Tutorial No: 2

Q1) Show than R^n forms a vector space over R.

→

Let, $\bar{u}, \bar{v} \in \mathbb{R}^n$

$\bar{u} = (x_1, x_2, \cdots, x_n)$

$\bar{v} = (y_1, y_2, \cdots y_n)$

i)

$\bar{u} + \bar{v} = (x_1 + y_1, \cdots x_n + y_n) \in \mathbb{R}^n$.

ii) Let, $\bar{u}, \bar{v}, \bar{w} \in \mathbb{R}^n$

$\bar{u} = (x_1, x_2, \cdots, x_n)$

$\bar{v} = (y_1, y_2, \cdots y_n)$

$\bar{w} = (z_1, z_2, \cdots z_n)$

$(\bar{u}+\bar{v})+\bar{w} = [(x_1+y_1), (x_2+y_2) \cdots (x_n+y_n)] + (z_1, z_2, \cdots z_n)$

$= [(x_1+y_1+z_1), (x_2+y_2+z_2) + \cdots (x_n+y_n+z_n)]$

$\bar{u}+(\bar{v}+\bar{w}) = (x_1, x_2, \cdots x_n) + [(y_1+z_1), (y_2+z_2) + \cdots (y_n+z_n)]$

$= (x_1+y_1+z_1) + (x_2+y_2+z_2) + \cdots (x_n+y_n+z_n)$

$\therefore (\bar{u}+\bar{v})+\bar{w} = \bar{u}+(\bar{v}+\bar{w})$

iii) Let $\bar{u} \in \mathbb{R}^n$

$\bar{u} = (x_1, x_2, \cdots x_n)$

$\bar{0} = (0, 0, \cdots 0)$

$\bar{u}+\bar{0} = (x_1+0, x_2+0, \cdots, x_n+0)$

$\bar{u}+\bar{0} = (x_1, x_2, \cdots x_n)$

$\therefore \bar{u}+\bar{0} = \bar{u}$

iv) Let $\bar{u} \in \mathbb{R}^2$

$\bar{u} = (x_1, x_2, \cdots x_n)$

$-\bar{u} = (-x_1, -x_2, \cdots, -x_n)$

$\bar{u}+(-\bar{u}) = (x_1-x_1, x_2-x_2, \cdots x_n-x_n)$

$\bar{u}+(-\bar{u}) = 0$.

v) $\bar{u}+\bar{v} = (a_1+b_1, a_2+b_2, \cdots, a_n+b_n)$

$= (b_1+a_1, b_2+a_2, \cdots, b_n+a_n)$

$= (b_1, b_2, \cdots b_n) + (a_1, a_2, \cdots a_n)$

$\bar{u}+\bar{v} = \bar{v}+\bar{u}$.

vi) Let, $\alpha \in \mathbb{R}^n$, $\bar{u} \in V$

$\alpha \cdot \bar{u} \in \mathbb{R}^n$

$\bar{u} = (a_1, a_2, \cdots a_n)$

$\alpha \bar{u} = (\alpha a_1, \alpha a_2, \cdots \alpha a_n) \in \mathbb{R}^n$.

vii) $\alpha, \beta \in \mathbb{R}$, $\bar{u} \in \mathbb{R}^n$

$(\alpha \cdot \beta)\bar{u} = \alpha(\beta \bar{u})$

$\bar{u} = (a_1, a_2, \cdots a_n)$.

$(\alpha \cdot \beta)\bar{u} = (\alpha\beta)(a_1, a_2, \cdots a_n)$.

$= (\alpha\beta a_1, \alpha\beta a_2, \cdots \alpha\beta a_n)$.

$\alpha(\beta\bar{u}) = \alpha(\beta a_1, \beta a_2, \cdots \beta a_n)$.

$= (\alpha\beta a_1, \alpha\beta a_2, \cdots \alpha\beta a_n)$

$(\alpha\beta)\bar{u} = \alpha(\beta\bar{u})$.

viii.) $(\alpha+\beta)\cdot\bar{u} = \alpha\bar{u} + \beta\bar{u}$

$\bar{u} = (a_1, a_2, --- a_n)$

$(\alpha+\beta)\bar{u} = [(\alpha a_1 + \beta a_1), (\alpha a_2 + \beta a_2) --- (\alpha a_n + \beta a_n)]$

$= [(\alpha a_1, \alpha a_2) + (\beta a_1, \beta a_2) --- (\ldots)]$

$(\alpha+\beta)\bar{u} = \alpha\bar{u} + \beta\bar{u}$.

ix.) $\alpha(\bar{u}+\bar{v}) = \alpha\bar{u} + \alpha\bar{v}$.

$\bar{u} = (a_1, a_2, -- a_n)$, $\bar{v} = (b_1, b_2, -- b_n)$

$\bar{u}+\bar{v} = (a_1+b_1, a_2+b_2, --- a_n+b_n)$

$\alpha(\bar{u}+\bar{v}) = \alpha[(a_1+b_1), (a_2+b_2) --- (a_n+b_n)]$

$= (\alpha a_1 + \alpha b_1, \alpha a_2 + \alpha b_2, --- \alpha a_n + \alpha b_n)$

$= \alpha(a_1, b_1) + \alpha(a_2, b_2) --- \alpha(a_n, b_n)$

$\alpha(\bar{u}+\bar{v}) = \alpha\bar{u} + \alpha\bar{v}$.

