College of Engineering Pune

Linear Algebra and Univariate Calculus(D.S.Y)

Tutorial 4

Linear Mappings, Kernel and image of a linear map, Rank nullity theorem

1. Let $T : V \to W$ be a linear transformation. Show that:

   (a) $T(0) = 0$.

   (b) $T(-v) = -T(v)$ for all $v \in V$

2. Determine which of the following mappings F are linear. If linear, then find its kernel and image space. Also find Nullity and rank and hence verify Rank-Nullity theorem.

   Onto (a) $F : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $F(x, y, z) = (x, z)$ Dim ker-1 img-2

   1-1; onto (b) $F : \mathbb{R}^4 \to \mathbb{R}^4$ defined by $F(x, y, z, w) = (-x, -y, -z, -w)$ Dim ker-0 img-4

   (c) $F : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $F(x, y, z) = (x, y, z) + (0, 1, 0)$

   1-1; onto (d) $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (x - y, 2y)$ Dim ker-0 img-2

   (e) $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (xy, x + y)$ Dim ker-0 img-2

   1-1; onto (f) $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (y, x)$ Dim ker-0 img-2

   (g) $F : \mathbb{R}^2 \to \mathbb{R}$ defined by $F(x, y, z) = xy$

   (h) $F : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $F(x, y) = (x, y + 1)$

   Onto (i) $F : \mathbb{R}^3 \to \mathbb{R}$ defined by $F(x, y, z) = 3x - 2y + z$ Dim ker-2 img-1

   (j) Let D be a derivative map from set of differentiable functions to set of differentiable functions i.e., $D(f) = \frac{df}{dx}$.

   (k) Let D^2 be a double derivative map from set of twice differentiable functions to set of twice differentiable functions i.e., $D^2(f) = \frac{d^2 f}{dx^2}$.

   (l) Let M be the space of all 2 x 2 matrices. Let, $P : M \to M$ be a map such that $P(A) = \frac{A + {}^t A}{2}$. Generalize to nxn matrices. Dim ker-1 img-3

   (m) Let M be the space of all 2 x 2 matrices. Let, $P : M \to M$ be a map such that $P(A) = \frac{A - {}^t A}{2}$. Generalize to nxn matrices. Dim ker-3 img-1

   Onto (n) Let M be the space of all 2 x 2 matrices. Let, $P : M \to M$ be a map such that $P(A) = trace(A)$. Generalize to n x n matrices.

   Dim ker-3 img-1

3. Using Kernel classify whether above functions are one-one or not. Further, using Rank-Nullity theorem conclude whether function is onto or not.

4. What is the dimension of the space of solutions of the following systems of lineat equations? In each case, find a basis for the space of solutions.

enter in the dimention etc cal without including the zeros

(a) Basis : {1, -2 , 0}
Dim : 1

$$2x + y - z = 0$$
$$2x + y + z = 0$$

(b) Solution set : {0,0,0}
Dim : 0

$$x + y + z = 0$$
$$x - y = 0$$
$$y + z = 0$$

(c) Dim : 0

$$2x - 3y + z = 0$$
$$x + y - z = 0$$
$$3x + 4y = 0$$
$$5x + y + z = 0$$

(d) Basis : {(-7+pi)/18, (2+pi)/9, 1
Dim : 1

$$4x + 7y - \pi z = 0$$
$$2x - y + z = 0$$

5. Let A be a fixed mxn matrix. Let. $T : \mathbb{R}^n \to \mathbb{R}^m$ be a map defined as: $T(X) = AX$ where X is a n x 1 vector in $\mathbb{R}^n$. Show that T is a linear transformation.

6. In above example, Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 7 \end{bmatrix}$. Find Null space of T, Image space of T. Hence conclude Nullity(T) and Rank(T). Further verify Rank-Nullity theorem. Dim ker-1 img-2

7. Take a 3 x 4 matrix of your choice and do the above things. (Don't take a null matrix :) . Also try to take distinct entries!)

8. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (2, 3)$. Find $T(a, b)$ for any $(a, b) \in \mathbb{R}^2$. Hence calculate image of $(3, 7)$. (17, 24)

9. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(1, 0, 0) = (1, 1, 0)$, $T(0, 1, 0) = (2, 3, 0)$ and $T(0, 0, 1) = (1, 0, 5)$. Find $T(a, b, c)$ for any $(a, b, c) \in \mathbb{R}^3$. Hence calculate image of $(3, 7, 1)$. (18,24,5)