Tutorial No - 2

Q. 1 Find the laplace Transforms of the following.

a) $(5e^{2t}-3)^2$

$\Rightarrow L\{(25e^{4t}-30e^{2t}+9)\}$

$= 25L\{e^{4t}\} - 30L\{e^{2t}\} + 9L\{1\}$

$= 25 \cdot \frac{1}{s-4} - 30 \cdot \frac{1}{s-2} + 9 \cdot \frac{1}{s}$

$$\boxed{= \frac{25}{s-4} - \frac{30}{s-2} + \frac{9}{s}}$$

b) $\sin 3t - 2\cos 5t$

$\Rightarrow L\{\sin 3t\} - 2L\{\cos 5t\}$

$= \frac{3}{s^2+9} - \frac{2s}{s^2+25}$

c) $\cosh at - \cos at$

$\Rightarrow L\{\cosh at\} - L\{\cos at\}$

$= \frac{s}{s^2-a^2} - \frac{s}{s^2+a^2}$

$= \frac{\cancel{s^3}+a^2s-\cancel{s^3}+a^2s}{s^4+\cancel{s^2a^2}-\cancel{s^2a^2}-a^4} = \boxed{\frac{2a^2s}{s^4-a^4}}$

d) $e^t(1+t)^2$

$\Rightarrow e^t(1+2t+t^2)$

$= e^t + e^t 2t + e^t t^2$

$= L\{e^t\} + L\{e^t 2t\} + L\{e^t t^2\}$

$= \frac{1}{s-1} + 2 \cdot \frac{1}{s^2} + \frac{2}{s^3}$

By using first shifting Theorem
put $s = (s-a)$

$= \frac{1}{(s-1)} + \frac{2}{(s-1)^2} + \frac{2}{(s-1)^3}$

Taking LCM

$= 1 \cdot (s-1)^5 + 2(s-1)^4 + 2(s-1)^3 \,/\, (s-1)^6$

$(s-1)^3 [(s-1)^2 + 2(s-1) + 2(1)] \,/\, (s-1)^{\cancel{6}\,3}$

$=$

$= \frac{[(s-1)^2 + 2s - 2 + 2]}{(s-1)^3}$

$= \frac{[s^2 - \cancel{2s} + 1 + \cancel{2s} - 2 + 2]}{(s-1)^3}$

$= \frac{s^2+1}{(s-1)^3}$

e) $f(t) = \begin{cases} t & , 0 < t < 1 \\ e^{1-t} & , t > 1 \end{cases}$

$\Rightarrow f(t) = t\{u(t-0) - u(t-1)\} + e^{1-t}\{u(t-1)\}$

Taking laplace transform on both sides

$L\{f(t)\} = L\{t[u(t-0) - u(t-1)]\} + L\{e^{1-t}[u(t-1)]\}$

using the formula

$L\{f(t)\, u(t-a)\} = e^{-as} L\{f(t-a)\}$

$F(s) = e^{-0s} L\{t\} - e^{-1s} . L\{t\} + e^{-1s} . L\{e^{1-t}\}$

$= 1 . \frac{1}{s^2} - e^{-s} . \frac{1}{s^2} + e^{-s}$

$= \frac{1}{s^2} [1 - e^{-s}$

f) $t^{7/2} e^{3t}$

g) $f(t) = t\cos at$

$\Rightarrow \quad L\{t f(t)\} = -\frac{d}{ds} L\{f(t)\}$

using this formula

$L\{f(t)\} = L\{\cos at\}$

$= \frac{s}{s^2+a^2}$

$L\{t f(t)\} = -\frac{d}{ds} \cdot \frac{s}{s^2+a^2}$

Taking derivative

$= -\left[s^2+a^2 \frac{d}{ds} s - s \frac{d}{ds} s^2+a^2 \right] / (s^2+a^2)^2$

$= -[s^2+a^2 \cdot 1 - s \cdot 2s + 0] / (s^2+a^2)^2$

$= \frac{-s^2 - a^2 + 2s^2}{(s^2+a^2)^2}$

$= \boxed{\frac{s^2-a^2}{(s^2+a^2)^2}}$

h) $L\{\sin^2 t\}$

$\Rightarrow$ Trignometric identity — $\sin^2 t = \frac{1-\cos 2t}{2}$

$= L\left\{ \frac{1}{2} [1 - \cos 2t] \right\}$

$= \frac{1}{2} [L\{1\} - L\{\cos 2t\}]$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

$$= \frac{1}{2s} - \frac{1}{2}\cdot\frac{s}{(s^2+4)} \qquad = \frac{1}{2s} - \frac{s}{2s^2+8}$$

$$= \frac{\cancel{2s^2}+8-\cancel{2s^2}}{4s^3+16}$$

$$= \frac{\cancel{8}\,2}{\cancel{4}s(s^2+4)}$$

$$= \frac{2}{s(s^2+4)}$$

i) $f(t) = \left\{\frac{e^{-at} - e^{-bt}}{t}\right\}$

2)2

$\Rightarrow \quad L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\, ds$ ---- division by t formula

Taking Laplace of $f(t) = e^{-at} - e^{-bt}$

$= L\{e^{-at}\} - L\{e^{-bt}\}$ --- by linearity property

$$= \frac{1}{s+a} - \frac{1}{s+b}$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \left[\frac{1}{s+a} - \frac{1}{s+b}\right] ds$$

