Feedback Control System

Introduction

Introduction

Control systems are an integral part of **modern society**.

Numerous applications are all around us:

The **rockets fire**, and the **space shuttle lifts off** to earth orbit; a **self-guided vehicle** delivering material to workstations in an aerospace assembly plant glides along the floor seeking its destination. These are just a few examples of the automatically controlled systems

The domestic applications are

an **air conditioner**, a **refrigerator**, a **bathroom toilet tank**, an **automatic iron**, **elevators**, **washing machines** and many processes within a car like **car braking system**, **fuel injection system**, and so on.

Introduction

Bio control systems

Within our own bodies we have numerous control systems, such as

the **pancreas**, which **regulates** our **blood sugar**.

In time of "**fight** or **flight**," our excitement increases along with our **heart rate**, causing **more oxygen** to be **delivered** to our **cells**.

Our **eyes** follow a **moving object** to keep it in **view**;

our **hands grasp** the **object** and **place** it **precisely** at a **predetermined location**.

Definition

A control system consists of **subsystems** and **processes** (or **plants**) assembled for the purpose of obtaining a **desired** output with **desired performance**, given a specified input.

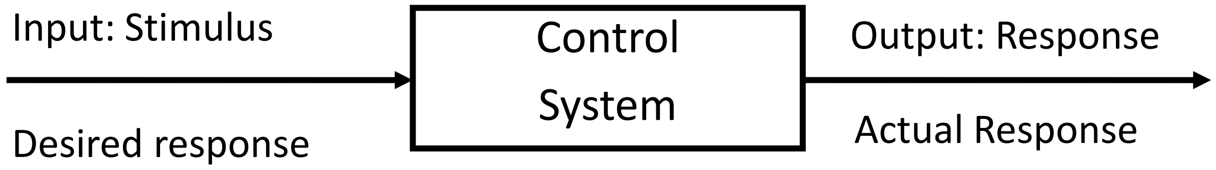
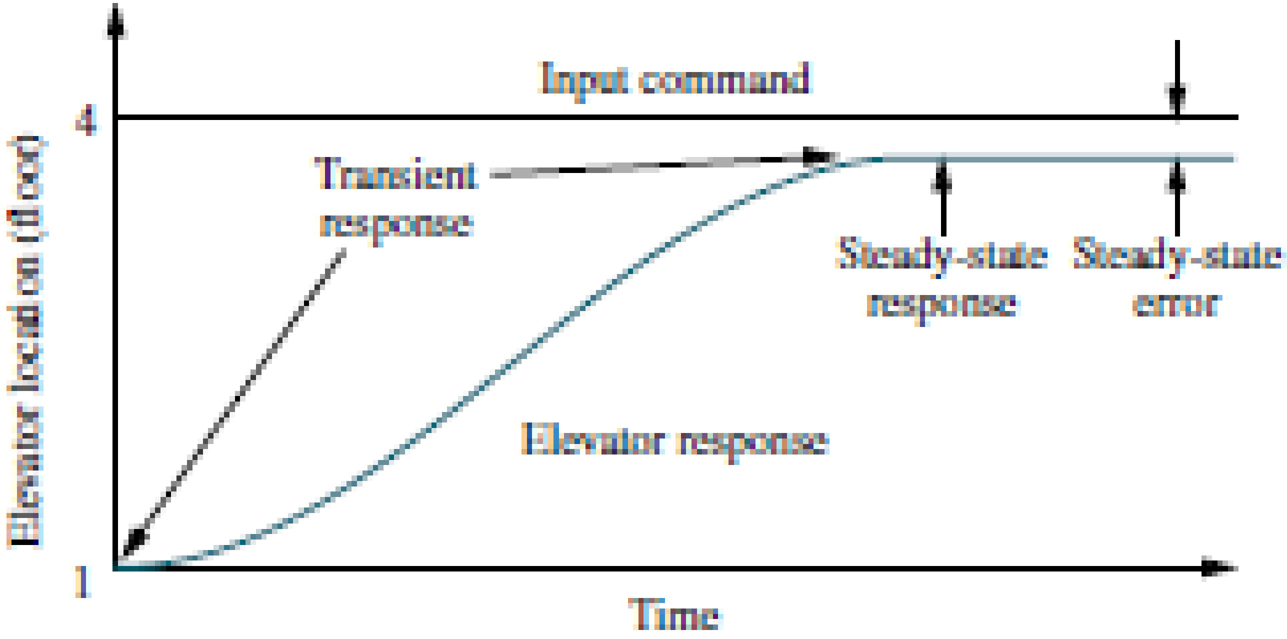


Figure shows a control system in its simplest form, where the input represents a desired output.

Definition

For example, consider an elevator.

- When the **fourth-floor** button is pressed on the **first floor**, the **elevator** rises to the **fourth floor** with a **speed** and **floor-leveling** accuracy **designed** for **passenger comfort**.
- The push of the **fourth-floor button** is an **input** that **represents** our **desired** output, shown as a **step function** in Figure.



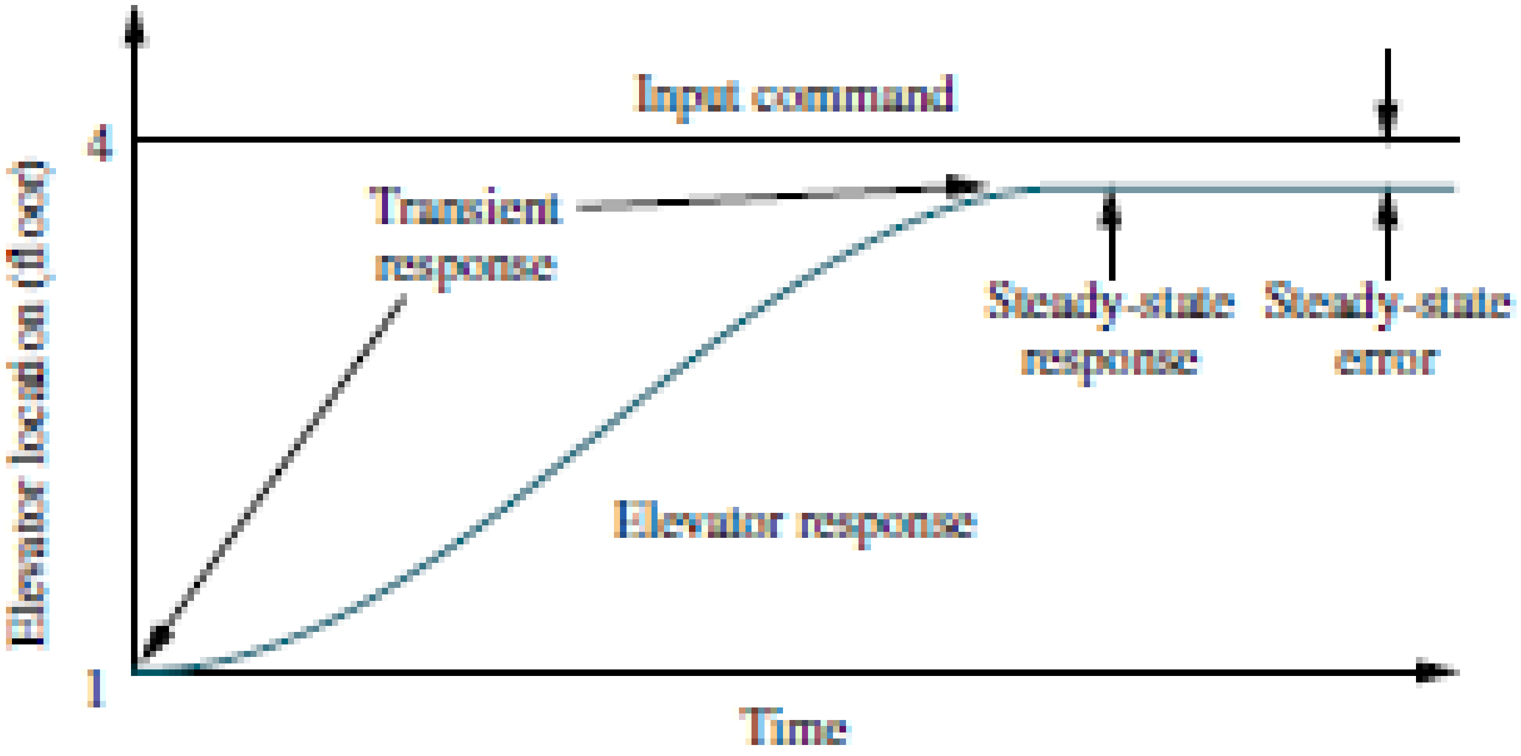
Definition

The performance of the elevator can be seen from the elevator response curve in the figure.

