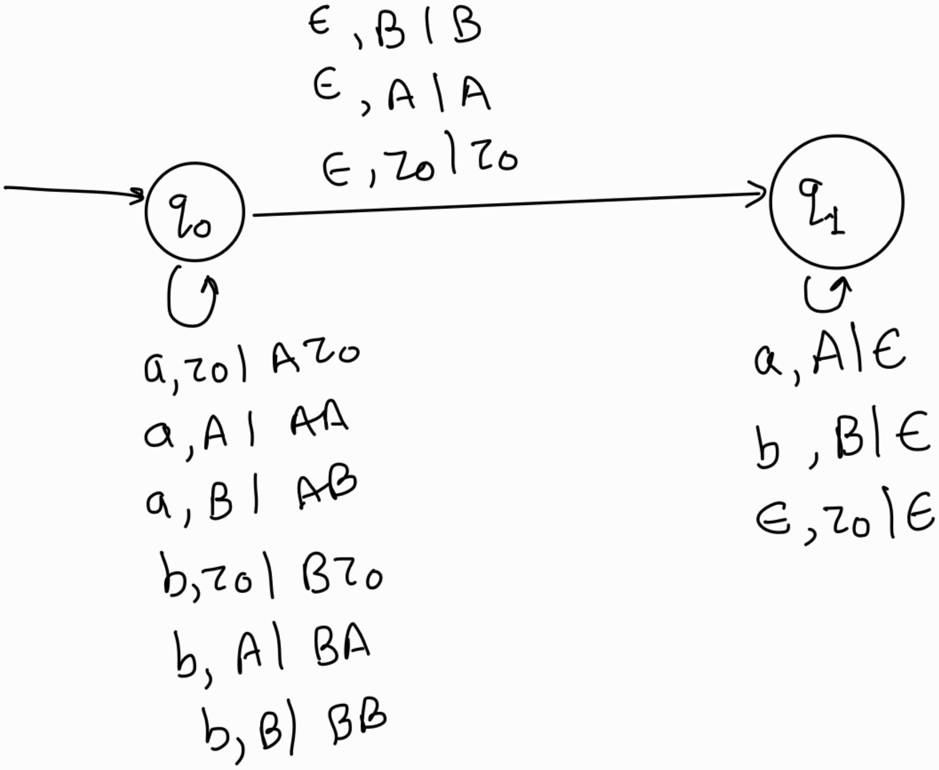
(d) $L \{ w \in \{0,1\}^* \mid w = w^R$ and length of w is odd.

