Reasoning with Uncertainty

- We briefly examined certainty factors earlier in the semester, but for the most part, we have only studied knowledge that is true/false or truth preserving
  - but the world is full of uncertainty, we need mechanisms to reason over that uncertainty
- We find two forms of uncertainty
  - unsure input
    - unknown – answer to a question is unknown
    - unclear – answer doesn't fit the question (e.g., not yes but 80% yes)
    - vague data – is a 100 degree temp a "high fever" or just "fever"?
    - ambiguous/noisy data – data may not be easily interpretable
  - non-truth preserving knowledge
    - most rules are *associational*, not truth preserving – for instance, "all men are mortal" is based on a class/subclass relationship whereas a more practical rule, "high fever means infection" is based on an association and the conclusion is not *guaranteed* to be true

Monotonicity

- Monotonicity – starting with a set of axioms, assume we draw certain conclusions
  - if we add new axioms, previous conclusions must remain true
    - the knowledge space can only increase, new knowledge should not rule out items previously thought to be true
  - example: assume that person X was murdered and through various axioms about suspects and alibis, we conclude person Y committed the murder
    - later, if we add new evidence, our previous conclusion that Y committed the murder *must* remain true
  - obviously, the real world doesn't work this way (assume for instance that we find that Y has a valid alibi and Z's alibi was a person who we discovered was lying because of extortion)

The Closed World Assumption

- In monotonic reasoning, if something is not explicitly known or provable, then it is false
  - this assumption in our reasoning can easily lead to faulty reasoning because its impossible to know everything
- – How can we resolve this problem?
  - we must either introduce all knowledge that is required to solve the problem at the beginning of problem solving
  - or we need another form of reasoning aside from monotonic logic
- – The logic that we have explored so far (first order predicate calculus with chaining or resolution) is monotonic (so is the Prolog system)
  - so now we turn to non-monotonic logic

Non-monotonicity

- Non-monotonic logic is a logic in which, if new axioms are introduced, previous conclusions *can* change
  - this requires that we update/modify previous proofs
    - this could be very computationally costly as we might have to redo some of our proofs
- We can enhance our previous algorithms
  - in logic, add M before a clause meaning "it is consistent with"
    - for all X: bright(X) & student(X) & studies(X,CSC) & M good_economy(time_of_graduation) → job(X, time_of_graduation)
      - a bright student who studies CSC will find a job at graduation if the economy is good – we may assume the CSC grad will find a job even if we don't know about the economy – that is, we are making an assumption in the face of a missing piece of knowledge
  - in a production system, add *unless* clauses to rules
    - if X is bright, X is a student and X studies computer science, then X will get a job at the time of graduation unless the economy is not good at that time
- These are forms of *assumption-based* reasoning

Dependency Directed Backtracking

- To reduce the computational cost of non-monotonic logic, we need to be able to avoid re-searching the entire search space when a new piece of evidence is introduced
  - otherwise, we have to backtrack to the location where our assumption was introduced and start searching anew from there
- In dependency directed backtracking, we move to the location of our assumption, make the change and propagate it forward from that point without necessarily having to re-search from scratch
  - as an example, you have scheduled a meeting on Tuesday at 12:15 because everyone indicated that they were available
  - but now, you cannot find a room, so you backtrack to the day and change it to Thursday, but you do not re-search for a new time because you assume if everyone was free on Tuesday, they will be free on Thursday as well

