We can write from equ. (2.25) that

$$V = \frac{Q}{4\pi\varepsilon_0 r} \tag{2.29}$$

2.7.1 Calculating the Potential from the Field

Let a charge move from an initial point 1 to final point 2 in an electric field along the path shown. As the charge moves a distance along this path, the electric field does an element of work dW on it.

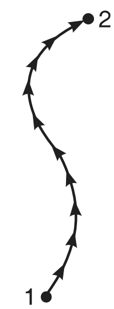

Fig. 2.7

$$dW = -\mathbf{F}\cdot dx = -q\mathbf{E}\cdot dx$$

∴ The total work done by the field $W_{12} = -q\int_1^2 \mathbf{E}\cdot dx$ (2.30)

∴ The electric potential difference $V_2 - V_1 = -\int_1^2 \mathbf{E}\cdot dx$

If the initial point 1 is taken to be at infinity, $V_1 = 0$ and writing $V_2 = V$, the above equation yields $V = -\int_1^2 \mathbf{E}\cdot dx$

In a more general way, we write the above expression as $V = -\int_1^2 \mathbf{E}\cdot dr$ (2.31)

2.7.2 Calculating the Field from the Potential

Let us consider a set of closely spaced equipotential surfaces perpendicular to the plane of the page and passing through it. Let the potential difference between each pair of adjacent surfaces be dV. Suppose a charge moves through a small distance dx from one equipotential surface to the adjacent equipotential surface. The work that the electric field does on the charge is given by equ. (2.26) as

$$dW = q\,dV$$

We can also express the work done is

$$dW = -\mathbf{F}\cdot dx = -q\mathbf{E}\cdot dx = -qE\cos\theta\,(dx)$$

Equating these two equations, we obtain

$$q\,dV = -q\,E\cos\theta\,(dx)$$

or

$$E\cos\theta = -\frac{dV}{dx} \tag{2.32}$$

As $E\cos\theta$ is the component of E in the x direction, we have to use partial derivative in the above equation. Thus,

$$E_x = -\frac{\partial V}{\partial x} \tag{2.33}$$

That is, the electric field is equal to the negative of the derivative of the electric potential with respect to some coordinate.

In general, the electric potential is a function of all three spatial coordinates. If V is given in terms of rectangular coordinates, and then

$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y} \tag{2.34}$$

$$E_z = -\frac{\partial V}{\partial z}$$

2.7.3 Potential Gradient

The rate of change of potential with distance is called the **potential gradient**. If the electric field is homogeneous and uniform, the potential gradient is given by

$$\text{Potential gardient} = \frac{dV}{dx} \tag{2.35}$$

where dV is the change in potential between two points separated by a distance x.

Example 2.7: *Two positive charges of 12×10^{-10} C and 8×10^{-10} C are placed 10 cm apart. Find the work done in bringing the charges 4 cm closer.*

Solution: The electrostatic force between the charges separated by a distance x, is given by

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 x^2}$$

$$\therefore \quad F = 9\times10^9 \times \frac{12\times10^{-10}\times8\times10^{-10}}{x^2} = \frac{8.64\times10^{-9}}{x^2}\ \text{N}$$

If the charge is moved through a small distance dx, the work done dW, in bringing the charges closer by distance dx, will be,

$$dW = F \cdot dx$$

$$\therefore \quad dW = \frac{8.64\times10^{-9}}{x^2}\times dx$$

Now total work done in moving the charges from 0.10 m to 0.06 m apart will be,

$$W = -\int_{0.1}^{0.06} dW = -8.64\times10^{-9}\int_{0.1}^{0.06}\frac{dx}{x^2}$$

$$= -8.64\times10^{-9}\left[\frac{1}{x}\right]_{0.1}^{0.06}$$

$$= -8.64\times10^{-9}\left[\frac{1}{0.06}-\frac{1}{0.1}\right]$$

$$\therefore \quad \mathbf{W = 5.76 \times 10^{-8}\ J.}$$

2.8 EQUIPOTENTIAL SURFACES

An **equipotential surface** is a surface on which the potential has the same value at all points. In other words, the potential difference between any two points on an equipotential surface is zero.

Properties of equipotential surfaces

(*i*) *Work done in moving a charged particle over an equipotential surface is zero*: Since the potential energy of a charged particle is the same at all points of a given equipotential surface, work done in moving a charged particle over an equipotential surface is zero.