Two major measures of performance are apparent:

(1) the **transient response** and

(2) the **steady-state error**.



Definition

In our example, passenger **comfort** and passenger **patience** are dependent upon the **transient response**.

If this **response** is **too fast**, passenger **comfort** is **sacrificed**;

if **too slow**, passenger **patience** is **sacrificed**.

The **steady-state error** is another important **performance specification** since passenger **safety** and **convenience** would be **sacrificed** if the **elevator** did not **properly level**.

Advantages of Control Systems

With **control systems** we can move **large equipment** with **precision** that would otherwise be **impossible**.

We can point **huge antennas** to pick up **faint/weak radio signals** from the universe; **controlling** these **antennas** by hand would be **impossible**.

Because of control systems, **elevators carry** us **quickly** to our **destination**, **automatically stopping** at the **right floor**.

We alone could not provide the **power** required for the **load** and the **speed**; **motors** provide the **power**, and **control systems regulate** the **position** and **speed**.

Advantages of Control Systems

We build control systems for four primary reasons:

1. **Power amplification**
2. **Remote control**
3. **Convenience** of **input** form
4. **Compensation** for **disturbances**

For example, a **radar antenna**, positioned by the **low-power rotation** of a **knob** at the **input**, requires a **large amount** of **power** for its **output rotation**. A control system can produce the **needed power amplification**, or **power gain**.

Robots designed by control system principles can compensate for human disabilities.

Control systems are also useful in remote or dangerous locations. For example, a remote-controlled robot arm can be used to pick up material in a radioactive environment.

Advantages of Control Systems

For example, in a **temperature** control system, the **input** is a **position** on a **thermostat**. The **output** is **heat**. Thus, a **convenient position** input yields a **desired thermal output**.

Another advantage of a control system is the **ability** to **compensate** for **disturbances**.

When the **rope** of **elevator** is **cut**, or the **speed** of **elevator increases** over the **rated speed safety brake** applied.

The system must be able to **yield** the **correct output** even with a **disturbance**.

For example, consider an antenna system that points in a commanded direction.

If **wind forces** the **antenna** from its **commanded position**, or if **noise enters internally**, the system must be **able** to **detect** the **disturbance** and **correct** the **antenna's position**.

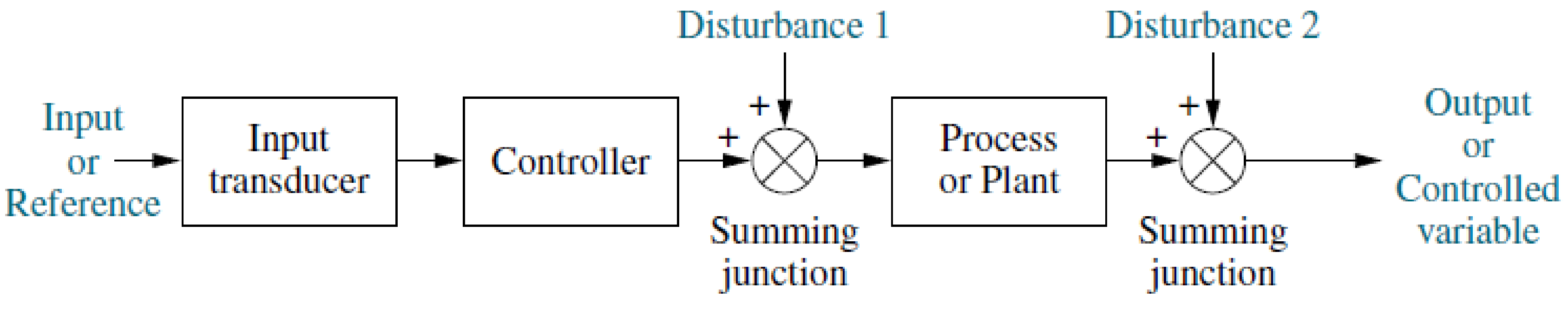
Advantages of Control Systems

Obviously, the system’s input will not change to make the correction.

Consequently, the **system** itself must **measure** the **amount** that the **disturbance** has repositioned the antenna and then return the antenna to the position commanded by the input.

Open Loop Control Systems

A generic open-loop system is shown in Figure.



(*a*)

It starts with a **subsystem** called an **input transducer**, which **converts** the **form** of the **input** to that **used** by the **controller**.

The controller drives a process or a plant.

The input is sometimes called the **reference**, while the **output** can be called the **controlled variable**.

Open Loop Control Systems

Other signals, such as **disturbances**, are shown added to the **controller** and **process outputs** via **summing junctions**, which **yield** the **algebraic sum** of their **input signals** using associated signs.

For example, the plant can be a furnace or air conditioning system, where the output variable is temperature.

The controller in a heating system consists of fuel valves and the electrical system that operates the valves.

The distinguishing characteristic of an open-loop system is that **it cannot compensate for any disturbances** that add to the controller's driving signal (Disturbance 1 in Figure).

Open Loop Control Systems

For example, if the controller is an electronic amplifier and Disturbance 1 is noise, then any additive amplifier noise at the first summing junction will also drive the process, corrupting the output with the effect of the noise.

The output of an open-loop system is corrupted not only by signals that add to the controller’s commands but also by disturbances at the output (Disturbance 2 in Figure). The system cannot correct for these disturbances, either.

Open Loop Control Systems

Open-loop systems **do not correct for disturbances** and are simply commanded by the input.

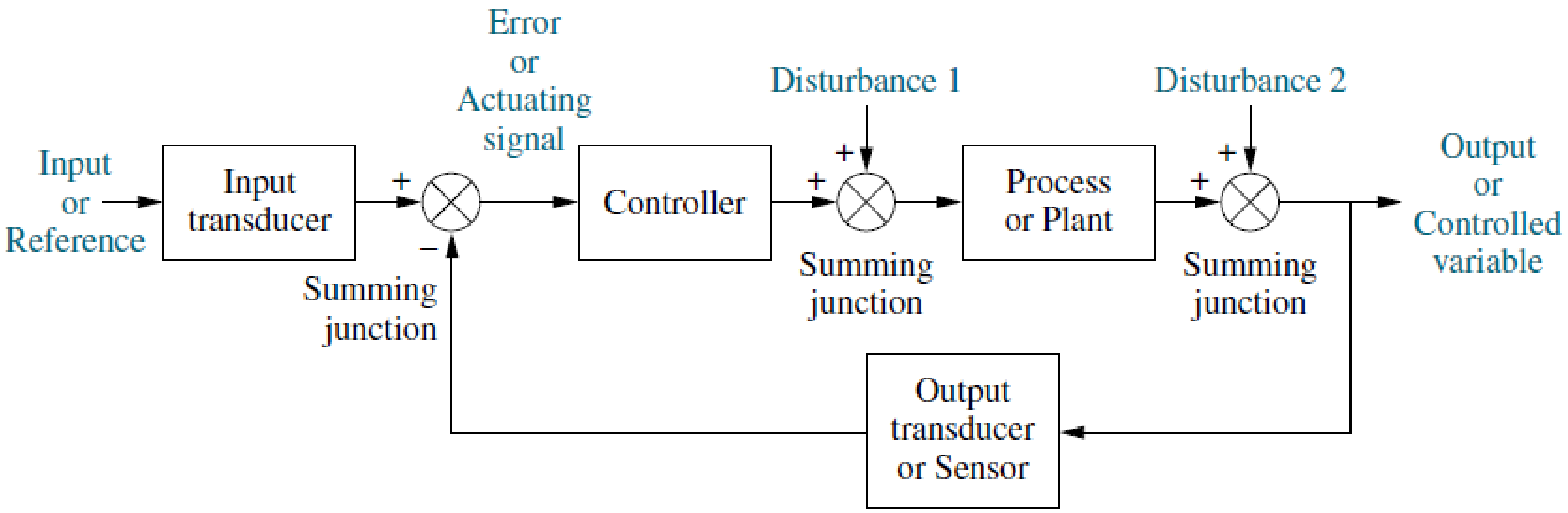
For example, **toasters** are open-loop systems.

