Tutorial I

Q.1) Suppose $\int_0^x f(t)\,dx = x^2 - 2x + 1$. Find $f(x)$

$\rightarrow$ $F(x) = \int_0^x f(t)\,dt$

$\therefore f(x) = x^2 - 2x + 1$ by FTC 1

Q.2) $f(4) = ?$ if $\int_0^x f(t)\,dt = x \cos \pi x$

$\rightarrow$ $F(x) = \int_0^x f(t)\,dt$

$\therefore f(x) = x \cos \pi x$

$= 4 \cos \pi 4$

$= 4(-1)4$

$= \underline{\underline{-16}}$

Q.3). Find volume of solid generated by revolving the regions bounded by the lines and curves about the x-axis.

a) $y = x^2,\ y = 0,\ x = 2$

Q.9) Find the length of following curves.

$\longrightarrow$

a) $y = \left(\frac{1}{3}\right)x^2 + 2$ from $x = 0$ to $x = 3$.

$y = f(x)$ $\quad$ $a = 0$ $\quad$ $b = 3$

$\longrightarrow$ by arc length formula,

$$L = \int_0^3 \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^3 \sqrt{1 + \left[\frac{2x}{3}\right)^2}\, dx = \int_0^3 \sqrt{1 + \frac{4x^2}{9}}\, dx$$

$$= \int_0^3 \sqrt{\frac{9 + 4x^2}{9}}\, dx = \frac{1}{3} \int_0^3 \sqrt{9 + 4x^2}\, dx$$

$$= \frac{2}{3} \int_0^3 \sqrt{x^2 + \left(\frac{3}{2}\right)^2}\, dx$$

Formula, $\sqrt{x^2 + a^2}\, dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \ln\left(x + \sqrt{x^2 + a^2}\right) + c$

$$= \frac{2}{3}\left[\frac{x\sqrt{x^2 + (3/2)^2}}{2} + \frac{(3/2)^2}{2} \ln\left(x + \sqrt{x^2 + (3/2)^2}\right) + c\right]_0^3$$

$$= \frac{2}{3}\left[\frac{3\sqrt{(3)^2 + \left(\frac{3}{2}\right)^2}}{2} + \frac{9/4}{2} \ln\left(3 + \sqrt{(3)^2 + (3/2)^2}\right) + c - \frac{9/4}{2} \ln\left(3/2\right)\right]$$

Page No. | Date

$$= \frac{2}{3}\left[1.67 + 1.12 \times 2.11 - 1.12 \times 0.40\right]$$

$$= \frac{2}{3}\left[4.33 - 0.448\right]$$

$$= 4.4368$$

(b). $y = x^{3/2}$ from $x=0$ to $x=4$

$\rightarrow$

$$L = \int_0^4 \sqrt{1+[F'(x)]^2}\, dx$$

$$= \int_0^4 \sqrt{1+[x^{3/2}]^2}\, dx = \int_0^4 \sqrt{1+\left(\frac{3\sqrt{x}}{2}\right)^2}$$

$$= \int_0^4 \sqrt{1+\frac{9x}{4}}\, dx = \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

put $u = 4+9x$ $\therefore \frac{du}{dx} = 9$

$$\therefore \int_0^4 \sqrt{u}\,\frac{du}{9} = \frac{1}{9}\int_0^4 \sqrt{u}$$

$$\therefore \frac{1}{9}\left[\frac{2u^{3/2}}{3}\right] \text{ .. power rule}$$

$$\therefore \left[\frac{2(9x+4)^{3/2}}{27}\right]_0^4 = \underline{\underline{18.1468}}$$

$$= \frac{1}{2} \times \underline{18.1468}$$

$$= 9.07$$

(c). $x = \left(\frac{y^3}{3}\right) + \left(\frac{1}{4y}\right)$ from $Y=1$ to $y=3$

Page No.			
Date			

→ $x = \frac{y^3}{3} + \frac{1}{4y}$, $1 \le y \le 3$

→ here, $g(y) = \frac{y^3}{3} + \frac{1}{4y}$, $c = 1, d = 3$

$L = \int_c^d \sqrt{1 + (g'(y))^2}\, dy$

$\therefore g'(y) = y^2 - \frac{1}{4y^2}$

$\therefore L = \int_1^3 \sqrt{1 + \underbrace{\left(y^2 - \frac{1}{4y^2}\right)^2}_{(a-b)^2}}\, dy$