(*ii*) *Electric field is always perpendicular to an equipotential surface*: The equipotential surface through any point will be perpendicular to the direction of electric field at that point, as shown in Fig. 2.8.

(*iii*) *The spacing between equipotential surfaces gives us indication of regions of strong and weak fields*: The region where the equipotential surfaces are crowded is the region of stronger field and the region where the surfaces are separated by larger distance is the region of weaker field.

(*iv*) *Equipotential surfaces never intersect each other*: If two equipotential surfaces could intersect, then at the point of intersection there would be two values of electric potential which is impossible.

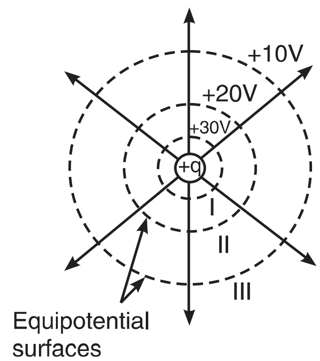

Fig. 2.8

2.9 ELECTRIC FIELD IS A CONSERVATIVE FIELD

A force is said to be **conservative**, if we can associate potential energy with it. If potential energy cannot be associated with a force, then the force is **nonconservative**. The gravitational force is conservative whereas frictional force is nonconservative.

We know that gravitational force does negative work on a body while the body is rising and an equal amount of positive work is done on its return trip. The total work done is zero for the round trip.

Definition: A field is conservative if the work done on a particle that moves through a round trip in the field is zero.

Or equivalently, a field is conservative if the work done by it on a particle between two points is the same for all paths connecting the two points.

Mathematically, a force field is said to be conservative if the line integral of the field along any closed path is zero. Therefore, an electric field is said to be conservative if

$$\oint \mathbf{E} \cdot \mathbf{dl} = 0 \tag{2.35}$$

Let a charge move from an initial point A to final point B in an electric field along the path shown in Fig. 2.9. As the charge moves a distance $d\mathbf{l}$ along this path, the electric field does an element of work dW on it.

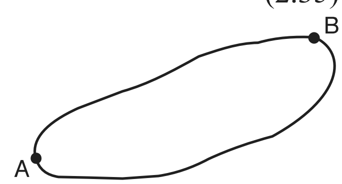

Fig. 2.9

$$dW = -\mathbf{F} \cdot d\mathbf{l} = -q\mathbf{E} \cdot d\mathbf{l}$$

$\therefore$ The total work done by the field in moving the charge from A to B is

$$W_{AB} = -q\int_A^B \mathbf{E} \cdot d\mathbf{l} \tag{2.36}$$

The integral in the above equation is called the **line integral**.

If the curve C forms a closed path, then the integral along the closed path is denoted by

$$\oint_C \mathbf{E} \cdot d\mathbf{l} \tag{2.37}$$

When the path is a closed curve, the line integral is referred to as "net circulation integral" for **E** around the chosen path. It is a measure of a vector field property called the curling up of field lines.

If the line integral around any closed path vanishes, that is $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$ then, the field is said to be **conservative**.

$$W_{AB} = -\frac{Qq}{4\pi\varepsilon_0}\int_{x_A}^{x_B}\frac{1}{x^2}dx = -\frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{x_A}-\frac{1}{x_B}\right]$$

It is seen from the above result that the work done depends only on the starting point x_A and the final point x_B and not on the path chosen to go from A to B. If now the charge returns from B to A through the same path or any other path the work done would be

$$W_{BA} = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{x_A}-\frac{1}{x_B}\right]$$

Therefore, the total work done on a point charge in the electric field over any closed path is

$$W_{\text{total}} = \oint dW = W_{AB} + W_{BA} = 0$$

$$\therefore \qquad \oint_C \mathbf{E}\cdot dl = 0$$

2.10 POTENTIAL AT A POINT DUE TO A GROUP OF POINT CHARGES

The potential due to a number of charges at a point can be found by taking the algebraic sum of potentials due to all charges. If point charges q_1, q_2, q_3, ... are at distances r_1, r_2, r_3, ... from a point P, then the resultant potential at P is given by

$$V = \frac{q_1}{4\pi\varepsilon_0 r_1}+\frac{q_2}{4\pi\varepsilon_0 r_2}+\frac{q_3}{4\pi\varepsilon_0 r_3}+\ldots\ldots$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}+\frac{q_3}{r_3}+\ldots\ldots\right]$$

or

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_n}{r_n} \qquad (2.38)$$

2.11 COMPUTATION OF ELECTRIC POTENTIAL IN SOME SPECIFIC CASES

1. Potential due to a charged sphere

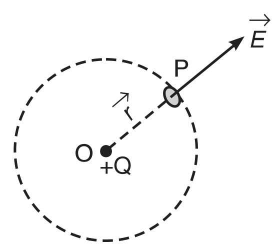

Fig. 2.10

Let us consider a charged conducting sphere of radius R and carrying a charge of Q which is uniformly distributed throughout the sphere. The entire charge may be assumed to be concentrated at the centre of the sphere. Then, the charged sphere can be treated as identical to a point charge Q located at the centre of the sphere O.