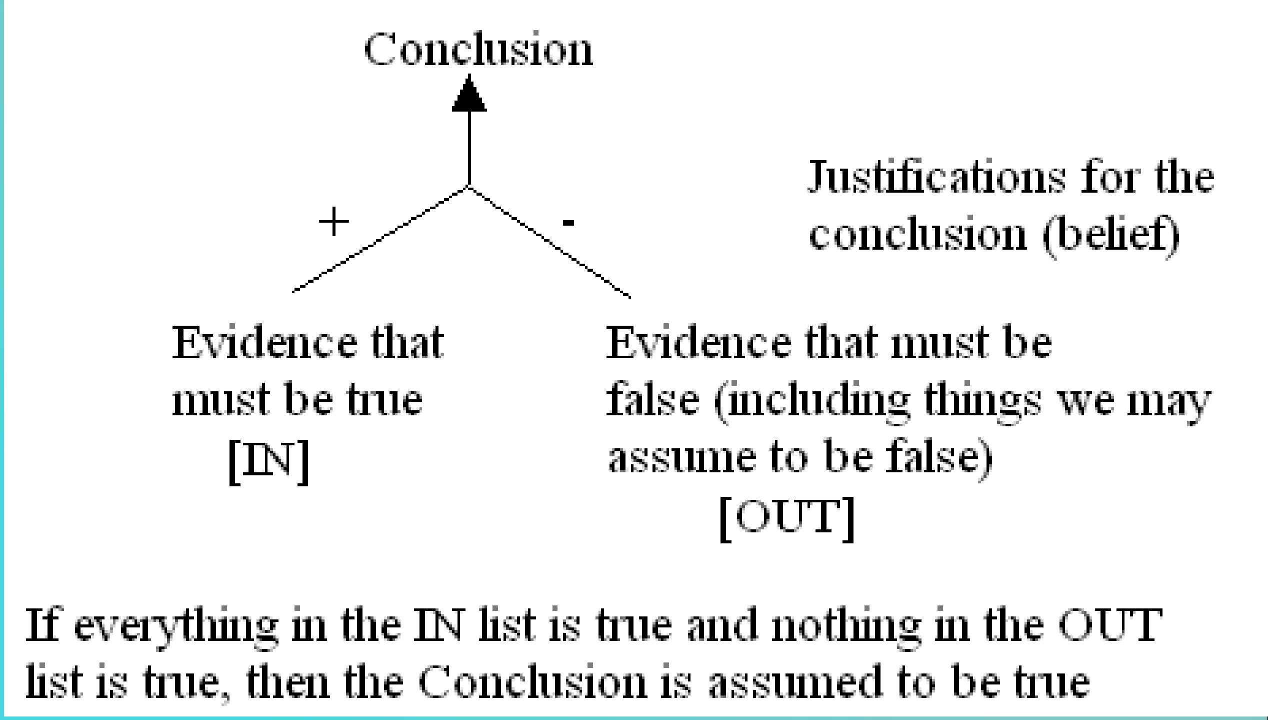
Truth Maintenance Systems

- In a TMS, inferences are supported by evidence
  - support is directly annotated in the representation so that new evidence can be mapped to conclusions easily
  - if some new piece of evidence is introduced which may overturn a previous conclusion, we need to know if this violates an assumption
  - if so, we negate the assumption and follow through to see what conclusions are no longer true
- The TMS is a graph-based representation to support dependency-directed backtracking
  - this simplifies how to make changes when new evidence is introduced or when an assumption is shown to be false
  - there are several forms of TMS, we will concentrate on the justification TMS (JTMS) but others include assumption-based TMS (ATMS), logic-based TMS (LTMS), and multiple belief reasoners (MBR)

Justification Truth Maintenance System

- The JTMS is a graph implementation whereby each inference is supported by evidence
  - an inference is supported by items that must be true (labeled as IN items) and those that must be false (labeled as OUT items), things we assume false will be labeled OUT

when a new piece of evidence is introduced, we examine the pieces of evidence to see if this either changes it to false or contradicts an assumption, and if so, we change any inferences that were drawn from this evidence to false, and propagate this across the graph



