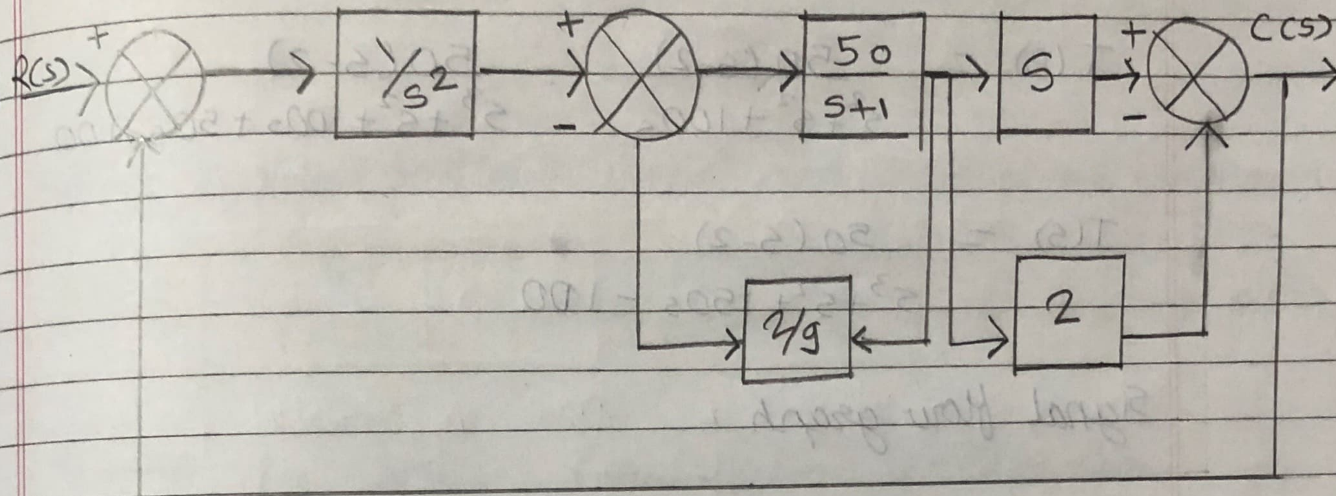
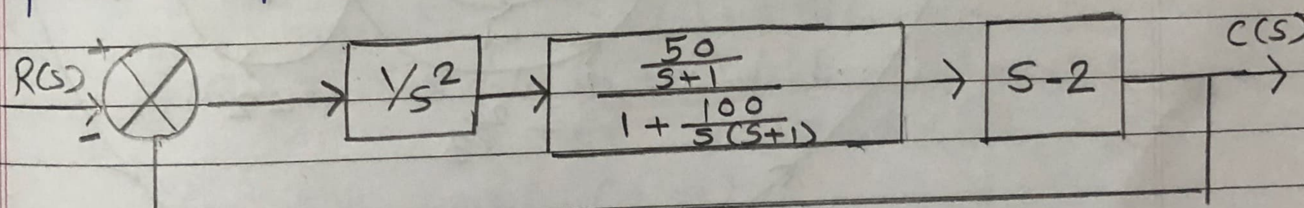
MIS 142103012

FCS Assignment - 4

A1]



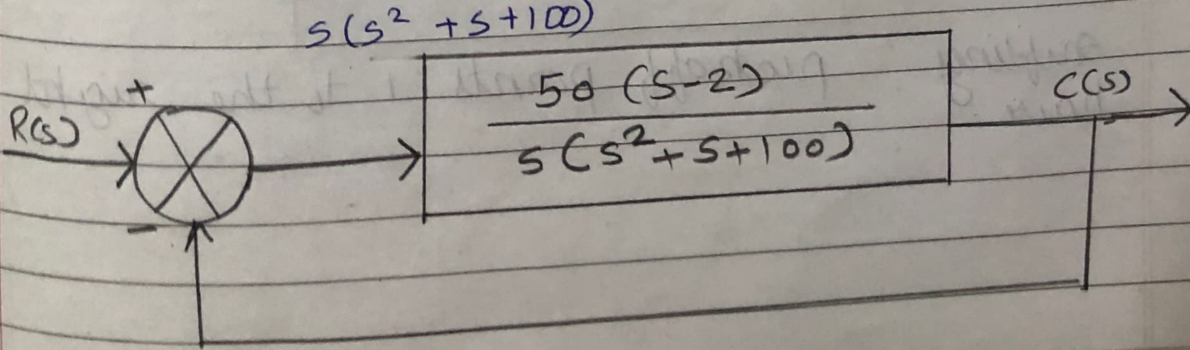
Resolving the negative feedback path and the parallel path we get



combining all paths in serises we have

$$G(s) = \frac{1}{s^2} \times \frac{50s}{100+s^2+s} \times (s-2)$$

$$G(s) = \frac{50(s-2)}{s(s^2+s+100)}$$



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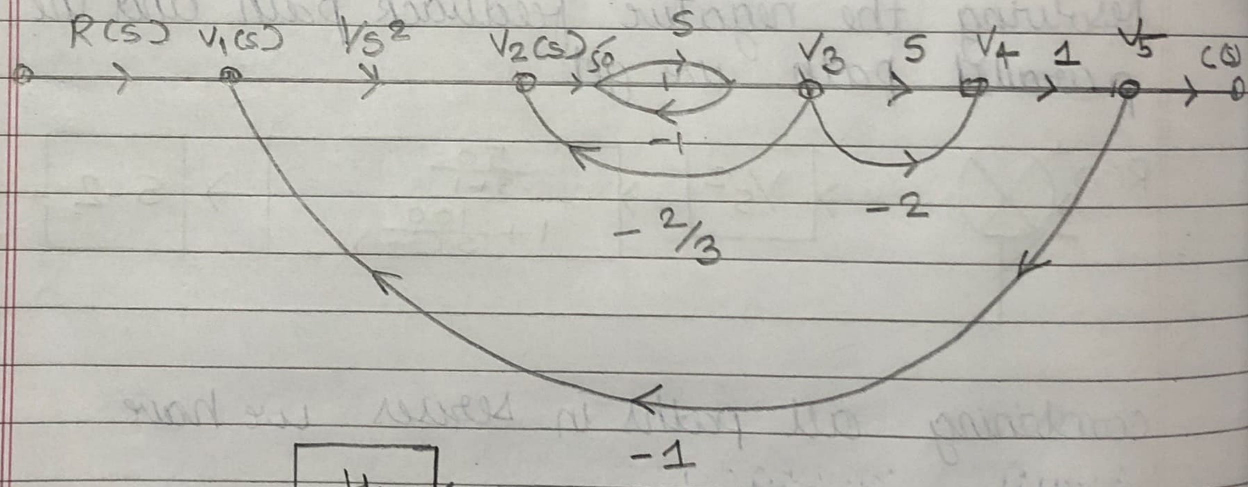
Finally applying formula for negative feedback we have transfer function

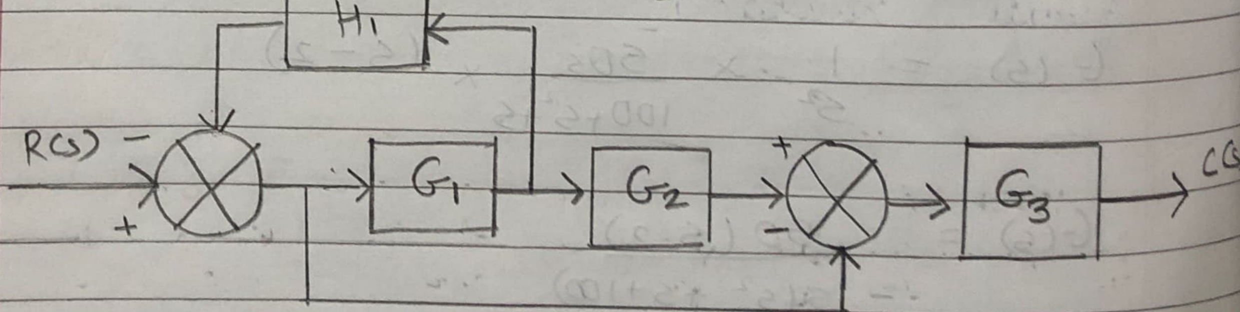
$$T(s) = \frac{50(s-2)}{s^3+s^2+100s} = \frac{50(s-2)}{s^3+s^2+100s+50s-100}$$

$$T(s) = \frac{50(s-2)}{s^3+s^2+150s-100}$$

Signal flow graph



A2



shifting pickoff points 1 to the right we have;

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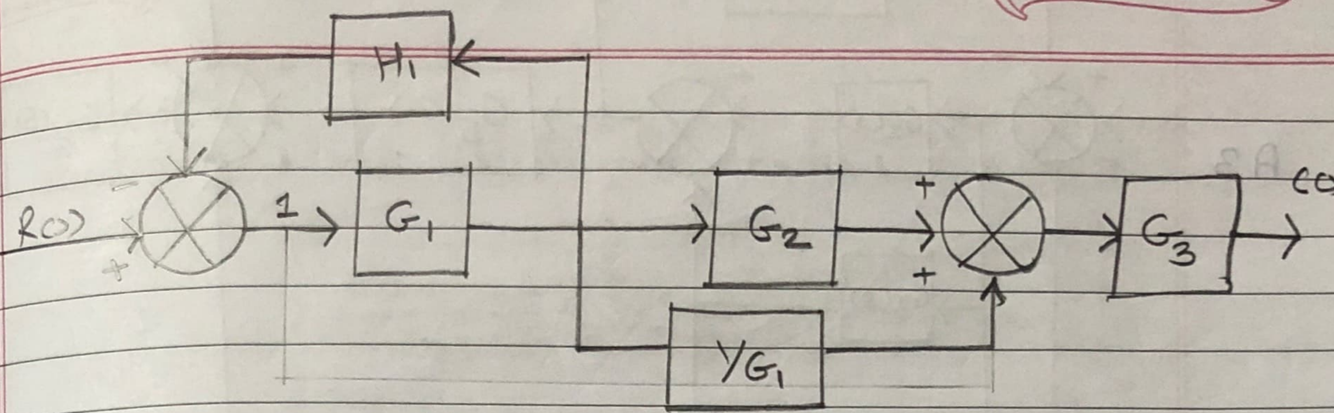
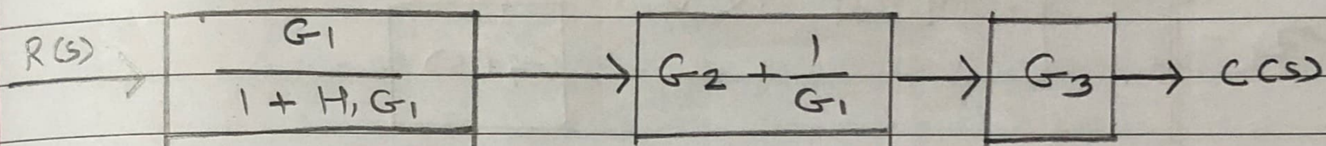


