Discrete Structures and Graph Theory

Computer Engineering
Semester III (Structure for Regular Students)

Sr. No.	Course Type	Course Name	Teaching Scheme			Credits
			L	T	P	
1	BSC	Ordinary Differential Equations and Multivariate Calculus	2	1	0	3
2	MLC	Professional Laws, Ethics, Values and Harmony	1	0	0	0
3	HSMC	Innovation and Creativity	1	0	0	1
4	SBC	Development Tools Laboratory	1	0	2	2
5	IFC	Feedback Control Systems	1	1	0	2
6	PCC	Data Structures and Algorithms – I	2	0	0	2
7	LC	Data Structures and Algorithms -I Laboratory	0	0	2	1
8	PCC	Digital Logic Design	3	0	0	3
9	LC	Digital Logic Design Laboratory	0	0	2	1
10	PCC	Discrete Structures and Graph Theory	2	1	0	3
11	PCC	Principles of Programming Languages	3	0	0	3
12	LC	Principles of Programming Languages Laboratory	0	0	2	1
		Total	16	3	8	**22**
			27			

Teaching Scheme
Lectures: 2 Hrs / Week
Tutorials: 1 hr / week

Examination Scheme:
Assignment/Quizzes : 40 marks
End Semester Exam : 60 marks

Course Outcomes

Students will be able to:

1. Explain formal logic and different proof techniques.
2. Recognize relation between different entities using sets, functions, and relations.
3. Use Chinese Remainder Theorem & the Euclidean algorithm for modular arithmetic.
4. Solve problems based on graphs, trees and related algorithms.
5. Relate, interpret and apply the concepts to various areas of computer science.

Course Content

Set Theory, Logic and Proofs : Propositions, Conditional Propositions, Logical Connectivity, Propositional calculus, predicates and Quantifiers, First order logic, Proofs: Proof Techniques, Mathematical Induction, Set, Combination of sets, Finite and Infinite sets, countable and Uncountable sets, Principle of inclusion and exclusion,

[8 Hrs]

Relations, Functions, Recurrence Relations: Definitions, Properties of Binary Relations, Equivalence Relations and partitions, Partial ordering relations and lattices, Chains and Anti chains. Theorem on chain, Warshall's Algorithm & transitive closure, Recurrence relations. Functions: Definition, Domain, Range, Image, etc. Types of functions: Surjection, Injection, Bijection, Inverse, Identity, Composition of Functions, Generating Function

[8 Hrs]

Number Theory: Basics of Modulo Arithmetic, Basic Prime Number Theory, GCD, LCM, Divisibility, Euclid's algorithm, Factorization, Congruences, inverse , multiplicative inverse, Chinese Remainder Theorem

[4 Hrs]

Counting: Basic Counting Techniques (sum, product, subtraction, division, exponent), Pigeonhole and Generalized Pigeonhole Principle with many examples, Permutations and Combinations and numerical problems, Binomial Coefficients Pascal's, Identity and Triangle

[6 Hrs]

Graphs & Trees: Basic terminology, multi graphs and weighted graphs, paths and circuits, shortest path Problems, Euler and Hamiltonian paths and circuits, factors of a graph, planar graph and Kuratowskis graph and theorem, independent sets, connectivity graph coloring. Trees, rooted trees, path length in rooted trees, binary search trees, spanning trees and, theorems on spanning trees, cut sets , circuits, minimum spanning trees, Kruskal’s and Prim’s algorithms for minimum spanning tree.

[8 Hrs]

Algebraic Systems: Algebraic Systems, Groups, Semi Groups, Monoids, Subgroups, Permutation Groups, Codes and Group codes, Isomorphism and Automorphisms, Homomorphism and Normal Subgroups, Ring, Field.

