is transmitted out into the air. After two reflections, the intensity will become insignificantly small. At each reflection, the intensity and hence the *amplitude of light wave* is divided into a reflected component and a refracted component. The reflected and refracted components travel along different paths and can be brought to overlap to produce interference. Therefore, the interference in thin films is called interference by division of amplitude. Newton and Robert Hooke first observed the thin film interference. However, Thomas Young gave the correct explanation of the phenomena. A thin film may be uniform or non-uniform in its structure. However, as long as its thickness lies within the specified limits, interference of light occurs.

6.8 PLANE PARALLEL FILM

A transparent thin film of uniform thickness bounded by two parallel surfaces is known as a *plane parallel thin film*.

When light is incident on a parallel thin film, a small portion of it gets reflected from the top surface and a major portion is transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it is transmitted from the lower surface of the film. Thin films transmit incident light strongly and reflect only weakly. After two reflections, the intensities of reflected rays drop to a negligible strength. Therefore, we consider the first two reflected rays only. These two rays are derived from the same incident ray but appear to come from two sources located below the film. The sources are virtual coherent sources (see Fig.6.12). The reflected waves 1 and 2 travel along parallel paths and interfere at infinity. This is a case of *two-beam* interference.

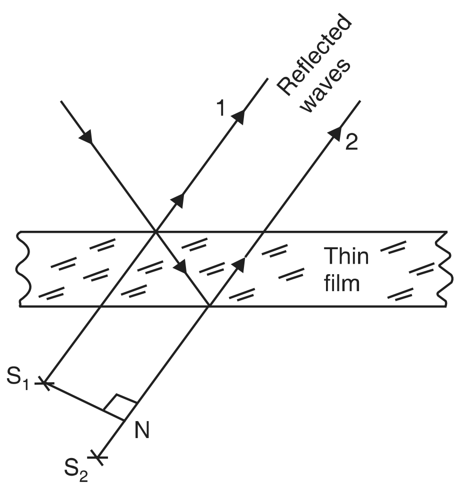

Fig. 6.12

The condition for maxima and minima can be deduced once we have calculated the optical path difference between the two rays at the point of their meeting.

6.8.1 Interference Due to Reflected Light

Let us consider a transparent film of uniform thickness '*t*' bounded by two parallel surfaces as shown in Fig.6.13. Let the refractive index of the material be μ. The film is surrounded by air on both the sides. Let us consider plane waves from a monochromatic source falling on the thin film at an angle of incidence '*i*'. Part of a ray such as AB is reflected along BC, and part of it is transmitted into the film along BF. The transmitted ray BF makes an angle '*r*' with the normal to the surface at the point B. The ray BF is in turn partly reflected back into the film along FD while a major part refracts into the surrounding medium along FK. Part of the reflected ray FD is transmitted at the upper surface and travels along DE. Since the film

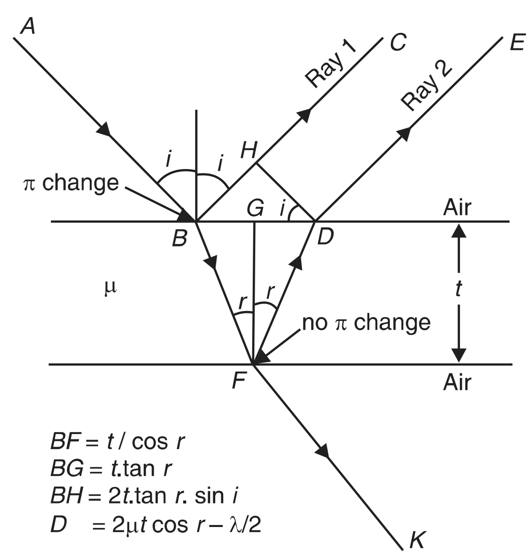

Fig. 6.13

boundaries are parallel, the reflected rays BC and DE will be parallel to each other. The waves travelling along the paths BC and BFDE are derived from a single incident wave AB. Therefore they are coherent and can produce interference if they are made to overlap by a condensing lens or the eye.

