Feedback Control System
Test 1

Instructions to candidates:

1. **All questions are compulsory.**
2. **All questions carry equal marks.**

1 Using the Laplace transform pairs and the Laplace transform theorems, derive the Laplace transforms for the following time functions. Specify the theorems.

a. $e^{-at}\cos(\omega t)\,u(t)$

b. $t^3 u(t)$

2 Find the inverse Laplace transform of

$$F(s) = \frac{(s+3)}{(s+2)^2(s+5)}$$

using partial fraction expansion.

3 Define the poles and zeros of the system described by the transfer function.

4 A system is represented by a relation given below

$$X(s) = R(s)\frac{100}{(s^2+2s+50)}$$

if $r(t) = 1.0$ unit, find the values of $x(t)$ when $t \to 0$.

5 Write the differential equation that is mathematically equivalent to the block diagram as shown. Assume that $r(t) = 3t^3$.

$$R(s) \longrightarrow \boxed{\frac{s^4+3s^3+2s^2+s+1}{s^5+4s^4+3s^3+2s^2+3s+2}} \longrightarrow C(s)$$

6 What are the steps involved in the design of control system?

7 Find the transfer function

$$G(s) = \frac{V_L(s)}{V(s)}$$

for network as shown in figure 1

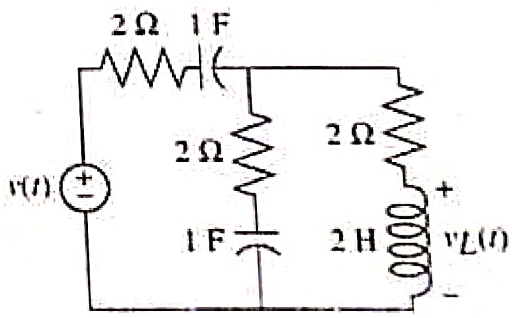

Figure 1

Feedback Control System
Test 1

8 Find the transfer function,

$$G(s) = \frac{X(s)}{F(s)}$$

For the translational mechanical system shown in Figure 2

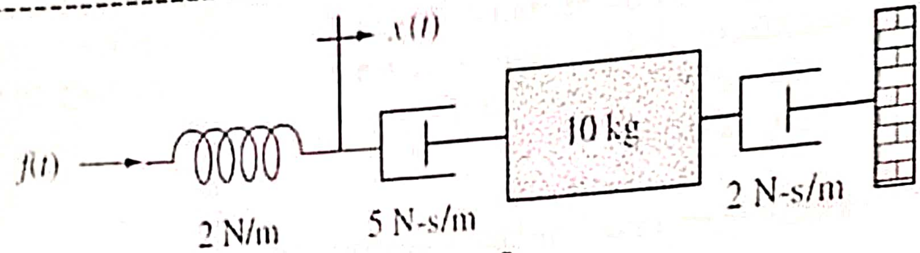

Figure 2

9 Find the output response, c(t), for the systems shown in Figure 3.
Also find the time constant, rise time, and settling time.

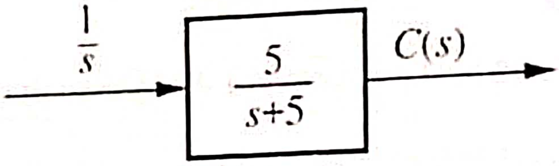

Figure 3

10 What is the type and order of following system. Justify your answer.

$$G(s) = \frac{s+1}{s^2(s+2)(s+3)}$$

College of Engineering, Pune-5

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Test-1

Feedback Control System

Duration: 01 hrs **Max Marks: 20**

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1 a. A closed loop control system is one in which the ________________ has an effect on the __________________. 5

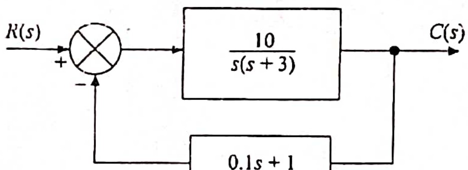
b. ________________ feedback is employed in control systems.

c. A signal which has zero value everywhere except at t=0 where the magnitude is infinity is called __________________.