Case 1 : *P lies outside the sphere*: Let P be at a distance r from the centre of the sphere. As the entire charge is concentrated at O, the potential at P will be

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Case 2 : *P lies on the surface of the sphere*: Let P be on the surface of the sphere. Therefore, $r = R$. Hence the potential at a point on the surface of sphere is

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

Case 3 : *P lies inside the sphere*: The potential due to a charged sphere at any point inside it is the same as on the surface of the sphere.

2. Potential due to an electric dipole

An electric dipole consists of two equal and opposite charges very close to each other. Let AB be an electric dipole of length $d = 2a$. Let P be a point where we would like to determine the potential due to the dipole. Let $OP = r$ and let θ be the angle between r and the dipole axis, AB (see Fig. 2.11).

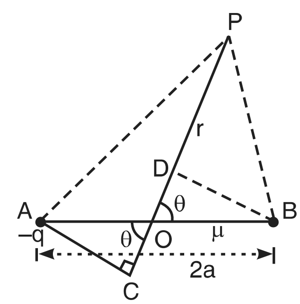

Fig. 2.11

From the Δ^{le} OAC, we get

$$\cos\theta = \frac{OC}{AO} = \frac{OC}{a}$$

$\therefore$ $$OC = a\cos\theta$$

Similarly, $$OD = a\cos\theta$$

If $r >> a$, we can write $$PA = PC = PO + OC = r + a\cos\theta$$

and $$PB = PD = PO - OD = r - a\cos\theta$$

The electric potential at the point P due to the electric dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{PB} - \frac{q}{PA}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2a\cos\theta}{r^2 - a^2\cos\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qa\cos\theta}{r^2 - a^2\cos\theta}$$

But $2qa = qd = \mu$, the electric dipole moment of the dipole.

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\mu\cos\theta}{r^2 - a^2\cos\theta} \quad (2.39)$$

At distances r far larger than the values of a, $a^2\cos^2\theta/r^2 << 1$ and the term can be neglected. The equation (2.39) then reduces to

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\mu\cos\theta}{r^2} \quad (2.40)$$

(*i*) When the point lies on the axial line of the dipole on the side of the positive charge, then $\theta = 0$ and $\cos\theta = 1$. Therefore,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\mu}{r^2}$$

(*ii*) If the point lies on the axial line of the dipole on the side of the negative charge, then $\theta = 180°$ and $\cos\theta = -1$. Therefore,

$$V = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{\mu}{r^2}$$

(*iii*) If the point lies on the equatorial line of the dipole, then $\theta = 90°$ and $\cos\theta = 0$. Therefore,

$$V = 0$$

Example 2.8: *A hollow sphere of radius 20 cm is charged with a charge of 30 × 10^{-9} C. Find the potential at a distance of 50 cm from the sphere centre.*

Solution: The potential at the surface of charged sphere of radius r is given by

$$V = \frac{q}{4\pi\varepsilon_0 r}.$$

So, the potential at a distance x will be $V = \dfrac{q}{4\pi\varepsilon_0 x} = 9\times10^9\times\dfrac{30\times10^{-9}}{0.5}$ = **0.539 kV**.

Example 2.9: *The potential due to an isolated point charge at a point 20 cm from the charge is 400 volts. Calculate magnitude of charge.*

Solution: The potential at any point at a distance x from the point charge will be

$$V = \frac{q}{4\pi\varepsilon_0 x}$$

$$\therefore \qquad 400 = 9\times10^9\times\frac{q}{0.2} \qquad \therefore \quad \boldsymbol{q = 8.9 \times 10^{-9}\,\mathrm{C}}$$

Example 2.10: *Calculate electrostatic potential at a point due to charge of 50 μC at a distance of 15 cm from it.*

Solution:

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

$$\therefore \qquad V = 9\times10^9\times\frac{50\times10^{-6}}{0.15} = \mathbf{3\times10^6\ V}$$

2.12 FLUX

Electric field is a vector field. The first important property that characterizes a vector field is *flux*. Michael Faraday made use of field lines for visualizing electric and magnetic fields. Gauss introduced the concept of flux to express the relation between a field and its source.