$= \int_1^3 \sqrt{1 + y^4 - \frac{1}{2} + \frac{1}{16y^4}}\, dy$

$= \int_1^3 \sqrt{y^4 + \frac{1}{16y^4} + \frac{1}{2}}\, dy$

$= \int_1^3 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}\, dy = \int_1^3 \left(y^2 + \frac{1}{4y^2}\right) dy$

$= \left[\frac{y^3}{3} - \frac{1}{4y}\right]_1^3 = \left[\frac{27}{3} - \frac{1}{12} - \frac{1}{3} + \frac{1}{4}\right]$

$= \left[9 - \frac{1}{12} - \frac{1}{12}\right] = 9 - \frac{1}{6} = \frac{53}{6}$

Page No.
Date

e) $y = \int_0^x \tan t \, dt, \quad 0 \leq x \, \pi/6$

$\rightarrow$ $y = \int_0^x \tan t (dt)$ $\quad a = 0 \quad b = \pi/6$

$0 \leq x \leq \pi/6$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx.$$

~~$f'(x) =$~~ but we have $y = \int_0^x \tan t \, dt.$

$\therefore$ by FTC, we ~~hay~~ have

$$\frac{dy}{dx} = \tan x$$

~~[illegible]~~ =

$$\therefore L = \int_0^{\pi/6} \sqrt{1 + (\tan x)^2} \, dx.$$

$$= \int_0^{\pi/6} \sqrt{\sec^2(x)} = \int_0^{\pi/6} \sec(x)$$

$$= \Big[\ln(|\tan(x) + \sec(x)|) + c \Big]_0^{\pi/6}$$

$= ~~1.30~~ \; 0.54 - 0$

$= \underline{\underline{0.54}}$

Q.11) →

b) $x = 1$, $1 \leq y \leq 2$, y-axis

→ $S = \int_c^d 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy = \int_1^2 2\pi \, 1 \sqrt{1+0}\, dy$

$= \int_1^2 2\pi = 0$

c) $x = 2\sqrt{(4-y)}$, $0 \leq y \leq 15/4$, y-axis.

$$\longrightarrow \quad S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_0^{15/4} 2\pi \cdot x \; 2\sqrt{4-y} \sqrt{1 + \left(F'(x)\right)^2}$$

$$\left(\frac{dx}{dy}\right) = -\frac{1}{\sqrt{4-y}}$$

$$= \int_0^{15/4} 2\pi 2\sqrt{4-y} \sqrt{1 + \left(-\frac{1}{\sqrt{4-y}}\right)^2}\, dy$$

$$= 4\cancel{2}\pi \int_0^{15/4} \sqrt{4-y} \sqrt{1 + \frac{1}{4-y}}\, dy = 4\pi \int_0^{15/4} \cancel{\sqrt{4-y}} \frac{\sqrt{4-y+1}}{\cancel{\sqrt{4-y}}}\, dy$$

$$= 4\pi \int_0^{15/4} \sqrt{5-y}\, dy$$

$$= 4\pi \left[-\frac{2(5-y)^{3/2}}{3} \right] + C$$

$$= 4\pi \left(\frac{(7.5)^{3/2}}{12} \right) = 4\pi \times 6.52$$

$$= 81.93$$

Q.3) Volume of solid about x-axis

→

ⓐ $y = x^2$, $y = 0$

| $x = 2$
| For lower limit put $y = 0$
| in eqn
| $\therefore x^2 = 0 \quad \therefore x = 0$
| $\therefore 0 \le x \le 2$

→

$$V = \int_a^b A(x)\,dx$$

$$= \int_a^b \pi (R(x))^2\,dx$$

$$= \int_0^2 \pi (x^2)^2\,dx = \int_0^2 \pi x^4\,dx = \pi \int_0^2 x^4\,dx$$

$$= \pi \left[\frac{x^5}{5}\right]_0^2 = \pi \left[\frac{2^5}{5} - \frac{0^5}{5}\right] = \frac{32}{5}\pi$$

lly

ⓑ $y = x^3$, $y = 0$ $x = 2$

→ $V = \int_0^2 \pi (x^3)^2\,dx = \pi \int_0^2 x^6\,dx = \pi \left[\frac{x^7}{7}\right]_0^2$

$$= \pi \left[\frac{2^7}{7} - \frac{0^7}{7}\right] = \frac{128}{7}\pi$$

ⓒ $y = \sqrt{(9 - x^2)}$, $y = 0$

$x^2 + y^2 = r^2$ ... circle for.

