Tutorial 3

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Q1 Define vector field & scalar field give an example of each.

→ vector field: in a vector calculus a vector field is an assignment of a vector to each point in a domain (subset) of space.
A vector field in the plane can be visualised as a collection of arrays with a given magnitude & direction.
eg vector field of electric current.

Scalar field: in mathematics & physics a scalar field associates a scalar value to every point in a space.
The scalar may be a mathematical number or physical quantity
e.g weight of body is 20kg

Q2 Define the derivative of a vector function? what is its significance in mechanics & in geometry.

→ A vector fun $v(t)$ is said to be differentiable at a point t if the following limits exist.

$$v'(t) = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

This vector $v'(t)$ is called derivative of $v(t)$

Q3 Define gradient of a scalar function, divergence & curl of a vector function & explain their physical significance.

→ gradient of scalar function:-
let concider function $f(x,y,z)$ is differentiable in a domain with cartesian co-ordinate x,y,z.

Then the gradient of that function is grad f or ∇f & notation is:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right] = \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

It also gives maximum rate of change of 'f'
It also gives the direction in which 'f' increases rapidly.

Q.4 Concider the pressure field given by $f(x,y) = 9x^2+4y^2$. Sketch the isobar for pressure 36. also find the region in which the pressure varies betn 36 & 144

→ $f(x,y) = 9x^2+4y^2$

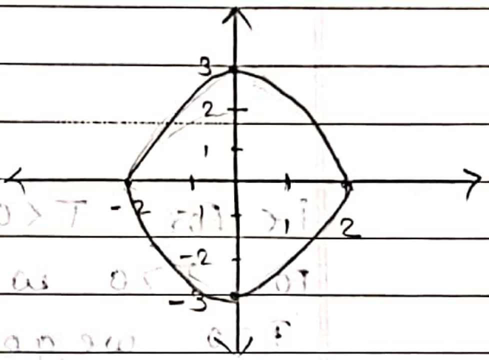
& $f(x,y) = 36$

∴ $9x^2+4y^2=36$

divide by 36 both side

$\frac{9x^2}{36}+\frac{4y^2}{36}=1$

$\frac{x^2}{4}+\frac{y^2}{9}=1$



$\frac{x^2}{2^2}+\frac{y^2}{3^2}=1$

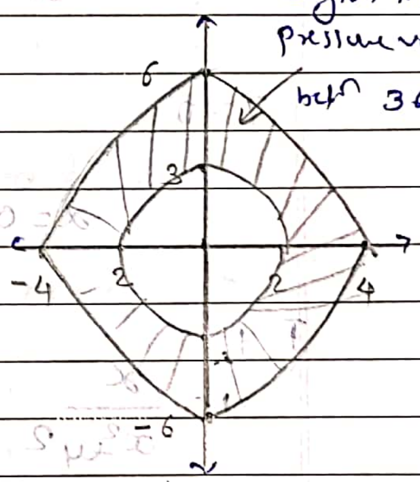
Now for 144

∴ $9x^2+4y^2=144$

$\frac{9x^2}{144}+\frac{4y^2}{144}=1$

$\frac{x^2}{16}+\frac{y^2}{36}=1$

$\frac{x^2}{4^2}+\frac{y^2}{6^2}=1$



Q5 Determine & sketch the isotherms of the temp field.

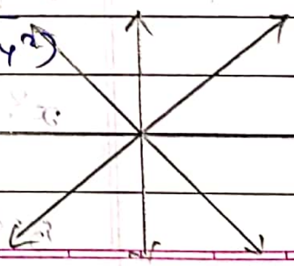
a) $T = x^2-y^2$ (b) $T = \frac{x}{(x^2+y^2)}$

→

a) $T = x^2-y^2$

at level 0 $= x^2-y^2$

$x^2 = y^2$

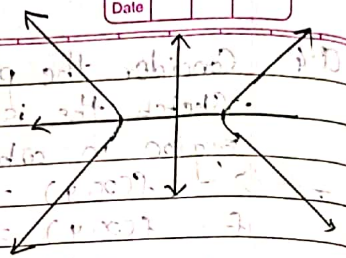


level 0

$T = 1$

$1 = x^2 - y^2$

$x^2 = 1 + y^2$



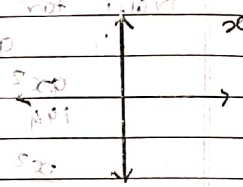
$T = -1$

$-1 = x^2 - y^2$

$x^2 = y^2 - 1$

i.e for $T<0$ we get vertical parabola.
For $T>0$ we get horizontal parabola
$T=0$ we get straight line.

