Nonparametric Statistics

1 Nonparametric Tests

Hypothesis-testing procedures can be based on the assumption that the random samples are selected from normal populations. Fortunately, most of these tests are still reliable when we experience slight departures from normality, particularly when the sample size is large. Traditionally, these testing procedures have been referred to as **parametric methods**. In this chapter, we consider a number of alternative test procedures, called **nonparametric** or **distribution-free methods**, that often assume no knowledge whatsoever about the distributions of the underlying populations, except perhaps that they are continuous.

Nonparametric, or distribution-free procedures, are used with increasing frequency by data analysts. There are many applications in science and engineering where the data are reported as values not on a continuum but rather on an **ordinal scale** such that it is quite natural to assign ranks to the data. In fact, the reader may notice quite early in this chapter that the distribution-free methods described here involve an *analysis of ranks*. Most analysts find the computations involved in nonparametric methods to be very appealing and intuitive.

For an example where a nonparametric test is applicable, consider the situation in which two judges rank five brands of premium beer by assigning a rank of 1 to the brand believed to have the best overall quality, a rank of 2 to the second best, and so forth. A nonparametric test could then be used to determine whether there is any agreement between the two judges.

We should also point out that there are a number of disadvantages associated with nonparametric tests. Primarily, they do not utilize all the information provided by the sample, and thus a nonparametric test will be less efficient than the corresponding parametric procedure when both methods are applicable. Consequently, to achieve the same power, a nonparametric test will require a larger sample size than will the corresponding parametric test.

As we indicated earlier, slight departures from normality result in minor deviations from the ideal for the standard parametric tests. This is particularly true for the t-test and the F-test. In the case of the t-test and the F-test, the P-value

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quoted may be slightly in error if there is a moderate violation of the normality assumption.

In summary, if a parametric and a nonparametric test are both applicable to the same set of data, we should carry out the more efficient parametric technique. However, we should recognize that the assumptions of normality often cannot be justified and that we do not always have quantitative measurements. It is fortunate that statisticians have provided us with a number of useful nonparametric procedures. Armed with nonparametric techniques, the data analyst has more ammunition to accommodate a wider variety of experimental situations. It should be pointed out that even under the standard normal theory assumptions, the efficiencies of the nonparametric techniques are remarkably close to those of the corresponding parametric procedure. On the other hand, serious departures from normality will render the nonparametric method much more efficient than the parametric procedure.

Sign Test

The procedures for testing the null hypothesis where $\mu = \mu_0$ is valid only if the population is approximately normal or if the sample is large. If $n < 30$ and the population is decidedly nonnormal, we must resort to a nonparametric test.

The sign test is used to test hypotheses on a population *median*. In the case of many of the nonparametric procedures, the mean is replaced by the median as the pertinent **location parameter** under test. The population counterpart, denoted by $\tilde{\mu}$, has an analogous definition. Given a random variable X, $\tilde{\mu}$ is defined such that $P(X > \tilde{\mu}) \leq 0.5$ and $P(X < \tilde{\mu}) \leq 0.5$. In the continuous case,

$$P(X > \tilde{\mu}) = P(X < \tilde{\mu}) = 0.5.$$

Of course, if the distribution is symmetric, the population mean and median are equal. In testing the null hypothesis H_0 that $\tilde{\mu} = \tilde{\mu}_0$ against an appropriate alternative, on the basis of a random sample of size n, we replace each sample value exceeding $\tilde{\mu}_0$ with a *plus* sign and each sample value less than $\tilde{\mu}_0$ with a *minus* sign. If the null hypothesis is true and the population is symmetric, the sum of the plus signs should be approximately equal to the sum of the minus signs. When one sign appears more frequently than it should based on chance alone, we reject the hypothesis that the population median $\tilde{\mu}$ is equal to $\tilde{\mu}_0$.

In theory, the sign test is applicable only in situations where $\tilde{\mu}_0$ cannot equal the value of any of the observations. Although there is a zero probability of obtaining a sample observation exactly equal to $\tilde{\mu}_0$ when the population is continuous, nevertheless, in practice a sample value equal to $\tilde{\mu}_0$ will often occur from a lack of precision in recording the data. When sample values equal to $\tilde{\mu}_0$ are observed, they are excluded from the analysis and the sample size is correspondingly reduced.