The controlled variable (output) of a toaster is the **color** of the **toast**. The device is designed with the **assumption** that the **toast** will be **darker** the **longer** it is **subjected** to **heat**.

The **toaster does not measure** the **color** of the toast; it does not **correct** for the **fact** that the **toast** is **rye**, **white**, or **sourdough**, **nor does** it **correct** for the fact that **toast comes** in **different thicknesses**.

Closed Loop Control Systems

The disadvantages of open-loop systems, namely **sensitivity** to **disturbances** and **inability** to **correct** for **these disturbances**, may be overcome in closed-loop systems.



The generic architecture of a closed-loop system is shown in Figure.

Closed Loop Control Systems

The input transducer converts the form of the input to the form used by the controller.

An **output transducer**, or **sensor**, **measures** the **output response** and **converts** it into the form used by the controller.

For example, if the controller uses **electrical signals** to operate the **regulators** of a temperature control system, the input **position** and the **output temperature** are converted to electrical signals.

The input **position** can be converted to a **voltage** by a **potentiometer**, a **variable resistor**, and the **output temperature** can be converted to a **voltage** by a **thermistor**, a device whose electrical resistance changes with temperature.

Closed Loop Control Systems

The **first summing** junction **algebraically** adds the **signal** from the **input** to the **signal** from the **output**, which arrives via the **feedback path**, the return path from the output to the summing junction.

In Figure, the **output** signal is **subtracted** from the **input signal**. The result is generally called the **actuating signal**.

However, in systems where both the input and output transducers have unity gain (that is, the transducer amplifies its input by 1), the **actuating signal's** value is **equal** to the **actual difference** between the **input** and the **output**. Under this condition, the **actuating signal** is called the **error**.

Closed Loop Control Systems

The closed-loop system **compensates** for **disturbances** by **measuring** the **output response**, **feeding** that **measurement** back through a **feedback path**, and **comparing** that **response** to the **input** at the **summing junction**.

If there is any difference between the two responses, the **system drives** the **plant**, via the **actuating signal**, to **make** a **correction**.

If there is no difference, the **system does not drive** the **plant**, since the **plant's response** is **already** the **desired response**.

Closed Loop Control Systems

Closed-loop systems have the obvious advantage of **greater accuracy** than open-loop systems.

They are less **sensitive** to **noise**, **disturbances**, and **changes** in the **environment**.

Transient response and **steady-state** error can be **controlled** more **conveniently** and **with greater flexibility** in closed-loop systems, often by a simple adjustment of gain (amplification) in the loop and sometimes by redesigning the controller.

We refer to the redesign as compensating the system and to the resulting hardware as a compensator.

Closed Loop Control Systems

Closed-loop systems are more **complex** and **expensive** than **open-loop** systems.

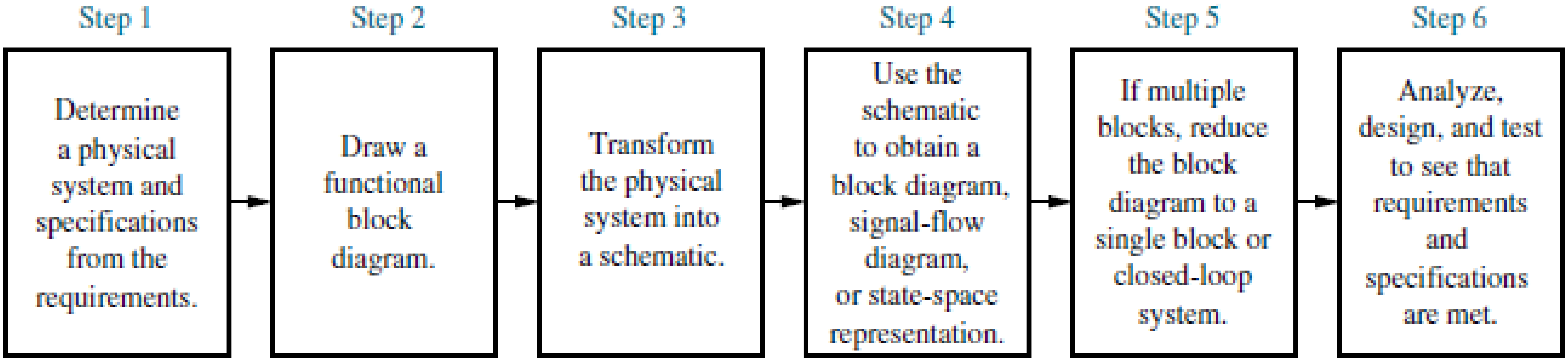
Consider the example of **open-loop** toaster: It is **simple** and **inexpensive**. A **closed-loop toaster** oven is more **complex** and more **expensive** since it has to **measure both color** (through **light reflectivity**) and **humidity inside** the **toaster oven**.

Thus, the control systems engineer must consider the trade-off between the simplicity and low cost of an open-loop system and the accuracy and higher cost of a closed-loop system.

In summary, systems that perform the previously described measurement and correction are called closed-loop, or feedback control, systems. Systems that do not have this property of measurement and correction are called open-loop systems

Design Procedure

Orderly sequence for the design of feedback control systems



Designed Procedure: Step 1

Step 1: **Transform Requirements Into a Physical System**

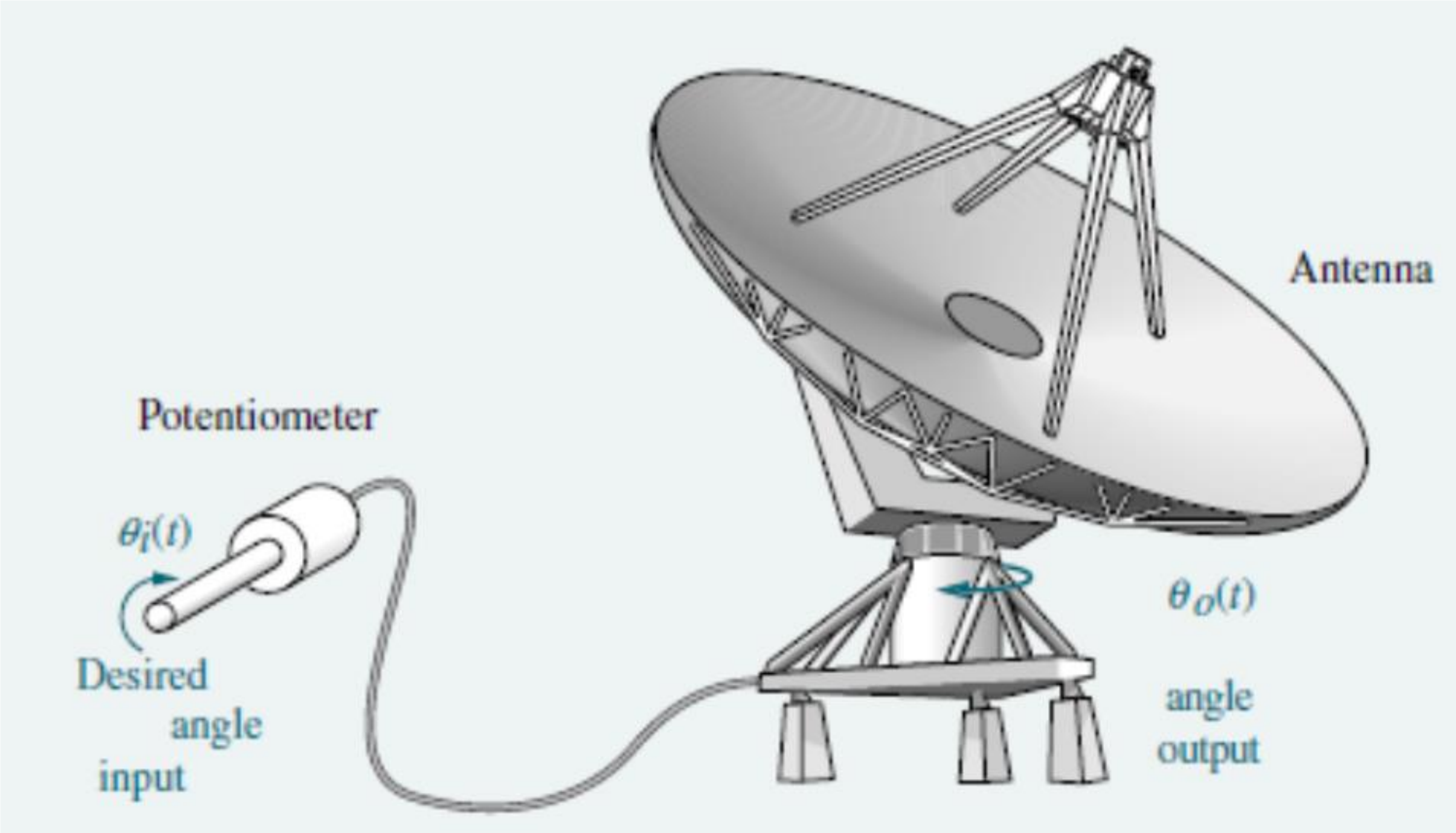
We begin by transforming the requirements into a physical system.

