Tutorial - 3.

Q.4. Find the partial derivative of following.

1. $f(x,y) = (xy-1)^2$

$$\frac{df}{dx} = \frac{d}{dx}(xy-1)^2$$

$$= \frac{d(xy)^2 - 2xy + 1}{dx}$$

$$\frac{df}{dx} = x\cancel{y^2}\,2xy^2 - 2y$$

$$\frac{df}{dy} = \frac{d}{dy}(x^2y^2 - 2xy + 1).$$

$$\frac{df}{dy} = 2yx^2 - 2x$$

2. $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right).$

$$\frac{df}{dx} = \frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right).$$

$$= \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot y \cdot \frac{-1}{x^2}.$$

$$= -\frac{y}{1+\frac{y^2}{x^2} \cdot x^2}$$

$$= \frac{-y}{\frac{x^2+y^2}{x^2}} \cdot \frac{1}{x^2}$$

$$\frac{df}{dx} = \frac{-y}{x^2+y^2}$$

$$\frac{df}{dy} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}$$

$$= \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x}$$

$$= \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{1}{x}$$

$$= \frac{x^2}{x^2+y^2} \cdot \frac{1}{x}$$

$$\frac{df}{dy} = \frac{x}{x^2+y^2}$$

3. $f(x \cdot y) = e^{-x}(\sin(x+y)$.

$$\frac{df}{dx} = e^{-x} \cdot \cos(x+y) + (\sin x+y) - (e^{-x})$$

$$= e^{-x}(\cos x+y - (\sin x+y)).$$

$$\frac{df}{dy} = e^{-x} \cdot \cos x(x+y) \cancel{+ \sin x+y}.$$

$$= e^{-x} \cdot \cos x(x+y)$$

4. $f(x,y) = \log(x+y)$.

$$\frac{df}{dx} = \frac{1}{x+y}.$$

$$\frac{df}{dy} = \frac{1}{x+y}.$$

5. $f(x,y) = e^{xy} \cdot \log y$

$$\frac{df}{dx} = \log y \cdot e^{xy} \cdot (+y).$$

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$$\frac{df}{dx} = e^{xy} \cdot \log y \, (y).$$

$$\frac{df}{dy} = e^{xy} \cdot \frac{1}{y} + \log y \cdot e^{xy} \cdot x$$

$$\frac{df}{dy} = \frac{e^{xy}}{y} + \log y \cdot x \cdot e^{xy}$$

6. $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$\frac{df}{dx} = \frac{1}{2\sqrt{x^2+y^2+z^2}} \cdot 2x$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}.$$

$$\frac{df}{dy} = \frac{1}{2\sqrt{x^2+y^2+z^2}} \cdot 2y$$

$$= \frac{y}{\sqrt{x^2+y^2+z^2}}.$$

$$\frac{df}{dz} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

7. $f(x,y,z) = \sin^{-1}(x,y,z).$

$$\frac{df}{dx} = \frac{1}{\sqrt{1-(xyz)^2}} \cdot yz$$

$$\frac{df}{dx} = \frac{yz}{\sqrt{1-xyz^2}}$$

$$\frac{df}{dy} = \frac{xz}{\sqrt{1-xyz^2}}$$

$$\frac{df}{dz} = \frac{xy}{\sqrt{1-xyz^2}}$$

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8. $f(x,y,z) = e^{-(x^2+y^2+z^2)}$

$\frac{df}{dx} = e^{-(x^2+y^2+z^2)} \cdot -2x$

$\frac{df}{dy} = e^{-(x^2+y^2+z^2)} \cdot -2y.$

$\frac{df}{dz} = e^{-(x^2+y^2+z^2)} \cdot -2z$

9. $f(x,y,z) = e^{-xyz}$.

$\frac{df}{dx} = e^{-xyz} \cdot -yz$

$\frac{df}{dx} = e^{-xyz} \cdot -xz.$

$\frac{df}{dz} = e^{-xyz} \cdot -xy.$

10. $g(u.v) = \cancel{u^2} v^2 \cdot e^{2u/v}$

$\frac{df}{du} = v^2 \cdot e^{2u/v} \cdot \frac{1}{v} \cdot 2$

$= v \cdot e^{2u/v} \cdot 2.$

$\frac{df}{dv} = v^2 \cdot e^{2u/v} \cdot \frac{2}{v} + e^{2u/v} \cdot 2v.$

$= v \cdot e^{2u/v} \cdot 2 + 2v \cdot e^{2u/v}.$

$= 2ve^{2u/v} + 2ve^{2u/v}$

$\frac{df}{dv} = 4ve^{2u/v}$

11. $h(p,\phi,\theta) = p \sin\phi \cos\theta$

$\frac{df}{dp} = \sin\phi \cos\theta$

$$\frac{df}{d\phi} = p \cdot \cos\theta \cdot \cos\phi.$$

$$\frac{df}{d\theta} = p \cdot \sin\phi \cdot -\sin\theta$$

11. $f(t, \alpha) = \cos(2\pi t - \alpha).$

$$\frac{df}{dt} = -\sin(2\pi t - \alpha) \cdot 2\pi.$$

$$\frac{df}{dt} = -2\pi \sin(2\pi t - \alpha).$$

$$\frac{df}{d\alpha} = -\sin(2\pi t - \alpha) - 1$$

$$\frac{df}{dt} = \sin(2\pi t - \alpha).$$

12. $g(r, \theta, z) = r(1 - \cos\theta) - z$

$$\frac{df}{dr} = (1 - \cos\theta).$$

$$\frac{df}{d\theta} = +\sin\theta \, r.$$

$$\frac{df}{dz} = -1.$$

Q.5. Find second order derivative

1. $f(x, y) = x + y + xy$

$\frac{df}{dx} = 1 + y.$	$\frac{df}{dy} = 1 + x$
$\frac{d^2f}{dx^2} = 0$	$\frac{d^2f}{dy^2} = 0.$