Taking derivative

$$= \left[\log(s+a) - \log(s+b)\right]_s^\infty$$

By the formula $\log(a) - \log(b) = \log \frac{a}{b}$

$$= \left[\log \frac{(s+a)}{(s+b)} \right]_s^\infty$$

$$= \left[\log \frac{\cancel{s}(1+a/s)}{\cancel{s}(1+b/s)} \right]_s^\infty$$

= puting limit

$$= \log \frac{(1+\frac{a}{\infty})}{(1+\frac{b}{\infty})} - \log \frac{(1+\frac{a}{s})}{(1+\frac{b}{s})} \quad \text{---} \quad \frac{a}{\infty} = 0$$

$$= \log 1 - \log \frac{(s+a/\cancel{s})}{(s+b/\cancel{s})}$$

$$= 0 - \log \frac{(s+a)}{(s+b)}$$

$$= \log \frac{(s+b)}{(s+a)}$$

k) $f(t) = \frac{\sin^2 t}{t}$

$\Rightarrow$ $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s)\, ds$ --- using the formula

$L\{f(t)\} = L\{\sin^2 t\}$ --- $\frac{1-\cos 2t}{2}$ --- Trig. identity

$$= \frac{1}{2} L\{1 - \cos 2t\}$$

$$= \frac{1}{2}\left[L\{1\} - L\{\cos 2t\}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

$$= \frac{1}{\cancel{2}\cancel{8}}\left[\frac{1}{s} - \frac{s}{\cancel{2}s^2+\cancel{8}4}\right] ds$$

Taking derivative

$$L\left\{\frac{\sin^2 t}{t}\right\} = \frac{1}{2}\int_s^{\infty}\left[\frac{1}{s} - \frac{s}{s^2+4}\right] ds$$

$$= \frac{1}{2}\left[\log(s) - \frac{1}{2} \cdot \frac{\cancel{2}\log(s^2+4)}{\cancel{\infty}}\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\frac{\log(s)}{(s^2+4)^{1/2}}\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log \frac{\cancel{s}\,1}{\cancel{s}(1+\frac{4}{s})^{1/2}}\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\log \frac{1}{(1+\frac{4}{\infty})^{1/2}} - \log \frac{1}{(1+\frac{4}{s})^{1/2}}\right]$$

$$= \frac{1}{2}\left[\log 1 - \log \frac{1}{(1+4/s^2)^{1/2}}\right]$$

$$= \frac{1}{2}\left[-\log \frac{1}{\left(\frac{s^2+4}{s^2}\right)^{1/2}}\right]$$

$$= \frac{1}{2}\left[-\log \left(\frac{s^2}{s^2+4}\right)^{1/2}\right]$$

$$= \frac{1}{2} \cdot \frac{1}{2} \log \frac{s^2+4}{s^2} = \boxed{\frac{1}{4} \log \frac{s^2+4}{s^2}}$$

j) $f(t) = \frac{\cos at - \cos bt}{t}$

$\Rightarrow$ using the formula $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,\frac{ds}{ds}$

$L\{f(t)\} = L\{\cos at\} - L\{\cos bt\}$

$F(s) = \frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}$

$L\{f(t)\} = \int_s^\infty \left[\frac{s}{s^2+a^2} - \frac{s}{s^2+b^2}\right] ds$

Taking derivative

$= \left[\frac{1}{2}\log(s^2+a^2) - \frac{1}{2}\log(s^2+b^2)\right]_s^\infty$

$= \frac{1}{2}\left[\log\frac{(s^2+a^2)}{(s^2+b)}\right]_s^\infty$

$= \frac{1}{2}\left[\log\frac{\cancel{s^2}(1+\frac{a^2}{s^2})}{\cancel{s^2}(1+\frac{b^2}{s^2})}\right]_s^\infty$

$= \frac{1}{2}\left[\log\frac{(1+\frac{a^2}{\infty})}{(1+\frac{b^2}{\infty})} - \log\frac{(1+\frac{a^2}{s^2})}{(1+\frac{b^2}{s^2})}\right]$

$= \frac{1}{2}\left[\log 1 - \log\frac{(1+\frac{a^2}{s^2})}{(1+\frac{b^2}{s^2})}\right]$

$= \frac{1}{2} - \log\frac{(1+\frac{a^2}{s^2})}{(1+\frac{b^2}{s^2})}$

$= \frac{1}{2} - \log\frac{(s^2+a^2/\cancel{s^2})}{(s^2+b^2/\cancel{s^2})} = \boxed{\frac{1}{2}\log\frac{(s^2+b^2)}{(s^2+a^2)}}$

l, m, o - remaining

n) $f(t) = t^2 \sin 2t$

$\Rightarrow$ using the formula. $L\{t^n f(t)\} = (-1)^n \frac{d^2}{ds^2} F(s)$

$L\{F(t)\} = L\{\sin 2t\} = \frac{2}{s^2+4}$

$L\{t^2 \sin 2t\} = (-1)^2 \cdot \frac{d^2}{ds^2} \frac{2}{s^2+4}$ — $(-1)^2 = 1$

$\frac{u}{v}$ -

$\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \left[s^2+4 \frac{d}{ds} 2 - 2 \frac{d}{ds} s^2+4\right] / (s^2+4)^2$

$= [s^2+4 \cdot 0 - 2 \cdot 2s] / (s^2+4)^2$

$= (0 - 4s) / (s^2+4)^2$

$\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = -\frac{4s}{(s^2+4)^2}$

$\frac{d^2}{ds^2}\left(\frac{2}{(s^2+4)}\right) = -4\left[(s^2+4)^2 \frac{d}{ds} s - s \frac{d}{ds} (s^2+4)^2\right] / (s^2+4)^2$

