Unit - III

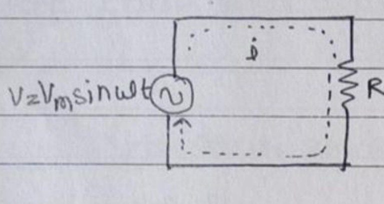
Single Phase AC Circuits

- The three basic elements of any ac circuit are Resistance (R), Inductance (L) and capacitance (C).
- The three basic circuits are.

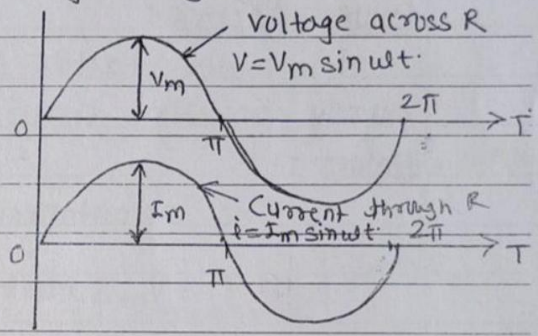
1. Purely resistive AC circuit-
2. Purely Inductive AC circuit-
3. Purely Capacitive AC circuit-

* AC circuit consisting of pure resistance:

- The purely resistive ac circuit is as shown in fig (a).
- It consists of an ac voltage source $v = V_m \sin \omega t$, and a resistor R connected across it.
- The instantaneous current flowing through the circuit is.

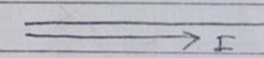


a) Purely resistive ac circuit



b) Voltage and current waveform:

Fig: Purely resistive AC circuit.



c) Phasor diag

Analysis of Purely resistive AC circuit:

Voltage and current:

Let instantaneous voltage be given by,

$$v = V_m \sin \omega t$$

Then by Ohm's law, instantaneous current

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R} = I_m \sin \omega t$$

$$\therefore I_m = \frac{V_m}{R}$$

PAGE No. :
DATE : / /

If rms voltage is V & rms current is I,

$$I = \frac{I_m}{\sqrt{2}} \text{ \& } V = \frac{V_m}{\sqrt{2}}$$

$$\therefore I = \frac{I_m}{\sqrt{2}} = \frac{V_m}{\sqrt{2}R} = \frac{V}{R} \quad \text{i.e } I = \frac{V}{R} \text{ (ohms law)}$$

Phase difference (ϕ):-

It is seen from above that when $v = V_m \sin \omega t$ & $i = I_m \sin \omega t$.

Therefore, 'V' and 'I' are in phase as shown in waveforms of fig 'b' & phasor diagram of fig 'c'.

$\therefore$ Phase difference $= \phi = 0$.

Power factor:

The power factor $= \cos\phi = \cos 0 = 1$ or unity for purely resistive circuit.

Power:

The instantaneous power 'P' is given by

$$P = v \cdot i = V_m \sin \omega t \cdot I_m \sin \omega t = V_m I_m \sin^2 \omega t$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

The eq^n shows that the waveform of power 'p' is a constant power $V_m I_m / 2$ with a double frequency sinusoidal power $\frac{V_m I_m}{2} \cos 2\omega t$ superimposed on it.

The average of 2nd term $\frac{V_m I_m}{2} \cos 2\omega t$ is zero

$\therefore$ The total average power, $P = \frac{V_m I_m}{2}$

$$\therefore P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI \qquad P = VI = \frac{V^2}{R} = I^2 R = VI\cos\phi$$

$$Q = VI \sin\phi = VI \times 0 = 0$$

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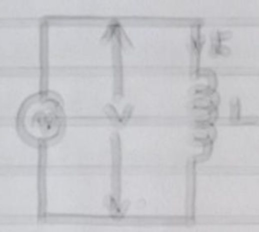
* AC Circuit Consisting of Pure inductance:

Inductor: It is a device which stores the energy in the magnetic form.

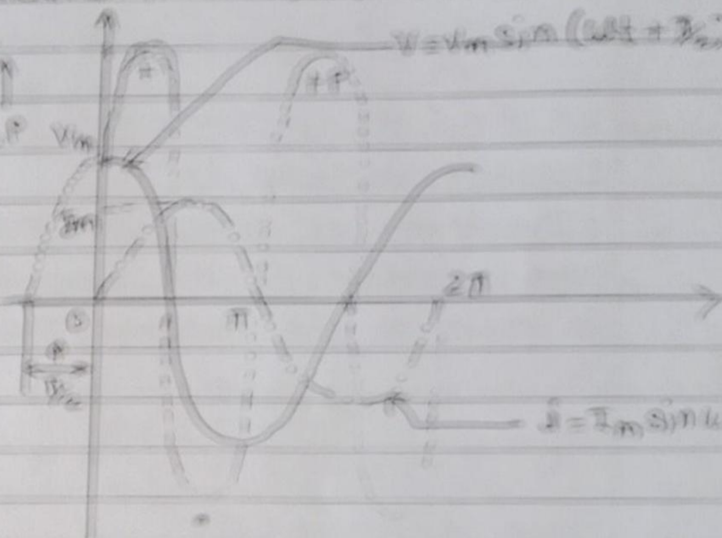
Construction: the inductor is formed by winding a coil the magnetic core

Inductance: It is defined as the property of the coil which an emf is induced in it when its current is changed w.r.t time

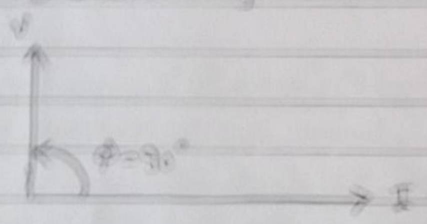
Circuit:



ⓐ Circuit diagram



ⓑ Waveforms



ⓒ Phasor Diagram.

- Fig. a shows the purely inductive circuit. The inductor L henry is connected across a sinusoidal r.m.s voltage V to pass r.m.s current I through it

Voltage and Current:-

Let instantaneous current be given by,

$i = I_m \sin\omega t$

Then the voltage across the inductance is given by,

PAGE No. :
DATE : / /

$v = -\text{emf } e = -\left(-L\frac{di}{dt}\right)$ by definition

$\therefore v = L\frac{d}{dt}(I_m \sin\omega t) = L \cdot I_m \cdot \omega \cos\omega t$

$= I_m \, \omega L \sin(\omega t + \pi/2)$

$v = V_m \sin(\omega t + \pi/2)$

$\therefore V_m = I_m \cdot \omega L$

$\therefore V = \frac{V_m}{\sqrt{2}} = \frac{I_m}{\sqrt{2}} \omega L = I \cdot \omega L$

i.e $V = I \cdot \omega L$ -- (ohm's law).

Inductive reactance (X_L):

- The term 'ωL' represents the opposition offered by the pure inductance to the flow of current. This opposition is known as the inductive reactance X_L.