The appropriate test statistic for the sign test is the binomial random variable X, representing the number of plus signs in our random sample. If the null hypothesis that $\tilde{\mu} = \tilde{\mu}_0$ is true, the probability that a sample value results in either a plus or a minus sign is equal to 1/2. Therefore, to test the null hypothesis that

$\tilde{\mu} = \tilde{\mu}_0$. we actually test the null hypothesis that the number of plus signs is a value of a random variable having the binomial distribution with the parameter $p = 1/2$. P-values for both one-sided and two-sided alternatives can then be calculated using this binomial distribution. For example, in testing

$$H_0: \ \tilde{\mu} = \tilde{\mu}_0,$$
$$H_1: \ \tilde{\mu} < \tilde{\mu}_0,$$

we shall reject H_0 in favor of H_1 only if the proportion of plus signs is sufficiently less than 1/2, that is, when the value x of our random variable is small. Hence, if the computed P-value

$$P = P(X \leq x \text{ when } p = 1/2)$$

is less than or equal to some preselected significance level α, we reject H_0 in favor of H_1. For example, when $n = 15$ and $x = 3$, we find from Table Binomial Probability Sums at the end of this chapter that

$$P = P(X \leq 3 \text{ when } p = 1/2) = \sum_{x=0}^{3} b\left(x; 15, \frac{1}{2}\right) = 0.0176,$$

so the null hypothesis $\tilde{\mu} = \tilde{\mu}_0$ can certainly be rejected at the 0.05 level of significance but not at the 0.01 level.

To test the hypothesis

$$H_0: \ \tilde{\mu} = \tilde{\mu}_0,$$
$$H_1: \ \tilde{\mu} > \tilde{\mu}_0,$$

we reject H_0 in favor of H_1 only if the proportion of plus signs is sufficiently greater than 1/2, that is, when x is large. Hence, if the computed P-value

$$P = P(X \geq x \text{ when } p = 1/2)$$

is less than α, we reject H_0 in favor of H_1. Finally, to test the hypothesis

$$H_0: \ \tilde{\mu} = \tilde{\mu}_0,$$
$$H_1: \ \tilde{\mu} \neq \tilde{\mu}_0,$$

we reject H_0 in favor of H_1 when the proportion of plus signs is significantly less than or greater than 1/2. This, of course, is equivalent to x being sufficiently small or sufficiently large. Therefore, if $x < n/2$ and the computed P-value

$$P = 2P(X \leq x \text{ when } p = 1/2)$$

is less than or equal to α, or if $x > n/2$ and the computed P-value

$$P = 2P(X \geq x \text{ when } p = 1/2)$$

is less than or equal to α, we reject H_0 in favor of H_1.

Whenever $n > 10$, binomial probabilities with $p = 1/2$ can be approximated from the normal curve, since $np = nq > 5$. Suppose, for example, that we wish to test the hypothesis

$$H_0\colon\ \tilde{\mu} = \tilde{\mu}_0,$$
$$H_1\colon\ \tilde{\mu} < \tilde{\mu}_0,$$

at the $\alpha = 0.05$ level of significance, for a random sample of size $n = 20$ that yields $x = 6$ plus signs. Using the normal curve approximation with

$$\tilde{\mu} = np = (20)(0.5) = 10$$

and

$$\sigma = \sqrt{npq} = \sqrt{(20)(0.5)(0.5)} = 2.236,$$

we find that

$$z = \frac{6.5 - 10}{2.236} = -1.57.$$

Therefore,

$$P = P(X \le 6) \approx P(Z < -1.57) = 0.0582,$$

which leads to the nonrejection of the null hypothesis.

Example 1: The following data represent the number of hours that a rechargeable hedge trimmer operates before a recharge is required:

$$1.5, 2.2, 0.9, 1.3, 2.0, 1.6, 1.8, 1.5, 2.0, 1.2, 1.7.$$

Use the sign test to test the hypothesis, at the 0.05 level of significance, that this particular trimmer operates a median of 1.8 hours before requiring a recharge.