[6 Hrs]

Text Books

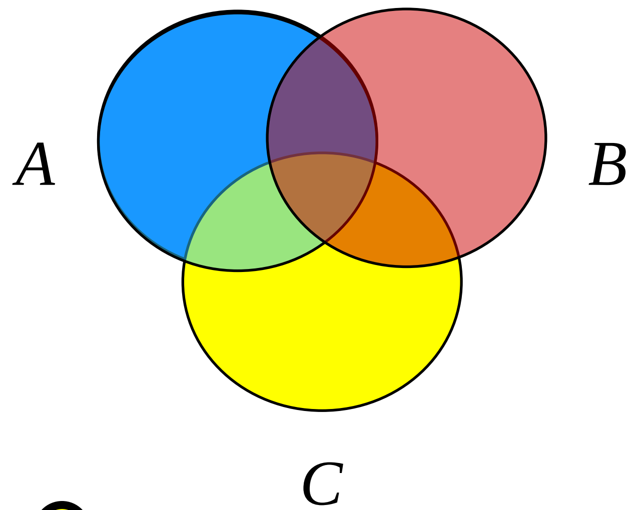
- “Discrete Mathematics and Its Applications”, Kenneth H. Rosen, 7th Edition, Tata McGraw-Hill, 2017, ISBN: 9780073383095.
- “Elements of Discrete Mathematics”, C. L. LIU, 4th Edition, Tata McGraw-Hill, 2017, ISBN-10: 1259006395 ISBN-13: 978125 9006395.

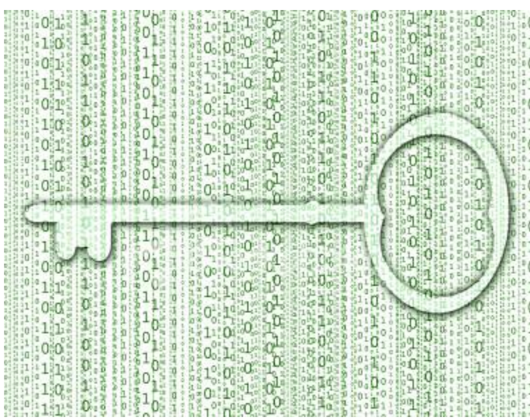
Reference Books

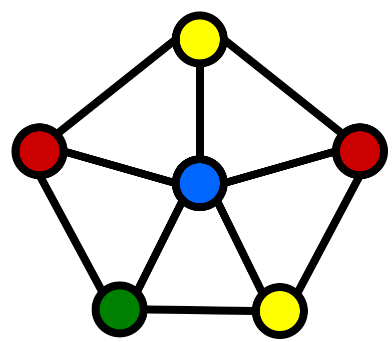
- ”Discrete Mathematical Structures”, G. Shanker Rao, 2 nd Edition2009, New Age International, ISBN-10: 8122426697, ISBN-13: 9788122426694
- “Discrete Mathematics”, Lipschutz, Lipson, 2nd Edition, 1999, Tata McGraw-Hill, ISBN: 007 463710X.
- “Graph Theory”, V. K. Balakrishnan, 1 st Edition, 2004, Tata McGraw-Hill , ISBN-10: 0- 07-058718-3, ISBN-13: 9780070587182.
- “Discrete Mathematical Structures”, B. Kolman, R. Busby and S. Ross, 4th Edition, Pearson Education, 2002, ISBN: 8178085569
- “Discrete Mathematical Structures with application to Computer Science”, J. Tremblay, R. Manohar, Tata McGraw-Hill, 2002, ISBN: 0070651426
- “Discrete Mathematics”, R. K. Bisht, H. S. Dhami, Oxford University Press, ISBN: 9780199452798

Introduction to Discrete Mathematics

$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geq \sqrt[n]{x1 \cdot x_2 \cdots x_n}$$







a = qb+r $\Rightarrow$ gcd(a,b) = gcd(b,r)

Why is discrete mathematics?