The ABC Murder Mystery

- As an example, we consider a murder
- Some pieces of knowledge are:
  - a person who stands to benefit from a murder is a suspect unless the person has an alibi
  - a person who is an enemy of a murdered person is a suspect unless the person has an alibi
  - an heir stands to benefit from the death of the donor unless the donor is poor
  - a rival stands to benefit from the death of their rival unless the rivalry is not important
  - an alibi is valid if you were out of town at the time unless you have no evidence to support this
  - a picture counts as evidence
  - a signature in a hotel registry is evidence unless it is forged
  - a person vouching for a suspect is an alibi unless the person is a liar

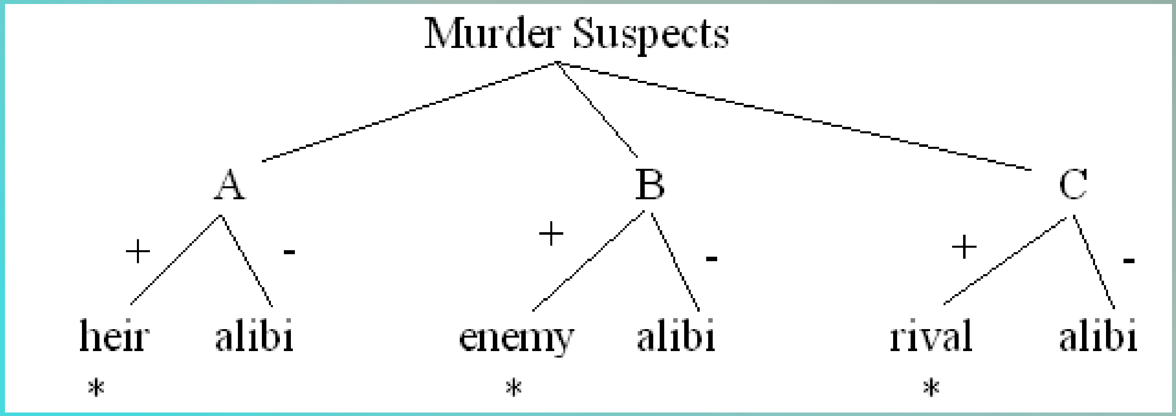
ABC Murder Mystery Continued

- Our suspects are
  - Abbott (A), an heir, Babbitt (B), a rival, Cabbott (C) an enemy
  - we do not know if the victim was wealthy or poor and we do not know if B's rivalry with the victim was important or not
- A claims to have been in Albany that weekend
- B claims to have been with his brother-in-law
- C claims to have been in the Catskills watching a ski meet
  - we have no evidence to back up A, B, or C's alibis, so they are all suspects

* denotes evidence directly supported by input

+ denotes IN evidence (must be true)

– denotes OUT evidence (assumed false)