$= -4\left[\frac{(s^2+4)^2 \cdot 1 - (s) 2 (s^2+4) \cdot 2s}{(s^2+4)^4}\right]$

$= -\frac{4(s^2+4)^2 - (4s)(2)(s^2+4)(2s)}{(s^2+4)^4}$ $(4s)(s) = 16s^2$

$= -\left[\frac{4(s^2+4)^2 (s^2+4) - (4s)^2}{(s^2+4)^4}\right]$

$= -4\left\{\frac{(s^2+4)\left[(s^2+4) - (4s^2)\right]}{(s^2+4)^4}\right\}$

$= -\frac{4(s^2+\cancel{4})(-3s^2+4)}{(s^2+4)^{\cancel{4}\,3}}$

$= -\frac{4(4-3s^2)}{(s^2+4)^3}$

Q.(2) Find the inverse Laplace transform of

a) $L^{-1}\left\{\frac{0.1s+0.9}{s^2+3.24}\right\}$

$\rightarrow = 0.1\, L^{-1}\left\{\frac{s}{s^2+3.24}\right\} + 0.9\, L^{-1}\left\{\frac{1}{s^2+3.24}\right\}$

$= 0.1\, L^{-1}\left\{\frac{s}{s^2+(1.8)^2}\right\} + \frac{0.9}{1.8}\, L^{-1}\left\{\frac{1.8}{s^2+(1.8)^2}\right\}$

$= \boxed{0.1 \cos 1.8t + 0.5 \sin 1.8t}$

d) $L^{-1}\left\{\frac{3s+1}{s^2+6s+13}\right\}$

$\Rightarrow (a^2+2ab+b^2) = \underset{a}{s^2} + 2\times \underset{b}{3} \times s + 13$

$= s^2+6s+(3)^2$

$= (s^2+6s+9)-9+13$

$= (s+3)^2+4$

$= (s+3)^2+(2)^2$

$L^{-1}\left\{\frac{3s+1}{(s+3)^2+(2)^2}\right\} = 3\left\{\frac{(s+3)-3+1}{(s+3)^2+2^2}\right\}$

$= 3L^{-1}\left\{\frac{(s+3)}{(s+3)^2+2^2}\right\} - L^{-1}\left\{\frac{2}{(s+3)^2+2^2}\right\}$ ... by first shifting theorem

consider ~~pact~~ $(s+3) = s$

$= 3e^{-3t}[\cos 2t] - [\sin 2t]$

$\boxed{= 3e^{-3t}(\cos 2t - \sin 2t)}$

f) $\frac{e^{-\pi s}}{s^2+9}$

$\Rightarrow$ $L^{-1}\{e^{-as} F(s)\} = \{f(t-a) \cdot u(t-a)\}$

--- Second shifting Theo

$F(s) = \frac{1}{s^2+9}$

$L^{-1}\{F(s)\} = \frac{1}{3} \cdot \frac{3}{s^2+3^2}$

$f(t) = \frac{\sin 3t}{3}$

$L^{-1}\left\{\frac{e^{-\pi s}}{s^2+9}\right\} = \frac{1}{3}\left\{\sin 3(t-\pi) \cdot u\,(t-\pi\right\}$

b) $\frac{-s-10}{s^2-s-20} = \frac{-s-10}{s^2+s-2s-20} = \frac{-s-10}{s(s+1)-2(s-10)}$

$\Rightarrow$ $\frac{-s-10}{(s+1)(s-2)} = \frac{A}{(s+1)} + \frac{B}{(s-2)}$

$-s-10 = A(s-2) + B(s+1)$

$-s-10 = As - 2A + Bs + B$

$A + B = -1$ — ①

$-2A + B = -10$ — ②

$3A = 9$

$A = 3$

put $A = 3$ in eqn ①

$3 + B = -1$

$B = -1 - 3$

$B = -4$

$\frac{-s-10}{(s+1)(s-2)} = \frac{3}{(s+1)} - \frac{4}{(s-2)}$

Taking inverse Laplace.

$= 3L^{-1}\left\{\frac{1}{s+1}\right\} - 4\left\{\frac{1}{s-2}\right\}$

$= 3e^{-t} - 4e^{2t}$

ing Theorem

Q.(3) Solve the following using laplace transform

b) $y'' + y = e^{-2t}\sin t$, $y(0) = y'(0) = 0$

$\Rightarrow$ $L\{y''\} + L\{y\} = L\{e^{-2t}\sin t\}$

$s^2 y(s) - s y(0) - y'(0) + y(s) = \frac{1}{(s+2)^2+1}$

$(s^2+1)y(s) - s(0) - 0 = \frac{1}{(s+2)^2+1}$

$y(s) = \frac{1}{[(s+2)^2+1](s^2+1)}$

$= \frac{1}{(s^2+4s+4+1)(s^2+1)} = \frac{1}{(s^2+4s+5)(s^2+1)}$

$\frac{1}{(s^2+4s+5)(s^2+1)} = \frac{As+B}{(s^2+4s+5)} + \frac{Cs+D}{(s^2+1)}$

$1 = As(s^2+1) + B(s^2+1) + Cs(s^2+4s+5) + D(s^2+4s+5)$

$1 = As^3 + As + Bs^2 + B + Cs^3 + 4Cs^2 + 5Cs + Ds^2 + 4Ds + 5D$

$= (A+C)s^3 + (B+4C+D)s^2 + (A+5C+4D)s + B + 5D$

$A + C = 0$ – (1) , $A = -C$

$B + 4C + D = 0$ – (2)

