of the substrate and the thin film on it are coated with a transparent metallic film of uniform thickness. A glass plate is also coated on one of its surfaces with the transparent metallic film. When the substrate and the glass plate are placed in contact and examined under monochromatic light, the reflected light shows a fringe system, as shown in Fig.6.31. A shift occurs in the fringes as we pass from the region occupied by thin film to the region where thin film is absent. The amount of displacement of one set of the fringes with respect to the second set of fringes is given by

$$s = 2t \quad \text{or} \quad t = s/2$$

where t is the thickness of the thin film. By measuring 's', t can be calculated.

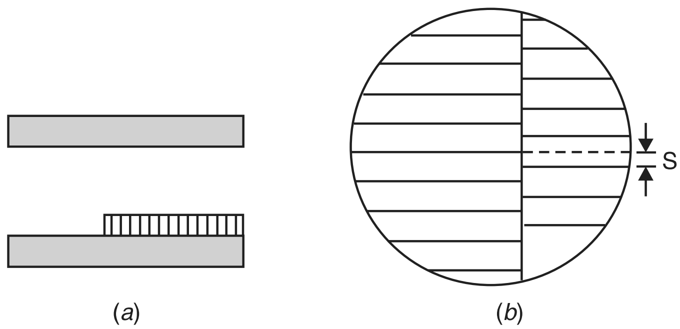

Fig. 6.31: Determination of the thickness of a thin coating.

6.12.3 Anti-Reflecting Coatings

Optical instruments such as telescopes and cameras use multicomponent glass lenses. When light is incident on the lens, part of the incident light is reflected away and that much amount of light is lost and wasted. When more surfaces are there, the number of reflections will be large and the quality of the image produced by a device will be poor. In case of solar cells, which operate on sunlight (daylight), the electrical energy produced will be less because of the loss of part of light energy due to reflection, at the cell surface. It is found that coating the surface with a thin transparent film of suitable refractive index can reduce such loss of energy due to reflections at surface. Such coatings are called **antireflection coatings**. Thus,

"Antireflection (AR) coatings are thin transparent coatings of optical thickness of one–quarter wavelength given on a surface in order to suppress reflections from the surface".

Alexander Smakula discovered in 1935 that the reflections from a surface can be reduced by coating the surface with a thin transparent dielectric film.

A thin film can act as an AR coating if it meets the following two conditions:

(*i*) **Phase condition:** The waves reflected from the top and bottom surfaces of the thin film are in *opposite phase* such that their overlapping leads to destructive interference, and

(*ii*) **Amplitude condition:** The waves have *equal* amplitudes.

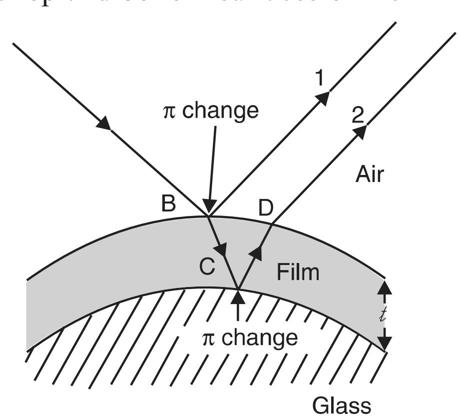

Fig. 6.32

The above conditions enable us determine respectively (a) the required thickness of the film and (b) the refractive index of the material to be used for forming the film.

(*i*) **Phase condition and minimum thickness of the film:** Let the thickness of the film be t and the refractive of the film-material be μ_f.The phase condition requires that the waves (ray 1 and ray 2) reflected from the top and bottom surfaces of thin film be 180° out of phase. It requires that the optical path difference between the two rays must equal one half-wave or an odd number of half-waves. Referring to Fig.6.32, the optical path difference between ray 1 and ray 2 is

$$\Delta = 2\mu_f t \cos r - \lambda/2 - \lambda/2$$

the first $\lambda/2$ corresponds to the π change at the top surface of the film (air-to-film boundary) and the second $\lambda/2$ to the π change that occurs at the film-to glass boundary because $\mu_f < \mu_g$. If we assume normal incidence of light, $\cos r = 1$ and the above equation reduces to

$$\Delta = 2\mu_f t - \lambda = 2\mu_f t$$

We wrote the above equality remembering that an *addition of a full wave or subtraction of a full wave from a train of waves does not affect the original phase relation*. The ray 1 and ray 2 interfere destructively if the optical path difference satisfies the condition that $\Delta = (2m + 1)\lambda/2$.