$\therefore X_L = \omega L = 2\pi f L$ ... ohms

& $V = I \cdot \omega L = I \cdot X_L$ --- volts.

Phase difference (ϕ):

It is seen that when $i = I_m \sin(\omega t + \pi/2)$

$\therefore$ v & i are out of phase as shown in waveforms of fig. b. and phasor diagram of fig. c.

$\therefore$ The phasor difference lagging $\phi = \pi/2$ rad $= 90°$ i.e the current i lags the voltage v by $90°$.

~~The phasor difference lagging $\phi = \pi/2$ rad $= 90°$ i.e the current i lags the voltage v by $90°$~~

Power factor: The p.f. $= \cos\phi = \cos 90° = 0$ lagging. This power factor is said to be zero lagging because current is lagging the voltage by $90°$.

PAGE No.
DATE: / /

Active Power (P): The instantaneous power P is given by

$$P = V \cdot i = V_m \sin(\omega t + \pi/2) \cdot I_m \sin \omega t$$
$$= V_m I_m \sin \omega t \cdot \sin(\omega t + \pi/2)$$

The waveform of power 'p' is as shown in fig. b.
It is clear that the average power or active power P=0 because positive half cycle of 'P' is equal to negative half cycle of 'P'.
∴ For purely inductive circuit, active power P=0 watts

Reactive Power (Q): The imaginary power $VI \sin\phi$ is known as the reactive power Q. It is measured in Volt-Amperes Reactive (VArs).

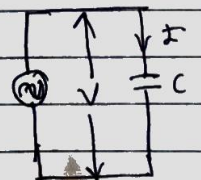
$$\therefore \text{Reactive Power, } Q = VI\sin\phi = VI\sin 90^\circ$$
$$= VI = \frac{V^2}{X_L} = I^2 X_L \text{ — VArs.}$$

* **AC circuit Consisting of Pure Capacitance:**

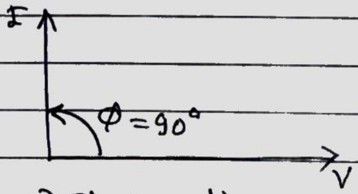
Capacitor - It is a device which stores the energy in the electric field.

Construction - A capacitor is defined as the property of the device by which it carries a current when its voltage is changed w.r.t. time.

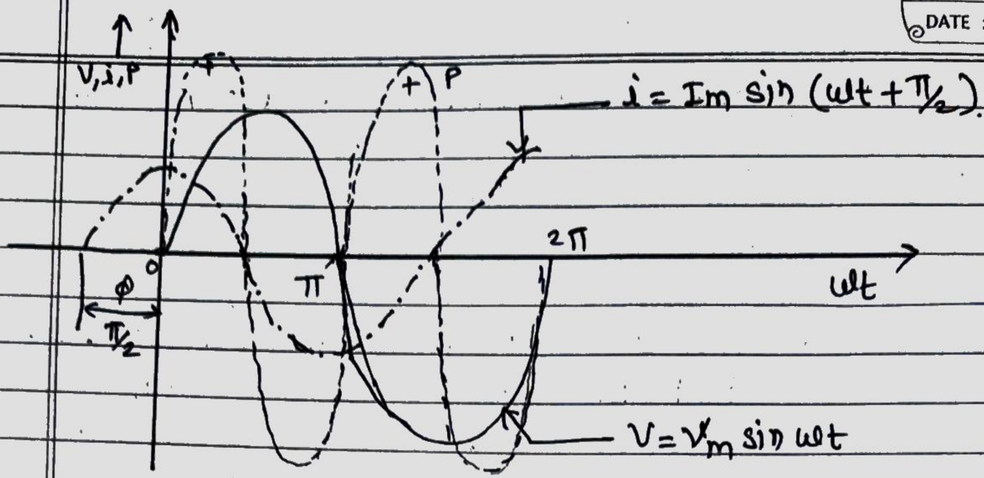
Circuit - fig 'a' shows the purely capacitive circuit. The Capacitance C farads is connected across a rms voltage V to take a rms current I.



a) Circuit diagram



c) Phasor diagram



b) Waveforms.

<u>Voltage and current</u>:

Let the instantaneous voltage be given by,

$$v = V_m \sin \omega t$$

Then the current 'i' is given by,

$$i = C\frac{dv}{dt} \quad \text{— by definition}$$

$$\therefore \quad i = C\frac{d}{dt}(V_m \sin \omega t)$$

$$i = CV_m \cdot \omega \cos \omega t = V_m \cdot \omega C \cdot \sin(\omega t + \pi/2)$$

$$= \frac{V_m}{\left(\frac{1}{\omega C}\right)} \sin(\omega t + \pi/2) = I_m \sin(\omega t + \pi/2)$$

$$\therefore I_m = \frac{V_m}{(1/\omega C)} \qquad \therefore \quad I = \frac{I_m}{\sqrt{2}} = \frac{V_m}{\sqrt{2}\left(\frac{1}{\omega C}\right)} = \frac{V}{(1/\omega C)}$$

i.e., $I = \dfrac{V}{\left(\frac{1}{\omega C}\right)}$ or $V = I \cdot \dfrac{1}{\omega C}$ — ohms law.

Capacitive Reactance (X_C):-

The term $(1/\omega C)$ represents the opposition offered by the purely capacitive circuit to the flow of current. Th opposition is known as the capacitive reactance X_C.

$$\therefore X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \text{ — Ohms.}$$

$$\& \quad V = I \cdot X_C = I \frac{1}{\omega C}$$

Phase difference (ϕ):-

It is seen that when $v = V_m \sin \omega t$,

$$i = I_m \sin(\omega t + \pi/2)$$

$\therefore$ v & i are out of phase as shown in the waveform o fig. 'b' and phasor diagram of fig. 'c'.

$\therefore$ The phasor difference, $\phi = \pi/2$ radians $= 90°$ leading

i.e the current i leads the voltage v by 90°.

Power factor :-

The P.f. $= \cos\phi = \cos 90° = 0$ leading.

This power factor is said to be zero leading because the current is leading the voltage by 90°.

Active Power (P):-

The instantaneous power 'P' is given by,

$$p = v \cdot i = V_m \sin\omega t \cdot I_m \sin(\omega t + \pi/2)$$

$$= V_m \cdot I_m \sin\omega t \cdot \sin(\omega t + \pi/2)$$

The waveform of power 'p' is shown in fig. b. It is clear that the average power or active power $P = 0$ because positive half cycle of 'P' is equal to negative ha cycle of 'P'.

$\therefore$ For purely capacitive circuit, active power $P = 0$ watts.

Reactive power (Q):

The imaginary Power $VI\sin\phi$ is known as the reactive power (Q).