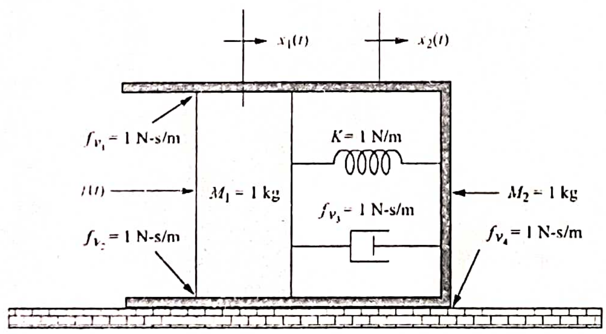
d. Traffic light system is the example of ________________ control system

e. Stability is very important characteristic of the ______________________ response of the system.

2 A positional control system with velocity feedback is shown in figure. What is the response of the system for a unit step input? 5



3 Find TF $X_2(s)/F(s)$ of the system 5



4 Discuss (i) feedback and feed-forward control system (ii) open loop and closed loop system 5

SY Computer
Feedback Control System
Test 2

Instructions to candidates:

1. **All questions are compulsory.**
2. **All questions carry equal marks.**
3. **Figures to right indicates full marks.**

1

i	For a second order underdamped system damping factor $\zeta = 0.5$ and natural frequency $\omega_n = 3$. Show these two factors graphically in s-plane	1
ii	Depending upon damping factor ζ classify the control systems	1
iii	The time response of a control system is the sum of the ________ and the ________.	1
iv	The ________ error is a measure of system accuracy.	1
v	If the characteristic equation of a system is $s^2 + 6s + 8 = 0$, the system is overdamp damped.	1

2 A unity feedback system whose open loop transfer function is given by $\frac{K}{s(sT+1)}$, when subjected to a unit step input gives underdamped response. If peak overshoot is 25% at 3 second, determine the value of K and T. By what factor K should be multiplied so that peak overshoot amplified from 25% to 75%? **10**

3 Determine the stability of system using Routh-Hurwitz criteria having characteristic equation $s^6 + 3s^5 + 5s^4 + 9s^3 + 8s^2 + 6s + 4 = 0$. Also find the closed loop pole locations. **5**

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END Semester Examination

Programme: B.Tech | **Semester: III**

Course Code: | **Course Name: Feedback Control System**

Branch: Computer | **Academic Year:** 2022-23

Duration: 3 hours | **Max Marks: 60**

Student PRN No.

Instructions:

1. **Figures in the Column M indicate the full marks.**
2. **Mobile phones and programmable calculators are strictly prohibited.**
3. **Writing anything on question paper is not allowed.**
4. **Exchange/Sharing of stationery, calculator etc. not allowed.**
5. **Write your PRN Number on Question Paper.**
6. **Perform all calculations up to 4 decimal points**
7. Solve any Five questions out of six.

			M	CO	PO
Q 1 A	i.	What mathematical model permits easy interconnection of physical systems?	6	1	1
	ii.	Define the transfer function.			
	iii.	What do we call the mechanical equations written in order to evaluate the transfer function?			
	iv.	If we understand the form the mechanical equations, what step do we avoid in evaluating the transfer function?			
	v.	Why do transfer functions for mechanical networks look identical to transfer functions for electrical networks?			
	vi.	Instability is attributable to what part of the total response			
B		Compare the open loop control system and closed loop control system on the basis of following points i. Accuracy ii. Power consumption iii. Complexity iv. Response to external disturbances v. Stability vi. Cost	6	1	1