(*i*) **Geometrical Path Difference:** Let DH be normal to BC. From points H and D onwards, the rays HC and DE travel equal path. The ray BH travels in air while the ray BD travels in the film of refractive index μ along the path BF and FD. The geometric path difference between the two rays is

$$BF + FD - BH.$$

(*ii*) **Optical Path Difference:**

$$\text{Optical path difference } \Delta_a = \mu L$$

$$\therefore \quad \Delta_a = \mu\,(BF + FD) - 1(BH) \tag{6.15}$$

In the ΔBFD, $\angle BFG = \angle GFD = \angle r$

$$BF = FD$$

$$BF = \frac{FG}{\cos r} = \frac{t}{\cos r}$$

$$\therefore \quad BF + FD = \frac{2t}{\cos r} \tag{6.16}$$

Also, $BG = GD$

$$\therefore \quad BD = 2BG$$

$$BG = FG \tan r = t \tan r$$

$$\therefore \quad BD = 2t \tan r$$

In the Δ^{le} BHD $\quad \angle HBD = (90 - i)$

$$\angle BHD = 90°$$

$$\therefore \quad \angle BDH = i$$

$$\therefore \quad BH = BD \sin i = 2\,t \tan r \sin i \tag{6.17}$$

From Snell's law,

$$\sin i = \mu \sin r$$

$$\therefore \quad BH = 2t \tan r\,(\mu \sin r) = \frac{2\mu t \sin^2 r}{\cos r} \tag{6.18}$$

Using the equations (6.17) and (6.16) into equ.(6.15), we get

$$\Delta_a = \mu\left[\frac{2t}{\cos r}\right] - \left[\frac{2\mu\, t \sin^2 r}{\cos r}\right]$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$= \frac{2\mu t}{\cos r}\cos^2 r$$

$$\therefore \quad \Delta_a = 2\mu\, t \cos r \tag{6.19}$$

(*iii*) **Correction on account of phase change at reflection:** When a ray is reflected at the boundary of a rarer to denser medium, a path-change of λ/2 occurs for the ray BC (see Fig.6.13). There is no path difference due to transmission at D. Including the change in path difference due to reflection in eqn. (6.19), the true path difference is given by

$$\Delta_t = 2\mu\, t \cos r - \lambda/2 \tag{6.20}$$

6.8.2 Conditions for Maxima (Brightness) and Minima (Darkness)

Maxima occur when the optical path difference $\Delta = m\,\lambda$. If the difference in the optical path between the two rays is equal to an *integral number of full waves*, then the rays meet each other in phase. The crests of one wave falls on the crests of the others and the waves *interfere constructively*. Thus, when

$$2\mu\, t \cos r - \frac{\lambda}{2} = m\lambda \tag{6.21}$$

the reflected rays undergo constructive interference to produce brightness or maxima at the point of their meeting.

$$2\mu\, t \cos r = m\lambda + \lambda/2$$

or $$2\mu\, t \cos r = (2m + 1)\lambda/2 \qquad \textbf{Condition for Brightness} \tag{6.22}$$

Minima occur when the optical path difference is $\Delta = (2m + 1)\,\lambda/2$. If the difference in the optical path between the two rays is equal to an *odd integral number of half-waves*, then the rays meet each other in opposite phase. The crests of one wave falls on the troughs of the others and *the waves interfere destructively*. Thus, when

$$2\mu\, t \cos r - \lambda/2 = (2m + 1)\lambda/2 \tag{6.23}$$

the reflected rays undergo destructive interference to produce darkness. Equ.(6.23) may be rewritten as

$$2\mu\, t \cos r = (m + 1)\lambda$$

The phase relationship of the interfering waves does not change if one full wave is added to or subtracted from any of the interfering waves. Therefore $(m + 1)\lambda$ can be as well replaced by $m\lambda$ for simplicity in expression. Thus,

$$2\mu\, t \cos r = m\lambda \qquad \textbf{Condition for Darkness} \tag{6.24}$$

6.8.3 Some Important Points

(*a*) It is seen that the conditions of interference depend on three parameters, namely μt, λ and r. In the case of constant thickness (parallel) film, μt is constant. When a parallel beam of light is incident on such a film, r also remains constant. Then the interference conditions solely depend on the wavelength, λ.

(*b*) When a parallel beam of monochromatic light is incident normal to the film, the whole film will appear *uniformly* dark or *uniformly* bright. The film will appear bright in reflected light, when the film is of $\lambda/4\mu$, $3\lambda/4\mu$, $5\lambda/4\mu$, ... thick and it appears dark when its thickness is $\lambda/2\mu$, λ/μ, $3\lambda/2\mu$, ... etc. (Fig.6.14). If the condition of constructive interference is satisfied, the film will show intense colour corresponding to the colour of the incident light.