→ $y^2 = 9 - x^2$ --- taking square on both sides.

$\therefore y^2 + x^2 = 9$ $\therefore r^2 = 9 \quad \therefore r = \pm 3$

$$V = \int_a^b \pi \left[R(x)\right]^2 dx = \int_{-3}^{3} \pi \left(\sqrt{9-x^2}\right)^2 dx$$

$$= \int_{-3}^{3} \pi \times 9 - x^2 \, dx = \pi \int_{-3}^{3} 9 - x^2 \, dx$$

$$= \pi \left[9x - \frac{x^3}{3} \right]_{-3}^{3} = \pi \left[9(3) - \frac{(3)^3}{3} \right] - \left[9(-3) - \frac{(-3)^2}{3} \right]$$

$$= \pi \left[27 - \frac{27}{3} \right] - \left[-27 + \frac{27}{3} \right]$$

$$= 36\pi$$

(d) $y = x - x^2 \quad y = 0$

$\longrightarrow$ put $x - x^2 = 0$

$\therefore x(1-x) = 0$

$\therefore x = 0$ & $x = 1$

$$V = \int_0^1 \pi (x - x^2)^2 dx = \int_0^1 \pi \left(x^2 - 2x(x)^2 + x^4\right) dx$$

$$= \pi \int_0^1 x^2 - 2x(x)^2 + x^4 = \pi \int_0^1 x^2 - 2x^3 + x^4 \, dx$$

$$= \pi \left[\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right]_0^1$$

$$= \pi\left[\frac{(1)^3}{3} - \frac{2(1)^4}{4} + \frac{(1)^5}{5}\right]_0^1 = \pi\left[\frac{1}{3} - \frac{2}{4} + \frac{1}{5}\right]$$

$$= \frac{\pi}{30}$$

(e) $y = \sqrt{\cos x}, \quad 0 \le x \le \pi/2 \quad y=0, \; x=0$

$$\longrightarrow \quad V = \int_a^b \pi\,(R(x))^2\,dx = \int_0^{\pi/2} \pi\left(\sqrt{\cos x}\right)^2 dx$$

$$= \int_0^{\pi/2} \pi \cdot \cos x \, dx = \pi \int_0^{\pi/2} \cos x \, dx$$

$$= \cancel{\pi[\cos\pi]} \quad \pi\left[\sin x\right]_0^{\pi/2}$$

$$= \pi\left[\sin\frac{\pi}{2} - \sin 0\right] = \underline{\pi}$$

$$\left.\frac{(-3)^2}{3}\right]$$

(f) $y = \sec x, \; y=0, \; x = -\pi/4, \; x = \pi/4$

$$\longrightarrow \quad V = \int_{-\pi/4}^{\pi/4} \pi(\sec x)^2 \, dx = \pi \int_{-\pi/4}^{\pi/4} (\sec^2 x)$$

$$\because \int_{-\pi/4}^{\pi/4} \sec^2 x = \left[\tan x\right]_{-\pi/4}^{\pi/4} \quad = \pi\left[\tan\frac{\pi}{4} - \tan\left(-\frac{\pi}{4}\right)\right]$$

$$= 2\pi$$

Page No.
Date

Q.4) revolving around y-axis.

→ (a) by $x = \sqrt{5}y^2$, $x=0$, $y=1$, $y=-1$

→ $V = \int_{-1}^{1} \pi (\sqrt{5}y^2)^2 \cdot dy = \pi \int_{-1}^{1} 5y^4 \, dy$

$= \pi \left[\frac{5y^5}{5}\right]_{-1}^{1} = \pi\left[\frac{5(1)^5}{5} - \frac{5(-1)^5}{5}\right]$

$= \pi\left[\frac{5}{5} + \frac{5}{5}\right] = \pi[1+1] = \underline{\underline{2\pi}}$

(b) $x = y^{3/2}$ $x=0$, $y=2$

→ ~~for~~ for lower limit, put ~~y=0~~ $x=0$

$\therefore$ $y^{3/2} = 0$ $\therefore \boxed{y=0}$

$V = \int_0^2 \pi \left(y^{3/2}\right)^2 dy = \pi \int_0^2 y^3 \, dy = \pi \left[\frac{y^4}{4}\right]_0^2$