Logic: artificial intelligence (AI), database, circuit design

Counting: probability, analysis of algorithm

Graph theory: computer network, data structures

Number theory: cryptography, coding theory

logic, sets, functions, relations, etc

Why is discrete mathematics?

GATE core subject

Competitive Exams

Learn Competitive Programming

It Improves:

- Mathematical thinking
- Problem solving ability
- Foundation of all subjects in computer Engineering

What are “discrete structures” ?

“***Discrete***” - Composed of distinct, separable parts. (Opposite of *continuous*.)
discrete:*continuous* :: *digital*:*analog*

“***Structures***” - Objects built up from simpler objects according to some definite pattern.

“***Discrete Mathematics***” - The study of discrete, mathematical objects and structures.

Lecture 1 Link

- https://web.microsoftstream.com/video/2c0044b2-bc32-4abe-bfa7-17ca741fa609

Logic, Proofs and Set Theory

https://www.youtube.com/watch?v=QmMnLxWVSGM

CAN YOU SOLVE THIS SIMPLE PUZZLE AND TELL,

WHICH CAR WAS STOLEN FROM THE SHOWROOM

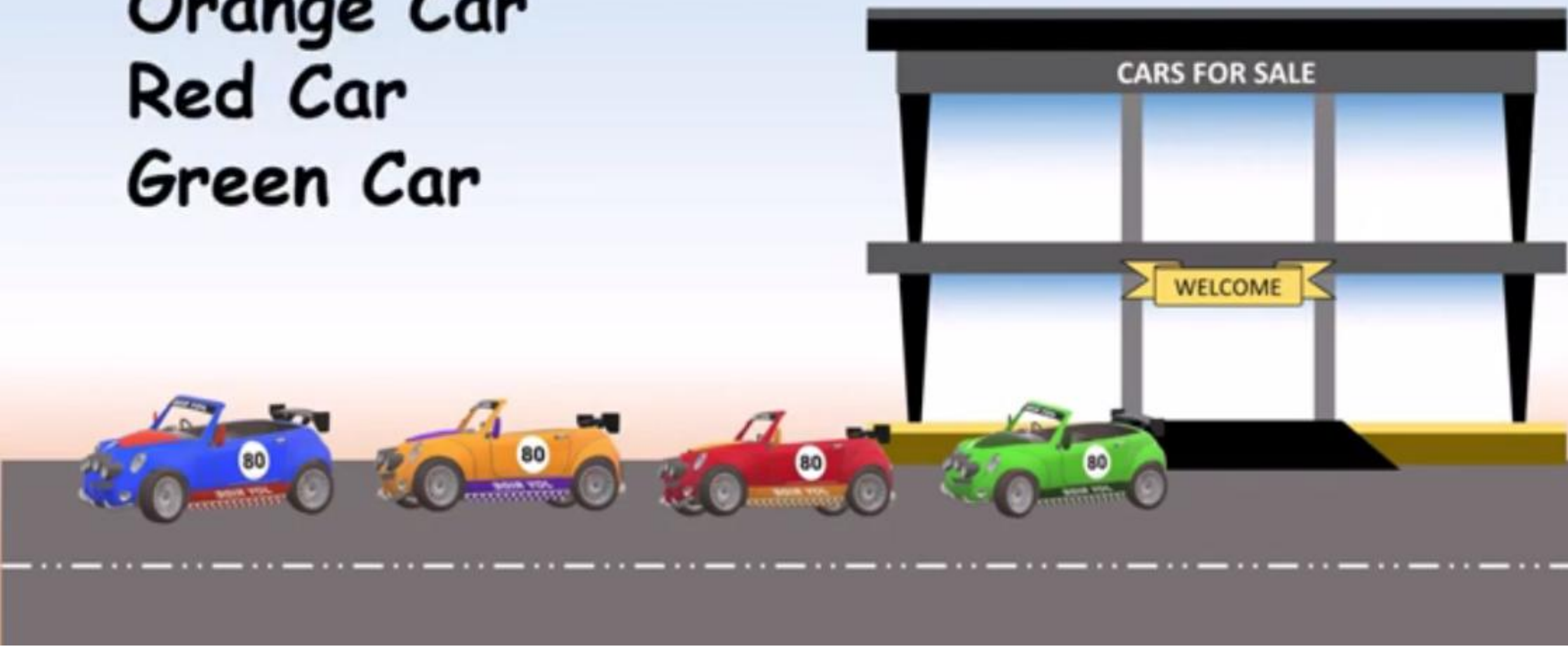
One day, 4 new cars went out of showroom