Solution:

1. $H_0\colon\ \tilde{\mu} = 1.8$.
2. $H_1\colon\ \tilde{\mu} \neq 1.8$.
3. $\alpha = 0.05$.
4. Test statistic: Binomial variable X with $p = \frac{1}{2}$.
5. Computations: Replacing each value by the symbol "+" if it exceeds 1.8 and by the symbol "−" if it is less than 1.8 and discarding the one measurement that equals 1.8, we obtain the sequence

$$- + - - + - - + - -$$

for which $n = 10$, $x = 3$, and $n/2 = 5$. Therefore, from Table Binomial Probability Sums the computed P-value is

$$P = 2P\left(X \le 3 \text{ when } p = \frac{1}{2}\right) = 2\sum_{x=0}^{3} b\left(x; 10, \frac{1}{2}\right) = 0.3438 > 0.05.$$

6. Decision: Do not reject the null hypothesis and conclude that the median operating time is not significantly different from 1.8 hours. ┘

We can also use the sign test to test the null hypothesis $\tilde{\mu}_1 - \tilde{\mu}_2 = d_0$ for paired observations. Here we replace each difference, d_i, with a plus or minus sign depending on whether the adjusted difference, $d_i - d_0$, is positive or negative. Throughout this section, we have assumed that the populations are symmetric. However, even if populations are skewed, we can carry out the same test procedure, but the hypotheses refer to the population medians rather than the means.

Example 2: A taxi company is trying to decide whether the use of radial tires instead of regular belted tires improves fuel economy. Sixteen cars are equipped with radial tires and driven over a prescribed test course. Without changing drivers, the same cars are then equipped with the regular belted tires and driven once again over the test course. The gasoline consumption, in kilometers per liter, is given in Table 1. Can we conclude at the 0.05 level of significance that cars equipped with radial tires obtain better fuel economy than those equipped with regular belted tires?

Table 1: Data for Example 2

Car	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Radial Tires	4.2	4.7	6.6	7.0	6.7	4.5	5.7	6.0
Belted Tires	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8
Car	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Radial Tires	7.4	4.9	6.1	5.2	5.7	6.9	6.8	4.9
Belted Tires	6.9	4.9	6.0	4.9	5.3	6.5	7.1	4.8

Solution: Let $\tilde{\mu}_1$ and $\tilde{\mu}_2$ represent the median kilometers per liter for cars equipped with radial and belted tires, respectively.

1. H_0: $\tilde{\mu}_1 - \tilde{\mu}_2 = 0$.
2. H_1: $\tilde{\mu}_1 - \tilde{\mu}_2 > 0$.
3. $\alpha = 0.05$.
4. Test statistic: Binomial variable X with $p = 1/2$.
5. Computations: After replacing each positive difference by a "+" symbol and each negative difference by a "−" symbol and then discarding the two zero differences, we obtain the sequence

$$+ \; - \; + \; + \; - \; + \; + \; + \; + \; + \; + \; + \; - \; +$$

for which $n = 14$ and $x = 11$. Using the normal curve approximation, we find

$$z = \frac{10.5 - 7}{\sqrt{(14)(0.5)(0.5)}} = 1.87,$$

and then

$$P = P(X \geq 11) \approx P(Z > 1.87) = 0.0307.$$

6. Decision: Reject H_0 and conclude that, on the average, radial tires do improve fuel economy.

Not only is the sign test one of the simplest nonparametric procedures to apply; it has the additional advantage of being applicable to dichotomous data that cannot be recorded on a numerical scale but can be represented by positive and negative responses. For example, the sign test is applicable in experiments where a qualitative response such as "hit" or "miss" is recorded, and in sensory-type experiments where a plus or minus sign is recorded depending on whether the taste tester correctly or incorrectly identifies the desired ingredient.

We shall attempt to make comparisons between many of the nonparametric procedures and the corresponding parametric tests. In the case of the sign test the competition is, of course, the t-test. If we are sampling from a normal distribution, the use of the t-test will result in a larger power for the test. If the distribution is merely symmetric, though not normal, the t-test is preferred in terms of power unless the distribution has extremely "heavy tails" compared to the normal distribution.

2 Signed-Rank Test

The reader should note that the sign test utilizes only the plus and minus signs of the differences between the observations and $\tilde{\mu}_0$ in the one-sample case, or the plus and minus signs of the differences between the pairs of observations in the paired-sample case; it does not take into consideration the magnitudes of these differences. A test utilizing both direction and magnitude, proposed in 1945 by Frank Wilcoxon, is now commonly referred to as the **Wilcoxon signed-rank test**.