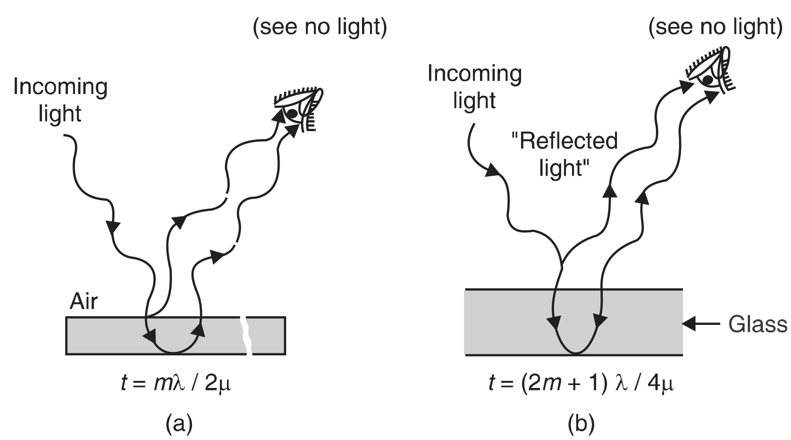

Fig. 6.14: Interference in film of constant thickness – (a) Interference is destructive and the film appears dark in reflected light for thickness satisfying $t = m\,\lambda/2\mu$ condition. (b) It is constructive and the film appears bright for film thickness satisfying the condition $t = (2m + 1)\lambda/4\mu$.

(*c*) A change in the angle of incidence of the rays leads to a change in the path difference. Consequently, if the inclination of the film with respect to the light beam is changed gradually, we find that it will appear dark and bright (or bright and dark) in succession.

(*d*) If a parallel beam of white light falls on a parallel film, those wavelengths for which the path difference is $m\lambda$, will be absent from the reflected light. The other colours will be reflected. Therefore, the film will appear uniformly coloured with one colour being absent.

Example 6.4: *A soap film of 5×10^{-7}m thick is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum, which will be absent from the reflected light. Given the refractive index of the film = 1.33.*

Solution: White light is incident on the film at an angle 30°. Let r be the angle of refraction of light into the film. r can be calculated from Snell's law $\mu = \dfrac{\sin i}{\sin r}$.

$$\therefore \qquad \sin r = \frac{\sin 30°}{1.33} \quad \text{or} \quad r = 25.55° \quad \text{and} \quad \cos r = 0.90.$$

The absence of certain wavelengths in reflected light is due to their undergoing destructive interference. The condition for destructive interference is

$$2\mu\, t \cos r = m\lambda$$

To find out the missing wavelengths, we have to use different m values into the above equation and find out which of them lie in the visible region 7000 to 4000 Å.

For $m = 1$, we get $\lambda_1 = 2 \times 1.33 \times 5 \times 10^{-7}\text{m} \times 0.90 = 12 \times 10^{-7}\text{m} = 12000$ Å.

For $m = 2$, we get $\lambda_2 = (2 \times 1.33 \times 5 \times 10^{-7}\text{m} \times 0.90) \div 2 = 6 \times 10^{-7}\text{m} = 6000$ Å.

For $m = 3$, we get $\lambda_3 = (2 \times 1.33 \times 5 \times 10^{-7}\text{m} \times 0.90) \div 3 = 4 \times 10^{-7}\text{m} = 4000$ Å.

For $m = 4$, we get $\lambda_4 = (2 \times 1.33 \times 5 \times 10^{-7}\text{m} \times 0.90) \div 4 = 3 \times 10^{-7}\text{m} = 3000$ Å.

It is clear that the first wavelength lies in the infra red and the last one lies in the UV region. The middle two wavelengths lie in the visible region. Hence, the absent wavelengths in the reflected light are 6000 Å and 4000 Å.

6.8.4 Restriction on Thickness of the Film

We know that interference colours are observed only in thin films but not in thick plates such as windowpanes or glass slabs. This is due to the fact that light waves can interfere only when both the conditions of temporal and spatial coherence are satisfied. In Fig. 6.14, we have assumed that a monochromatic wave of infinite length is incident on the film. In reality, the incident light consists of wave trains of finite length and coherence extends over the length of each wave train only. Interference can occur only when parts of the same group of wave trains overlap. Superposition of different wave trains cannot produce interference because they will be incoherent and do not maintain any constant phase relationship with each other.

Fig. 6.15 shows the real situation. Wave trains 1,2,3 of finite length are incident in succession on a thin film. Portions of each wave train are reflected by the top and bottom surfaces of the film. Each wave train is divided into two reflected wave trains (U_1, L_1, U_2, L_2 and U_3, L_3). In Fig.6.15 (a) the film is thin and the difference in the optical path lengths of U_1 and L_1 is small compared to the length of the wave train. Their superposition produces interference, as U_1 and L_1 are parts of the same wave train 1 and hence are coherent. In Fig. 6.15 (b) the film is thicker and the optical path difference between U_1 and L_1 is large than the coherence length. Consequently, superposition takes place between parts of different wave trains, U_2 and L_1 and U_3 and L_2. Therefore interference does not take place.