Blue Car

Orange Car

Red Car

Green Car



3 out of the 4 cars which went out were driven by Showroom staff

But the 4^{th} car was driven by a thief and was stolen

You have to Find out,

which car was stolen,

based on the clues,

which are:

1) Owner of the Showroom went home for Lunch in Blue car

2) Mechanic drove one car, but that was not the Green Car

3) Salesman took one car for Test Drive, but that was not Green or Orange Car

Based on these clues,

Can you tell which car

was stolen?

Lets see what is the Answer

Which car was stolen?

				
Owner				
Mechanic				
Salesman				
Thief				


Which car was stolen?
Owner
Mechanic
Salesman
Thief
1) Owner of the Showroom went home for Lunch in Blue car
This means, no one else took the Blue Car

Which car was stolen?

				
Owner	✓	✗	✗	✗
Mechanic	✗			✗
Salesman	✗			
Thief	✗			

1) Owner of the Showroom went home for Lunch in Blue car

2) Mechanic drove one car, but that was not the Green Car

This means, mechanic drove either Orange or Red Car

Which car was stolen?

				
Owner	✓	✗	✗	✗
Mechanic	✗			✗
Salesman	✗	✗	✓	✗
Thief	✗			

1) Owner of the Showroom went home for Lunch in Blue car
2) Mechanic drove one car, but that was not the Green Car
3) Salesman took one car for Test Drive, but that was not Green or Orange Car

This means, salesman drove the Red Car

Which car was stolen?

				
Owner	✓	✗	✗	✗
Mechanic	✗		✗	✗
Salesman	✗	✗	✓	✗
Thief	✗		✗	

1) Owner of the Showroom went home for Lunch in Blue car
2) Mechanic drove one car, but that was not the Green Car
3) Salesman took one car for Test Drive, but that was not Green or Orange Car

And no one else drove the red car

Which car was stolen?

				
Owner	✓	✗	✗	✗
Mechanic	✗	✓	✗	✗
Salesman	✗	✗	✓	✗
Thief	✗		✗	

1) Owner of the Showroom went home for Lunch in Blue car
2) Mechanic drove one car, but that was not the Green Car
3) Salesman took one car for Test Drive, but that was not Green or Orange Car

Which means, mechanic drove the Orange car

Which car was stolen?

				
Owner	✓	✗	✗	✗
Mechanic	✗	✓	✗	✗
Salesman	✗	✗	✓	✗
Thief	✗	✗	✗	

1) Owner of the Showroom went home for Lunch in Blue car
2) Mechanic drove one car, but that was not the Green Car
3) Salesman took one car for Test Drive, but that was not Green or Orange Car

And the Thief Stole the Green Car

Which car was stolen?

				
Owner	✓	✗	✗	✗
Mechanic	✗	✓	✗	✗
Salesman	✗	✗	✓	✗
Thief	✗	✗	✗	✓

GREEN CAR WAS STOLEN

Statements/ Proposition

- Proposition or Statement or An Assertion
- Primary (Primitive, atomic) statements
- Set of Declarative sentences which cannot be further broken down into simpler sentences.
- Those who have one and only one of two possible values called "Truth Values".
- True and False or T and F or 1 and 0
- Two-valued logic
- Some statements can be assertion but not the propositions
    - Ex. "This statement is false"

Statement (Proposition)

A *Statement* is a sentence that is either **True** or **False**

Examples:

$2 + 2 = 4$ True

$3 \times 3 = 8$ False

787009911 is a prime

Non-examples:

$x+y>0$

$x^2+y^2=z^2$

They are true for some values of x and y
but are false for some other values of x and y.