$A + 5C + 4D = 0$ (3)

$B + 5D = 1$ – (4)

put $A = -C$ in eqn (3)

$(-C) + 5C + 4D = 0$

$4C + 4D = 0$

$C + D = 0$ – (5)

$D = -C$

put $D = -C$ in eqn (2)

$B + 4C + (-C) = 0$

$B + 4C - C = 0$

$B + 3C = 0$ – (6)

put $D = -C$ in eqn (1)

$B + 5(-C) = 1$

$B - 5C = 1$ – (7)

Adding eq^n (6) & (7)

$B + 3C = 0 \quad - (6)$

$-B \mp 5C = -1 \quad - (7)$

$8C = -1$

$$\boxed{C = \frac{-1}{8}}$$

As $A = -C$ so $A = -\left(\frac{-1}{8}\right)$

$$\boxed{A = \frac{1}{8}}$$

put $C = \frac{-1}{8}$ in eq^n (5)

$B + D = 0$

$\frac{-1}{8} = -D$

$$\boxed{D = \frac{1}{8}}$$

put $C = \frac{-1}{8}$ in eq^n (6)

$B + 3\left(\frac{-1}{8}\right) = 0$

$B - \frac{3}{8} = 0$

$$\boxed{B = \frac{3}{8}}$$

$$\frac{1}{(s^2+4s+5)(s^2+1)} = \frac{\frac{1}{8}s + \frac{3}{8}}{(s^2+4s+5)} + \frac{\left(-\frac{1}{8}\right)s + \frac{1}{8}}{(s^2+1)}$$

$$= \frac{1}{8}\left[\frac{s+2+1}{(s+2)^2+1}\right] - \frac{1}{8}\left[\frac{s-1}{(s^2+1)}\right]$$

$$= \frac{1}{8}\left\{\frac{s+2}{(s+2)^2+1} + \frac{1}{(s+2)^2+1}\right\} - \frac{1}{8}\cdot\frac{s}{s^2+1} + \frac{1}{8}\cdot\frac{1}{s^2+1}$$

$$= \frac{1}{8}\left\{e^{-2t}(\cos t + \sin t)\right\} - \frac{1}{8}\cos t + \frac{1}{8}\sin t$$

Ans = $$\boxed{y = \frac{1}{8}\left[\sin t - \cos t + e^{-2t}(\sin t + \cos t)\right]}$$

d) $y'' + 2y' + 5y = e^{-t} \sin t$, $y(0) = 0$, $y'(0) = 1$

$\Rightarrow L\{y''\} + 2\{y'\} + 5\{y\} = L\{e^{-t} \sin t\}$

$$[s^2 y(s) - s y(0) - y'(0)] + 2[s y(s) - y(0)] + 5[y(s)] = \frac{1}{(s+1)^2+1}$$

$$(s^2+2s+5) y(s) - s(0) - (1) - 2(0) = \frac{1}{(s+1)^2+1}$$

$$(s^2+2s+5) y(s) = \frac{1}{(s+1)^2+1} + 1$$

$$(s^2+2s+5) y(s) = \frac{s^2+2s+2+1}{s^2+2s+2} = \frac{s^2+2s+3}{s^2+2s+2}$$

$$y = \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{As+B}{(s^2+2s+2)} + \frac{Cs+D}{(s^2+2s+5)}$$

$$s^2+2s+3 = As(s^2+2s+5) + B(s^2+2s+5) + Cs(s^2+2s+2) + D(s^2+2s+2)$$

$$= As^3 + 2As^2 + 5As + Bs^2 + 2Bs + 5B + Cs^3 + 2Cs^2 + 2Cs + Ds^2 + 2Ds + 2D$$

$A + C = 0$ - (1) $2A + B + 2C + D = 1$ - (2), $5A + 2B + 2C + 2D = 2$ - (3)

$5B + 2D = 3$ - (4)

$A = -C$ $\therefore$ put $A = -C$ in eq^n (2)

$2(-C) + B + 2C + D = 1$

$-\cancel{2C} + \cancel{2C} + B + D = 1$

$B + D = 1$ - (5)

Adding eq^n (4) & (5)

$5B + 2D = 3$ - (4)

$B + D = 1$ - (5)

multiplying by 2 to eq^n (5)

$2B + 2D = 2$ - (6)

$-5B + \cancel{2D} = 3$ - (4)

$-3B = -1$

$\boxed{B = \frac{1}{3}}$

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put $B = \frac{1}{3}$ in eq^n (5)

$\frac{1}{3} + D = 1$

$D = 1 - \frac{1}{3}$

$D = \frac{3-1}{3}$

$\boxed{D = \frac{2}{3}}$

$2A + B + 2C + D = 1$

$\cancel{2A+}$

$2(-C) + \frac{1}{3} + 2C + \frac{2}{3} = 1$

put B & D value in eq^n (3)