For example, in the antenna position control system, the requirements would state the desire to position the antenna from a remote location and describe such features as weight and physical dimensions.

Using the requirements, design specifications, such as desired transient response and steady-state accuracy, are determined.

Designed Procedure: Step 1

Step 1



Designed Procedure: Step 2

Step 2: **Draw a Functional Block Diagram**

The designer now **translates** a **qualitative** description of the system into a **functional block diagram** that describes the component parts of the system (that is, function and/or hardware) and shows their interconnection. Figure is an example of a functional block diagram for the antenna position control system. It indicates functions such as input transducer and controller, as well as possible hardware descriptions such as amplifiers and motors. At this point the designer may produce a detailed layout of the system, such as that shown in Figure, from which the next phase of the analysis and design sequence, developing a schematic diagram, can be launched

Designed Procedure: Step 2

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Designed Procedure: Step 2

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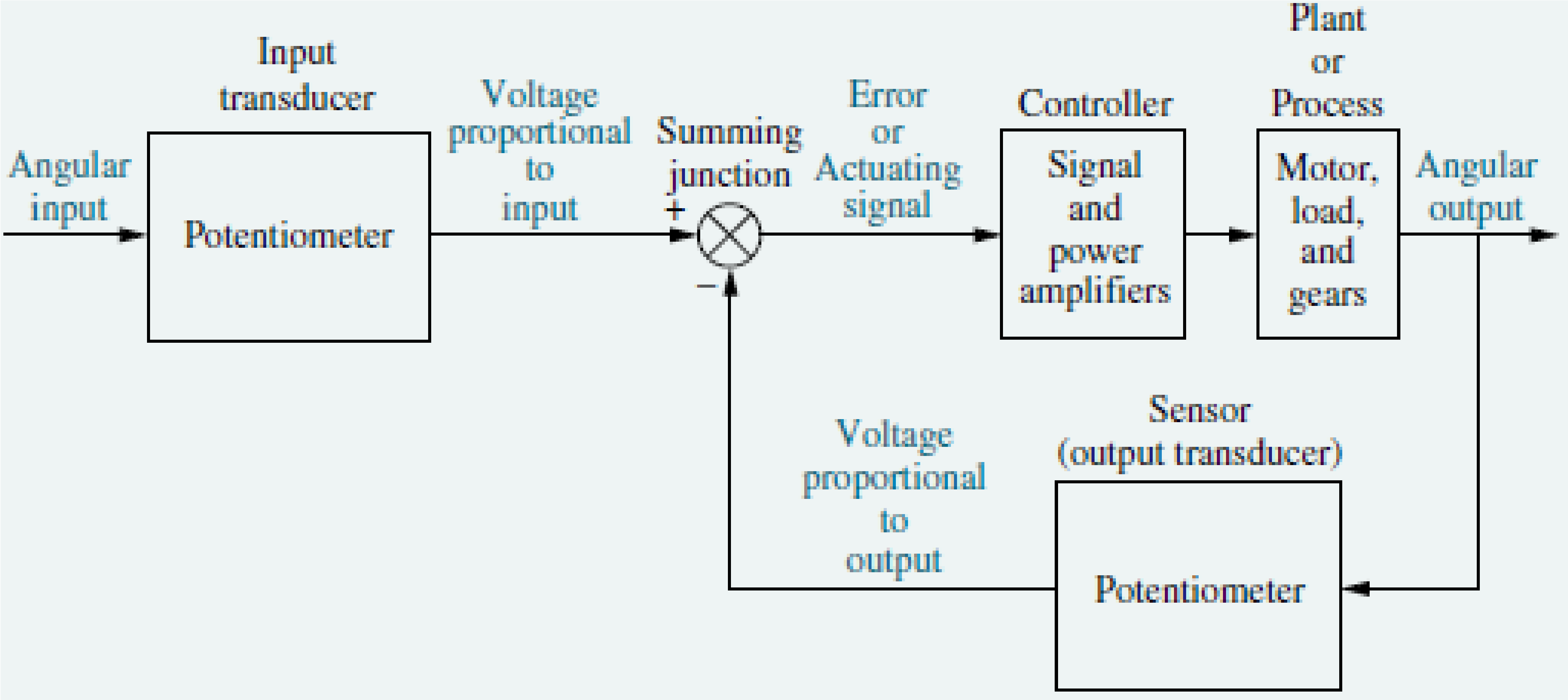

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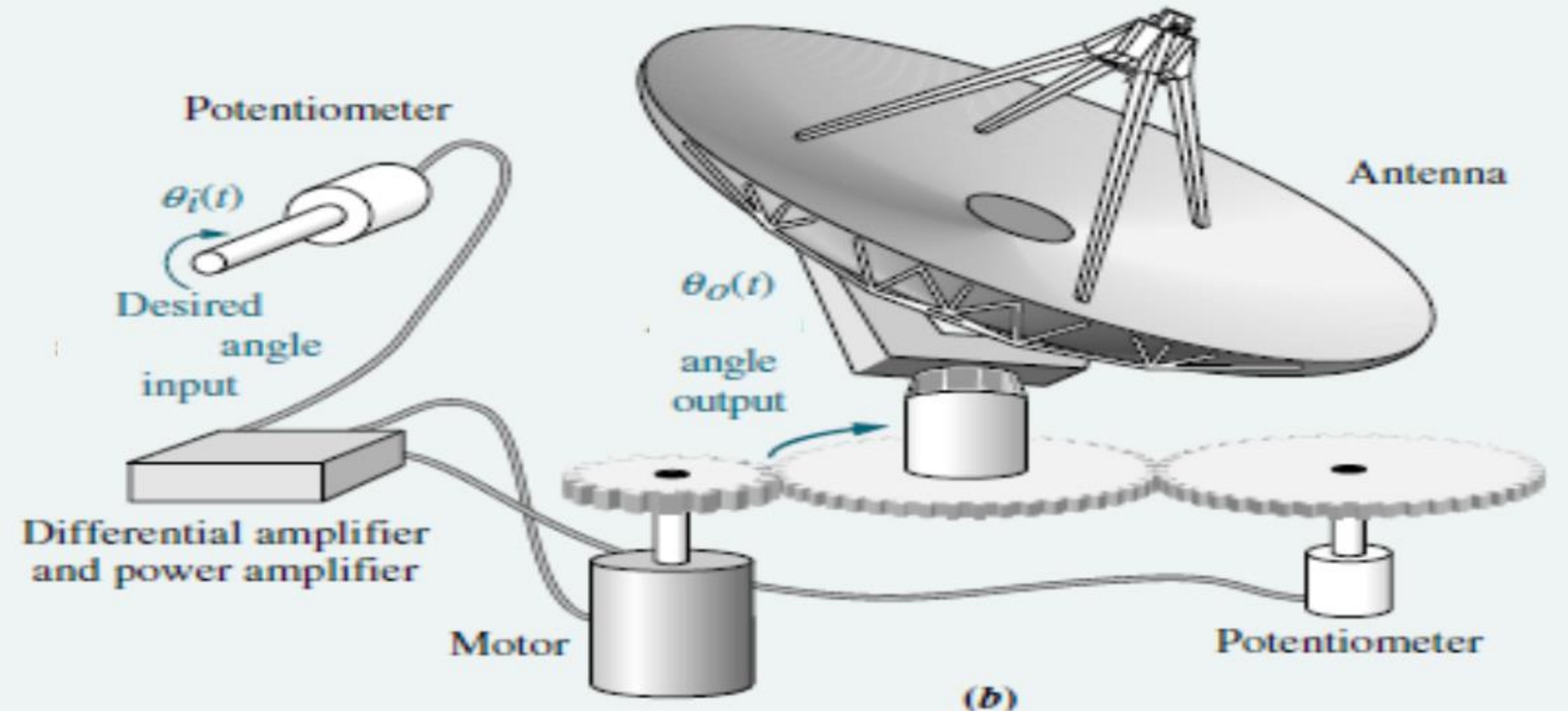


Designed Procedure: Step 3

Step 3: **Create a Schematic**

As we have seen, position control systems consist of **electrical**, **mechanical**, and **electromechanical** components.