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2. $f(x,y) = \sin(x,y)$.

$\frac{df}{dx} = \cos xy \cdot y$ | $\frac{df}{dy} = \cos xy \cdot x$.

$\frac{d^2f}{dx^2} = -y^2 \sin xy$ | $\frac{d^2f}{dy^2} = -x^2 \sin xy$

3. $h(x,y) = xe^y + y + 1$

$\frac{dh}{dx} = e^y$ | $\frac{dh}{dy} = x \cdot e^y + 1$.

$\frac{d^2h}{dx^2} = 0$ | $\frac{d^2h}{d^2y} = x \cdot e^y$.

4. $S(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$

$\frac{ds}{dx} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{y}{-x^2}$

$= \frac{-y}{\frac{1+y^2}{x^2}} \cdot \frac{1}{x^2}$

$= \frac{-y}{\frac{x^2+y^2}{\cancel{x^2}}} \cdot \frac{1}{\cancel{x^2}}$

$\frac{ds}{dx} = \frac{-y}{x^2+y^2}$

$\frac{d^2s}{dx^2} = -y \cdot \frac{-1}{(x^2+y^2)^2} \cdot 2x$

$\frac{d^2s}{dx^2} = \frac{2xy}{(x^2+y^2)^2}$

$\frac{ds}{dy} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}$

$= \frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{1}{x}$

$\frac{ds}{dy} = \frac{x}{x^2+y^2}$

$\frac{d^2s}{dy^2} = x \cdot \frac{-1}{(x^2+y^2)^2} \cdot 2y$

$\frac{d^2s}{dy^2} = \frac{-2xy}{(x^2+y^2)^2}$

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5. $r(x, y) = \log(x+y)$

$\frac{dr}{dx} = \frac{1}{(x+y)}$

$\frac{d^2r}{dx^2} = \frac{-1}{(x+y)^2} \cdot 1$

$\frac{d^2r}{dx^2} = \frac{-1}{(x+y)^2}$

$\frac{dr}{dy} = \frac{1}{(x+y)}$

$\frac{d^2r}{dx^2} = \frac{-1}{(x+y^2)}$

$\frac{d^2r}{dx^2} = \frac{-1}{(x+y)^2}$

Q.6. Verify that $f_{xy} = f_{yx}$.

(1). $f(x,y) = e^x + x\log y + y\log x$

$f_x = e^x + \log y + \frac{y}{x}$

$f_{xy} = \frac{1}{y} + \frac{1}{x} = \frac{x+y}{xy}$.

$f_y = x \cdot \frac{1}{y} + y \frac{\log x}{x}$

$= \frac{x}{y} + \frac{\log x}{x}$

$f_{yx} = \frac{1}{y} + \frac{1}{x} = \frac{x+y}{xy}$. $\therefore f_{xy} = f_{yx}$

(2). $f(x,y) = xy^2 + x^2y^3 + x^3y^4$.

$f_x = y^2 + 2x \cdot y^3 + 3x^2 \cdot y^4$

$f_{xy} = 2y + 2x \cdot 3y^2 + 3x^2 \cdot 4y^3$.

$f_{xy} = 2y + 6xy^2 + 12x^2y^3$

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$fy = x \cdot 2y + x^2 \cdot 3y^2 + x^3 \cdot 4y^3$

$fyx = 2y + 2x \cdot 3y^2 + 3x^2 \cdot 4y^3$
$fyx = 2y + 6xy^2 + 12x^2y^3$

$f_{xy} = fyx$

Q.11. Find derivate $\frac{dw}{dt}$ using chain Rule.

1. $w = x^2 + y^2$. $x = \cos t$ $y = \sin t$ $t = \pi$

$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt}$.

$= 2x \cdot (-\sin t) + 2y \cos t$.

$\frac{dw}{dt} = -2x \sin t + 2y \cos t$.

$= -2(\cos t) \cdot \sin t + 2y \sin t \cdot \cos t$

$= -2 \cos\pi \cdot \sin\pi \cdot + 2 \sin\pi \cdot \cos\pi$

$= -2 \cdot 1 \cdot (0) + 2(0)$

$\frac{dw}{dt} = 0$.

2. $w = x^2 + y^2$, $x = \cos t + \sin t$. $y = \cos t - \sin t$ $t=$

$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{dt}$

$= 2x \cdot (-\sin t) + (\cos t) + 2y \cdot (-\sin t) - (\cos t)$.

