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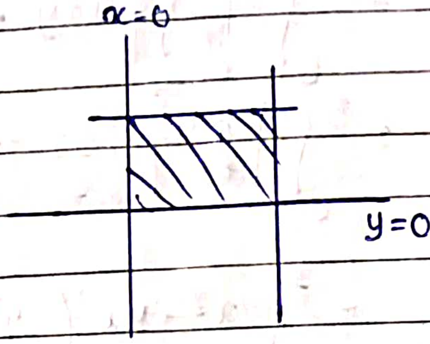
Tutorial - 4.

① a). Sketch the region and evaluate

a). $\int_0^3 \int_0^2 (4-y^2)\, dy\, dx$



$y=0 \qquad y=2$

$x=0 \qquad x=3$

Now.

$$\int_0^3 \left[\int_0^2 (4-y^2)\, dy \right] dx$$

$$\int_0^3 \left[4y - \frac{y^3}{3} \right]_0^2 dx$$

$$\int_0^3 \left[8 - \frac{8}{3} \right] dx$$

$$\int_0^3 \left[\frac{24-8}{3} \right] dx$$

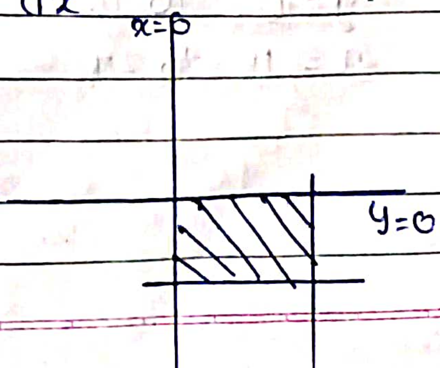
$$\int_0^3 \frac{16}{3}\, dx$$

$$\frac{16}{3} \int_0^3 1\, dx$$

$$\frac{16}{3} \cdot \left| x \right|_0^3$$

$$\frac{16 \times 3}{3} = \frac{48}{3} = \cancel{14}\ \underline{16}$$

b). $\int_0^3 \int_{-2}^0 (x^2y - 2xy)\, dy\, dx$



$x=0 \qquad x=3$

$y=-2 \qquad y=0.$

$$= \int_0^3 \int_{-2}^0 (x^2y - 2xy)\, dy\, dx$$

$$= \int_0^3 \left[\int_{-2}^0 (x^2y - 2xy)\, dy \right] dx$$

$$= \int_0^3 \left[x^2 \cdot \frac{y^2}{2} - \cancel{2}x \cdot \frac{y^2}{\cancel{2}} \right]_{-2}^{0} dx$$

$$= \int_0^3 \cancel{x^2 - 2x \left(\frac{y^2}{2} - \frac{y^2}{2}\right)}.$$

$$\int_0^3 \left[\frac{x^2y^2}{2} - xy^2 \right]_{-2}^{0}$$

$$= \int_0^3 \left[0 - \left(\frac{4x^2}{2} - x4 \right) \right] dx$$

$$= \int_0^3 \left[-\frac{\cancel{4}^{2}x^2}{\cancel{2}} + 4x \right] dx$$

$$= \left[-\frac{2x^3}{3} + \frac{4x^2}{2} \right]_0^3$$

$$= \left(-\frac{2 \times 27}{3} + \frac{4 \times 9}{2} \right)$$

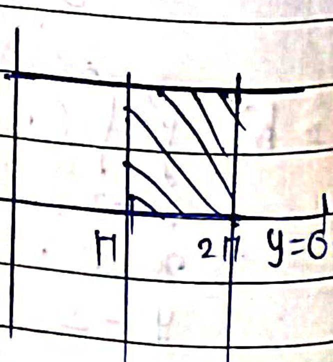
$$= \left(-\frac{54}{3} + \frac{36}{2} \right)$$

$$= (18 - 18) = 0.$$

c). $\int_{\pi}^{2\pi} \int_0^{\pi} (\sin x + \cos y)\, dy\, dx$

$y = 0$ to π

$x = \pi$ to 2π.



$$= \int_{\pi}^{2\pi} \left[\int_{0}^{\pi} (\sin x + \cos y)\, dy \right] dx$$

$$= \int_{\pi}^{2\pi} \left[\cancel{-\cos x}\ \sin xy + \sin y \right]_{0}^{\pi} dx$$

$$= \int_{\pi}^{2\pi} [-\sin x\pi + \sin \pi] - [\sin 0 + \sin 0]$$

$$= \int_{\pi}^{2\pi} [\sin x\pi + 0] - [0 + 0]\, dx$$

$$= \int_{\pi}^{2\pi} [\sin \pi x]\, dx$$

$$= \pi . [-\cos x]_{\pi}^{2\pi}$$

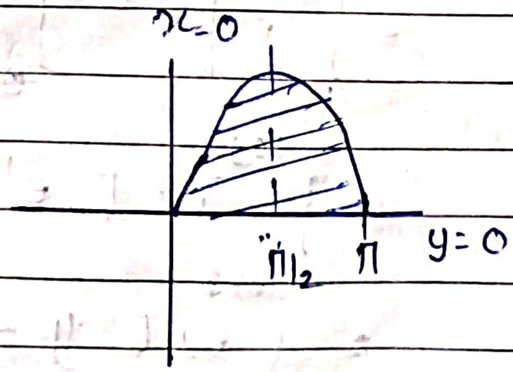
$$= -\pi [\cos 2\pi] - [\cos \pi]$$

$$= -\pi [1] - [-1]$$

$$= -\pi . (+2)$$

$$= -2\pi .$$

d) $$\int_{0}^{\pi} \int_{0}^{\sin x} y\, dy\, dx$$



$x = 0$ to π

$y = 0$ to $\sin x$

$$\int_{0}^{\pi} \left[\int_{0}^{\sin x} y\, dy \right] dx$$

$$\int_{0}^{\pi} \left[\frac{y^2}{2} \right]_{0}^{\sin x} dx = \int_{0}^{\pi} \left[\frac{\sin^2 x^2}{2} \right] dx$$

$$= \frac{1}{2}\int_0^{\pi} \frac{1-\cos 2x}{2}\, dx$$

$$= \frac{1}{4}\int_0^{\pi}\left[x - \frac{\sin 2x}{2}\right] dx$$

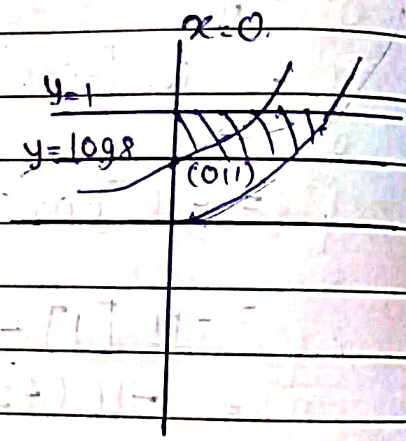
$$= \frac{1}{4}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$$

$$= \frac{1}{4}\left[\pi - \sin 2\pi\right]$$

$$= \frac{1}{4}\left[\pi - 0\right]$$

$$= \frac{\pi}{4}$$

e) $\displaystyle\int_1^{\ln 8}\int_0^{\log y} e^{x+y}\, dy\, dx$



$x = 0 \qquad x = \log y$

$y = 1 \qquad y = \log 8$

$$\int_1^{\ln 8}\left[\int_0^{\log y} e^{x+y}\, dx\right] dy$$

$$\int_1^{\log 8}\left[\int_0^{\log y} e^{x}\cdot e^{y}\, dx\right] dy$$

$$= \int_1^{\log 8} e^{y}\left[e^{x}\right]_0^{\log y} dy$$

$$= \int_1^{\log 8} e^{y}\left[e^{\log y} - e^{0}\right] dy$$

$$= \int_1^{\log 8} e^{y}\left[y - 1\right] dy$$

$$= \int_1^{\log 8}\left[y e^{y} - e^{y}\right] dy$$

$$= \int_1^{\log 8} \cancel{e^1 + e} \quad y\int e^y - \int 1 \cdot e^y - e^y$$

$$= \left[(ye^y - e^y) - (e^y)\right]_1^{\log 8}$$

$$= \left[(\log 8\, e^{\log 8} - e^{\log 8}) - (e^{\log 8})\right] - \left[(1e^1 - e^1) - (e^1)\right]$$

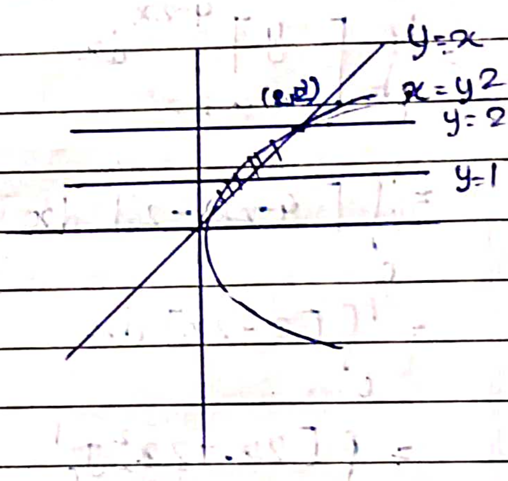
$$= [\ln 8 \times 8 - 8) - (8)] - (e - e - e).$$

$$= (\ln 8 \times 8 - 8) - (8)] - \cancel{e} + e + \cancel{e})$$

$$= (8 \log 8 - 8 - 8 + e)$$

$$= 8(\ln 8 - 2) + e$$

f). $\int_1^2 \int_y^{y^2} dy\, dx$



$x = y \qquad x = y^2$

$y = 1 \qquad y = 2$

$$\int_1^2 [x]_y^{y^2} \, d\cancel{x}y.$$

$$\int_1^2 [y^2 - y]\, dy.$$

$$\int_1^2 \left[\frac{y^3}{3} - \frac{y^2}{2}\right]_1^2$$

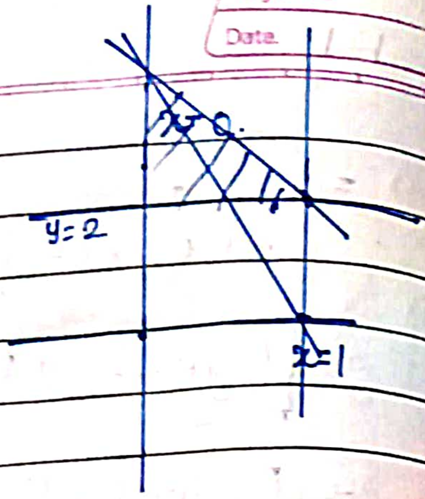
$$\left[\frac{8}{3} - \frac{4}{2}\right] - \left[\frac{1}{3} - \frac{1}{2}\right]$$

$$= \left[\frac{8}{3} - 2\right] - \left[\frac{1}{3} - \frac{1}{2}\right]$$

$$= \left[\frac{8-6}{3}\right] - \left[\frac{2-3}{6}\right] \quad = \left[\frac{2}{3}\right] - \left[\frac{-1}{6}\right]$$

