College of Engineering Pune

Linear Algebra and Univariate Calculus(D.S.Y)

Tutorial 5

Matrices associated to a linear map, Eigenvalues and Eigenvectors.

1. Find the matrix associated with the following linear maps with respect to standard basis.

   (a) $F : \mathbb{R}^4 \to \mathbb{R}^2$ given by $F(x_1, x_2, x_3, x_4) = (x_1, x_2)$.(the projection.)

   (b) The projection from $\mathbb{R}^4$ to $\mathbb{R}^3$.

   (c) $F : \mathbb{R}^2 \to \mathbb{R}^2$ given by $F(x, y) = (3x, 3y)$.

   (d) $F : \mathbb{R}^n \to \mathbb{R}^n$ given by $F(X) = 7X$.

   (e) $F : \mathbb{R}^4 \to \mathbb{R}^2$ given by $F(X) = cX$, where $c \in \mathbb{R}$.

   (f) Find matrices with respect to standard basis for the tansformations given in Question 1 of tut 4.

2. Let V be the vector space generated by the three functions $f_1(t) = 1, f_2(t) = t, f_3(t) = t^2$. Let $D : V \to V$ be the derivative. What is the matrix of D with respect to the basis $\{f_1, f_2, f_3\}$.

3. Let V be the vector space generated by two functions $f_1(t) = cos\ t$ and $f_2(t) = sin\ t$. Let D be the derivative. What is the matrix of D with respect to the basis $\{f_1, f_2\}$.

4. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$. Compute eigenvalues and eigenvectors of $A - 7I$. How are they related to those of A.

5. Verify that sum of eigenvalues of A(above) is equal to trace of A and product of eigenvalues of A is equal to determinant of A. Is this true in general?

6. Prove that eigenvalues of a matrix and its transpose are always same.

7. Prove that similar matrices have same eigenvalues. What can you say about eigenvectors?

8. If a matrix M has λ as an eigenvalue then what can say about eigenvalue of M^{-1}. What about eigenvectors of M and M^{-1}?

9. If a matrix M has λ as an eigenvalue then what can say about eigenvalue of kM where k is some real number. What about eigenvectors of M and kM?

10. Consider a 2×2 matrix whose trace is 5 and determinant is 6. Find its eigenvalues.

11. For the following matrices:

    (a) Compute real eigenvalues and eigenvectors.

    (b) Write down algebraic and geometric multiplities for each eigenvalues.

    (c) Are the matrices diagonalizable? Justify. Further write down the diagonal matrix D and the invertible matrix P.

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $(Rotation)$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $(Projection)$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $(Reflection)$

(d) $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ $(stretching\ in\ x - direction)$

(e) $\begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

(f) $\begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix}$

(g) $\begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$

(h) $\begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

(i) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$