∴ Reactive power $Q = VI\sin\phi$

$$= VI\sin 90^\circ$$

$$= VI = \frac{V^2}{X_C}$$

$$= I^2 X_C \text{ — VArs.}$$

* Series R-L Circuit :-

- The series R-L circuit is as shown in figure. This ac voltage source of instantaneous voltage $V = V_m \sin\omega t$ is connected across the series combination of L & R.
- Assume the resistance to be of value R ohms and inductance to be a pure inductance of value L Henries.

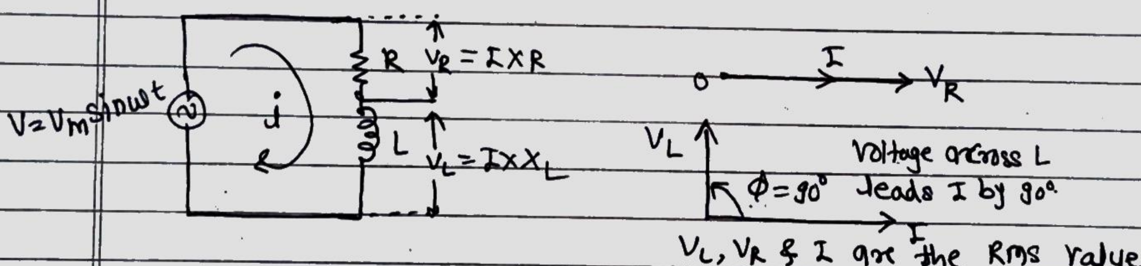


Fig.1 R-L series circuit.

- Assume that the current flowing through L and R is I amp. Where I is the rms value of the instantaneous current i.
- Due to this current the voltage drops across the L & R are given by,

Voltage drop across R,

$$V_R = I \cdot R \quad (V_R \text{ is in phase with } I)$$

Voltage drop across L,

$$V_L = I \cdot X_L \quad (V_L \text{ leads } I \text{ by } 90^\circ)$$

Phasor diagram:-

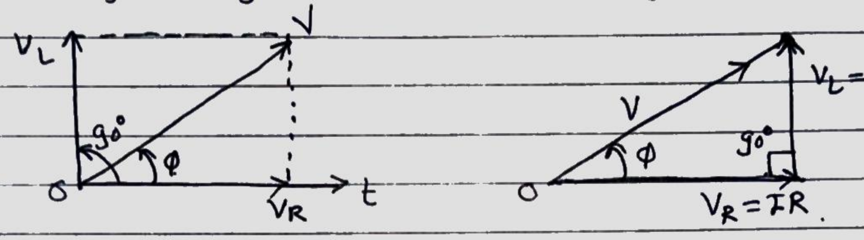
- The applied voltage V is equal to the phasor addition of V_R & V_L,

$$\bar{V} = \bar{V}_R + \bar{V}_L \quad \text{(Phasor addition)}$$

Substituting $\bar{V}_R = \bar{I}R$ & $\bar{V}_L = \bar{I}X_L$ we get,

$$\bar{V} = \bar{I}R + \bar{I}X_L$$

- This addition is shown as phasor diagram in fig 2.(a) the voltage triangle is as shown in fig. 'b'



(a) Phasor diagram of L-R circuit

(b) voltage triangle.

fig. 2

- Impedance of the L-R series circuit (Z) can be obtained from the voltage triangle of fig 2 as

Resultant voltage,

$$V = \text{Vector sum of } V_R \text{ \& } V_L$$

$$\therefore \quad V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = \sqrt{I^2(R^2 + X_L^2)}$$

$$= I\sqrt{(R^2 + X_L^2)}$$

- It can be seen that current lags voltage by angle ϕ

$$\therefore \quad v(t) = V_m \sin \omega t \quad \&$$

$$i(t) = I_m \sin(\omega t - \phi)$$

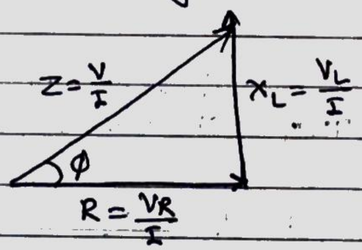
* Impedance and Impedance Triangle :-

- Impedance is defined as the opposition of circuit to flow of alternating current. It is denoted by Z and its unit is ohms.
- For the R-L series circuit, it can be observed from the phasor diagram that the current lags behind the applied voltage by an angle ϕ from the voltage triangle, we can write,

$$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R} \quad ; \quad \cos\phi = \frac{V_R}{V} = \frac{R}{Z}$$

$$\sin\phi = \frac{V_L}{V} = \frac{X_L}{Z}$$

- If all the sides of the voltage triangle are divided by current, we get a triangle called Impedance triangle as shown in the fig.



- From this impedance triangle, we can see that the x component of impedance is R & is given by, $R = Z\cos\phi$ and Y component of impedance is X_L and is given by, $X_L = Z\sin\phi$
- In Rectangular form the impedance is denoted as,

$$\boxed{Z = R + jX_L}\ \Omega$$

- While in polar form, it is denoted as,

$$Z = |Z| \angle \phi\ \Omega$$

$$|Z| = \sqrt{R^2 + X_L^2} \quad , \quad \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

* Power and Power Triangle

- The expression for the current in the series R-L circuit is,

$$i = I_m \sin(\omega t - \phi) \quad \text{as current lags voltage.}$$

- The power is product of instantaneous values of voltage and current,

$$\therefore \quad P = V \times i$$

$$= V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m \cdot I_m [\sin(\omega t) \cdot \sin(\omega t - \phi)]$$

$$= V_m \cdot I_m \left[\frac{\cos(\phi) - \cos(2\omega t - \phi)}{2} \right]$$

$$= \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

- Now, the second term is cosine term whose average value over a cycle is zero.
- Hence, average power consumed is,

$$P_{av} = \frac{V_m I_m}{2} \cos\phi - \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\therefore \quad \boxed{P = VI\cos\phi} \text{ Watts}$$

Where V & I are r.m.s. values

- If we multiply voltage equation by current I, we get the power equation-

$$\overline{VI} = \overline{V_R I} + \overline{V_L I} = \overline{V\cos\phi}, + \overline{V\sin\phi},$$

- From this eqn, Power triangle can be obtained as shown in the fig.

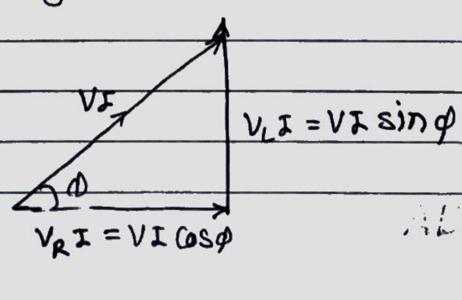


fig. Power triangle

PAGE No. :
DATE : / /

So, three sides of this triangle are,
1) VI 2) $VI\cos\phi$ 3) $VI\sin\phi$

1. Apparent Power (S) -
- It is defined as the product of r.m.s. value of voltage (V) and current (I). It is denoted by S

$$\therefore \quad \boxed{S = VI \quad VA}$$

- It is measured in unit volt-amp (VA) or kilo volt-amp (KVA).