$= \pi\left[\frac{2^4}{4} - \frac{0^4}{4}\right] = 4\pi$

(c) $x = \sqrt{2\sin 2y}$, $0 \le y \le \pi/2$, $x=0$

→ $V = \int_0^{\pi/2} \pi \left(\sqrt{2\sin 2y}\right)^2 dy$

$= \pi \int_0^{\pi/2} \cancel{\sqrt{\;}}\, 2\sin 2y$

$$= 2\pi \int_0^{\pi/2} \sin 2y \cdot dy = 2\pi \left[\frac{-\cos 2y}{2} \right]_0^{\pi/2}$$

$$= 2\pi \left[\frac{-\cos 2(\pi/2)}{2} + \frac{\cos 2(0)}{2} \right] = 2\pi \left[\frac{1}{2} + \frac{1}{2} \right]$$

$$= 2\pi$$

(d) $x = \sqrt{\cos(\pi y/4)} \quad -2 \le y \le 0 \quad , x = 0$

$$\rightarrow \quad V = \int_{-2}^{0} \pi \left(\sqrt{\cos\left(\frac{\pi y}{4}\right)} \right)^2 dy$$

$$= \pi \int_{-2}^{0} \cos\left(\frac{\pi y}{4} \right) dy$$

let $u = \frac{\pi y}{4} \Rightarrow \therefore \frac{du}{dy} = \frac{\pi}{4} \quad \therefore dy = \frac{4}{\pi} du$

$$\therefore \frac{4}{\pi} \int \cos(u)\, du$$

$$= \sin(u)$$

$$\therefore \frac{4 \sin(u)}{\pi}$$

$$\therefore \frac{4 \sin(\pi y/4)}{\pi} \quad dy$$

$$\pi \left[\frac{4 \sin(\pi y/4)}{\pi} + c \right]_{-2}^{0}$$

$$= \pi \times \frac{4}{\pi} \left[\sin \pi(0) - \sin (\pi(-2)) \right]$$

$$= \underline{\underline{4}} \quad \text{or} \quad \underline{\underline{0}}$$

(e) $x = 2/(y+1)$ $\quad x=0, \; y=0, \; y=3$

$\rightarrow$

$$V = \int_0^3 \pi \left(\frac{2}{(y+1)} \right)^2 dy$$

$$= 4\pi \int_0^3 (y+1)^{-2} \, dy$$

$$= 4\pi \left[\frac{(y+1)^{-1}}{-1} \right]_0^3 = 4\pi \left[\frac{-1}{(y+1)} \right]_0^3$$

$$= 4\pi \left[\frac{-1}{(1+3)} + \frac{1}{(1+0)} \right] = 4\pi \left[-\frac{1}{4} + \frac{1}{1} \right]$$

$$= 4\pi \left[\frac{-1+4}{4} \right] = 4\pi \left[\frac{3}{4} \right] = \underline{\underline{3\pi}}$$

Q.6) Washer's method.

→

ⓐ $y = x \quad , \quad y = 1 \qquad x = 0.$

→ Here, outer radius :- $R(x) = 1$

inner radius :- $r(x) = x$

& $x = 0 \quad x = 1$

$$\therefore \; V = \int_0^1 \pi \left[(R(x))^2 - (r(x))^2 \right] dx$$

$$= \pi \int_0^1 \left[(1)^2 - (x)^2 \right] dx \qquad = \pi \int_0^1 1 - x^2 \, dx$$

$$= \cancel{\pi \left[\frac{2}{3} - \frac{x^3}{3} \right]_0^1} \qquad = \pi \left[x - \frac{x^3}{3} \right]_0^1$$

$$= \cancel{\pi \left[\frac{2}{3} - \frac{1}{3} \right] - \left[\frac{2}{3} - \frac{0}{3} \right]}$$

$$= \cancel{\pi \left[\frac{1}{3} - \frac{2}{3} \right]} \qquad = \pi \left[1 - \frac{1}{3} \right] - \left[0 - 0 \right]$$

$$= \cancel{\pi - 1} \qquad = \pi \left[\frac{3 - 1}{3} \right] - 0$$

$$= \cancel{-\pi}$$

$$= \boxed{\pi \frac{2}{3}}$$

ⓑ $y = 2\sqrt{x} \quad , \quad y = 2 \qquad x = 0$

→ $\therefore \; 2\sqrt{x} = 2$

$\therefore \; \sqrt{x} = 1$

$\therefore \; \boxed{x = 1}$

Page No.
Date

$\rightarrow$ $\cancel{f(x)=}$ $R(x) = 2$

$r(x) = 2\sqrt{x}$.