Solving negative feedback path and parallel pat we have



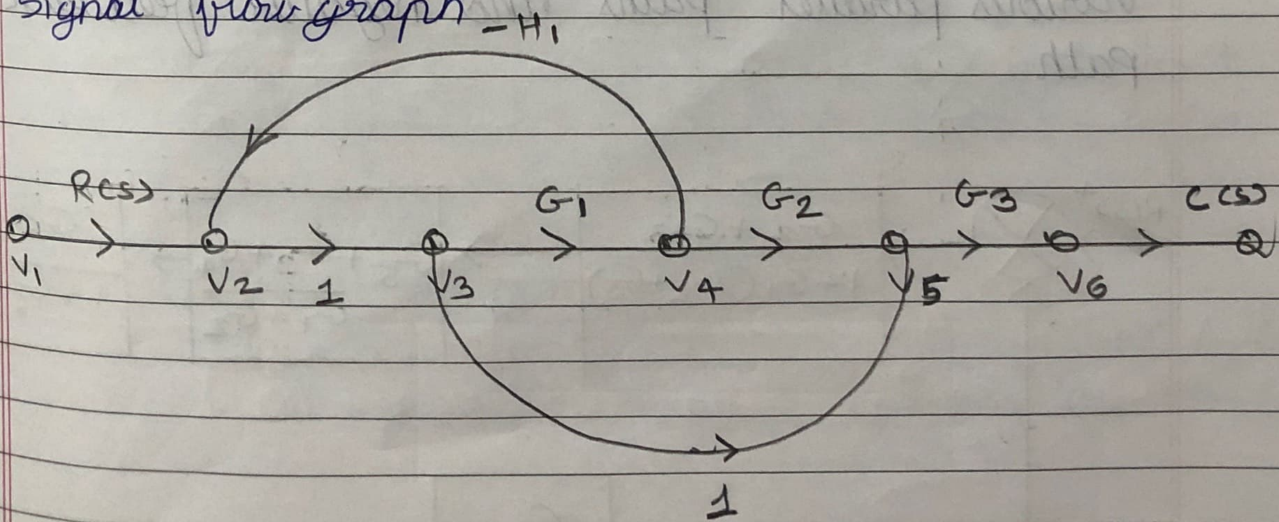
solving the seriese combination we have our transfer function as:

$$T(s) = G_3\left(G_2 + \frac{1}{G_1}\right)\left(\frac{G_1}{1+H_1G_1}\right)$$

$$T(s) = G_3\left(\frac{G_1G_2+1}{G_1}\right)\left(\frac{G_1}{1+G_1H_1}\right)$$

$$T(s) = \frac{G_3G_1(G_1G_2+1)}{(1+G_1H_1)}$$

Signal flow graph



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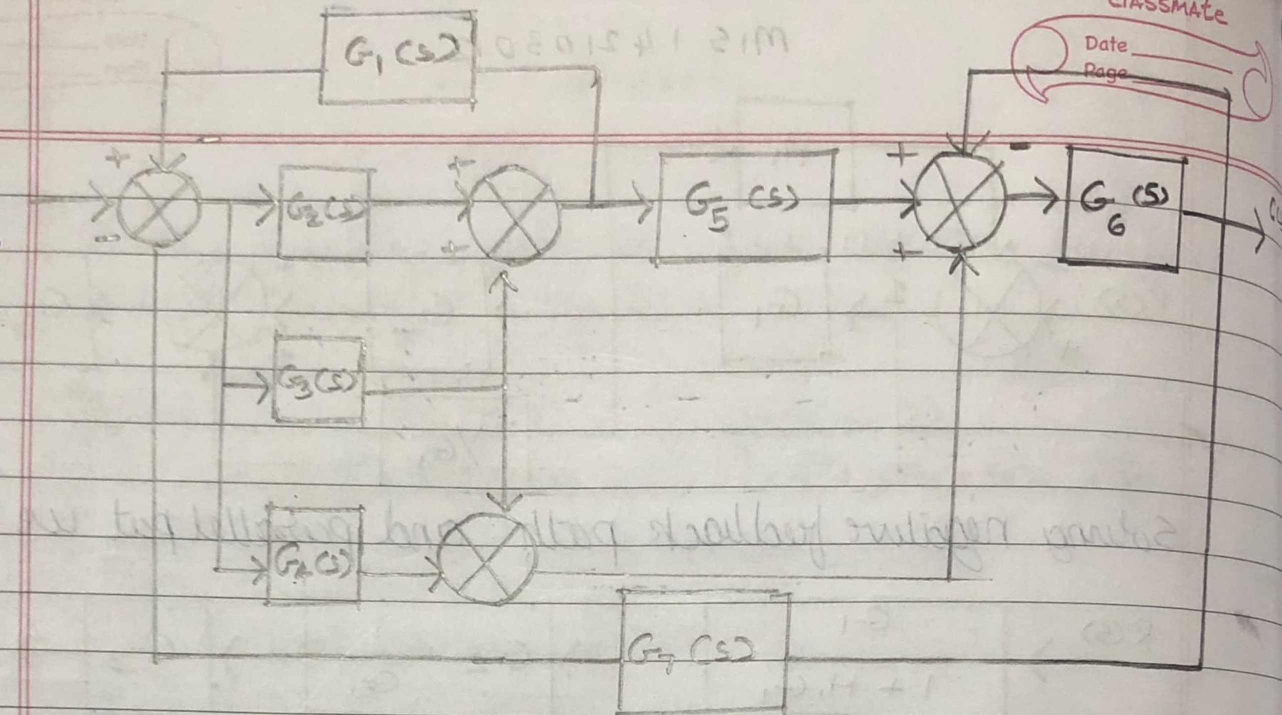
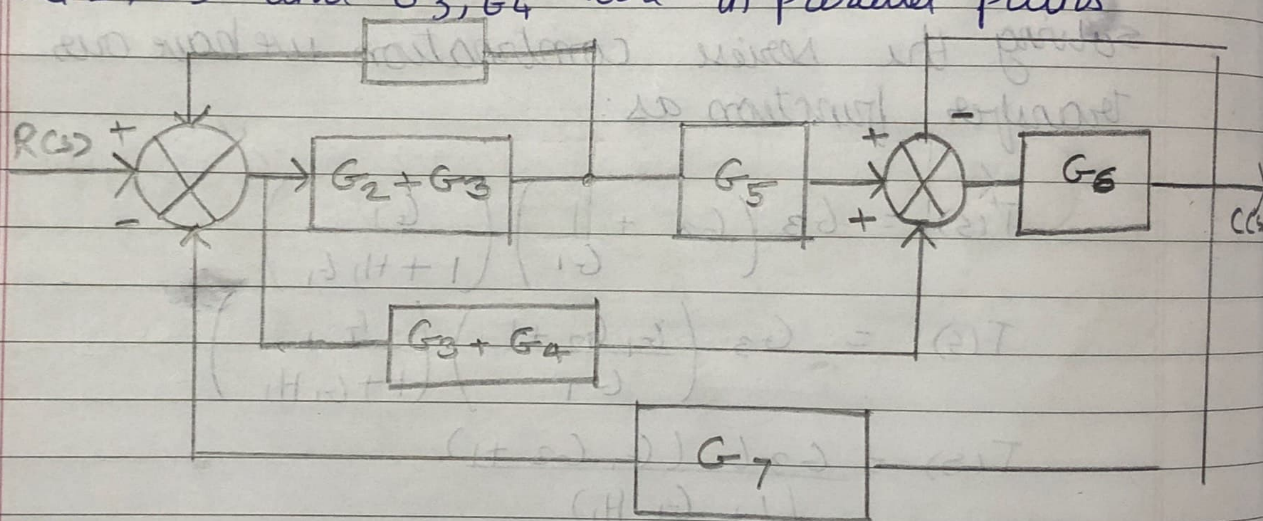
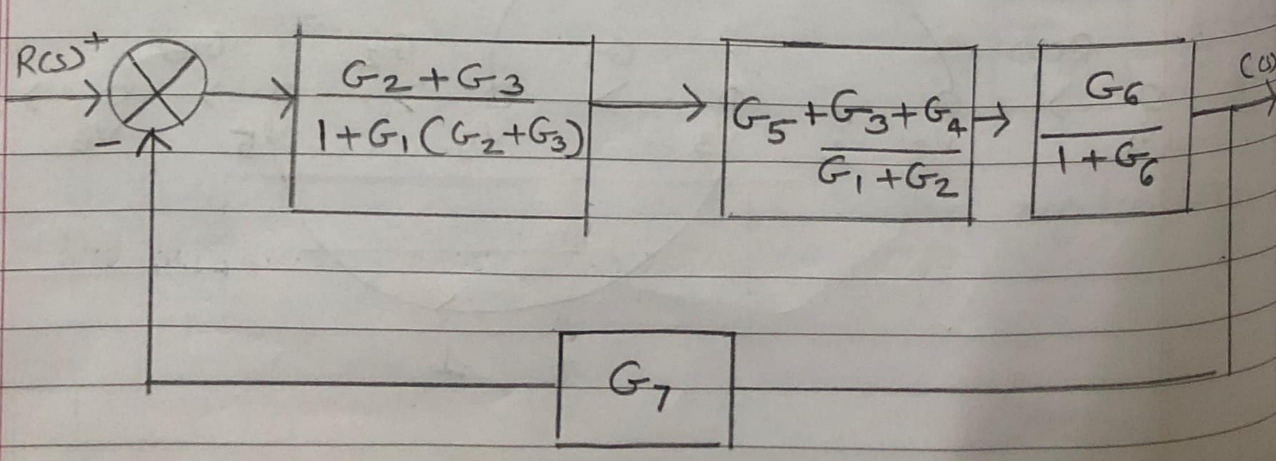
A3



G_2, G_3 and G_3, G_4 are in parallel paths



Shifting pickoff point P is to the right, and resolving various parallel paths and negative feedback path



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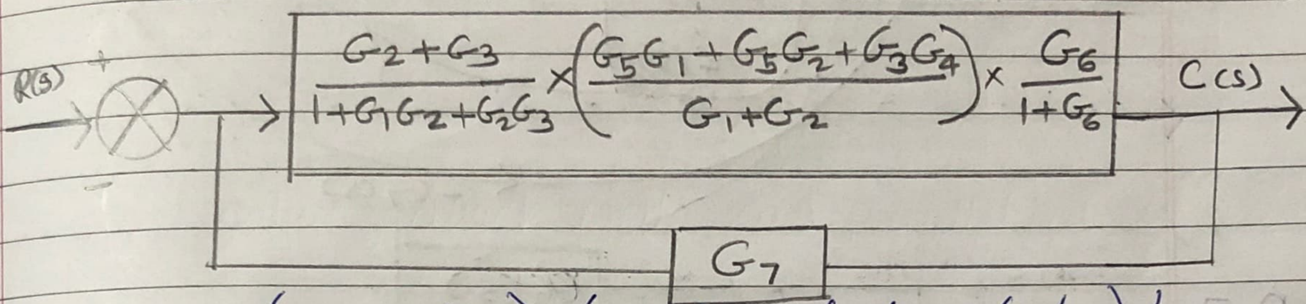
Combining the blocks in series we have,



$$G(s) = \frac{(G_2+G_3)(G_5G_1+G_5G_2+G_5G_3)G_6}{(1+(G_1G_2)+(G_2G_3))(G_1+G_2)(1+G_6)}$$

Now combining G(s) with negative feedback path, we have

$$T(s) = \frac{\dfrac{(G_2+G_3)(G_5G_1+G_5G_2+G_5G_3)G_6}{(1+G_1G_2+G_2G_3)(G_1+G_2)(1+G_6)}}{1+\dfrac{G_2G_6(G_2+G_3)(G_5G_1+G_5G_2+G_5G_3)}{(1+G_1G_2+G_2G_3)(G_1+G_2)(1+G_3)}}$$

$$T(s) = \frac{G_6G_4+G_6G_3+G_6G_5G_3+G_6G_5G_2}{1+G_6+G_3G_1+G_2G_1+G_7G_6G_4+G_7G_6G_3+G_7G_6G_5G_3+G_6G_2G_1+G_5G_3G_2G_7+G_6G_3G_1}$$

$$T(s) = \frac{G_6G_4+G_6G_3+G_6G_5G_3+G_6G_5G_2}{1+G_6+G_2G_1+G_1G_3+G_6G_3G_1+G_6G_2G_1+G_7G_6G_4+G_7G_6G_3+G_7G_6G_5G_3+G_5G_6G_2G_7}$$