$$= \left[\frac{2}{3} + \frac{1}{6}\right] = \frac{5}{6}.$$

g). $\displaystyle\int_0 \int_2^{4-2x} dy\ dx$



$y=2 \qquad y=4-2x$

$x=0 \qquad x=1$

$y=2 \qquad y=4-2x$

$2=4-2x$

$2x=4-2$

$x=2/2$

$\boxed{x=1}$

NOW. $x=0$

$y=4-20$

$\boxed{y=4}$

$$\int_0^1 \int_2^{4-2x} 1\ dy\ dx$$

$$= \int_0^1 \left[\ y\ \right]_2^{4-2x} dx$$

$$= \int_0^1 [4-2x-2]\ dx$$

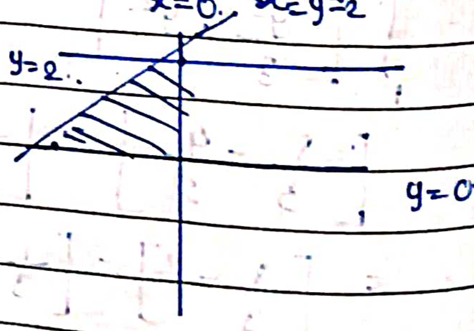
$$= \int_0^1 [2-2x]\ dx$$

$$= \int \left[2x - \frac{2x^2}{2}\right]_0^1$$

$$= [2-1]$$

$$= \underline{\underline{1}}$$

h). $\displaystyle\int_0^2 \int_{y-2}^0 dy\ dx$



$x=y-2 \qquad x=0$

$y=0 \qquad y=2$

$$\int_0^2 \int_{y-2}^0 1\ dx\ dy.$$

$$= \int_0^2 \left[x \right]_{y-2}^{0} dy$$

$$= \int_0^2 \left[0-(y-2) \right] dy.$$

$$= \int_0^2 \left[-y+2 \right] dy.$$

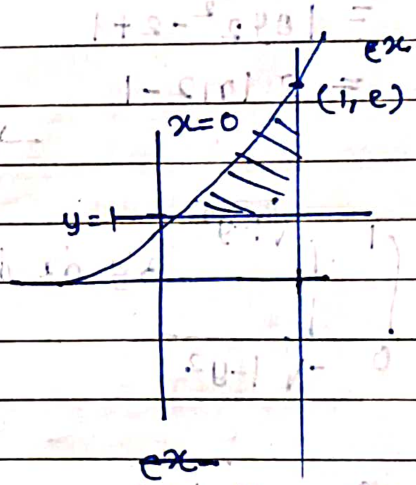
$$= \left[-\left(\frac{y^2}{2} - 2y \right) \right]_0^2$$

$$= -\left(\frac{4}{2} - 4 \right)$$

$$= -(2-4)$$

$$= \underline{\underline{2}}$$

i). $\int_0^1 \int_1^{e^x} dx\, dy$



$y = e^x \quad x=1$

$y = e^1 \quad \boxed{y=e.}$

$$\int_0^1 \int_1^{e^x} 1\, dy\, dx$$

$$= \int_0^1 \left[y \right]_1^{e^x} dx$$

$$= \int_0^1 \left[e^x - 1 \right] dx$$

$$= \int_0^1 \left[e^x - 1 \right] dx$$

$$= \left[e^x - x \right]_0^1$$

$$= \left[e^1 - 1 \right] - \left[e^0 - 0 \right]$$

$$= \left[e-1 \right] - \left[1 \right]$$

$$e - 1 - 1$$

$$= \underline{\underline{e-2}}$$

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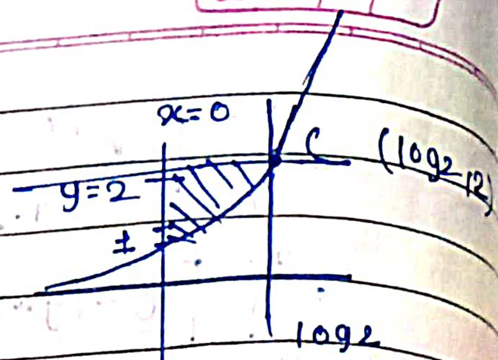
J) $\int_0^{\log 2} \int_{e^x}^{2} dy\, dx$



$= \int_0^{\log 2} \left[y \right]_{e^x}^{2} dx$

$= \int_0^{\log 2} \left[2 - e^x \right] dx$

$= \left[2x - e^x \right]_0^{\log 2}$

$= (2\log 2 - e^{\log 2}) - (0 - e^0)$

$= (\log 2^2 - 2) - (-1)$

$= \log 2^2 - 2 + 1$

$= 2\ln 2 - 1$

log 2

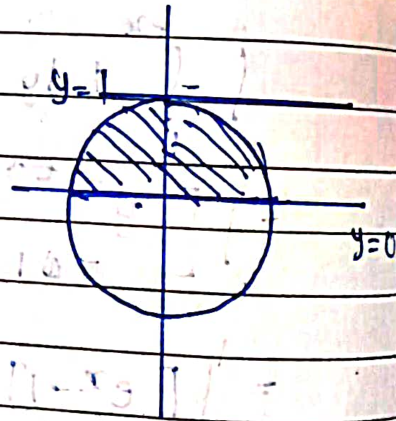
$x = \log 2$

$\cancel{\log 1 = 0}$

$y = 2. \quad e^x = y$

—x—

k). $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\, dx\, dy$



$x = -\sqrt{1-y^2} \qquad x = \sqrt{1-y^2}$

$y = 0$ to $y = 1$

$\int_0^1 \left[3yx \right]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}$

$= \int_0^1 3y \left(\sqrt{1-y^2} + \sqrt{1-y^2} \right) dy$

$= \int_0^1 3y\, 2\sqrt{1-y^2}\, dy$

$x = \sqrt{1-y^2}$

$x = -\sqrt{1-y^2}$

$y = 1$

$y = 0$

$x^2 = 1 - y^2$

$x^2 + y^2 = 1$

$\cancel{x^2 = 1 + y^2}$

$= \int_0^1 6y\sqrt{1-y^2}\,dy$

$6\int_0^1 y\sqrt{1-y^2}\,dy$

put $1-y^2=t$

lim H
$y=1 \quad t=0$
$y=0 \quad t=1.$

$0-2y\,dy=dt$

$\boxed{y\,dy = \frac{-dt}{2}}$

$=6\int_1^0 \sqrt{t}\cdot\frac{dt}{2}$

$= -3\int_1^0 \sqrt{t}\,dt$

$= -3\left[\frac{t^{1/2+1}}{1/2+1}\right]_1^0$

$= -\cancel{3}\left[\frac{t^{3/2}}{\cancel{3}/2}\right]_1^0$

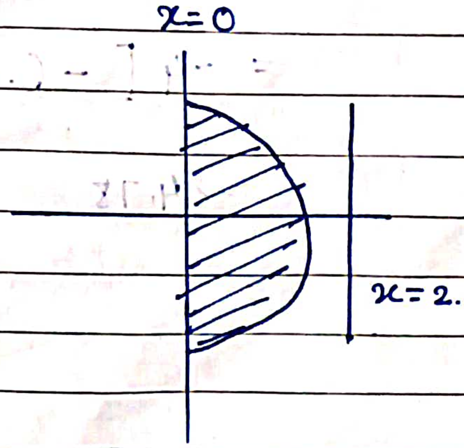
$= -2\left[t^{3/2}\right]_1^0$

$= -2\left[0-1^{3/2}\right]$

$= -2[-1]$

$= \underline{\underline{2}}$

4) $\int_0^2\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 6x\,dy\,dx$



$y=\sqrt{1-x^2}$ $\quad x=\underline{0}$

$y^2=1-x^2$ $\quad x=\underline{2}$

$x^2+y^2=1.$

OR

-1

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$$= \int_0^2 6x \left(y \right)_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= \int_0^2 6x \left(\sqrt{1-x^2} + \sqrt{1-x^2} \right) dx$$

$$= \int_0^2 \left(12\, x \sqrt{1-x^2} \right) dx$$

put $1-x^2 = t$

$0 - 2x\, dx\; dt$

$x\,dx = \frac{dt}{-2}$

when

$x \to 0 \quad t=1$

$x \to 2 \quad t=-3$

$$= \int_1^{-3} \cancel{12}^{6} \cancel{x} \sqrt{t}\, \frac{dt}{-\cancel{2}}$$

$$= -6 \int_1^{-3} \sqrt{t}\, dt$$

$$= -6 \int_1^{-3} t^{1/2}\, dt$$

$$= -6 \left[\frac{t^{3/2}}{3/2} \right]_1^{-3}$$

$$= -\cancel{6}^{2} \frac{\times 2}{\cancel{3}} \left[t^{3/2} \right]_1^{-3}$$

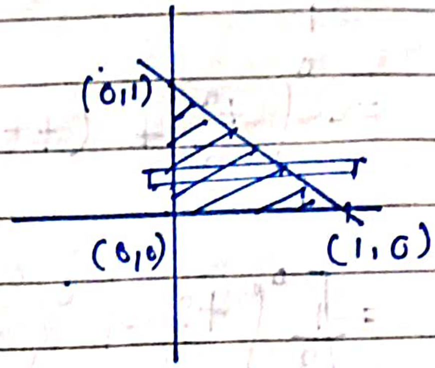
$$= -4 \left[-3^{3/2} - 1^{3/2} \right]$$

$$= -4 \left[-6.19 \right]$$

$$= 24.78$$

2. Integrate f over region

① $f(x,y) = x^2+y^2$ over triangular vertices $(0,0)\ (0,1)\ (1,0)$.



Now.

~~$x = 0$ to 1~~

$y =$

~~Slope = $\frac{1-0}{0-1} = \frac{1}{-1}$~~

$$\text{slope} = \frac{1-0}{0-1} = \frac{1}{-1} = -1$$

Now, $y = mx+c$

$\therefore\ y = -1x+1$

$y = -x+1$

$$\boxed{x = 1-y}$$

limit

$x = 0 \qquad x = 1-y.$

$y = 0 \qquad$ to $y = 1.$

$$\int_0^1 \int_0^{1-y} (x^2+y^2)\, dx\, dy$$

$$= \int_0^1 \left[\frac{x^3}{3} + y^2 x\right]_0^{1-y} dy$$

$$= \int_0^1 \left[\frac{(1-y)^3}{3} + y^2(1-y)\right] dy$$

put $1-y = t$

$-y\,dy = dt$

$$\boxed{dy = -dt}$$

$1-t = y$

$$\boxed{y^2 = 1-t^2}$$

limit

$y \to 0 \qquad t = 1$

$y \to 1 \qquad t = 0.$

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$$\int_1^0 \left[\frac{t^3}{3} + (1-t^2)t\right](-dt)$$

$$= -\int_1^0 \frac{t^3}{3} + (1-2t+t^2)(t)\, dt$$

$$= -\int_1^0 \frac{t^3}{3} + (t-2t^2+t^3)\, dt$$

$$= -\left[\frac{1}{3}\int_1^0 t^3 + \int_1^0 t - 2t^2 + t^3\, dt\right]$$

$$= -\left[\frac{1}{3}\left(\frac{t^4}{4}\right)_1^0 + \left[\frac{t^2}{2} - \frac{2t^3}{3} + \frac{t^4}{4}\right]_1^0\right]$$

$$= -\left[\frac{-1}{12}\right]\left(-\frac{1}{2} + \frac{2}{3} - \frac{1}{4}\right)$$

$$= -\left[\left[\frac{-1}{12}\right] - \frac{1}{2} + \frac{2}{3} - \frac{1}{4}\right]$$

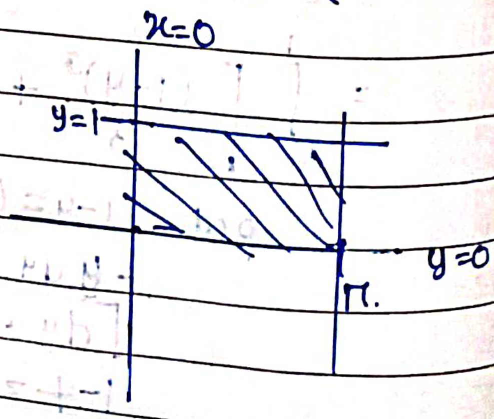
$$= \frac{1}{12} + \frac{1}{2} - \frac{2}{3} + \frac{1}{4}$$

$$= \frac{1}{6}$$

b). $f(x,y) = y\cos xy$ over rectangle $0 \leq x \leq \pi$
$0 \leq y \leq 1$

$x = 0 \quad x = \pi$

$y = 0 \quad y = 1$



$$= \int_0^1 \int_0^\pi y \cos xy \, dx \, dy.$$

$$= \int_0^1 \cancel{y} \frac{(+\sin xy)}{\cancel{y}}\Bigg|_0^\pi dy.$$

$$= \int_0^1 -\cancel{y}(\sin \pi y) - (\sin 0) \, dy.$$

$$= \int_0^1 \sin \pi y \, dy.$$

$$= \frac{1}{\pi}(-\cos \pi y)_0^1$$

$$= \frac{1}{\pi} \cancel{-\cos \pi - (-\cos 0)}.$$

$$= \cancel{\cos 1 + 1}$$

$$= \cancel{2}$$

$$= \frac{1}{\pi}(-\cos \pi y)_0^1$$

$$= \frac{1}{\pi}(-\cos \pi - (-\cos 0)).$$

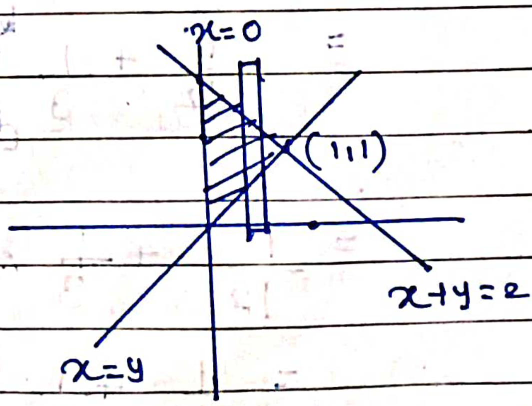
$$\frac{1}{\pi}(1+1)$$

$$= \frac{2}{\pi}.$$

3 $Z = x^2 + y^2$.