Q4.) Which of the following forms subspaces ?

a.) $S_1 = \{(x, y) \in \mathbb{R}^2 \mid x = y\}$.

$\rightarrow$ i.e. $S_1 = \{(x, x) \in \mid x \in \mathbb{R}\}$.

$\bar{u}, \bar{v} \in S_1$.

$\bar{u} = \begin{pmatrix} a \\ a \end{pmatrix}$, $\bar{v} = \begin{pmatrix} b \\ b \end{pmatrix}$

$\bar{u}+\bar{v} = \begin{pmatrix} a+b \\ a+b \end{pmatrix} \in S_1$

$\alpha\cdot\bar{u} = \begin{pmatrix} \alpha a \\ \alpha a \end{pmatrix} \in S$

$S \underset{\text{subset}}{\subset} \mathbb{R}^2$.

b.) $S_2 = \{(x, y) \in \mathbb{R}^2 \mid x = 2y\}$

$\rightarrow$ i.e. $S_2 = \left\{\begin{pmatrix} 2y \\ y \end{pmatrix} \middle| y \in \mathbb{R}\right\}$

i.)

$\bar{u}, \bar{v} \in S_2$

$\bar{u} = \begin{pmatrix} 2a \\ a \end{pmatrix}$, $\bar{v} = \begin{pmatrix} 2b \\ b \end{pmatrix}$

$\bar{u}+\bar{v} = \begin{pmatrix} 2(a+b) \\ a+b \end{pmatrix} \in S_2$.

$u + v \in S_2$

ii.) $\alpha\cdot\bar{u} = \begin{pmatrix} 2\alpha a \\ \alpha a \end{pmatrix}$

$\alpha\cdot\bar{u} = \cancel{2/\alpha a} \begin{pmatrix} 2(\alpha a) \\ \alpha a \end{pmatrix} \in S_2$.

$\therefore \alpha\cdot\bar{u} \in S_2$ $\quad S_2 \underset{\text{subset}}{\subset} \mathbb{R}^2 \mid x = 2y$.

c.) $S_3 = \{(x, y) \in \mathbb{R}^2 \mid x = cy, c \in \mathbb{R})0\}$.

i.e. $S_3 = \left\{\begin{pmatrix} cy \\ y \end{pmatrix} \mid y \in \mathbb{R}, c \in \mathbb{R}\right\}$.

→ $\bar{u}, \bar{v} \in S_3$.

$\bar{u} + \bar{v} = \begin{pmatrix} c(a+b) \\ a+b \end{pmatrix} \in S_3$.

$\alpha \cdot \bar{u} = \begin{pmatrix} \alpha c a \\ a \end{pmatrix} = \begin{pmatrix} c(\alpha a) \\ \alpha a \end{pmatrix} \in S_3$.

S_3 forms subspace.

d.) $S_4 = \{(x, y) \in \mathbb{R}^2 \mid x = y + 1\}$. -(as constant is present, not a subspace)

i.e. $S_4 = \left\{\begin{pmatrix} \cancel{x} y+1 \\ y \end{pmatrix} \mid y \in \mathbb{R}\right\}$.

→ $\bar{u}, \bar{v} \in \mathbb{R}$,

$\bar{u} = \begin{pmatrix} a+1 \\ a \end{pmatrix}$, $\bar{v} = \begin{pmatrix} b+1 \\ b \end{pmatrix}$

$\bar{u} + \bar{v} = \begin{pmatrix} a+b+2 \\ a+b \end{pmatrix} \notin S_4$

(eg:) $\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} \in S_4 \Rightarrow \begin{pmatrix} 3 \\ 1 \end{pmatrix} \notin S_4$

Hence, S_4 is not a subspace.

e.) $S_5 = \left\{\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x = y + c, c \in \mathbb{R})0\right\}$.

i.e. $S_5 = \left\{\begin{pmatrix} y+c \\ y \end{pmatrix} \mid y \in \mathbb{R}\right\}$.

→ $\bar{u} + \bar{v} = \begin{pmatrix} \cancel{2}x + y + \cancel{2}2c \\ x+y \end{pmatrix} \notin S_5$.

Hence, not a subspace.

f.) $S_6 = \left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + y + z = 0\right\}$.

→ $S_6 = \left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid x+y+z=0\right\}$, $S_7 = \left\{\begin{pmatrix} a \\ b \\ c \end{pmatrix} \mid a+b+c=0\right\}$.

$\bar{u} + \bar{v} = \begin{pmatrix} x+a \\ y+b \\ z+c \end{pmatrix} \in S_6$

$\alpha \cdot \bar{u} = \begin{pmatrix} \alpha x \\ \alpha y \\ \alpha z \end{pmatrix}$

$\alpha \cdot \bar{u} = \in S_6$.

∴ S_6 forms a subspace.

g.) $S_7 = \{(x, y, z) \mid \mathbb{R}^3 \mid x = y,\ 2y = z\}$.

$\longrightarrow$ $S_7 = \left\{ \begin{pmatrix} x \\ x \\ 2x \end{pmatrix} \middle| x \in \mathbb{R} \right\}$.