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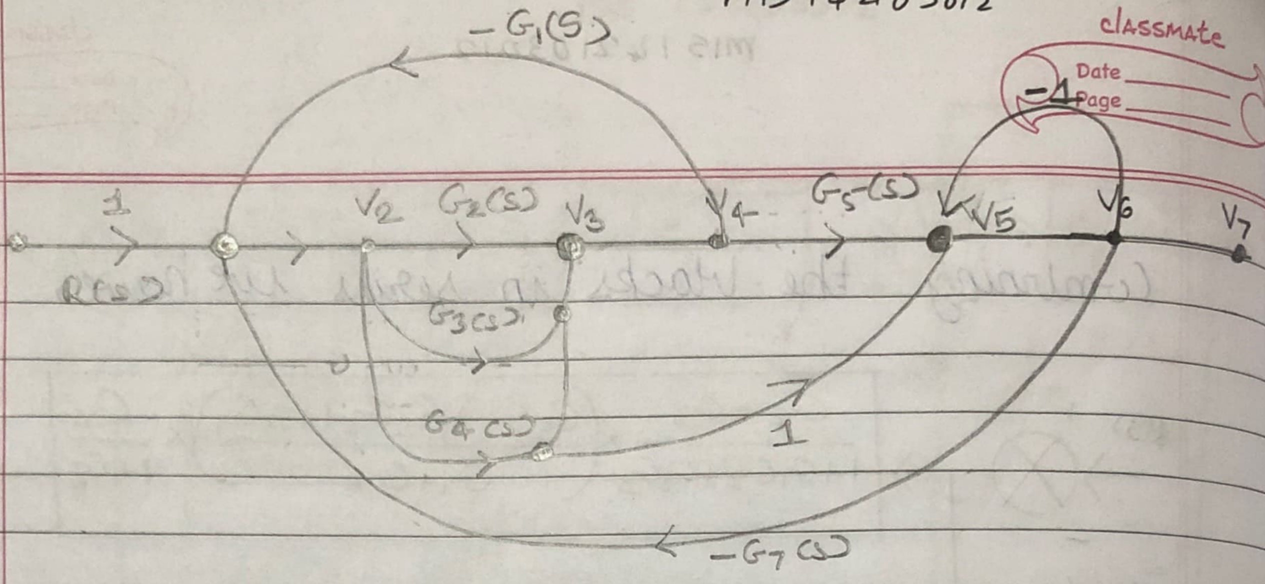
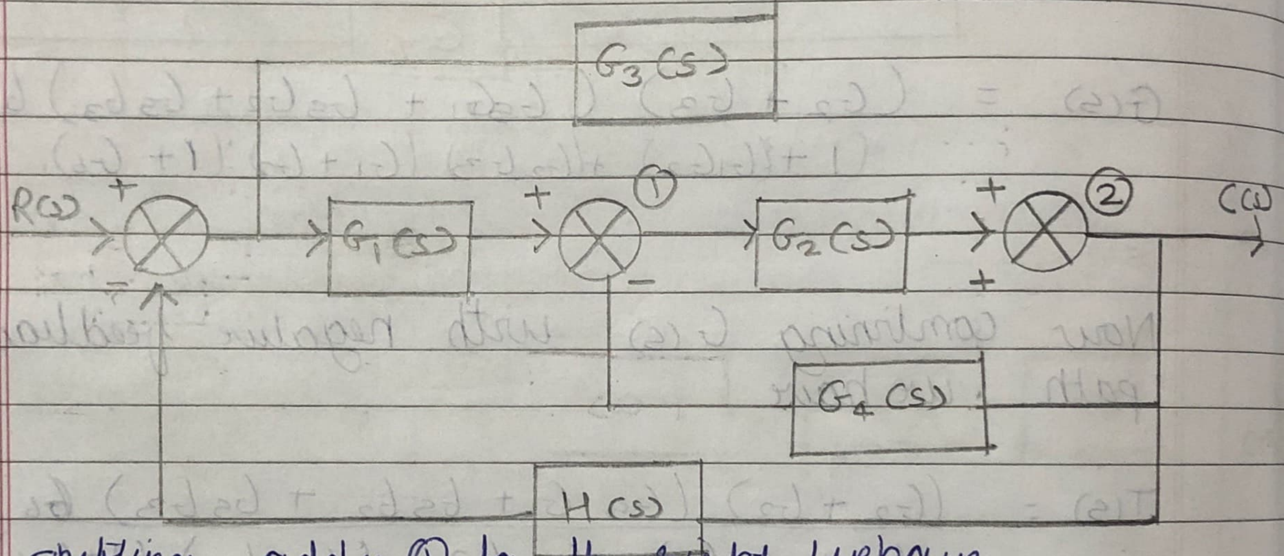
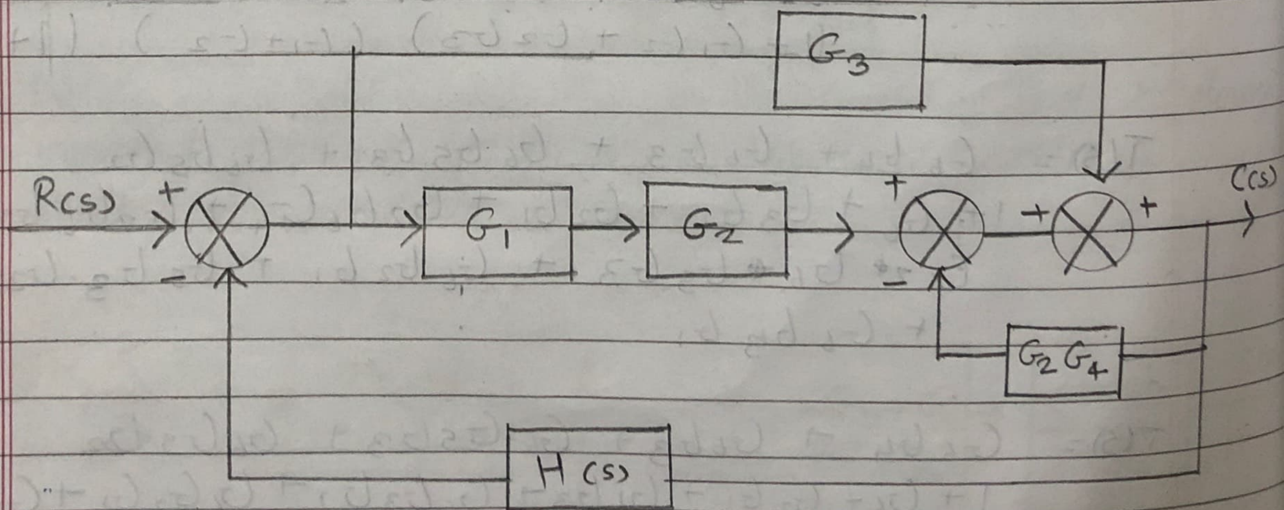


A4]



shifting adder (1) to the right we have



Combining the two adders we have

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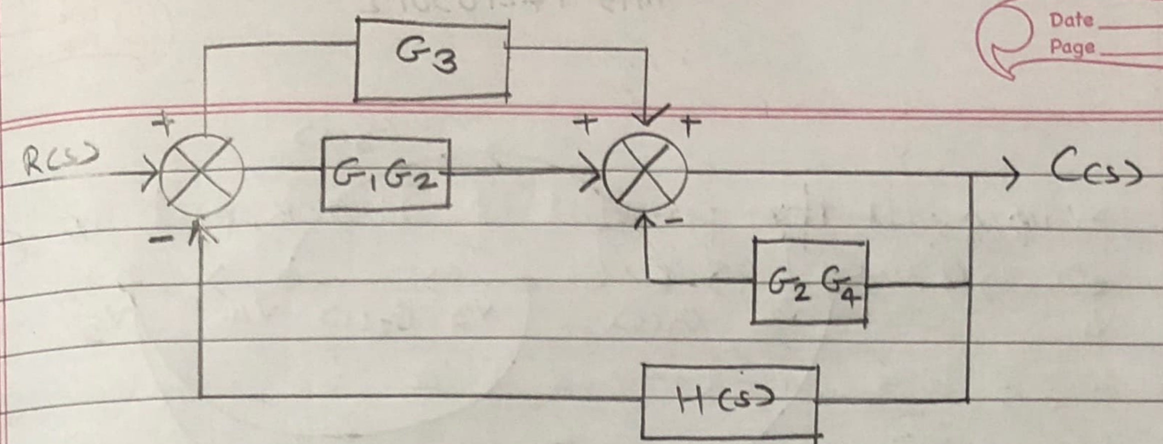
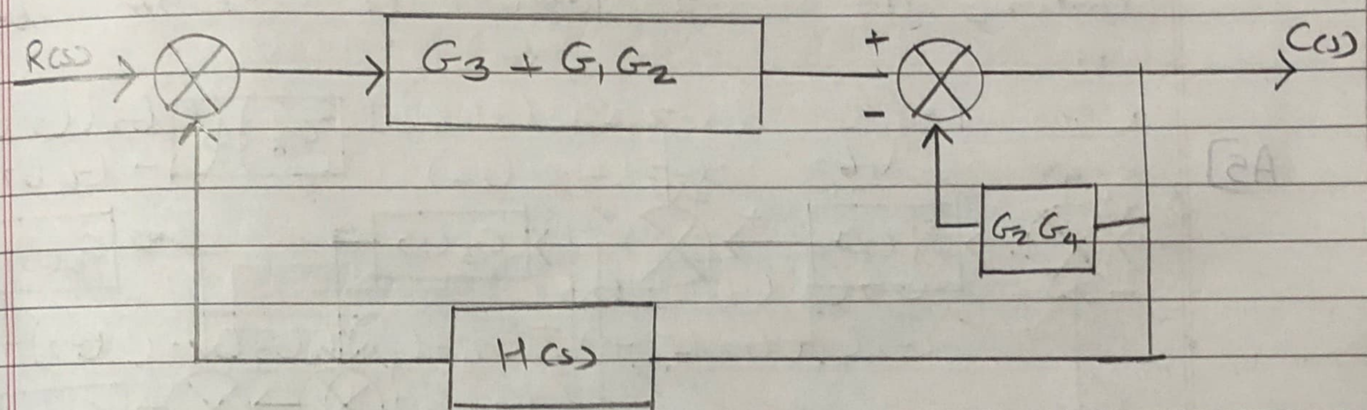
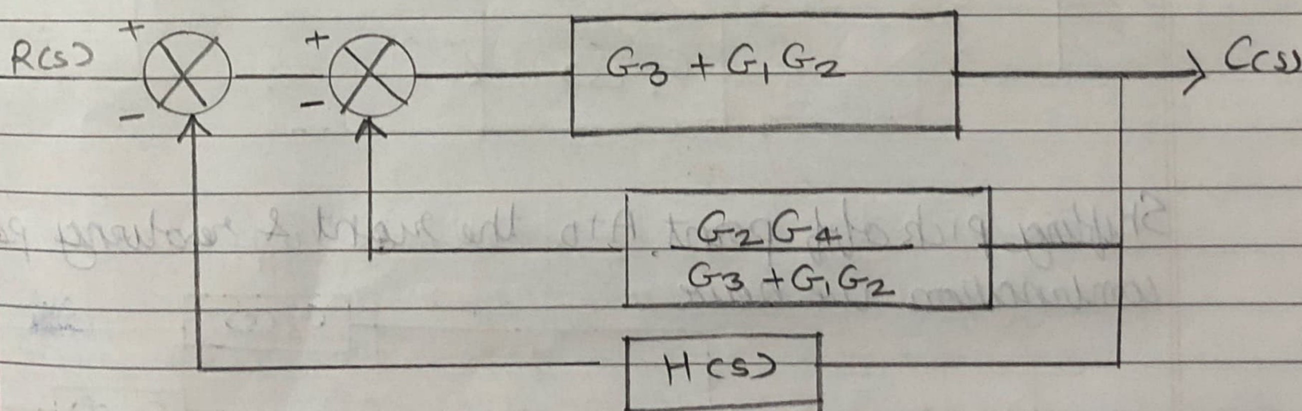