$y = x$ $\quad x = 0$

$x + y = 2$.



$x + y = 2$

$y = 2$. if $x = 0$

$x = 2$

Now limits Strip parallel to Y axis.

$y = x$ $\quad y = 2 - x$

$\boxed{x = 0}$ $\quad x = y \quad x + y = 2$

$\boxed{x = 1}$ $\quad y = 1 \quad x = 1$.

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$$= \int_0^1 \int_x^{2-x} x^2+y^2 \, dy \, dx$$

$$= \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_x^{2-x} dx$$

$$= \int_0^1 \left[x^2 (2-x) + \frac{(2-x)^3}{3} \right] - \left[x^2 x + \frac{x^3}{3} \right] dx$$

$$= \int_0^1 [2x^2 - x^3] + \frac{(2-x)^3}{3}] - [x^3 + \frac{x^3}{3}] dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^1 + \frac{1}{3} \int_0^1 (2-x)^3 - \left[\frac{x^4}{4} + \frac{x^4}{12} \right]_0^1$$

$$= \left[\frac{2}{3} - \frac{1}{4} \right] - \left[\frac{1}{4} + \frac{1}{12} \right] + \frac{1}{3} \int_0^1 (2-x)^3$$

$$= \frac{2}{3} - \frac{1}{4} - \frac{1}{4} - \frac{1}{12} + \frac{1}{3} \int_0^1 (2-x)^3 \, dx$$

$$= \frac{1}{12} + \frac{1}{3} \left[\frac{(2-x)^4}{4 \times (-1)} \right]_0^1$$

$$= \frac{1}{12} + \frac{1}{3} \times \left(\frac{1}{-4} + \frac{16}{-4} \right)$$

$$= \frac{1}{12} + \frac{1}{3} \left(\frac{15}{4} \right)$$

$$= \frac{4}{3}.$$

$x^2(y)$

$x^2[(2-x^2)-(x)]$

4. $Z = x^2$.

$y = 2-x^2$ line $y = x$ in xy plane

$y = x$ | $y = 2-x^2$

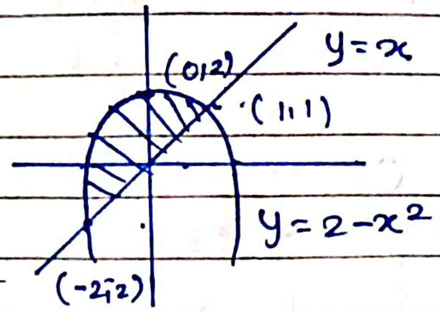
$x = 2-x^2$

$x^2+x-2=0$

$x = 2$. $x = 1$

$\underline{x=-2}$ or $\underline{\underline{x=1}}$

-2, $2 \; -1$



$x = (-2,-2)$

$x = (1,1)$

Strip parallel to x axis.

$y = x$ $\quad$ $y = 2-x^2$

$x = -2$ $\quad$ $x = 1$

$$= \int_{-2}^{1}\int_{x}^{2-x^2} x^2 \, dy \, dx$$

$$= \int_{-2}^{1} \left[x^2 y \right]_{x}^{2-x^2} dx$$

$$= \int_{-2}^{1} \left(x^2(2-x^2) - x^3\right) dx$$

$$= \int_{-2}^{1} \cancel{x^2}\left(2x^2 - x^4 - x^3\right) dx.$$

$$= \left[\frac{2x^3}{3} - \frac{x^5}{5} - \frac{x^4}{4} \right]_{-2}^{1}$$

$$= \left(\frac{2}{3} - \frac{1}{5} - \frac{1}{4}\right) - \left(\frac{-16}{3} - \frac{(-32)}{5} - \frac{16}{4}\right).$$

$$= \frac{63}{20}.$$

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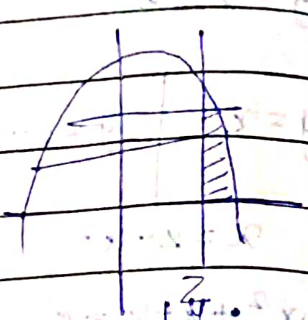
5. $x=3$ and $z = 4-y^2$

In 1st quadrant

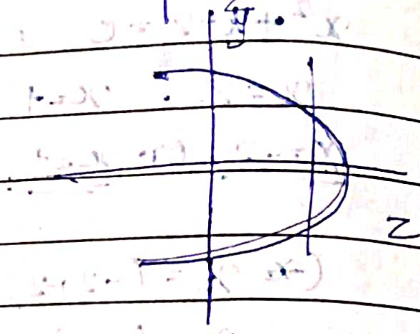
$z=0$



$0 = 4-y^2$

$y^2 = \pm 2$

$x = 0$ to 3



$0 \le y \le 2$

$0 \le x \le 3$

z 0 to $4-y^2$

$$V = \int_0^3 \int_0^2 \int_0^{4-y^2} dz\, dy\, dx$$

$$= \int_0^3 \int_0^2 [z]_0^{4-y^2} dy\, dx$$

$$= \int_0^3 \int_0^2 [4-y^2]\, dy\, dx$$

$$= \int_0^3 \left[4y - \frac{y^3}{3}\right]_0^2 dx$$

$$= \int_0^3 \left[4\times 2 - \frac{8}{3}\right] dx$$

$$= \int_0^3 \left[8 - \frac{8}{3}\right] dx$$

$$= \int_0^3 \left[\frac{24-8}{3}\right] dx$$

$\iint f(x)\, dx\, dy$

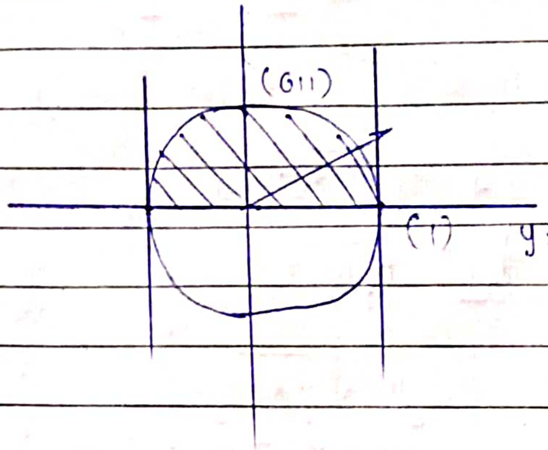
$$= \frac{16}{3} \int_0^{\cancel{2}} \boxed{x}\, dx \, [x]_0^3$$

$$= \frac{16}{\cancel{3}} (\cancel{3})$$

$$= \underline{\underline{16}}$$

7. change carteesian into polar co-ordinates

a) $$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} dy\, dx$$



$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$\boxed{x^2 + y^2 = 1}$

$y = 0$.

$x = 1 \quad x = -1$.

Change into polar

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = 1$

$r^2\cos^2\theta + r\sin^2\theta = 1$

$r^2(\sin^2\theta + \cos^2\theta) = 1$

$r^2 = 1$

$\boxed{r = 1}$

limit $r = 0$ to 1

Now $\theta = 0$ to π

$$= \int_0^{\pi} \int_0^{1} r\, dr\, d\theta$$

$$= \int_0^{\pi} \left[\frac{r^2}{2}\right]_0^1 d\theta$$

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$$= \int_0^{\pi} \left[\frac{1^2}{2}\right] d\theta$$

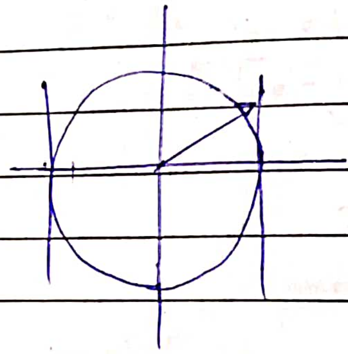
$$= \int_0^{\pi} \frac{1}{2} \, d\theta$$

$$= \frac{1}{2} (\theta)_0^{\pi}$$

$$= \frac{1}{2}\pi - \frac{1}{2} \times 0$$

$$= \frac{1}{2}\pi$$

2) $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy \, dx$



$y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$\boxed{x^2 + y^2 = 1}$

$0 \leq r \leq 1$

$0 \leq \theta \leq 2\pi$

$$I = \int_0^{2\pi} \int_0^{1} r \, dr \, d\theta$$

$$= \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^1 d\theta$$

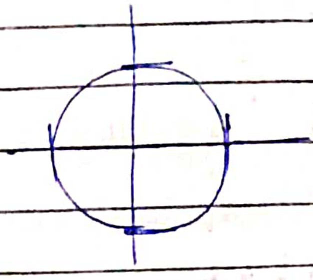
$$= \int_0^{2\pi} \left[\frac{1}{2}\right] d\theta$$

$$= \frac{1}{2} (\theta)_0^{2\pi}$$

$$= \frac{1}{2} (2\pi - 0)$$

$$= \frac{1}{\cancel{2}} \cancel{2}\pi = \underline{\underline{\pi}}$$

3) $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\,dx$



$y=\sqrt{a^2-x^2}$

$y^2+x^2=a^2$

$0 \leq r \leq a$

$0 \leq \theta \leq 2\pi$

$$I=\int_0^{2\pi}\int_0^{a} r\,dr\,d\theta = \int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^a \cancel{dr}\,d\theta$$

$$=\int_0^{2\pi}\left[\frac{a^2}{2}\right]d\theta$$

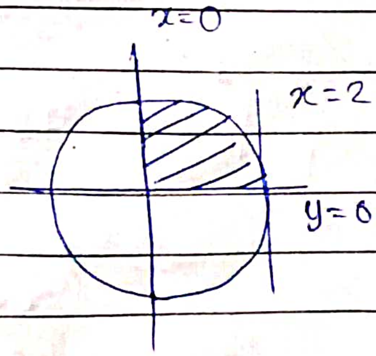
$$\cancel{=\frac{1}{2}\left[\frac{a^3}{3}\right]_0^{2\pi}} - \cancel{\frac{1}{3}\left[2\pi^3-0\right]}$$

$$=\frac{a^2}{2}\int_0^{2\pi} d\theta$$

$$=\frac{a^2}{2}\left[\theta\right]_0^{2\pi} = \frac{a^2}{\cancel{2}}\left[\cancel{2}\pi\right]$$

$$=\underline{\pi a^2}$$

4) $\int_0^{2}\int_0^{\sqrt{4-x^2}} (x^2+y^2)\,dx\,dy$



$y=0 \quad y=\sqrt{4-x^2}$

$y^2+x^2=2$

$x=0 \quad y=2$

$0 \leq r \leq 2$

$0 \leq \theta \leq \pi/2$

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$$= \int_0^{\pi/2} \int_0^{2} (x^2+y^2)\ \frac{\cancel{dx\,dy}}{r}\ dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{2} (r^2)\ \sin^2\theta + \cos^2\theta\ r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{2} r^2\, r\ dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{2} r^3\ dr\, d\theta$$