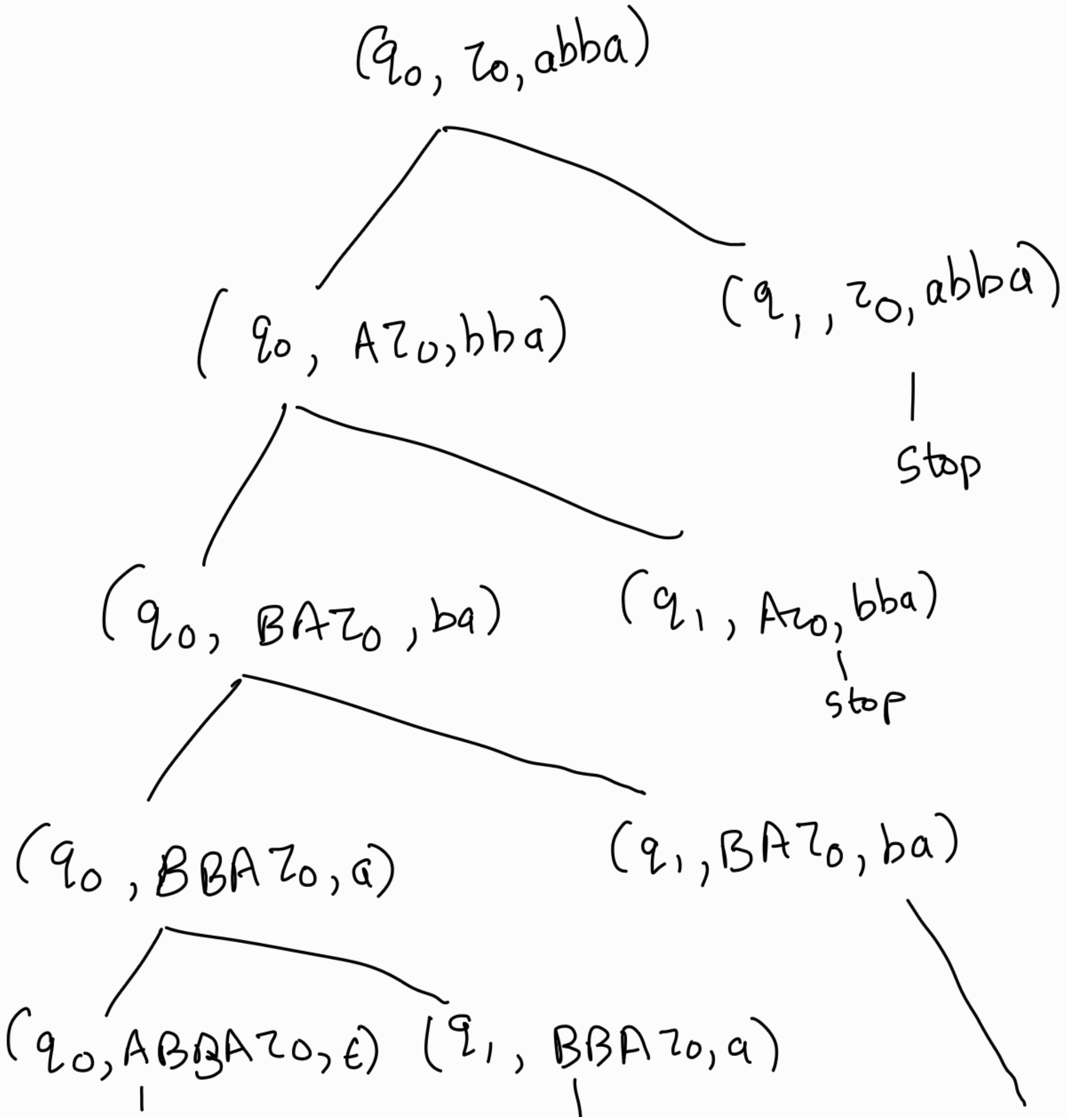
$L = \{ a, b, aba, bab, aabaa, ababa \}$





(e) L $\{ w \in \{0,1\}^* \mid w = w^R$ and length of w is any $\}$





$(q_1, ABBAZ_0, \epsilon)$

|

Stop

|

Stop

|

(q_1, AZ_0, a)

|

(q, Z_0, a)

|

$(q_1, \epsilon, \epsilon)$

|

Accepted

when we say PDA stops where there are no 'δ' mapping for current state, does it stop at current state or just goes to some undefined dead state?

③ $G = \{V, \Sigma, R, S\}$ ← Start variable.

$V = \{S, T, X\}$

$\Sigma = \{a, b\}$

<u>R:</u>
Rules

$S \rightarrow aTXb$

$T \rightarrow XTS \mid \epsilon$

$X \rightarrow a \mid b$

abb

Soln:

$(q, \epsilon, S) = \{(q, aTXb)\}$

$(q, \epsilon, T) = \{(q, XTS), (q, \epsilon)\}$

$(q, \epsilon, X) = \{(q, a), (q, b)\}$

$(q, a, a) = \{(q, \epsilon)\}$

$(q, b, b) = \{(q, \epsilon)\}$

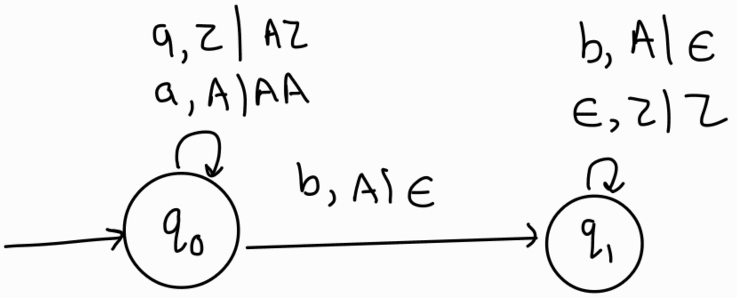
$(q, abb, S) \vdash (q, aTXb, abb) \vdash (q, TXb, bb)$

