College of Engineering Pune

Tutorial on Ordinary Differential Equation

1. Verify that the following functions are solutions of the corresponding differential equations.

   (a) $y = \sin^{-1} xy, \quad xy' + y = y'\sqrt{1 - x^2y^2}$

   (b) $x^2 + y^2 = 1, \quad x + yy' = 0$

   (c) $y = ce^{-x} + x^2 - 2x, \quad y' + y = x^2 - 2$

   (d) $y^2 - 2x^2 = c, \quad yy' = 2x$

   (e) $y = e^{x^2} \int_0^x e^{-t^2} dt, \quad y' = 2xy + 1$

2. Obtain the general solution of each of the following differential equations:

   (a) $y' = \dfrac{1}{(x+1)(x^2+1)}$

   (b) $y' = \dfrac{y^2 - xy}{x^2 + xy}$

   (c) $xy' = y + x\cos^2(y/x)$

   (d) $y' = \dfrac{-x + 2y - 1}{4x - 3y - 6}$

   (e) $(2x^2 + 3y^2 - 7)xdx = (3x^2 + 2y^2 - 8)ydy$

3. Find the perticular solution of each of the following differential equations :

   (a) $x^3(\sin y)y' = 2; \quad y(x) \to \pi/2$ as $x \to +\infty$

   (b) $y' = y(y^2 - 1); \quad y(0) = 2$

   (c) $(x+2)y' - xy = 0; \quad y(0) = 1$

   (d) $y' + \dfrac{y - x}{y + x} = 0; \quad y(1) = 1.$

   (e) $y' = (y - x)^2; \quad y(0) = 2$

   (f) $e^x y' = 2(x+1)y^2; \quad y(0) = 1/6.$

4. Show that the following differential equations are exact and hence obtain their general solution.

   (a) $3x(xy - 2)dx + (x^3 + 2y)dy = 0$

   (b) $(\cos x \cos y - \cot x)dx - \sin x \sin y dy = 0$

   (c) $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$

   (d) $(\sin x \cosh y)dx - (\cos x \sinh y)dy = 0.$

   (e) $(\dfrac{\cos y}{x+3})dx - (\sin y \ln(5x + 15) - 1/y)dy = 0$

5. Verify that $1/x^2, 1/y^2, 1/xy, 1/(x^2 + y^2)$ are integrating factors of the differential equation

$$-ydx + xdy = 0$$

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6. Verify that $(y+1)/x^4$ is an integrating factor of the differential equation $3(y+1)dx = 2xdy$. Solve it by using this IF and otherwise.

7. Solve the following differential equations:

   (a) $(2\cos y + 4x^2)dx = x \sin y dy$

(b) $ye^{x/y}dx + (y - xe^{x/y})dy = 0$

(c) $(2x + e^y)dx + xe^y dy = 0$

(d) $(x + y^2)dy - dx = 0$

(e) $(x + y)^2 y' = 1$

(f) $y' - x^{-1}y = x^{-1}y^2$.

(g) $xy' = y(\ln y - \ln x)$

(h) $(xy + x^3y^3)\frac{dy}{dx} = 1$

(i) $e^y y' - e^y = 2x - x^2$

(j) $y' + 4xy = e^{-2x^2}; y(0) = -4$

(k) $6y^2dx - x(2x^3 + y)dy = 0$

(l) $\cos y \sin 2x dx + (\cos^2 y - \cos^2 x)dy = 0$

8. For a differential equation $Mdx + Ndy = 0$, if $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ is a function of x alone i.e f(x), then prove that $e^{\int f(x)dx}$ is an IF of the given equation.

9. Obtain the solutions of following differential equations :

(a) $xdy = (y + x^2 + 9y^2)dx$

(b) $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$

(c) $y - xy' = y'y^2e^y$

(d) $y' = cosecx - ycotx$

(e) $2(y + 1)y' - \dfrac{2}{x}(y + 1)^2 = x^4$

(f) $(4xy + 3y^4)dx + (2x^2 + 5xy^3)dy = 0$

10. Verify that the given functions are linearly independent solutions of of the given differential equation. Hence write the general solution and solve the given initial value problem.

(a) $y'' + 9y = 0, y(0) = 4, y'(0) = 6, y_1 = \cos 3x, y_2 = \sin 3x$

(b) $4x^2y'' - 3y = 0, y(1) = 3, y'(1) = 2.5, y_1 = x^{-1/2}, y_2 = x^{3/2}$

11. Using the method of reduction of order, obtain the second linearly independent solution of the following differential equations :

(a) $xy'' + 2y' + xy = 0, y_1 = (\sin x)/x$

(b) $(1 - x^2)y'' - 2xy' + 2y = 0, y_1 = x$

12. Obtain a general solution of the following differential equations:

(a) $25y'' + 40y' + 16y = 0$ (b) $y'' + 4y' + (4 + \omega^2)y = 0$

(c) $y'' - k^2y = 0$ (d) $2y'' - 9y' = 0$

(e) $y'' - 2\sqrt{2}y' + 2.5y = 0$ (f) $4y'' + 16y' + 17y = 0$

(g) $(9D^2 + 6D + 1)y = 0$ (h) $(D^2 + \pi(\pi - 1)D - \pi^3)y = 0$

~~13. Solve the following boundary value problems:~~

~~(a)$y'' - 25y = 0, y(-2) = y(2) = cosh10$ (b) $y'' + 2y' + 2y = 0; y(0) = 1, y(\pi/2) = 0$~~

14. Solve the following initial value problems:

(a) $x^2y'' - 2xy' + 2y = 0, y(1) = 1.5, y'(1) = 1$

(b) $x^2y'' + xy' + 9y = 0, y(1) = 2, y'(1) = 0$

(c) $(x^2D^2 + 3xD + 1)y = 0, y(1) = 3, y'(1) = -4$

15. Using the method of undetermined coefficients, obtain a real general solution of following nonhomogeneous differential equations:

(a) $y'' - y' - 2y = 3e^{2x}$ (b) $3y'' + 10y' + 3y = 9x + 5\cos x$

(c) $y'' + 6y' + 9y = 50e^{-x}\cos x$ (d) $y'' + 2y' + 10y = 25x^2 + 3$

(e) $y'' + 4y' + 4y = 18coshx$ (f) $y'' + y' = 2 + 2x + x^2$

16. Using the method of variation of parameters, obtain a real general solution of following nonhomogeneous differential equations:

(a) $y'' - 4y' + 4y = e^{2x}/x$ (b) $y'' + 9y = sec3x$

(c) $y'' - 4y' + 5y = e^{2x}cosecx$ (d) $(D^2 + 6D + 9)y = 16e^{-3x}/(x^2 + 1)$

(e) $y'' + 4y' + 4y = e^{-2x}/x^2; x > 0$

17. Solve the following differential equations:

(a) $(D^4 + 4D^3 + 8D^2 + 8D + 4)y = 0$

(b) $(D^4 + 10D^2 + 9)y = 0$

(c) $(D^5 - 3D^4 + 3D^3 - D^2)y = 0$

(d) $y''' - y' = 2x^2e^x$

(e) $(D^3 + 3D^2 + 3D + 1)y = e^{-x}\sin x$