$$= \int_0^{\pi/2} \left(\frac{r^4}{4}\right)_0^2 d\theta$$

$$= \int_0^{\pi/2} \frac{16}{4}\ d\theta$$

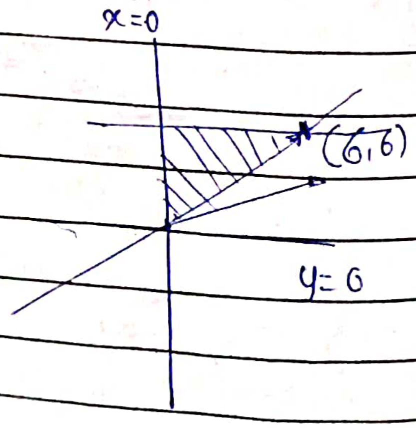
$$= \int_0^{\pi/2} 4\ d\theta$$

$$= 4\left[\theta\right]_0^{\pi/2}$$

$$= 4\left(\frac{\pi}{2} - 0\right)$$

$$= 2\pi$$

5) $\int_0^6 \int_0^y x\, dx\, dy$



$x=0 \quad x=y$

$y=0 \quad y=6$

~~r=~~ $y=6$

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$r \sin\theta = 6$

$r = \operatorname{cosec} 6$

$r = 6 \operatorname{cosec}\theta$

$r = 0$ to $6 \operatorname{cosec}\theta$

$\tan\theta = \frac{6-0}{6-0} = \frac{6}{6} = 1$

$\theta = \frac{\pi}{4}$ to $\frac{\pi}{2}$

$$I = \int_{\pi/2}^{\pi/2} \int_{0}^{6\operatorname{cosec}} x r^2 \cancel{\sin\theta} \cos\theta \, dr \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left[\frac{r^3}{3} \cos\theta \right]_0^{6\operatorname{cosec}\theta} \cancel{d\theta}$$

$$= \frac{1}{3} \int_{\pi/4}^{\pi/2} \left[6^3 \operatorname{cosec}\theta^3 - 0 \right] \cos\theta \, d\theta$$

$$= \frac{1}{3} \times 6^3 \int_{\pi/4}^{\pi/2} \operatorname{cosec}^2\theta \cdot \cot\theta \, d\theta$$

put $\cot\theta = t$ when

$-\operatorname{cosec}^2\theta \, d\theta = dt$

$\theta = \pi/4 \quad t = 0$

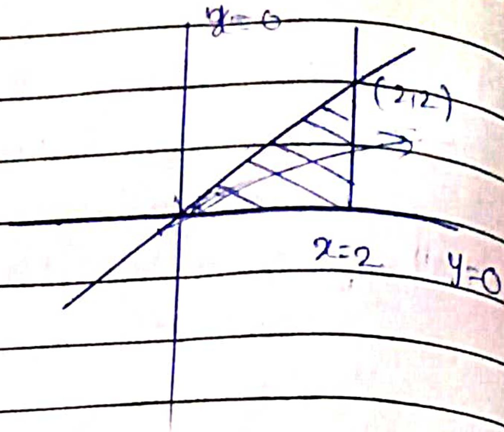
$\theta = \pi/2 \quad t = 1$

$$= \frac{216}{3} \int_{1}^{0} -t \, dt$$

$$= \frac{-216}{3} \left[\frac{t^2}{2} \right]_1^0$$

$$= \frac{-216}{3} \left[-\frac{1}{2} \right] = -72 \times \frac{1}{2} = \underline{\underline{36}}$$

6) $\int_0^2 \int_0^x y \, dy \, dx$



$y=0 \quad y=x$

$x=0 \quad x=2$

Now

$\boxed{r=0}$ to $x=2$

$r\cos\theta = 2$

$r = \sec\theta$

$\boxed{r = 2\sec\theta}$

Now $\boxed{\theta = 0}$ to $\tan\theta = \frac{2-0}{2-0} = 1$

$\tan\theta = 1$

$\boxed{\theta = \frac{\pi}{4}}$

$$I = \int_0^{\pi/4}\int_0^{2\sec\theta} r^2 \sin\theta \, dr \, d\theta$$

$$= \int_0^{\pi/4} \sin\theta \left(\frac{r^3}{3}\right)_0^{2\sec\theta} d\theta$$

$$= \int_0^{\pi/4} \sin\theta \cdot \frac{2^3 \sec^3\theta}{3} \, d\theta$$

$$= \frac{8}{3}\int_0^{\pi/4} \sin\theta \cdot \sec^3\theta \, d\theta$$

$$= \frac{8}{3}\int \cancel{\sin\theta} \sec^2\theta \cdot \tan\theta \, d\theta$$

put $\tan\theta = t$

$\sec^2\theta \cdot d\theta = dt$

when $\theta \to 0 \quad t=0$

$\theta \to \pi/4 \quad t=1$

$$= \frac{8}{3}\int_0^1 t\,dt$$

$$= \frac{8}{3}\left(\frac{t^2}{2}\right)_0^1$$

$$= \frac{\cancel{8}^{4}}{3}\left(\frac{1}{2}\right)$$

$$\boxed{I = \frac{4}{3}}$$

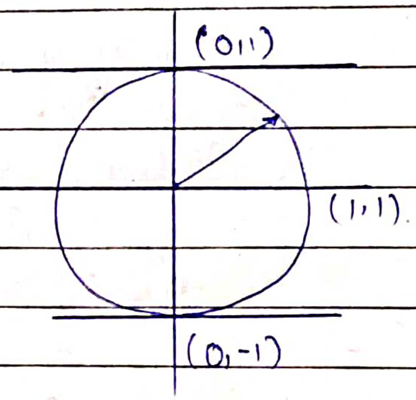
7) $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \log(x^2+y^2+1)\,dx\,dy$$

$\cancel{r}\,x\,y = \sqrt{1-x^2}$ $\quad y^2+x^2=1$

$xy = \sqrt{1-x^2}$

$xy = 1$

$xy = -1$



$r = 0$ to 1

$\theta = 0$ to 2π

$$\int_0^{2\pi}\int_0^{1} \log(x^2+y^2+1)\,\cancel{dr}\,dx\,dy$$

$\cancel{x^2+y^2+1}$ $\quad x = r\cos\theta$

$y = r\sin\theta$

$\boxed{x^2+y^2 = r^2}$

$$= \int_0^{2\pi}\int_0^{1} \log(r^2+1)\,r\,dr\,d\theta$$

$$= \int_0^{2\pi}$$

$r^2+1 = t$

$2r\,dr = dt$

$r\,dr = \frac{dt}{2}$

when

$r \to 0 \quad t = 1$

$r \to 1 \quad t = 2$

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$$= \int_0^{2\pi} \int_1^2 \left(\log(t) \cdot \frac{dt}{2} \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \left(\int_1^2 \log t \, dt \right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \left[\log t \cdot (t) - \int \frac{1}{\cancel{t}} \cdot \cancel{t} \, dt \right]_1^2 d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \left[t \log t - t \right]_1^2 d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \left[2 \log 2 - 2 \right] \cancel{d\theta} - \left[0 - 1 \right] d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \left[2 \log 2 - 2 \right] + 1 \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{\cancel{2}} \cancel{2} (\log 2 - 1) + \frac{1}{2} \, d\theta$$

$$= \int_0^{2\pi} (\log 2 - 1) + \frac{1}{2} \, d\theta$$

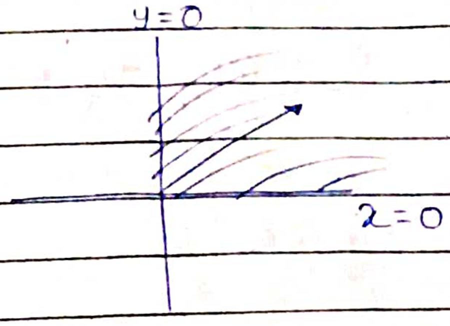
$$= \int_0^{2\pi} \cancel{\log 2 + 1 \, d\theta} \; \log 2 - 1 + \frac{1}{2}$$

$$= \log 2 \, (\theta)_0^{2\pi} \mp \frac{1}{2} (\theta)_0^{2\pi}$$

$$= \log 2 \, (2\pi - 0) \mp \pi$$

$$= \log 2 \, (2\pi) \mp \pi$$

8) $\displaystyle\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$



I

$0 \leq r \leq \infty$

$0 \leq \theta \leq \pi/2$

$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\sin^2\theta + \cos^2\theta)$

$\boxed{x^2+y^2 = r^2}$

$\displaystyle\int_0^{\pi/2}\int_0^{\infty} e^{-(r^2)}\, r\,dr\,d\theta.$

$= \quad r^2 = t$ | when

$2r\,dr = dt$ | $r=0 \quad t=0.$

$r\,dr = \frac{dt}{2}$ | $r=\infty \quad t=\infty$

$= \displaystyle\int_0^{\pi/2}\left[\int_0^{\infty} e^{-t}\,\cancel{dr}\frac{dt}{2}\right]d\theta$

$= \displaystyle\int_0^{\pi/2}\frac{1}{2}\left[\frac{e^{-t}}{-1}\right]_0^{\infty} d\theta$

$= \displaystyle\frac{1}{2}\int_0^{\pi/2} -\left[e^{-\infty} - e^{0}\right] d\theta$

$= \displaystyle\frac{1}{2}\int_0^{\pi/2} -[0-1]\,d\theta$

$= \displaystyle\frac{1}{2}\int_0^{\pi/2} 1\,d\theta$

$= \displaystyle\frac{1}{2}(\theta)_0^{\pi/2} \quad = \frac{1}{2}\left(\frac{\pi}{2}-0\right)$

$= \dfrac{\pi}{4}$

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11 Volume integrals

$x = 0$ to 1

$y = 0$ to 2

$z = 0$ to 3.

+ first octant

① $V = \int_0^3 \int_0^2 \int_0^1 f(x,y)\, dx\, dy\, dz$

② $V = \int_0^2 \int_0^1 \int_0^3 f(x,y)\, \cancel{dy}\, dz\, dx\, dy$

③ $V = \int_0^1 \int_0^3 \int_0^2 f(x,y)\, dy\, dz\, dx$

Similarly we can write remaining 3 Integral

$$V = \int_0^1 \int_0^2 \int_0^3 1\, dz\, dy\, dx$$

$$= \int_0^1 \int_0^2 (z)_0^3\, dy\, dx$$

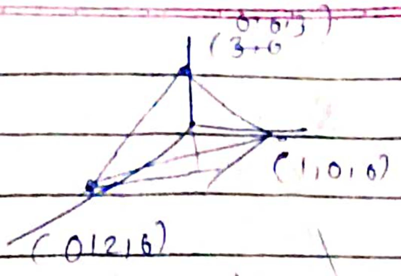
$$= \int_0^1 \int_0^2 (3)\, dy\, dx$$

$$= \int_0^1 3 \left[y \right]_0^2 dx$$

$$= \int_0^1 3(2-0)\, dx$$

$$= 6 \int_0^1 dx$$

$$= 6 \left[1-0 \right] \quad = \underline{\underline{6}}$$



plane

12 Tetrahedron equation

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

first ~~quar~~ octant so lower limit $\underline{\underline{0}}$

$$6x+3y+2z=6$$

$0 \le 6x \le 6-3y+2z$	$0 \le x \le 1-\frac{1}{2}y+\frac{2z}{3}$
$0 \le 3y \le 6-2z-6x$	
$0 \le 2z \le 6-6x-3y$	

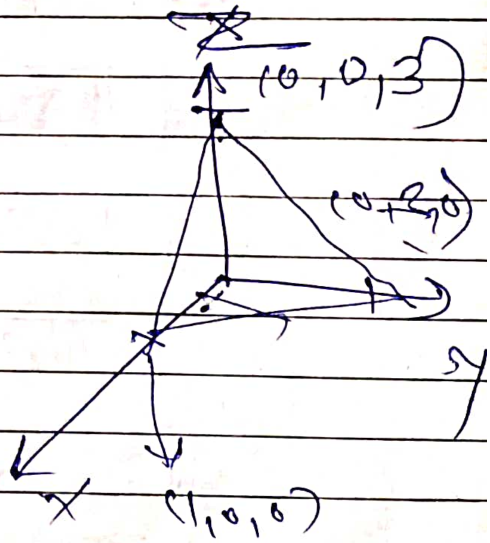
Now ~~equati~~ equation equation

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\frac{6x+3y+2z}{6}=\frac{6}{6}$$

$$x+\frac{1}{2}y+\frac{1}{3}z=1$$

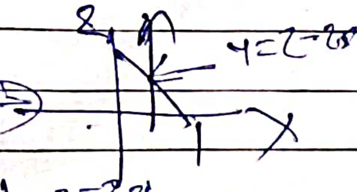
$$\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$$