The analyst can extract more information from the data in a nonparametric fashion if it is reasonable to invoke an additional restriction on the distribution from which the data were taken. The Wilcoxon signed-rank test applies in the case of a **symmetric continuous distribution**. Under this condition, we can test the null hypothesis $\tilde{\mu} = \tilde{\mu}_0$. We first subtract $\tilde{\mu}_0$ from each sample value, discarding all differences equal to zero. The remaining differences are then ranked without regard to sign. A rank of 1 is assigned to the smallest absolute difference (i.e., without sign), a rank of 2 to the next smallest, and so on. When the absolute value of two or more differences is the same, assign to each the average of the ranks that would have been assigned if the differences were distinguishable. For example, if the fifth and sixth smallest differences are equal in absolute value, each is assigned a rank of 5.5. If the hypothesis $\tilde{\mu} = \tilde{\mu}_0$ is true, the total of the ranks corresponding to the positive differences should nearly equal the total of the ranks corresponding to the negative differences. Let us represent these totals by w_+ and w_-, respectively. We designate the smaller of w_+ and w_- by w.

In selecting repeated samples, we would expect w_+ and w_-, and therefore w, to vary. Thus, we may think of w_+, w_-, and w as values of the corresponding random variables W_+, W_-, and W. The null hypothesis $\tilde{\mu} = \tilde{\mu}_0$ can be rejected in favor of the alternative $\tilde{\mu} < \tilde{\mu}_0$ only if w_+ is small and w_- is large. Likewise, the alternative $\tilde{\mu} > \tilde{\mu}_0$ can be accepted only if w_+ is large and w_- is small. For a two-sided alternative, we may reject H_0 in favor of H_1 if either w_+ or w_-, and hence w, is sufficiently small. Therefore, no matter what the alternative hypothesis may be, we reject the null hypothesis when the value of the appropriate statistic W_+, W_-, or W is sufficiently small.

Two Samples with Paired Observations

To test the null hypothesis that we are sampling two continuous symmetric populations with $\tilde{\mu}_1 = \tilde{\mu}_2$ for the paired-sample case, we rank the differences of the paired observations without regard to sign and proceed as in the single-sample case. The various test procedures for both the single- and paired-sample cases are summarized in Table 2.

Table 2: Signed-Rank Test

H_0	H_1	Compute
$\tilde{\mu} = \tilde{\mu}_0$	$\tilde{\mu} < \tilde{\mu}_0$	w_+
	$\tilde{\mu} > \tilde{\mu}_0$	w_-
	$\tilde{\mu} \neq \tilde{\mu}_0$	w
$\tilde{\mu}_1 = \tilde{\mu}_2$	$\tilde{\mu}_1 < \tilde{\mu}_2$	w_+
	$\tilde{\mu}_1 > \tilde{\mu}_2$	w_-
	$\tilde{\mu}_1 \neq \tilde{\mu}_2$	w

It is not difficult to show that whenever $n < 5$ and the level of significance does not exceed 0.05 for a one-tailed test or 0.10 for a two-tailed test, all possible values of $w+$, $w-$,or w will lead to the acceptance of the null hypothesis. However, when $5 \leq n \leq 30$, Table Critical Values for the Signed-Rank Test shows approximate critical values of W_+ and $W\_$ for levels of significance equal to 0.01, 0.025, and 0.05 for a one-tailed test and critical values of W for levels of significance equal to 0.02, 0.05, and 0.10 for a two-tailed test. The null hypothesis is rejected if the computed value w_+, $w\_$, or w is **less than or equal to** the appropriate tabled value. For example, when $n = 12$, Table Critical Values for the Signed-Rank Test at the end of this chapter shows that a value of $w_+ \leq 17$ is required for the one-sided alternative $\tilde{\mu} < \tilde{\mu}_0$ to be significant at the 0.05 level.

Example 3: Rework Example 1 by using the signed-rank test.