b) $T = \dfrac{x}{x^2+y^2}$



$T=0$ $\quad 0 = \dfrac{x}{x^2+y^2}$

$x = 0$

$T=1$

$1 = \dfrac{x}{x^2+y^2}$

$x^2+y^2 = x$

$x^2 - x + y^2 = 0$

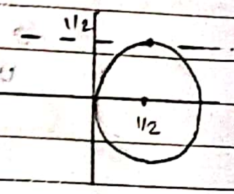
3rd term $\left(\frac{1}{2} \times \text{coeff. of } x\right)^2$

$\therefore \left(\frac{1}{2}\times 1\right)^2 = \frac{1}{4}$

$\therefore x^2 - x + \frac{1}{4} - \frac{1}{4} + y^2$

$x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$



circle with center $(\frac{1}{2}, 0)$ & radius $= \frac{1}{2}$.

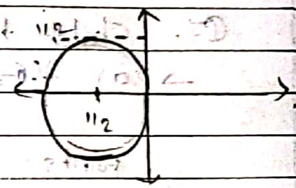
$T = -1$

$-1 = \dfrac{x}{x^2+y^2}$

$x^2+y^2 = -x$

$x^2+y^2+x = 0$

$x^2 + x + y^2 = 0$



$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$x^2 + x + \frac{1}{4} + y^2 = \frac{1}{4}$

$\left(x + \frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$

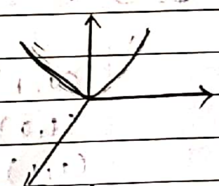
circle with center $(-\frac{1}{2}, 0)$ & radius $= \frac{1}{2}$

Q6 Determine the level surfaces of the scalar field

a) $f = x^2 + y^2 - z$

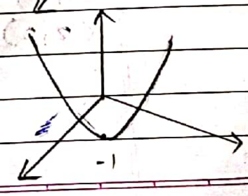
$\rightarrow$ $0 = x^2 + y^2 - z$

$x^2 + y^2 = z$



$1 = x^2 + y^2 - z$

$x^2 + y^2 - 1 = z$



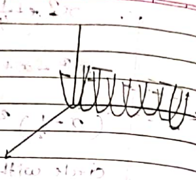
$-i = \leftarrow$ $+i = \rightarrow$

$-j = \downarrow$ $+j = \uparrow$

b) $f = y^2 - z$

$0 = y^2 - z$

$y^2 = z$

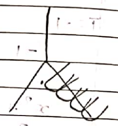


$1 = y^2 - z$

$1 + z = y^2$

$z = y^2 - 1$

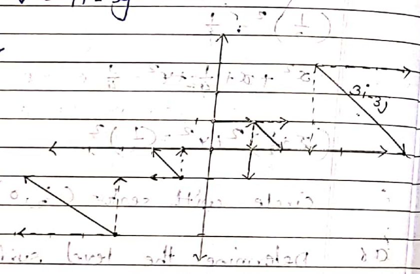
i.e $z = -1$



Q7. Sketch the vector fields given by the vector fun.

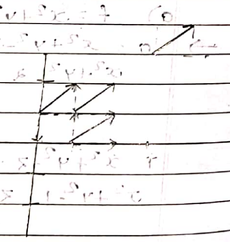
→ a) ~~$f(x^2+$~~ $\vec{v} = y\hat{i} - x\hat{j}$

points	vector
(0, 1)	$\hat{i}$
(1, 0)	$-\hat{j}$
(1, 1)	$i - \hat{j}$
(3, 3	$3\hat{i} - 3j$
(-1, -1)	$-i + j$
(-3, -2)	$-3i + 2j$



b $\vec{v} = i + j$

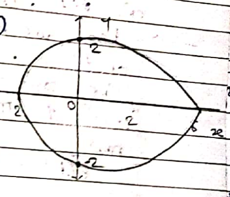
(0, 1)	$i + j$
(1, 0)	$i + j$
(1, 1)	$1 + j$
(3, 3)	$1 + j$