Let us consider a uniform electric field, a shown in Fig. 2.12. Let the field lines (lines of force) penetrate a plane rectangular surface of area, A, which is perpendicular to the field. The number of field lines per unit area is proportional to the magnitude of the electric field. Therefore, the number of field lines penetrating the area A is proportional to the product EA. The **electric flux** is defined as the product of the magnitude of the electric field and surface area, A, perpendicular to the field.

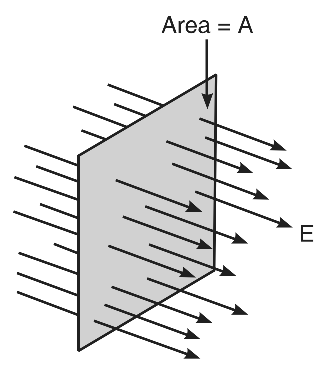

Fig. 2.12

$$\Phi = EA$$

When the surface is not perpendicular to the field lines, then the component of E along the normal to the surface is to be multiplied by the area. Thus,

$$\Phi = (E\cos\theta)\,A$$

We may express the above relation as the scalar product of vectors **E** and **A**, as

$$\Phi = \mathbf{E}\cdot\mathbf{A} \qquad (2.41)$$

In more general situations, the surface is of arbitrary shape. To know the flux passing through the surface, we have to divide the surface into a large number of small elements, each of area dA. The area $d\mathbf{A}$ of surface element is defined as a vector whose magnitude represents the area of the element and the direction is indicated by the outward normal to the elemental surface. The electric flux through this small element is

$$\Delta\Phi = \mathbf{E}_i \cdot \Delta\mathbf{A}_i$$

By adding the contributions of all elements, we obtain the total flux through the entire surface. Thus,

$$\Phi = \int_{\text{Surface}} \mathbf{E} \cdot d\mathbf{A}$$

Therefore, the flux of an electric field $\mathbf{E}$ through an *open surface* S is given by

$$\Phi = \int_{S} \mathbf{E} \cdot d\mathbf{A} \tag{2.42}$$

If the surface is closed, we call it a closed surface integral and denote it by a circle on the integration symbol. Thus,

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{A} \tag{2.43}$$

2.13 SOLID ANGLE

The concept of solid angle is an extension of the concept of a two-dimensional angle that is measured in radians or degrees. Let us consider the small area element dS at a distance r from the point O. Let $\mathbf{n}$ be the unit vector perpendicular to the surface element dS. When every point of the boundary of dS is joined to O, a cone is formed.

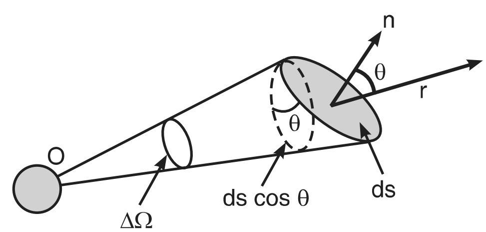

Fig. 2.13

The vector $\mathbf{OA} = \mathbf{r}$ makes an angle θ with the unit vector $\mathbf{n}$. The projection of the area dS on a plane normal to $\mathbf{r}$ is given by

$$\mathbf{r} \cdot \mathbf{n}\, dS = \mathbf{r} \cdot dS \cos\theta$$

The solid angle $d\Omega$ subtended by the area element $d\mathbf{S}$ at a point O is defined as

$$d\Omega = \frac{\text{Projection of } d\mathbf{S} \text{ perpendicular to } \mathbf{r}}{r^2} = \frac{dS\cos\theta}{r^2} = \frac{\mathbf{r}\cdot d\mathbf{S}}{r^2} \tag{2.44}$$

Solid angle has no dimensions but is represented by the unit *steradian*.

The total solid angle subtended by the surface of a sphere at O is given by

$$\frac{\text{Total surface area}}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi \tag{2.45}$$

2.14 GAUSS' LAW OF ELECTROSTATICS IN FREE SPACE

Integral laws are called *global laws* since they indicate what happens over a wide range. Gauss law is one of such powerful global laws. It gives the relationship between the integral component of the electric field over a closed surface and the total charge enclosed by the surface. This law relates the electric flux through any closed surface to the net amount of charge within the surface.