$\therefore V = \pi \int_0^1 (2)^2 - (2\sqrt{x})^2 \, dx$

$= \pi \int_0^1 4 - 4x \, dx$

$= 4\pi \int_0^1 1 - x \, dx$

$= 4\pi \left[x - \frac{x^2}{2} \right]_0^1$ $\cancel{dx}$

$= 4\pi \left[1 - \frac{1}{2} \right] = 4\pi \frac{1}{2} = \underline{2\pi}$

(c) $y = x^2 + 1$, $y = x + 3$

$\rightarrow$ $\therefore x^2 + 1 = x + 3$

$\therefore x^2 - x - 2 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{+1 \pm 3}{2}$

$\frac{1+3}{2} = \frac{4}{2} = 2$ $\quad \frac{1-3}{2} = \frac{-2}{2} = -1$

$\therefore x = +2$ $\quad \therefore V = \pi \int_{-1}^{2} (x+3)^2 - (x^2+1)^2 \, dx$

$\& \; x = -1$

Page No.
Date

$$= \pi \int_{-1}^{2} x^2 + 6x + 9 - (x^4 + 2x^2 + 1)\, dx$$

$$= \pi \int_{-1}^{2} x^2 + 6x + 9 - x^4 - 2x^2 - 1\, dx$$

$$= \pi \int_{-1}^{2} [-x^4 - x^2 + 6x + 8]\, dx$$

$$\therefore \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} + \frac{6x^2}{2} + 8x \right]_{-1}^{2} dx$$

$$\pi \left[\frac{1}{5} + \frac{1}{3} + \frac{6}{2} - 8 \right] - \left[\frac{-32}{5} - \frac{8}{3} + \frac{24}{2} + 16 \right]$$

$$\pi \left[\frac{-32}{5} - \frac{8}{3} + \frac{24}{2} + 16 - \frac{1}{5} - \frac{1}{3} - \frac{6}{2} + 8 \right]$$

$$\pi \left[\frac{-33}{5} - \frac{9}{3} + \frac{18}{2} + 24 \right] = \pi \left[\frac{-33}{5} - 3 + 9 + 24 \right]$$

$$\pi \left[-\frac{33}{5} - 3 + 33 \right] = \pi \left[\frac{-33 - 15}{5} + 33 \right]$$

little calc mistake but ans is $\frac{117\pi}{5}$

$$= \pi \frac{-48}{5} + 33 \qquad \frac{117}{5}\pi$$

$$\frac{-33}{5} - 3 + 33 \qquad \frac{-32}{5} - \frac{1}{5} - \frac{8}{3} - \frac{1}{3} + \frac{24}{2} - \frac{6}{2} + 16 + 8$$

$$\frac{-33}{5} - \frac{9}{3} + \frac{18}{2} + 24 = \frac{-33}{5} - 3 + 9 + 24 \qquad 30$$

ⓓ $y = 4 - x^2$, $y = 2 - x$

$\longrightarrow$ $\therefore \; 4 - x^2 = 2 - x$

$4 = 2 - x + x^2$

$x^2 - x + 2 - 4 = 0$

$x^2 - x - 2 = 0$

$\therefore x = 2 \;\; \& \;\; x = -1$

$$V = \pi \int_{-1}^{2} (F(x)^2 - (f x)^2) dx$$

$$V = \pi \int_{-1}^{2} \cancel{(4-x)^2} \; (4 - x^2)^2 - (2-x)^2$$

$$V = \pi \int_{-1}^{2} 16 - 8x^2 + x^4 - [4 - 4x + x^2]$$

$$= \pi \int_{-1}^{2} 16 - 8x^2 + x^4 - 4 + 4x - x^2$$

$$= \pi \int_{-1}^{2} x^4 - 9x^2 + 4x + 12$$

$$= \pi \left[\frac{x^5}{5} - \frac{9x^3}{3} + \frac{4x^2}{2} + 12x \right]_{-1}^{2}$$

$$= \cancel{\left[\frac{32}{5} - \frac{72}{3} + \frac{32}{2} + 24 \right] + \frac{32}{5} - \frac{9}{3} + \frac{8}{2} + 12}$$