So $a=1$
$b=2$
$z=3$

$$0 \le x \le 1$$

$z=\frac{6-6x-3y}{2}$ $\quad y \le 2$
$z \le 3$



$$6x+3y=6$$

$$y=\frac{6-6x}{3}$$

$$=2-2x$$

$$\int_0^1\int_0^{2-2x}\int_0^{\frac{6-6x-3y}{2}} dz\,dy\,dx$$

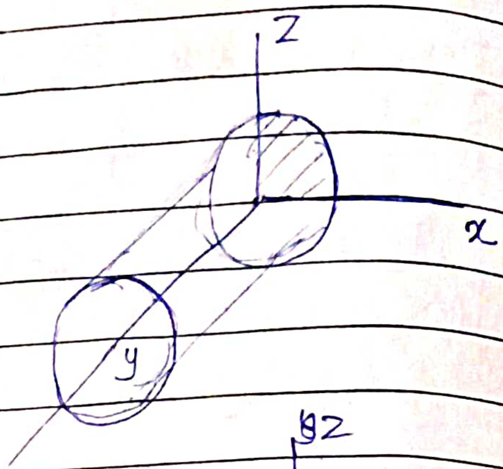
$x^2+z^2=4.$

13). Now

$0 \leq y \leq 3$

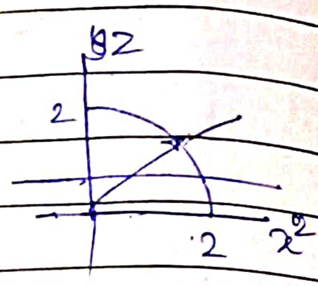
$0 \leq x \leq \sqrt{4-z^2}$

$0 \leq z \leq 2$



Now, Volume

$$V = \int_0^2 \int_0^3 \int_0^{\sqrt{4-z^2}} 1 \, dx \, dy \, dz$$



$$= \int_0^2 \int_0^3 (x)_0^{\sqrt{4-z^2}} dy \, dz$$

$$= \int_0^2 \int_0^3 (\sqrt{4-z^2}) \, dy \, dz$$

$$= \int_0^2 \left[\sqrt{4-z^2} \, (y)_0^3 \right] dz$$

$$= \int_0^2 \left[\sqrt{4-z^2} \, (3-0) \right] dz$$

$$= \int_0^2 \left[\sqrt{4-z^2} \, (3) \right] dz$$

$$= 3 \int_0^2 \sqrt{4-z^2} \, dz$$

$$= 3 \int_0^2 (4-z^2)^{1/2} \, dz$$

$$= 3 \left[\frac{(4-z)^{3/2}}{3/2} \right]_0^2$$

$$= \cancel{3} \times \frac{2}{\cancel{3}} \left[(4-z)^{3/2} \right]_0^2$$

$$= 2 \left[(4-2)^{3/2} - 4^{3/2} \right]$$

$$= 2\left[(2)^{3/2} - 4^{1/2}\, 2^{3}\right]$$

$$= 2\left[\qquad - 8\right]$$

$$=$$

$$= 3\int_0^2 \sqrt{2^2 - z^2}\, dz.$$

$\int$ formula $\sqrt{a^2-x^2} = \dfrac{x}{2}\sqrt{a^2-x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x}{a}\right)$

$$= 3\left[\frac{z}{2}\sqrt{4-z^2} + \frac{4}{2}\sin^{-1}\left(\frac{z}{2}\right)\right]_0^2$$

$$= 3\left[\frac{2}{2}\sqrt{4-4} + \frac{4}{2}\sin^{-1}\left(\frac{2}{2}\right)\right]$$

$$- \left[0 + \cancel{5112}\, 2\sin^{-1}(0)\right]$$

$$= 3\left[1\sqrt{0} + 2\sin^{-1}(1) - 0 - 2\sin^{-1}(0)\right]$$

$$= 3\left[0 + \cancel{2}\frac{\pi}{\cancel{2}} - 2(0)\right]$$

$$= \underline{3\pi}$$

14) $z = 8 - x^2 - y^2 \qquad z = x^2 + y^2$

equate equation

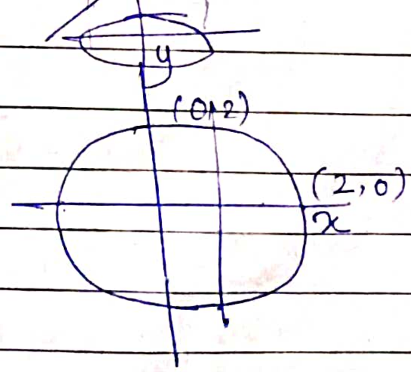
$8 - x^2 - y^2 = x^2 + y^2$

$8 = x^2 + x^2 + y^2 + y^2$

$8 = 2x^2 + 2y^2$

$8 = 2(x^2 + y^2)$

$\boxed{4 = x^2 + y^2}$



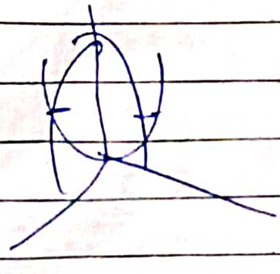
so, $r = 2$

Now

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

$x = -2$ to 2



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(17)

$$V = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{8-x^2-y^2}^{x^2+y^2} 1 \, dz \, dy \, dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left(z \right)_{8-x^2-y^2}^{x^2+y^2} dy \, dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left(x^2+y^2 - 8 + x^2 + y^2 \right) dy \, dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left(2x^2 + 2y^2 - 8 \right) dy \, dx$$

$$= 2 \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left(x^2 + y^2 - 4 \right) dy \, dx$$

$$= 2 \int_{-2}^{2} \left(x^2 y + \frac{y^3}{3} - 4y \right)_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$

$$= 2 \int_{-2}^{2} \left(x^2 \sqrt{4-x^2} + \frac{\left(\sqrt{4-x^2}\right)^3}{3} - 4\sqrt{4-x^2} \right) - \left(x^2 - \sqrt{4-x^2} + \frac{\left(-\sqrt{4-x^2}\right)^3}{3} - 4\sqrt{4-x^2} \right.$$

$$= 2 \int_{-2}^{2} 2x^2 \sqrt{4-x^2} + \frac{2}{3} \left(\sqrt{4-x^2} \right)^3 - 8\sqrt{4-x^2} \, dx$$

15. Evaluate

a) $\int_0^1 \int_0^1 \int_0^1 (x^2+y^2+z^2)\, dz\, dy\, dx$

$\Rightarrow \int_0^1 \int_0^1 \left[x^2 z + y^2 z + \frac{z^3}{3} \right]_0^1 dy\, dx$

$= \int_0^1 \int_0^1 \left(x^2 + y^2 + \frac{1}{3} \right) dy\, dz$

$= \int_0^1 \int_0^1 \left(x^2 y + \frac{y^3}{3} + \frac{1}{3} y \right)_0^1 dz$

$= \int_0^1 \left(x^2 + \frac{1}{3} + \frac{1}{3} \right) dx$

$= \int_0^1 \left(\frac{x^3}{3} + \frac{1}{3} x + \frac{1}{3} x \right)_0^1$

$= \cancel{\frac{x^2 + 1 + 1}{3 \quad 3}} \quad \underline{\underline{1}}$

b) $\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dx\, dy\, dz$

$= \int_1^e \int_1^e \left| \frac{\log x}{yz} \right|_1^e dy\, dz$

$= \int_1^e \int_1^e \frac{1}{yz} \left| \log e - \log 1 \right| dy\, dz$

$= \int_1^e \int_1^e \frac{1}{yz} \left| \log_e \right| dy\, dz$

$= \int_1^e \int_0^e \frac{1}{z} (\log y)_1^e\, dz$

$$= \int_1^e \frac{1}{z} (\log e)\, dz$$

$$= \int_1^e \frac{1}{z}\, dz$$

$$= (\log z)_1^e$$

$$= \log e - \log 1$$

$$= \underline{\underline{1}}$$

c). $\int_0^{2\pi} \int_0^{\theta/2\pi} \int_0^{\sqrt{3+2+r^2}}$

$$\int_0^1 \int_0^\pi \int_0^\pi y \sin z \, dx\, dy\, dz$$

$$= \int_0^1 \int_0^\pi y \sin z\, (x)_0^\pi\, dy\, dz$$

$$= \int_0^1 \int_0^\pi y \sin z\, (\pi - 0)\, dy\, dz$$

$$= \int_0^1 \int_0^\pi y \sin z \pi\, dy\, dz$$

$$= \int_0^1 \sin z \pi \left(\frac{y^2}{2}\right)_0^\pi dz$$

$$= \int_0^1 \sin z \pi \left(\frac{\pi^2}{2}\right) dz$$

$$= \frac{\pi}{2} \int_0^1 \pi^2 \sin z\, dz$$

$$= \frac{\pi^3}{2} (-\cos z)_0^1$$

$$= \frac{\pi^3}{2} (-\cos 1 + \cos 0)$$

$$= \frac{\pi^3}{2} (-\cos 1 + 1)$$

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$$= \frac{\pi^3}{2}(1-\cos 1)$$

d). $$\int_0^2 \int_{\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz\, dy\, dx$$

$$= \int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz\, dx\, dy$$

$$= \int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (z)_0^{2x+y}\, dx\, dy$$

$$= \int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} (2x+y)\, dx\, dy$$

$$= \int_0^2 \left(\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} 2x+y\, dx \right) dy$$

$$= \int_0^2 \left(\frac{\cancel{2}x^2}{\cancel{2}} + yx \right)_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} dy$$

$$= \int_0^2 \left((\sqrt{4-y^2})^2 + y(\sqrt{4-y^2}) - \left[(\sqrt{4-y})^2 + y(-\sqrt{4-y^2}) \right] \right) dy$$

$$= \int_0^2 \left(\cancel{(4-y^2)} + y\sqrt{4-y^2} \right) - \cancel{(4-y^2)} + y\cdot\sqrt{4-y^2})\, dy$$

$$= \int_0^2 2y\sqrt{4-y^2}\, dy$$

put $4-y^2 = t$

$0 - 2y\, dy = dt$

$-2y\, dy = dt$

$2y\, dy = -dt$

when

$y \to 0 \quad t = 4$

$y \to 2 \quad t = 0.$

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$$= \int_4^0 \sqrt{t} \; -dt$$

$$= -\int_4^0 t^{1/2}\, dt$$

$$= -\int_4^0 \frac{t^{3/2}}{3/2}\; dt$$

$$= \frac{-2}{3}\left(t^{3/2}\right)_4^0$$

$$= \frac{-2}{3}\left(-4^{3/2}\right)$$

$$= \frac{-2}{3}\left(-4^{1/2}\right)^3$$

$$= \frac{+2}{3} \times 8$$

$$= \frac{+16}{3}.$$

16) Evaluate cylindrical co-ordinates

a) $$\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} dz\, r\, dr\, d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} r\; dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 r\,(z)_r^{\sqrt{r^2-r^2}}\; dr\, d\theta.$$

$$= \int_0^{2\pi}\int_0^1 \left(\sqrt{2-r^2} - r\right) r\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(r\sqrt{2-r^2} - r^2\right) dr\, d\theta$$