Shifting adder to left we get



$$X(s) = H(s) + \frac{G_2 G_4}{G_3 + G_1 G_2} = \frac{H(s) G_3 + H(s) G_1 G_2 + G_2 G_4}{G_3 + G_1 G_2}$$

combining with negative feedback path,

$$T(s) = \frac{G_3 G_1 G_2}{1 + H(s) G_3 + H(s) G_1 G_2 + G_2 G_4}$$

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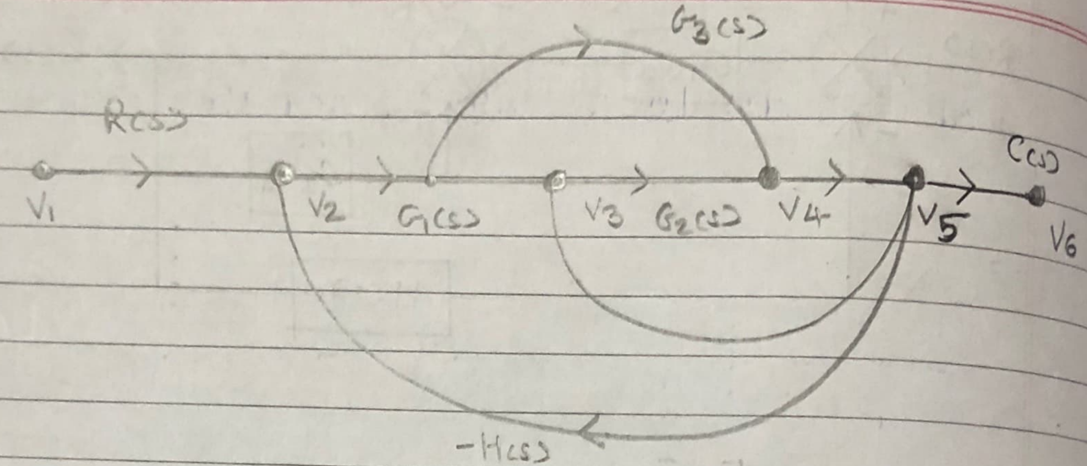
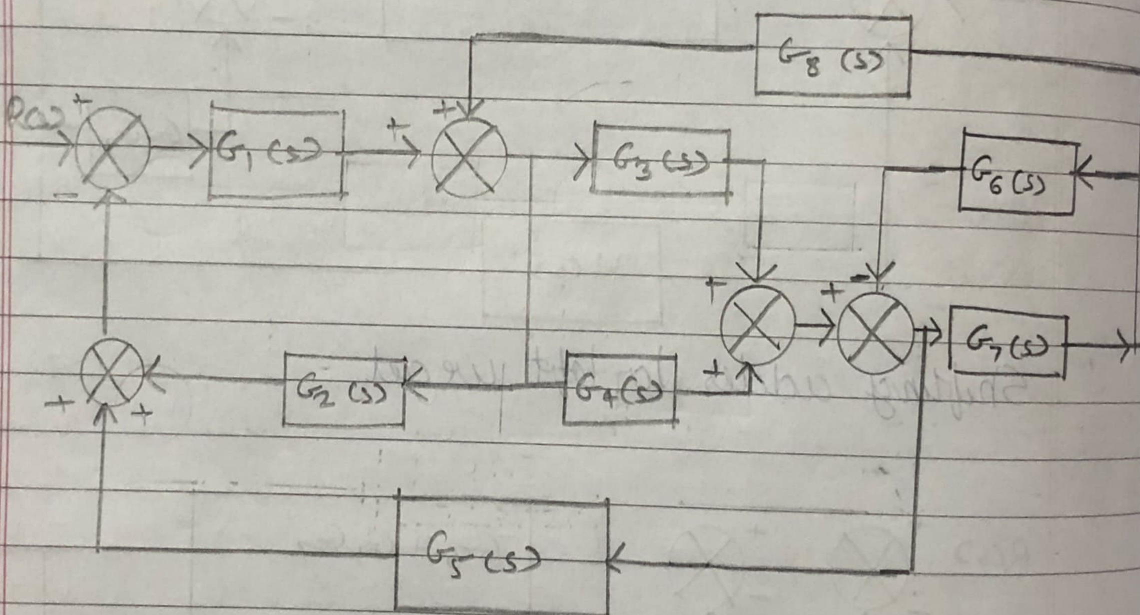
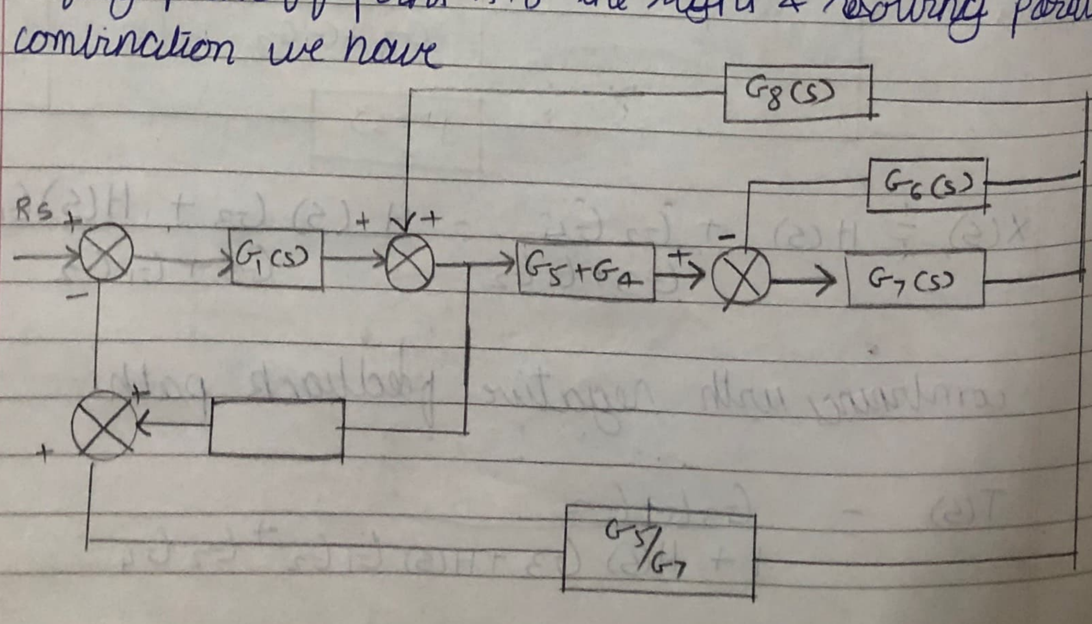


A5]



Shifting pick off point A to the right & resolving parallel combination we have



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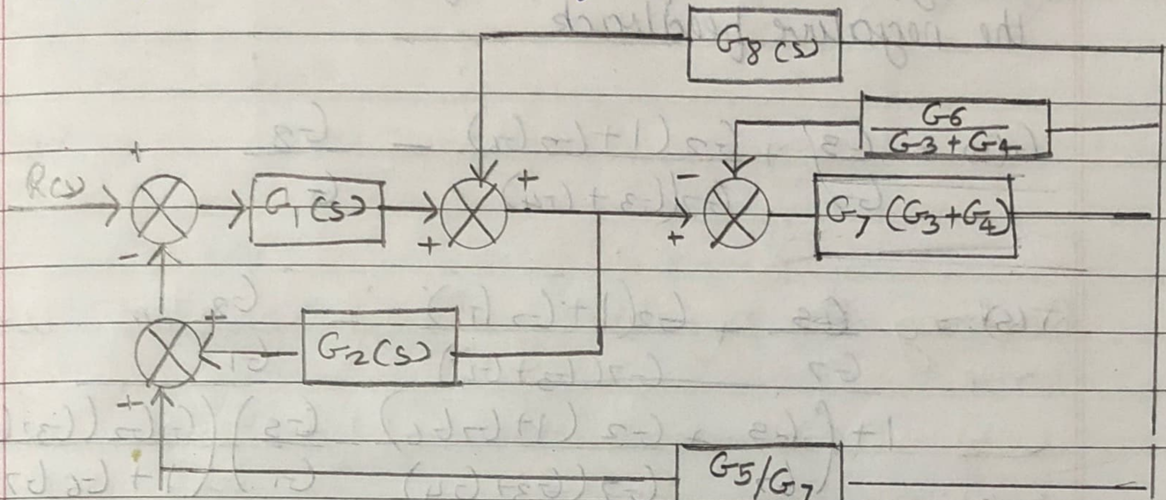
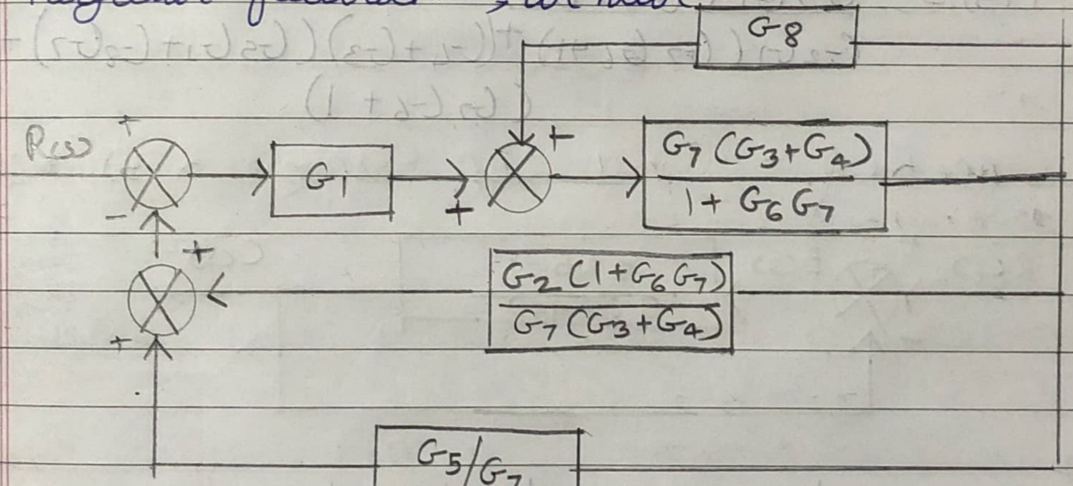
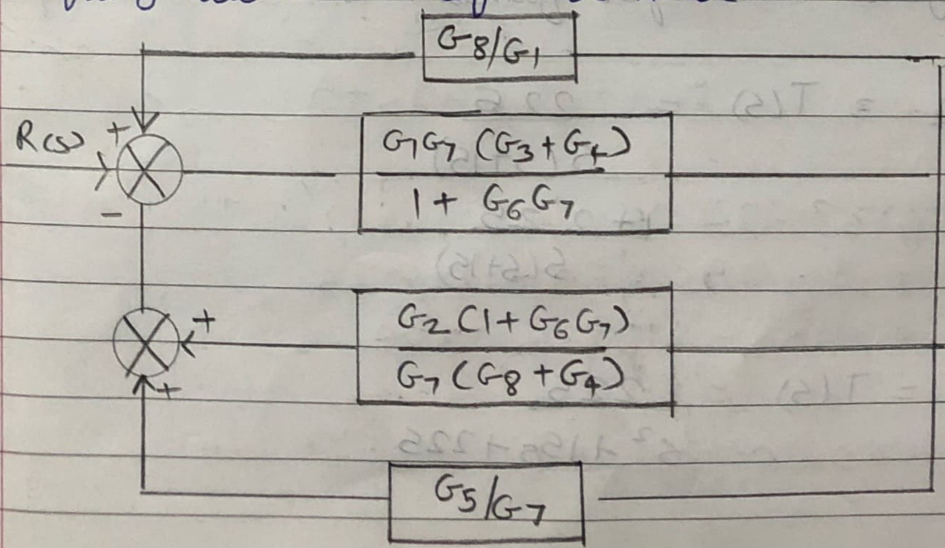
Shifting adder to the left, we have:



shifting pickoff point to the right resolving negative feedback, we have:



shifting adder to left we have



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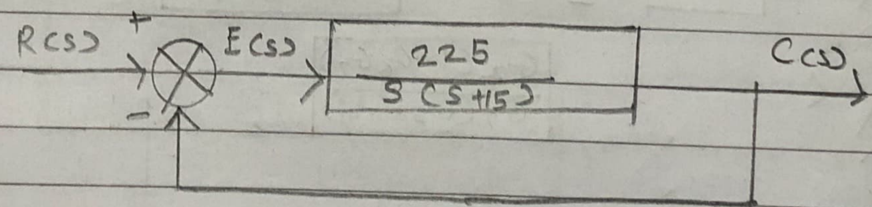
Adding all the parallel feedback path to get the negative feedback

$$G(s) = \frac{G_5}{G_7} + \frac{G_2(1+G_7G_6)}{G_7(G_3+G_4)} - \frac{G_8}{G_1}$$

$$T(s) = \frac{\frac{G_5}{G_7} + \frac{G_2(1+G_7G_6)}{G_7(G_3+G_4)} - \frac{G_8}{G_1}}{1+\left(\frac{G_5}{G_7} + \frac{G_2(1+G_7G_6)}{G_7(G_3+G_4)} - \frac{G_5}{G_1}\right)\left(\frac{G_1G_2(G_3+G_4)}{1+G_6G_7}\right)}$$

$$T(s) = \frac{G_7G_1(G_4+G_3)}{G_2G_1(G_7G_6+1)+(G_4+G_3)(G_5G_1+G_8G_7)+(G_7G_6+1)}$$