$= -2x \sin t + \cos t \cancel{+} - 2y \sin t - \cos t$.

$= -2(\cos t + \sin t) \cdot \sin t + \cos t - 2(\cos t - \sin t) \cdot \sin t - \cos t$

$= -2(1+0) \cdot 0 + 1 - 2(1-0) \cdot 0 - 1$

$= -2 \cdot (1) \cdot -2(1) \cdot -1$.

$= -2 + 2 = 0$.

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3. $\omega = \frac{x}{z} + \frac{y}{z}$ $x = \cos^2 t$ $y = \sin^2 t$ $z = 1/t$ $t = 3$

$$\frac{d\omega}{dt} = \frac{d\omega}{\partial x} + \frac{\partial x}{\partial t} + \frac{\partial \omega}{\partial y} - \frac{dy}{dt} + \frac{d\omega}{\partial z} \frac{\partial z}{\partial t}$$

$$= \frac{1}{z}(-2\sin t \cdot \cos t) + \frac{1}{2}(2\sin t \cdot \cos t) +$$

$$\frac{-1}{z^2} x + \frac{-1}{z^2} \cdot y \cdot \frac{-1}{t^2}$$

$$= \frac{-1}{z}\cancel{(2\sin t \cdot \cos t)} + \frac{1}{2}\cancel{(2\sin t \cdot \cos t)} + \cancel{\frac{-x}{z^2}}$$

$$+ \frac{-x-y}{z^2} \cdot \frac{-1}{t^2}$$

$$= \frac{-x-y}{z^2} \cdot \frac{+1}{t^2}$$

$$= \frac{+(\sin^2 t + \cos^2 t)}{(1/t)^2} \cdot \frac{1}{3^2}$$

$$= \frac{1}{(1/3)^2} \cdot \frac{1}{3^2}$$

$$= \frac{1}{1/9} \cdot \frac{1}{9}$$

$$\frac{d\omega}{dt} = 1$$

4. $\omega = 2ye^x - \log z$ $x = \log(t^2+1)$. $y = \tan^{-1}(t)$.
$z = e^t$. $t = 1$.

$$\frac{d\omega}{dt} = \frac{d\omega}{dx} \cdot \frac{\partial x}{dt} + \frac{d\omega}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial \omega}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$$= 2y \cdot e^x \frac{2t}{\cancel{\log}(t^2+1)} + 2 \cdot e^x \cdot \frac{1}{1+t^2} + \frac{-1}{z} \cdot e^t$$

$$= 2 \cdot \tan^{-1}(1) \cdot (2) \cdot \frac{2 \times 1}{\cancel{\log}(2)} + 2(2) \cdot \frac{1}{1+1} - \frac{1}{\cancel{e^t}} \cdot \cancel{e^t}$$

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$= \frac{\cancel{2}\times\pi}{\cancel{4}} \cdot \frac{\cancel{4}}{\log(2)} + \frac{4^2-1}{2}$

$= \frac{2\pi + 2 - 1}{\pi + 2 - 1}$

$= \frac{2\pi + 1}{\pi + 1}$

Q:12. Find partial derivative $\frac{d}{du}$, $\frac{d}{dv}$ as function of u and v.

1. $z = 4e^x \log y$ $x = \log(u\cos v)$. $y = u\sin v$

$(u, v) = (2, \pi/4)$

$\frac{dz}{du} = \frac{dz}{dx} \cdot \frac{dx}{du} + \frac{dz}{dy} \cdot \frac{dy}{du}$

$= 4 \cdot e^x \log y \cdot \frac{1}{u\cos v} \cdot (\cos v) + 4 \cdot e^x \frac{1}{y} \cdot \sin v$

$= 4 \cdot e^x \log y \cdot \frac{\cancel{\cos v}}{u \cdot \cancel{\cos v}} + 4 \cdot e^x \cdot \frac{1}{y} \cdot \sin v$

$= 4 \cdot e^x \log y \cdot \frac{1}{u} + 4e^x \frac{1}{y} \cdot \sin v$

$= 4e^x \left(\frac{\log y}{u} + \frac{\sin v}{y} \right)$

$= 4 \cdot (u\cos v) \left(\frac{\log(u\sin v)}{u} + \frac{\sin \pi/4}{u(\sin v)} \right)$

$= 4 (2/\cos \pi/4) \left(\frac{\log(2\sin \pi/4)}{2} + \frac{\cancel{\sin \pi/4}}{2 \cdot \cancel{\sin \pi/4}} \right)$

$= 4\left(2 \cdot \frac{1}{\sqrt{2}}\right) \left(\frac{\log(2 \times 1/\sqrt{2})}{2} \right) + \frac{1}{2}$