Put $2-r^2 = t$	limit
$-2r\,dr = dt$	$r=0 \quad t=2$
$r\,dr = \dfrac{dt}{-2}$	$r=1 \quad t=1$

$$\int_0^{2\pi}\left[\int_2^1 \frac{dt}{-2}\sqrt{t}\,\cancel{dt} - \int_0^1 r^2\,dr\right] d\theta$$

$$= \int_0^{2\pi}\left[\frac{1}{-2}\int_2^1 \sqrt{t}\,dt\right] - \left[\frac{r^3}{3}\right]_0^1 d\theta$$

$$= \int_0^{2\pi}\left[\frac{1}{-2}\left(\frac{t^{3/2}}{3/2}\right)_2^1 - \left[\frac{1}{3} - \frac{0}{3}\right]\right] d\theta$$

$$= \int_0^{2\pi}\left[\frac{1}{-2}\times\frac{2}{3}\left(t^{3/2}\right)_2^1 - \left[\frac{1}{3}\right]\right] d\theta$$

$$= \int_0^{2\pi}\left(\frac{-1}{3}\right)\left(1^{3/2} - 2^{3/2}\right) - \frac{1}{3}\, d\theta$$

$$= \int_0^{2\pi}\frac{-1}{3}\left(1 - \sqrt{8}\right) - \frac{1}{3}\, d\theta$$

$$= \int_0^{2\pi}\frac{-1}{3} + \frac{\sqrt{8}}{3} - \frac{1}{3}\, d\theta$$

$$= \int_0^{2\pi}\frac{-2}{3} + \frac{\sqrt{8}}{3}\, d\theta$$

$$= \int_0^{2\pi}\frac{-2+\sqrt{8}}{3}\, d\theta$$

$$= \frac{-2+\sqrt{8}}{3}(2\pi) = 2\pi\times\frac{\sqrt{8}-2}{3}$$

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b) $$\int_0^{2\pi}\int_0^3\int_{z=r^2/3}^{\sqrt{18-r^2}} dz\, r\, dr\, d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_0^3 r\left[\sqrt{18-r^2}-\frac{r^2}{3}\right] dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^3\left(r\sqrt{18-r^2}-\frac{r^3}{3}\right) dr\, d\theta$$

NOW,

$$\int_0^{2\pi}\left[\int_0^3 \left(r\sqrt{18-r^2}\right) dr - \int_0^3 \frac{r^3}{3}\, dr\right] d\theta$$

put $18-r^2=t$
$0-2r\,dr=dt$
$r\,dr=\frac{-dt}{2}$

when
$r\to 0 \quad t=18$
$r\to 3 \quad t=9$

$$\int_0^{2\pi}\left[\int_{18}^{9}\left(-\sqrt{t}\,\frac{dt}{2}\right)-\frac{1}{3}\left(\frac{r^4}{4}\right)_0^3\right] d\theta$$

$$\int_0^{2\pi}\left[\frac{-1}{2}\left(\frac{t^{3/2}}{3/2}\right)_{18}^{9} - \left(\frac{r^4}{12}\right)_0^3\right] d\theta$$

$$= \int_0^{2\pi} \frac{-1}{\cancel{2}}\times\frac{\cancel{2}}{3}\left(9^{3/2}-18^{3/2}\right)-\frac{1}{12}\left(81-0\right) d\theta$$

$$= \int_0^{2\pi} \frac{-1}{3}\left(27-18\sqrt{18}\right) \frac{-1}{12}(81)\, d\theta$$

$$= \int_0^{2\pi} \left(\frac{-1}{3}\left(27-18\sqrt{18}\right)-\frac{81}{12}\right) d\theta$$

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$$= \frac{-1}{3}\int_0^{2\pi}\left(\cancel{18}\,27 - 18\times3\sqrt{2} - \frac{81}{12}\right)d\theta$$

$$= \frac{-1}{3}\int_0^{2\pi}\left(27 - 54\sqrt{2} - \frac{81}{4}\right)d\theta$$

$$= \frac{-1}{3}\left(27\theta - 54\sqrt{2}\,\theta - \frac{81}{4}\theta\right)_0^{2\pi}$$

$$= \frac{-1}{3}\left(27\cdot 2\pi - 54\sqrt{2}\,(2\pi) - \frac{81}{4}(2\pi)\right)$$

$$= \frac{-1}{3}\int_0^{2\pi}\left(\frac{108 - 216\sqrt{2} - 81}{4}\right)d\theta$$

$$= \frac{-1}{3}\int_0^{2\pi}\frac{(27 - 216\sqrt{2})}{4}\,\theta\Big|_0^{2\pi}$$

$$= \frac{-1}{\cancel{12}}\,\cancel{27}^{9}(1 - 8\sqrt{2})\,\cancel{2}\pi$$

$$= \frac{-9}{2}\pi(1 - 8\sqrt{2})$$

$$= \frac{9}{2}\pi(8\sqrt{2} - 1)$$

c) $\int_0^{2\pi}\int_0^{\theta/2\pi}\int_0^{\sqrt{3+24r^2}} dz\, r\, dr\, d\theta$

$dz =$

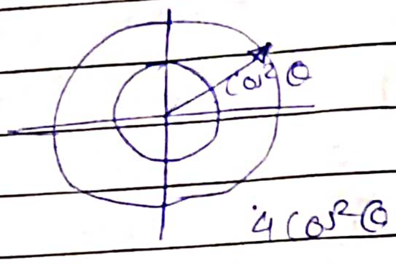
—/—

d) $\int_0^{\pi}\int_0^{a/\pi}\int_{\sqrt{4-y^2}}^{3\sqrt{4-y^2}} z \, dz \, r \, dr \, d\theta$

$I = \int_{\theta=0}^{\pi}\int_{r=0}^{a/\pi}\int_{\sqrt{4-r^2\sin\theta}}^{3\sqrt{4-r^2\sin\theta}} zr \, dz \, dr \, d\theta$

Complicated.

7) $x^2+y^2=r^2$



① circle

$x^2+y^2 = \cos^2\theta$

② circle

$x^2+y^2 = r^2 4\cos^2\theta$

Now,

$\cos\theta \leq r \leq 2\cos\theta$

$0 \leq \theta \; 2\pi$

$0 \leq z \leq 3-y$

$0 \leq z \leq 3 - r\sin\theta$

$I = \int_0^{2\pi}\int_{\cos\theta}^{2\cos\theta}\int_0^{3-r\sin\theta} (1)\, r \, dz \, dr \, d\theta$

$= \int_0^{2\pi}\int_{\cos\theta}^{2\cos\theta} r.(1)\Big|_0^{3-r\sin\theta} dr \, d\theta$

$= \int_0^{2\pi}\int_{\cos\theta}^{2\cos\theta} r(3 - r\sin\theta) \, dr \, d\theta$

$= \int_0^{2\pi}\int_{\cos\theta}^{2\cos\theta} (3r - r^2\sin\theta) \, dr \, d\theta$

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$$= \int_0^{2\pi} \left[\frac{3r^2}{2} - \frac{r^3}{3} \sin\theta \right]_{\cos\theta}^{2\cos\theta} d\theta$$

$$= \int_0^{2\pi} \frac{3\,\cancel{4}^{2}\cos^2\theta}{\cancel{2}} - \frac{8\cos^3\theta}{3}\sin\theta -$$

$$\frac{3}{2}\cos^2\theta - \frac{1}{3}\cos^3\theta\sin\theta)\,d\theta$$

$$= \int_0^{2\pi} 6\cos^2\theta - \frac{8}{3}\cos^3\theta\sin\theta - \frac{3}{2}\cos^2\theta + \frac{1}{3}\cos^3\theta \sin\theta$$

$$= \int_0^{2\pi} \left(\frac{9}{2}\cos^2\theta - \frac{7}{3}\cos^3\theta \cdot \sin\theta \right) d\theta$$

$$= \int_0^{2\pi} \cos^2\theta \left[\frac{9}{2} - \frac{7}{3}\cos\theta \cdot \sin\theta \right] d\theta$$

$$= \int_0^{2\pi} \frac{1+\cos 2\theta}{2} \left[\frac{9}{2} - \frac{7}{3}\frac{\sin 2\theta}{2} \right] d\theta$$

$$= \int_0^{2\pi} \frac{1+\cos 2\theta}{2} \left[\frac{9}{2} - \frac{7\sin 2\theta}{2\times 3} \right] d\theta$$

$$= \frac{1}{4} \int_0^{2\pi} (1+\cos 2\theta)\left(9 - \frac{7\sin 2\theta}{3} \right) d\theta$$

$$= \frac{1}{4} \int_0^{2\pi} \left(9 - \frac{7\sin 2\theta}{3} + 9\cos 2\theta - \frac{7}{3}\cos 2\theta \cdot \sin 2\theta \right)$$

$$= \frac{1}{4} \int_0^{2\pi} \left(9 - \frac{7}{3}\sin 2\theta + 9\cos 2\theta - \frac{7}{3}\frac{\sin 4\theta}{2} \right).$$

$$= \frac{1}{4} \int_0^{2\pi} \left(9 - \frac{7}{3}\sin 2\theta + 9\cos 2\theta - \frac{7}{6}\sin 4\theta \right).$$

$$= \frac{1}{4} \left[9\theta + \frac{7}{3}\frac{\cos 2\theta}{2} + \frac{9\sin 2\theta}{2} + \frac{7}{6}\frac{\cos 4\theta}{4} \right).$$

$$= \frac{1}{4} \left[18\pi + \frac{7}{6}(1) + \frac{9}{2} + \frac{7}{24}(1) \right)$$

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$$-\left(0 + \frac{7}{6} + \frac{9}{2} + \frac{7}{24}(\pi)\right)$$

$$= \frac{19}{4}\cdot\frac{1}{4}(18\pi)$$

$$= \frac{9}{2}\pi$$

18. Evaluate spherical co-ordinate Integral.

(1). $$\int_0^{\pi/2}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin^2\phi \, d\rho \, d\phi \, d\theta$$

θ - 0 to $\pi/2$

ϕ - 0 to π

ρ - 0 to $2\sin\phi$

Now

$$\int_0^{\pi/2}\int_0^{\pi}\left[\int_0^{\sin\phi} \rho^2 \, d\rho\right] \sin^2\phi \, d\phi \, d\theta$$

$$\int_0^{\pi/2}\int_0^{\pi}\left[\frac{\rho^3}{3}\right]_0^{2\sin\phi} \sin^2\phi \, d\phi \, d\theta$$

$$\int_0^{\pi/2}\left[\int_0^{\pi}\frac{8\sin^{5}\phi}{3} \, d\phi\right] d\theta$$

$$\int_0^{\pi/2}\frac{8}{3}\left[\frac{5\sin 6\phi}{6\cos\phi}\right]_0^{\pi}$$

$$\int_0^{\pi/2}\frac{8}{3}\left[\frac{\sin^6\pi}{6\cos\pi} - 0\right] d\theta$$

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$$\int_0^{\pi/2} 0 \, d\theta$$

$$\int_0^{\pi/2}\int_0^{\pi}\left[\frac{8}{3}\sin^3\phi \cdot \sin^2\phi\right] d\phi \, d\theta$$

$$\int_0^{\pi/2}\int_0^{\pi}\left[\frac{8}{3}\right]\left[\sin^4\phi \cdot \sin\phi\right] d\phi \, d\theta$$

$$\int_0^{\pi/2}\int_0^{\pi}\frac{8}{3}\left[(1-\cos^2\phi)^2 \cdot \sin\phi \, d\phi \, d\theta\right.$$