$\bar{u} + \bar{v} = \begin{pmatrix} a+b \\ a+b \\ 2(a+b) \end{pmatrix} \in S_7$

$\alpha \cdot \bar{u} = \begin{pmatrix} \alpha a \\ \alpha b \\ \alpha(a+b) \end{pmatrix}$

$= \alpha \begin{pmatrix} x \\ x \\ 2x \end{pmatrix}$

$\alpha \cdot \bar{u} = \alpha \bar{u} \quad \in S_7$.

h.) $S_8 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| x + y = 3z \right\}$.

$\longrightarrow$ $S_8 = \left\{ \begin{pmatrix} x \\ y \\ \frac{x+y}{3} \end{pmatrix} \middle| x, y \in \mathbb{R} \right\}$.

$\bar{u} = \begin{pmatrix} a \\ b \\ \frac{a+b}{3} \end{pmatrix}$, $\bar{v} = \begin{pmatrix} c \\ d \\ \frac{c+d}{3} \end{pmatrix}$

$\bar{u} + \bar{v} = \begin{pmatrix} a+c \\ b+d \\ \frac{a+b+c+d}{3} \end{pmatrix} \in S_8$.

$\alpha \cdot \bar{u} = \begin{pmatrix} \alpha a \\ \alpha b \\ \frac{\alpha a + \alpha b}{3} \end{pmatrix} \in S_8$.

$\alpha \cdot \bar{u} = \alpha \begin{pmatrix} a \\ b \\ \frac{a+b}{3} \end{pmatrix}$

Hence, forms subspace.

i.) $S_9 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \middle| x = 0 \right\}$.

$\longrightarrow$ let, $\bar{u} = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \middle| a, b, c \in \mathbb{R} \right\}$.

$\bar{v} = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

$\bar{u} + \bar{v} = \left\{ \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix} \right\} = \begin{pmatrix} 0 \\ b+e \\ c+f \end{pmatrix} \in S_9$.

$\alpha \cdot \bar{u} = \begin{pmatrix} \alpha a \\ \alpha b \\ \alpha c \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha b \\ \alpha c \end{pmatrix} \in S_9$.

Thus, S_9 forms subspace.

Q5.) Which of the following forms a subspace for $M_{n\times n}(\mathbb{R})$

a.) Set of upper triangular matrices.

→ Let, $S_1 = \{A \mid a_{ij} = 0,\ i > j\}$.

$A, B \in S_1$, $A = [a_{ij}]$, $B = [b_{ij}]$

$a_{ij} = b_{ij} = 0$, $i > j$.

$a_{ij} + b_{ij} = 0 \quad \forall\ i > j \quad \in S_1$.

Since, $a_{ij} = 0$, $b_{ij} = 0 \quad \to i > j$

$A + B \in S_1$.

$\alpha A = [\alpha \cdot a_{ij}]$

$= \alpha \cdot 0 \quad i > j$.

$\alpha A = 0 \quad \in S_1$.

Hence, set of upper triangular matrix forms subspace.

b.) Set of lower triangular matrices:

→ $S_2 = \{A \mid a_{ij} = 0,\ j > i\}$.

$A, B \in S$ $\quad A = [a_{ij}]$, $B = [b_{ij}]$

$a_{ij} = b_{ij} = 0 \quad j > i$.

$a_{ij} + b_{ij} = 0 \quad j > i$,

$\therefore A + B \in S_2$.

$\alpha A = \alpha(a_{ij})$

$= \alpha \cdot 0 \quad - j > i$.

$\alpha \cdot A = 0 \quad \in S_2$

Hence, lower triangular matrix forms subspace.

c.) Set of diagonal matrices:

→ ~~$S_3 = \begin{bmatrix} a_{11} & 0 \\ 0 & a_n \end{bmatrix}$~~. $\quad S_3 = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M_{2\times 2} \in \mathbb{R} \;\middle|\; a, b \in \mathbb{R} \right\}$

Let, $\bar{u}, \bar{v} \in S_3$.

$\bar{u} = \left\{ \begin{bmatrix} a_1 & 0 \\ 0 & b_1 \end{bmatrix} \;\middle|\; a_1, b_1 \in \mathbb{R} \right\}$.

$\bar{v} = \left\{ \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix} \;\middle|\; a_2, b_2 \in \mathbb{R} \right\}$.

$\bar{u} + \bar{v} = \left\{ \begin{bmatrix} a_1 + a_2 & 0 \\ 0 & b_1 + b_2 \end{bmatrix} \right\}$. $\quad \in S_3$.

$\alpha \cdot \bar{u} = \left\{ \begin{bmatrix} \alpha a_1 & 0 \\ 0 & \alpha b_1 \end{bmatrix} \right\} \quad \in S_3$.

Hence, S_3 forms subspace.

d.) Set of scalar matrices :

$\longrightarrow$ $S_4 = \left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \in M_{2\times 2} \in \mathbb{R} \;\middle|\; a \in \mathbb{R} \right\}$.

Let, $\bar{u}, \bar{v} \in S_4$.

$\bar{u} = \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} b_1 & 0 \\ 0 & b_1 \end{bmatrix}$

$\bar{u} + \bar{v} = \begin{bmatrix} a_1+b_1 & 0 \\ 0 & a_1+b_1 \end{bmatrix} \in S_4$

$\alpha \bar{u} = \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \end{bmatrix} \alpha$

$\alpha . \bar{u} = \begin{bmatrix} \alpha . a_1 & 0 \\ 0 & \alpha . a_1 \end{bmatrix} \in S_4$.