Statement

Gauss law states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed.

Proof

Let us consider a closed surface S surrounding a charge q, as shown in Fig. 2.14. Let $d\mathbf{S}$ be an element of area around the point P on the surface and $\hat{\mathbf{n}}$ an outward unit vector normal to it. Let θ be the angle between the electric field at P and the unit vector $\hat{\mathbf{n}}$. The electric flux through the element of area $d\mathbf{S}$ is

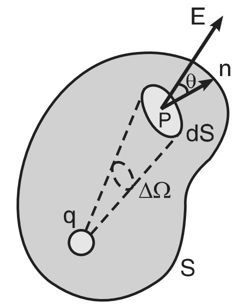

Fig. 2.13

$$d\phi = \mathbf{E} \cdot d\mathbf{S} = \text{E} \cos\theta \, d\text{S}$$
$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \cos\theta \, dS$$

where r is the distance of the surface element $d\mathbf{S}$ from charge q.

Now $\frac{d\mathbf{S}\cos\theta}{r^2} = d\Omega$ is the solid angle subtended by $d\mathbf{S}$ at q. Therefore,

$$d\phi = \frac{1}{4\pi\varepsilon_0} q \, d\Omega$$

$\therefore$ Total flux through the entire surface S is

$$\phi = \int d\phi = \oint \mathbf{E} \cdot dS = \frac{1}{4\pi\varepsilon_0} q \oint d\Omega$$

The total solid angle subtended by S at O is 4π .

$$\therefore \qquad \phi = \oint \mathbf{E} \cdot dS = \frac{q}{\varepsilon_0} \tag{2.46}$$

Gauss' law becomes very useful in calculation of electric field in cases where Coulomb's law or principle of superposition becomes tedious.

2.15 DIVERGENCE OF ELECTRIC FIELD

The **divergence** of vector field **E** is defined as the limiting value of the ratio of the closed surface integral and the volume enclosed by the surface over which integration is carried out, when the volume element tends to zero.

$$\text{div } \mathbf{E} = \underset{\Delta V \to 0}{\text{Lt}} \frac{\oint_S \mathbf{E} \cdot dS}{\Delta V} \tag{2.47}$$

It is common practice to denote div **E** as $\nabla \cdot \mathbf{E}$. The divergence in rectangular coordinates is found to be given by

$$\text{div } \mathbf{E} = \nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \tag{2.48}$$

$\nabla \cdot \mathbf{E}$ can be taken as a measure of the spreading out of the field **E**. If a vector function **E** spreads out from a point, then it has a *positive* divergence at that point and the point acts as a **source** of the field **E**. On the other hand, if the field converges to a point, then $\nabla \cdot \mathbf{E}$ would be negative at that point and the point acts as a **sink** for the field **E**. If the vector field neither converges nor diverges, then $\nabla \cdot \mathbf{E} = 0$. For a field at a point to have finite divergence means that an equivalent to a source must exist there. If the divergence of a field is zero for a small volume, it means that all the flux that enters the volume also leaves it; this region of space is then free of a source. The divergence of a vector field can be considered to be a measure of scalar sources of the field. Further, at the points of the field where the divergence of **E** is positive, we have the sources of the field (positive charges), while at the points where it is negative, we have sinks (negative charges). The field lines emerge from the field sources and terminate at the sinks.

2.16 DIFFERENTIAL FORM OF GAUSS'S LAW

The integral form of Gauss's law relates the net flux out of a finite volume to the net amount of charge enclosed in that volume. In contrast to (2.46), the differential form of the Gauss theorem establishes the relation between the volume charge density and the changes in the field intensity **E** in the vicinity of a given point in space. Thus, the differential law is a *local law*, which tells us what happens at a given point. The differential form of Gauss's law can be found by applying Gauss's integral law to an infinitesimally small volume surrounding a point. The integrals will then transform to differentials in the limit as the volume goes to zero, and we will obtain a point relationship of Gauss's law involving derivatives only. The differential form of Gauss's law is more general and will be very useful since derivatives are easier to calculate compared to integrals.