$\cos\phi = t$	when
$-\sin\phi \, d\phi = dt$	$\phi - 0 \quad t = 1$
$\sin\phi \, d\phi = -dt$	$\phi - \pi \quad t = -1$

$$\int_0^{\pi/2}\int_{1}^{-1} -\frac{8}{3}(1-t^2)^2 \, dt \, d\theta$$

$$\int_0^{\pi/2} -\frac{8}{3}\int_1^{-1}(1-2t^2+t^4)\, dt \, d\theta$$

$$\int_0^{\pi/2} -8/3\left(t - \frac{2t^3}{3} + \frac{t^5}{5}\right)_1^{-1} d\theta$$

$$\int_0^{\pi/2} -\frac{8}{3}\left(-1+\frac{2}{3}-\frac{1}{5}\right)-\left(1-\frac{2}{3}+\frac{1}{5}\right) d\theta$$

$$\int_0^{\pi/2} -\frac{8}{3}\left(-1+\frac{2}{3}-\frac{1}{5}-1+\frac{2}{3}-\frac{1}{5}\right) d\theta$$

$$-\frac{8}{3}\int_0^{\pi/2} -2+\frac{4}{3}-\frac{2}{5} \, d\theta$$

$$\int_0^{\pi/2}\left(\frac{16}{3}-\frac{32}{9}+\frac{16}{15}\right) d\theta$$

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$$\int_0^{\pi/2} \frac{128}{45}\, d\theta$$

$$= \frac{128}{45}\theta \Big|_0^{\pi/2}$$

$$= \frac{64}{45}\pi.$$

(2) $$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} (\rho\cos\phi)\,\rho^2 \sin\phi \; d\rho\, d\phi\, d\theta.$$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} \rho^3 \cos\phi \cdot \sin\phi \; d\rho\, d\phi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi/4} \left[\frac{\rho^4}{4}\right]_0^2 \sin\phi \cdot \cos\phi \cdot d\phi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi/4} \left[\frac{16}{4}=4\right] \sin\phi \cdot \cos\phi \cdot d\phi\, d\theta$$

$$\frac{4}{2}\int_0^{2\pi}\int_0^{\pi/4} \frac{\sin 2\phi}{\cancel{2}}\, d\phi\, d\theta$$

$$2\int_0^{2\pi} \cancel{\int_0^{\pi/4}} \left(-\frac{\cos 2\phi}{2}\right)\Big|_0^{\pi/4} \cancel{d\phi}\, d\theta$$

$$= 1 \int_0^{2\pi} \left(-\cos 2\,\pi/4 + \cos 2\times 0\right) d\theta$$

$$= 1 \int_0^{2\pi} (-0+1)\, d\theta$$

$$= 1 \int_0^{2\pi} (1)\, d\theta.$$

$$= \theta \Big|_0^{2\pi}$$

$$= \underline{2\pi}$$

(3) $$\int_0^{2\pi}\int_0^{\pi}\int_0^{(1-\cos\phi)/2} \rho^2 \sin^2\phi \, d\rho \, d\phi \, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi}\left[\int_0^{(1-\cos\phi)/2} \rho^2 \, d\rho\right] \sin^2\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho^3}{3}\right]_0^{(1-\cos\phi)/2} \sin^2\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\frac{1}{8}\int_0^{\pi}\frac{\left(\frac{(1-\cos\phi)}{2}\right)^3}{3} \sin^2\phi \, d\phi \, d\theta$$

$$\int_0^{2\pi}\frac{1}{24}\int_0^{\pi}\left((1-\cos\phi)^3 \cdot \sin^2\phi \, d\phi \, d\theta\right.$$

To internet.

$$\int_0^{2\pi}\frac{1}{24\cancel{8}}\cdot\left(\frac{7\pi}{8}\right) d\theta$$

$$\frac{7}{24\times8}\pi\left(\theta\right)_0^{2\pi}$$

$$\frac{7\pi}{192}\times 2\pi.$$

$$= \frac{14\pi^2}{192}$$

$$= \frac{7}{96}\pi^2.$$

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(4). $$\int_0^{3\pi/2}\int_0^{\pi}\int_0^{1} 5\rho^3 \sin^3\phi \, d\rho\, d\phi\, d\theta$$

$$\int_0^{3\pi/2}\int_0^{\pi} 5\sin^3\phi \left[\frac{\rho^4}{4}\right]_0^1 d\phi\, d\theta$$

$$= \int_0^{3\pi/2}\int_0^{\pi} \frac{5}{4}\sin^3\phi \, d\phi\, d\theta$$

$$= \int_0^{3\pi/2} \frac{5}{4}\int_0^{\pi} \sin^3\phi \, d\phi\, d\theta$$

$$\int_0^{3\pi/2} \frac{5}{4}\int_0^{\pi} (1-\cos^2\phi)\cdot \sin\phi \, d\phi\, d\theta$$

put $\cos\phi = t$ | $\phi \to 0 \quad 1$

$-\sin\phi\, d\phi = dt$ | $\phi \to \pi \quad -1$

$\sin\phi\, d\phi = -dt$

$$\int_0^{3\pi/2} \frac{5}{4}\int_1^{-1} (1-t^2)\, -dt\, d\theta$$

$$\int_0^{3\pi/2} \frac{-5}{4}\int \left(\cancel{1-2t^2+t}\right)\left(t-\frac{t^3}{3}\right)_1^{-1} d\theta$$

$$\int_0^{3\pi/2} \frac{-5}{4}\left(-1+\frac{1}{3}\right)-\left(1-\frac{1}{3}\right) d\theta$$

$$= \int_0^{3\pi/2} \frac{-5}{4}\left(-1+\frac{1}{3}-1+\frac{1}{3}\right) d\theta$$

$$= \int_0^{3\pi/2} \frac{-5}{4}\left(-2+\frac{2}{3}\right) d\theta$$

$$= \int_0^{3\pi/2} \frac{-5}{4}\left(\frac{-6+2}{3}\right) d\theta$$

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$$= \int_0^{3\pi/2} \frac{-5}{4} \cdot \frac{-4}{3} \, d\theta$$

$$= \int_0^{3\pi/2} \cdot \frac{20}{12} \, d\theta$$

$$\frac{20}{12} \left(\theta\right)_0^{3\pi/2}$$

$$= \frac{\cancel{20}^{10}}{1\cancel{2}4} \cdot \frac{3\pi}{\cancel{2}}$$

$$= \frac{10}{4} \pi.$$

$$= \frac{5}{2} \pi$$

19. Evaluate the integral

$$\int_0^4 \int_{y/2}^{y/2+1} \frac{2x-y}{2} \, dx \, dy.$$

$$u = \frac{2x-y}{2} \qquad v = y/2$$

$$\boxed{y = 2v}$$

$$u = \frac{2x}{2} - \frac{y}{2}$$

$$u = x - y/2$$

$$u = x - v$$

$$\boxed{u + v = x}$$

$$\text{find } |J| = \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = \underline{\underline{2}}$$

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xy plane $\qquad\qquad$ uv plane

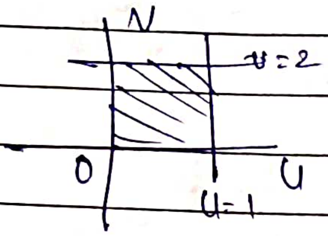
~~$x = u+v$~~

$x = y/2 \longrightarrow$ ~~$y=$~~ $u=0$

$x = y/2+1 \longrightarrow u=1$

$y=0 \longrightarrow v=0.$

$y=4. \longrightarrow v=2.$



Now. $I = \iint_R f(x,y) = \iint_{uv} u.2\, dx\,dy$

$= \int_0^1 \int_0^2 2u\, \cancel{dx\,dy}\, du\, dv$

$= \int_0^1 2u \left| v \right|_0^2$

$= \cancel{4u}\Big|_0^1 \int_0^1 4u\, du$

$= 4 \frac{u^2}{2}\Big|_0^1$

$= 4 \cdot \frac{1}{2}$

$= \underline{\underline{2}}.$

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26 $u = x - y$ $\qquad$ $v = 2x + y$

$\boxed{\cancel{x = u + y}}$ $\qquad$ $\cancel{= 2(u+y)}$

$\boxed{y = x - u}$ $\qquad$ $v = 2x + y$

$\cancel{v - 2x = 2}$

$\boxed{\cancel{y = 2x - v}}$

$\cancel{2x - v = x - u}$

$\cancel{2x - x = -u + v}$

$\cancel{x = -u + v}$

$\boxed{\cancel{x = v - u}}$

$u = x - y$ $\qquad$ $v = 2x + y$

$y = x - u$ $\qquad$ $\boxed{v - 2x = y}$

$v - 2x = x - u$ $\qquad$ $y = v - 2\left(\frac{v+u}{3}\right)$

$v + u = x + 2x$

$\boxed{v + u = 3x}$ $\qquad$ $= v - \frac{2v - 2u}{3}$

$\boxed{x = \frac{v+u}{3}}$

$= \frac{3v - 2v - 2u}{3}$

$\boxed{y = \frac{v - 2u}{3}}$

first line

$y = -2x + 4$

$y + 2x = 4$

$\boxed{v = 4}$

for second line

$y = -2x + 7$

$y + 2x = 7$

$\boxed{v = 7}$

for 3 line

$y = x - 2$

$2 = x - y$

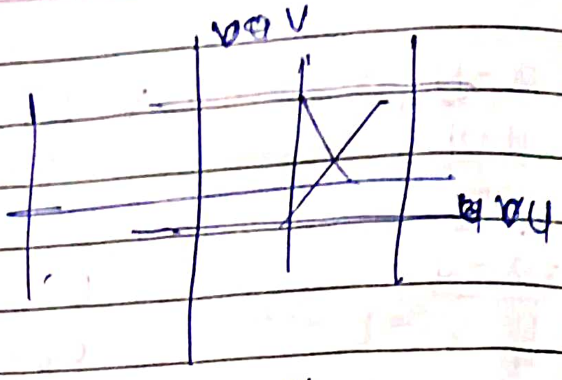
$\boxed{u = 2}$

$y = x + 1$

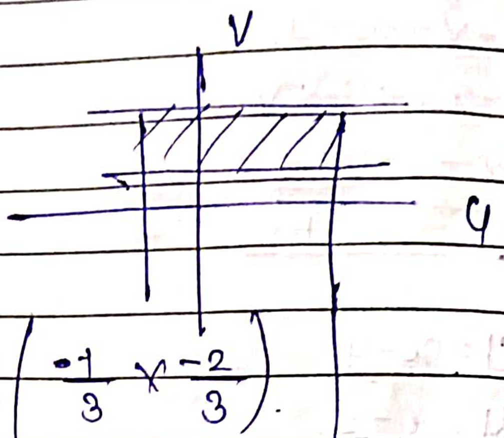
$x - y = -1$

$\boxed{u = -1}$

$$|J| = \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix}$$



$$\begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{-2}{3} & \frac{1}{3} \end{vmatrix}$$



$$= \left| \left(\frac{1}{3} \times \frac{1}{3}\right) - \left(\frac{1}{3} \times \frac{-2}{3}\right) \right|$$

$$= \left(\frac{1}{9}\right) + \left(\frac{2}{9}\right)$$

$$= \frac{3}{9} = \left|\frac{1}{3}\right|$$

$$\iint_R (2x^2 - xy - y^2)\, dx\, dy$$

$$= \int_{-1}^{2}\int_{4}^{7} (2x+y)(x-y)\, dx\, dy$$

$$= \int_{-1}^{2}\int_{4}^{7} v\,(u) \left|\frac{1}{3}\right| du\, dv$$

$$= \int_{-1}^{2} \left[\frac{v}{3} \left(\frac{u^2}{2}\right) \right]_4^7 dv$$

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$$\int_{1}^{2} \frac{v}{3}\left(\frac{49}{2} - 8\right) dv.$$

$$\frac{33}{6}\frac{v^2}{2}\Big|_{1}^{2}$$

$$\frac{33}{6}\left(\frac{4}{2} - \frac{1}{2}\right)$$

$$= \frac{33}{6}\left(\frac{\cancel{5}3}{2}\right)$$

$$= \frac{\cancel{165}\,99}{12}$$

$$= 33/4$$

22. $\cancel{u = 2x - 3y}$ $\qquad$ $\cancel{v = x + y}$

$\cancel{u + 3y = 2x}$ $\qquad$ $x = y - v.$

$\cancel{u + 3(y - v}$ $\qquad$ $\cancel{y = x + v.}$

$\cancel{u + 3y = 3(2y - v)}$

$\cancel{u + 3y = 3y - v.}$

$\cancel{u + 3y - 3y + v = 0}$

$\boxed{\cancel{u = -v}}$

$\cancel{3y = 2x - u.}$

$\cancel{3(x + v) = x - u.}$

$\cancel{3x + 3v = x - u.}$

$\cancel{3x - x = u - 3v}$

$-u - 3v$

$\cancel{2x = -u - 3v}$

$\boxed{\cancel{x = \frac{-u - 3v}{2}}}$

$= \cancel{-(u}$

$\cancel{y = \frac{-u - 3v}{2} + v.}$

$= \cancel{\frac{-u - 3v + 2v}{2}}$

$\boxed{\cancel{y = \frac{-u - v}{2}}}$

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22. $u = 2x - 3y$ $\qquad$ $v = -x + y$

$3y = 2x - u$

$y = \frac{2x - u}{3}$

$v + x = \frac{2x - u}{3}$

$3(v + x) = 2x - u$

$3v + 3x = 2x - u$

$\boxed{x = -u - 3v}$

$\cancel{u}\ v + x = y$

$v - u - 3v = y$

$\boxed{-u - 2v = y}$

$$|J| = \begin{vmatrix} dx/du & dx/dv \\ dy/du & dy/dv \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -3 \\ -1 & -2 \end{vmatrix}$$

$= 2 - 3$

$= -1$

① $x = -3$

$-u - 3v = -3$

$-u = -3 + 3v$

$\boxed{u = 3 - 3v}$

② $x = 0$

$-u - 3v = 0$

$-u = +3v$

$\boxed{u = -3v}$

③ $y = x$

$\cancel{y} = -\cancel{u} - 3v = -\cancel{u} - 2v$

$-3v = -2v$

$-3v + 2v = 0$

$\boxed{-v = 0}$

④ $y = x + 1$

$-\cancel{u} - 2v = -\cancel{u} - 3v + 1$

$-2v + 3v - 1 = 0$

$v - 1 = 0$

$\boxed{v = 1}$

$(2x-3y)(-x+y)$

$-2x^2+2xy+3xy+3y^2$

$-2x^2+5xy+3y^2=0$.