$\therefore$ Scalar matrix form subspace.

e.) Set of matrices whose determinant is non-zero.

$\longrightarrow$ $S_5 = \{ A \mid |A| \neq 0 \}$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$A + B = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \notin S_5$.

Hence, matrices whose determinant is non-zero doesn't form subspace.

f.) Set of matrices whose determinant is zero.

$\longrightarrow$ $S_6 = \left\{ \begin{bmatrix} a_1 & b_1 \\ b_1 & a_1 \end{bmatrix} \in M_{2\times 2} \in \mathbb{R} \;\middle|\; a_1 = b_1 \right\}$.

let, $\bar{u} = \begin{pmatrix} 1 & +1 \\ +1 & 1 \end{pmatrix}$, $\bar{v} = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$,

$|\bar{u}| = 0$, $|\bar{v}| = 0$.

$\bar{u} + \bar{v} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$

$|\bar{u} + \bar{v}| = 9 - 1 = 8$.

~~$\alpha \bar{u} = \alpha(1)$~~

Hence, matrix whose determinant is zero forms subspace.

g.) Set of matrices whose trace (sum of diagonal entries is zero.

$\longrightarrow$ $S_7 = \left\{ \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} \in M_{2\times 2} \in \mathbb{R} \;\middle|\; a \in \mathbb{R} \right\}$

$\bar{u} + \bar{v} \in S_7$

$\bar{u} = \begin{bmatrix} a_1 & 0 \\ 0 & -a_1 \end{bmatrix}$

$\bar{v} = \begin{bmatrix} b_1 & 0 \\ 0 & -b_1 \end{bmatrix}$

$\bar{u} + \bar{v} = \begin{bmatrix} a_1+b_1 & 0 \\ 0 & -a_1-b_1 \end{bmatrix}$

$\bar{u} + \bar{v} = \begin{bmatrix} a_1+b_1 & 0 \\ 0 & -(a_1+b_1) \end{bmatrix} \in S_7.$

$\alpha \cdot \bar{u} = \begin{bmatrix} \alpha a_1 & 0 \\ 0 & -\alpha a_1 \end{bmatrix} \in S_7.$

Hence, S_7 forms subspace.

h.) Set of matrices whose trace ~~is~~ is non-zero.

$\longrightarrow S_8 = \left\{ \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} m_{2\times 2} \in \mathbb{R} \middle| a \in \mathbb{R} \right\}.$

let $\bar{u}, \bar{v} \in S_8$.

$\bar{u} = \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} a_2 & 0 \\ 0 & a_2 \end{bmatrix}$

$\bar{u} + \bar{v} = \begin{bmatrix} a_1+a_2 & 0 \\ 0 & a_1+a_2 \end{bmatrix}$

~~$\bar{u}+\bar{v}$~~ But, a_1 and a_2 can be zero or non-zero number.

Hence, S_8 doesn't form subspace.

i.) Set of symmetric matrices:

$\longrightarrow S_9 = \left\{ A \middle| m_{2\times 2} \in \mathbb{R} \middle| A = A^t \right\}$

Let $\bar{u}, \bar{v} \in S_9$.

$\bar{u} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, $\bar{v} = \begin{bmatrix} d & e \\ e & f \end{bmatrix}$

$\bar{u} + \bar{v} = \begin{bmatrix} a+d & b+e \\ b+e & c+f \end{bmatrix} \in S_9.$

$\alpha \bar{u} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha b & \alpha c \end{bmatrix} \in S_9.$

$\therefore S_9$ forms a subspace.

j.) Set of all skew-symmetric matric.

$\longrightarrow S_{10} = \left\{ A \middle| m_{2\times 2} \in \mathbb{R} \middle| A = -A^t \right\}$

$\bar{u} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $\bar{v} = \begin{bmatrix} d & e \\ -e & f \end{bmatrix}$

$\bar{u} + \bar{v} = \begin{bmatrix} 0 & b+e \\ -(c+e) & a+d \end{bmatrix}$

$\alpha \cdot \bar{u} = \alpha \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ $\alpha \bar{u}$

$\therefore \alpha L \bar{u} \in S_{10}$

Hence, S_{10} forms a subspace.

Q7.) What of the following are subspaces of $\mathbb{R}^\infty$

(a) Write all the sequence $(1, 0, 1, 0, 1, \ldots)$ i.e. zero at even ~~posit~~ positions.

$\longrightarrow$ All sequence $= (1, 0, 1, 0, 1, \ldots)$
$\because$ zero at even positions.

$\bar{u} = \{a_1, 0, a_2, 0, a_3, 0 \ldots \mid \cancel{a_1, b, c}\ a_1, a_2, a_3 \in \mathbb{R}\}$
$\bar{v} = \{b_1, 0, b_2, \cancel{v_2}\ 0, b_3, \cancel{v_3}\ 0 \ldots \mid b_1, b_2, b_3 \in \mathbb{R}$

$\bar{u} + \bar{v} = \{a_1 + b_1, 0, a_2 + b_2, \ldots\} \in S_1$

~~$\bar{u} + \bar{v} =$~~ $\alpha \bar{u} = \{\alpha(a_1 \cancel{+v_1}), 0, \alpha(a_2 \cancel{+v_2}) + \ldots\}$
$\in S_1$

$\therefore$ It forms sub-space.

b.) All sequences $(x_1, x_2, x_3 \ldots)$ with $x_j = 0$ from some point onwards.

