COLLEGE OF ENGINEERING, PUNE
(An Autonomous Institute of Government of Maharashtra.)

(MA-) Linear Algebra and Univariate Calculus

Test I

Course: Direct Second Year , Semester III
Academic Year: 2022-2023 Max. Marks: 20
Duration: 1 Hour Date: 16/12/2022

Instructions: **Student MIS NO. :** ☐☐☐☐☐☐☐☐☐

(1) All questions are compulsory.
(2) Figures to the right indicates full marks.
(3) Mobile phones and programmable calculators are strictly prohibited.
(4) Writing anything on question paper is not allowed.
(5) Exchange/Sharing of stationery, calculator etc. not allowed.
(6) Write your MIS Number on Question Paper.

Attempt the following questions.

(1) Using elementary row transformations, find the inverse of following matrix: [3M](CO4)

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

(2) Express the given vector X as a linear combination of the given vectors A and B and find the coordinates of X with respect to A, B where

$$X = {}^t(2,1),\ A = {}^t(1,-1),\ B = {}^t(1,1). \tfrac{3}{2}, \tfrac{1}{2}$$

[2M](CO2)

(3) True or false with Justification: [4M](CO3)
   (a) If A and B are symmetric then AB is symmetric.
   (b) If A and B are invertible then BA is invertible.

(4) S is the set of all skew symmetric matrices of order 3. Is S a subspace of $M_{3\times3}(\mathbb{R})$. [3M](CO2)

(5) Determine the values of a and b for which the system

$$\begin{aligned} x + 2y + 3z &= 6 \\ x + 3y + 5z &= 9 \\ 2x + 5y + az &= b \end{aligned}$$

has (i) No solution (ii) Infinite number of solutions (iii) Unique solution. [4M](CO3)

(6) Which of the following forms subspaces. Prove or provide counterexamples:
   (a) $S = \{(x, y) \in \mathbb{R}^2 : x = y + 1\}$. [2M](CO2)
   (b) $T = \{(x, y, z) \in \mathbb{R}^3 : x = y \text{ and } 2y = z\}$. [2M](CO3)

COLLEGE OF ENGINEERING, PUNE

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(MA-20002) Linear Algebra and Univariate Calculus

Test II

Course: Direct Second Year, Semester III
Academic Year: 2022-2023
Duration: 1 Hour

Max. Marks: 20
Date: 20/01/2023

Instructions:

Student MIS NO. :

(1) All questions are compulsory.
(2) Figures to the right indicates full marks and course outcomes.
(3) Mobile phones and programmable calculators are strictly prohibited.
(4) Writing anything on question paper is not allowed.
(5) Exchange/Sharing of stationery, calculator etc. not allowed.
(6) Write your MIS Number on Question Paper.

Attempt the following.

Q1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$. Find Nullity(A) and Rank(A). Also verify rank-nullity theorem. [3](CO2)

Q2. Check whether the following matrix is diagonalizable or not, using the steps given below: (CO3)

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

(a) Find the eigenvalues of the matrix A. [1]
(b) Find the corresponding eigenvectors. [1.5]
(c) Write down algebraic multiplicity and geometric multiplicity for each eigenvalue. [2]
(d) Is the matrix A diagonalizable? If yes, then write down the matrix P and a diagonal matrix D such that $D = P^{-1}AP$. [1.5]

Q3. Let $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = -x^3 + 6x^2 - 9x + 3$. Then (CO3)
(a) Find all critical points. [1]
(b) Find the extreme values of f, if exist. [2]
(c) Find the intervals where f is increasing or decreasing. [2]
(d) Find all inflection points. [1]
(e) Find the intervals where f is concave up or concave down. [2]
(f) Sketch the graph of f. [1]

Q4. Prove that similar matrices have same eigenvalues. [2](CO4)

** Good Luck **

COLLEGE OF ENGINEERING, PUNE
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(MA-20002) Linear Algebra and Univariate Calculus

End Semester Exam

Course: Direct Second Year, Semester III
Academic Year: 2022-2023
Duration: 3 Hours

Max. Marks: 60
Date: 29/01/2023

Instructions:

Student MIS NO. : 142203013

(1) All questions are compulsory.
(2) Figures to the right indicates full marks and course outcomes.
(3) Mobile phones and programmable calculators are strictly prohibited.
(4) Writing anything on question paper is not allowed.
(5) Exchange/Sharing of stationery, calculator etc. not allowed.
(6) Write your MIS Number on Question Paper.