$$= \pi \left[\frac{x^5}{5} - 3x^3 + \frac{4x^2}{2} + 12x \right]_{-1}^{2}$$

Page No.
Date

$$= \pi \left[\frac{2^5}{5} - 3(2)^3 + 2(2)^2 + 12(2) \right] - \left[\frac{\cancel{x}(-1)^5}{5} - \frac{3(-1)^3}{} + 2(-1)^2 + 12(-1) \right]$$

$$= \pi \left[\frac{32}{5} - 24 + 8 + 24 + \frac{1}{5} - 3 - 2 + 12 \right]$$

$$= \pi \left[\frac{33}{5} - 24 + 39 \right] \qquad \frac{-87}{5} + 39$$

$$= \frac{108}{5}\pi \qquad \frac{33}{5} - 15$$

(e) $y = \sec x, \quad y = \sqrt{2} \quad -\pi/4 \leq x \quad \pi/4$

$$\rightarrow \quad V = \int_{-\pi/4}^{\pi/4} \pi \left[(\sqrt{2})^2 - (\sec x)^2 \right]$$

$$= \pi \int_{-\pi/4}^{\pi/4} 2 - \sec^2 x$$

$$= \pi \int_{-\pi/4}^{\pi/4} 2 - \int_{-\pi/4}^{\pi/4} \sec^2 x = \pi \left[2x - \tan(x) \right]_{-\pi/4}^{\pi/4}$$

$$= \pi(\pi - 2)$$

(P) $y = \sec x$, $y = \tan x$ $x = 0$, $x = 1$

$\longrightarrow$

$$V = \int_0^1 \pi (\sec^2 x - \tan^2 x)$$

$$= \pi \int_0^1 \sec^2 x - \tan^2 x \ dx$$

$$= \pi \int_0^1 1 \ dx = \pi \Big[(x) + c \Big]_0^1$$

$$= \pi$$

Q.8) $y = \frac{x^2}{2}$ $\therefore x = \sqrt{2y}$ $y = 0$ to $y = 5$ around y-axis.

$\longrightarrow$ $\cancel{V = \int_0^5 \pi \left(\frac{x^2}{2}\right)^2 dx = \pi \int_0^5 \frac{x^4}{4}}$

$$V = \int_0^5 \pi (\sqrt{2y})^2 \, dy = \pi \int_0^5 2y \ dy$$

$$= \pi \left[\frac{2y^2}{2} \right]_0^5 = \pi \left[y^2 \right]_0^5 = \pi (5^2 - 0^2)$$

$$= 25\pi$$

Page No.
Date

Q(13) Improper integrals.

(a) $$\int_0^{\infty} \frac{dx}{x^2+1}$$

→ Type 1 improper integral.

$$\therefore \int_0^{\infty} \frac{1}{x^2+1}\,dx = \lim_{B\to\infty} \int_0^{B} \frac{1}{x^2+1}\,dx$$

$$\therefore \lim_{B\to\infty} \left[\tan^{-1}(x)\right]_0^B$$

$$= \lim B\to\infty \left[\tan^{-1}(B)\right] - \left[\tan^{-1}(0)\right]$$

$$= \pi/2$$

(b) $$\int_{-\infty}^{\infty} \frac{2x\,dx}{(x^2+1)^2}$$

→ ∴ Type 1 ∴ rewrite as limits of definite improper integral.

$$\int_{-\infty}^{\infty} \frac{2x}{(x^2+1)^2} = \int_{-\infty}^{C} \frac{2x}{(x^2+1)^2}\,dx + \int_{C}^{\infty} \frac{2x}{(x^2+1)^2}\,dx$$

$$= \lim_{B\to\infty} \int_{-B}^{C} \frac{2x}{(x^2+1)^2} + \lim_{B\to\infty} \int_{C}^{B} \frac{2x}{(x^2+1)^2}\,dx$$

We can choose any appropriate value of c, let $c=0$.

$$= \lim_{B\to\infty} \int_{-B}^{0} \frac{2x}{(x^2+1)^2}\,dx + \lim_{B\to\infty} \int_{0}^{B} \frac{2x}{(x^2+1)^2}\,dx$$

$$=$$

$$\lim_{B\to\infty} \int_{-B}^{0} \frac{2x}{(x^2+1)^2}\,dx$$

$\therefore$ put $u = x^2+1$ $\therefore \frac{du}{dx} = 2x$ $\therefore dx = \frac{du}{2x}$

$$= \lim_{B\to\infty} \int_{-B}^{0} \frac{\cancel{2x}}{(x^2+1)^2}\frac{du}{\cancel{2x}} = \lim_{B\to\infty} \int_{-B}^{0} \cancel{\frac{1}{(x^2+1)^2}} \frac{1}{(u)^2}\,dx$$

