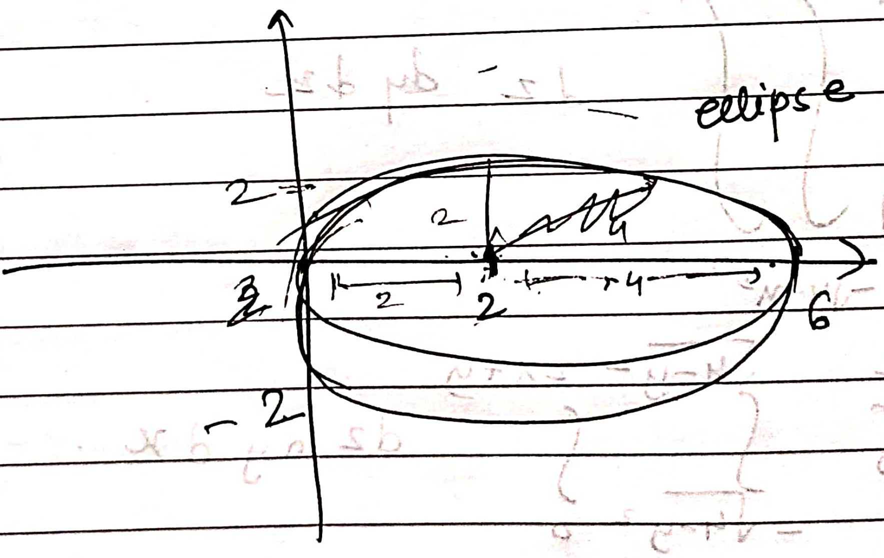
$-\pi/2 \cos\theta$ 0

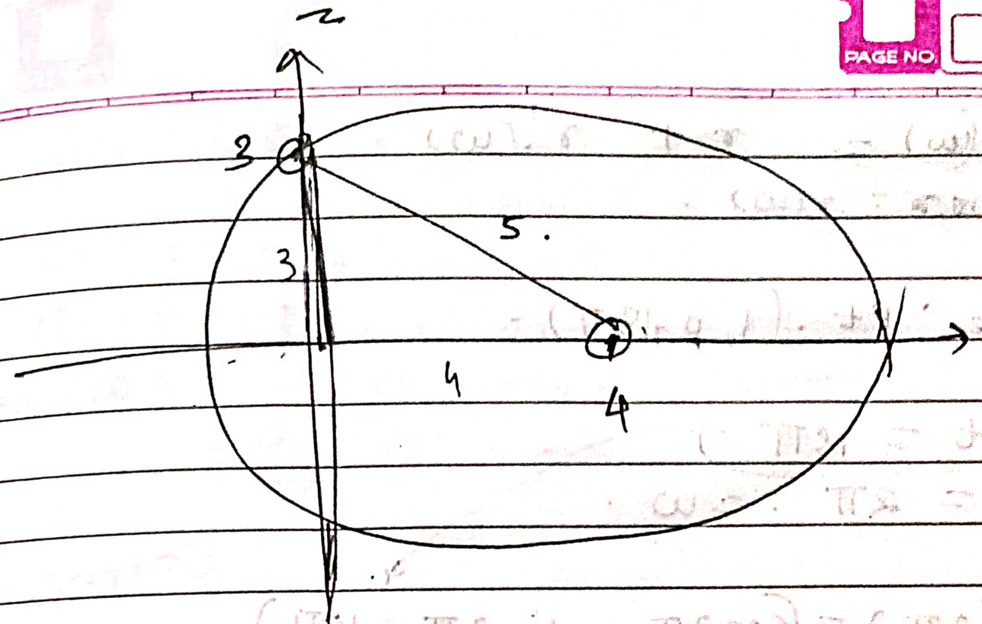
Tut - 5

Q 4) (a) $\overline{r(t)} = [2 + 4\cos t, 2\sin t, 0]$.



(b)

Q5)



$r(t) = [0, \; 4+5\cos t, \; \cancel{3}+5\sin t]$

Q6) $x^2+y^2=25$.

