College of Engineering Pune
MA 20001 : Ordinary Differential Equations and Multivariate Calculus
S.Y. B.Tech. Semester III (All Branches)
Academic Year: 2022-23
Tutorial 1
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1. Write a note on unifying power of mathematics.
2. Define general solution, particular solution and singular solution of an ordinary differential equation and explain the difference between them with an example.
3. Solve the following :
   (i) $y''' = e^{-0.2x}$
   (ii) $y' = sec^2 y$
   (iii) $\frac{dr}{dt} = -2tr; r(0) = r_0$
   (iv) $y' = (x + y - 2)^2; y(0) = 2$
   (v) $xy' = x + y$
4. Problem 24, page 19
5. A thermometer reading 10^0C is brought into a room whose temperature is 27^0C. One minute later the thermometer reading is 15^0C. How long does it take for the thermometer reading to become 26.99^0C?
6. Solve the following :
   (i) $x^3dx + y^3dy = 0$
   (ii) $3(y + 1)dx = 2xdy$
   (iii) $e^x(cosy\ dx - siny\ dy) = 0$
   (iv) $(3xe^y + 2y)dx + (x^2e^y + x)dy = 0$
   (v) $(a + 1)ydx + (b + 1)xdy = 0; y(1) = 1$
7. Under what conditions for the constants is the following exact? Solve it.

$$(ax + by)dx + (px + qy)dy = 0$$

8. Solve the following :

   (i) $y' + 2y = 4cos2x; y(\pi/4) = 3$

   (ii) $xy' + 4y = 8x^4; y(1) = 2$

   (iii) $y' + y = \frac{-x}{y}$

   (iv) $y' + \frac{y}{2} = y^3; y(0) = 0$

   (v) $2xy'' = 3y'$

9. Find the orthogonal trajectories of the family $y = \sqrt{x + c}$

10. Problem no. 12, page 38

11. Solve $xy'' + 2y' + xy = 0$ by reduction of order given that $y_1 = \frac{cosx}{x}$ is a solution.

12. Show that x and lnx are linearly independent(LI) solutions of $x^2y'' - xy' + y = 0$. Hence solve the IVP $y(1) = 1; y'(1) = 2$

13. Verify that e^{-4x}, xe, x^2e^{-4x} are LI solutions of $y'''+12y''+48y'+64y = 0$.

14. Find a 2nd order homogeneous LDE for which following are LI solutions. Hence solve the IVP. $e^{-kx}cos\pi x, e^{-kx}sin\pi x; y(0) = 1; y'(0) = -k - \pi$

15. Solve the following :

   (i) $y'' + 4y' + (\pi^2 + 4)y = 0$

   (ii) $4y'' - 4y' - 3y = 0$

   (iii) $y'' + 2k^2y' + k^4y = 0$

   (iv) $y'' - 2y' - 3y = 0; y(-1) = e, y'(-1) = -e/4$

   (v) $9y''–30y' + 25y = 0; y(0) = 3.3, y'(0) = 10$

   (vi) $(D^3 - D^2 - D + I)y = 0$

(vii) $y^{(5)} - 5y''' + 4y' = 0; y(0) = 3, y'(0) = -5, y''(0) = 11, y'''(0) = -23, y^{(4)}(0) = 47$

(viii) $(D^2 + 3D + 2.5I)y = 0$

(ix) $(x^2D^2 - xD + 5I)y = 0$

(x) $(9x^2D^2 + 3xD + I)y = 0$

16. Solve the following non-homogeneous LDE using the method of undetermined coefficients:

(i) $y'' - 4y' = 8cos\pi x$

(ii) $y'' + 6y' + 9y = e^{-x}cos2x; y(0) = 1, y'(0) = -1$

(iii) $(2D^2 - 3D - 2I)y = 13 - 2x^2$

(iv) $(D^2 - I)y = sinhx$

(v) $y'' - 4y' + 3y = 10cosx; y(0) = 1, y'(0) = -1$

17. Solve using variation of parameters:

(i) $y'' + 4y = cos2x$

(ii) $(D^2 + 2D + 2I)y = 4e^{-x}sec^3x$

(iii) $(D^3 + 4D)y = sinx$

18. Solve by converting to a system of ODEs $y'' + 4y' + 3y = 0$

19. Find a real general solution of the following system of ODEs:

(i) $y_1' = 6y_1 + 9y_2 \ ; \ y_2' = y_1 + 6y_2$

(ii) $y_1' = -4y_1 + 5y_2 \ ; \ y_2' = -y_1 + y_2; y_1(0) = 0, y_2(0) = 4$

20. Problem number 18, page 147.

Additional application problems for Tutorial 1

If in a culture of yeast the rate of growth $y'(t)$ is proportional to the amount $y(t)$ present at time t, and if $y(t)$ doubles in 1 day, how much yeast can be expected after 3 days at the same rate of growth ? After 1 week ? (Ans: $8\ y_0,\ 128\ y_0$)

If the growth rate of the amount of yeast at any time t is proportional to the amount present at that time and doubles in 1 week, how much yeast can be expected after 2 weeks? After 4 weeks?

Experiments show the rate of inversion of cane sugar in dilute solution is proportional to the the concentration $y(t)$ of unaltered sugar. Let the concentration be $1/100$ at $t = 0$ and $1/300$ at $t = 4\ hrs$. Find $y(t)$. (Ans: $0.01\ e^{-0.275\ t}$)

A body originally at 80^0C cools down to 60^0C in 20 minutes, the temperature of the air being 40^0C. What will be the temperature of the body after 40 minutes from the original ? (Ans: 50^0C)

A tank contains 5000 liters of fresh water. Salt water which contains 100 gms of salt per liter flows into it at the rate of 10 liters per minute and the mixture kept uniform by stirring, runs out at the same rate. When will the tank contain $200,000$ gms of salt?

A tank contains 1000 gallons of water in which 200 lb of salt, are dissolved. Fifty gallons of brine, each containing $(1 + \cos t)$ lb of dissolved salt, runs into the tank per minute, the mixture kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time t.
(Ans: $y(t) = 1000 + 2.494\cos t + 49.88\sin t - 802.5\ e^{-0.05t}$)