trial

$\vdash (q, \epsilon Xb, bb) \vdash (q, Xb, bb) \vdash (q, bb, bb)$

$\vdash (q, b, b) \vdash (q, \epsilon, \epsilon)$ .. accepted.

④



$Q = \{q_0, q_1\}$

$S = S$

$V = A, Z, B$

$T = \{a, b\}$

1. $\delta(q_0, a, Z) = \{(q_0, AZ)\}$

2. $\delta(q_0, a, A) = \{(q_0, AA)\}$

3. $\delta(q_0, b, A) = \{(q_1, \epsilon)\}$

4. $\delta(q_1, b, A) = \{(q_1, \epsilon)\}$

5. $\delta(q_1, \epsilon, Z) = \{(q_1, Z)\}$

$N = \{ S, [\underline{q_0}, Z_0, q_0]$

OR(→) q_1, A, q_1

~~$S \rightarrow [q_0, Z, q_0]$~~ $q_0 Z q_0$ is non-terminating.

$S \rightarrow [q_0, Z, q_1]$ ✓✓

3. $\delta(q_0, b, A) = (q_1, \epsilon)$

$$[q_0, A, q_1] \longrightarrow b$$

4. $\delta(q_1, b, A) = (q_1, \epsilon)$

$$[q_1, A, q_1] = b$$

5. $\delta(q_1, \epsilon, Z) = (q_1, Z)$

$$[q_1, Z, q_1] \longrightarrow \epsilon\, [q_1, Z, q_1]$$

1. $\delta(q_0, a, Z) = (q_0, AZ)$

~~$[q_0, Z, q_0] \longrightarrow a[q_0, A, q_0][q_0, Z, q_0]$~~

{$[q_1, Z, q_0]$ not possible}

~~$[q_0, Z, q_0] \longrightarrow a[q_0, A, q_1][q_1, Z, q_0]$~~

~~$[q_0, Z, q_1] \longrightarrow a\underbrace{[q_0, A, q_0]}_{\text{non terminating}}[q_0, Z, q_1]$~~

$$[q_0, Z, q_1] \longrightarrow a[q_0, A, q_1][q_1, Z, q_1]$$

2. $\delta(q_0, a, A) = (q_0, AA)$

does not terminate

$\cancel{[q_0, A, q_0] \rightarrow a[q_0, A, q_0][q_0, A, q_0]}$

$\cancel{[q_0, A, q_0] \rightarrow a[q_0, A, q_1][q_1, A, q_0]}$

$\cancel{[q_0, A, q_1] \rightarrow a\underbrace{[q_0, A, q_0]}_{\text{Removed}}[q_1, A, q_1]}$

$[q_0, A, q_1] \rightarrow a[q_0, A, q_1][q_1, A, q_1]$

<u>Removing useless:</u>

① Remove the rules which are non-terminating

① $q_0 2 q_0 \rightarrow q_0, A, q_0$ (Non terminating)

$\therefore$ CFG :