2. Real or True Power (P) -
- It is defined as the product of the applied voltage and the active component of the current.
- It is real component of the apperent power. It is measured in unit Watts (W) or kilowatts (KW).

$$\boxed{P = VI\cos\phi \quad \text{Watts}}$$

3. Reactive Power (Q):
- It is defined as imaginary component of the apparent power. It is represented by 'Q' and it is measured in unit volt-amp reactive (VAR) or kilovolt-amp reactive (KVAR).

$$\boxed{Q = VI\sin\phi \quad VAR}$$

* Power factor ($\cos\phi$):
- It is defined as factor by which the apparent power must be multiplied in order to obtain the true power.
- It is the ratio of true power to apparent Power.

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$$\text{Power Factor} = \frac{\text{True Power}}{\text{Apparent Power}}$$

$$= \frac{VI\cos\phi}{VI} = \cos\phi$$

- The numerical value of cosine of the phase angle between the applied voltage and the current drawn from the supply voltage gives the power factor. It cannot be greater than 1.
- It is also defined as the ratio of resistance to the impedance.

$$\boxed{\cos\phi = \frac{R}{Z}}$$

- The nature of power factor is always determined by position of current with respect to the voltage.
- If current lags voltage power factor is said to be lagging. If current leads voltage power factor is said to be leading.
- So, for pure inductance, the power factor is cos (90°) i.e zero lagging while for pure capacitance the power factor is cos (90°) i.e zero but leading.
- For purely resistive circuit voltage and current are in phase i.e $\phi = 0$.
- Therefore, power factor is $\cos(0°) = 1$. Such circuit is called unity power factor circuit.

$$\text{Power Factor} = \cos\phi$$

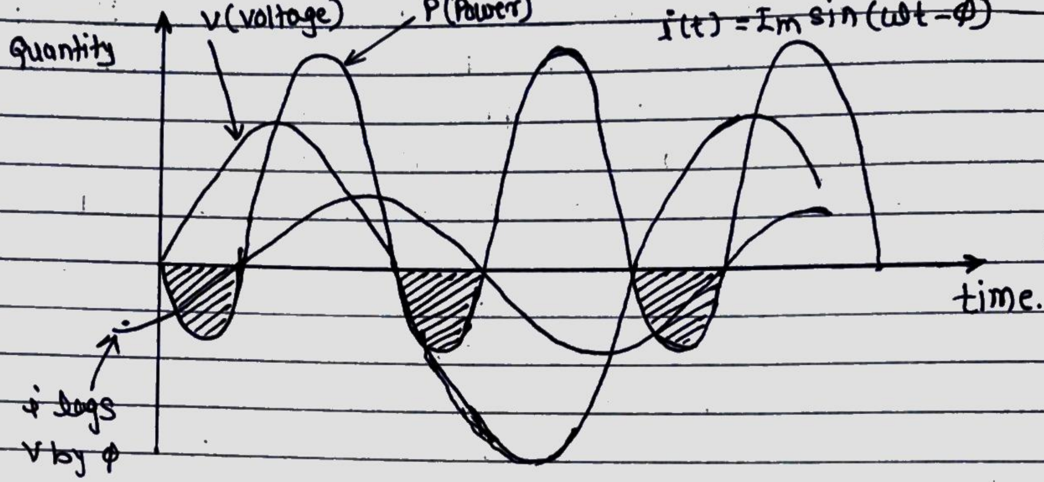
ϕ is the angle between supply voltage and current
- Nature of power factor always tells position of current with respect to voltage.

* Waveforms of voltage, current and Power
- The waveforms are shown in fig.

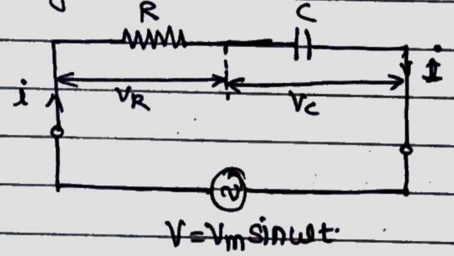
$$\boxed{P = VI\cos\phi \ \text{W}}$$

$v(t) = V_m \sin \omega t$

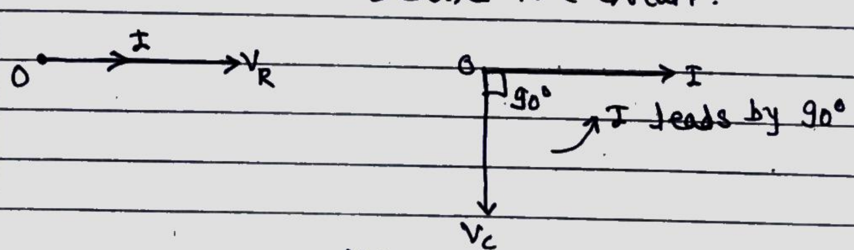
$i(t) = I_m \sin(\omega t - \phi)$



* <u>Series R-C circuit</u>:
- consider a circuit consisting of pure resistance R-ohms and connected in series with a pure capacitor of C-farads as shown in fig.



a) series R-C circuit.



(b)

- The series combination is connected across a.c. supp given by

$$V = V_m \sin \omega t$$

circuit draws a current I, then there are two voltage drops.

a) Drop across pure resistance, $V_R = I \times R$

b) Drop across pure capacitance, $V_C = I \times X_C$

Where $X_C = \frac{1}{2\pi f C}$

& I, V_R, V_C are the r.m.s values

The kirchhoff's voltage law can be applied to get,

$$V = \bar{V}_R + \bar{V}_C$$

$$= \bar{I}R + \bar{I}X_C \quad \text{—— (Phasor Addition)}$$

- Phasor Diagram :-

- Let us draw the phasor diagram. Current I is taken reference as it is common to both the elements

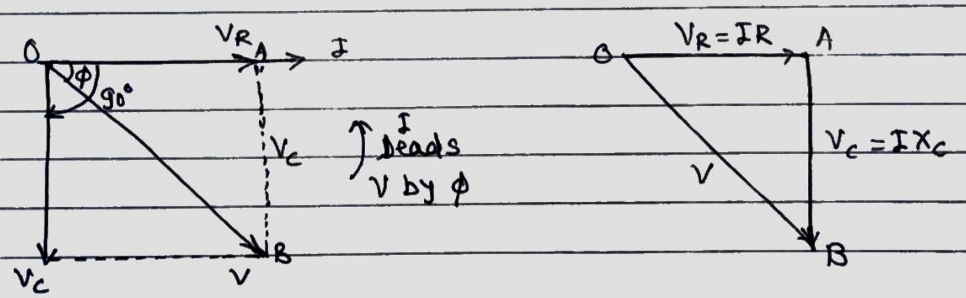


Fig.: Phasor diagram and voltage triangle.