$z = \tan\left(\frac{y}{x}\right)$.

$r(t) = [5\cos t, 5\sin t, t]$

Q7) $r(t) = [\cos t, \sin t, 9t]$

$\Rightarrow r'(t) = [-\sin t, \cos t, 9]$.

Unit tangent vector $= \dfrac{r'(t)}{|r'(t)|}$

$|r'(t)| = \sqrt{\sin^2 + \cos^2 t + 81}$

$|r'(t)| = \sqrt{1+81} = \sqrt{82}$

$\therefore$ Unit tangent $=$ ~~cost~~ $\left[\dfrac{-\sin t}{\sqrt{82}}, \dfrac{\cos t}{\sqrt{82}}, \dfrac{9}{\sqrt{82}}\right]$.

$q(\omega) = r + r'(\omega)$

where $r = r(\omega)$.

~~z~~ ' pt : $(1, 0, 18\pi)$.

$9t = 18\pi$

$t = 2\pi = \omega$

$\therefore r(2\pi) = [\cos 2\pi, \sin 2\pi, 18\pi]$

~~$\mathfrak{z}$~~ $r(2\pi) = [1, 0, 18\pi]$.

$r'(2\pi) = [0, 0, 1$

Q8) $f = xy - yz$.

$\vec{v} = [2y, 2z, 4x + 2]$.

$\bar{\omega} = [3z^2, 2x^2 - y^2, y^2$

(a) div (grad f).

$\Rightarrow \nabla f = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$

$\nabla f = [y\hat{i}, x - z\hat{j}, -y\hat{k}]$

$\text{div}(\cancel{F}) = \frac{\partial v_1}{\partial x}\hat{i} + \frac{\partial v_2}{\partial y}\hat{j} + \frac{\partial v_3}{\partial z}\hat{k}$

$\text{div}(\nabla f) = \bar{0}\hat{i} + \bar{0}\hat{j} + \bar{0}\hat{k}$

$\text{div}(\nabla f) = 0$

(b) di grad(div $\bar{\omega}$).

$= \partial iv(\bar{\omega}) = 0 + -2y + 0.$

$\partial iv(\bar{\omega}) = -2y.$

$grad(\bar{\omega}) = [0, -2, 0].$

$= 0\hat{i} + -2\hat{j} + 0\hat{k}.$

(c). div(curl $\bar{v}$).

$\Rightarrow curl(\bar{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & 2z & 4x+z \end{vmatrix}.$

$= (0-2)\hat{i} - (4-0)\hat{j} + (0+2)\hat{k}$

$= -2\hat{i} - 4\hat{j} + 2\hat{k} = [-2, -4, 2].$

$\partial iv(curl\ \bar{v}) = 0.$

(e) $[(curl\ \bar{v}) \times \bar{\omega}] \cdot \bar{\omega}.$

$\Rightarrow curl(\bar{v}) = [-2, -4, 2].$

$curl(\bar{v}) \times \bar{\omega} = [-2, -4, 2] \times [3z^2, 2x^2-y^2, y^2]$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -4 & -2 \\ 3z^2 & 2x^2-y^2 & y^2 \end{vmatrix} = [-4y^2 - \{4x^2-2y^2\}]\hat{i} - [-2y^2+6z^2]\hat{j} + [-4x^2+2y^2+12z^2]\hat{k}$

$= (-4y^2 - 4x^2 + 2y^2)\hat{i} - [6z^2 - 2y^2]\hat{j} + (-4x^2 + 2y^2 + 12z^2)\hat{k}$

$= (-4x^2 - 2y^2)\hat{i} - (6z^2 - 2y^2)\hat{j} + (-4x^2 + 2y^2 + 12z^2)\hat{k}$

Q10) Angle betⁿ surfaces =

$$\theta = \cos^{-1}\left(\frac{\nabla f \cdot \nabla g}{|\nabla f| \cdot |\nabla g|}\right)$$