$$= \left[-\frac{1}{u}\right]_{-B}^{0} \therefore \lim_{B\to\infty}\left[-\frac{1}{x^2+1}\right]_{B}^{0}$$

$$= \lim_{B\to\infty}\left[-\frac{1}{x^2+1}\right]_{-B}^{0} + \lim_{B\to\infty}\left[-\frac{1}{(x^2+1)}\right]_{0}^{B}$$

$$= \frac{-1}{(-B)^2+1} + \frac{1}{(0)^2+1} - \frac{1}{(B)^2+1} + \frac{1}{(0)^2+1}$$

$$= -\infty + 1 + \infty + 1$$

$$= \infty$$

9) $\int_{-1}^{\infty} \frac{dx}{x^2+5x+6}$

$\rightarrow \int \frac{dx}{x^2+5x+6} = \int \frac{1}{x^2+5x+6} dx$

$= \int \frac{1}{(x+2)\cdot(x+3)} dx = \frac{A}{(x+2)} + \frac{B}{(x+3)}$ — ①

$\therefore \frac{1}{(x+2)\ (x+3)} = \frac{A(x+3)+B(x+2)}{(x+2)(x+3)}$

$1 = A(x+3)+B(x+2)$

$1 = Ax+3A+Bx+2B$

$1 = x(A+B)+3A+2B$

$\therefore A+B=0 \quad \therefore A=-B$

$\therefore -3B+2B=1$	$3A+2B=1$
$-1B=1$	$3A-2=1$
$\boxed{B=-1}$	$3A=3 \quad \therefore \boxed{A=1}$

$\therefore$ from ① $\lim_{B\to\infty}\left[\int_{-1}^{\infty}\frac{1}{x+2} - \int_{-1}^{B}\frac{1}{x+3}\right]$

$\therefore \lim_{B\to\infty}\left[\ln(|x+2|)\right]_{-1}^{B} - \lim_{B\to\infty}\left[\ln(x+3)\right]_{-1}^{B}$

$\therefore \ln(B+2)-\ln(-1+2)-\ln(B+3)+\ln(-1+3)$

$\ln(B+2)-\ln(1)-\ln(B+3)+\ln(2)$

$\therefore \ln(B+2) - \ln(B+3) + \ln(2)$

$$\therefore \lim_{B\to\infty} \left[\ln\left(\frac{B+2}{B+3}\right)\right] + \ln(2)$$

$$\therefore \ln\left[\frac{1+\frac{2}{\infty}}{1+\frac{3}{\infty}}\right] + \ln(2)$$

$\therefore \ln(1) + \ln(2)$

$\therefore 0 + \ln(2)$

$\therefore \underline{\underline{\ln(2)}}$

b) $$\int_0^\infty \frac{dx}{(x+1)(x^2+1)}$$

$\longrightarrow$ $$\frac{1}{(x+1)(x^2+1)} = \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+1)} \quad \text{--- (1)}$$

$$1 = A(x^2+1) + (x+1)(Bx+C)$$
$$1 = Ax^2 + A + Bx^2 + Bx + Cx + C$$
$$1 = (A+B)x^2 + (B+C)x + A + C$$

$\therefore A+B=0 \longrightarrow A=-B$

$B+C=0 \longrightarrow B=-C \quad \therefore C=-B$

$A+C=1$

$\Rightarrow \therefore 2A=1 \quad \therefore A=\frac{1}{2} \quad \therefore B=-\frac{1}{2}, \quad C=\frac{1}{2}$

from ①,

$$\int_0^\infty \frac{1}{(x+1)(x^2+1} = \int \frac{1/2}{(x+1)} + \frac{(-\frac{1}{2})x+\frac{1}{2}}{(x^2+1)}$$

$$= \int_0^\infty \frac{1}{2(x+1)} + \frac{\frac{-2x+2}{4}}{(x^2+1)}\,dx$$

$$= \int_0^\infty \frac{1}{2(x+1)}\,dx + \int_0^\infty \frac{-x+1}{2(x^2+1)}\,dx$$

$$\therefore \lim_{B\to\infty} \int_0^B \frac{1}{2(x+1)}\,dx + \lim_{B\to\infty}\int_0^B \frac{-x+1}{2(x^2+1)}\,dx$$