$= \frac{8}{\sqrt{2}} \left(\frac{\log \sqrt{2}}{2} + \frac{1}{2} \right)$

$= \frac{8}{\sqrt{2}} \left(\frac{\log \sqrt{2}}{2} + \frac{1}{2} \right)$

$$= \frac{4}{\cancel{2}} \times \cancel{2}\frac{1}{\sqrt{2}} (\log \sqrt{2} + 1).$$

$$\frac{dz}{du} = 2\sqrt{2} (\log \sqrt{2} + 1).$$

$$\frac{dz}{dv} = \frac{dz}{dx} \cdot \frac{dx}{dv} + \frac{dz}{dy} \cdot \frac{dy}{dv}.$$

$$= 4 \cdot e^{x} \log y \cdot \frac{1}{u\cos v} (-u \sin v) + 4e^{x} \frac{1}{y} u \cos v.$$

$$= 4 \cdot e^{x} \log y \cdot \frac{\cancel{u}(-\sin v)}{\cancel{u}(\cos v)} + 4e^{x} \cdot \frac{u\cos v}{y}.$$

$$= 4u\cos v \log (u \sin v) \cdot (-\tan v) + 4\cancel{e^{x}} u\cos v \cdot \frac{u\cos v}{y}$$

$$= 4u\cos v \left(\log(u\sin v) (-\tan v) + \frac{u\cos v}{y} \right)$$

$$= 4\left(2\cos\frac{\pi}{4}\right)\left(\log\left(2\sin\frac{\pi}{4}\right) - \tan\frac{\pi}{4} + \frac{\cancel{2}\cos \pi/4}{\cancel{2}\sin \pi/4} \right)$$

$$= 4\left(2 \cdot \frac{1}{\sqrt{2}}\right)\left(\log\left(2 \cdot \frac{1}{\sqrt{2}}\right) \cdot (-1) + 1\right).$$

$$= 4\sqrt{2}\left(-\log\sqrt{2} + 1\right).$$

$$\frac{dz}{dv} = 4\sqrt{2} (1 - \log\sqrt{2}).$$

2. $Z = \tan^{-1}\left(\frac{x}{y}\right)$ $\quad x = u\cos v \quad y = u\sin v.$

$(u, v) = (1 \cdot 3 \cdot \pi/6).$

$$\frac{dz}{du} = \frac{dz}{dx} \cdot \frac{dx}{du} + \frac{dz}{dy} \cdot \frac{dy}{du}.$$

$$= \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} (\cos v) + \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{x}{-y^2} \cdot (\sin v).$$

$$= \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{\cos v}{y} + \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{x}{-y^2}(\sin v)$$

$$= \frac{y^2}{y^2+x^2} \cdot \frac{\cos v}{y} + \frac{y^2}{y^2+x^2} \cdot \frac{x(\sin v)}{-y^2}$$

$$= \frac{y}{y^2+x^2} \cdot \cos v + \frac{1}{x^2+y^2} \cdot -x(\sin v)$$

$$= \frac{1}{x^2+y^2}\left(y \cdot \cos v - x \cdot (\sin v)\right)$$

$$= \frac{1}{(u\cos v)^2+(u\sin v)^2}\left(u\sin v \cdot \cos v - u\cos v \cdot \sin v\right)$$

$$= \frac{1}{u(1)} \times (0)$$

$$= 0$$

$$\frac{dz}{dv} = \frac{dz}{dx} \cdot \frac{dx}{dv} + \frac{dz}{dy} \cdot \frac{dy}{dv}$$

$$= \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} \cdot (u \cdot -\sin v) + \frac{1}{1+\left(\frac{x}{y}\right)^2} \frac{x}{-y^2} u\cos v$$

$$= \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{1}{y}(-\sin v u) + \frac{1}{\frac{y^2+x^2}{y^2}} \cdot \frac{x}{-y^2}(u\cos v)$$

$$= \frac{y^{\cancel{2}}}{y^2+x^2} \cdot \frac{1}{\cancel{y}}(-u\sin v) + \frac{\cancel{y^2}}{y^2+x^2} \cdot \frac{x}{-\cancel{y^2}}(u\cos v)$$

$$= \frac{y}{y^2+x^2}(-u\sin v) + \frac{x}{y^2+x^2}(-u\cos v)$$

$$= \frac{u\sin v}{(u\sin v)^2+(u\cos v)^2}(-u\sin v) + \frac{u\cos v}{(u\sin v)^2+(u\cos v)^2}(-u\cos v)$$

$$= \frac{-(u\sin v)^2}{u^2(1)} + \frac{-(u\cos v)^2}{u^2(1)}$$

$$= \frac{1}{u^2}\left(-u^2\sin v^2 - u^2\cos v^2\right)$$

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$= \frac{1}{u^2} - (u^2 \sin v^2 + u^2 \cos^2 v)$

$= \frac{1}{u^2} - (u^2(1))$

$= -1$.

3. $\omega = \log|x^2+y^2+z^2|$ $x = ue^v \sin u$.

$y = u.e^v \cos u$.

$z = u.e^v$

$(u,v) = (-2,0)$.

$\frac{d\omega}{du} = \frac{d\omega}{dx} \cdot \frac{dx}{du} + \frac{d\omega}{dy} \cdot \frac{dy}{du} + \frac{d\omega}{dz} \cdot \frac{dz}{du}$.

$= \frac{1}{x^2+y^2+z^2} \cdot 2x \cdot e^v (u.\cos u + \sin u) + \frac{2y}{x^2+y^2+z^2}$

$e^v . (u.\cancel{\cos u} + \cancel{\sin u})^{-\sin u + \cos u} + \frac{2z}{x^2+y^2+z^2} . e^v$.