$\Rightarrow$ $f \Rightarrow x^2 + y^2 + z^2 = 9$

$g \Rightarrow x^2 + y^2 - 3 - z$

$\nabla f = (2x, 2y, 2z)$

$\nabla g = (2x, 2y, -1)$

$\nabla f \cdot \nabla g = (4x^2 + 4y^2 - 2z)$

$= 4(2)^2 + 4(-1)^2 - 2(2)$

$= 16 + 4 - 4 = 16$

$|\nabla f| = \sqrt{4x^2 + 4y^2 + 4z^2} = \cancel{2\sqrt{1+3}}$

$p+ = 2, -1, 2.$

$= \sqrt{4[(2)^2 + (-1)^2 + (2)^2)} =$

$2\sqrt{4+1+4} = 2\sqrt{9} = 2 \times 3 = 6.$

$|\nabla g| = \sqrt{4x^2 + 4y^2 + (-1)^2}$

$= \sqrt{4(4) + 4(-1)^2 + 1}$

$= \cancel{\sqrt{8+2+1}} = \sqrt{16+4+1} = \sqrt{21}$

$|\nabla f| \cdot |\nabla g| = 6 \sqrt{21}$

$\therefore \theta = \cos^{-1}\left(\frac{16}{6\sqrt{21}}\right).$

12.22

$\frac{8\sqrt{21}}{3}$

Q.1)(a) $[xy, 2xy, 0] = v$

$\text{curl}(v) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2xy & 0 \end{vmatrix}$

$= (0-0)\hat{i} - (\cancel{2xy} - 0)\hat{j} + (2y - x)\hat{k}$

$= (2y - x)\hat{k}.$

As $\cancel{\ }$ curl $\neq 0$, potential f doesn't exist.

(b). $[x^2 - yz,\ y^2 - zx,\ z^2 - xy]$

$$\Rightarrow \text{curl} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2-yz & y^2-zx & z^2-xy \end{vmatrix}$$

$$= \cancel{(-x+x)\hat{i} - (-y-0)\hat{j} + (-2y-2)\hat{k}}$$

$$= (-x+x)\hat{i} - (-y+y)\hat{j} + (-z+z)\hat{k}$$

$$= 0$$

$\therefore$ f exists.

$$\frac{\partial f}{\partial x} = x^2 - yz \qquad \frac{\partial f}{\partial y} = y^2 - zx, \quad \frac{\partial f}{\partial z} = z^2 - xy$$

$$\int \frac{\partial f}{\partial x} = \int x^2 - yz \, dx$$

$$\Rightarrow f = \frac{x^3}{3} + c(y,z) - yzx \quad \to (1)$$

$$\frac{df}{dy} = \frac{\partial}{\partial y}\left(\frac{x^3}{3} - yzx + c(y,z)\right)$$

$$y^2 - \cancel{z}x = 0 - \cancel{z}x + \frac{\partial (c(y,z))}{\partial y}$$

$$\therefore \frac{\partial}{\partial y}(c(y,z)) = y^2$$

$$= \int \frac{\partial}{\partial y} (c(y,z)) = \int y^2$$

$$\Rightarrow c(y,z) = \frac{y^3}{3} + c(z) \quad \to (2)$$

Put ② in ①:

$$f = \frac{x^3}{3} - yzx + \frac{y^3}{3} + c(z)$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[\frac{x^3}{3} - yzx + \frac{y^3}{3} + c(z)\right].$$

$$z^2 - xy = -xy + \frac{\partial}{\partial z}(c(z)).$$

$$\therefore \frac{\partial}{\partial z}(c(z)) = z^2.$$

$$\int \frac{\partial}{\partial z}(c(z)) = \int z^2 \, dz$$

$$= c(z) = \frac{z^3}{3} + k. \quad \to ③.$$

$$\therefore \text{Ans} \Rightarrow \frac{x^3}{3} - yzx + \frac{y^3}{3} + \frac{z^3}{3} + k.$$

Q13. curl $= 0 \Rightarrow$ irrotational.
div $= 0 \Rightarrow$ incompressible.