$\longrightarrow$ $S_2 = \{(x_1, x_2, x_3, \ldots) \mid x_j = 0$ from some point onwards$\}$

Let, $\bar{u}, \bar{v} \in S_2$.

$\bar{u} = \{a_1, a_2, a_3, \ldots a_n, 0, 0 \ldots\}$.
$n < m$
$\bar{v} = \{b_1, b_2, b_3, \ldots b_m, 0, 0 \ldots\}$

$\bar{u} + \bar{v} = \{(a_1 + b_1), (a_2 + b_2), \ldots (a_n + b_n) + b \ldots$
$\ldots b_m, 0, 0 \ldots\}$.

$\bar{u} + \bar{v} \in S_2$.

And,
Let $\alpha \in \mathbb{R}$,
$\alpha \cdot \bar{u} = \alpha(a_1, a_2, a_3, \ldots a_n, 0, 0 \ldots\}$.
$= (\alpha a_1, \alpha a_2, \alpha a_3, \ldots \alpha a_n, 0, 0 \ldots\}$.

$\alpha \cdot \bar{u} \in S_2$.
S_2 forms a subspace.

c.) All decreasing sequences : $x_{j+1} \leq x_j$ for each j

$\longrightarrow$ $S_3 = \{(x_1, x_2, x_3, \ldots) \mid x_1 > x_2 > x_3 \ldots\}$

Let, $\bar{u}, \bar{v} \in S_3$.

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$\bar{u} = (1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$

$\bar{v} = (-1, -2, -3, -4, \ldots)$

$\bar{u} + \bar{v} = (0, -\frac{3}{2}, -\frac{8}{3}, -\frac{15}{4})$

$\therefore \bar{u} + \bar{v} \in S_3$.

Now, $\alpha \in \mathbb{R}$.
$\alpha = -1$.

$\therefore \alpha \cdot \bar{v} = (-1)(-1, -2, -3, \ldots)$
$= (1, 2, 3, \ldots)$
$\alpha \cdot \bar{v} \notin S_3$.

$\therefore$ It doesn't form subspace.

[Similarly, set of all increasing sequence do not form subspace]

Q8) If U and W are subspaces of a vector space V then show that $U \cap W$ and $U + W$ are also subspaces of V. What can you say about $U \cap W$, does it form a subspace in general?

$\cap \Rightarrow$ intersection (common)
$\cup \Longrightarrow$ union (take all)

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(a)

i.) Let $\bar{u}, \bar{v} \in U \cap W$.
$\therefore \bar{u} = U$ and $\bar{u} = W$
$\bar{v} = U$ and $\bar{v} = W$

$\bar{u} + \bar{v} = U$ (U is subspace of V)
$\bar{u} + \bar{v} = W$ (W is subspace of V)
$\bar{u} + \bar{v} = U \cap W$

ii.) Let, $\bar{u} \in U \cap W$.
$\alpha \in \mathbb{R}$
claim: $\alpha \cdot \bar{u} = U \cap W$.

$\alpha \cdot \bar{u} = U$
$\bar{v} = W$.

$\alpha \bar{u} \in U$
$\alpha \bar{v} \in W$.

$\therefore \alpha \cdot \bar{u} \in U \cap W$.

(b.) Now $U \cup W$,
let, $\bar{u} = \left\{ \begin{pmatrix} x \\ 0 \end{pmatrix} \middle| x \in \mathbb{R} \right\}$.

$\bar{v} = \left\{ \begin{pmatrix} 0 \\ y \end{pmatrix} \middle| y \in \mathbb{R} \right\}$

$U \cup W = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| \begin{array}{l} \text{either} \\ x = 0 \text{ or } y = 0 \end{array} \right\}$

or

$U \cup W = \left\{ \begin{pmatrix} x \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ y \end{pmatrix} \middle| x, y \in \mathbb{R} \right\}.$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \in U \cup W$; $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \in U \cup W$

but $\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin U \cup W$.

Hence, it doesn't form subspace.

Q9.) Construct a subset of the x-y plane in $\mathbb{R}^2$ that is:

a.) Closed under vector addition and subtraction but not under scalar multiplication.

i.) $X = \left\{ \begin{pmatrix} m \\ n \end{pmatrix} \middle| m, n \in \mathbb{Z} \right\} \subseteq \mathbb{R}^2$.

Let $\bar{u}, \bar{v} \in X$.
then $\bar{u} = \begin{pmatrix} a \\ b \end{pmatrix}$, $\bar{v} = \begin{pmatrix} c \\ d \end{pmatrix}$.

$\bar{u} + \bar{v} = \begin{pmatrix} a+c \\ b+d \end{pmatrix} \in X$

ii.) Let $u = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \in X$ and

Let $\alpha = \frac{1}{2} \in \mathbb{R}$.

$\alpha \cdot \bar{u} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix} \notin X$.

b.) Closed under scalar multiplication but not under vector addition:

i.) $W = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \middle| \begin{matrix} \text{either } x = 0 \\ \text{or } y = 0 \end{matrix} \right\}$.

Let, $\bar{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\bar{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\bar{u} + \bar{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \notin W$

W is not closed under vector addition.

ii.) $\bar{u} = \begin{pmatrix} a \\ b \end{pmatrix} \in W$.

where, $a = 0$ or $b = 0$.