$$\lim_{B\to\infty} \frac{1}{2}\int_0^B \frac{1}{(x+1)} + \lim_{B\to\infty} \frac{1}{2}\int_0^B \frac{-x+1}{x^2+1}$$

$$\lim_{B\to\infty} \frac{1}{2}\Big[\ln(|x+1|)\Big]_0^B - \lim_{B\to\infty} \frac{1}{2}\int_0^B \frac{x}{x^2+1}\,dx + \frac{1}{2}\int_0^B \frac{1}{x^2+1}$$

$$\therefore \lim_{B\to\infty} \frac{1}{2}\Big[\ln(|x+1|)\Big]_0^B - \frac{1}{2}\left[\Big[\frac{1}{2}\ln(x^2+1)\Big]_0^B + \frac{1}{2}\Big[\tan^{-1}(x)\Big]_0^B\right]$$

$$\frac{1}{2}\Big[\ln(B+1) - \ln(0+1)\Big] - \frac{1}{2}\Big[\frac{1}{2}\ln(B^2+1) + \frac{1}{2}\tan^{-1}(B)$$

$$- \frac{1}{2}\ln(0^2+1) + \frac{1}{2}\tan^{-1}(0)\Big]$$

$\therefore$ ~~log~~ ~~log~~

$= \frac{1}{2}[\infty - 0] - \frac{1}{2}\left[\infty + \frac{1}{2}\left(\frac{\pi}{2}\right)\right] - \frac{1}{2} \times 0 + 0$

$= \underline{\underline{\infty}}$

(b). $\int_0^4 \frac{dx}{\sqrt{(4-x)}}$.

→ This is type 2 improper integral.
$\therefore$ rewrite as limits of definite integral.

$\therefore \int_0^4 \frac{dx}{\sqrt{(4-x)}} = \int_0^1 \frac{dx}{\sqrt{(4-x)}} + \int_1^4 \frac{1}{\sqrt{(4-x)}} dx$

$= \lim_{k \to 1^-} \int_0^k \frac{1}{(4-x)^{1/2}} dx + \lim_{k \to 1^+} \int_k^4 \frac{1}{(4-x)^{1/2}} dx$

$= \lim_{k \to 1^-} \left[-2(4-x)^{1/2}\right]_0^k + \lim_{k \to 1^+} \left[-2(4-x)^{1/2}\right]_k^4$

$= -2(4-1)^{1/2} + 2(4-0)^{1/2} + -2(4-4)^{1/2} + 2(4-0)^{1/2}$

$= -2\sqrt{3} + 2(2) - 2(0) + 2\sqrt{3}$

$= \underline{\underline{4}}$

f) $\int_{-1}^{4} \frac{dx}{|x|}$

→ Solve $\int \frac{dx}{|x|}$

$\therefore \int \frac{dx}{x}, x \geq 0$ and $\int \frac{dx}{-x}, x \leq 0$

$= \ln(|x|) + c, x \geq 0$ and $-\ln(|x|) + c, x \leq 0$

$\lim_{k \to 0^-} \int_{-1}^{k} \frac{1}{x} dx + \lim_{k \to 0^+} \int_{k}^{4} -\frac{1}{x} dx$

$= \lim_{k \to 0^-} [\ln(|x|)]_{-1}^{k} + \lim_{k \to 0^+} [-\ln(|x|)]_{k}^{4}$

$= \cancel{\log} \ln(k) - \ln(-1) + (-\ln[(4)] - \ln(k))$

$= \cancel{\infty} \; \underline{\underline{\infty}}$

c) $\int_{-1}^{1} \frac{dx}{x^{2/3}}$

→ $\lim_{k \to 0} \int_{-1}^{0} \frac{1}{x^{2/3}} dx + \lim_{k \to 0} \int_{0}^{1} \frac{dx}{x^{2/3}} dx$

$-\frac{2}{3}+1$

$\frac{-2+3}{3}$

$\frac{1}{3}$

$3\sqrt[3]{x}$

$= \lim_{k \to 0^-} [3\sqrt[3]{x}]_{-1}^{k} + \lim_{k \to 0^+} [3\sqrt[3]{x}]_{k}^{1}$

$= 3\sqrt[3]{0} - 3(-1) + 3 - 0$

$= 0 + 3 + 3 - 0$

$= \underline{\underline{6}}$

d) $\int_{0}^{1} \frac{dx}{\sqrt{(1-x^2)}} = [\sin^{-1}(x)]_{0}^{1} + c$