Q8 Sketch the following curves & identify them.

a) $\vec{E(t)} = [2 + 4\cos t,\ 0 + 2\sin t,\ 0]$
$x \qquad y \qquad z$

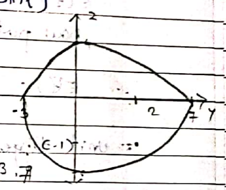
→ elipse having center (2, 0)
on xy plane $z = 0$



here
$-2 < x < 2$ & $-2 \leq y \leq 2$

b) $\vec{E(t)} = [-2,\ 2 + 5\cos t,\ -1 + 5\sin t]$

→ circle on yz plane
with radius 5 at
point (2, -1)
& $x = -2$



$2 + 0,\ 2 - 5,\ 2 + 5$ i.e $2, -3, 7$

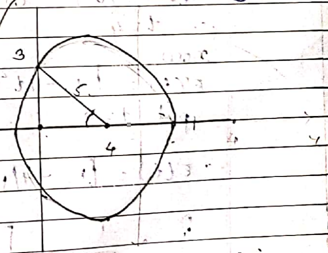
$-1 + 0,\ -1 - 5,\ -1 + 5$ i.e $-1, -6, 4$

∴ $-3 \leq y \leq 7$ & $-6 \leq z \leq 4$

Q9 Find the parametric representation of the circle in yz plane with center (4, 0) & passing through (0, 3) Sketch it

→ radius of circle
is $\sqrt{3^2 + 4^2} = \sqrt{9 + 16}$
$= 25$

$\boxed{E = 5}$



∴ $x = 0$

$y = 4 + 5\cos\theta$

$z = 0 + 5\sin\theta$

$\sin\theta = \frac{3}{5}$ $z = 5\sin\theta$

$\cos\theta = \frac{4}{5}$ $y = 5\cos\theta$

Q10

Q11. Find the tangent of the unit tangent vector to the given curve at the given point

a) $\vec{E(t)} = [\cos t, \sin t, 9t]$ point $P(1, 0, 18\pi)$

→ here we have need to ~~find need~~ value of 't'.

i.e $\cos t = 1$, $\sin t = 0$, $9t = 18\pi$

∴ $t = 2\pi$

now

$\vec{E(t)} = [\cos t, \sin t, 9t]$

$E'(t) = [-\sin t, \cos t, 9]$

now put $t = 2\pi$ in derivative

∴ $E(t) = [-\sin(2\pi), \cos(2\pi), 9] = [0, 1, 9]$

∴ unit tangent vector $(\hat{r}) = \frac{1}{\sqrt{0^2+1^2+9^2}} [0, 1, 9]$

$= \frac{1}{\sqrt{82}} [0, 1, 9]$

b) $\vec{E(t)} = [t, 4/t, 0]$, $P(4, 1, 0)$.

→ $[t, 4/t, 0] = [4, 1, 0]$

∴ $t = 4$

now

$E'(t) = [1, -4t^{-2}, 0]$

put $t = 4$

∴ $E'(t) = [1, -4/16, 0] = [1, -1/4, 0]$.

$\hat{r} = \frac{1}{\sqrt{1+\frac{1}{16}+0}} [1, -1/4, 0]$

$= \frac{1}{\sqrt{1.06}} [1, -1/4, 0]$

Q12. Find the length of the above curves from $t = 0$ to $t = 2\pi$ & $t = 1$ to $t = 2$ resp.

a) $\vec{E(t)} = [\cos t, \sin t, 9t]$

→ length of curve from 0 to 2π is

$\int_a^b E'(t)\,dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

we can put here

$\int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + (9)^2}\,dt = \int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t + 81}\,dt$

$(0)^2 + (1)^2 + (9)^2$

$\int_0^{2\pi} \sqrt{1+81}\,dt = \int_0^{2\pi} \sqrt{82}\,dt = \sqrt{82}\,[2\pi]$