$$S \rightarrow [q_0, Z, q_1]$$

$$[q_0, Z, q_1] \rightarrow a[q_0, A, q_1][q_1, Z, q_1]$$

$$[q_0, A, q_1] \rightarrow a[q_0, A, q_1][q_1, A, q_1]$$

$$[q_0, A, q_1] \rightarrow b$$

$$[q_1, A, q_1] \rightarrow b$$

$$[q_1, Z, q_1] \rightarrow \epsilon \quad [q_1, Z, q_1]$$

let

$[q_0, Z, q_1]$ be A

$[q_0, A, q_1]$ be B

$[q_1, Z, q_1]$ be C

$[q_1, A, q_1]$ be D

$\therefore$ CFG will be

$$S \rightarrow A$$

$$A \rightarrow aBC$$

$$B \rightarrow aBD \mid b$$

$$C \rightarrow \epsilon$$

$$D \rightarrow b$$

Removing unit productions

$S \to A$

$A \to aBE$

$B \to aBb \mid b$

$\otimes C, D$

$\Rightarrow$

$S \to aB$

$B \to aBb \mid b$

6.3.3

Convert the PDA $P = (\{p, q\}, \{0, 1\}, \{X, Z_0\}, \delta, q, Z_0)$ to a CFG if δ is:

1. $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$
2. $\delta(q, 1, X) = \{(q, XX)\}$
3. $\delta(q, 0, X) = \{(p, X)\}$
4. $\delta(q, \epsilon, X) = \{(q, \epsilon)\}$
5. $\delta(p, 1, X) = \{(p, \epsilon)\}$
6. $\delta(p, 0, Z_0) = \{(q, Z_0)\}$

$N = \{ S, \quad q \; z_0 \; q \}$

$\quad\quad\quad\quad // \; P \; X \; P$

$S \to [q, z_0, q]$

$S \to [q, z_0, p]$

4. $\delta(q, \epsilon, X) = (q, \epsilon)$

$[q, X, q] \to \epsilon$

5. $\delta(p, 1, X) = \{(p, \epsilon)\}$

$[p, X, p] \to 1$

1. $\delta(q, 1, z_0) = (q, x z_0)$

$\cancel{[q, z_0, q] \to 1 [q, x, q][q, z_0, q]}$

$[q, z_0, q] \to 1 [q, x, p][p, z_0, q]$

$[q, z_0, p] \to 1 [q, x, q][q, z_0, p]$

$[q, z_0, p] \to 1 [q, x, p][q, z_0, p]$

2. $\delta(q, 1, x) = (q, x x)$

✓ $[q, x, q] \to 1 [q, x, q][q, x, q]$

$\cancel{[q, x, q] \to 1 [q, x, p][p, x, q]}$

$[q, x, p] \to 1 [q, x, q][q, x, p]$

$[q, x, p] \to 1 [q, x, p][p, x, p]$

3. $\delta(q, 0, X) = (p, X)$

$[q, X, p] \rightarrow 0\,[q, X, q]\,[q, X, p]$

$[q, X, p] \rightarrow 0\,[q, X, p]\,[p, X, p]$

$[q, X, q] \rightarrow 0\,[q, X, q]\,[q, X, q]$

$[q, X, q] \rightarrow 0\,[q, X, p]\,[p, X, q]$

6. $\delta(p, 0, Z_0) = \{q, Z_0\}$ //No operation stack top didn't change

$[p, Z_0, q] \rightarrow 0\,[q, Z_0, q]$

$[p, Z_0, p] \rightarrow 0\,[q, Z_0, p]$

$M = \{\{p, q\}, \{0, 1\}, \{x, z\}, \delta, q, z\}$

1. Start Symbol, 'S':

$S \rightarrow [q, Z_0, q]$

$S \rightarrow [q, Z_0, p]$

1st production:

push operation

$\delta(q, 1, z) = (q, xz)$

✓ $[q, z, q] \rightarrow 1\ [q, X, q][q, z, q]$

✓ $[q, z, q] \rightarrow 1\ [q, X, p][p, z, q]$

✓ $[q, z, p] \rightarrow 1\ [q, X, q][q, z, p]$

✓ $[q, z, p] \rightarrow 1\ [q, X, p][p, z, p]$

2nd Production:

Push operation

$$\delta(q, 1, x) = (q, xx)$$

✓ $[q, x, q] \rightarrow 1\,[q, x, p]\,[p, x, q]$

✓ $[q, x, q] \rightarrow 1\,[q, x, q]\,[q, x, q]$

✓ $[q, x, p] \rightarrow 1\,[q, x, p]\,[p, x, p]$

✓ $[q, x, p] \rightarrow 1\,[q, x, q]\,[q, x, p]$

3rd Transition

pop operation

$$\delta(q, \epsilon, x) = (q, \epsilon)$$

✓ $[q, X, q] \rightarrow \epsilon$

4th transition

No operation, read but don't touch stack.