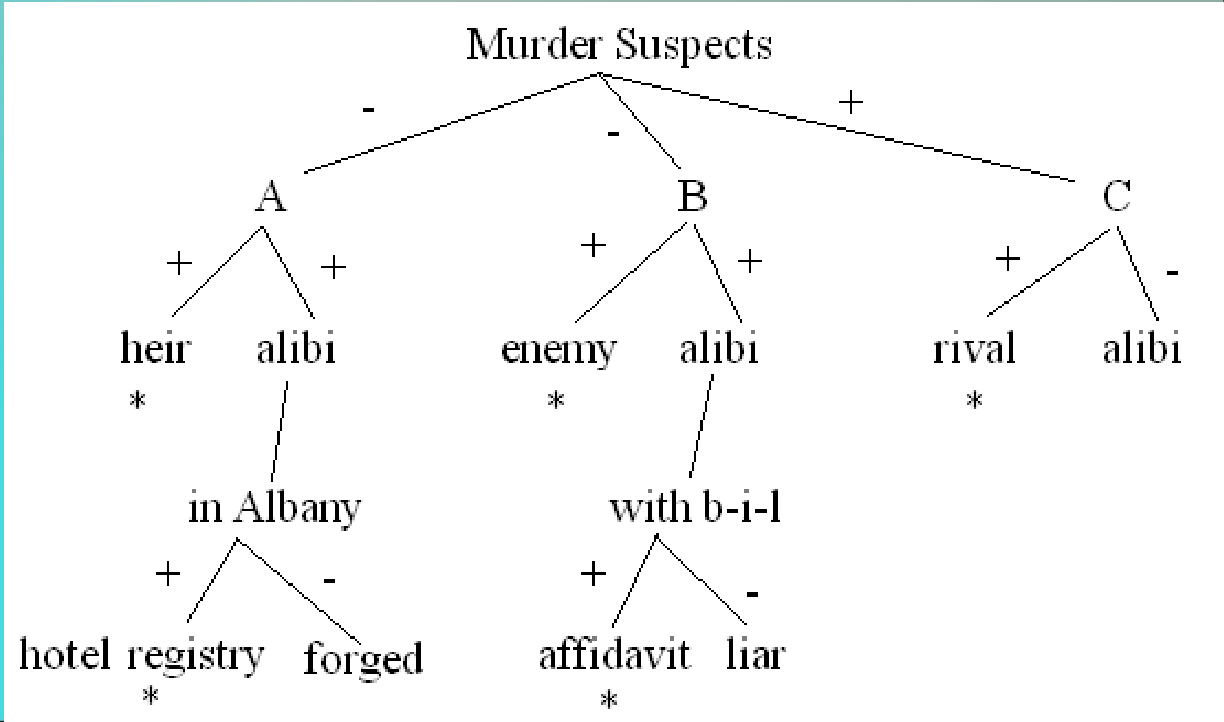
Since we have no evidence of an alibi for any of A, B, C, and because each is a known heir/enemy/rival, we conclude all three are suspects

New Evidence Comes To Light

- Abbott produces evidence that he was out of town
  - his signature is found in the hotel registry of a respectable hotel in Albany, NY
- Babbitt's brother-in-law signs an affidavit stating that Babbitt did in fact spend the weekend with him
  - B has an alibi (not in town) and is no longer a suspect

We have an alibi for A changing the assumption to true and therefore ruling him out as a suspect