From the voltage triangles,

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{(R)^2 + (X_C)^2}$$

$$\therefore \quad V = IZ$$

PAGE No. :
DATE : / /

Where,
$$\boxed{Z = \sqrt{(R)^2 + (X_c)^2}}$$

- It can be seen that current leads voltage by angle ϕ hence

$$\boxed{v(t) = V_m \sin \omega t} \quad \& \quad \boxed{i(t) = I_m \sin(\omega t + \phi)}$$

* Impedance and Impedance Triangle :
- similar to R-L series circuit, in this case also, the impedance is nothing but opposition to the flow of alternating current.
- It is measured in ohms given by $Z = \sqrt{(R)^2 + (X_c)^2}$
Where
$$X_c = \frac{1}{2\pi f C} \ \Omega \text{ called capacitive reactance.}$$

- In R-C series circuit, current leads voltage by angle ϕ or supply voltage V lags current I by angle ϕ as shown in the phasor diagram in fig. of voltage phasor diagram.
- From voltage triangle, we can write,
$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R},$$

$$\cos\phi = \frac{V_R}{V} = \frac{R}{Z}, \quad \sin\phi = \frac{V_C}{V} = \frac{X_C}{Z}$$

- If all the sides of the voltage triangle are divided by the current, we get a triangle called Impedance triangle.
- Two sides of the triangle are 'R' and 'X_c' and the third side is impedance 'Z'.
- The X & Y components are given as
$$R = Z\cos\phi \quad \& \quad X_c = Z\sin\phi$$

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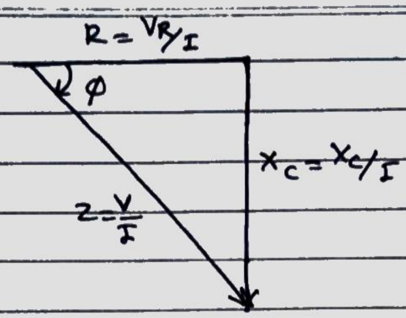


Fig.: Impedance triangle.

- But, as direction of the X_C is the negative Y direction the rectangular form of the impedance is denoted as,

$$\boxed{Z = R - jX_C} \ \Omega$$

While in polar form, it is denoted as,

$$Z = |Z| \angle -\phi \ \Omega$$

$$|Z| = \sqrt{R^2 + X_C^2} \ , \ \phi = \tan^{-1}\left[\frac{-X_C}{R}\right]$$

* Power and Power Triangle :-

- The current leads voltage by angle ϕ hence its expression is,

$$i = I_m \sin(\omega t + \phi) \quad \text{as current leads voltage}$$

- The power is the product of instantaneous values of voltage and current.

$$\therefore P = V \times i = V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin(\omega t) \cdot \sin(\omega t + \phi)]$$

$$= V_m I_m \left[\frac{\cos(-\phi) - \cos(2\omega t + \phi)}{2}\right]$$

$$P = \frac{V_m I_m \cos\phi}{2} - \frac{V_m I_m}{2} \cos(2\omega t + \phi)$$

as $\cos(-\phi) = \cos\phi$

- Now, second term is cosine term whose average value over a cycle is zero.

Hence, average power consumed by the circuit is,

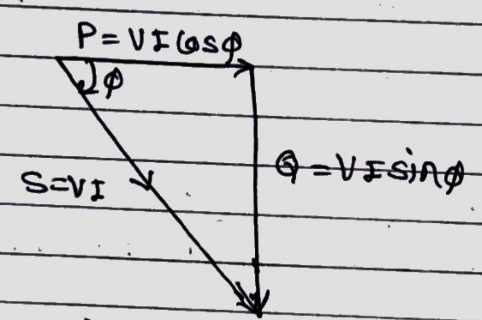
$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

∴ $\boxed{P = VI\cos\phi}$ watts Where V & I are r.m.s. values.

- If we multiply vtg equation by current I, we get the power equation,

$$\overline{VI} = \overline{V_R I} + \overline{V_C I} = \overline{VI\cos\phi} + \overline{VI\sin\phi}$$

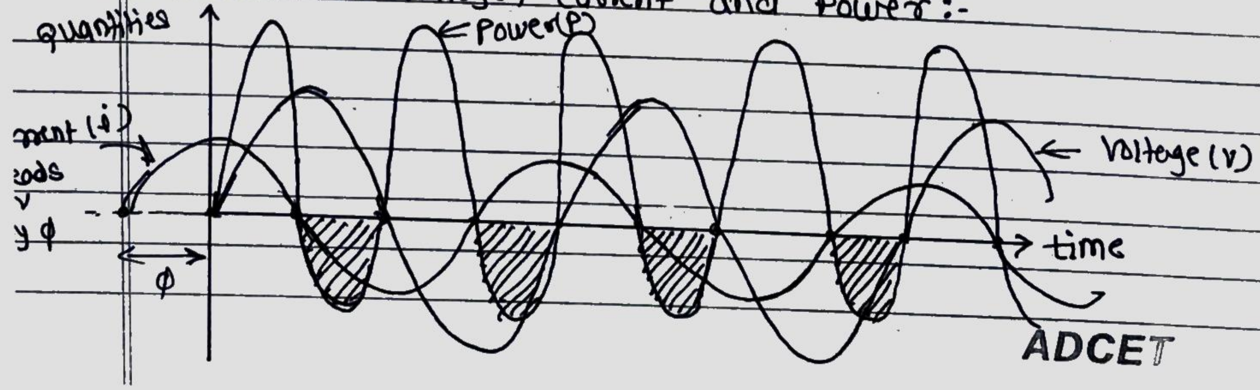
- Hence, the power triangle can be shown as in the fig.



Thus, the various powers are,

Apparent power, $S = VI$ VA
True or average power, $P = VI\cos\phi$ W
Reactive power, $Q = VI\sin\phi$ VAR

* Waveforms of voltage, current and power :-



* Series R-L-C Circuit :-

- Consider a circuit consisting of resistance R ohms pure inductance L henries and capacitance C farads connected series with each other across a.c. supply.
The circuit is shown in figure.