Let us represent the charge q in the volume V enclosed by a closed surface S as $q_{\text{int}} = \langle \rho \rangle V$, where $\langle \rho \rangle$ is the volume charge density, averaged over the volume V. Using this into equ.(2.46) and dividing the equation with V, we obtain

$$\frac{1}{V}\oint E \cdot ds = \frac{\langle \rho \rangle}{\varepsilon_0} \tag{2.49}$$

We now make the volume V to tend to zero by contracting it to the point we are interested in. Then, $\langle \rho \rangle$ will tend to the value of ρ at the given point of the field and hence the ratio on the R.H.S. of equ. (2.49) will tend to ρ/ε_0. When V tends to zero, the quantity on the L.H.S. is called the **divergence of the field E**. By definition

$$\nabla \cdot \mathbf{E} = \underset{V \to 0}{\text{Lt}} \frac{1}{V} \oint_S \mathbf{E} \cdot ds$$

Consequently, the above relation transforms into

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{2.50}$$

Equ. (2.50) is the Gauss theorem expressed in the differential form. It shows that the divergence of the field **E** at a given point depends only on the electric charge density ρ at this point.

2.17 DERIVATION OF COULOMB'S LAW FROM GAUSS LAW

Coulomb's law can be deduced from Gauss law. Let us consider an isolated point charge q, as shown in Fig. 2.15. Let us consider any imaginary spherical surface r centered on the charge. Such an imaginary closed surface enclosing a charge is called a *Gaussian surface*. The advantage of the spherical surface is that **E** is normal to it at all points and has the same magnitude.

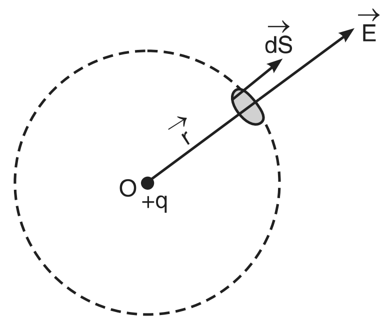

Fig. 2.15

In Fig. 2.15 both **E** and $d\mathbf{S}$ are directed radially outward at any point on the surface. The angle between them is zero.

$$\therefore \qquad \mathbf{E} \cdot d\mathbf{S} = \mathbf{E}d\mathbf{S}$$

Gauss law therefore reduces to

$$\varepsilon_0 \oint \mathbf{E} \cdot d\mathbf{S} = \varepsilon_0 \oint \mathbf{E}\, d\mathbf{S} = q$$

Since **E** is constant for all points on the sphere, it can be taken out of the integral.

$$\therefore \qquad \varepsilon_0\, \mathbf{E} \oint d\mathbf{S} = q$$

$$\oint d\mathbf{S} = \text{Area of the sphere, } 4\pi r^2$$

$$\therefore \qquad \varepsilon_0 \mathbf{E}(4\pi r^2) = q$$

Or

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

The above equation gives the magnitude of the electric field strength **E** at any point at a distance r from the isolated charge q. If a second charge q_1 is kept at any point on the spherical surface, it experiences a force

$$\mathbf{F} = q_1 \mathbf{E}$$

Using the expression for **E** into the above equation, we obtain

$$\mathrm{F} = \frac{1}{4\pi\varepsilon_0} \frac{q q_1}{r^2} \qquad (2.51)$$

which is the Coulomb's law.

2.18 APPLICATIONS OF GAUSS'S LAW

1. Electric field due to a solid charged sphere

Let us consider an isolated sphere of charge Q having radius R. Let us consider a point P at a distance r from the centre O of the sphere.

Case 1: *Point P lies outside the charged sphere*: Let us draw the Gaussian surface through point P so that it encloses the sphere of charge. In this the Gaussian surface is a spherical surface of radius r and is centered on O, as shown in Fig. 2.16.

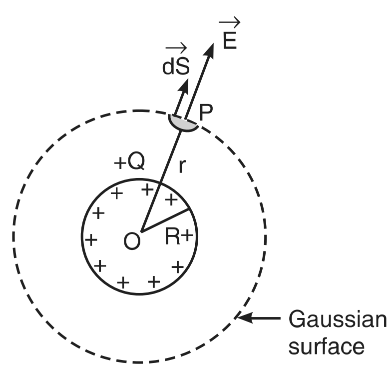

Fig. 2.16

Let E be electric field at point P due to the sphere of charge Q. The field is spherically symmetrical. Therefore, E is along the normal to the spherical surface and has the same magnitude at all points on the surface of the sphere.

Total flux through the Gaussian surface is

$$\oint \mathbf{E} \cdot dS = \oint \mathbf{E}\, dS = E \oint dS = E(4\pi r^2)$$

According to Gauss's theorem,