College of Engineering Pune
Ordinary Differential Equations and Multivariate Calculus
Tutorial-1 (2020-21)

1. Define Ordinary differential equation, General solution, Particular solution, Singular solution of an ODE, Exact differential equation and Linear differential equation.

2. Verify that the given function is a solution of corresponding differential equation (a, b, c are arbitrary constants)

   a) $y = a\ cos\ \pi x + b\ sin\ \pi x, \quad y'' + \pi^2 y = 0$
   b) $y = 5\ e^{-2x} + 2\ x^2 + 2x + 1, \quad y' + 2y = 4\ (x+1)^2$
   c) $y = -sin\ x + ax^2 + bx + c, \quad y''' = cos\ x$
   d) $y = \sin^{-1} xy, \quad xy^{'} + y = y^{'}\sqrt{1 - x^2y^2}$
   e) $x^2 + y^2 = 1, \quad x + yy^{'} = 0$
   f) $y = e^{x^2} \int_0^x e^{-t^2} dt, \quad y' = 2\ xy + 1$

3. Obtain the general solution (or particular solution) of the following differential equations.

   a) $y^{'} = 2 \sec 2y$
   b) $x^{'} = \cos(x+y)$
   c) $x\ dy - y\ dx = x\sqrt{x^2+y^2}\ dx, \qquad y(1) = 1$
   d) $y^{'} = (y + 9x)^2$
   e) $x^2y^{'} + xy + \sqrt{1 - x^2y^2} = 0$
   f) $y' = \dfrac{y - x}{y - x + 2}$
   g) $y\ y' = (x-1)\ e^{-y^2}, \qquad y(0) = 1$
   h) $x^2y' = 3\ (x^2 + y^2)\ \tan^{-1}\left(\dfrac{y}{x}\right) + xy$
   i) $y' = \dfrac{y + 2}{x + y + 1}$
   j) $(y/x)\, y' = \sqrt{1 + x^2 + y^2 + x^2y^2}$

k) $x^3(\sin y)\, y^{'} = 2, \qquad y(x) \to \dfrac{\pi}{2}$ as $x \to \infty$

l) $xy^{'} = y + 4x^5 \cos^2\left(\dfrac{y}{x}\right), \qquad y(2) = 0$

4. If in a culture of yeast the rate of growth $y'(t)$ is proportional to the amount $y(t)$ present at time t, and if $y(t)$ doubles in 1 day, how much yeast can be expected after 3 days at the same rate of growth ? After 1 week ? (Ans: $8\ y_0,\ 128\ y_0$)

5. If the growth rate of the amount of yeast at any time t is proportional to the amount present at that time and doubles in 1 week, how much yeast can be expected after 2 weeks? After 4 weeks?

6. Experiments show the rate of inversion of cane sugar in dilute solution is proportional to the the concentration $y(t)$ of unaltered sugar. Let the concentration be $1/100$ at $t = 0$ and $1/300$ at $t = 4\ hrs.$ Find $y(t)$. (Ans: $0.01\ e^{-0.275\ t}$)

7. A thermometer, reading 10^0C, is brought into a room whose temperature is 23^0C. Two minutes later the thermometer reading is 18^0C. How long will it take until the reading is 22.8^0C. (Ans : 8.73 mins.)

8. A body originally at 80^0C cools down to 60^0C in 20 minutes, the temperature of the air being 40^0C. What will be the temperature of the body after 40 minutes from the original ? (Ans: 50^0C)

9. A tank contains 5000 liters of fresh water. Salt water which contains 100 gms of salt per liter flows into it at the rate of 10 liters per minute and the mixture kept uniform by stirring, runs out at the same rate. When will the tank contain $200,000$ gms of salt?

10 A tank contains 1000 gallons of water in which 200 lb of salt, are dissolved. Fifty gallons of brine, each containing $(1 + \cos t)$ lb of dissolved salt, runs into the tank per minute, the mixture kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time t.
(Ans: $y(t) = 1000 + 2.494 \cos t + 49.88 \sin t - 802.5\ e^{-0.05t}$)

11 State and prove the necessary and sufficient condition of exactness for $M(x, y)\ dx + N(x, y)\ dy = 0$.

12. Define integrating factor and then prove the following theorem:
If $\dfrac{Q_x - P_y}{P}$ is a function of y alone, say $\phi(y)$, then $e^{\int \phi(y)\ dy}$ is an integrating factor of a differential equation $P(x, y)\ dx + Q(x, y)\ dy = 0$.

13. Verify that $\dfrac{y+1}{x^4}$ is an integrating factor of the differential equation $3(y + 1)\ dx = 2x\ dy$, and then solve using this I.F and otherwise.

14. Find the constant $\mathbf{n}$ such that $y^{\mathbf{n}}$ is an integrating factor of $y(2x^2y + e^x)\ dx - (e^x + y^3)\ dy = 0$.

15. Test for exactness, if exact then solve. If not so, then find the **I.F** and then solve. Given the initial conditions, find the particular solution.

a) $\left(\dfrac{\cos\ y}{x+3}\right)\ dx - \left(\sin y\ \ln(5x + 15) - \dfrac{1}{y}\right)\ dy = 0$

b) $3x(xy - 2)\ dx + (x^3 + 2y)\ dy = 0$

c) $(\cos x \cos y - \cot x)\ dx - \sin x \sin y\ dy = 0$

d) $(y^2\ e^{xy^2} + 4x^3)\ dx + (2xy\ e^{xy^2} - 3y^2)\ dy = 0$

e) $(\sin x \cosh y)\ dx - (\cos x \sinh y)\ dy = 0$

f) $e^{-y}\ dx + e^{-x}(1 - e^{-y})\ dy = 0$

g) $(2xy\ dx + dy)\ e^{x^2} = 0, \qquad y(0) = 2$

h) $(2x \ln x - xy)\ dy + 2y\ dx = 0$

16. Solve the following linear / non-linear differential equations:

a) $x(1-x^2)\,\dfrac{dy}{dx} + (2x^2-1)\;y = x^3$

b) $e^{-y}\;\sec^2 y\;dy = dx + x\;dy$

c) $\dfrac{dy}{dx} = x^3y^3 - xy$

17. Find the orthogonal trajectories of the following family of curves:

a) $y^2 = \dfrac{x^3}{c-x}$

b) $x^2 + y^2 = 2ax$

18. Show that the family of curves $x^2 + 4y^2 = c_1$ and $y = c_2x^4$ are orthogonal to each other.

19. Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self orthogonal.

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