$= \frac{2x. e^v (u\cos u + \sin u)}{x^2+y^2+z^2} + \frac{2y\ e^v(u\cancel{\cos u} + \cancel{\sin u})^{+\sin u \cos u}}{x^2+y^2+z^2}$

$+ \frac{2z}{x^2+y^2+z^2} . e^v$.

$= \frac{e^v}{x^2+y^2+z^2} \left(2x (u\cos u + \sin u) + 2y (u\cancel{\cos u + \sin u})^{-\sin u + \cos u} + 2z \cdot \right)$.

$= \frac{e^v}{x^2+y^2+z^2} (2 \cdot (u.e^v.\sin u)(u\cos u + \sin u) + 2y.$
$(u e^v.\cos u) (u \cdot -\sin u + \cos u) + 2z)$.

$= \frac{2e^v}{x^2+y^2+z^2} (\cancel{2 (-2 + 0)} (\div 2 \times 1 \times \sin(-2)$
$(-2\cos(-2) + \sin(-2) +$

Time waste ☺

Q.13

a) $u = \dfrac{p-q}{q-r}$ $\qquad p = x+y+z \qquad q = x-y+z$

$r = x+y-z \qquad (x,y,z) = \sqrt{3}, 2, 1$

$$\frac{du}{dx} = \frac{du}{dp}\cdot\frac{dp}{dx} + \frac{du}{dq}\cdot\frac{dq}{dx} + \frac{du}{dr}\cdot\frac{dr}{dx}$$

$$= \frac{1}{q-r}(1)\cdot(1) + \frac{[(q-r)\cdot(-1) - (p-q)\cdot(1)]}{(q-r)^2}\cdot(1)$$

$$+ (p-q)\cdot\frac{-1}{(q-r)^2}\cdot(-1)\cdot(1)$$

$$= \frac{1}{q-r} + \frac{[(-q+r) - (p-q)]}{(q-r)^2} + p-q \cdot \frac{1}{(q-r)^2}$$

$$= \frac{1}{q-r}\left[1 + \frac{(r-q) - (p+q)}{(q-r)} + \frac{p-q}{q-r}\right]$$

X $$= \frac{1}{q-r}\left[1 + \frac{r-\cancel{q}+p+\cancel{q}}{(q-r)} + \frac{p-q}{q-r}\right]$$

$$= \frac{1}{q-r}\left[1 + \frac{p+r}{q-r} + \frac{p-q}{q-r}\right]$$

$$= \frac{1}{q-r}\left[1 + \frac{p+r+p-q}{(q-r)^2}\right]$$

$$= \frac{1}{q-r}\left[1 + \frac{p(r-q)}{(q-r)^2}\right]$$

$$= \frac{1}{\cancel{x}-y+z-\cancel{x}-y+z}\left[1 + \frac{x+y+z(\cancel{x}+y-z-\cancel{x}+y-z)}{(\cancel{x}-y+z-\cancel{x}-y+z)^2}\right]$$

$$= \frac{1}{-2+1-2+1}\left[1 + \frac{\sqrt{3}+2+1(2-1+2-1)}{(-2+1-2+1)^2}\right]$$

$$= \frac{1}{-4+2}\left[1 + \frac{\sqrt{3}+3(4-2)}{(-4+2)^2}\right]$$

$$= \frac{1}{-2}\left[\frac{4+\sqrt{3}(2)}{2}\right]$$

$$= \frac{1}{q-r} + \left[\frac{-q+r-p+q}{(q-r)^2}\right] + \frac{p-q}{(q-r)^2}$$

$$= \frac{1}{(q-r)^2}\left[\cancel{q}-\cancel{r}+-\cancel{q}+\cancel{r}-\cancel{p}+\cancel{q}+\cancel{p}-\cancel{q}\right]$$

$$= \frac{1}{(q-r)^2}$$

$$= \frac{1}{q^2-2qr+r^2}$$

$$= \frac{1}{(x+y+z)^2-2(x-y+z)(x+y-z)+(x+y-z)^2}$$

$$= \frac{1}{x^2+y^2+\cancel{z^2}-2\cancel{x}+2\cancel{y}-2\cancel{z}-2x-2\cancel{y}+2\cancel{z}+x^2+y^2-\cancel{z^2}}$$

$$= \frac{1}{x^2+y^2+x^2+y^2-2x-2x}$$

$$= \frac{1}{\quad}$$

$$\frac{1}{(\cancel{x}-y+\cancel{z}-\cancel{x}-y+z)^2}$$

$$= \frac{1}{(-2+1-2+1)^2} = \frac{1}{-4+2}$$

$$= \frac{1}{(-2y+2z)^2} = \frac{1}{(-2\times2+2\times1)^2} = \frac{1}{(-4+2)^2}$$

$$= \frac{1}{(-2)^2} = \frac{1}{4}.$$

Same procedure for $\frac{du}{dy}$ and $\frac{du}{dz}$

coordinate $(1, \log 2, 0)$

$x = \cos t$
$y = \log(t+2)$
$z = t$

16. $\omega = x^2 \cdot e^{2y} \cdot \cos 3z$

$$\frac{d\omega}{dt} = \frac{d\omega}{dx}\cdot\frac{dx}{dt} + \frac{d\omega}{dy}\cdot\frac{dy}{dt} + \frac{d\omega}{dz}\cdot\frac{dz}{dt}$$

