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Tutorial No : 1

Q1) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Compute A^2, A^3, A^4 and $B^2; B^3$. Generalize A and B to 4x4 matrix.

$\rightarrow$ $A^2 = A \cdot A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0+0 & 1+1+0 & 1+1+1 \\ 0+0+0 & 0+1+0 & 0+1+1 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A$

$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0+0 & 1+2+0 & 1+2+3 \\ 0+0+0 & 0+1+0 & 0+1+2 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$

$\therefore A^3 = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

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$A^4 = A^3 \cdot A$

$$= \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 1+3+0 & 1+3+6 \\ 0+0+0 & 0+1+0 & 0+1+3 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$\therefore A^4 = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

— (Diagonal and below diagonal are zero, are called as strictly upper triangular matrises).

$B^2 = B \cdot B$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{bmatrix}$$

$$\therefore B^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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$B^3 = B^2 \cdot B$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{bmatrix}$$

$$\therefore B^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Generalize format of (4×4) matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}, \quad B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times4}$$

where, $m = 4$ and $n = 4$

Q2.) Let A be a diagonal matrix with diagonal elements $a_1, a_2, \ldots, a_n$. What is A^2, A^3, A^K for any positive integer K.

→

$$A = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & \ddots & a_n \end{bmatrix}$$

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$$A^2 = \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_n \end{bmatrix} \cdot \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_n \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} a_1^2 & 0 & 0 & 0 \\ 0 & a_2^2 & 0 & 0 \\ 0 & 0 & a_3^2 & 0 \\ 0 & 0 & 0 & a_n^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a_1^2 & 0 & 0 & 0 \\ 0 & a_2^2 & 0 & 0 \\ 0 & 0 & a_3^2 & 0 \\ 0 & 0 & 0 & a_n^2 \end{bmatrix} \cdot \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 \\ 0 & 0 & a_3 & 0 \\ 0 & 0 & 0 & a_n \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a_1^3 & 0 & 0 & 0 \\ 0 & a_2^3 & 0 & 0 \\ 0 & 0 & a_3^3 & 0 \\ 0 & 0 & 0 & a_n^3 \end{bmatrix}$$

$\therefore$ In general, for any positive integer k, we can write:

$$A^k = \begin{bmatrix} a_1^k & 0 & 0 & 0 \\ 0 & a_2^k & 0 & 0 \\ 0 & 0 & a_3^k & 0 \\ 0 & 0 & 0 & a_n^k \end{bmatrix}$$

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Q3.) Let A be a square matrix.

a.) If $A^2 = 0$, show that $I-A$ is investible.

→ Let us consider,

$I^2 - A^2 = (I-A)\cdot(I+A)$

as $A^2 = 0$, —(given)

$\therefore (I-A)(I+A) = I^2$

$\therefore (I-A)(I+A) = I$ —($I^2 = I$)

It means that there exists a matrix $(I+A)$ which makes $(I-A)$ equal to I.

$\therefore$ By defination,

$(I-A)$ is investible.

b.) If $A^3 = 0$, show that $I-A$ is investible.

→ Let us consider,

$I^3 - A^3 = (I-A)\cdot(I^2 + IA + A^2)$

As $A^3 = 0$ —(given)

$\therefore (I-A)\cdot(I+A+A^2) = \cancel{(I-A^3)}\ I$ —($I\cdot A = A$ & $I^2 = I$)

It means that their exists a matrix $(I+A+A^2)$ which makes $(I-A)$ equal to I.

$\therefore$ By defination,

$(I-A)$ is investible.

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c.) In general, if $A^n = 0$ for some positive integer n, show that $I-A$ is invertible.

→ Let us consider

$I^n - A^n = (I-A)(I^{n-1} + I^{n-2}.A + I^{n-3}.A^2 + --- A^{n-1})$

But $A^n = 0$ and $I^n = I$.

Hence, there exist a matrix which makes the $(I-A)$ equal to I.

∴ By defination,

$(I-A)$ is invertible.

[Thm: If A^n is 0, then $I-A$ is always invertible]

Q4.) If the inverse of A^2 is B, show that the inverse of A is AB (Thus A is invertible whenever A^2 is invertible).

→ Inverse of $A^2 = B$.

i.e. $A^2 . B = I$.

$(A.A)B = I$.

By associvity property,

$A(AB) = I$.

∴ $A = AB^{-1}$

∴ It means that A is inverse of AB and vice versa.

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Q5.) a.) If A is invertible and if $AB = AC$, then prove that $B = C$.

→ $AB = \overset{A}{\cancel{B}}C$ —— (i) —— (given).

and A is invertible —— (given). -(Hence, $AB \neq \cancel{B}C$).