$2\pi\sqrt{82}$

b) $\vec{E(t)} = [t, 4/t, 0]$

→

$\int_1^2 \sqrt{\left(\frac{dt}{dt}\right)^2 + \left(\frac{d\,4/t}{dt}\right)^2 + 0}\,dt = \int_1^2 \sqrt{1^2 + (-4t^{-2})^2 + 0}\,dt$

~~put~~ here

$\int_1^2 \sqrt{1 + \frac{16}{t^4}}\,dt = \cancel{\int_1^2 \sqrt{\frac{t^4+16}{t^4}}\,dt}$

$\cancel{\int_1^2 \sqrt{1^2 +}}$ $\int_1^2 \sqrt{\left(\frac{1}{16} + \frac{1}{t^4}\right)16}\,dt$

$4\int_1^2 \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{t^2}\right)^2}\,dt = 4\int_1^2 \sqrt{\left(\frac{1}{4}\right)^2 + (t^{-2})^2}\,dt$

use formula $\sqrt{a^2+x^2}\,dx =$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x+\sqrt{a^2+x^2}\right| + c$$

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$$4\left[\frac{t^{2}}{2}\sqrt{\frac{1}{16}+(t)^{4}} + \frac{1}{32}\ln\left|t^2+\sqrt{\frac{1}{16}+t^4}\right|\right]$$

$$4\left\{\left[\frac{1}{8}\sqrt{\frac{1}{16}+\frac{1}{16}} + \frac{1}{32}\ln\left|\frac{1}{8}+\sqrt{\frac{1}{16}+\frac{1}{16}}\right|\right] - \left[\frac{1}{2}\sqrt{\frac{1}{16}+1} + \frac{1}{32}\ln\left|1+\sqrt{\frac{1}{16}+1}\right|\right]\right\}$$

$$4\left[\frac{1}{8}\sqrt{\frac{1}{8}} + \frac{1}{32}\ln\left|\frac{1}{8}+\sqrt{\frac{1}{8}}\right|\right] - \left[\frac{1}{2}\sqrt{\frac{17}{16}} + \frac{1}{32}\ln\left|1+\sqrt{\frac{17}{16}}\right|\right]$$

$$4\left[\frac{0.35}{8} + \frac{1}{32}\ln\left|\frac{1}{8}+0.35\right|\right] - \left[\frac{1}{2}(1.030) + \frac{1}{32}\ln|1+1.030|\right]$$

$$4\left[\left[0.043 + \frac{1}{32}\ln|0.475|\right] - \left[0.515 + \frac{1}{32}\ln|2.030|\right]\right]$$

$4[(0.043+(-0.023)) - (0.515+0.022)]$

$4[0.043 - 0.023 - 0.515 - 0.022]$

$4[-0.517]$

-2.068

$\therefore 2.068$ Check This Answer

For grad (, ,)

For div() (+ +)

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Q19 Let $f = xy - yz$, $\vec{v} = [2y, 2z, 4x+z]$

$\vec{\omega} = [3z^2, 2x^2-y^2, y^2]$ find

a) div (∇f)

$\rightarrow$ $\nabla f = \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

$= y, x-z, -y$

$\therefore \nabla f = (y, x-z, -y)$

Now

$$\text{div}(\nabla f) = \left(\frac{\partial y}{\partial x} + \frac{\partial (x-z)}{\partial y} + \frac{\partial (-y)}{\partial z}\right)$$

$\therefore (0+0+0)$

$\text{div}(\nabla f) = 0$

b) grad $(\text{div}\,\vec{\omega})$

$\rightarrow$ $\vec{\omega} = [3z^2, 2x^2-y^2, y^2]$

$$\text{div}(\vec{\omega}) = \frac{\partial}{\partial x}(3z^2) + \frac{\partial}{\partial y}(2x^2-y^2) + \frac{\partial}{\partial z}(y^2)$$

$= 0 - 2y + 0$

$= -2y$

Now

$$\text{grad}(\text{div}(\vec{\omega})) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$= 0 - \frac{\partial}{\partial y}2y + 0$$

$\therefore \text{grad}(\text{div}(\vec{\omega})) = -2\hat{j}$ i.e. $0\hat{i} + 2\hat{j} + 0\hat{k}$

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