$$= 2x \cdot e^{2y} \cdot \cos 3z \,(-\sin t) + x^2 \cdot \cos 3z \cdot e^{2y} \cdot 2 \cdot \frac{1}{(t+2)} + x^2 \cdot e^{2y} \cdot (-\sin 3z) \cdot 3 \cdot (1)$$

$$= 2x \cdot e^{2y} \cdot \cos 3z \,(-\sin t) + x^2 \cdot \cos 3z \cdot e^{2y} \cdot \frac{2}{(t+2)} + x^2 e^{2y} (-\sin 3z)(3).$$

$$= e^{2y}\left(2x \cdot \cos 3z \,(-\sin t) + x^2 \cdot \cos 3z \cdot \frac{2}{t+2} + 3x^2 \cdot (-\sin 3z)\right).$$

$$= e^{2(\log(t+2)}\left(\cos^2 t \cdot (1) \cdot \frac{2}{2}\right)$$

$$= e^{\log(t+2)^2} \left(\cos^2(0)\right)$$

$$= (0+2)^2 \,(1).$$

$$\frac{d\omega}{dt} = 4.$$

17. find gradient of function.

1. $f(x, y) = y - x \qquad (2, 1).$

$$\Rightarrow \nabla f = \frac{\partial f}{\partial x}\,\bar{i} + \frac{\partial f}{\partial y}\,\bar{j}$$

$$= -1\,i + 1\,j$$

2. $f(x, y) = \log(x^2 + y^2). \qquad (1, 1)$

$$\nabla f = \frac{\partial f}{\partial x}\,\bar{i} + \frac{\partial f}{\partial y}\,\bar{j}$$

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$$= \frac{1}{x^2+y^2} \cdot 2x\, i + \frac{2y}{x^2+y^2}\, j$$

$$= \frac{2x}{x^2+y^2}\, i + \frac{2y}{x^2+y^2}\, j$$

$$\nabla f \big|_{(1,1)} = \frac{2}{1+1}\, i + \frac{2}{1+1}\, j$$

$$= \frac{2}{2}\, i + \frac{2}{2}\, j$$

$$= 1i + 1j$$

3. $f(x,y) = y - x^2 \quad (-1,0)$

$$\nabla f = \frac{df}{dx}\, i + \frac{df}{dy}\, j$$

$$= -2x\, i + 1j$$

$$\nabla f \big|_{(-1,0)} = -2(-1)\, i + 1j$$

$$= 2i + 1j$$

4. $f(x,y) = \frac{x^2}{2} - \frac{y^2}{2} \quad (\sqrt{2}, 1).$

$$\nabla f = \frac{\partial f}{\partial x}\, i + \frac{\partial f}{\partial y}\, j$$

$$= \frac{1}{\cancel{2}} \cdot \cancel{2}x\, i - \frac{1}{\cancel{2}}\, \cancel{2}y\, j$$

$$= x i - y j.$$

$$\nabla f \big|_{\sqrt{2},1} = \sqrt{2}\, i - 1j$$

~~18~~.

19. $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4.$

a).

$f_x = 2x + y + 3 = 0$

$f_y = x + 2y - 3 = 0$

$4x + 2y + 6 = 0$

$x + 2y - 3 = 0.$

$4x + \cancel{2y} = -6$

$- \quad x + \cancel{2y} = -3$

$- \quad - \quad -$

$3x = -9$

$x = -9/3$

$\boxed{x = -3}$

$4(-3) + 2y + 6 = 0.$

$-12 + 2y + 6 = 0$

$2y - 6 = 0$

$y = 6/2$

$\boxed{y = 3}$

at point $(-3,3)$.

$f_{xx} = 2 = A$

$f_{xy} = \cancel{2} 1 = 1 = B$

$f_{yy} = 2 = C.$

$\Delta = AC - B^2$

$= 2 \times 2 - 1^2$

$= 4 - 1$

$= 3 > 0.$

$\Delta > 0, \quad A > 0.$

$\therefore$ $(-3,3)$ is absolute minimum and local minimum

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b) $f(x,y) = y.\sin x$

$f_x = y.\cos x$
$f_y = \sin x$

$f_x = y\cos x = 0$
$f_y = \sin x = 0$

$x = \sin^{-1}(0)$ $\quad$ $y=0$ or $\cos^{-1}0 = x$

$\boxed{x = 0}$ $\quad$ $y=0.$ $\quad$ $x = \pi/2$

$\boxed{y=0}$ $\quad$ $(\frac{\pi}{2}, 0).$

point $(0,0)$.