$\alpha \cdot \bar{u} = \begin{pmatrix} \alpha a \\ \alpha b \end{pmatrix}$.

if $a = 0$, then $\alpha a = 0$.
if $b = 0$, then $\alpha b = 0$.

$\therefore$ W is closed under scalar multiplication.

Q.10) Express the given vector X as a linear combination of given vectors, $A \cdot B$ and find the coordinates X with respect to $A \cdot B$.

a.) $X = \begin{pmatrix}1\\0\end{pmatrix}^t$, $A = \begin{pmatrix}1\\1\end{pmatrix}^t$, $B = \begin{pmatrix}0\\1\end{pmatrix}^t$

$X = \begin{pmatrix}1\\0\end{pmatrix}$, $A = \begin{pmatrix}1\\1\end{pmatrix}$, $B = \begin{pmatrix}0\\1\end{pmatrix}$

$c_1 \cdot A + c_2 B = X$

$c_1 \begin{pmatrix}1\\1\end{pmatrix} + c_2 \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$\therefore \begin{pmatrix}c_1\\c_1\end{pmatrix} + \begin{pmatrix}0\\c_2\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

$c_1 = 1$

$c_1 + c_2 = 0$

$\therefore c_2 = -1$

b.) ii) $X = \begin{pmatrix}2\\1\end{pmatrix}$, $A = \begin{pmatrix}1\\-1\end{pmatrix}$, $B = \begin{pmatrix}1\\1\end{pmatrix}$

$c_1 \begin{pmatrix}1\\-1\end{pmatrix} + c_2 \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

$\begin{pmatrix}c_1\\-c_1\end{pmatrix} + \begin{pmatrix}c_2\\c_2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$

$c_1 + c_2 = 2$

$-c_1 + c_2 = 1$

$2c_2 = 3$

$c_2 = \frac{3}{2}$

$c_1 = 2 - \frac{3}{2}$

$c_1 = \frac{1}{2}$

c.) $X = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $A = \begin{pmatrix}1\\1\\1\end{pmatrix}$, $B = \begin{pmatrix}-1\\1\\0\end{pmatrix}$, $C = \begin{pmatrix}1\\0\\-1\end{pmatrix}$

$c_1 \begin{pmatrix}1\\1\\1\end{pmatrix} + c_2 \begin{pmatrix}-1\\1\\0\end{pmatrix} + c_3 \begin{pmatrix}1\\0\\-1\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$

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$$\begin{pmatrix} c_1 \\ c_1 \\ c_1 \end{pmatrix} + \begin{pmatrix} -c_2 \\ c_2 \\ 0 \end{pmatrix} + \begin{pmatrix} c_3 \\ 0 \\ -c_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$c_1 - c_2 + c_3 = 1$.
$c_1 + c_2 = 0$.
$c_1 - c_3 = 0$
$\underline{c_1 = c_3}$.

$c_1 - c_2 + c_3 = 1$
$\therefore \underline{c_2 = -1}$

$c_1 + c_2 = 0$.
$c_1 - 1 = 0$.
$\underline{c_1 = 1 = c_3}$

d.) $X = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $A = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$$c_1 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$0 + c_2 + c_3 = 1$
$c_1 + c_2 + 0 = 1$
$-c_1 + 0 + 2c_3 = 1$

$c_2 + c_3 = 1$
$\underline{-c_1 + c_2 = 1}$
$-c_1 + c_3 = 0$

$-c_1 + c_3 = 0$.
$\therefore \underline{c_3 = c_1}$

$-c_1 + 2(c_1) = 1$
$-c_1 + 2c_1 = 1$
$\therefore \underline{c_1 = 1 = c_3}$

$1 + c_2 = 1$
$\underline{c_2 = 0}$.

Q.11) Check linear independence and dependence of following vectors:

a.) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$c_1 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$c_1 + c_3 = 0$.
$2c_1 = 0$.
$3c_1 = 0$.
$\therefore \underline{c_1 = 0}$
$\therefore \underline{c_3 = 0}$.

Here, $\underline{c_2}$ can be any number which can be non-zero.

$\therefore$ Vectors are linearly dependent.

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b.) $\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\-1\end{pmatrix}$

$c_1\begin{pmatrix}1\\1\\0\end{pmatrix} + c_2\begin{pmatrix}1\\1\\1\end{pmatrix} + c_3\begin{pmatrix}0\\1\\-1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$c_1 + c_2 = 0$
$c_1 + c_2 + c_3 = 0$
$c_2 - c_3 = 0$. $\quad c_2 = c_3$. $\quad \therefore \underline{c_2 = 0}$
$c_1 + c_2 + c_2 = 0$. $\quad \underline{c_3 = 0}$
$c_1 + 2c_2 = 0$.
$\underline{c_1 = 0}$.