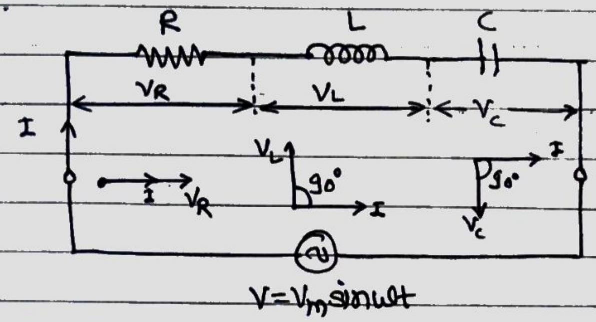


Fig.: R-L-C series circuit

- The a.c. supply is given by,

$$V = V_m \sin \omega t$$

- The circuit draws a current I.
- Due to current I, there are different voltage drops across R, L & C which are given by,

∴ Drop across Resistance R is, $V_R = IR$

Drop across inductance L is, $V_L = I \cdot X_L$

Drop across capacitance C is, $V_C = I \cdot X_C$

- The values of I, V_R, V_L & V_C are r.m.s values.
- The characteristics of three drops are,

∴ V_R is in phase with current I

V_L leads current I by 90°

V_C lags current I by 90°.

- According to Kirchhoff's laws, we can write,

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{— Phasor addition}$$

Phasor Diagram:-

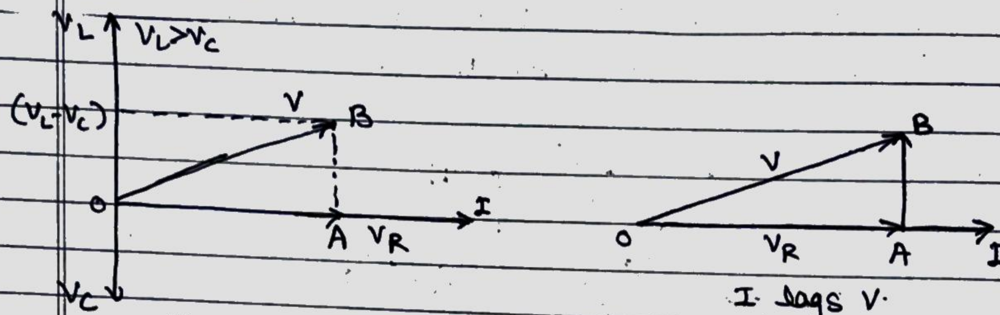


Fig : Phasor diagram and voltage triangle for $X_L > X_C$

- Let us see the phasor diagram. Current I is taken as reference as it is common to all the elements.
- The phasor diagram depends on the conditions of the magnitudes of V_L & V_C which ultimately depends on the values of X_L & X_C.
- Let us consider the different case -

- $X_L > X_C$
- When $X_L > X_C$, obviously IX_L i.e V_L is greater than IX_C i.e V_C. So, resultant of V_L & V_C will be directed towards V_L i.e leading current I. Current I will lag the resultant of V_L & V_C i.e $(V_L - V_C)$.
- The circuit is said to be inductive in nature. The phasor sum of V_R & $(V_L - V_C)$ gives the resultant supply voltage, V. This is shown in above fig.
- From the voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{(R)^2 - (X_L - X_C)^2}$$

$$\therefore \quad = IZ$$

Where $Z = \sqrt{(R)^2 - (X_L - X_C)^2}$

- So, if $V = V_m \sin \omega t$, then $i = I_m \sin(\omega t - \phi)$ as cur lags voltage by angle ϕ for $X_L > X_C$.

- $X_L < X_C$

- When $X_L < X_C$, obviously, IX_L i.e V_L is less than IX_C i.e V_C. So, the resultant of V_L & V_C will be directed towards V_C. Current I will lead $(V_C - V_L)$.
- The circuit is said to be capacitive in nature. The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage V. This is shown in following fig.

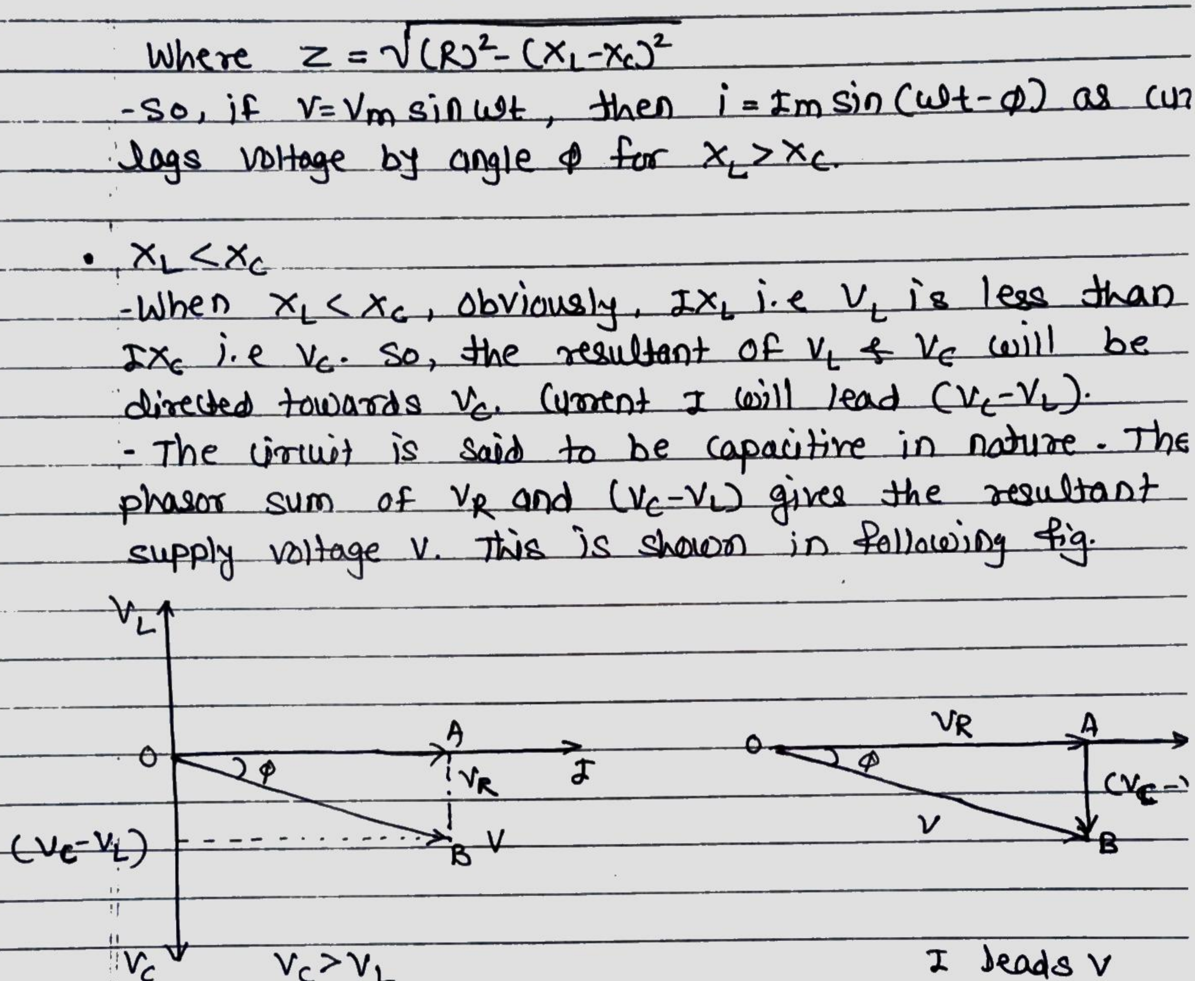


Fig: Phasor diagram and voltage triangle for $X_L < X$

- from voltage triangle,

$$V = \sqrt{(V_R)^2 + (V_C - V_L)^2}$$

$$V = \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$= I\sqrt{(R)^2 + (X_C - X_L)^2}$$

$\therefore$ $V = IZ$

Where, $Z = \sqrt{(R)^2 + (X_C - X_L)^2}$

- So, if $V = V_m \sin\omega t$, then $i = I_m \sin(\omega t + \phi)$ as current leads voltage by angle ϕ for $X_L < X_C$.