Solution:

1. H_0: $\tilde{\mu} = 1.8$.
2. H_1: $\tilde{\mu} \neq 1.8$.
3. $\alpha = 0.05$.
4. Critical region: Since $n = 10$ after discarding the one measurement that equals 1.8, Table Critical Values for the Signed-Rank Test shows the critical region to be $w \leq 8$.
5. Computations: Subtracting 1.8 from each measurement and then ranking the differences without regard to sign, we have

d_i	-0.3	0.4	-0.9	-0.5	0.2	-0.2	-0.3	0.2	-0.6	-0.1
Ranks	5.5	7	10	8	3	3	5.5	3	9	1

Now $w_+ = 13$ and $w_- = 42$, so $w = 13$, the smaller of w_+ and w_-.

6. Decision: As before, do not reject H_0 and conclude that the median operating time is not significantly different from 1.8 hours. ┛

The signed-rank test can also be used to test the null hypothesis that $\tilde{\mu}_1 - \tilde{\mu}_2 = d_0$. In this case, the populations need not be symmetric. As with the sign test, we subtract d_0 from each difference, rank the adjusted differences without regard to sign, and apply the same procedure as above.

Example 4: It is claimed that a college senior can increase his or her score in the major field area of the graduate record examination by at least 50 points if he or she is provided with sample problems in advance. To test this claim, 20 college seniors are divided into 10 pairs such that the students in each matched pair have almost the same overall grade-point averages for their first 3 years in college. Sample problems and answers are provided at random to one member of each pair 1 week prior to the examination. The examination scores are given in Table 3.

Table 3: Data for Example 4

	Pair									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
With Sample Problems	531	621	663	579	451	660	591	719	543	575
Without Sample Problems	509	540	688	502	424	683	568	748	530	524

Test the null hypothesis, at the 0.05 level of significance, that sample problems increase scores by 50 points against the alternative hypothesis that the increase is less than 50 points.

***Solution*:** Let $\tilde{\mu}_1$ and $\tilde{\mu}_2$ represent the median scores of all students taking the test in question with and without sample problems, respectively.

1. H_0: $\tilde{\mu}_1 - \tilde{\mu}_2 = 50$.
2. H_1: $\tilde{\mu}_1 - \tilde{\mu}_2 < 50$.
3. $\alpha = 0.05$.
4. Critical region: Since $n = 10$, Table A.16 shows the critical region to be $w_+ \leq 11$.
5. Computations:

	Pair									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
d_i	22	81	−25	77	27	−23	23	−29	13	51
$d_i - d_0$	−28	31	−75	27	−23	−73	−27	−79	−37	1
Ranks	5	6	9	3.5	2	8	3.5	10	7	1

Now we find that $w_+ = 6 + 3.5 + 1 = 10.5$.

6. Decision: Reject H_0 and conclude that sample problems do not, on average, increase one's graduate record score by as much as 50 points. ┛

Normal Approximation for Large Samples

When $n \geq 15$, the sampling distribution of W_+ (or W_-) approaches the normal distribution with mean and variance given by

$$\mu_{W_+} = \frac{n(n+1)}{4} \text{ and } \sigma^2_{W_+} = \frac{n(n+1)(2n+1)}{24}.$$

Therefore, when n exceeds the largest value in Table A.16, the statistic

$$Z = \frac{W_+ - \mu_{W_+}}{\sigma_{W_+}}$$

can be used to determine the critical region for the test.

Exercises

1 The following data represent the time, in minutes, that a patient has to wait during 12 visits to a doctor's office before being seen by the doctor:

17	15	20	20	32	28
12	26	25	25	35	24

Use the sign test at the 0.05 level of significance to test the doctor's claim that the median waiting time for her patients is not more than 20 minutes.

2 The following data represent the number of hours of flight training received by 18 student pilots from a certain instructor prior to their first solo flight:

9	12	18	14	12	14	12	10	16
11	9	11	13	11	13	15	13	14

Using binomial probabilities from Table A.1, perform a sign test at the 0.02 level of significance to test the instructor's claim that the median time required before his students' solo is 12 hours of flight training.

3 A food inspector examined 16 jars of a certain brand of jam to determine the percent of foreign impurities. The following data were recorded:

2.4	2.3	3.1	2.2	2.3	1.2	1.0	2.4
1.7	1.1	4.2	1.9	1.7	3.6	1.6	2.3

Using the normal approximation to the binomial distribution, perform a sign test at the 0.05 level of significance to test the null hypothesis that the median percent of impurities in this brand of jam is 2.5% against the alternative that the median percent of impurities is not 2.5%.