Pre-multiplying equ (i) by A^{-1},

∴ $A^{-1}(AB) = A^{-1}(AC)$

By property of associvity,

$(A^{-1}.A)B = (A^{-1}.A)C$

$I.B = I.C$

∴ $B = C$ —— $(IB = B)$.

Hence proved.

b.) If A is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, find an example with $AB = BC$ but $B \neq C$.

→ $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$|A| = 0$.

Hence, A^{-1} doesn't exist.

Let, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 7 & 8 \\ 3 & 4 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}$ —— (i)

$$AC = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 7 & 8 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix} \text{ --- (ii)}$$

From (i) and (ii),

$AB = AC$.

and $B \neq C$ Hence proved.

Q 6.) Give examples of A and B such that:

a.) A + B is not invertible although A and B are invertible.

→ Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$|A| = 1 - 0 = 1$

$|B| = 1 - 0 = 1$

∴ Inverse of A and B exists.

Now, $A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$|A + B| = 0$.

∴ Inverse of A + B doesn't exists.

b.) A + B is invertible although A and B are not invertible.

→ Let, $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$|A| = 4 - 4 = 0$

$|B| = 9 - 9 = 0$.

$A + B = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$A + B = \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$

∴ $|A + B| = 25 - 1 = 24$

Thus, A^{-1} and B^{-1} doesn't exists, but, $(A+B)^{-1}$ exist.

c.) All the A, B and A + B are ~~ver~~ invertible.

→ Let, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

$|A| = 4 - 6 = -2$

$|B| = 2 - 0 = 0$ ∴ $|B| = 2$

Thus, by $|B^{-1}|$ are A ree transforms ∴

Q7) (a) Show that for any square matrix, the matrix $A+A^t$ is symmetric.

→ We know that for symmetric matrix,
$X = X^t$.

$A+A^t = (A+A^t)^t$
$= A^t + (A^t)^t$
$A+A^t = A^t + A$.
∴ LHS = RHS.
∴ matrix $(A+A^t)$ is symmetric matrix.

b.) Show that for any square matrix, the matrix $A-A^t$ is skew symmetric.

→ We know that for skew-symmetric matrix,
$X = -X^t$.

$A-A^t = -(A-A^t)^t$
$= -(A^t-(A^t)^t)$
$= -(A^t - A)$
$= (-)(-)(A-A^t)$
$= A-A^t$.
∴ LHS = RHS.
∴ matrix $(A-A^t)$ is skew-~~m~~ symmetric matrix.

c.) If a matrix is skew-symmetric then what can you say about its diagonal entries.

→ For a skew symmetric matrix, diagonal entries are always zero.

$a_{ii} = -a_{ii}$ — (when $i=j$)
$a_{ii} + a_{ii} = 0$
$2a_{ii} = 0$
∴ $a_{ii} = 0$.

~~b~~d.) Show that any square matrix can always be written as sum of symmetric and skew-symmetric matrix.

→ Let, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

∴ $A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

To prove: $A = \frac{A+A^t}{2} + \frac{A-A^t}{2}$

$\frac{A+A^t}{2} = \frac{1}{2}\begin{bmatrix} 2 & 5 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 5/2 \\ 5/2 & 4 \end{bmatrix}$

$\frac{A-A^t}{2} = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix}$

RHS $= \frac{A+A^t}{2} + \frac{A-A^t}{2}$

$= \begin{bmatrix} 1 & 5/2 \\ 5/2 & 4 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= A$. Hence proved.

RHS = LHS.

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Q.8) Let A be a skew symmetric matrix with odd order, then what can you say about it's determinant?

→ Given: $A = -A^t$.

$n \to$ odd.

$|A| = ?$

$A = -A^t$ —(given).

$|A| = |-A^t|$

By property, $|\alpha \cdot A^t| = \alpha^n |A^t|$.

$\therefore |A| = (-1)^n \cdot |A^t|$

By property: $|A^t| = |A|$.

$\therefore |A| = (-1)^n \cdot |A|$

but, n = odd number,

$\therefore |A| = -|A|$.

$|A| + |A| = 0$.

$2|A| = 0$.

$\therefore |A| = 0$.

$\therefore$ Determinant of A is zero.

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Q9) True or false, with reason if true or counter example if false:

a.) If A and B are symmetric then AB is symmetric.

→ Given: $A = A^t$

$B = B^t$.

To prove: $AB = (AB)^t = ?$

RHS: $(AB)^t$

$= B^t \cdot A^t$

$\cancel{AB} = B \cdot A$ — ($B^t = B$, $A^t = A$ given).