$\Rightarrow$

$$\bar{v} = [x, y, -z].$$

$$\text{div}(\bar{v}) = 1 + 1 - 1 = 1$$

$\therefore \text{div}(\bar{v}) \neq 0.$

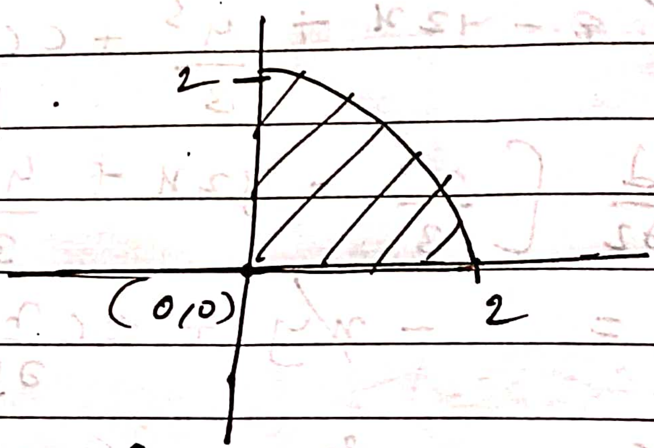
$\therefore$ It is compressible.

$$\text{curl}(\bar{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & -z \end{vmatrix} = (0-0)\hat{i} - (0-0)\hat{j} + (0-0)\hat{k} = \bar{0}.$$

$\text{curl}(\bar{v}) = 0 \quad \therefore$ It is irrotational.

Q18) $\int_C \bar{F}(r) \cdot d\bar{r} = \int f(r(t)) \cdot r'(t)\, dt$.

$\Rightarrow$ (a) $\bar{F} = [xy, x^2y^2]$.



$r(t) = (2\cos t, 2\sin t)$.
$0 \le t \le \pi/2$

~~$f(r(t)) = (2\cos t\, 2\sin t, 2\cos$~~

$r'(t) = (-2\sin t, 2\cos t)$

$f(r(t)) = [4\cos t \cdot \sin t, 16\cos^2 t \cdot \sin^2 t]$

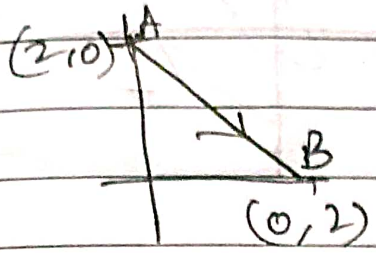
$\therefore \cancel{\int_C F(r) \cdot dr} = \int_0^{\pi/2}$

$f(r(t)) \cdot r'(t) = (-8\cos t \cdot \sin^2 t + 32\cos^3 t \cdot \sin^2 t)$

$\therefore \int_C F(r)\,dr = \int_0^{\pi/2} -8\cos t \cdot \sin^2 t + 32\cos^3 t \cdot \sin^2 t\, dt.$

(b). $\bar{F} = [xy, x^2y^2]$.

c is straight line from $(2,0), (0,2)$.



$r(t) = (1-t)A + Bt$

$= (1-t)(2,0) + (0,2)t$

$\therefore r(t) = (2-2t, 2t)$

$r'(t) = (-2, 2)$

$F(r(t)) = \cancel{2t}((2-2t)2t, (2-2t)^2(2t)^2)$

$= (4t-4t^2, (4-8t+4t^2)(4t^2))$

$= (4t-4t^2, 16t^2-32t^3+16t^4)$

$F(r(t)) \cdot r'(t) = (4t-4t^2, 16t^2-32t^3+16t^4) \cdot (-2, 2)$

$= (-8t+8t^2 + 32t^2 - 64t^3 + 32t^4)$

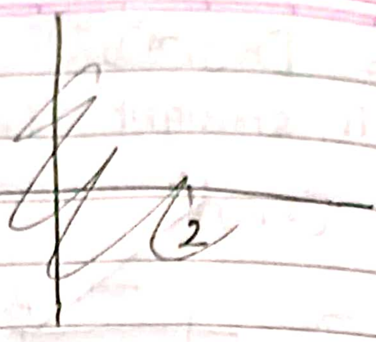
$\therefore \int_C F(\bar{r}) dr = \int_0^1 -8t+8t^2+32t^2-64t^3+32t^4 \, dt$

$= \left. -\frac{8t^2}{2} + \frac{8t^3}{3} + \frac{32t^3}{3} - \frac{64t^4}{4} + \frac{32t^5}{5} \right|_0^{\pi/2}$

$= \left. -4t^2 + \frac{8t^3}{3} + \frac{32t^2}{3} - 16t^4 + \frac{32t^5}{5} \right|_0^{\pi/2}$

(c) $\bar{F} = [x-y, y-z, z-x]$.