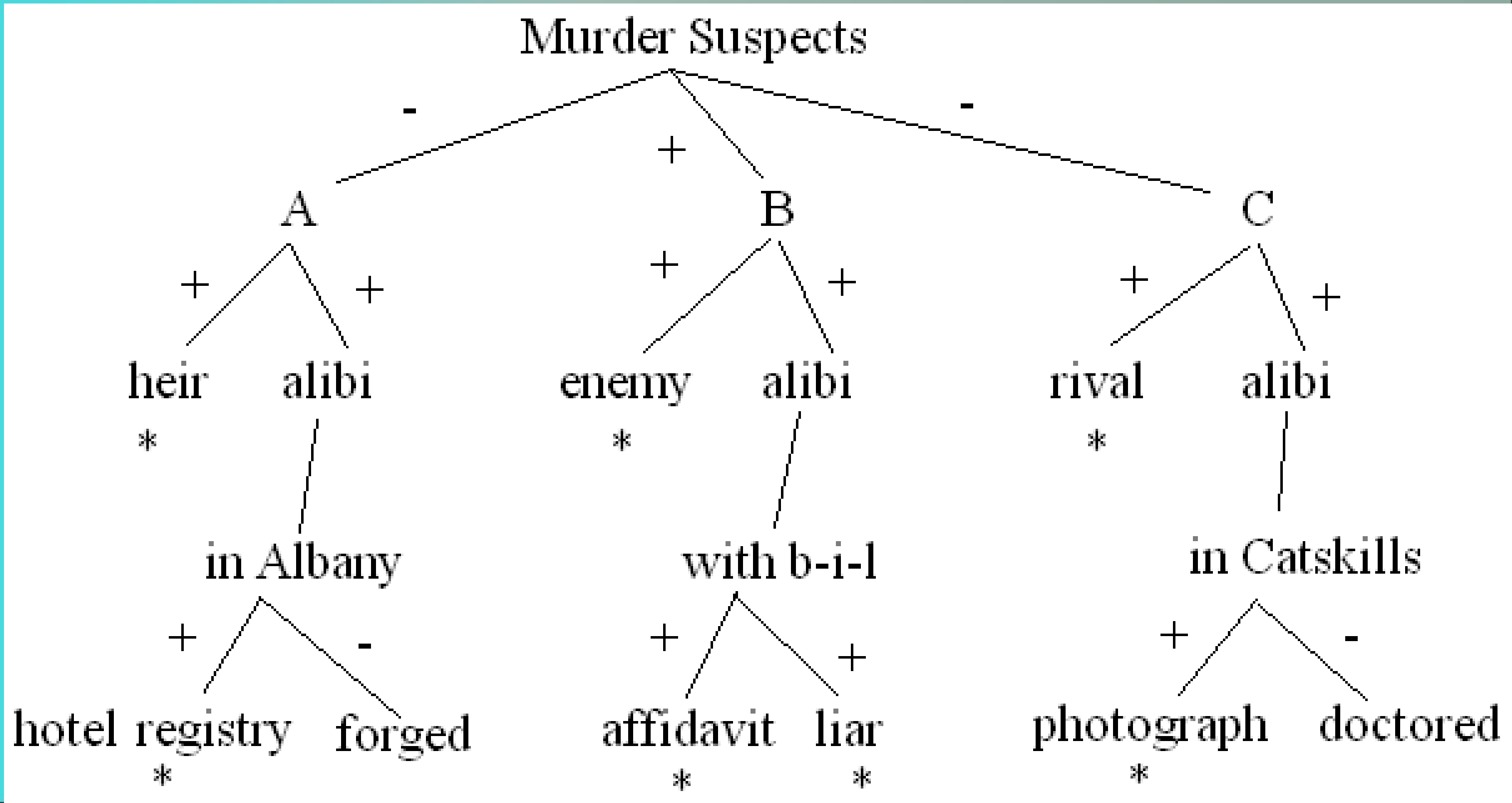
Similarly for B, but there is no change made to C, so C remains a suspect



But Then…

- B’s brother-in-law has a criminal record for perjury, so he is a known liar
  - thus, B’s alibi is not valid and B again becomes a suspect
- A friend of C’s produces a photograph of C at the meet, shown with the winner
  - the photograph supports C’s claim that he was not in town and therefore is a valid alibi, C is no longer a suspect

With these final modifications, B becomes our only suspect



Abduction

- In traditional logic, Modus Ponens tell us that if we have
  - A → B
  - A
  - we conclude B
- In abduction, we have instead
  - A → B
  - B
  - we conclude A
- The idea here is that we are saying “A can cause B”, “B happened”, we conclude “A was its cause”
  - this form of reasoning is useful for diagnosis (as an example) but it is not truth-preserving
  - consider that we know that if the battery has lost its charge then the car won’t start
    - if the car doesn’t start, we can conclude that the battery lost its charge
    - the reason this isn’t truth preserving is because there are *other* possible causes for the car not starting (bad starter, no fuel, etc)

How Abduction Can be Truth-Preserving

- We can still use abduction, but it now takes more work:
  - assume there are several causes for B:
    - A1 → B, A2 → B, A3 → B, A4 → B
  - if we can rule out A1, A2 and A3 (that is, we introduce ~A1, ~A2, ~A3) then we conclude A4
- Diagnosis is commonly performed through abduction
  - although in the case of a medical doctor
    - the possible causes A1, A2, A3, A4 are not ruled out
    - instead the doctor assigns plausibility values (likelihoods) to each of A1, A2, A3 and A4 so that if A1, A2 and A3 are very unlikely, A4 is the best explanation
  - how do we get these plausibility values?