After producing the description of a physical system, the control systems engineer transforms the physical system into a schematic diagram. The control system designer can begin with the physical description, to derive a schematic.

The engineer must make approximations about the system and neglect certain phenomena, or else the schematic will be unwieldy, making it difficult to extract a useful mathematical model during the next phase of the analysis and design sequence.

Designed Procedure: Step 3

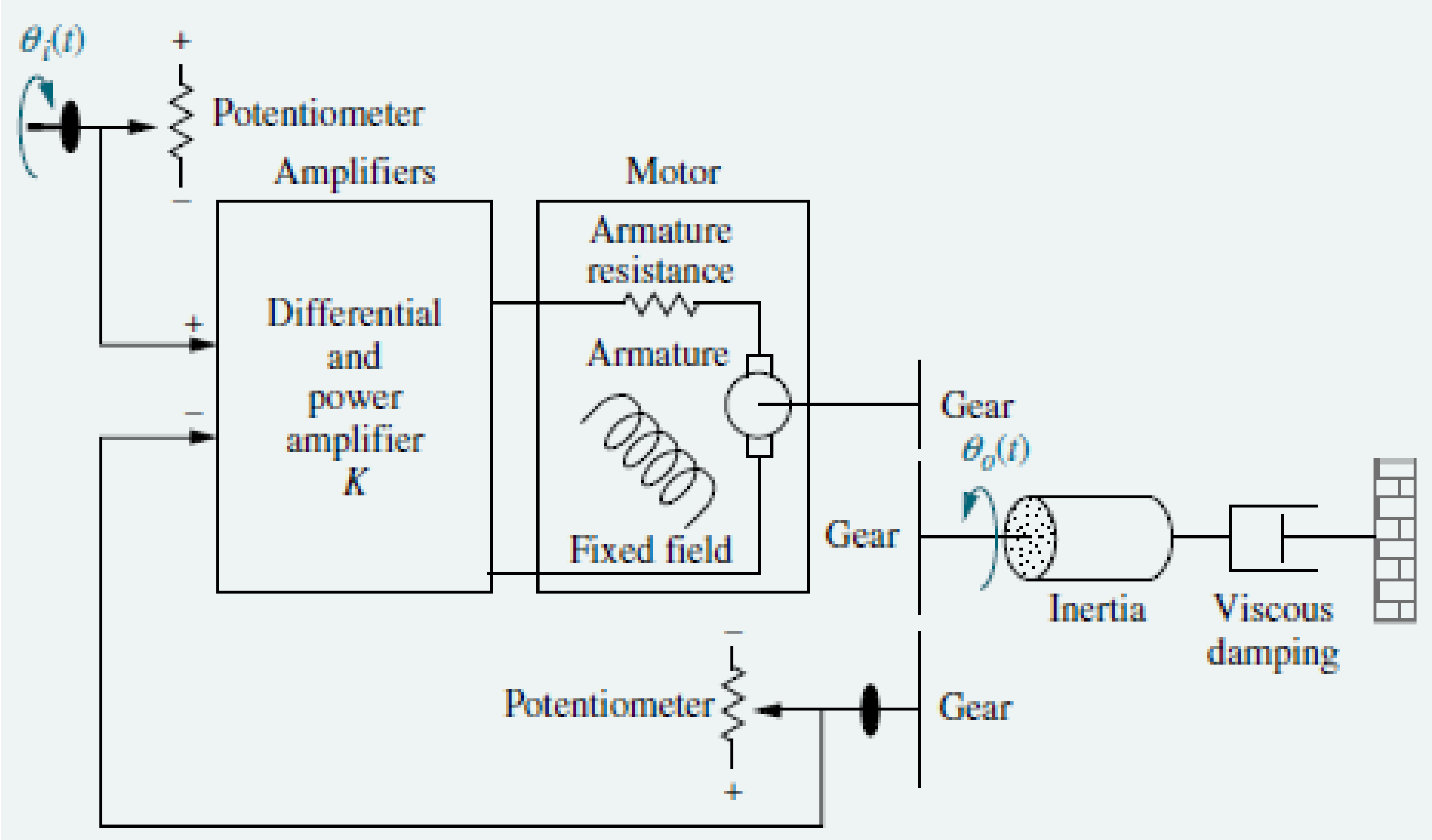
Step 3: Create a Schematic

The designer starts with a simple schematic representation and, at subsequent phases of the analysis and design sequence, checks the assumptions made about the physical system through analysis and computer simulation.

If the schematic is too simple and does not adequately account for observed behavior, the control systems engineer adds phenomena to the schematic that were previously assumed negligible.

A schematic diagram for the antenna position control system is shown in following figure.

Step 3: Create a Schematic



Designed Procedure: Step 3

Step 3: Create a Schematic

When we draw the potentiometers, we make our first simplifying assumption by neglecting their friction or inertia.

These mechanical characteristics yield a dynamic, rather than an instantaneous, response in the output voltage.

We assume that these mechanical effects are negligible and that the voltage across a potentiometer changes instantaneously as the potentiometer shaft turns.

A differential amplifier and a power amplifier are used as the controller to yield gain and power amplification, respectively, to drive the motor.

Designed Procedure: Step 3

Step 3: **Create a Schematic**

Again, we assume that the dynamics of the amplifiers are rapid compared to the response time of the motor; thus, we model them as a pure gain, K .

A dc motor and equivalent load produce the output angular displacement. The speed of the motor is proportional to the voltage applied to the motor’s armature circuit. Both inductance and resistance are part of the armature circuit.

We assume the effect of the armature inductance is negligible for a dc motor. The designer makes further assumptions about the load.

The load consists of a rotating mass and bearing friction. Thus, the model consists of inertia and viscous damping whose resistive torque increases with speed, as in an automobile’s shock absorber or a screen door damper.

Designed Procedure: Step 3

Step 3: Create a Schematic

The decisions made in developing the schematic stem from knowledge of the physical system, the physical laws governing the system's behavior, and practical experience.

Designed Procedure: Step 4

Step 4: **Develop a Mathematical Model (Block Diagram)**

Once the schematic is drawn, the designer uses physical laws, such as **Kirchhoff's** laws for **electrical** networks and **Newton's** law for **mechanical** systems, along with simplifying assumptions, to **model** the system **mathematically**. These laws are

Kirchhoff's voltage law: The sum of voltages around a closed path equals zero.

Kirchhoff's current law: The sum of electric currents flowing from a node equals zero.

Newton's laws: The sum of forces on a body equals zero; the sum of moments on a body equals zero.

Kirchhoff's and Newton's laws lead to mathematical models that describe the relationship between the input and output of dynamic systems. One such model is the linear, time-invariant differential equation,

Designed Procedure: Step 4

Step 4: **Develop a Mathematical Model (Block Diagram)**

$$a_n \frac{d^n c(t)}{dt^n} + a_{n-1} \frac{d^{n-1} c(t)}{dt^{n-1}} + \cdots + a_0 c(t) = b_m \frac{d^m r(t)}{dt^m} + b_{m-1} \frac{d^{m-1} r(t)}{dt^{m-1}} + \cdots + b_0 r(t)$$

Many systems can be approximately described by this equation, which relates the **output**, **c(t)**, to the **input**, **r(t)**, by way of the system parameters, $\mathbf{a_i}$ and $\mathbf{b_j}$.

In **addition** to the **differential** equation, the **transfer function** is another way of **mathematically modeling** a **system**.

The model is derived from the **linear**, **time-invariant differential** equation using what we call the **Laplace transform**. Although the **transfer function** can be used **only** for **linear systems**.