$$\delta(q, 0, x) = (p, x)$$

✓ $[q, X, q] \rightarrow 0\,[p, X, q]$

✓ $[q, x, p] \rightarrow 0\,[p, x, p]$

5^{th} transition

$$\delta(p, 1, x) = (p, \epsilon)$$

✓ $[p, x, p] \rightarrow 1$

6^{th} transition :

No operation

$$\delta(p, 0, z) = (q, z)$$

✓ $[p, z, p] \rightarrow 0\,[q, z, p]$

✓ $[p, z, q] \rightarrow 0\,[q, z, q]$

let,

transition name	new name
q, z, q	A
q, z, p	B
p, z, q	C
p, z, p	D
q, x, q	E
q, x, p	F
p, x, q	G
p, x, p	H

∴ CFG will be :

$S \rightarrow A \mid B$

$A \rightarrow 1EA \mid 1FC$

$B \rightarrow 1EB \mid 1FD$

$C \rightarrow 0A$

$D \rightarrow 0B$

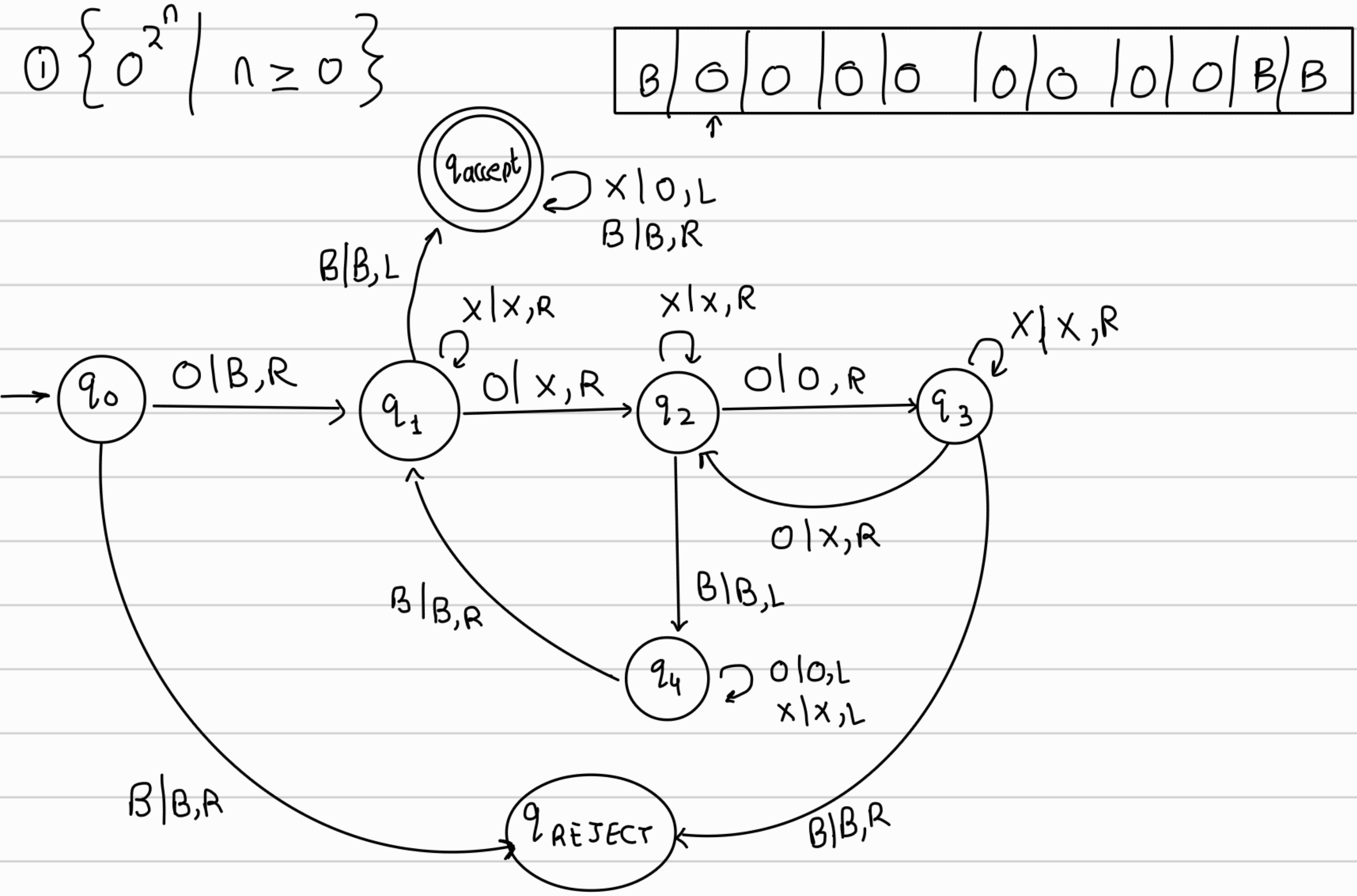
$E \rightarrow 0G \mid \epsilon \mid 1EE \mid 1FG$