4 A paint supplier claims that a new additive will reduce the drying time of its acrylic paint. To test this claim, 12 panels of wood were painted, one-half of each panel with paint containing the regular additive and the other half with paint containing the new additive. The drying times, in hours, were recorded as follows:

	Drying Time (hours)	
Panel	New Additive	Regular Additive
1	6.4	6.6
2	5.8	5.8
3	7.4	7.8
4	5.5	5.7
5	6.3	6.0
6	7.8	8.4
7	8.6	8.8
8	8.2	8.4
9	7.0	7.3
10	4.9	5.8
11	5.9	5.8
12	6.5	6.5

Use the sign test at the 0.05 level to test the null hypothesis that the new additive is no better than the regular additive in reducing the drying time of this kind of paint.

5 It is claimed that a new diet will reduce a person's weight by 4.5 kilograms, on average, in a period of 2 weeks. The weights of 10 women were recorded before and after a 2-week period during which they followed this diet, yielding the following data:

Woman	Weight Before	Weight After
1	58.5	60.0
2	60.3	54.9
3	61.7	58.1
4	69.0	62.1
5	64.0	58.5
6	62.6	59.9
7	56.7	54.4
8	63.6	60.2
9	68.2	62.3
10	59.4	58.7

Use the sign test at the 0.05 level of significance to test the hypothesis that the diet reduces the median weight by 4.5 kilograms against the alternative hypothesis that the median weight loss is less than 4.5 kilograms.

6 Two types of instruments for measuring the amount of sulfur monoxide in the atmosphere are being compared in an air-pollution experiment. The following readings were recorded daily for a period of 2 weeks:

	Sulfur Monoxide	
Day	**Instrument *A***	**Instrument *B***
1	0.96	0.87
2	0.82	0.74
3	0.75	0.63
4	0.61	0.55
5	0.89	0.76
6	0.64	0.70
7	0.81	0.69
8	0.68	0.57
9	0.65	0.53
10	0.84	0.88
11	0.59	0.51
12	0.94	0.79
13	0.91	0.84
14	0.77	0.63

Using the normal approximation to the binomial distribution, perform a sign test to determine whether the different instruments lead to different results. Use a 0.05 level of significance.

7 The following figures give the systolic blood pressure of 16 joggers before and after an 8-kilometer run:

Jogger	Before	After
1	158	164
2	149	158
3	160	163
4	155	160
5	164	172
6	138	147
7	163	167
8	159	169
9	165	173
10	145	147
11	150	156
12	161	164
13	132	133
14	155	161
15	146	154
16	159	170

Use the sign test at the 0.05 level of significance to test the null hypothesis that jogging 8 kilometers increases the median systolic blood pressure by 8 points against the alternative that the increase in the median is less than 8 points.

8 Analyze the data of Exercise 1 by using the signed-rank test.

9 Analyze the data of Exercise 2 by using the signed-rank test.

10 The weights of 5 people before they stopped smoking and 5 weeks after they stopped smoking, in kilograms, are as follows:

	Individual				
	1	**2**	**3**	**4**	**5**
Before	66	80	69	52	75
After	71	82	68	56	73

Use the signed-rank test for paired observations to test the hypothesis, at the 0.05 level of significance, that giving up smoking has no effect on a person's weight against the alternative that one's weight increases if he or she quits smoking.

11 Rework Exercise 5 by using the signed-rank test.

12 The following are the numbers of prescriptions filled by two pharmacies over a 20-day period:

Day	Pharmacy *A*	Pharmacy *B*
1	19	17
2	21	15
3	15	12
4	17	12
5	24	16
6	12	15
7	19	11
8	14	13
9	20	14
10	18	21
11	23	19
12	21	15
13	17	11
14	12	10
15	16	20
16	15	12
17	20	13
18	18	17
19	14	16
20	22	18

Use the signed-rank test at the 0.01 level of significance to determine whether the two pharmacies, on average, fill the same number of prescriptions against the alternative that pharmacy *A* fills more prescriptions than pharmacy *B*.

13 Rework Exercise 7 by using the signed-rank test.

14 Rework Exercise 6 by using the signed-rank test.