Designed Procedure: Step 4

Step 4: Develop a Mathematical Model (Block Diagram)

We will be able to change system parameters and rapidly sense the effect of these changes on the system response. The transfer function is also useful in modeling the interconnection of subsystems by forming a block diagram with a mathematical function inside each block.

Designed Procedure: Step 4

Step 4: Develop a Mathematical Model (Block Diagram)

Another model is the **state-space representation**.

One advantage of **state space** methods is that they can also be used for systems that **cannot be described** by **linear differential equations**.

Further, state-space methods are used to **model systems** for **simulation** on the **digital computer**.

Basically, this **representation turns** an **n^{th}** order **differential equation** into **n simultaneous first-order differential equations**.

Designed Procedure: Step 4

Step 4: **Develop a Mathematical Model (Block Diagram)**

Finally, we should mention that to produce the mathematical model for a system, we need to have **knowledge** of the **parameter values**, such as **equivalent resistance**, **inductance**, **mass**, and **damping**, which is often not easy to obtain.

Analysis, measurements, or specifications from vendors are sources that the control systems engineer may use to obtain the parameters.

Designed Procedure: Step 5

Step 5: **Reduce the Block Diagram**

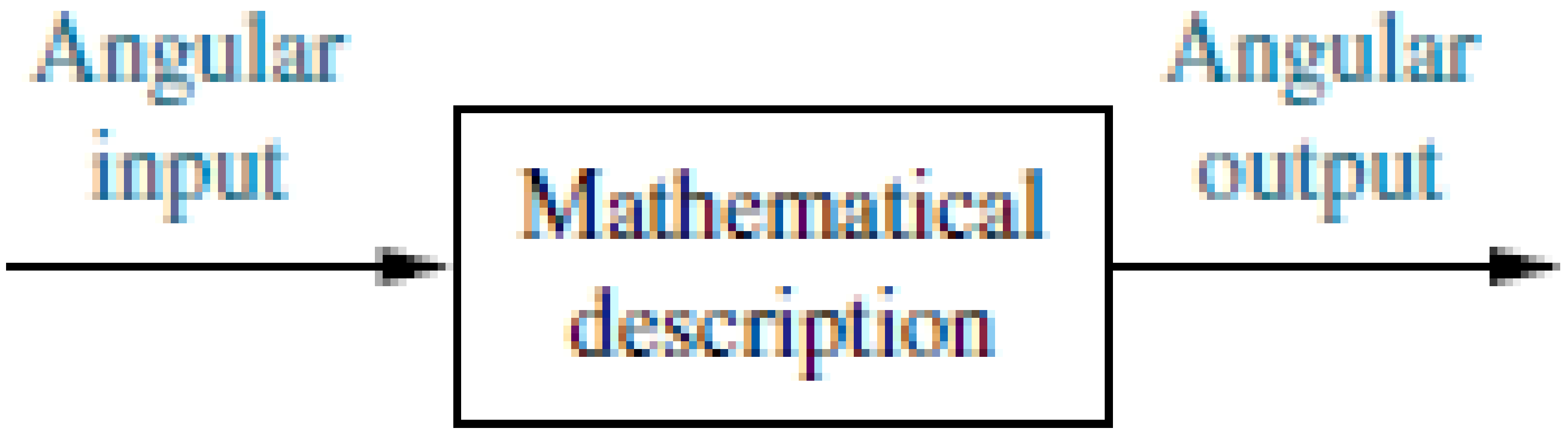
Subsystem models are interconnected to form block diagrams of larger systems where each block has a mathematical description.

Notice that many signals, such as proportional voltages and error, are internal to the system. There are also two signals – angular input and angular output – that are external to the system.

In order to evaluate system response in this example, we need to reduce this large system's block diagram to a single block with a mathematical description that represents the system from its input to its output, as shown in following figure. Once the block diagram is reduced, we are ready to analyze and design the system.

Designed Procedure: Step 5

Step 5: Reduce the Block Diagram



Designed Procedure: Step 6

Step 6: **Analyze and Design**

The next phase of the process, following block diagram reduction, is **analysis** and **design**.

If we are interested only in the performance of an **individual subsystem**, we can **skip** the **block diagram reduction** and **move** immediately into **analysis** and **design**.

In this phase, the engineer analyzes the system to see if the response specifications and performance requirements can be met by simple adjustments of system parameters.

If specifications cannot be met, the designer then designs additional hardware in order to get a desired performance.

Designed Procedure: Step 6

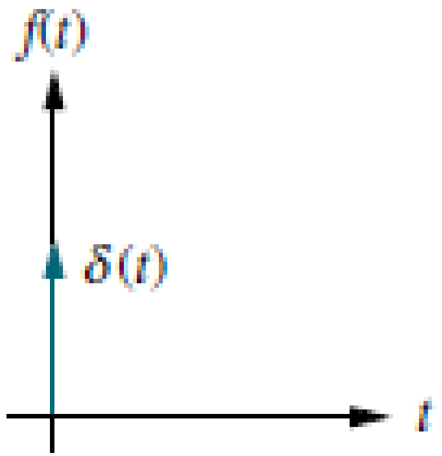
Step 6: **Analyze and Design**

Test input signals are used, both **analytically** and **during testing**, to **verify** the **design**.

It is **neither** necessarily **practical nor illuminating** to **choose complicated input signals** to **analyze** a **system's performance**.

Thus, the engineer usually selects standard test inputs.

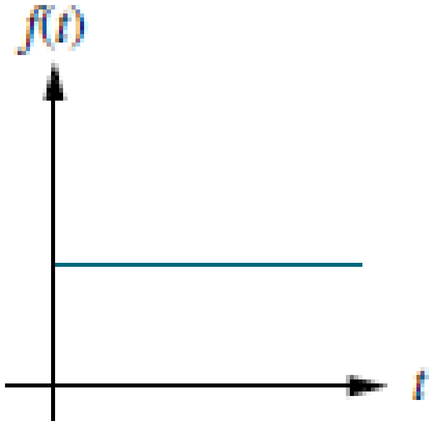
Designed Procedure: Step 6

I/P	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $-0 < t < +0$ $= 0$ elsewhere $\int_{-0}^{+0} \delta(t)dt = 1$		Transient response Modelling

An **impulse** is **infinite** at **t = 0** and **zero elsewhere**.

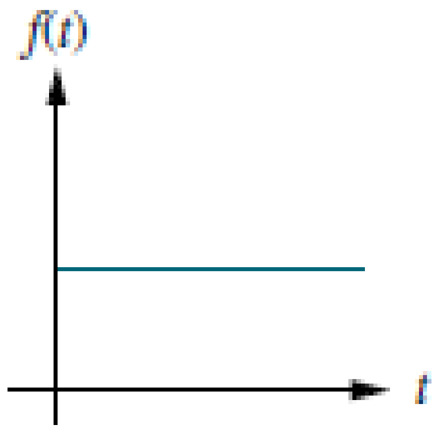
The **area** under the **unit impulse** is **1**. An approximation of this type of waveform is used to **place initial energy** into a **system** so that the **response** due to that **initial energy** is only the **transient response** of a system. From this response the designer can derive a mathematical model of the system.

Designed Procedure: Step 6

I/P	Function	Description	Sketch	Use
Step	u(t)	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error

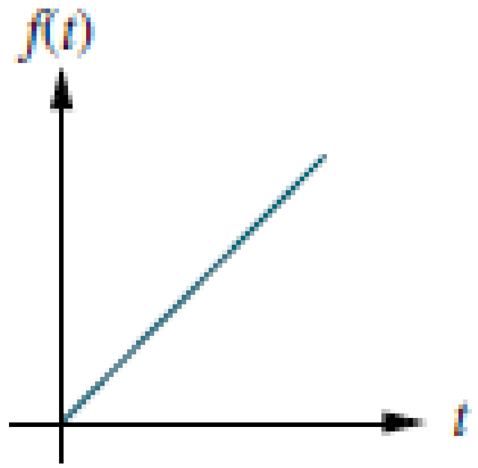
A step input represents a constant command, such as **position**, **velocity**, or **acceleration**. Typically, the step input command is of the same form as the output. For example, if the system's output is **position**, as it is for the antenna position control system, the **step input** represents a **desired position**, and the **output represents** the **actual position**.