The Statement/Proposition Game

- “Elephants are bigger than ant.”

Is this a proposition? **yes**

What is the truth value of the proposition? **true**

The Statement/Proposition Game

- “520 < 111”

Is this a proposition? **yes**

What is the truth value of the proposition? **false**

The Statement/Proposition Game

- “Please do not fall asleep.”

Is this a statement? **no**

It’s a request.

Is this a proposition? **no**

Only statements can be propositions.

Examples of statements/ Propositions

All the following declarative sentences are propositions.

1. Washington, D.C., is the capital of the United States of America.
2. Toronto is the capital of Canada.
3. $1 + 1 = 2$.
4. $2 + 2 = 3$.

Propositions 1 and 3 are true, whereas 2 and 4 are false.

Examples

- Consider the following sentences.

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$.
4. $x + y = z$.

Sentences 1 and 2 are not propositions because they are not declarative sentences.

Sentences 3 and 4 are not propositions because they are neither true nor false.

Note that each of sentences 3 and 4 can be turned into a proposition if we assign values to the variables

Class Assignement

- Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts.
b) Miami is the capital of Florida.
c) $2 + 3 = 5$.
d) $5 + 7 = 10$.
e) $x + 2 = 11$.
f) Answer this question.
g) Do not pass go.
h) What time is it?
i) There are no black flies in Maine.
j) $4 + x = 5$.
k) The moon is made of green cheese.
l) $2n \geq 100$

Class Assignement

- Which of these sentences are propositions? What are the truth values of those that are propositions?

a) Boston is the capital of Massachusetts. T
b) Miami is the capital of Florida.
c) $2 + 3 = 5$. T
d) $5 + 7 = 10$. F
e) $x + 2 = 11$.
f) Answer this question.
g) Do not pass go.
h) What time is it?
i) $4 + x = 5$.
j) The moon is made of green cheese. F
k) $2n \geq 100$

Lecture 2

- https://web.microsoftstream.com/video/2d775ffa-9508-4141-a3af-08b35c8d8073

Operators/ Connectives

- An *operator* or *connective* combines one or more *operand* expressions into a larger expression.
- Two types of declarative sentences
- First is Primitive or primary or atomic statement
- Denoted by letters A,B,C…..P,Q,R…or a,b,c,…p,q,r…

  P: London is capital of India.

  A:Ram is poor.
- Second types are obtained from primitives using **connectives and parenthesis,** Called molecular or compound statements
- Like statements connective also denoted by symbol

Examples

e.g.

1. India is country and Mumbai is capital of India.

    P:India is country

    Q:Mumbai is capital of India.

    P and Q $P \wedge Q$

2. Ram is poor but he is clever.

    A: Ram is poor.

    B: Ram is clever.

    A and B

Connectives

1. Negation (Not)
2. Conjunction (and)
3. Disjunction (or)
4. Conditional (if…then) /implication
5. Bi-conditional (if and only if)

Connectives' Symbols

Formal Name	**Nickname**	**Property**	**Symbol**
Negation operator	NOT	Unary	$\neg$
Conjunction operator	AND	Binary	$\wedge$
Disjunction operator	OR	Binary	$\vee$
Exclusive-OR operator	XOR	Binary	$\oplus$
Implication operator	IMPLIES	Binary	$\rightarrow$
Biconditional operator	IFF	Binary	$\leftrightarrow$

Lecture 3

- https://web.microsoftstream.com/video/eaf401a0-8259-4d61-af55-87efd46b1b92
- https://web.microsoftstream.com/video/6ef37cbd-170d-440c-ac61-cfe331ac5816