$\neq AB$ —(general)

$\therefore$ RHS $\neq$ LHS

$\therefore$ The given statement is False.

Counterexample: If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

and A and B are symmetric with $AB = BA$, then AB is also symmetric matrix.

b.) If A and B are invertible, then BA is invertible.

→ Given: $A \cdot A^{-1} = I$.

$B \cdot B^{-1} = I$.

To prove: $BA \cdot (BA)^{-1} = I$. i.e. $|BA| \neq 0$.

As A and B are invertible,

$|A| \neq 0$ and $|B| \neq 0$.

If BA is invertible,
then $|BA| \neq 0$.

we know,
$|BA| = |B||A|$
$\neq 0$ — as multiplying two non-zero no., result is non-zero no.

$\therefore |BA| \neq 0$.
$\therefore BA$ is invertible.
Hence, given statement is true.

Q10) Let A and B be two matrices of the same size. We say that A is similar to B if there exists an invertible matrix T such that $B = TAT^{-1}$. Suppose this is the case.
Prove:

a) B is similar to A.

→

When $A \sim B$,
$B = TAT^{-1}$.

$\therefore$ If $B \sim A$,
then prove: $A = Q \cdot B \cdot Q^{-1}$

$B = TAT^{-1}$.
$T^{-1} \cdot B = T^{-1} \cdot T \cdot A \cdot T^{-1}$
$T^{-1} B = A \cdot T^{-1}$ — $(T^{-1} \cdot T = I)$

$T^{-1} \cdot B \cdot T = A \cdot T^{-1} \cdot T$
$T^{-1} \cdot B \cdot T = A$ — $(T^{-1} \cdot T = I)$

$\therefore A = T^{-1} B T$.
Let,
$T^{-1} = Q$. — (i)
$\therefore Q^{-1} = (T^{-1})^{-1}$
$\therefore Q^{-1} = T$. — (ii)

$\therefore A = Q \cdot B \cdot Q^{-1}$ — (from (i), (ii))

As we can write, A in format $Q \cdot B \cdot Q^{-1} = A$,
Hence, $B \sim A$.

b) A is invertible if B is invertible.

→

$|B| \neq 0$, B^{-1} exist — (given).

$B = T \cdot A \cdot T^{-1}$
multiplying b-s by B^{-1},

$B \cdot B^{-1} = T \cdot A \cdot T^{-1} \cdot B^{-1}$
$I = A \cdot B^{-1} \cdot I$ — $(T \cdot T^{-1} = I)$.

$\therefore A(B^{-1} \cdot I) = I$.
Their exist a matrix which makes A equal to I.
$\therefore A$ is invertible.

c.) A^t is similar to B^t :

To prove: $A^t \sim B^t$.

$B = T \cdot A \cdot T^{-1}$.

$B^t = (T \cdot A \cdot T^{-1})^t$

$A = T^{-1} \cdot B \cdot T$.

Let, $\varphi = T^{-1}$,

$\therefore A = \varphi \cdot B \cdot \varphi^{-1}$.

$A^t = (\varphi \cdot B \cdot \varphi^{-1})^t$

$\therefore \underline{A^t \sim B^t}$.

d.) Suppose $A^n = 0$ and B is an invertible matrix of same size as A. Show that $(BAB^{-1})^n = 0$.

Given: $|B| \neq 0$,

$A^n = 0$.

To show : $(BAB^{-1})^n = 0$

Let,

$B = X$.

multiplying b-s by A^n.

$BA^n = XA^n$.

As, $A^n = 0$,

$\therefore B \cdot A^n = 0$.

multiplying both sides by B^{-1},

$\therefore B \cdot A^n \cdot B^{-1} = 0$.

$\underline{(B \cdot A \cdot B^{-1})^n = 0}$.

Hence proved.

Q.12.) Determine the values of a and b for which the system has (i) No solution, ii.) Infinite number of solutions, (iii) Unique solution.

a.) $x + 2y + 3z = 6$.
$x + 3y + 5z = 9$
$2x + 5y + az = b$.

$$\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ b \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ b \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & a-6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ b-12 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & a-8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ b-15 \end{bmatrix}$$

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$x+2y+3z=6$

$y+2z=3$

$(a-8)z=b-15$

i) Let $a=8$ and $b\neq 15$

$0=$ Non-zero number,

which is not possible.

$\therefore$ System has no solution.

ii) Let $a\neq 8$ and $b=15$

$\therefore$ System has unique solution.

iii) Let $a=8$ and $b=15$

$x+2y+3z=6$

$y+2z=3$.