Hence, vector is linearly independent.

c.) $\begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}0\\2\\1\end{pmatrix}, \begin{pmatrix}1\\5\\3\end{pmatrix}$

$c_1\begin{pmatrix}0\\1\\1\end{pmatrix} + c_2\begin{pmatrix}0\\2\\1\end{pmatrix} + c_3\begin{pmatrix}1\\5\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$\underline{c_3 = 0}$
$c_1 + 2c_2 + 5c_3 = 0$. $\quad \cancel{c_1 - 2c_2 = 0}$
$c_1 + c_2 + 3c_3 = 0$ $\quad c_1$ $-c_2 + 2c_2 + 0 = 0$.
$c_1 + c_2 + 0 = 0$ $\quad \underline{c_2 = 0}$.
$c_1 = -c_2$. $\quad \therefore \underline{c_1 = 0}$

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vector is linearly independent.

d.) $\begin{pmatrix}1\\1\\2\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}2\\2\\4\end{pmatrix}$

$c_1\begin{pmatrix}1\\1\\2\end{pmatrix} + c_2\begin{pmatrix}1\\2\\3\end{pmatrix} + c_3\begin{pmatrix}2\\2\\4\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$c_1 + c_2 + 2c_3 = 0$.
$c_1 + 2c_2 + 2c_3 = 0$.
$2c_1 + 3c_2 + 4c_3 = 0$.

$-c_2 = 0$.
$\therefore c_2 = 0$.

$c_1 + 2c_3 = 0$
$c_1 = -2c_3$

$\cancel{+c_3 + 0 + c_3 = 0}$
$\cancel{\therefore c_3 = 0}$

Hence, vector is L.D.

e.) $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 0\\3 & 0\end{bmatrix}, \begin{bmatrix}1 & 0\\3 & 1\end{bmatrix}$

$c_1\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} + c_2\begin{bmatrix}0 & 0\\3 & 0\end{bmatrix} + c_3\begin{bmatrix}1 & 0\\3 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\begin{bmatrix}c_1 + c_3 & 2c_1\\3c_2 + 3c_3 & c_1 + c_3\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$C_1 + C_3 = 0$ $\therefore C_3 = 0$

$2C_1 = 0$ $\therefore C_1 = 0$

$3C_2 + 3C_3 = 0$

$C_1 + C_3 = 0$

$3C_2 + 0 = 0$

$\therefore C_2 = 0$

Vector is linearly independent.

f.) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

$$C_1 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} C_1 & 2C_1 & 3C_1 \\ 3C_2 & C_1 & 4C_1 \\ 2C_2 & C_2 & C_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_1 = 0$

$3C_2 = 0$

$\therefore C_3 = 0$

Hence, vector is L.I.

Q.12) Show that v_1, v_2, v_3 are independent but v_1, v_2, v_3, v_4 are dependent:

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

To prove: i) $v_1 + v_2 + v_3 = 0$.

ii) $v_1 + v_2 + v_3 + v_4 \neq 0$.

i) $C_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$C_1 + C_2 + C_3 = 0$

$C_2 + C_3 = 0$

$C_3 = 0$

$\therefore C_2 = 0$

$\therefore C_1 = 0$

Hence, v_1, v_2, v_3 forms linearly independent vector.

ii) $C_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + C_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + C_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_4 \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$C_1 + C_2 + C_3 + 2C_4 = 0$

$C_2 + C_3 + 3C_4 = 0$ $C_2 - 4C_4 + 3C_4 = 0$

$C_3 + 4C_4 = 0$ $C_2 - C_4 = 0$

$\therefore C_3 = -4C_4$ $C_2 = C_4$

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$c_1 + c_4 - 4c_4 + \cancel{3}\,2c_4 = 0$

$c_1 + 3c_4 - 4c_4 = 0$

$c_1 - c_4 = 0$

$c_1 = c_4$

Hence, vectors v_1, v_2, v_3, v_4 are L.D.

Q.13) If w_1, w_2, w_3 are independent vectors, show that the differences $v_1 = w_2 - w_3$, $v_2 = w_1 - w_3$, and $v_3 = w_1 - w_2$ are dependent.

→ $c_1 \cdot v_1 + c_2 \cdot v_2 + c_3 \cdot v_3 = 0$. —(given)

$c_1(w_2 - w_3) + c_2(w_1 - w_3) + c_3(w_1 - w_2) = 0$.

$c_1 \cdot w_2 - c_1 w_3 + c_2 \cdot w_1 - c_2 \cdot w_3 + c_3 \cdot w_1 - c_3 \cdot w_2 = 0$.

$w_1(c_2 + c_3) + w_2(c_1 - c_3) + w_3(-c_1 - c_2) = 0$.

$c_2 + c_3 = 0$. $\quad c_2 = -c_3$

$c_1 - c_3 = 0$. $\quad c_1 = c_3$

$-c_1 - c_2 = 0$. $\quad \therefore c_1 = -c_2$

Hence, V_1, V_2, V_3 are L.D.

Q.14) If w_1, w_2, w_3 are independent vectors, show that the sum $v_1 = w_2 + w_3$, $v_2 = w_1 + w_3$ and $v_3 = w_1 + w_2$ are linearly independent.

→ $c_1 \cdot v_1 + c_2 \cdot v_2 + c_3 \cdot v_3 = 0$.

$c_1(w_2 + w_3) + c_2(w_1 + w_3) + c_3(w_1 + w_2) = 0$.

$c_1 w_2 + c_1 \cdot w_3 + c_2 \cdot w_1 + c_2 \cdot w_3 + c_3 \cdot w_1 + c_3 \cdot w_2 = 0$.

$w_1(c_2 + c_3) + w_2(c_1 + c_3) + w_3(c_1 + c_2) = 0$.

