College of Engineering Pune
Ordinary Differential Equations and Multivariate Calculus
Tutorial-3 (2019-2020)

1. Find the steady state and transient state motion of the mass-spring system with mass $4\ kg$, damping constant $c = 8\ kg/sec$, spring constant $k = 3\ kg/sec^2$, and driving force $r(t) = 425\ \sin 2t\ newton$, where $y(0) = -16$ and $y^{'}(0) = -26$.

2. Find the steady state and transient state motion of the mass-spring system with mass $m = 4\ kg$, damping constant $c = 4\ kg/sec$, spring constant $k = 17\ kg/sec^2$, and the driving force $r(t) = 202\ \cos 3t\ newton$.

3. In $L-R-C$ circuit the charge Q on the plate is given by $L\ \dfrac{d^2Q}{dt^2} + R\ \dfrac{dQ}{dt} + \dfrac{Q}{C} = E\ \sin pt$. The circuit tuned to resonance so that $p^2 = \dfrac{1}{LC}$. If initially the current $i(t)$ and the charge $Q(t)$ be zero, then show that for small values of $\dfrac{R}{L}$, the current in time t is given by $\dfrac{E\ t}{2\ L}\ \sin pt$.

4. Find the current in $L - R - C$ circuit when $L = 0.1\ H$, $R = 20\ \Omega$, $C = 2 \times 10^{-4}\ F$ and $E(T) = 110\ \sin 314t\ V$.

5. State the theorem on conversion of an n^{th} order ODE to a system of equations.

6. Find the general solution of the given ODE by **first converting it to a system of equations**.

   a) $y^{''} - 4y = 0$
   b) $y^{''} + 2y^{'} - 24y = 0$
   c) $y^{''} + 15y^{'} + 50y = 0$

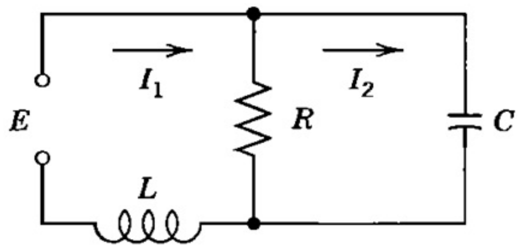
7. Find the real general solution of the following system of homogeneous and non-homogeneous differential equations / initial value problems:

   a) $y_1^{'} = 9y_1 + 13.5y_2\ ,\quad y_2^{'} = 1.5y_1 + 9y_2$
   b) $y_1^{'} = y_2\ ,\quad y_2^{'} = 6y_1 - 5y_2$
   c) $y_1^{'} = 8y_1 - y_2\ ,\quad y_2^{'} = y_1 + 10y_2$
   d) $y_1^{'} = 2y_1 + 8y_2 - 4y_3\ ,\quad y_2^{'} = -4y_1 - 10y_2 + 2y_3\ ,\quad y_3^{'} = -4y_1 - 4y_2 - 4y_3$
   e) $y_1^{'} = 4y_2 + 9t\ ,\quad y_2^{'} = -4y_1 + 5$
   f) $y_1^{'} = 4y_1 + y_2 + \sin t\ ,\quad y_2^{'} = -4y_1 + y_2$
   g) $y_1^{'} = y_1 - 2y_2 - \sin t\ ,\quad y_2^{'} = -3y_1 - 4y_2 - \cos t$
   h) $y_1^{'} = y_1 + 2y_2 + t^2\ ,\quad y_2^{'} = 2y_1 + y_2 - t^2$
   i) $y_1^{'} = -2y_2 + 4t\ ,\quad y_2^{'} = 2y_1 - 2t\ ,\quad y_1(0) = 4\ ,\quad y_2(0) = 0.5$
   j) $y_1^{'} = y_1 + 2y_2 + e^{2t} - 2t\ ,\quad y_2^{'} = -y_2 + 2t + 1\ ,\quad y_1(0) = 1\ ,\quad y_2(0) = -4$

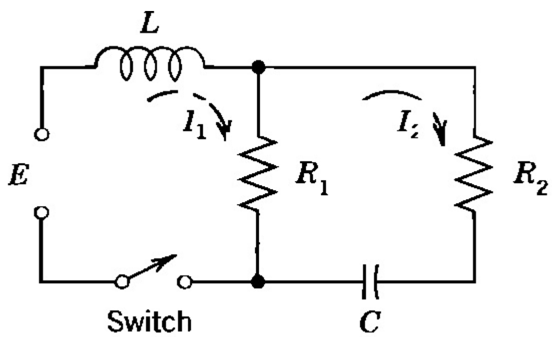
8. Solve the followingby the method of variation of parameters:

$y_1' = -3y_1 + y_2 - 6e^{-2t}\ , \qquad y_2' = y_1 - 3y_2 + 2e^{-2t}$

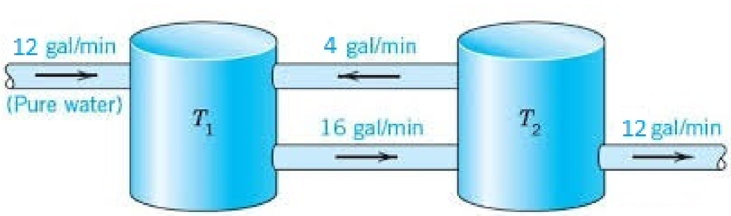
9. Find the currents in the electrical network when $R = 2.5\ \Omega,\ L = 1\ H,\ C = 0.04\ F,$ $E(t) = 845 \sin t\ V,$ and $I_1(0) = 0,\ I_2(0) = 0.$



10. Find the currents in the electrical network when $R_1 = 2\ \Omega,\ R_2 = 8\ \Omega,\ L = 1\ H,$ $C = 0.5\ F,\ E(t) = 200\ V.$



11. In given Fig each of the two tanks contains 200 gal of water, in which initially 100 lb of fertilizer in Tank T_1 and 200 lb of fertilizer in Tank T_2 are dissolved. The inflow and outflow circulation are as shown in fig. The mixture is kept uniform by stirring, then find the fertilizer contents $\mathbf{y_1(t)}$ in T_1 and $\mathbf{y_2(t)}$ in T_2.



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