Designed Procedure: Step 6

I/P	Function	Description	Sketch	Use
Step	u(t)	u(t) = 1 for t > 0 = 0 for t < 0		Transient response Steady-state error

If the system's output is velocity, as is the spindle speed for a video disc player, the step input represents a constant desired speed, and the output represents the actual speed. The designer uses step inputs because both the transient response and the steady-state response are clearly visible and can be evaluated.

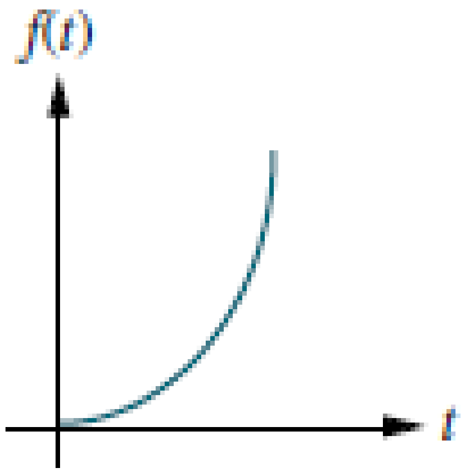
Designed Procedure: Step 6

I/P	Function	Description	Sketch	Use
Ramp	t u(t)	t u(t) = 1 for $t \geq 0$ = 0 otherwise		Steady-state error

The ramp input represents a linearly increasing command.

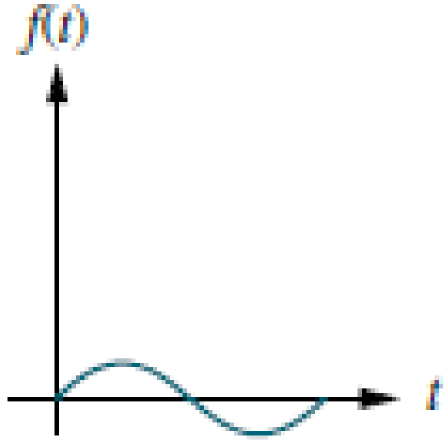
For example, if the system's output is position, the input ramp represents a linearly increasing position, such as that found when tracking a satellite moving across the sky at constant speed.

Designed Procedure: Step 6

I/P	Function	Description	Sketch	Use
Parabola	$\frac{1}{2} t^2 u(t)$	$\frac{1}{2} t^2 u(t) = \frac{1}{2} t^2$ for $t \geq 0$ $= 0$ otherwise	 	Steady-state error

If the system's output is velocity, the input ramp represents a linearly increasing velocity. The response to an input ramp test signal yields additional information about the steady-state error. The previous discussion can be extended to parabolic inputs, which are also used to evaluate a system's steady-state error.

Designed Procedure: Step 6

I/P	Function	Description	Sketch	Use
Sinusoid	sin ωt	sin ωt		Transient response Modeling Steady-state error

Sinusoidal inputs can also be used to test a physical system to arrive at a mathematical model.

Laplace Transform

A system represented by a differential equation is difficult to model as a block diagram. By using the Laplace transform, with which we can represent the input, output, and system as separate entities.

Further, their interrelationship will be simply algebraic.

Let us first define the Laplace transform and then show how it simplifies the representation of physical systems

The Laplace transform is defined a

$$L[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

Where $s = \sigma + j\omega$ is a complex variable.

Laplace Transform

The notation for the lower limit (**0_**) means that even if **f(t)** is **discontinuous** at **t = 0**, we can **start** the **integration prior** to the **discontinuity as long as** the **integral converges**. Thus, we can find the Laplace transform of impulse functions.

This **property** has distinct **advantages** when **applying** the **Laplace** transform to the **solution** of **differential equations** where the **initial conditions** are **discontinuous** at **t = 0**.

Using differential equations, we have to solve for the initial conditions after the discontinuity knowing the initial conditions before the discontinuity

Laplace Transform

The inverse Laplace transform is defined a

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds = f(t)u(t)$$

Where u(t) = 1; t > 0

= 0; t < 0

is the unit step function. T

Multiplication of **f(t)** by **u(t)** yields a **time function** that is **zero** for **t < 0**.

Laplace Transform Table

SN	f(t)	F(s)
1	$\delta(t)$	1
2	$u(t)$	$1/s$
3	$t\ u(t)$	$1/s^2$
4	$t^n\ u(t)$	$n!\ /(s^n + 1)$
5	$e^{-at}\ u(t)$	$1/(s+a)$
6	$\sin \omega t\ u(t)$	$\omega/(s^2 + \omega^2)$
7	$\cos \omega t\ u(t)$	$s/(s^2 + \omega^2)$

Laplace Transform Theorems

SN	Theorem	Name
1	$L[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2	$L[kf(t)] = kF(s)$	Homogeneity
3	$L[f_1(t) + f_1(t)] = F_1(s) + F_2(s)$	Additivity
4	$L[af_1(t) + bf_1(t)] = aF_1(s) + bF_2(s)$	Linearity
5	$L[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
6	$L[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
7	$L[f(at)] = \left(\frac{1}{a}\right)F(s/a)$	Scaling theorem
8	$L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0-)$	Differentiation theorems

Laplace Transform Theorems

SN	Theorem	Name
9	$L\left[\frac{d^2 f(t)}{dt^2}\right] = s^2 F(s) - sf(0-) - f'(0-)$	Differentiation theorems
10	$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0-)$	Differentiation theorems
11	$L\left[\int_{0-}^{t} f(\tau)d\tau\right] = F(s)/s$	Integration theorem
12	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem
13	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem

Laplace Transform

Find the Laplace transform of $f(t) = Ae^{-at}u(t)$.

Laplace Transform

Find the Laplace transform of $f(t) = Ae^{-at}u(t)$.

Since the time function does not contain an impulse function, we can replace the lower limit of 0- with 0.

Hence,

$$F(s) = \int_0^\infty f(t)e^{-st}dt = \int_0^\infty Ae^{-at}e^{-st}dt$$

$$= A\int_0^\infty e^{-(s+a)t}dt = -\frac{A}{s+a}e^{-(s+a)t}\Big|_0^\infty = \frac{A}{s+a}$$

Laplace Transform

Find the inverse Laplace transform of $F_1(s) = 1/(s+3)^2$.

Laplace Transform

Find the inverse Laplace transform of $F_1(s) = 1/(s+3)^2$.

We know the frequency shift theorem

$$L[e^{-at} f(t)] = F(s+a)$$

the Laplace transform of $f(t) = tu(t)$ is $1/s^2$,

If the inverse transform of $F(s) = 1/s^2$ is $tu(t)$,

the inverse transform of $F(s + a) = 1/(s + a)^2$ is $e^{-at}\, t\, u(t)$.

Hence, $f_1(t) = e^{-3t}\, t\, u(t)$

Laplace Transform: Partial-Fraction Expansion

To find the inverse Laplace transform of a complicated function,

Convert the function to a sum of simpler terms

The result is called a partial-fraction expansion.

If F1(s) = N(s)/D(s),

If the order of N(s) is less than the order of D(s), then a partial-fraction expansion can be made.

If the order of N(s) is greater than or equal to the order of D(s), then N(s) must be divided by D(s) successively until the result has a remainder whose numerator is of order less than its denominator.

Laplace Transform: Partial-Fraction Expansion

For example, if $F_1(s) = \frac{s^3+2s^2+6s+7}{s^2+s+5}$

we must perform the indicated division until we obtain a remainder whose numerator is of order less than its denominator. Hence $F_1(s) = s + 1 + \frac{2}{s^2+s+5}$

Taking the inverse Laplace transform, using the above Tables we obtain $f_1(t) = \frac{d\delta(t)}{dt} + \delta(t) + L^{-1}\left[\frac{2}{s^2+s+5}\right]$ Using partial-fraction expansion, we will be able to expand functions like $F(s) = \frac{2}{s^2+s+5}$ into a sum of terms and then find the inverse Laplace transform for each term

Case 1. Roots of the Denominator of F(s) Are Real and Distinct

Let $F(s) = \frac{2}{(s+1)(s+2)}$

The roots of the denominator are distinct, since each factor is raised only to unity power. We can write the partial-fraction expansion as a sum of terms where each factor of the original denominator forms the denominator of each term, and constants, called residues, form the numerators. Hence, $F(s) = \frac{2}{(s+1)(s+2)} = \frac{K_1}{(S+1)} + \frac{K_2}{(S+2)}$

To find K_1, we first multiply the equation by (s + 1), which isolates K_1. Thus,

$$\frac{2}{(s+1)(s+2)} = K_1 + \frac{K_2(s+1)}{(S+2)}$$

Let s → -1 eliminates the last term and gives K1 = 2. Similarly, K2 can be found by multiplying the equation by (s + 2) and then letting s → -2; hence, K2 = -2.

