COLLEGE OF ENGINEERING, PUNE

(An Autonomous Institute of Government of Maharashtra.)

END Semester Examination

Subject Code : CT 16006 **Subject Name : Discrete Structures and Graph Theory**

Course : S.Y. B.Tech **Semester : III**

Branch : Computer Engineering **Academic Year : 2019-20**

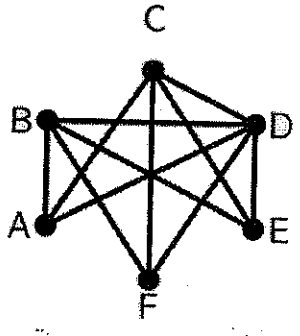
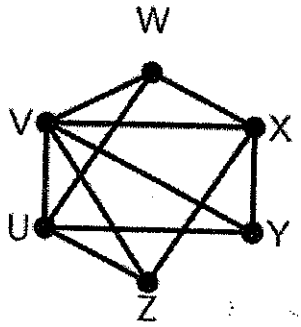
Date :30th Nov 2019 **Max Marks : 60** **Duration : 3 Hours**

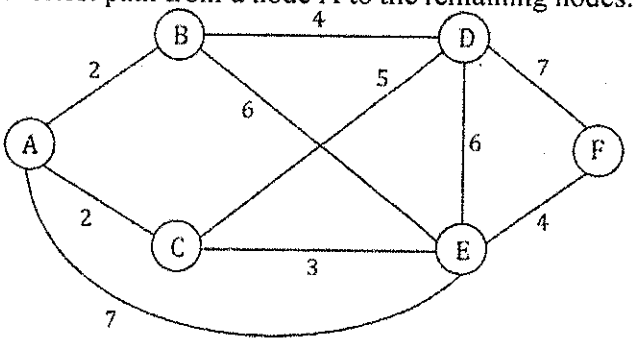
Instructions **Student MIS No.** | | | | | | | | | |

1. Figures to the right indicate the full marks.
2. Mobile phones and programmable calculators are strictly prohibited.
3. Writing anything on question paper is not allowed.
4. Exchange/Sharing of stationery, calculator etc. not allowed.
5. Write your MIS Number on Question Paper
6. Assume any suitable data if needed.

Section A

		Marks	CO	PO
Q 1.				
A)	Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\neg Q(x) \vee S(x))$, $\forall x(R(x) \rightarrow \neg S(x))$, and $\exists x \neg P(x)$ are true, then $\exists x \neg R(x)$ is true.	**2**	1,2	4
B)	A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the B is telling the truth then so is the C; the C and the G cannot both be telling the truth; the G and the H are not both lying; and if the H is telling the truth then the C is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain your reasoning.	**3**	1,2	4
C)	Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives). **a)** $\neg \forall y \forall x(P(x, y) \vee Q(x, y))$ **b)** $\neg(\exists x \exists y \neg P(x, y) \wedge \forall x \forall y Q(x, y))$ **c)** $\neg \forall x(\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$	**3**	1,2	4
Q 2.				
A)	Use the extended Euclidean algorithm to express gcd(26, 91) as a linear combination of 26 and 91.	**4**		4
B)	Use the method of back substitution to find all integers x such that $x \equiv 1 \pmod 5$, $x \equiv 2 \pmod 6$, and $x \equiv 3 \pmod 7$.	**4**		4

C)	Consider the graphs given below:  G1  G2 Determine if two Graphs are Isomorphic. What is a reason(s) for why these graphs could be isomorphic? Or What is a reason(s) for why these graphs could NOT be isomorphic?	2	4	4
Q 3. A)	i. Let A and B are two multisets defined as $A = \{2 \bullet a, 3 \bullet b, 1 \bullet c\}$ and $A = \{1 \bullet a, 4 \bullet b, 1 \bullet d\}$ respectively, then , determine a) $A \cup B$ b) $A \cap B$ c) $A - B$ d) $B - A$	2	1	7
	ii. How many positive integers not exceeding 100 are divisible either by 4 or by 6?	2		
B)	Solve the difference recurrence equation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ satisfying the conditions $a_0 = 0, a_1 = 6$.	3	1,3	5
	OR			
B)	If a relation R={(1,2), (2,3), (3,4), (2,1)} on set A={1,2,3,4} Find transitive closure of R using Warshall's Algorithm.	3	1	5,7
C)	Let $f: Z \to Z$ be a function defined as $f(x) = x + 5$. Determine whether the function is invertible or not. If it is invertible then find its inverse.	2	1	5
D)	If a relation on A={1,2,3,4,5} is defined as matrix $M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ Check that the relation is a POSET or not? If yes draw a hasse diagram?	3	1,5	
	Section B			
Q 4.				
A)	In how many ways a garland maker can stich the flowers to make a garland with 4 red and 10 white roses all with different sizes, so that a) No two red roses come together. b) All red roses come together.	2 2	5	5,7

B)	i. If a given binomial is $(2x + 3y)^{15}$. Find a) Last term. b) Coefficient of term x^7y^8. c) Draw a pascal's triangle of coefficients for first 5 powers for $(2x + 3y)$	**2** **2** **2**	5	5,7
	ii. If a shopkeeper has 50 bicycles in his shop. A shopkeeper wanted to colour them. If he used 7 colours to colour those bicycles. Calculate the minimum number of same coloured bicycles.	**2**	5	5,7
Q 5.				
A)	How many vertices and how many edges do these graphs have? For which values of ***n*** are following graph bipartite? a) K_n b) C_n c) $K_{m,n}$	**3**	4	7
B)	Prove that for any connected planar graph G=(V,E) with e≥3, v−e+r=2, where v=|V|, e=|E|, and is the number of regions in the graph by using Mathematical Induction.	**3**	4	7
C)	Implement Dijkstra's algorithm to the following graph and find the shortest path from a node A to the remaining nodes.  	**4**	4	5,7
Q 6.				
A)	Complete the following operation table to obtain a semi group. | * | a | b | c | | a | c | a | b | | b | a | b | _ | | c | _ | _ | _ |	**2**	5	8
B)	If S= {1,2,3,6,12}, where $a * b$ is defined as GCD(a, b). a) Construct an operation table b) Is it a semigroup? if yes Determine it is commutative. c) Is it algebraic structure a monoid? if yes specify identity element.	**2** **2** **2**	5	8
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