Thus, it requires that $$2\mu_f t = (2m + 1)\lambda/2$$

For the film to be transparent, its thickness should be a minimum, which happens when $m = 0$.

$$2\mu_f t_{\min} = \lambda/2$$

$$\therefore \quad t_{\min} = \frac{\lambda}{4\mu_f} \quad (\mu_f < \mu_g) \tag{6.68}$$

It means that the optical thickness of the AR coating should be of one-quarter wavelength. Such quarter-wavelength coatings suppress the reflections and cause the light to pass into the transmitted component.

(*ii*) **Amplitude condition:** The amplitude condition requires that the amplitudes of reflected rays, ray 1 and ray 2 are equal. That is,

$$E_1 = E_2 \tag{6.69}$$

It requires that

$$\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2 \tag{6.70}$$

where μ_a, μ_f, and μ_g are the refractive indices of air, thin film and glass substrate respectively. As $\mu_a = 1$, the above expression may be rewritten as

$$\left[\frac{\mu_f - 1}{\mu_f + 1}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

Expanding the above equation, we get

$$\frac{\mu_f^2 - 2\mu_f + 1}{\mu_f^2 + 2\mu_f + 1} = \frac{\mu_g^2 - 2\mu_g\mu_f + \mu_f^2}{\mu_g^2 + 2\mu_g\mu_f + \mu_f^2}$$

$$4\mu_f^3\mu_g + 4\mu_f\mu_g = 4\mu_f^3 + 4\mu_f\mu_g^2$$

Dividing by $4\mu_f$ and rearranging the terms

$$\mu_f^2 - \mu_g\mu_f + \mu_g^2 - \mu_g = 0$$

$$\mu_f^2 = \mu_g\left(1 + \mu_f^2 - \mu_g\right)$$

$$\therefore \qquad \mu_f^2 \cong \mu_g \text{ (as } \mu_f \approx \mu_g)$$

$$\therefore \qquad \mu_f = \sqrt{\mu_g} \tag{6.71}$$

It implies that the refractive index of thin film should be less than that of the substrate and possibly nearer to its square root.

In case of glass, if we take $\mu_g = 1.5$, $\mu_f = \sqrt{\mu_g} = 1.22$.

The materials which have refractive index nearer to this value are magnesium fluoride, MgF_2 (μ = 1.38) and cryolite, $3NaF.AlF_3$ (μ = 1.36). Apart from the refractive index, the material should possess some more additional properties. The film should adhere well, should be durable, scratch proof and insoluble in ordinary solvents. MgF_2 and cryolite satisfy these requirements. However, among the two, magnesium fluoride is cheaper and is hence widely used as AR coating.

It may be noted that the condition (6.68) is satisfied only at one particular wavelength. The wavelength normally chosen is 5500 Å for which the eye is most sensitive. This wavelength is located in the yellow-green portion of the spectrum. Consequently, the reflection of red and violet light will be larger when white light is incident on the component such as a camera lens. Hence, the component shows *purple hue* in reflected light.

Example 6.11: *A glass microscope lens (μ = 1.5) is coated with magnesium fluoride (μ_f = 1.38) film to increase the transmission of normally incident light λ = 5800 Å. What minimum film thickness should be deposited on the lens?*

Solution:

$$t_{\min} = \frac{\lambda}{4\mu_f} = \frac{5800\times10^{-10}\,\text{m}}{4\times1.38} = 1051\ \text{Å}$$

Example 6.12: *Can a thin film of water (μ_f = 1.33) formed on a glass window pane (μ_f = 1.52) act as a non-reflecting film? If so, how thick should be the water film?*

Solution: A film of refractive index μ_f can act as a non-reflecting film on a substrate having refractive index μ, if $\mu_f = \sqrt{\mu}$.

Here, $$\sqrt{\mu} = \sqrt{1.52} = 1.233.$$

As the refractive index of water is 1.33, it is nearer to water film $\sqrt{\mu}$ can act as a non-reflecting film on glass.

The minimum thickness of the film is given by

$$t_{\min} = \frac{\lambda}{4\mu_f}.$$

As human eye is more sensitive to green, it may be assumed that λ = 5500 Å.

$$\therefore \qquad t_{\min} = \frac{5500\times10^{-10}\,\text{m}}{4\times1.33} = \mathbf{1034\ Å}.$$

Multilayer AR coatings:

A single layer AR coating is effective only at one particular wavelength. A much wider coverage across the spectrum is possible with multiple coatings, called *multilayers*. In practice three layer coatings are widely used and are highly effective over most of the visible spectrum. The central layer is half-wave (λ / 2) thick and is of high refractive index materials