COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)

END Semester Exam

(MA-16006) Multivariate Calculus and Differential Equations

Course: S.Y.B.Tech , Semester IV
Academic Year: 2017-2018
Duration: 3 Hours
Branches: All

Max.Marks:60
Date: 03/05/2018

Instructions:

Student MIS NO. : | | | | | | | | | |

(1) All questions are compulsory.
(2) Figures to the right indicate the full marks.
(3) Mobile phones and programmable calculators are strictly prohibited.
(4) Writing anything on question paper is not allowed.
(5) Exchange/Sharing of stationery. calculator etc. not allowed.
(6) Write your MIS Number on Question Paper.

Attempt the following questions:

Q1. (a) Find the value of $\frac{\partial z}{\partial x}$ at the point $(1,1,1)$ if the equation $xy + z^3x - 2yz = 0$ defines z implicitly as a function of the two independent variables x and y. [4]

(b) Use the transformation $u = x + 2y$, $v = x - y$ to evaluate the integral [5]

$$\int_0^{2/3}\int_y^{2-2y}(x+2y)e^{(y-x)}\,dx\,dy$$

by first writing it is an integral over a region G in the $uv-$plane.

(c) Evaluate [3]

$$\int_0^{\pi/2}\int_0^{\pi}\int_0^{2\sin\phi}\rho^2\sin^2\phi\, d\rho\, d\phi\, d\theta.$$

(d) Attempt **any One:** [3]
(i) Define the following terms:
(A) Interior point
(B) Boundary point
(C) Open Set

OR

(ii) Express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ as functions of u and v both by using the Chain Rule $w = xy + yz + xz,\ x = u + v,\ y = u - v,\ z = uv$.

Q2. (a) For $f(x, y) = x^2 - y^2$ and $g(x, y) = e^{x+y}$ verify [4]

$$div(f\nabla g) - div(g\nabla f) = f\nabla^2 g - g\nabla^2 f.$$

(b) State the following: [3]

(i) Gauss-Divergence theorem.

(ii) Stoke's theorem.

(c) Using Green's theorem, find area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [3]

(d) Attempt **any One**: [3]

(i) Verify Gauss-Divergence theorem for the vector field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

OR

(ii) Find the flux of $\vec{F} = yz\vec{j} + z^2\vec{k}$ outward through the surface S cut from the cylinder $y^2 + z^2 = 1$, $z \geq 0$, by the planes $x = 0$ and $x = 1$.

(e) Find a normal vector of the surface $x^2 - y^2 + 4z^2 = 67$ at the point $P : (-2, 1, 4)$. [2]

Q3. (a) **State** the choice of $y_p(x)$ for the following differential equations using Method of Undetermined coefficients. [4]

(i) $y'' + 4y = 19cos(2x)$.

(ii) $y'' + 9y = cosx + \frac{sin(3x)}{3}$.

(iii) $y''' - y'' = x^2$.

(iv) $y'' + 4y' + 4y = e^{2x}cos(5x)$.

(b) Suppose $y = x^3$ is one solution of $x^2y'' - 6y = 0$. Find another linearly independent solution. [2]

(c) Solve the following using Variation of Parameter method. (**Any One**) [4]

(i) $y'' - 4y' + 4y = x^2e^x$

OR

(ii) $y'' - 3y' + y = e^x sinx$.

Q4. (a) Express the following in terms of Unit step function and find its Laplace:

(i) $f(t) = sin(3t); 0 < t < \pi$. [2]

(ii) $f(t) = t; 1 < t < 2$. [2]

(b) Solve **any One:** [4]

(i) $y'' + 3y' + 2y = r(t)$ where $r(t) = 1.$ if $0 < t < 1$ and $r(t) = 0.$ if $t > 1$ and $y(0) = 0, y'(0) = 0.$

OR

(ii) $y'' + y = r(t)$ where $r(t) = t,$ if $0 < t < 1$ and $r(t) = 0,$ if $t > 1$ and $y(0) = 0, y'(0) = 0.$

(c) Is it true that Laplace of product of 2 fuctions is equal to product of laplaces? Justify! [2]

Q6. (a) Solve: [3]

$$y'' + 2y' + 2y = 5\delta(t-2), y(0) = 0, y'(0) = 1$$

(b) Attempt **any One:** [2]

(i) Find laplace of $f(t) = 1, 0 \leq t < 2$ and $f(t) = -1, 2 \leq 2 \leq 4$ and $f(t+4) = f(t)$.

OR

(ii) Solve using convolution theorem:

$$y'' + y = sint,\ y(0) = 0,\ y'(0) = 0$$

(c) State the standard assumptions and derive the one-dimensional wave equation which governs the transverse vibrations of an elastic string of Length L. [5]

⁂ Good Luck ⁂