$5A + 2B + 2C + 2D = 2$

$5(-C) + 2 \times \frac{1}{3} + 2C + 2 \times \frac{2}{3} = 2$

$-5C + \frac{2}{3} + \frac{4}{3} + 2C = 2$

$-3C + 2 = 2$

$-3C = 0$

$\boxed{C = 0}$

$A + C = 0$

$A + 0 = 0$

$\boxed{A = 0}$

$$\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{0(s)+\frac{1}{3}}{(s^2+2s+2)} + \frac{0(s)+\frac{2}{3}}{(s^2+2s+5)}$$

$$= \frac{1}{3}\left[\frac{1}{(s+1)^2+1}\right] + \frac{1}{3}\left[\frac{2}{(s+1)^2+2^2}\right]$$

$$= \frac{1}{3}\left[e^{-t}\left(\sin t + \sin 2t\right)\right]$$

$$\boxed{y = e^{-t}(\sin t + \sin 2t)/3}$$

c) $y'' + 2y' + 5y = 50t - 150$, $y(3) = -4$, $y'(3) = 14$

$\Rightarrow L\{y''\} + 2\{y'\} + 5\{y\} = L\{50t - 150\}$

$[s^2 y(s) - s y(0) - y'(0)] + 2[s y(s) - y(0)] + 5[y(s)] = \frac{50}{s^2}$

The eq^n is given as with cond^n. $y'(-3) = 14$,

The initial cond^n are pivoting at $t = 3$
$y(0) = -4$, & $y'(0) = 14$

$y(t) = 4(t-3)$

Taking Laplace transform

$(s^2 + 2s + 5) y(s) - s(-4) - (14) - (-4) = \frac{50}{s^2}$

$(s^2 + 2s + 5) y(s) + 4s - 6 = \frac{50}{s^2}$

$(s^2 + 2s + 5) y(s) = \frac{50}{s^2} - 4s + 6$

$(s^2 + 2s + 5) y(s) = \frac{50 - 4s^3 + 6s^2}{s^2}$

$y(s) = \frac{50 - 4s^3 + 6s^2}{s^2(s^2 + 2s + 5)}$

$\therefore \frac{50 - 4s^3 + 6s^2}{s^2(s^2 + 2s + 5)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{(s^2 + 2s + 5)}$

Multiply through by the common denominator of $s^2(s^2 + 2s + 5)$

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$50-4s^3+6s^2 = A(s(s^2+2s+5)) + B(s^2+2s+5) + Cs(s)^2 + D(s)^2$

$= A(s^3+2s^2+5s) + Bs^2 + 2Bs + 5B + Cs^3 + Ds^2$

$= As^3 + 2As^2 + 5As + Bs^2 + 2Bs + 5B + Cs^3 + Ds^2$

$A + C = -4$ — ① $2A + B + D = 6$ — ② $5A + 2B = 0$ — ③

$5B = 50$ — ④

$B = \cancel{50}/5 \; 10$

$\boxed{B = 10}$

put $B = 10$ in eqⁿ ③

$5A + 2(10) = 0$

$5A + 20 = 0$

$A = -\cancel{20}/5 \; 4$ $\therefore A = \boxed{-4}$

put $A = -4$ & $B = 10$ in eqⁿ ②

$2(-4) + 10 + D = 6$

$-8 + 10 + D = 6$

$2 + D = 6$

$D = 6 - 2$

$\boxed{D = 4}$

put $A = 4$ in eqⁿ ①

$A + C = -4$

$-4 + C = -4$

$\boxed{C = 0}$

$$\frac{50-4s^3+6s^2}{s^2(s^2+2s+5)} = \frac{-4}{s} + \frac{10}{s^2} + \frac{4}{(s^2+2s+5)}$$

$$y(s) = \frac{-4}{s} + \frac{10}{s^2} + 2 \times \frac{2}{(s^2+1)^2+2^2}$$

Taking inverse laplace,

$y = -4 + 10t + 2e^{-t}\sin 2t$

calculating value of $y(t)$

Q. ①

m)

⟹

Q. ②

$Cs(s)^2 + D(s)^2$
Ds^2
Ds^2

$2B = 0$ — ③

$y(t) = y(t-3)$

$y = -4 + 10(t-3) + 2 \cdot e^{-(t-3)} \cdot \sin(2(t-3))$

$$\boxed{y = 10(t-3) - 4 + 2e^{-(t-3)} \sin(2(t-3))}$$

Q. ①

m) $\delta(t-3)\, u(t-3)$

$\Rightarrow$ using the formula

$L\{f(t-a)\, u(t-a)\} = e^{-as} F(s)$

$$\boxed{L\{(t-3)\, u(t-3)\} = e^{-3s}}$$

Q. ② h) $F(s) = \cot^{-1} \frac{s}{\omega}$

$\Rightarrow$

i) $\frac{1}{2} \ln\left(\frac{s^2-a^2}{s^2}\right)$

$\Rightarrow = \frac{1}{2} \ln. (s^2-a^2) - \ln(s^2)$

$= \frac{1}{2}\left[\frac{d}{ds}\ln(s^2-a^2) - \frac{d}{ds}\ln(s^2)\right]$

$= \frac{1}{2}\left[\frac{1}{s^2-a^2}\cdot 2s - \frac{1}{s^2}\cdot 2s\right]$

$= \frac{\cancel{2}s}{\cancel{2}(s^2-a^2)} - \frac{\cancel{2}s}{\cancel{2}(s^2)}$