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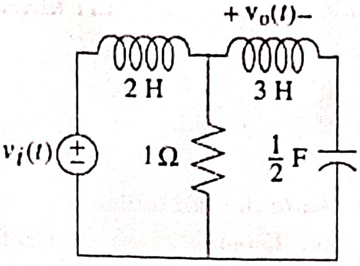
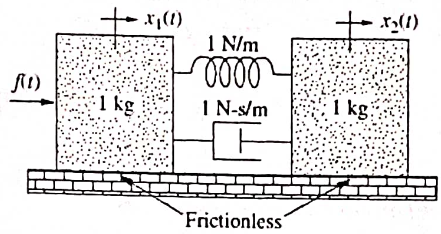
Q 2 A Find the transfer function, $G(s) = \frac{V_o(s)}{V_i(s)}$, for following network. Solve the problem using mesh analysis. 6 2 2,3



Q 2 B Find the transfer function, $G(s) = \frac{X_2(s)}{F(s)}$, for the translational mechanical network shown in Figure. 6 2 2,3



Q 3 A Consider a unity feedback system with closed transfer function 3 3 1,2

$$\frac{C(s)}{R(s)} = \frac{Ks + b}{s^2 + as + b}$$

Determine the open loop transfer function $G(s)$. Find static error constant and steady state error for unit ramp input.

B A unity feedback system is characterised by an open loop transfer function 5 3 1,2

$$G(s) = \frac{K}{s(s+10)}$$

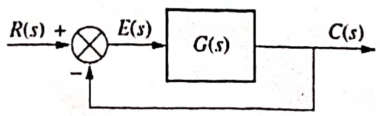
Determine the gain K so that the system will have a damping ratio of 0.5. For this value of K determine the settling time, peak overshoot and time to peak overshoot for unit step input

C For the unity feedback system shown in Figure, where 4 3 1,2

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

Find the steady-state errors for the following test inputs:

$25\,u(t), 37\,t\,u(t), and\ 47t^2u(t)$.



Q 4	A	By means of Routh criterion, determine the stability of the system represented by the following characteristics equations. For system found to be unstable, determine the number of roots of the characteristics equation in the right half of the s plane. $s^6 + 2s^5 + s^4 + 2s^3 + 3s^2 + 4s + 5 = 0.$	4	3	1,2
	B	Determine the stability of system using Routh-Hurwitz criteria having characteristic equation $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0.$ Also find the closed loop poles.	5	3	1,2
	C	Determine the range of K for the system to be stable using Routh criterion. $s^3 + 3Ks^2 + (K + 2)s + 4 = 0.$	3	3	1,2
Q 5	A	Sketch the root locus for $G(s)H(s) = \dfrac{K}{(s+2)(s+1)(s+8)}$ Find K for stability	8	4	1,2
	B	Find K for the above system (described in Q 5 A) when it is operating with damping factor 0.6.	4	4	1,2
Q 6	A	Sketch the root locus for $G(s)H(s) = \dfrac{K(s+3)}{s^2(s+9)}$ Find K for stability	8	4	1,2
	B	Find dominant pole for the above system (described in Q 6 A) when the system is operating with 0.5 damping factor.	4	4	1,2

Q 1 A (6)

i. What mathematical model permits easy interconnection of physical systems?

Transfer Function

ii. Define the transfer function.

It the ratio of Laplace transform of output to the Laplace transform of input assuming all initial conditions zero.

iii. What do we call the mechanical equations written in order to evaluate the transfer function?

Equation of motion

iv. If we understand the form the mechanical equations take, what step do we avoid in evaluating the transfer function?

Free body diagram

v. Why do transfer functions for mechanical networks look identical to transfer functions for electrical networks

There are direct analogies between the electrical variables and components and the mechanical variables and components.

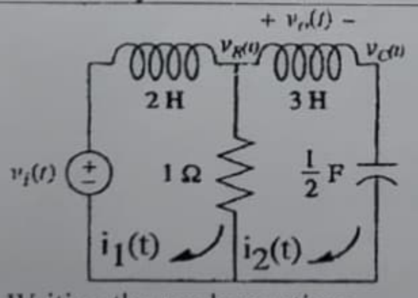
vi. Instability is attributable to what part of the total response?