A6]



Resolving the negative feedback loop we have the final transfer function as

$$\frac{C(s)}{R(s)} = T(s) = \frac{\frac{225}{s(s+15)}}{1+\frac{225}{s(s+15)}}$$

$$\frac{G(s)}{R(s)} = T(s) = \frac{225}{s^2+15s+225}$$

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Comparing this to equation for transfer function

$$T(s) = \frac{\omega n^2}{s^2 + 2\zeta\omega n s + \omega n^2}$$

$\omega n^2 = 225$; $\omega n = 15$ rad/s

$2\zeta\omega n = 15$

$$\zeta = \frac{15}{2 \times 15} = 0.5$$

① $\% OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100$

$$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = -1.813852571$$

$e^{-1.813852571} \times 100 = 0.163 \times 100 = \underline{16.3\ \%}$

② Settling time $= \dfrac{4}{\zeta\omega n} = \dfrac{4}{0.5 \times 10} = \underline{0.5333 \text{ secs}}$

③ Peak time $(T_p) = \dfrac{\pi}{\omega n\sqrt{1-\zeta^2}} = \dfrac{\pi}{15 \times \sqrt{1-(0.5)^2}}$

$= 0.24184$ secs

Since damping lies between 0 & 1 $0 < \zeta < 1$ and poles of the system consist of real part & imaginary part, we can say that the response is underdamped

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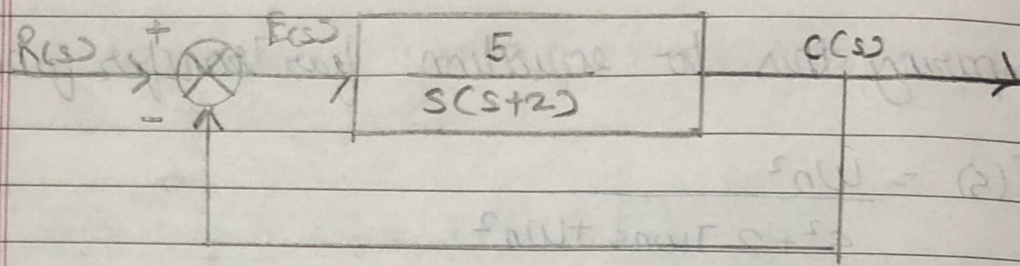
Q7)



Resolving the negative feedback system, we have

$$\frac{C(s)}{R(s)} = T(s) = \frac{5}{s(s+2)} = 1 + \frac{5 \times 1}{s(s+2)}$$

$$T(s) = \frac{5}{s^2 + 2s + 5}$$

Since R(s) = Input step response $R(s) = \frac{1}{s}$

$C(s) = T(s)\,R(s)$

$$C(s) = \frac{5}{s(s^2+2s+5)}$$

By using partial fraction, we have

$$\frac{5}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs + C}{s^2+2s+5}$$

$$5 = A(s^2+2s+5) + s(Bs+C)$$

$$5 = As^2 + 2As + 5A + Bs^2 + Cs$$

$$5 = (A+B)s^2 + (2A+C)s + 5A$$

Comparing coefficients

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$5A = 5, \; A = 1$

$2A + C = 0 \; ; \; C = -2A = -2 \; , \; C = -2$

$A + B = 0 \; ; \; A = -B \; , \; B = -1$

$$C(s) = \frac{1}{s} + \frac{-s-2}{s^2+2s+5}$$

$$C(s) = \frac{1}{s} - \frac{(s+2)}{(s+1)^2+(2)^2}$$

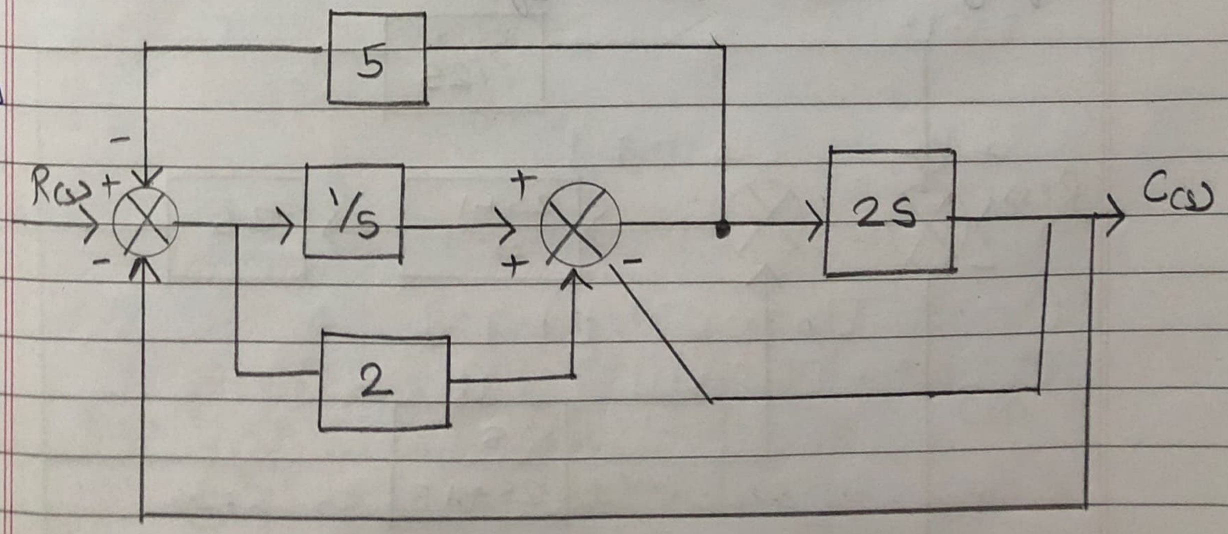
$$C(s) = \frac{1}{s} - \frac{(s+1)}{(s+1)^2+(2)^2} - \frac{2}{(s+1)^2+(2)^2} \times \frac{1}{2}$$

$$C(s) = \frac{1}{s} - \frac{(s+1)}{(s+1)^2+(2)^2} - \frac{1}{2} \times \frac{2}{(s+1)^2+(2)^2}$$

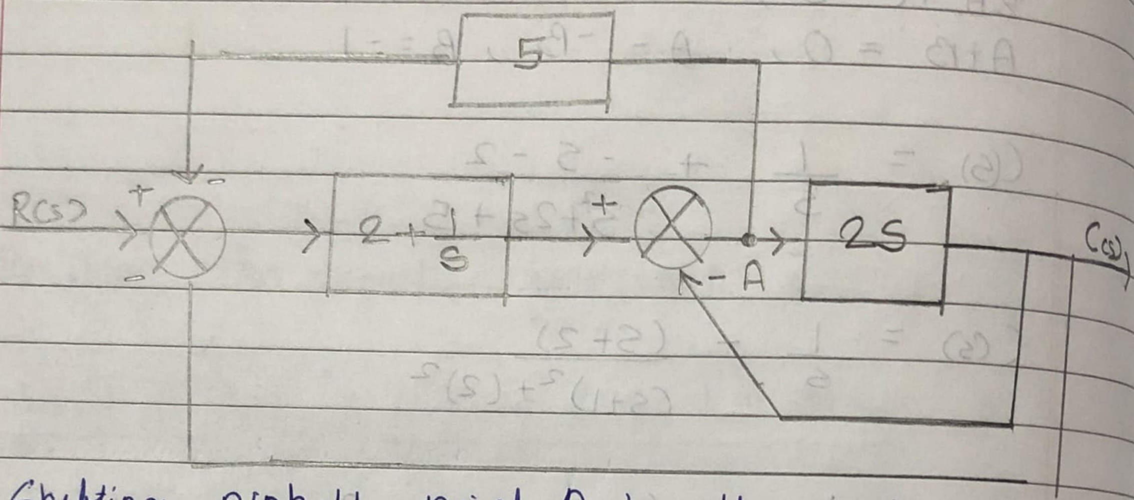
$$c(t) = 1 - e^{-t}\cos 2t - \frac{1}{2} e^{-t} \sin 2t$$

$$c(t) = 1 - e^{-t}\left(\cos 2t + \frac{\sin 2t}{2} \right)$$

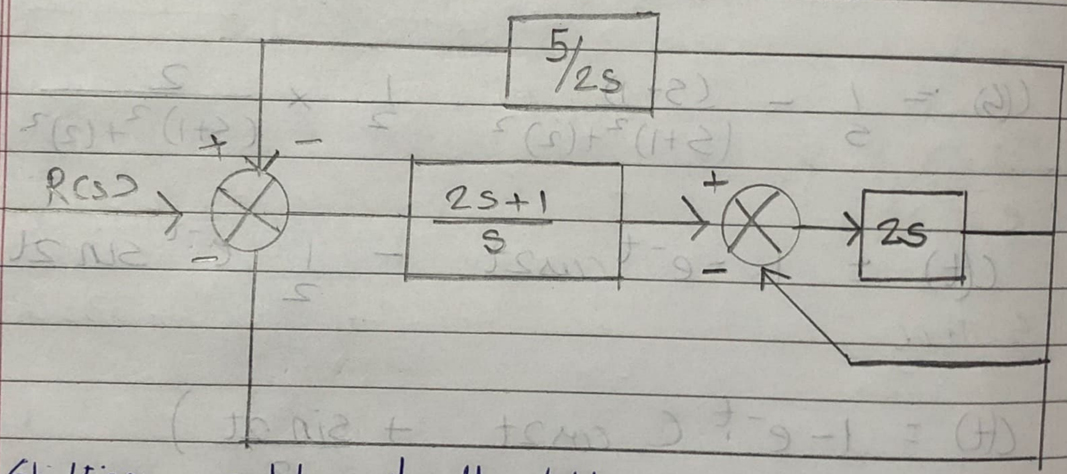
A3]