$= \frac{s}{s^2-a^2} - \frac{1}{s}$

$f(t) = -\frac{1}{t} L^{-1}\left\{\frac{s}{s^2-a^2} - \frac{1}{s}\right\}$

$= -\frac{1}{t} \times \cosh at - 1$

$= \frac{-\cosh at + 1}{t}$

$\boxed{f(t) = \frac{1-\cosh at}{t}}$

j) $\ln\sqrt{\frac{s^2+b^2}{s^2+a^2}}$

$\Rightarrow = \ln\sqrt{s^2+b^2} - \ln\sqrt{s^2+a^2}$

$= \ln(s^2+b^2)^{1/2} - \ln(s^2+a^2)^{1/2}$

$= \frac{1}{2}\ln(s^2+b^2) - \frac{1}{2}\ln(s^2+a^2)$

$= \frac{1}{2}\left[\frac{d}{ds}\ln(s^2+b^2) - \frac{d}{ds}\ln(s^2+a^2)\right]$

$$= \frac{1}{2}\left[\frac{1}{s^2+b^2}\,2s - \frac{1}{s^2+a^2}\,2s\right]$$

$$= \frac{\cancel{2}s}{\cancel{2}(s^2+b^2)} - \frac{\cancel{2}s}{\cancel{2}(s^2+a^2)}$$

$$= \frac{s}{(s^2+b^2)} - \frac{s}{(s^2+a^2)}$$

$$f(t) = \frac{-1}{t} \times L^{-1}\left\{\frac{s}{s^2+b^2} - \frac{s}{s^2+a^2}\right\}$$

$$= \frac{-1}{t} \times \cos bt - \cos at$$

$$= \frac{-\cos bt + \cos at}{t}$$

$$= \frac{\cos at - \cos bt}{t}$$

Tutorial No - 3

Q 1) Describe the function's domain, find the function's range & also describe the functions level curve's.

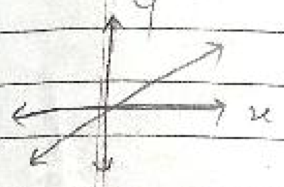
a) $f(x,y) = y - x$

$\Rightarrow$ Domain = Entire plane or $\mathbb{R} \times \mathbb{R}$ or $\mathbb{R}^2$

$= \{(x,y) \in \mathbb{R} \times \mathbb{R}\}$

or

$\{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$



Range = $[-\infty, \infty]$ — All Real numbers

b) $f(x,y) = \sqrt{y-x}$

$\Rightarrow$ Domain $= \{(x,y) \in \mathbb{R}^2 \mid y - x \geq 0\}$

$= \{(x,y) \in \mathbb{R}^2 \mid y \geq x\}$

$y - x \geq 0$

$y \geq x$

$\underline{\underline{y = x}}$

Range = $[0, \infty]$ (all non-negative numbers)

c) $f(x,y) = 4x^2 + 9y^2$

$\Rightarrow$ Domain $= \{(x,y) \in \mathbb{R}^2\}$

$= \{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$

Range = $[0, \infty]$ (all non-negative numbers)

d) $f(x,y) = x^2 - y^2$

$\Rightarrow$ Domain $= \{(x,y) \in \mathbb{R}^2\}$ – Entire plane

Range $= [-\infty, \infty]$ – All Real numbers

e) $f(x,y) = xy$

$\Rightarrow$ Domain $= \{(x,y) \in \mathbb{R}^2\}$ – Entire plane
$\{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\}$

Range $= [-\infty, \infty]$ — All Real numbers

f) $f(x,y) = y/x^2$

$\Rightarrow$ Domain $= \{(x,y) \in \mathbb{R}^2 \mid x > 0$ and $x \neq 0$

Range $= [-\infty, \infty]$ – All Real numbers

g) $f(x,y) = \frac{1}{\sqrt{16 - x^2 - y^2}}$

$\Rightarrow$ Domain $= \{ (x,y) \in \mathbb{R}^2 : \sqrt{16 - x^2 - y^2} \neq 0$ and $x^2 - y^2 \leq 16 \}$

$\sqrt{16 - x^2 - y^2} > 0$ — It should not be equal to 0.

$16 \geq x^2 + y^2$

Suppose, $x = 2$ & $y = 5$

$\sqrt{16 - (2)^2 - (5)^2}$

$= \sqrt{16 - (4 - 25)}$

$= \sqrt{16 - (-21)}$ --- $(x^2 - y^2) \leq 16$

$= \sqrt{16 + 21}$

$= \sqrt{37}$

$16 \geq x^2 + y^2$

Range $= [\frac{1}{4}]$

h) $f(x,y) = \sqrt{9-x^2-y^2}$

$\Rightarrow$ $9-x^2-y^2 \geq 0$

$9 \geq x^2+y^2$

Domain $= \{x,y \in \mathbb{R}^2 : x^2+y^2 \leq 9\}$

Range $= [0, 3]$

i) $f(x,y) = \ln(x^2+y^2)$

$\Rightarrow$ $x^2+y^2 > 0$ — $\log(0)$ - undefined

Domain $= \{x,y \in \mathbb{R}^2 : x^2+y^2 > 0\}$

Range $= [-\infty, \infty]$ — All real numbers

[log of some function can be -ve in certain cases

9 = (9 - 9)

② ③ Rem

Q. ④ Find the following limits

a) $\lim_{(x,y)\to(0,\pi/4)} \sec x \tan y$

$\Rightarrow \lim_{(x,y)\to(0,\pi/4)} \sec(0) \cdot \tan\left(\frac{\pi}{4}\right)$

$= \frac{1}{\cos 0} \cdot \tan\left(\frac{\pi}{4}\right)$ — $\cos(0) = 1$, $\tan\left(\frac{\pi}{4}\right) = 1$