Natural response

B Compare the open loop control system and closed loop control system on the basis of following points. (6)

Accuracy	Inaccurate and unreliable	Accurate and reliable
Power consumption	consume less power	consume more power
Complexity	Simple	Complex
Response to external disturbances	The changes in output due to external disturbances are not corrected automatically	The changes in output due to external disturbances are corrected are automatically
Stability	they are generally stable	efforts are needed to design a stable system
Cost	Economical	Costlier

Q 2 A Find the transfer function, $G(s) = \frac{V_o(s)}{V_i(s)}$, for following network. Solve the problem using mesh analysis. (6)



Writing the mesh equations,

$$(2s+1)I_1(s) - I_2(s) = V_i(s)$$
$$-I_1(s) + (3s+1+2/s)I_2(s) = 0$$

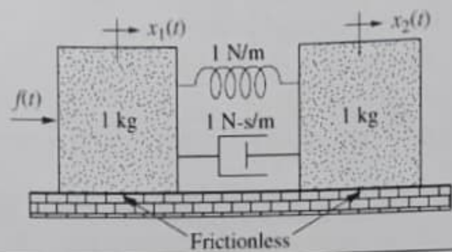
Solving for $I_2(s)$,

$$I_2(s) = \frac{\begin{vmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2s+1 & -1 \\ -1 & \frac{3s^2+s+2}{s} \end{vmatrix}}$$

But $V_o(s) = 3s\, I_2(s)$

$$\frac{I_2(s)}{V_i(s)} = \frac{s}{6s^3 + 5s^2 + 4s + 2}$$
$$\therefore I_2(s) = V_o(s)/3s$$
$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{3s^2}{6s^3 + 5s^2 + 4s + 2}$$

Q 2 B Find the transfer function, $G(s) = \frac{X_2(s)}{F(s)}$, for the translational mechanical network shown in Figure. **6**



Writing the equation of motion

$$(s^2 + s + 1)X_1(s) - (s + 1)X_2(s) = F(s)$$
$$-(s + 1)X_1(s) + (s^2 + s + 1)X_2(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} (s^2 + s + 1) & F(s) \\ -(s + 1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + s + 1) & -(s + 1) \\ -(s + 1) & (s^2 + s + 1) \end{vmatrix}}$$
$$= \frac{(s + 1)F(s)}{s^2((s^2 + s + 1)}$$
$$\frac{X_2(s)}{F(s)} = \frac{(s + 1)}{s^2((s^2 + 2s + 2)}$$

Q 3 A Consider a unity feedback system with closed transfer function **3**

$$\frac{C(s)}{R(s)} = \frac{Ks + b}{s^2 + as + b}$$

Determine the open loop transfer function $G(s)$. Find the steady state error with unit ramp input

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{Ks + b}{s^2 + as + b}$$

Cross multiplying the above equation

$$(s^2 + as + b)G(s) = (1 + G(s))(Ks + b)$$
$$(s^2 + as + b)G(s) = (Ks + b) + (Ks + b)G(s)$$
$$[(s^2 + as + b) - (Ks + b)]G(s) = (Ks + b)$$
$$G(s) = \frac{(Ks + b)}{[(s^2 + as + b) - (Ks + b)]} = \frac{(Ks + b)}{s^2 + (a - K)s} = \frac{(Ks + b)}{s[s + (a - K)]}$$

The velocity error constant

$$K_v = \lim_{s \to 0} sG(s)$$
$$= \lim_{s \to 0} s \times \frac{(Ks + b)}{s[s + (a - K)]} = \frac{b}{a - K}$$