$c = [2\cos t, t, 2\sin t]$



$r(t) = (2\cos t, t, 2\sin t)$

$0 \le t \le 2\pi$

$r'(t) = (-2\sin t, 1, 2\cos t)$.

$F(r(t)) = \cancel{-2\sin t - 2\cos t, 2\cos}$

$= (2\cos t - t, t - 2\sin t, 2\sin t - 2\cos t)$

$F(r(t)).r'(t) = (\cancel{-4\cos t\sin t} + 2t\sin t + t - 2\sin t + \cancel{4\cos t\sin t} - 4\cos^2 t)$

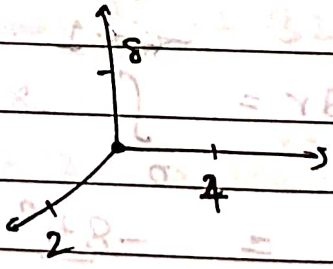
$= 2t\sin t + t - 2\sin t - 4\cos^2 t$

$\int_C F(\bar{r})\, dr = \int_0^{2\pi} 2t\sin t + t - 2\sin t - 4\cos^2 t\, dt$.

(d) $\bar{F} = [\cosh x, \sinh y, e^z]$

$r(t) = [t, t^2, t^3]$.

$0 \le t \le 2$.



$r'(t) = (1, 2t, 3t)$

$f(r(t)) = [\cosh t, \sinh t^2, e^{t^3}]$

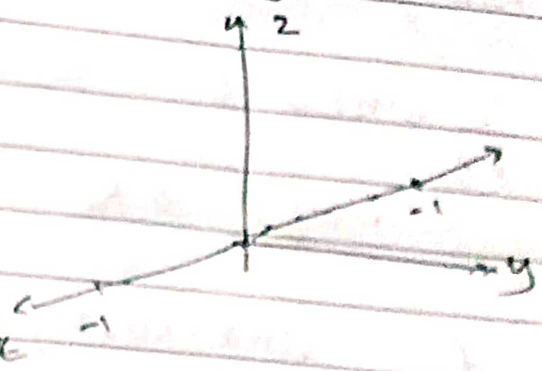
$f(r(t)).r'(t) = \cosh t + 2t\sinh t^2 + 3t.e^{t^3}$

$\int_C F(\bar{r})\, dr = \int_0^2 [\cosh t + 2t\sinh t^2 + 3t.e^{t^3}\, dt$

(e) $F = [ze^{xz}, 2\sinh(2y), xe^{xz}]$

$y = x,\ z = x^2$

$-1 \leq x \leq 1.$



Q9 (a) $\displaystyle\int_{(\pi/2,\,-\pi)}^{(\pi/4,\,0)} (\cos x \cos 2y\, dx - 2\sin x \sin 2y\, dy).$

$\Rightarrow f_1 = \cos x \cos 2y \quad ; \quad f_2 = -2\sin x \sin 2y$

$\therefore F = [F_1, F_2, 0]$

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & 0 \end{vmatrix}$$

$$= (0-0)\hat{i} - (0-0)\hat{j} + (-2\cos x \cdot \sin 2y + 2\cos x \sin 2y)\hat{k}$$

$$= 0$$

As curl $= 0$ $\therefore$ it is path independent.