$= \frac{1}{1} \cdot 1 = \underline{\underline{1}}$

b) $\lim_{(x,y)\to(1,1)} \ln|1 + x^2y^2|$

$\Rightarrow$ put the limits

$\lim_{(x,y)\to(1,1)} \ln|1 + (1^2)(1)^2|$

$= \ln|2|$

$= \underline{\underline{0.3010}}$

c) $\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}$

$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{e^{(0)} \sin(0)}{0}$

can
ses)

4) $\lim\limits_{(x,y)\to(\pi/2,0)} \dfrac{\cos y + 1}{y - \sin x}$

$\Rightarrow$ put limits $(x,y) = \left(\frac{\pi}{2}, 0\right)$

$= \cancel{\dfrac{\cos\left(\frac{\pi}{2}\right) + 1}{\left(\frac{\pi}{2}\right) - \sin(}$ $= \dfrac{\cos(0) + 1}{(0) - \sin\left(\frac{\pi}{2}\right)}$

$= \dfrac{1+1}{0-1} = \dfrac{2}{-1}$

$= -2$

5) $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^3 - y^3}{x - y}$

$\Rightarrow = \dfrac{\cancel{(x-y)}\,(x^2 + xy + y^2)}{\cancel{(x-y)}}$

$= x^2 + xy + y^2$

put limits $(x,y) = (1,1)$

$= (1)^2 + (1)(1) + (1)^2$

$= 1 + 1 + 1$

$= 3$

f) $\lim_{(x,y,z)\to(\pi,0,3)} ze^{-2y}\cos 2x$

$\Rightarrow$ put limits $(x,y,z) = (\pi,0,3)$

$= (3)e^{-2(0)} \cos 2(\pi)$

$= 3e^{0} \cos(360°)$ — $e^{0} = 1$

$= 3 \cdot 1$ $\cos(360°) = 1$

$= \underline{\underline{3}}$

g) $\lim_{(x,y,z)\to(1,-1,-1)} \frac{2xy+yz}{x^2+z^2}$

$\Rightarrow$ put the limits. $(x,y,z) = (1,-1,-1)$

$= \frac{2(1)(-1)+(-1)(-1)}{(1)^2+(-1)^2}$

$= \frac{-2+1}{1+1} = \frac{-1}{2}$

$= \underline{\underline{\frac{-1}{2}}}$

h) $\lim_{(x,y)\to(0,0)} \frac{x-y+2\sqrt{x}-2\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

Q. (1) Find the first order partial derivatives with respect to each variables.

a) $f(x,y) = (xy-1)^2$

$\Rightarrow \dfrac{\partial f}{\partial x} = \dfrac{\partial}{\partial x}(xy-1)^2$

$= 2(xy-1)\dfrac{\partial}{\partial x}(xy-1)$ — keep y constant

$= 2(xy-1)(y)$

$\dfrac{\partial f}{\partial y} = \dfrac{\partial}{\partial y}(xy-1)^2$

$= 2(xy-1)\dfrac{\partial}{\partial y}(xy-1)$

$= 2(xy-1)\cdot\dfrac{\partial}{\partial y}xy - \dfrac{\partial}{\partial y}1$

$= 2(xy-1)x$ — keep x constant

b) $f(x,y) = \tan^{-1}(y|x)$

$\Rightarrow \dfrac{\partial f}{\partial x} = \dfrac{\partial}{\partial x}\tan^{-1}\left(\dfrac{y}{x}\right)\cdot\dfrac{\partial}{\partial x}\dfrac{y}{x}$

$= \dfrac{1}{1+\left(\dfrac{y}{x}\right)^2}\cdot -\dfrac{1}{x^2}\cdot y$ — keep y constant

$= \dfrac{1}{1+\dfrac{y^2}{x^2}} - \dfrac{y}{x^2}$

$= \dfrac{1}{\dfrac{x^2+y^2}{x^2}} - \dfrac{y}{x^2}$

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$$= \frac{x^2}{x^2+y^2} - \frac{y}{\cancel{x^2}}$$

$$\frac{\partial f}{\partial x} = \frac{-y}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial x} \tan^{-1}\left(\frac{y}{x}\right) \cdot \frac{\partial}{\partial x} \frac{y}{x}$$

$$= \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{1}{x}$$ — keep x constant

$$= \frac{\frac{x^2}{x^2+y^2}}{x^2} \cdot \frac{1}{x}$$

$$= \frac{x^{\cancel{2}}}{x^2+y^2} \cdot \frac{1}{\cancel{x}}$$

$$= \frac{x}{x^2+y^2}$$

c) $f(x,y) = e^{-x}\sin(x+y)$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} e^{-x}\sin(x+y) \cdot \frac{\partial}{\partial x}\sin(x+y)$$

$$= e^{-x}\frac{d}{dx}\sin(x+y) + \sin(x+y)\frac{d}{dx}e^{-x}$$

$$= e^{-x}\cos(x+y) + \sin(x+y)e^{-x}\cdot(-1)$$

$$= e^{-x}\cos(x+y) - \sin(x+y)e^{-x}$$

$$\frac{\partial f}{\partial x} = -e^{-x}\sin(x+y) + e^{-x}\cos(x+y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} e^{-x} \sin(x+y) \cdot \frac{\partial}{\partial x} (x+y)$$

put e^{-x} as constant

$$= e^{-x} \cos(x+y) \cdot 1$$

$$\frac{\partial f}{\partial y} = \underline{\underline{e^{-x} \cos(x+y)}}$$

d) $f(x,y) = \ln(x+y)$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \ln(x+y) \cdot \frac{\partial}{\partial x} (x+y)$$