Therefore, the steady state error for a unit ramp input is

$$e(\infty) = \frac{1}{K_v} = \frac{a-K}{b}$$

B A unity feedback system is characterised by an open loop transfer function **5**

$$G(s) = \frac{K}{s(s+10)}$$

Determine the gain K so that the system will have a damping ratio of 0.5. For this value of K determine the settling time, peak overshoot and time to peak overshoot for unit step input

The closed loop transfer function of the given unity feedback system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}} = \frac{K}{s^2+10s+K}$$

Compare with standard form of transfer function of a second order system

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+10s+K} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n^2 = K \quad \therefore \omega_n = \sqrt{K} \text{ and}$$

$$2\zeta\omega_n = 10$$

$$2 \times 0.5 \times \omega_n = 10 \qquad \therefore \omega_n = 10$$

$$\therefore K = \omega_n^2 = 100$$

The settling time for 5% criterion is

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5 \times 10} = 0.8\ seconds$$

The peak overshoot is

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = e^{-\pi \times 0.5/\sqrt{1-0.5^2}} = 0.163$$

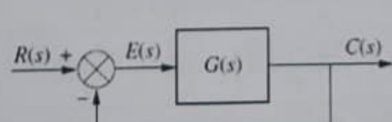
The peak time is

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{10\sqrt{1-0.5^2}} = 0.363\ seconds$$

C For the unity feedback system shown in Figure, where **4**

$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

Find the steady-state errors for the following test inputs:
$25\,u(t), 37\,t\,u(t), and\ 47t^2u(t).$



$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)} = \lim_{s\to 0}\frac{sR(s)}{1+\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}}$$

For step input $R(s) = 25/s$

$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}} = 0$$

For ramp input $R(s) = 37/s^2$

$$e(\infty) = \lim_{s\to 0}\frac{sR(s)}{1+\frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}} = 6.075 \times 10^{-2}$$

	For parabolic input $R(s) = 47/s^3$ $$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1 + \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}} = \infty$$
Q 4 A	By means of Routh criterion, determine the stability of the system represented by the following characteristics equations. For system found to be unstable, determine the number of roots of the characteristics equation in the right half of the s plane. $s^6 + 2s^5 + s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$.

s^6	1	1	3	5
s^5	2	2	4	
s^4	$\frac{2\times 1 - 1\times 2}{2} = 0$	$\frac{2\times 3 - 1\times 4}{2} = 1$	$\frac{2\times 5 - 1\times 0}{2} = 5$	
s^3	∞			

The first element in the s^4 row its zero, whereas there are some non-zero elements in the same row. So the system is unstable. To find the location of the roots replace the first zero element by a small positive number epsilon and proceed with formation of root table

s^6	1	1	3	5
s^5	2	2	4	
s^4	ε	$\frac{2\times 3 - 1\times 4}{2} = 1$	$\frac{2\times 5 - 1\times 0}{2} = 5$	
s^3	$\frac{\varepsilon\times 2 - 2\times 1}{\varepsilon}$	$\frac{4\varepsilon - 10}{\varepsilon}$		
s^2	$\frac{-4\varepsilon^2 + 12\varepsilon - 2}{\varepsilon}$	5		
s^1	$\frac{\left(\frac{-4\varepsilon^2+12\varepsilon-2}{\varepsilon}\right)\left(\frac{4\varepsilon-10}{\varepsilon}\right) - \left(\frac{2\varepsilon-2}{\varepsilon}\right)\times 5}{\frac{-4\varepsilon^2+12\varepsilon-2}{\varepsilon}}$	0		
s^0	5			

As $\varepsilon \to 0$, there are 2 sign changes in the elements of first column of Routh array. Therefore, there are 2 roots of the characteristic equation in the right half of the s plane. So the system is unstable.