$\therefore F = \nabla f$

$\therefore \dfrac{\partial f}{\partial x} = \cos x \cos 2y \quad , \quad \dfrac{\partial f}{\partial y} = -2\sin x \sin 2y$

$\Rightarrow \displaystyle\int \frac{df}{\partial x} = \int \cos x \cos 2y$

$f = \sin x \cdot \cancel{\sin 2} \cos 2y + c(y).$

$\dfrac{\partial f}{\partial y} = -2\sin x \cdot \sin 2y + \dfrac{\partial}{\partial y}(c(y)).$

$$-2\sin x \cancel{\cos} 2y = -2\sin x\cancel{\cos}2y + \frac{\partial}{\partial y}(c(y)).$$

$$\frac{\partial}{\partial y}(c(y)) = 0$$

$$\therefore c(y) = k$$

$$\therefore f = \sin x \cdot \cos 2y + k$$

$$= \sin\frac{\pi}{4}\cdot\cos 0 - \sin\frac{\pi}{2}\cdot\cos(-\pi)$$

$$= \frac{\sqrt{2}}{2} - 1.$$

$$= \frac{\sqrt{2}-2}{2}$$

(b) $\displaystyle\int_{(0,0,0)}^{(1,1,1)} ye^{z}\,dy - ze^{y}\,dz.$

$\Rightarrow F_1 = ye^{z}$, $F_2 = -ze^{y}$.

$F = [0, F_1, F_2]$

$$\text{curl}(f) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & ye^{z} & -ze^{y} \end{vmatrix}$$

$$\Rightarrow (-ze^{y} - ye^{z})\hat{i} - (0-0)\hat{j} + (0-0)\hat{k}$$

$$= (-ze^{y} - ye^{z})\hat{i}$$

As curl $\neq 0$, $\therefore$ It is path dependent.

(c) $\int_{(\pi, \frac{\pi}{2}, 2)}^{(0, \pi, 1)} -z\sin(xz)\,dx + \cos y\,dy - x\sin(xz)\,dz$

$\Rightarrow F_1 = -z\sin(xz)$, $F_2 = \cos y$ $F_3 = -x\sin(xz)$

$F = [F_1, F_2, F_3]$.

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$-x\sin(xz)$

$-x \cdot \cos(xz) \cdot z - \sin(xz)$.

$-z\sin(xz)$.

$= -z \cdot \cos(xz) \cdot x - \sin(xz)$.

$= (0-0)\hat{i} - \cancel{(-\cos xz \cdot z + \cos xz \cdot x)}$

$[(-zx \cdot \cos(xz) - \sin(xz)) + zx\cos(xz) + \sin(xz)]\hat{j}$

$+ (0-0)\hat{k}$

$= 0$.

As $\text{curl}(F) = 0$, ~~the~~ it is path independent.

$\frac{\partial f}{\partial x} = -z \cdot \sin(xz)$, $\frac{\partial f}{\partial y} = \cos y$, $\frac{\partial f}{\partial z} = -x\sin(xz)$

$\int \frac{\partial f}{\partial x} = \int -z\sin(xz) \cdot dx$

$f = \frac{+\cancel{z}\cos(xz)}{\cancel{z}} = \cos(xz) + c(y,z)$

$\frac{\partial f}{\partial y} = 0 + \frac{\partial c(y,z)}{\partial y}$

$\therefore \cos y = \frac{\partial}{\partial y} c(y,z)$.

$\int \cos y\,dy = \int \frac{\partial}{\partial y} c(y,z) = \sin y = c(y,z)$

$\therefore c(y,z) = \sin y + c(z)$

$\therefore f = \cos(xz) + \sin y + C(z)$.

$\therefore \frac{\partial f}{\partial z} = -x \sin xz \cdot \cancel{x} + \frac{\partial C(z)}{\partial z}$

$\cancel{-x \cdot \sin(xz)} = \frac{\partial}{\partial} - \cancel{x \sin(xz)} + \frac{\partial}{\partial z} C(z)$

$\therefore \frac{\partial}{\partial z} C_1(z) = 0 + k \cdot = k$.

$\therefore f = \cos(xz) + \sin y + k$.

$= \cos(0 \times 1) + \sin \pi - \cos(2\pi) + \sin(\frac{\pi}{2})$.

$= \cancel{1} + 0 - \cancel{1} - 1$.

$= -1$.