$F \rightarrow 1FH \mid 1EF \mid 0H$

$H \rightarrow 1$

5) Design TM:

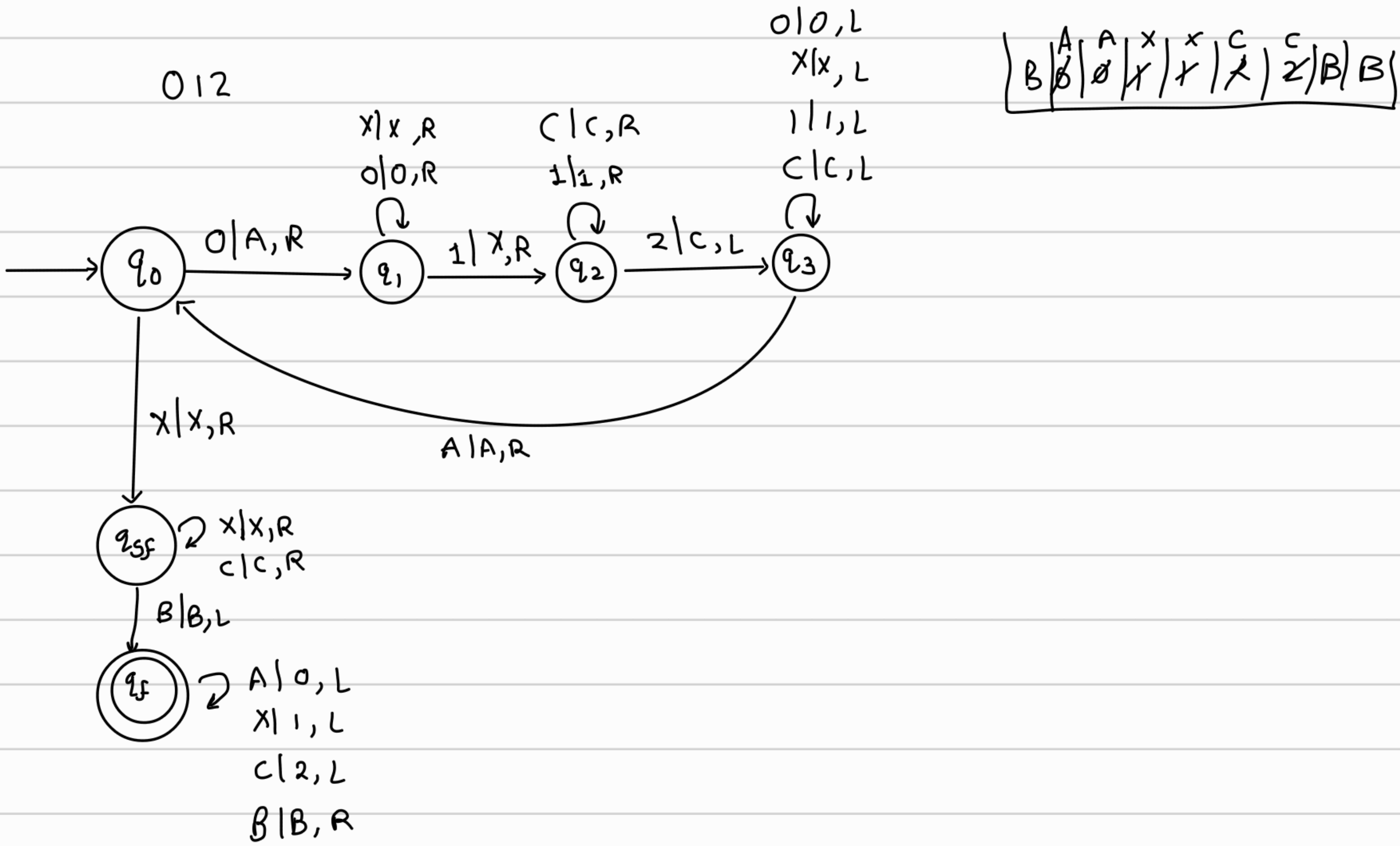
① $\{0^{2^n} \mid n \geq 0\}$



Transition table:

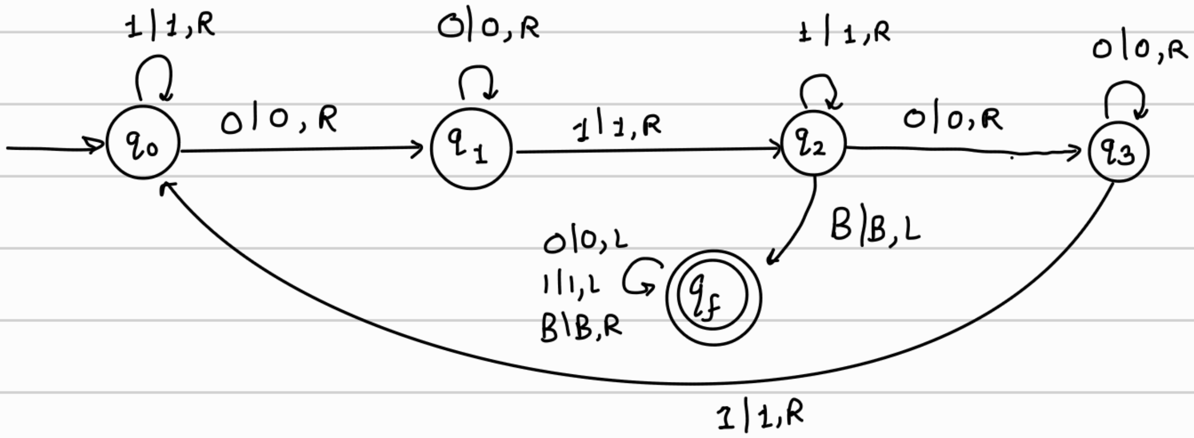
	0	B	X
q_0	$[B, R, q_1]$	$[B, R, q_{reject}]$	$\emptyset$
q_1	$[X, R, q_2]$	$[B, L, q_{accept}]$	$[X, R, q_1]$
q_2	$[0, R, q_3]$	$[B, L, q_4]$	$[X, R, q_2]$
q_3	$[X, R, q_2]$	$[B, R, q_{reject}]$	$[X, R, q_3]$
q_4	$[0, L, q_4]$	$[B, R, q_1]$	$[X, R, q_4]$
q_{accept}	$\emptyset$	$[B, R, q_{accept}]$	$[0, L, q_{accept}]$
q_{reject}	$\emptyset$	$\emptyset$	$\emptyset$

② $L = \{0^n 1^n 2^n \mid n \geq 1\}$



③ All strings over $\{0,1\}$ in which number "01" pairs is odd.

110010101



(d) $q_0 W \vdash q_f WW^R$ ∴ assumption $q_0 W$ derives $q_f WW^R$