B	By means of Routh criterion, ddetermine the stability of system using Routh-Hurwitz criteria having characteristic equation $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$. Also find the closed loop poles.	5

s^5	1	8	7
s^4	4	8	4
s^4	1	2	1

s^3	$\frac{1\times 8-1\times 2}{1}=6$	$\frac{1\times 7-1\times 1}{1}=6$	
s^2	$\frac{6\times 2-1\times 6}{6}=1$	$\frac{6\times 1-1\times 0}{6}=1$	
s^1	$\frac{1\times 6-6\times 1}{1}=0$	0	
s^0			

All the elements in the s^1 are zeros. So, the Routh's test fails. The system is unstable. To complete the Routh table, form an auxiliary equation using the coefficients of the row s^2 (the row just above the row of zeroes)

$$A(s) = s^2 + 1 = 0$$

Taking the first derivative of auxiliary equation

$$\frac{d}{ds}A(s) = \frac{d}{ds}(s^2 + 1) = 0$$

$$\therefore 2s + 0 = 0$$

Replace the row of zeros by the elements of first derivative of the auxiliary equation and process with the formation of Routh table

s^5	1	8	7
s^4	1	2	1
s^3	6	6	
s^2	1	1	
s^1	2	0	
s^0	1		

Since all the elements in the first column of the Routh array positive, there are no roots of characteristics equation in the right half of the s plane. Still the system is unstable due to existence of the row of zeros, which means that there must be roots on imaginary axis of the s plane. To determine the roots on imaginary axis, solve the auxiliary equation. To determine the other roots divide characteristics equation by auxiliary equation.

$$s^2 + 1 = 0, \qquad s^2 = -1 \qquad i.e.\ \ s = \pm j1$$

To find the other poles factorize the characteristic equation

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = (s^2 + 1)(s^3 + 4s^2 + 7s + 4)$$

$$= (s^2 + 1)(s + 1)(s^2 + 3s + 4)$$

$$= (s^2 + 1)(s + 1)(s + 1.5 + j1.3229)(s + 1.5 - j1.3229)$$

Therefore, the poles are at

$$s = \pm j1, \quad s = -1, \quad s = -1.5 \pm j1.3229$$

C	Determine the range of K for the system to be stable by using Routh array. $s^3 + 3Ks^2 + (K + 2)s + 4 = 0$.	3

$$\infty > K > 0.526 \quad or \quad 0.526 < K < \infty$$

Q 5 A	Sketch the root locus for $$G(s)H(s) = \frac{K}{(s+2)(s+4)(s+8)}$$ Find K for stability	8
	Number of poles 3 number of zeros 0 Number of root locus branches = 3 Centroid = (-2-4-8)/3 = -14/3 = -4.6667 Angle $\theta = \frac{(2q+1)180^0}{P-Z}; \qquad q = 0, 1, 2, \ldots, (P-Z)-1.$ $\theta_1 = \frac{(2\times 0+1)180^0}{3-0} = 60^0; \ \theta_2 = \frac{(2\times 1+1)180^0}{3-0} = 180^0;$ $\theta_3 = \frac{(2\times 2+1)180^0}{3-0} = 300^0;$ Break away point = -2.9 $j\omega$ axis crossing point = $\pm 7.48j$ K < 718	
B	Find K for the above system (described in Q 5 A) when it is operating with damping factor 0.6.	4
	K = 65	
Q 6 A	Sketch the root locus for	8

Find K for stability $$G(s)H(s) = \frac{K(s+3)}{s^2(s+9)}$$		
Number of poles 3 number of zeros 1 Number of root locus branches = 2 Centroid = -4.5 Angle $\theta = \frac{(2q+1)180^0}{P-Z}; \qquad q = 0, 1, 2, \ldots, (P-Z)-1.$ $\theta_1 = \frac{(2\times 0+1)180^0}{3-1} = 90^0; \ \theta_2 = \frac{(2\times 1+1)180^0}{3-1} = 270^0;$ Break away point = NIL $j\omega$ axis crossing point = Nil K > 0		
Find dominant pole for the above system (described in Q 6 A) when the system is operating with 0.5 damping factor.	4	
No dominant pole		