No. of unknown. > No. of equations,

$\therefore$ System has infinitely many solⁿ.

iv) Let $a\neq 8$ and $b\neq 15$

let $a=9$ and $b=16$

$(9-8)z=16-15$

$\underline{z=1}$

$\therefore y=-1$

and $x+2(-1)+3(1)=6$.

$x+1=6$.

$x=5$.

$\therefore$ System has a unique solution.

$\therefore$ We can take any values of a and b except 8 and 15 respectively.

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b.) $2x+3y+5z=9$.

$7x+3y-2z=8$

$2x+3y+az=b$.

$$\longrightarrow \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ b \end{bmatrix}$$

$R_2 \rightarrow R_2 - \frac{7}{2}R_1$

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & -\frac{15}{2} & -\frac{39}{2} \\ 2 & 3 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -47/2 \\ b \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 2 & 3 & 5 \\ 0 & -15/2 & -39/2 \\ 0 & 0 & a-5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -47/2 \\ b-9 \end{bmatrix}$$

$2x+3y+5z=9$.

$-\frac{15}{2}y-\frac{39}{2}z=-\frac{47}{2}$ $\Rightarrow \therefore -15y-39z=-47$.

$(a-5)z=b-9$.

i) $a\neq 5$ and $b=9$

This system has unique solution.

ii) $a=5$ and $b\neq 9$

$0=$ Non-zero no.

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This is not possible,
hence, system has no solution.

iii.) $a = 5$ and $b = 9$

$2x + 3y + 5z = 9$

$-15y - 39z = -47$

No. of unknowns > No. of eqn^s.

$\therefore$ System has infinitely many soln.

iv.) $a \neq 5$ and $b \neq 9$

Let $a = 6$ and $b = 10$.

$\therefore z = 1$

$-15y = -47 + 39(1)$

$= -8$.

$y = \frac{8}{15}$.

Also, $2x = 9 - 3y - 5z$

$= 9 - 3\left(\frac{8}{15}\right) - 5(1)$

$= \frac{20-8}{5} = \frac{12}{5}$.

$x = \frac{6}{5}$.

$\therefore$ System has unique solution.

Q.13) Find inverse of following matrix if exists:

a.) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

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$\rightarrow$ Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

$|A| = 1(-12-12) - 1(-4-6) + 3(-4+6)$

$= -24 + 10 + 6$

$= -8$

$\therefore |A| \neq 0$

A^{-1} exists.

$A \cdot A^{-1} = I$.

$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_2 \Longrightarrow R_2 - R_1$

$R_3 \longrightarrow R_3 + 2R_1$

$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$R_2 \longrightarrow \frac{1}{2} R_2$

$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & -2 & 2 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$R_3 \longrightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & -4 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_1 \longrightarrow R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & -4 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 3/2 & -1/2 & 3 \\ -1/2 & 1/2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_3 \longrightarrow \frac{-1}{4} \cdot R_3$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 3/2 & -1/2 & 3 \\ -1/2 & 1/2 & 0 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$R_2 \longrightarrow R_2 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 3/2 & -1/2 & 3 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 6R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Here, $A = I$,

$$\therefore A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

b.) $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$\longrightarrow$ Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

$|A| = 2(12-2) - 3(16-1) + 4(8-3)$
$= 20 - 45 + 20$
$= 40 - 45$
$|A| = -5$

$\therefore |A| \neq 0$ $\therefore$ Inverse of A exist, i.e. A^{-1} exist.

$A \cdot A^{-1} = I$

$$\therefore \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Interchanging rows :

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 4 \\ 4 & 3 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

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$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -4 \\ 4 & 3 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 4R_1$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -4 \\ 0 & -5 & -15 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & -4 \end{bmatrix}$$

$R_2 \to (-1) \cdot R_2$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & -5 & -15 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & -4 \end{bmatrix}$$

$R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ -5 & 1 & 6 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 2 & 0 & -3 \\ -1 & 0 & 2 \\ -5 & 1 & 6 \end{bmatrix}.$$

$R_3 \to \frac{1}{5} \cdot R_3$

$$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 2 & 0 & -3 \\ 1 & 0 & 2 \\ -1 & 1/5 & 6/5 \end{bmatrix}$$

$R_2 \to R_1 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} -2 & 4/5 & 9/5 \\ -1 & 0 & 2 \\ -1 & 1/5 & 6/5 \end{bmatrix}$$

$R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} -2 & 4/5 & 9/5 \\ -3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix}$$

Now, $A = I$.

$$\therefore A^{-1} = \begin{bmatrix} -2 & 4/5 & 9/5 \\ -3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix}.$$

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