Attempt the following.

(1) (a) Consider the following system:

$$-2x + 3y + z + 4w = 0,\ x + y + 2z + 3w = 0,\ 2x + y + z - 2w = 0.$$

(i) Write the matrix form of the above system and identify whether it is homogeneous or non-homogeneous. 0 [2][CO1]

(ii) Find the solution for the above system using Gauss Elimination method. 2 [3][CO2]

(b) Are the vectors ${}^t(1,1,2)$, ${}^t(1,2,3)$, ${}^t(2,2,4)$ linearly independent? Justify. 3 [3][CO3]

(c) Let A be a square matrix. 2 [4][CO4]

(i) If $A^2 = 0$ show that $I - A$ is invertible.

(ii) If $A^3 = 0$ show that $I - A$ is invertible.

(iii) In general, If $A^n = 0$ for some positive integer n, show that $I - A$ is invertible.

(2) (a) Express the vector $X = {}^t(1,1,1)$ as a linear combination of vectors $A = {}^t(0,1,-1)$, $B = {}^t(1,1,0)$, $C={}^t(1,0,2)$. Furhter find the coordinates of X with respect to A, B, C. 4 [4][CO3]

(b) Show that $S = \{(x,y,z) \in \mathbb{R}^3 | x + y + z = 0\}$ forms a subspace of $\mathbb{R}^3$. What is the dimension of S? 4 [4][CO2]

(c) Construct a subset of $\mathbb{R}^3$ that is closed under vector addition but not under scalar multiplication. 2 [2][CO5]

(3) (a) Let A be a square matrix of order n. Prove that A and A^t have same eigenvalues. [2][CO4]
2

(b) Check whether the matrix A is diagonalizable or not. If yes, then write down the matrix P and a diagonal matrix D such that $D = P^{-1}AP$. [6][CO3]

$$A = \begin{pmatrix} 2 & -2 & 1 \\ -1 & -3 & -1 \\ 2 & -4 & 3 \end{pmatrix}$$

(4) (a) Show that the function $f(x) = x^3 + 3x + 1$ has exactly one root in the interval $[-1, 0]$. [3][CO3]

(b) Define critical points. Find the set of critical points and determine the local extreme values for the function $f(x) = x^{2/3}(x+2)$. [3][CO1,CO2]

(c) Show that $x = 7$ is a critical point of the function $f(x) = 2 + (x-7)^3$ but f does not have a local extreme value at $x = 7$. [2][CO3]

(5) Solve **any two**: [6][CO3]

(a) Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \le x \le 4$.

(b) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis.

(c) Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant about the y-axis.

(6) (a) For what values of p, does the integral $\displaystyle\int_1^\infty \frac{dx}{x^p}$ converges? What is the value of the integral, when it converges? [4][CO4]

(b) Check the convergence of the following integrals (**any two**) and hence evaluate the integral (if possible). [4][CO3]

(i) $\displaystyle\int_1^\infty \frac{sin^2 x}{x^2} dx$ (ii) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^2 - 0.1}} dx$. (iii) $\displaystyle\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

(c) Define Gamma function, and hence evaluate the integral $\displaystyle\int_0^\infty x^4 e^{-x^4} dx$. [4][CO1,CO3]

(7) Let $I_n = \displaystyle\int_0^{\pi/2} x^n \sin x \, dx$. Prove that $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$. Hence find $\displaystyle\int_0^{\pi/2} x^3 \sin x \, dx$. [4][CO4]

** Good Luck **