- $X_L = X_C$
  - When $X_L = X_C$, Obviously, $V_L = V_C$. So, V_L and V_C will cancel each other and their resultant is zero.
  - So, $V_R = V$ in such case and overall circuit is purely resistive in nature. The phasor digram is shown in fig.

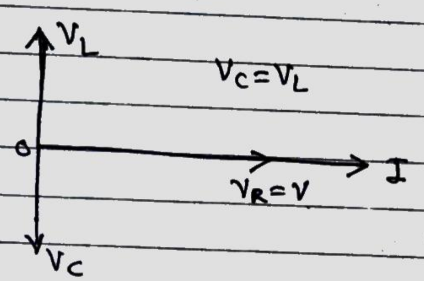


Fig. : Phasor diagram for $X_L = X_C$

from phasor diagram,

$$V = V_R = IR$$

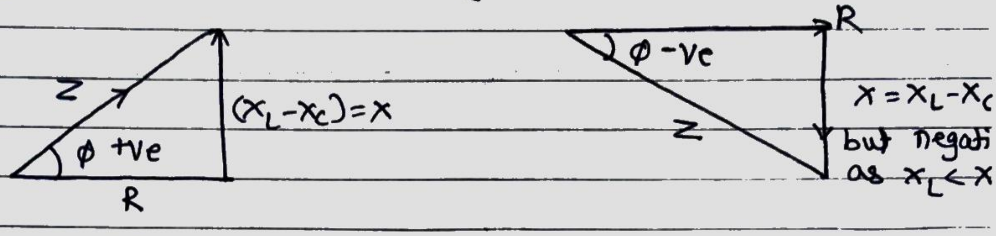
$\therefore$ $V = IZ$ where $Z = R$.

* Impedance Triangle :

- The impedance is expressed as,

$\therefore \boxed{Z = R + jX}$ where $\boxed{X = X_L - X_C}$

- for $X_L > X_C$, ϕ is positive and the impedance triangle is as shown in fig. (a).



(a) (b)

Fig. : Impedance triangles

- for $X_L < X_C$, $X_L - X_C$ is -ve, so ϕ is -ve and t impedance triangle is as shown in fig. (b).
- In both the cases,

$$R = Z\cos\phi \quad \& \quad X = Z\sin\phi$$

* Power and Power Triangle -

- The average power consumed by the circuit is,

P_{av} = Average Power consumed by R + Average Power consu by L + Average Power consumed by C.

- But, pure L & C never consume any power.

$\therefore$ P_{av} = Power taken by R $= I^2R = I(IR) = I \cdot V_R$

But, $V_R = V\cos\phi$ in both the cases

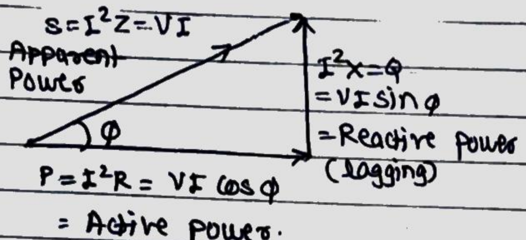
$\therefore \boxed{P = VI\cos\phi}$ W

PAGE No.:
DATE:

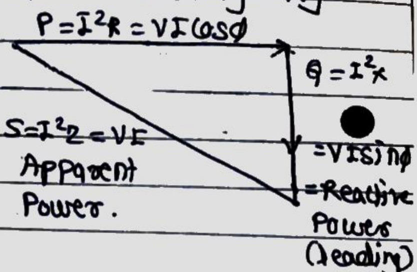
- Thus, for any condition, $X_L > X_C$ or $X_L < X_C$, the power can be expressed as,

P = Voltage x component of current in phase with voltage.

- The power triangle are shown in the following fig.



(a) $X_L > X_C$



b) $X_L < X_C$

Fig. : Power triangles.

* Resonance in series R-L-C circuit -

- We know that both X_L & X_C are the functions of frequency f. When f is varied both X_L and X_C also get verified.
- At a certain frequency, X_L becomes equal to X_C. Such a condition when $X_L = X_C$ for a certain frequency is called series resonance.
- At resonance the reactive part in the impedance of RLC series circuit is zero.
- The frequency at which the resonance occurs is called resonant frequency denoted as ω_r rad/sec or f_r Hz.

- **Characteristics of series Resonance:**

- In a series resonance, the voltage applied is consta[nt] and frequency is variable. Hence following parameters o[f] series RLC circuit get affected due to change in frequency:

1) X_L 2) X_C 3) Total reactance X 4) Impedance
5) I 6) $\cos\phi$

- As $X_L = 2\pi f L$, as frequency is changed from 0 to [∞] X_L increases linearily and graph of X_L against f i[s] straight line passing through origin.
- As $X_C = \frac{1}{2\pi f C}$, as frequency is changed from 0 [to ∞] X_C reduces and the graph of X_C against f is opposite to X_L hence graph of X_L Vs f is shown i[n] the first quadrant while X_C Vs f is shown in thi[rd] quadrant.
- At $f = f_r$, the value of $X_L = X_C$ at this frequency
- As $X = X_L - X_C$, the graph of X against f is sh[own] in the first quadrant while X_C Vs f is shown in the third quadrant. in fig.
- For $f < f_r$, the $X_C > X_L$ and net reactance X is capacitive while for $f > f_r$ the $X_L > X_C$ and net reactance X is inductive.
- $Z = R + jX = R + j(X_L - X_C)$ but at $f = f_r$, $X_L = X_C$ $X = 0$ hence the net impedance $Z = R$ which is purely resistive. so impedance is minimum & purel[y] resistive at series resonance.
- The graph of Z against f is also shown in th[e] figure.
- The Power factor $\cos\phi = R/Z$ & at $f = f_r$ as $Z = R$, t[he] power factor is unity & at its maximum at series resonance. For $f < f_r$ it is leading in nature while fo[r] $f > f_r$ it is lagging in nature.