$c_2 + c_3 = 0$. $\quad \therefore c_2 + c_2 = 0$.

$c_1 + c_3 = 0$. $\quad \therefore 2c_2 = 0$

$c_1 + c_2 = 0$. $\quad \therefore c_2 = 0 = c_3$

$c_3 - c_2 = 0$. $\quad c_1 + 0 = 0$

$\therefore c_3 = c_2$. $\quad \therefore c_1 = 0$

Hence, V_1, V_2, V_3 are L.I.

Q.15) Suppose v_1, v_2, v_3, v_4 are vectors in R^3.

a.) These four vectors are dependent because...

→ v_1, v_2, v_3, v_4 will be dependent if

$c_1 V_1 + c_2 V_2 + c_3 V_3 + c_4 V_4 = 0$.

but, either c_1, c_2, c_3, c_4 must be non zero number.

Let $V_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $V_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$, $V_3 = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$, $V_4 = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$c_1 + 2c_2 + 3c_3 + 4c_4 = 0$.

$3c_3 + 4c_4 = 0$. $\quad \therefore c_3 = -\frac{4}{3} c_4$

As one of the vector is non-zero ($c_3 \neq 0$)
hence, v_1, v_2, v_3, v_4 are L.D.

b.) The two vectors v_1 and v_2 will be dependent if ...

Vectors v_1 and v_2 will be dependent if
$c_1 \cdot v_1 + c_2 \cdot v_2 = 0$
~~and~~ but either c_1 or c_2 or both $\neq 0$.

eg: $\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}2\\2\end{pmatrix}$

$c_1 + 2c_2 = 0$ $\therefore$ $c_1 = -2c_2$
$c_1 + 2c_2 = 0$
Hence, v_1, v_2 are L.D.

c.) The vectors v_1 and $(0,0,0)$ are dependent because ...

$c_1 \cdot v_1 + c_2 \cdot (\bar{0}) = 0$.
and either c_1 or $c_2 \neq 0$.

$\therefore c_1 \begin{pmatrix}1\\1\end{pmatrix} + c_2 \begin{pmatrix}0\\0\end{pmatrix} = 0$.

$c_1 = 0$

but. c_2 can be any non-zero number

hence, v_1 and $(0,0,0)$ is L.D.

Q.16) True or False, Justify:

a.) Subset of linearly independent set is linearly independent.

$\longrightarrow$ Let, $J = \{v_1, v_2, ---v_n\}$.

$c_1 \cdot v_1 + c_2 \cdot v_2 + ---- c_n \cdot v_n = 0$.
where, $c_1 = 0, c_2 = 0, --- c_n = 0$.

Now, consider subset of J as I. let I $\rightarrow$ Linearly independent
$I \subseteq J$.

$I = \{v_1, v_2, --- v_k\}$ $-(k \leq n)$

$c_1 \cdot v_1 + c_2 \cdot v_2 + ---- c_k \cdot v_k + 0\, v_{k+1} + 0 \cdot v_{k+2} + --- + 0 \cdot v_n = 0$.

$\therefore$ As I is subset of J,
$c_1, c_2, --- c_k = 0$.

$\therefore$ Subset of linearly independent set is linearly independent.
Hence, given statement is true.

b.) ~~Subset~~ Superset of linearly independent set is linearly independent.

$\longrightarrow$ let, $A = \left\{ \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix} \right\}$

$c_1\begin{pmatrix}0\\1\end{pmatrix}+c_2\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$c_2=0$
$c_1=0$

Hence, A is L.I.

~~#~~ Now, $B=\left\{\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix}\right\}$.

$\therefore A\subset B.$

$\therefore c_1\begin{pmatrix}0\\1\end{pmatrix}+c_2\begin{pmatrix}1\\0\end{pmatrix}+c_3\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$c_1=0$
$c_2=0$

but here, c_3 can be any non-zero number
Hence, B is L.D.

$\therefore$ set of L.I set is L.D.

Hence, given statement is False.

c.) Superset of linearly independent set is linearly independent.

$\rightarrow$ let, $I=\{(v_1, v_2 \ldots v_k)\}$ $\quad k\leq n$.

$J=\{v_1, v_2, \ldots v_n\}$.

$I\subseteq J$.

$c_1.v_1+c_2.v_2+\ldots+c_k.v_k=0.$
where one of the $c_1, c_2 \ldots c_k$ is non-zero.

Now,
$c_1.v_1+c_2.v_2+\ldots c_k.v_k+0.v_{k+1}+\ldots v_n=0.$
which is L.C of v_1 to v_n.

But, one of $c_1, c_2 \ldots c_k$ is non-zero.
$\because J$ is a L.D set.
$\therefore$ Superset of LD set is L.D. $\rightarrow$ True.

d.) Subset of linearly dependent is linearly dependent.

$\rightarrow$ Let, $I=\{(0,1),(0,0)\}$.

$c_1.v_1+c_2.v_2=0.$

$c_1\begin{pmatrix}0\\1\end{pmatrix}+c_2\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$\therefore c_1=0$.

but c_2 can be any non-zero no.
hence I is L.D.

let, $J\subseteq I$.
$J=\begin{pmatrix}0\\1\end{pmatrix}$

$$c_1 \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\therefore$ $c_1 = 0$

$\therefore$ J is independent set.

$\therefore$ Subset of L.D set is L.D. $\longrightarrow$ False.

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