Set Covering

- In diagnosis, there may be *multiple* contributing factors or multiple causes of the symptoms
- Assume that the following malfunctions (H1-H5, which we will call our hypotheses) can cause the symptoms (observations, O1-O5) as shown
  - H1 → O1, O2, O3 H2 → O1, O4
  - H3 → O2, O3, O5 H4 → O5
  - H5 → O2, O4, O5
- O1, O2 and O5 are observed, and we find H1-H5 to be all plausible (say "likely"), what is our best explanation?
  - {H1, H4} explains them all but includes O3 (not observed)
  - {H2, H5} explains them all but includes O4 (twice) (not observed)
  - {H1, H3} explains them all but includes O3 (twice)
  - {H1, H4, H5} explains them all but H4 is superfluous
- Mathematically, this problem is known as *set covering*

Controlling Abduction

- Set covering is an NP-complete problem
  - it is computationally expensive because it requires trying all combinations of subsets (of H's) until we have a cover
  - it should be apparent that while diagnosticians use abduction, they do not resort to complete set covering, that is, they solve the problem in less amount of time
- Factors involved in set covering/abduction
  - minimal explanation – the explanation with the fewest hypotheses
  - parsimonious explanation – no superfluous parts
  - highest rated explanation – the explanation should contain the most highly evaluated hypotheses (if we evaluate them)
    - these first three combined are known as *cost-based abduction*
  - consistent explanation – the explanation should not include hypotheses that contradict each other
    - this last one is known as *coherence-based abduction*

Forms of Abduction

- Aside from trying to build a complete and consistent explanation without superfluous parts, we often want to select the explanation that *best* explains the data
  - this requires that we somehow gage the hypotheses in terms of their plausibility
- How?
  - many different approaches have been taken
    - structured matching
    - certainty factors
    - Bayesian probabilities
    - fuzzy logic
    - neural networks