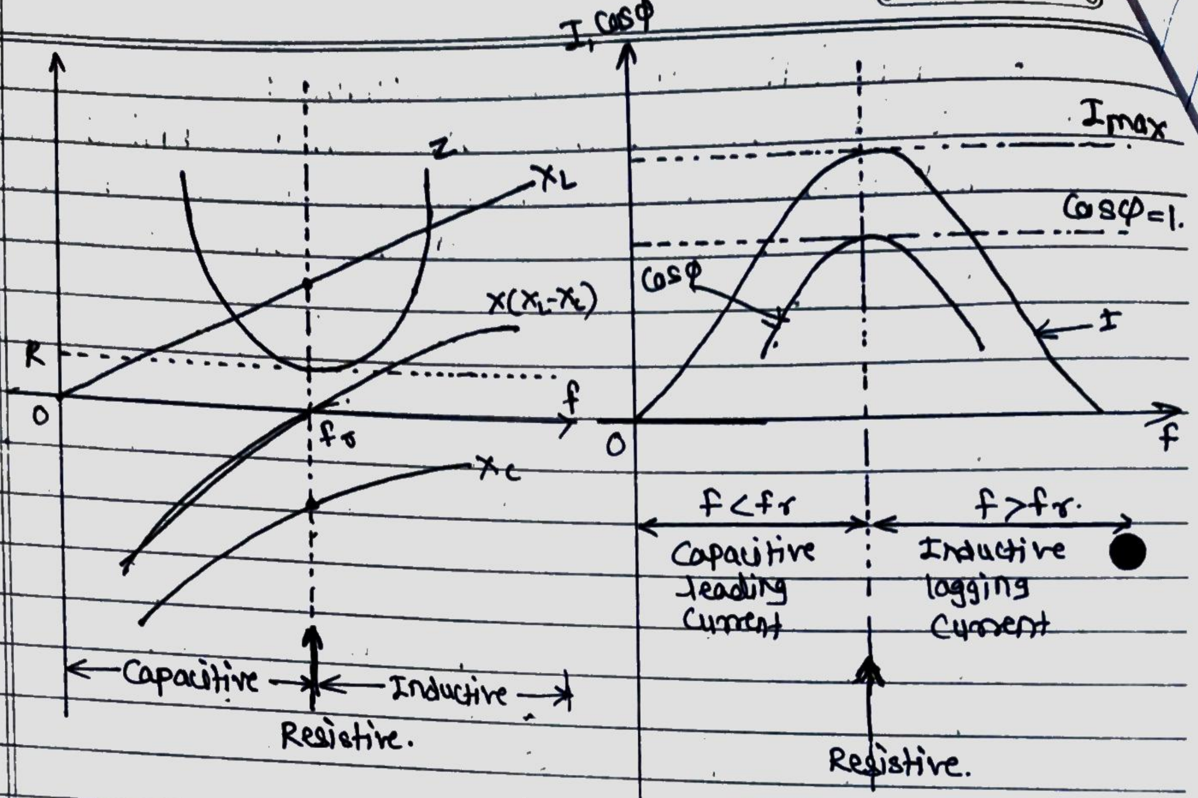


Fig : Characteristics of series resonance.

- Expression for resonance frequency-

Let f_r be the resonant frequency in Hz at which,

$$X_L = X_C$$

i.e. $2\pi f_r L = \frac{1}{2\pi f_r C}$ ------ series resonance

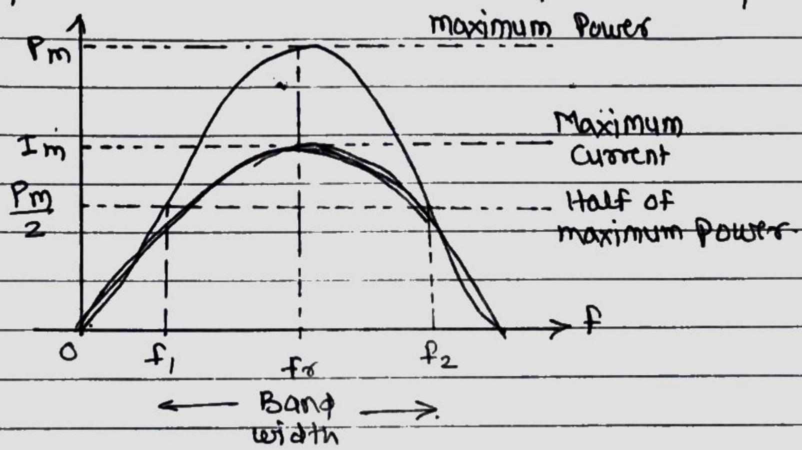
i.e. $(f_r)^2 = \frac{1}{4\pi^2 LC}$

i.e $\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$ Hz

$\therefore$ $\omega_r = \frac{1}{\sqrt{LC}}$ rad/sec.

• Bandwidth of series R-L-C circuit:

- At series resonance, current is maximum and impeda
Z is minimum.
- Now power consumed in a circuit is proportional to square of the current as $P = I^2R$.
- So at series resonance as current is maximum, pow is also at its maximum i.e P_m.
- In fig. it can be observed that at two frequenci f_1 & f_2 the power is half of its maximum value. These frequencies are called half power frequencies



- The difference between the half power frequencies and f_2 at which power is half of its maximum is called bandwidth of the series R-L-C circuit.

$$\boxed{B.W. = f_2 - f_1}$$

PAGE No. :
DATE : / /

$$\bar{I} = \bar{V} \times \left(\frac{1}{Z_1}\right) + \bar{V} \times \left(\frac{1}{Z_2}\right) + \bar{V} \times \left(\frac{1}{Z_3}\right)$$

$$\overline{VY} = \overline{VY_1} + \overline{VY_2} + \overline{VY_3}$$

$$\therefore \quad \bar{Y} = \bar{Y_1} + \bar{Y_2} + \bar{Y_3}$$

Where Y is the admittance of the total circuit.

- The three impedances connected in parallel can be replaced by an equivalent circuit, where these admittances are connected in series, as shown in fig.

Components of Admittance :

- Consider an impedance given as, $Z = R \pm jX$
- Positive sign for inductive & negative for capacitive circuit.

Admittance , $Y = \frac{1}{Z} = \frac{1}{R \pm jX}$

- Rationalising the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)} = \frac{R \mp jX}{R^2 + X^2}$$

$$= \left(\frac{R}{R^2+X^2}\right) \mp j\left(\frac{X}{R^2+X^2}\right) = \frac{R}{Z^2} \mp j\frac{X}{Z^2}$$

$$\therefore \quad \boxed{Y = G \mp jB}$$

In above expression.

$$G = \text{Conductance} = \frac{R}{Z^2}$$

$$\& \quad B = \text{Susceptance} = \frac{X}{Z^2}$$