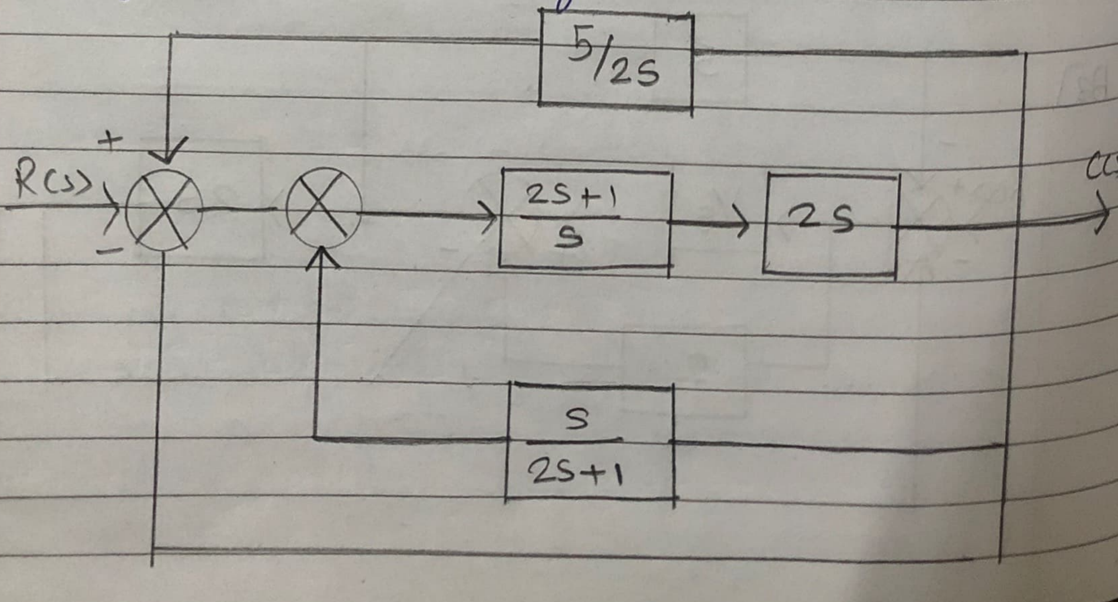
Resolving parallel combination we have



Shifting pickoff point A to the right



Shifting adder to the left



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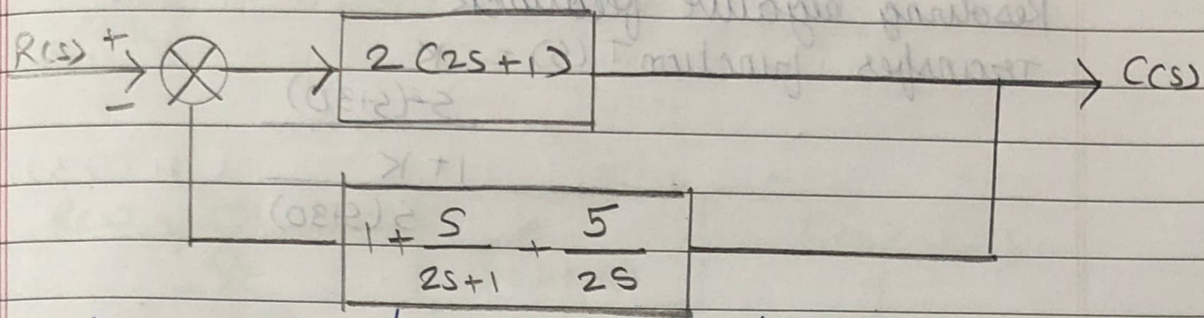


Resolving negative feedback, we have.

$$T(s) = \frac{2(2s+1)}{1+2(2s+1)\left(1+\frac{s}{2s+1}+\frac{5}{2s}\right)}$$

$$T(s) = \frac{2(2s+1)}{1+2(2s+1)+2s+\frac{5(2s+1)}{s}}$$

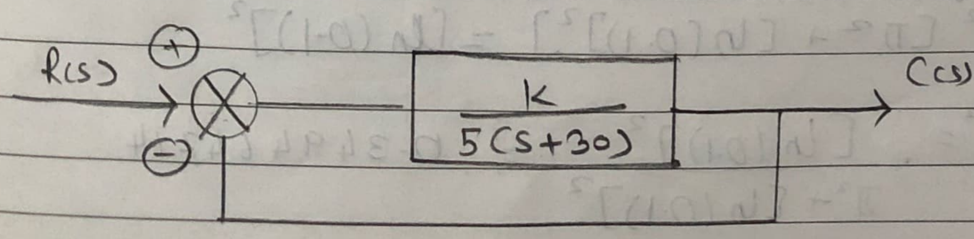
$$T(s) = \frac{2s(2s+1)}{s+2s(2s+1)+2s^2+10s+5} = \frac{2s(2s+1)}{s+4s^2+2s+2s^2+10s+5}$$

$$T(s) = \frac{2s(2s+1)}{6s^2+13s+5} = \frac{2s(2s+1)}{6s^2+10s+3s+5}$$

$$= \frac{2s(2s+1)}{(2s+1)(3s+5)}$$

Poles at $a = -\frac{1}{2}, -\frac{5}{3}$

A9]



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Resolving negative feedback

Transfer function $T(S) = \dfrac{\dfrac{K}{S(S+30)}}{1+\dfrac{K}{S(S+30)}}$

$$\frac{C(S)}{R(S)} = T(S) = \frac{K}{S^2+30S+K}$$

comparing T(S) with eq,

$$T(S) = \frac{\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$$

$\omega_n^2 = K$, $\underline{\omega_n = \sqrt{K}}$

$2\zeta\omega_n = 30$

$\zeta = \dfrac{15}{\sqrt{K}}$

% OS = 10 %

$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 10$ $= 0.1$

$$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = \ln(0.1)$$

$$+\frac{\zeta^2\pi^2}{1-\zeta^2} = [\ln(0.1)]^2$$

$$\zeta^2\pi^2 = [\ln(0.1)]^2[1-\zeta^2]$$

$$\zeta^2[\pi^2+[\ln(0.1)]^2] = [\ln(0.1)]^2$$

$$\zeta^2 = \frac{[\ln(0.1)]^2}{\pi^2+[\ln(0.1)]^2} = 0.349464274$$

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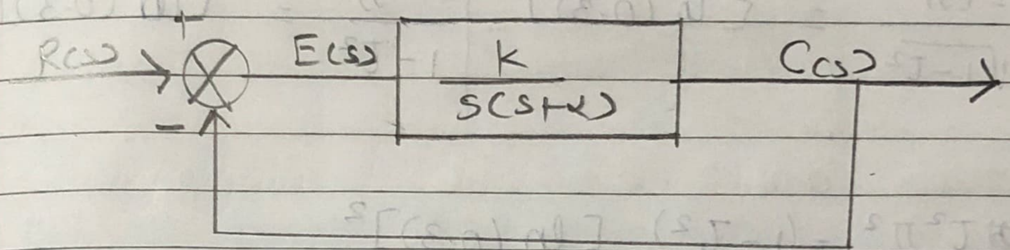
$\frac{225}{k} = 0.34946 \Rightarrow k = 643.8$

A10]



Resolving negative feedback system, we have,

$$T(s) = \frac{\frac{k}{s(s+2)}}{1 + \frac{k}{s(s+2)}}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s^2 + 2s + k} = T(s)$$

comparing $T(s)$ with eq

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\omega_n = \sqrt{k}$

$2\zeta\sqrt{k} = \alpha$

$\zeta = \frac{\alpha}{2\sqrt{k}}$

Settling time $T_s = \frac{4}{\zeta\omega_n} = 0.15$

$\frac{4}{0.15} = \frac{\alpha}{2\sqrt{k}} \cdot \sqrt{k}$

$\alpha = \frac{8}{0.15} = 53.33$

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% overshoot = 30 %

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = \frac{30}{100} = 0.3$$

$$\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = [\ln(0.3)] \quad \Big| \quad \frac{\zeta^2\pi^2}{1-\zeta^2} = [\ln(0.3)]^2$$

$$\zeta^2\pi^2 = (1-\zeta^2)[\ln(0.3)]^2$$

$$\zeta^2[\pi^2 + [\ln 0.3)^2] = [\ln(0.3)]^2$$

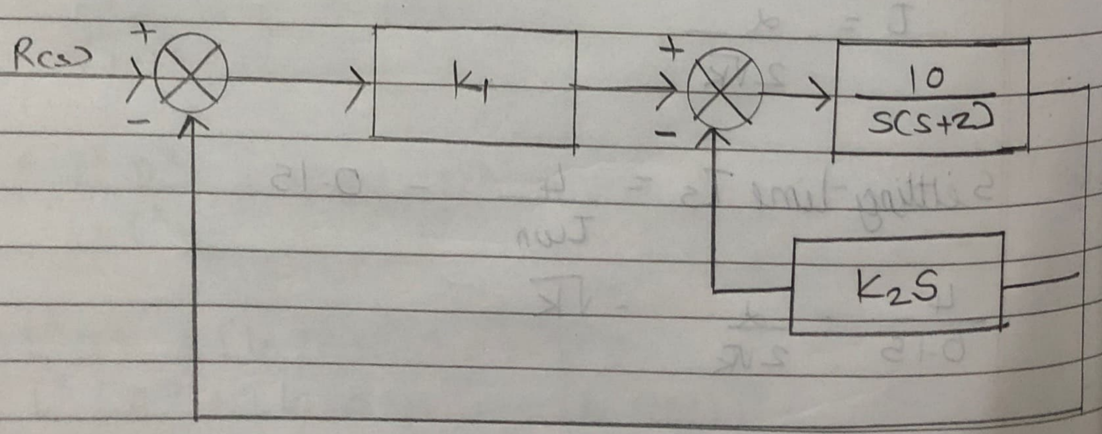
$$\zeta^2 = \frac{[\ln(0.3)]^2}{[\pi^2 + (\ln 0.3)^2]}$$

$$\frac{\alpha^2}{4K} = 0.128061725$$

$$\frac{(53.33)^2}{4(0.128061725} = K$$

$$\boxed{K = 5549}$$

A11]



Shifting adder to the left we have

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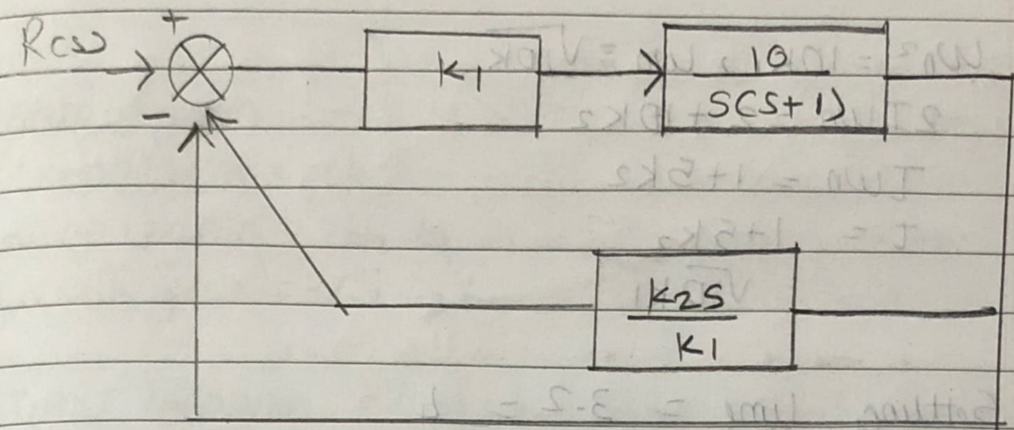
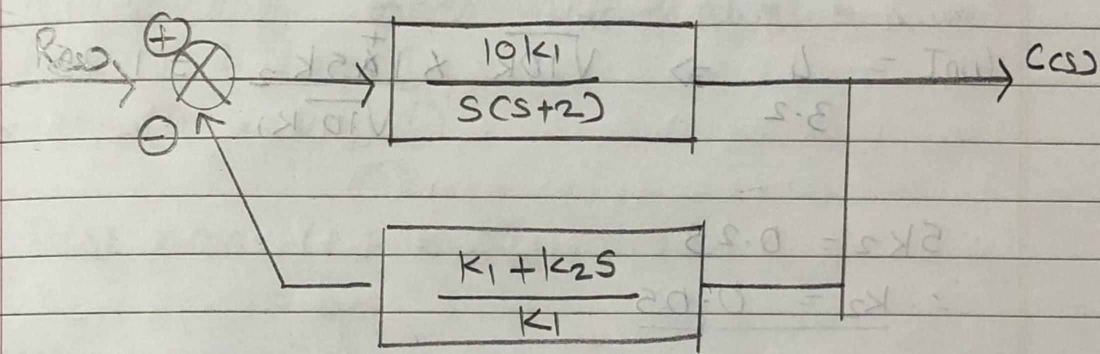