$f_{xx} = y.(-\sin x). \quad = y(-\sin 0) = 0 \quad = A$

$f_{xy} = \cos x \quad = \cos(0) = 1 \quad = B$

$f_{yy} = 0. \quad = C$

$\Delta = AC - B^2$
$= 0 \times 0 - 1^2$
$= -1 < 0.$
saddle point.

c) $f(x,y) = x^3 + 3xy + y^3$

$f_x = 3x^2 + 3y \quad = 0$
$f_y = 3x + 3y^2 \quad = 0.$

$3x^2 + 3y = 0.$
$3x + 3y^2 = 0.$

$x^2 + y = 0$ –
$x + y^2 = 0.$ –
$x^2 = -y$
$\cancel{x = \sqrt{-y}}$ In eqn $\quad x + (-x^2)^2 = 0$
$x + x^4 = 0.$

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$x(1+x^3)=0$

$x=0$ or $y=1+x^3=0$

$x=0$ or $x^3=-1$

$x=0$. $x=-1$

put $x=0$

$x+y^2=0$

$0+y^2=0$

$y=0$

$(0,0)$.

put $x=-1$

$-1+y^2=0$

$y^2=1$

$y=\pm 1$

Points are $(0,0)$ $(0,+1)$ $(0,-1)$ $(-1,0)$ $(-1,1)$ $(-1,-1)$.

d) $f(x,y)=e^{2x}.\cos y$.

$f_x = \cos y . e^{2x}. 2.$

$f_y = e^{2x}. (-\sin y)$

$\cos y . e^{2x}.2=0.$

$e^{2x}(-\sin y)=0.$

$\cos y = 0$

$y=\cos^{-1}(0)$

$\boxed{y=\frac{\pi}{2}}$

$e^{-2x}.\sin y=0.$

$\sin y=0$

$y=\sin^{-1}(0)$

$\boxed{y=0}$

No critical point exist

e). $f(x,y,z) = x^3 - xy + y^2 + yz + z^2 - 2z$

~~fx~~

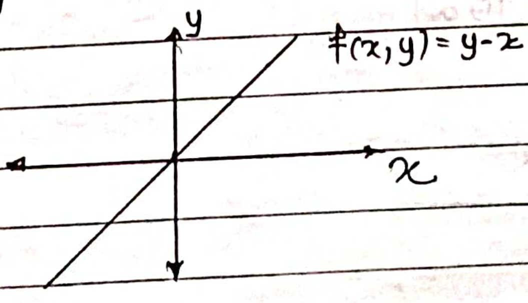
Q.21

a). False
Saddle or Absolute minimum Absolute minimum exist so statement is false.

b) True
It exist also other point exist as it has critical point at derivative 0.

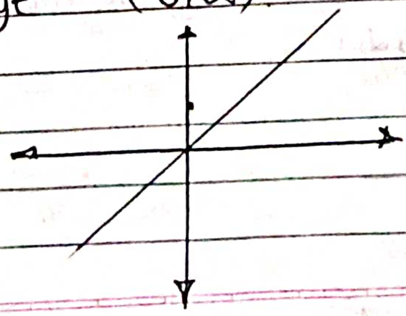
Q.1 Find function, Domain, range and draw level curves.

a) $f(x,y) = y - x$
Domain - xy plane
Range - $(-\infty, \infty)$



b) $f(x,y) = \sqrt{y-x}$
Domain - xy plane s.t. $y \geq x$
Range - $(0, \infty)$

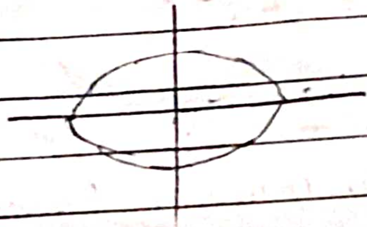
$\sqrt{y-x} = 1$
$y - x = 0$
$\boxed{y = x}$