Case 1. Roots of the Denominator of F(s) Are Real and Distinct

Let $s \rightarrow -1$ eliminates the last term and gives $K_1 = 2$. Similarly,

K_2 can be found by multiplying the equation by $(s + 2)$ and then letting $s \rightarrow -2$; hence,

$K_2 = -2$.

Therefore, $F(s) = \frac{2}{s+1} - \frac{2}{s+2}$

By referring the tables above

$f(t) = (2e^{-t} - 2e^{-2t})\, u(t)$

Case 1. Roots of the Denominator of F(s) Are Real and Distinct

Given the following differential equation, solve for y(t) if all initial conditions are zero. Use the Laplace transform. $\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$

Substitute the corresponding Laplace transform for each term in above equation, using Table, and the initial conditions of $y(t)$ and $\frac{dy(t)}{dt}$ given by $y(0-) = 0$ and $\frac{dy(0-)}{dt} = 0$, respectively. Hence, the Laplace transform of equation is

$$s^2Y(s) + 12sY(s) + 32Y(s) = \frac{32}{s}$$

Solving for the response Y(s) we have $Y(s) = \frac{32}{s(s^2+12s+32)} = \frac{32}{s(s+4)(s+8)}$

Now y(t) is obtained by taking the inverse Laplace transform, for that we need to partial fraction method.

Case 1. Roots of the Denominator of F(s) Are Real and Distinct

Therefore, $Y(s) = \frac{32}{s(s+4)(s+8)} = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+8}$

$$K_1 = \left.\frac{32}{(s+4)(s+8)}\right|_{s\to 0} = 1$$

$$K_2 = \left.\frac{32}{s(s+8)}\right|_{s\to -4} = -2$$

$$K_3 = \left.\frac{32}{s(s+4)}\right|_{s\to -8} = 1$$

Case 1. Roots of the Denominator of F(s) Are Real and Distinct

Hence $Y(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+8}$

This is the simplest form, and we can easily find the LT of each term

$$y(t) = (1 - 2e^{-4t} + e^{-8t})u(t)$$

The u(t) shows that the response is zero until t = 0.

Thus, output responses will also be zero until t = 0. For convenience, we will leave off the u(t) notation. Accordingly, we write the output response as

$$y(t) = (1 - 2e^{-4t} + e^{-8t})$$

Case 2. Roots of the Denominator of F(s) Are Real and Repeated

Let $F(s) = \frac{2}{(s+1)(s+2)^2}$

The roots of $(s + 2)^2$ in the denominator are repeated, since the factor is raised to an integer power higher than 1. In this case, the denominator root at 2 is a multiple root of multiplicity 2

We can write the partial-fraction expansion as a sum of terms, where each factor of the denominator forms the denominator of each term. In addition, each multiple root generates additional terms consisting of denominator factors of reduced multiplicity. For example, if $F(s) = \frac{2}{(s+1)(s+2)^2} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)^2} + \frac{K_3}{(s+2)}$

Case 2. Roots of the Denominator of F(s) Are Real and Repeated

For K_1

$$\frac{2}{(s+2)^2} = K_1 + \frac{(s+1)K_2}{(s+2)^2} + \frac{(s+1)K_3}{(s+2)}, \ \therefore K_1{=}2$$

K_2 can be isolated by multiplying by $(s + 2)^2$, yielding

$\frac{2}{s+1} = (s+2)^2\frac{K_1}{(s+1)} + K_2 + (S+2)K_3$. Letting s → -2 ; K_2 = - 2.

To find K_3 we differentiate above equation with respect to s

$\frac{-2}{(s+1)^2} = \frac{(s+2)K_1}{(s+1)^2} + K_3$, K_3 is isolated and can be found by s → -2 Hence, K3 = -2

$$\therefore F(s) = \frac{2}{(s+1)(s+2)^2} = \frac{2}{(s+1)} - \frac{2}{(s+2)^2} - \frac{2}{(s+2)}$$

$$\therefore f(t) = 2e^{-t} - 2te^{-2t} - 2e^{-2t}$$

Case 3. Roots of the Denominator of F(s) Are Complex or Imaginary

Let $F(s) = \frac{3}{s(s^2+2s+5)}$ *this*

This function can be expanded in the following form

$$\frac{3}{s(s^2+2s+5)} = \frac{K_1}{s} + \frac{K_2 s + K_3}{(s^2+2s+5)}$$

K_1 is found in the usual way to be 3/5. K_2 and K_3 can be found by first multiplying the above equation by the lowest common denominator, $s(s^2 + 2s + 5)$, and clearing the fractions. After simplification with $K_1 = 3/5$, we obtain

$$3 = \left(K_2 + \frac{3}{5}\right)s^2 + \left(K_3 + \frac{6}{5}\right)s + 3$$

Case 3. Roots of the Denominator of F(s) Are Complex or Imaginary

Balancing coefficients, $(K_2 + 3/5) = 0$ and $(K_3 + 6/5) = 0$. Hence $K_2 = -3/5$ and $K_3 = -6/5$. Thus,

Let $F(s) = \frac{3}{s(s^2+2s+5)} = \frac{3/5}{s} - \frac{3}{5}\frac{s+2}{(s^2+2s+5)}$

The last term can be shown to be the sum of the Laplace transforms of an exponentially damped sine and cosine. Using $L[\sin(\omega t)\,u(t)] = \frac{\omega}{(s^2+\omega^2)}$; $L[\cos\omega t\,u(t)] = \frac{s}{(s^2+\omega^2)}$, $L[kf(t)] = kF(s)$; and $L[e^{-at}f(t)] = F(s+a)$

$L[\mathrm{A}e^{-at}\cos \omega t] = \frac{A(s+a)}{(s+a)^2+\omega^2}$ and $L[\mathrm{B}e^{-at}\sin \omega t] = \frac{B\omega}{(s+a)^2+\omega^2}$

If we add these two equations

Case 3. Roots of the Denominator of F(s) Are Complex or Imaginary

If we add these two equations

$$L[\mathrm{A}e^{-at}\cos \omega t + \mathrm{B}e^{-at}\sin \omega t] = \frac{A(s+a)+B\omega}{(s+a)^2+\omega^2}$$

Now rearrange $F(s) = \frac{3}{s(s^2+2s+5)} = \frac{3/5}{s} - \frac{3}{5}\frac{s+2}{(s^2+2s+5)}$ by completing the squares in the denominator and adjusting terms in the numerator without changing its value

$$F(s) = \frac{3/5}{s} - \frac{3}{5}\frac{s+1+(1/2)(2)}{(s+1)^2+2^2}$$

$$\therefore f(t) = \frac{3}{5} - \frac{3}{5}e^{-t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)$$

Case 3. Roots of the Denominator of F(s) Are Complex or Imaginary

The alternative method is

$$F(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{3}{s(s + 1 + j2)(s + 1 - j2)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s + 1 + j2} + \frac{k_3}{s + 1 - j2}$$

$$K_2 = \left.\frac{3}{s(s + 1 - j2)}\right|_{s\to-1-j2} = -\frac{3}{20}(2 + j1)$$

We know k1 = 3/5; and K3 is complex conjugate of K2.

Case 3. Roots of the Denominator of F(s) Are Complex or Imaginary

Hence

$$F(s) = \frac{3/5}{s} - \frac{3}{20}\left(\frac{2+j1}{S+1+j2} + \frac{2-j1}{S+1-j2}\right)$$

$$\therefore f(t) = \frac{3}{5} - \frac{3}{20}e^{-t}\left((2+j1)e^{-(1+j2)t} + (2-j1)e^{-(1-j2)t}\right)$$

$$= \frac{3}{5} - \frac{3}{20}e^{-t}\left[4\left(\frac{e^{j2t}+e^{-j2t}}{2}\right) + 2\left(\frac{e^{j2t}+e^{-j2t}}{2j}\right)\right]$$

$$= \frac{3}{5} - \frac{3}{5}e^{-t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)$$