College of Engineering Pune

Linear Algebra and Univariate Calculus(D.S.Y)

Application of derivatives.

1. Find absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graphs where the absolute extreme occurs.

   (a) $f(x) = \frac{-1}{x+3}, -2 \le x \le 3$

   (b) $f(x) = x^{1/3}, -1 \le x \le 8$

   (c) $f(x) = -3x^{2/3}, -1 \le x \le 1$

   (d) $f(x) = \sqrt{4 - x^2}, -2 \le x \le 1$

   (e) $f(\theta) = sin\theta, -\pi/2 \le \theta \le 5\pi/6$

   (f) $3x^4 - 16x^3 + 18x^2, -1 \le x \le 4$

2. Identify the largest possible domain of the following functions. Find the extreme values of the functions and where they occur.

   (a) $f(x) = 2x^2 - 8x + 9$

   (b) $f(X) = x^3 - 2x + 4$

   (c) $f(x) = \sqrt{x^2 - 1}$

   (d) $f(x) = 1/\sqrt{x^2 - 1}$

   (e) $f(x) = x/(x^2 + 1)$

   (f) $f(X) = e^x$

3. Find the set of critical points and determine the local extreme values.

   (a) $f(x) = x^{2/3}(x + 2)$

   (b) $f(x) = x^2\sqrt{3 - x}$

   (c) $f(x) = x|x| - x$

   (d) $f(x) = \{ \begin{array}{cc} 3 - x & if\ x > 0 \\ 3 + 2x - x^2 & if\ x \le 0 \end{array}$

4. Show that equation $x^3 + x - 1 = 0$ has exactly one real root.

5. Show that the 5 is a critical point of the function $f(x) = 2 + (x - 5)^3$ but f does not have a local extreme value at 5.

6. Sketch a graph of a function

   (a) has local maximum at 2 and is differentiable at 2.

   (b) has local maximum at 2 and it is continuous but not differentiable at 2.

   (c) has local maximum at 2 but not continuous at 2.

7. Use LMVT to conclude that the given function f which satisfies all the conditions do not exists: $f''(x) > 0, \forall x \in \mathbb{R}$ and $f'(0) = 1, f'(1) = 1$.

8. Assume that f is continuous on $[a, b]$ and differentiable on (a, b). Also assume that $f(a)$ and $f(b)$ have opposite sign and $f' \neq 0$ between a and b. Show that $f(x) = 0$ exactly once between a and b.

9. Use the Mean Value theorem to prove $|Sin\ a - Sin\ b| \leq |a - b|, \forall a, b \in \mathbb{R}$.

10. Suppose that $f(0) = -3$ and $f'(x) = -5$ for all values of x. How large can $f(2)$ possibly be?

11. Two runners start the race at the same time and finish in a tie. Prove that at some time during the race they have the same velocity.

12. Let $a > 0$ and f be continuous on $[-a, a]$. Suppose that $f'(x) \leq 1, \forall x \in (-a, a)$, if $f(a) = a$ and $f(-a) = -a$. Show that $f(0) = 0$.

13. Prove the following inequalities

    (a) $\frac{b-a}{\sqrt{1-a^2}} < Sin^{-1}b - Sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ for $0 < a < 1$ and $0 \leq \theta \leq 2\pi$

    (b) $tan\ x > x$ for $0 < x < \pi/2$

    (c) $\frac{x}{1+x} < log(1 + x) < x; x > 0$

14. As x moves from left to right through point $c = 2$ is the graph of $f(x) = x^3 - 3x + 2$ rising or falling?

15. For the following functions:

    (a) Find Critical points.

    (b) Find Extreme values of f.

    (c) Find intervals where f is increaing or decreasing.

    (d) Find inflection points.

    (e) Find intervals where f is concave up or concave down.

    (f) Sketch the graph of f.

    (a) $f(x) = 4x^3 - x^4$.

    (b) $f(x) = -x^3 + 6x^2 - 3$

    (c) $f(x) = x^3 - 3x - 3$

    (d) $f(x) = x(6 - 2x)^2$

(e) $f(x) = -2x^3 + 6x^2 - 3$

(f) $f(x) = 1 - 9x - 6x^2 - x^3$

(g) $f(X) = (x-2)^3 + 1$

(h) $f(x) = x^4 - 2x^2$