College of Engineering Pune

Linear Algebra and Univariate Calculus(D.S.Y)

Tutorial 3

Spanning Set, Basis, Dimension, Rank of matrix, Application to system of linear equations

1. Describe the subspace spanned by:
   (a) vector $(1,1) \in \mathbb{R}^2$.
   (b) vector $(1,0)$ and $(1,1) \in \mathbb{R}^2$.
   (c) the two vectors $(1,1,-1)$ and $(-1,-1,1) \in \mathbb{R}^3$
   (d) the three vectors $(0,1,1)$, $(1,1,0)$ and $(0,0,0)$.
   (e) the vector $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
   (f) the vectors $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.
   (g) the vector $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
   (h) the vectors $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.
2. What is the dimension of the following spaces.
   (a) Subspace spanned by $B = \{(1,1,0),(0,1,1),(1,0,1)\}$ in $\mathbb{R}^3$ 3
   (b) Subspace spanned by $B = \{(1,1),(2,1)\}$ in $\mathbb{R}^2$ 2
   (c) Subspace spanned by $B = \{(1,1,0,0),(0,0,1,1),(1,0,1,0)\} \subset \mathbb{R}^4$ 3
   (d) all 2x2 matrices. 4
   (e) all mxn matrices.
   (f) all 2x2 symmetric, skew-symmetric, upper triangular, lower triangular,trace 0, scalar, diagonal matrices. Generalize this to nxn matrices. n(n+1)/2 n(n+1)/2 n2-1 2
3. Find a basis for each of these subspaces of $\mathbb{R}^4$:

(a) All vectors whose components are equal.

(b) All vectors whose components add to zero.

4. Find rank of the following matrix:

(a) $\begin{bmatrix} 2 & 1 & 3 \\ 7 & 2 & 0 \end{bmatrix}$ 2

(b) $\begin{bmatrix} -1 & 2 & -2 \\ 3 & 4 & -5 \end{bmatrix}$ 2

(c) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{bmatrix}$ 3

(d) $\begin{bmatrix} 2 & 0 & 0 \\ -5 & 1 & 2 \\ 3 & 8 & -7 \end{bmatrix}$ 3