Resolving parallel and series combination we have



Resolving negative feedback, we have

$$T(s) = \frac{\frac{10K_1}{s(s+2)}}{1+\frac{10K_1}{s(s+2)}\cdot\frac{(K_1+K_2 s)}{K_1}}$$

$$T(s) = \frac{10K_1}{s^2+2s+10(K_1+K_2 s)}$$

$$T(s) = \frac{10K_1}{s^2+(2+K_2\times 10)s+10K_1} = \frac{10K_1}{s^2+(10K_2+2)s+10K_1}$$

Comparing with eq $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\omega_n^2 = 10K_1, \ \omega_n = \sqrt{10K}$

$2\zeta\omega_n = 2 + 10K_2$

$\zeta\omega_n = 1 + 5K_2$

$\zeta = \dfrac{1+5K_2}{\sqrt{10}K_1}$

Settling time $= 3.2 = \dfrac{4}{\omega_n\zeta}$

$\omega_n\zeta = \dfrac{4}{3.2} \Rightarrow \sqrt{10K} \times \dfrac{1+5K_2}{\sqrt{10}K_1} = 1.25$

$5K_2 = 0.25$

$\therefore \underline{K_2 = 0.05}$

Peak time $= 1.5 = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$(1.5)^2 = \dfrac{\pi^2}{\omega_n^2(1-\zeta^2)}$

$(1.5)^2 \times 10K_1 = \dfrac{\pi^2}{1 - \dfrac{(1+5K_2)^2}{10K_1}}$

$K_1 = \dfrac{\pi^2 + ((1.5)\times(1.25))^2}{(1.5)^2 \times 10}$

$\boxed{K_1 = 0.594}$

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A12]

Loop ① gain $= -1$
Loop ② gain $= -G_4$
Loop ③ gain $= -G_3 G_4$
Loop ④ gain $= -G_2 G_3 G_4$

Total loop gain $= -1 - G_4 - G_3 G_4 - G_2 G_3 G_4$

Non touching loops taken 2 at a time
loop ① & loop ②
loop ③ & loop ①

Total gain (Non touching loops)
$= G_4 + G_3 G_4$

$\Delta = 1 - \Sigma$ loop gains $+ \Sigma$ non touching loops taken 2 at a time

$\Delta = 1 + 1 + G_4 + G_3 G_4 + G_2 G_3 G_4 + G_4 + G_3 G_4$

$\Delta = 2 + 2G_4 + 2 G_3 G_4 + G_2 G_3 G_4$

$\Delta_1 = 1$

$T_1 =$ foward path gain $= G_1 G_2 G_3 G_4$

$$T(s) = \frac{G_1(s)\, G_2(s)\, G_3(s)\, G_4(s)}{2 + 2G_4 + 2 G_3 G_4 + G_2 G_3 G_4}$$

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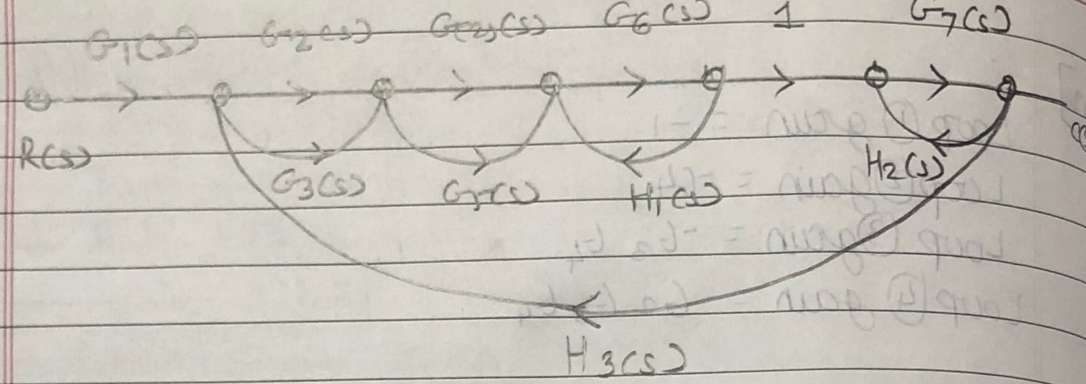
A13)



Forward path gains

$T_1 = G_1(S)\,G_2(S)\,G_4(S)\,G_6(S)\,G_7(S)$

$T_2 = G_1(S)\,G_3(S)\,G_5(S)\,G_6(S)\,G_7(S)$

$T_3 = G_1(S)\,G_3(S)\,G_6(S)\,G(7)(S)$

$T_4 = G_1(S)\,G_2(S)\,G_5(S)\,G_6(S)\,G_7(S)$

Closed Loop gains

Loop ① gain $= G_2 G_5 G_6 G_7 H_3$

Loop ② gain $= G_3 G_2 G_6 G_7 H_3$

Loop ③ gain $= G_6 H_2$

Loop ⑤ gain $= G_2 G_4 G_6 G_7 H_3$

Loop ④ gain $= G_7 H_2$

Loop ⑥ gain $= G_3 G_5 G_6 G_7 H_3$

Non touching loops taken two at a time

Gain $= G_6 H_1 G_7 H_2$

$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = 1$

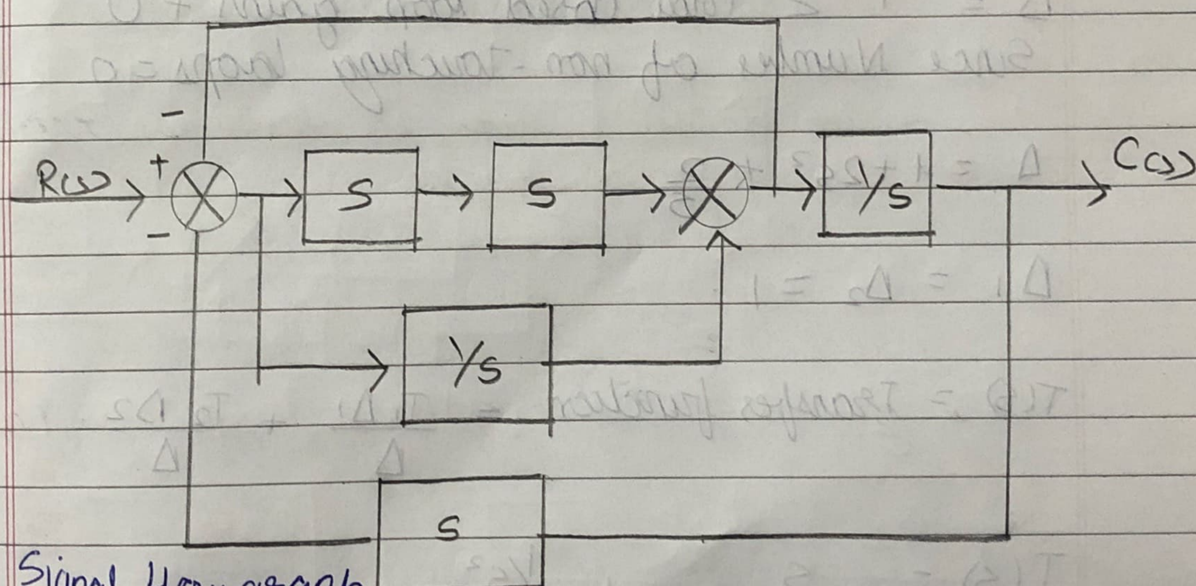
$$T(s) = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

$$\Delta = 1 - (G_2G_5G_6G_7H_3 + G_3G_4G_6G_7H_3 + G_2G_4G_6G_7H_3 + G_6H_1 + G_7H_2) + G_6H_1G_7H_2 - G_3G_5G_6G_7H_3$$

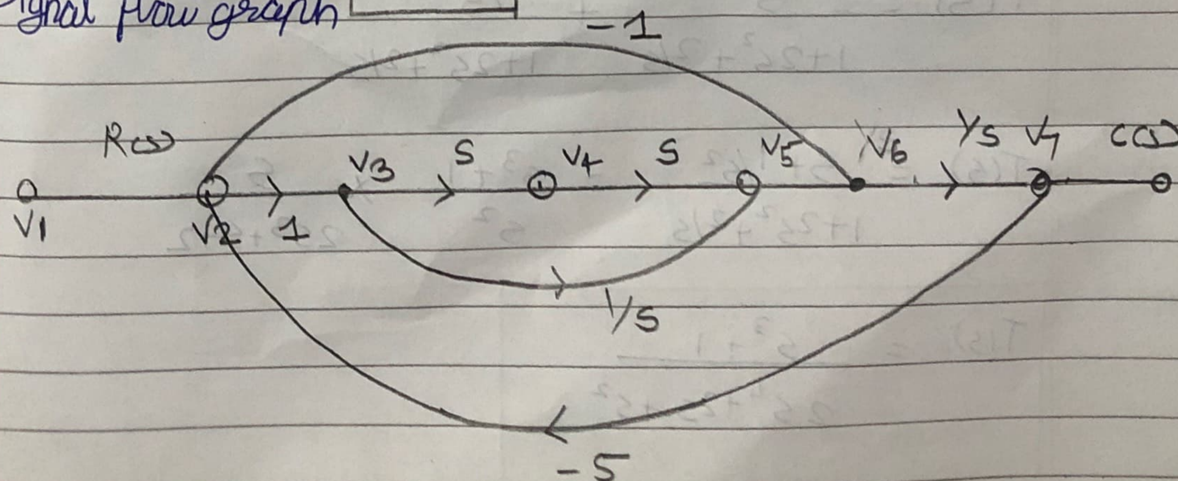
$$T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4$$
$$= G_1G_2G_4G_7G_6 + G_1G_3G_5G_6G_7 + G_1G_3G_4G_6G_7 + G_1G_2G_5G_6G_7$$

$$T(s) = \frac{G_1G_2G_4G_7G_6 + G_1G_3G_5G_6G_7 + G_1G_3G_4G_6G_7 + G_1G_2G_5G_6G_7}{(1 - G_2G_5G_6G_7H_3 - G_2G_4G_6G_7H_3 - G_6H_1 - G_7H_2 - G_3G_5G_6G_7H_3 + G_6H_1G_7H_2)}$$

A14]



Signal flow graph



Forward path gains

Path ① = $V_1V_2V_3V_4V_5V_6V_7 = s^2 \times \frac{1}{s} = s\ (T_1)$

Path ② $= V_1V_2V_3V_5V_6V_7 = \frac{1}{s} \times \frac{1}{s} = \frac{1}{s^2} = T_2$

Loop gains

Loop① $\rightarrow V_2V_3V_4V_5V_6V_7 = -s^2$
Loop② $\rightarrow V_2V_3V_4V_5V_6V_7V_2 = -s^2$
Loop③ $\rightarrow V_2V_3V_5V_6V_7 = 1/s$
Loop ④ $\rightarrow V_2V_3V_5V_6V_7V_2 = \frac{1}{s^2}(-s) = -1/s$

$\Delta = 1 - \Sigma$ Total closed loop gains $+ 0$
Since Number of non-touching loops $= 0$