$(2x^2+6x)y$

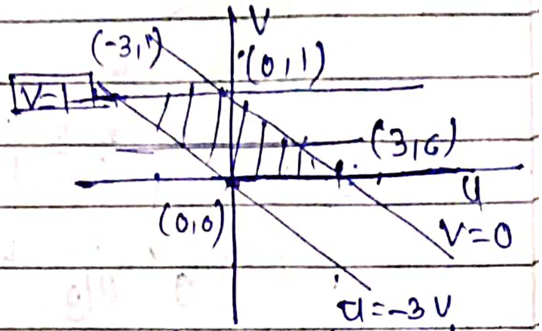
$3x-2$

$=6$

$6 \quad 1$

$$\cancel{\iint_R 2(x-y)\,dx\,dy}.$$

$$\cancel{\iint (2x-3y)(-x+y)}$$



Strip parallel to V.

$$\int_0^1 \int_{-3V}^{3-3V} 2\underbrace{(-v)}_{|J|}\,\cancel{du\,dv}$$

$$\cancel{\int_0^1 -2\left[\frac{v^2}{2}\right]_{-3V}^{3-3V} dv},$$

$$\cancel{\int_0^1 \frac{-2}{2}\left[(3-3V)^2-(3V)^2\right] dv}$$

$$\int_0^1 -2V\,(u)_{-3V}^{3-3V}\,dv.$$

$$\int_0^1 +2v\,(3-\cancel{3v}+\cancel{3v})\,dv$$

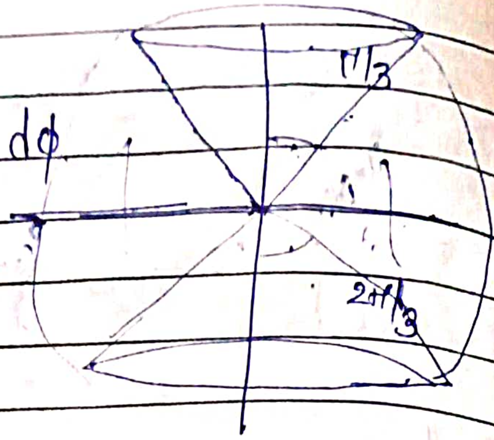
$$\int_0^1 +6v\,dv$$

$$+6\left(\frac{v^2}{2}\right)_0^1$$

$$6\left(\frac{1}{2}\right)$$

$$=3.$$

23. $$V = \int_{\theta=0}^{2\pi} \int_{\frac{2\pi}{3}}^{2\pi/3} \int_{0}^{a} 1 \, d\rho \, d\theta \, d\phi$$



$$\therefore V = \int_{0}^{2\pi} \int_{\pi/3}^{2\pi/3} \left[\int_{0}^{a} \rho^2 \sin\phi \, d\rho \right] d\theta \, d\phi$$

$$= \int_{0}^{2\pi} \int_{\pi/3}^{2\pi/3} \left[\frac{\rho^3}{3} \right]_{0}^{a} \sin\phi \, d\theta \, d\phi$$

$$= \int_{0}^{2\pi} \int_{\pi/3}^{2\pi/3} \left[\frac{a^3}{3} \right] \sin\phi \, d\phi \, d\theta$$

$$= \int_{0}^{2\pi} \int_{\pi/3} \frac{a^3}{3} \left(-\cos\phi \right)_{\pi/3}^{2\pi/3}$$

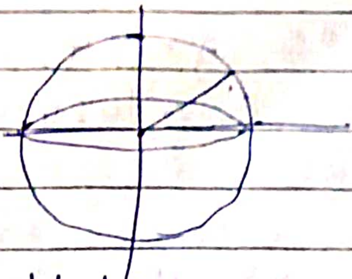
$$= \frac{-a^3}{3} \int_{0}^{2\pi} \left(\frac{-1}{2} - \frac{1}{2} \right) d\theta$$

$$= \frac{+a^3}{3} \int_{0}^{2\pi} 1 \, d\theta$$

$$= \frac{a^3}{3} (2\pi)$$

$$V = \frac{2\pi a^3}{3}$$

24. $\rho = 0$ to a.
$\phi = 0$ to $\pi/6$
$\theta = 0$ to $2\pi/2$



$$V = \int_0^{2\pi/2}\int_0^{\pi/6}\int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi/2}\int_0^{\pi/6} \left(\frac{\rho^3}{3}\right)_0^a \sin\phi \, . d\phi \, d\theta$$

$$= \int_0^{2\pi/2}\int_0^{\pi/6} \left(\frac{a^3}{3}\right) \sin\phi \, d\phi \, d\theta$$

$$= \int_0^{2\pi/2} \frac{-a^3}{3} \left(\cos\phi\right)_0^{\pi/6} d\theta$$

$$= \int_0^{2\pi/2} \frac{-a^3}{3} \left(\cos\frac{\pi}{6} - \cos 0\right) d\theta$$

$$= \int_0^{2\pi/2} \frac{-a^3}{3} \left(\frac{\sqrt{3}}{2} - 1\right) d\theta$$

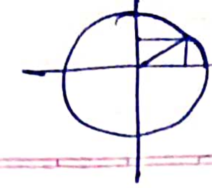
$$= \frac{-a^3}{3}\left(\frac{\sqrt{3}-2}{2}\right) . \left(\theta\right)_0^{2\pi/2}$$

$$= \frac{-a^3(\sqrt{3}-2)}{6} \cdot \cancel{2}\pi/2$$

$$= \frac{-a^3(\sqrt{3}-2)}{6\cdot 2}$$

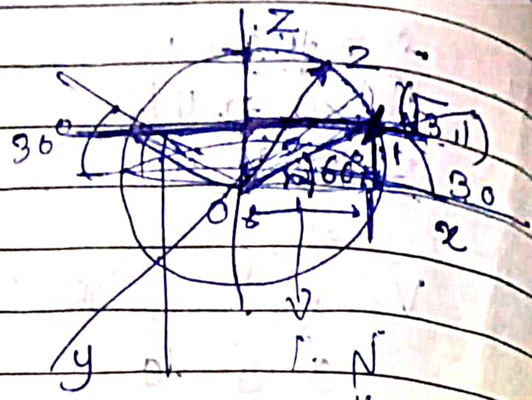
$$= \frac{-a^3(\sqrt{3}-2)\pi}{12}$$

$$= \frac{a^3(2-\sqrt{3})\pi}{12}$$



25. $\rho = 0$ to 2

$\theta = 0$ to 2π



$2^2 = 1^2 + a^2$

$4 = 1 + a^2$

$4 - 1 = a^2$

$3 = a^2$

$\boxed{\sqrt{3} = a}$

$\tan\alpha = \dfrac{\sqrt{3} - 0}{1 - 0}$

$\tan\alpha = \sqrt{3}$

$\boxed{\alpha = 60^\circ}$

$\boxed{\alpha = 30}$

$\boxed{\cancel{\tan} B = 30^\circ}$

$\theta = 0$ to 2π

$\phi = 0$ to $\cancel{\pi/6}$ $\pi/3$

$\rho = 0$ to 2

$z = 1$

$\rho \sin\phi = 1$

$\boxed{\rho = \sec}$

$$= \int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/3} \sin\phi \left(\frac{\rho^3}{3}\right)_{\sec\phi}^{2} d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/3} \frac{\sin\phi}{3} \left(8 - \sec^3\phi\right) d\phi \, d\theta$$

$r\cos\phi = 1$

$r = \frac{1}{\cos\theta}$

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$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi/3}\left(8\sin\phi - \frac{\sin\phi}{\cos^3\phi}\right) d\phi\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi/3}\left(8\sin\phi - \frac{\tan\phi}{\cos^2\phi}\sec^2\phi\right) d\phi\, d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi/3} 8\sin\phi - \left(\int_0^{\pi/3}\tan\phi\sec^2\phi\, d\phi\right) d\theta$$

put $\tan\phi = t$ | $\phi \to 0$ 0

$\sec^2\phi\, d\phi = dt$ | $\phi \to \pi/3$ $\sqrt{3}$

$$= \frac{1}{3}\int_0^{2\pi}\left[\int_0^{\pi/3} 8(-\cos\theta)\Big|_0^{\pi/3} - \int_0^{\sqrt{3}} t\, dt\right] d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\left[-8\left(\frac{1}{2}-1\right) - \frac{t^2}{2}\Big|_0^{\sqrt{3}}\right] d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\left(4 - \frac{1}{2}\left((\sqrt{3})^2 - 0\right)\right) d\theta$$

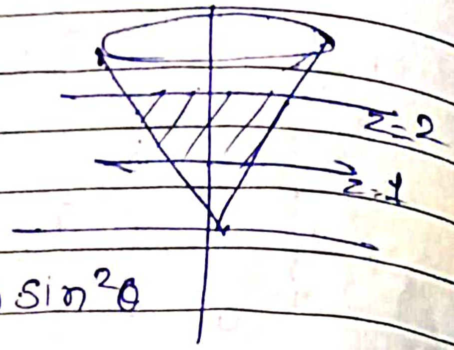
$$= \frac{1}{3}\int_0^{2\pi}\left(4 - \frac{3}{2}\right) d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\frac{5}{2}\, d\theta$$

$$= \frac{1}{3}\int_0 (\theta)\Big|_0^{2\pi} \quad \frac{1}{3}\times\frac{5}{2}\,\theta\Big|_0^{2\pi}$$

$$= \frac{5}{6}\times 2\pi = \frac{5\pi}{3}$$

26. $z = \sqrt{x^2+y^2}$



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$\rho^2 \cos^2\phi = \rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta$

$\rho^2 \cos^2\phi = \rho^2 \sin^2\phi \, (1)$

$\cancel{\tan^2\phi}$

$\tan^2\phi = 1$

$\tan\phi = 1$

$\boxed{\phi = \pi/4}$

$\phi = 0$ to $\pi/4$

$\theta = 0$ to 2π.

$z = 1$ $\qquad\qquad$ $z = 2$

$\rho\cos\phi = 1$ $\qquad\qquad$ $\rho\cos\phi = 2$

$\boxed{\rho = \sec\phi}$ to $\boxed{\rho = 2\sec\phi}$

$$V = \int_0^{2\pi}\int_0^{\pi/4}\int_{\sec\phi}^{2\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \left[\sin\phi \left(\frac{\rho^3}{3}\right)_{\sec\phi}^{2\sec\phi}\right] d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{1}{3}\left[\sin\phi \, (8\sec^3\phi - \sec^3\phi)\right) d\phi \, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi/4} \frac{7}{3}\left[\sin\phi \cdot \sec^3\phi\right]$$

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$$= \int_0^{2\pi} \frac{7}{3} \int_0^{\pi/4} \tan\phi \cdot \cancel{\cos^2\phi}\, \sec^2\phi \, d\phi \, d\theta$$

$\tan\phi = t$ | $\phi \to \pi/4$ 1

$\sec^2\phi \, d\phi = dt$ | $\phi \to 0$ 0

$$= \int_0^{2\pi} \frac{7}{3} \int_0^{1} t \, dt \, d\theta$$

$$= \int_0^{2\pi} \frac{7}{3} \left[\frac{t^2}{2}\right]_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{7}{36} (1) \, d\theta$$

$$= \frac{7}{6} (\theta)_0^{2\pi}$$

$$= \frac{7}{6} (2\pi)$$

$$= \frac{7}{3}\pi$$