c) $f(x,y) = 4x^2 + 9y^2$

Domain - xy plane

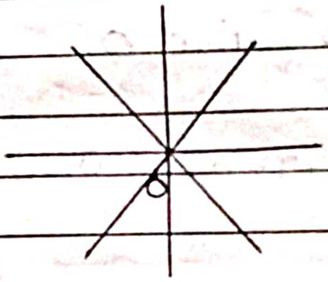
Range - $(0, \infty)$



d) $f(x,y) = x^2 - y^2$

D = xy plane

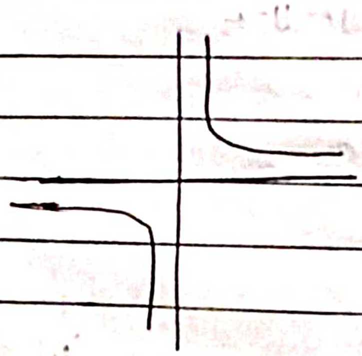
R = $-\infty$ to ∞



e) $f(x,y) = x \cdot y$

D = xy plane

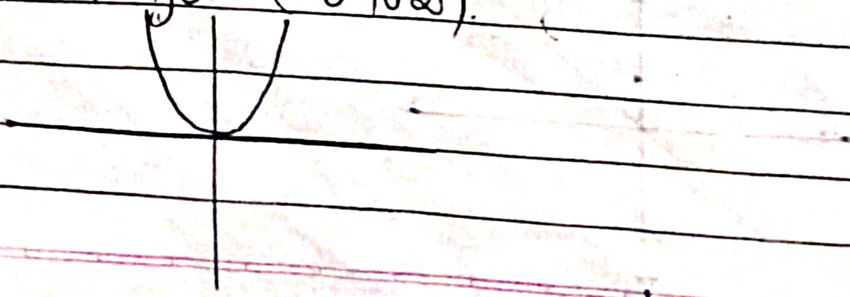
R = $(-\infty$ to $\infty)$



f) $f(x,y) = y/x^2$

Domain $(-\infty$ to $\infty)$ when $x \neq 0$

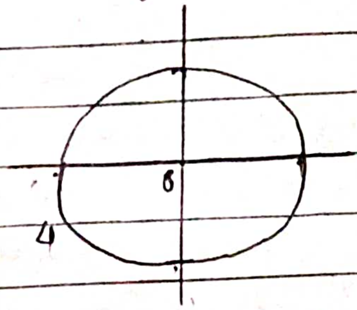
Range. $(-\infty$ to $\infty)$.



g) $f(x,y) = \dfrac{1}{\sqrt{16-x^2-y^2}}$ $\quad 16 > x^2+y^2$

Domain – xy plane $\quad 16 > x^2+y^2$

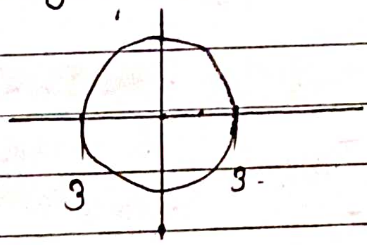
Range – ~~0 to 4~~ 0 to 4



f) $f(x,y) = \sqrt{9-x^2-y^2}$

Domain – x $\quad 9 \geq x^2+y^2$

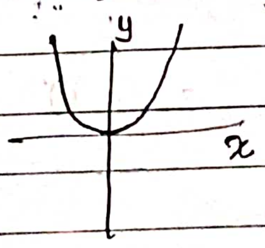
Range – 0 to 3.

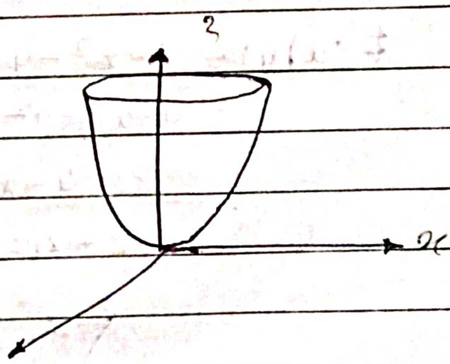


Q.2 Sketch the surface.

a) $f(x,y) = y^2$

$x = y^2$

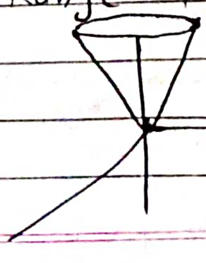




b). $f(x,y) = x^2+y^2$

Domain – xy plane

Range – $f \in \mathbb{R}$ $\quad R > f \geq 0$.



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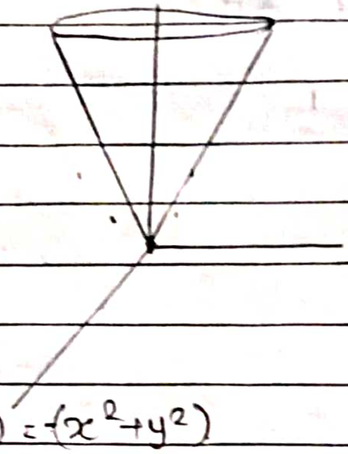
c) $f(x,y) = \sqrt{x^2+y^2}$ $\qquad x^2+y^2=r^2$

Domain + xy plane

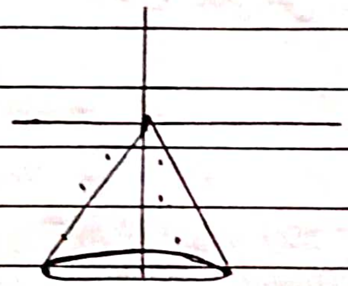
Range - $f \in R$ $\quad f \geq 0$



d). $f(x,y) = -(x^2+y^2)$

Domain - xy plain

Range - $f \in R$, $f \leq 0$

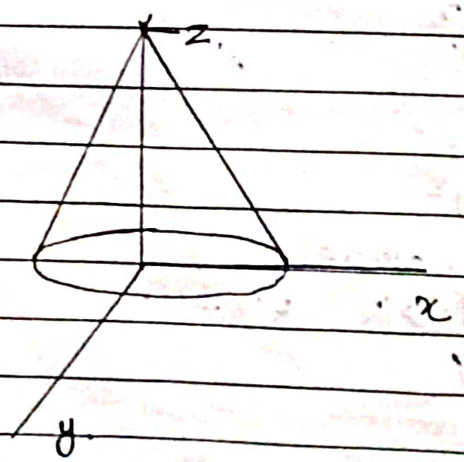


e) $f(x,y) = 4-x^2-y^2$ $\qquad$ Domain - xy plane

$= 4 = x^2+y^2$ $\qquad$ Range - 0 to 4

$z = 4-x^2-y^2$

$-z+4 = x^2+y^2$ $\qquad z=(0,1)$



f) $f(x,y) = 1-|x|-|y|$.

Domain - xy plane

Range - $(1 \text{ to } \infty)$

No. graph.