$\Delta = 1 + 2s^2 + \frac{2}{s}$

$\Delta_1 = \Delta_2 = 1$

$T(s)$ = Transfer function $= \frac{T_1\Delta_1}{\Delta} + \frac{T_2\Delta_2}{\Delta}$

$$T(s) = \frac{s}{1+2s^2+2/s} + \frac{1/s^2}{1+2s^2+2/s}$$

$$T(s) = \frac{s+1/s^2}{1+2s^2+2/s} = \frac{s^3+1}{s^2} \times \frac{s}{2s^3+s+2}$$

$$\boxed{T(s) = \frac{s^3+1}{2s^4+2s+s^2}}$$

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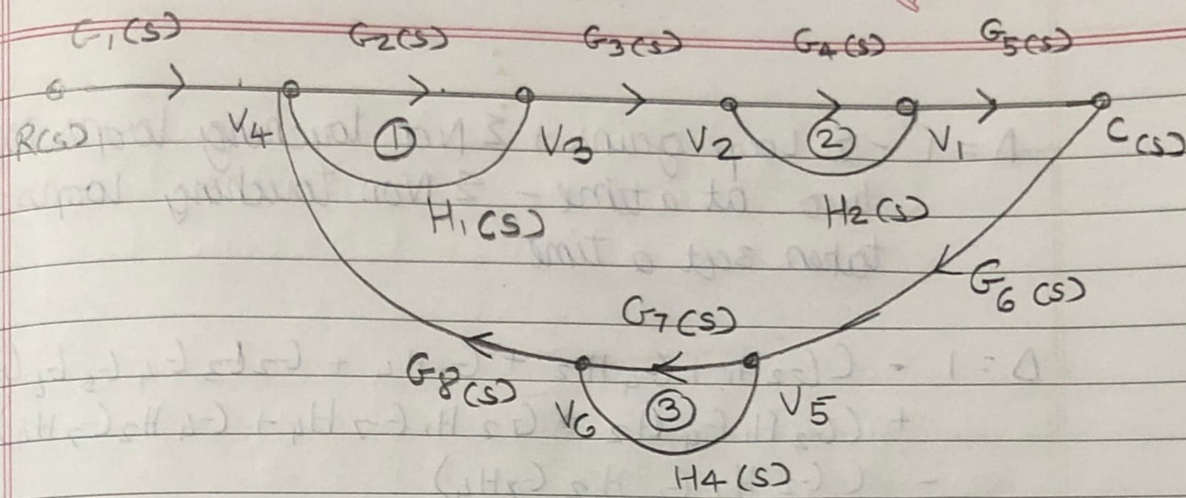
A15]



Forward path gain

$T_1 = G_1 G_2 G_3 G_4 G_5$

Closed loop gains

1. $G_2 H_1$
2. $G_4 H_2$
3. $G_7 H_4$
4. $G_2 G_3 G_4 G_5 G_6 G_7 G_8$

Gains of Non-touching loops taken 2 at a time

1 and 2 $\Rightarrow G_2 H_1 G_4 H_2$
1 & 3 $\Rightarrow G_2 H_1 G_7 H_4$
2 & 3 $\Rightarrow G_4 H_2 G_7 H_4$

Gains of Non touching loop taken 3 at a time.
1, 2 and 3 $\Rightarrow G_2 H_1 G_4 H_2 G_7 H_4$

Δ_1 is obtained by eliminating from Δ the forward path

$\Delta_1 = 1 - G_2 H_4$

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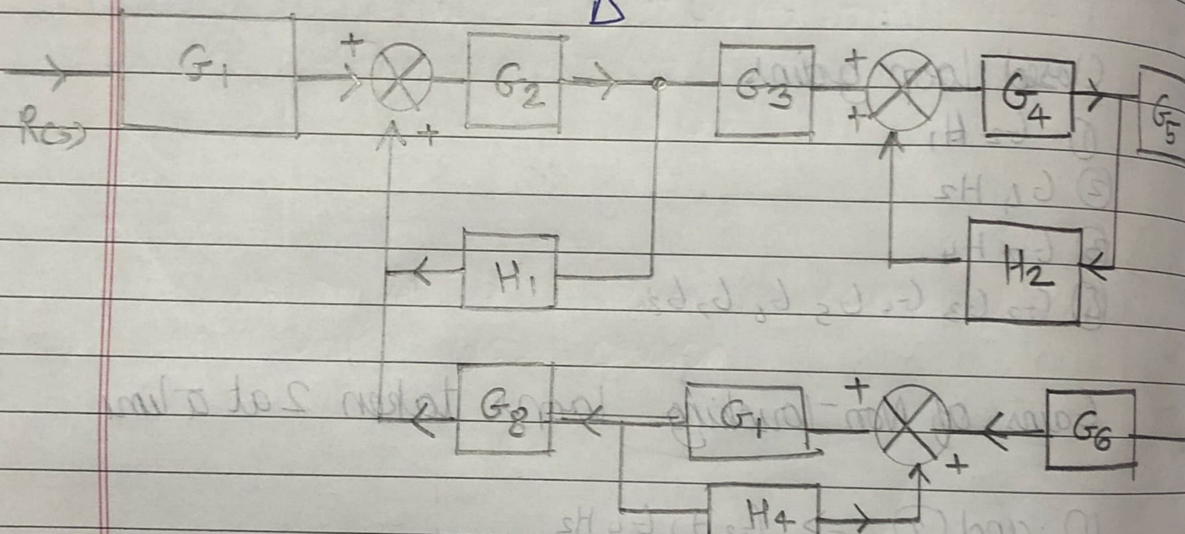
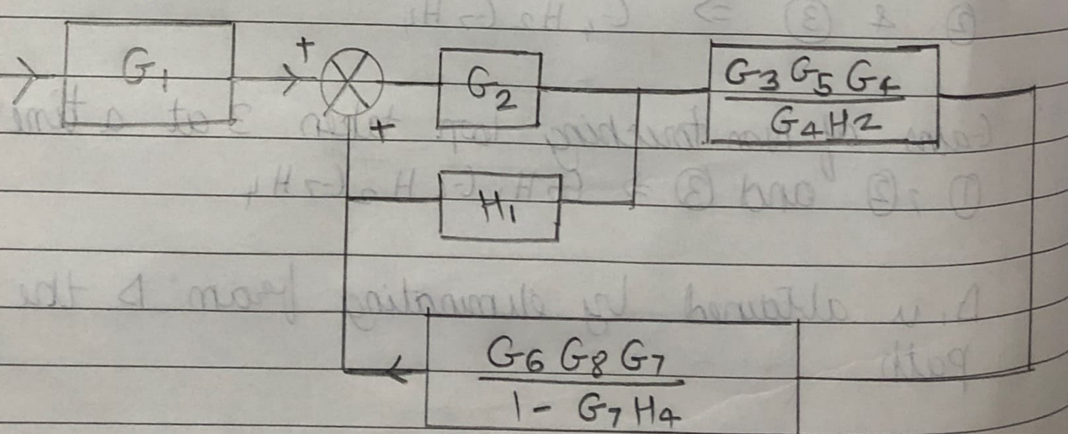
$\Delta = 1 - \Sigma$ Loop gains $+ \Sigma$ Non touching loops taken two at a time $- \Sigma$ Non touching loops taken 3 at a time

$$\Delta = 1 - (G_2H_1 + G_4H_2 + G_7H_4 + G_2G_3G_4G_5G_6G_7G_8$$
$$+ (G_2H_1G_4H_2 + G_2H_1G_7H_4 + G_4H_2G_7H_4)$$
$$- (G_2H_1G_4H_2G_7H_4)$$

$$T(S) = \frac{T_1\, G_1G_2G_3G_4G_5(1-G_7H_4)}{\Delta}$$



Resolving all positive feedback loops we have



Shifting pickoff point to the right

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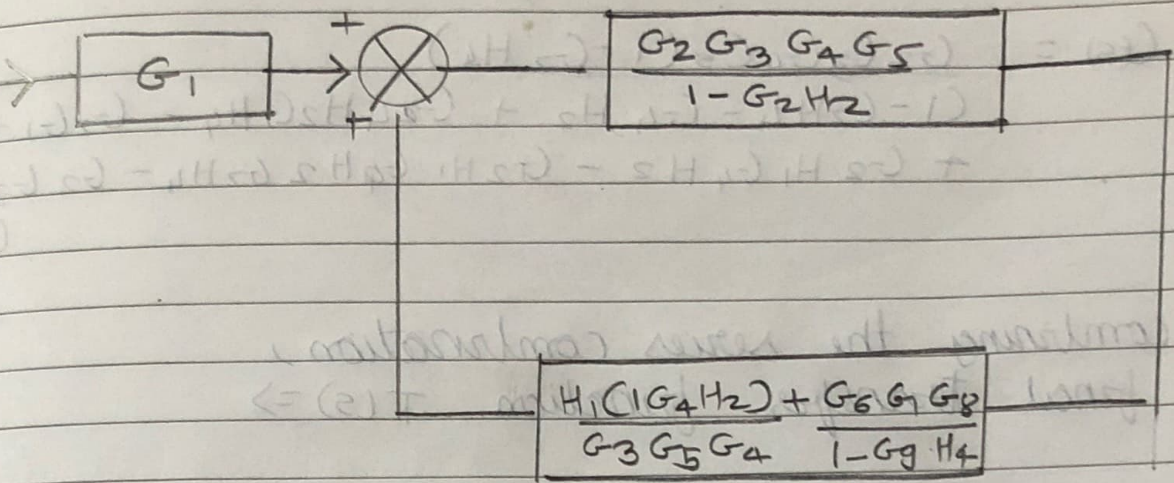

Resolving positive feedback we have

$$G(s) = \frac{\frac{G_2 G_3 G_4 G_5}{1 - G_4 H_2}}{1 - \left(\frac{G_2 G_3 G_4 G_5}{1 - G_4 G H_2}\right)\left(\frac{H_1(1 - G_4 H_2)}{G_3 G_4 G_5} + \frac{G_6 G_7 G_8}{1 - G_7 H_4}\right)}$$

$$G(s) = \frac{\frac{G_2 G_3 G_4 G_5}{1 - G_4 H_2}}{1 - G_2 H_1 - \frac{G_2 G_3 G_4 G_5 G_6 G_7 G_8}{(1 - G_4 H_2)(1 - G_7 H_4)}}$$

$$G(s) = \frac{G_2 G_3 G_4 G_5 (1 - G_7 H_4)}{(1 - G_4 H_2)(1 - G_7 H_4)(1 - G_2 H_1) - G_2 G_3 G_4 G_5 G_6 G_7 G_8}$$

$$G(s) = \frac{G_2 G_3 G_4 G_5 \quad (1 - G_7 H_4)}{(1 - G_2 H_1)(1 - G_7 H_4 - G_4 H_2 + G_4 H_2 G_7 H_4) - G_2 G_3 G_4 G_5 G_6 G_7 G_8}$$

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$$C(s) = \frac{G_2 G_3 G_4 G_5 (1 - G_7 H_4)}{(1 - G_7 H_4 - G_4 H_2 + G_4 H_2 G_7 H_4 - G_2 G_1 + G_2 H_1 G_7 H_4 + G_2 H_1 G_4 H_2 - G_2 H_1 G_4 H_2 G_7 H_4 - G_2 G_3 G_4 G_5 G_6 G_7 G_8)}$$

combining the series combination, final transfer function $T(s) \Rightarrow$

$$T(s) = \frac{G_1 G_2 G_3 G_4 G_5 (1 - G_7 H_4)}{(1 - G_7 H_4 - G_4 H_2 + G_4 H_2 G_7 H_4 + G_2 H_1 G_7 H_4 + G_2 H_1 G_4 H_2 - G_2 G_3 - G_2 H_1 G_4 H_2 G_7 H_4 - G_2 G_3 G_4 G_5 G_6 G_7 G_8)}$$

As you can see the Answer is same as obtained from signal flow graph