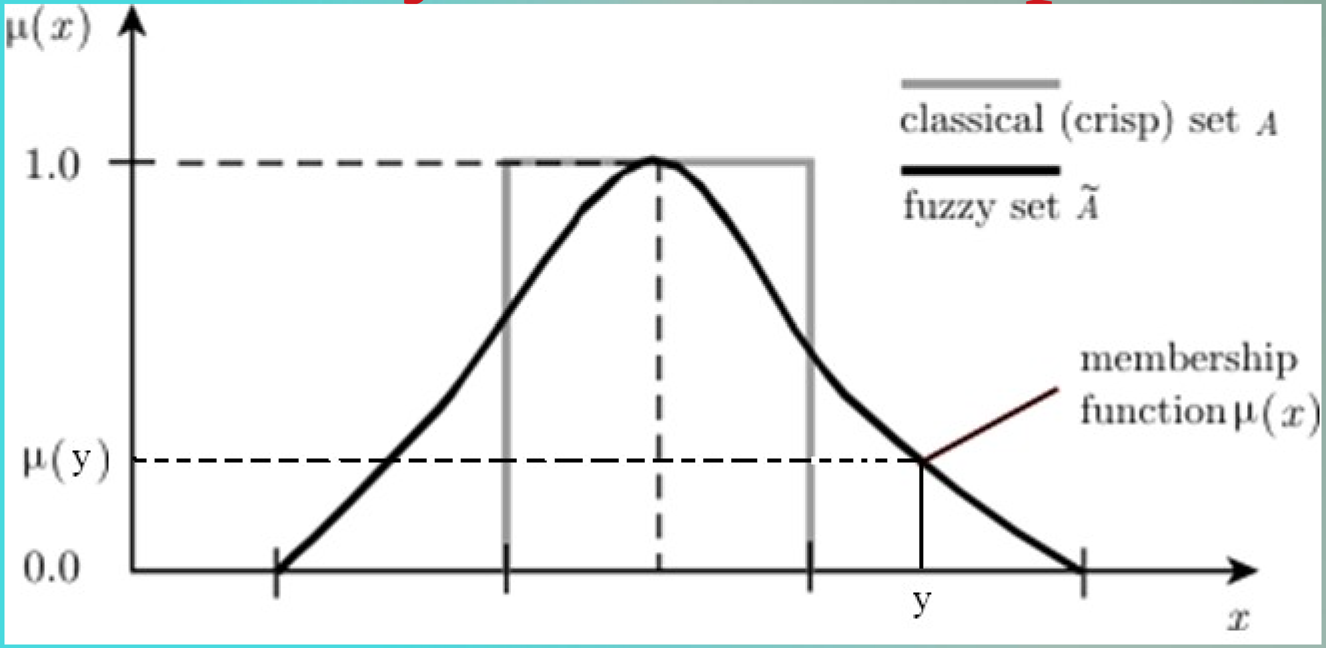
Fuzzy Logic

- Prior to CFs, Zadeh introduced fuzzy logic (FL) as a means to represent "shades of grey" into logic
  - traditional logic is two-valued, true or false only
  - FL allows terms to take on values in the interval [0, 1] (that is, real numbers between 0 and 1)
- Being a logic, Zadeh introduced the algebra to support logical operators of AND, OR, NOT, →
  - X AND Y = min(X, Y)
  - X OR Y = max(X, Y)
  - NOT X = (1 – X)
  - X → Y = X * Y
- Where the values of X, Y are determined by where they fall in the interval [0, 1]

Fuzzy Set Theory

- Fuzzy sets are to normal sets what FL is to logic
  - fuzzy set theory is based on fuzzy values from fuzzy logic but includes *set operators* (is an element of, subset, union, intersection) instead of logic operations
- The basis for fuzzy sets is defining a fuzzy membership function for a set
  - a fuzzy set is a set of items in the set along with their membership values which denote how closely each individual item is to being in that set
- Example: the set *tall* might be denoted as
  - tall = { x | f(x) = 1.0 if height(x) > 6'2", .8 if height(x) > 6', .6 if height(x) > 5'10", .4 if height(x) > 5'8", .2 if height(x) > 5'6", 0 otherwise}
  - so we can say that a person is tall at .8 if they are 6'1" or we can say that the set of tall people are {Anne/.2, Bill/1.0, Chuck/.6, Fred/.8, Sue/.6}

Fuzzy Membership Function



- Typically, a membership function is a continuous function (often represented in a graph form like above)
  – given a value y, the membership value for y is u(y), determined by tracing the curve and seeing where it falls on the u(x) axis
- How do we define a membership function?
  – for instance, is our fuzzy set for Tall realistic?
  – defining membership functions remains an open question

Using Fuzzy Logic/Sets

- 1. fuzzify the input(s) using fuzzy membership functions
- 2. apply fuzzy logic rules to draw conclusions
  - we use the previous rules for AND, OR, NOT, →
- 3. if conclusions are supported by multiple rules, combine the conclusions
  - like CF, we need a combining function, this may be done by computing a "center of gravity" using calculus
- 4. defuzzify conclusions to get specific conclusions
  - defuzzification requires translating a numeric value into an actionable item
- FL is often applied to domains where we can easily derive fuzzy membership functions and require *few* rules
  - fuzzy logic begins to break down when we have more than a dozen or two rules
  - we visit a complete example in the on-line notes

Using Fuzzy Logic

- The most common applications for FL are for controllers
  - devices that, based on input, make minor modifications to their settings – for instance
    - air conditioner controller that uses the current temperature, the desired temperature, and the number of open vents to determine how much to turn up or down the blower
    - camera aperture control (up/down, focus, negate a shaky hand)
    - a subway car for braking and acceleration
- FL has been used for expert systems
  - but the systems tend to perform poorly when more than just a few rules are chained together
    - in our previous example, we just had 5 stand-alone rules
    - when we chain rules, the fuzzy values are multiplied (e.g., .5 from one rule * .3 from another rule * .4 from another rule, our result is .06)