$$= \frac{1}{x+y} \cdot 1$$

$$= \underline{\underline{\frac{1}{x+y}}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \ln(x+y) \cdot \frac{\partial}{\partial x} (x+y)$$

$$= \frac{1}{x+y} \cdot x$$

$$= \underline{\underline{\frac{1}{x+y}}}$$

e) $f(x,y) = e^{xy} \ln y$

$$\Rightarrow \frac{\partial f}{\partial x} = e^{xy} \frac{\partial}{\partial x} \ln y + \ln y \frac{\partial}{dx} e^{xy} \cdot \frac{\partial}{d} xy$$

$$= \cancel{e^{xy} \cdot \frac{1}{y}} + \ln y \cdot e^{xy} \cdot 1y$$

$$= \frac{e^{xy}}{y} + y \ln y \cdot e^{xy}$$

$\frac{1}{f)}$

$\Rightarrow$

$$\frac{\partial f}{\partial y} = e^{xy} \frac{\partial}{\partial y} \ln y + \ln y \frac{\partial}{\partial y} e^{xy} \cdot \frac{\partial}{\partial y} xy$$

$$= e^{xy} \frac{1}{y} + \ln y \cdot e^{xy} \cdot x$$

$$= \frac{e^{xy}}{y} + x \ln(y) e^{xy}$$

$$\frac{\partial f}{\partial y} = \frac{e^{xy}}{y} + e^{xy} x \ln(y)$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} e^{xy} \ln y$$

$$= e^{xy} \ln y \frac{\partial}{\partial x}(xy)$$

$$= e^{xy} \ln(y) \cdot 1\, y$$

$$= e^{xy}\, y \ln(y)$$

f) $f(x, y, z) = \sqrt{x^2+y^2+z^2}$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \sqrt{x^2+y^2+z^2} \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)$$

$$= \frac{1}{2\sqrt{x^2+y^2+z^2}} \cdot 2x$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\sqrt{x^2+y^2+z^2} \cdot \frac{\partial}{\partial y} x^2+y^2+z^2$$

$$= \frac{1}{2\sqrt{x^2+y^2+z^2}} \cdot 2y$$

$$= \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$= \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\sqrt{x^2+y^2+z^2} \cdot \frac{\partial}{\partial z} x^2+y^2+z^2$$

$$= \frac{1}{\cancel{2}\sqrt{x^2+y^2+z^2}} \cdot \cancel{2}z$$

$$= \frac{z}{\sqrt{x^2+y^2+z^2}}$$

g) $f(x,y,z) = \sin^{-1}(xyz)$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\sin^{-1}(xyz) \cdot \frac{\partial}{\partial x} xyz$$

$$= \frac{1}{\sqrt{1-(xyz)^2}} \cdot 1 \cdot yz$$

$$\frac{\partial f}{\partial x} = \frac{yz}{\sqrt{1-(xyz)^2}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \sin^{-1}(xyz) \cdot \frac{\partial}{\partial y} xyz$$

$$= \frac{1}{\sqrt{1-(xyz)^2}} \cdot xz$$

$$= \frac{xz}{\sqrt{1-(xyz)^2}}$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} \sin^{-1}(xyz) \cdot \frac{\partial}{\partial z} xyz$$

$$= \frac{1}{\sqrt{1-(xyz)^2}} \cdot xy$$

$$= \frac{xy}{\sqrt{1-(xyz)^2}}$$

h) $f(x,y) = e^{-(x^2+y^2+z^2)}$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} e^{-(x^2+y^2+z^2)} \cdot \frac{\partial}{\partial x} -(x^2+y^2+z^2)$$

$$= e^{-x^2-y^2-z^2}(-2x)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} e^{-x^2-y^2-z^2} \cdot \frac{\partial}{\partial y} -x^2-y^2-z^2$$

$$= e^{-x-y^2-z^2} \cdot (-2y)$$

i) $f(x,y,z) = e^{-(xyz)}$

$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} e^{-(xyz)} \cdot \frac{\partial}{\partial x} xyz$

$= -e^{-(xyz)} 1 \cdot (yz)$

$= -e^{-xyz} yz$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} e^{-(xyz)} \cdot \frac{\partial}{\partial y} xyz$

$= -e^{-xyz} 1 \cdot (xz)$

$= -e^{-xyz} xz$

$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} e^{-(xyz)} \frac{\partial}{\partial z} xyz$

$= -e^{-xyz} 1 \cdot (xy)$

$= -e^{-xyz} xy$

j) $g(u,v) = v^2 e^{2u/v}$

$\Rightarrow \frac{\partial g}{\partial u} = \frac{\partial}{\partial u} v^2 e^{2u/v}$

keep v^2 constant

d) $f(t, \alpha) = \cos(2\pi t - \alpha)$

$$\Rightarrow \frac{\partial f}{\partial t} = \frac{\partial}{\partial t} \cos(2\pi t - \alpha) \; \frac{\partial}{\partial t} (2\pi t - \alpha)$$

$$= -\sin(2\pi t - \alpha) \cdot 2\pi$$

$$\frac{\partial f}{\partial \alpha} = \frac{\partial}{\partial \alpha} \cos(2\pi t - \alpha) \cdot \frac{\partial}{\partial \alpha} (2\pi t - \alpha)$$

$$= -\sin(2\pi t - \alpha) \cdot -1$$

$$= \sin(2\pi t - \alpha)$$