ProblemSet6

November 4, 2019

1 Expectation-Maximization-Implementation

```
Implementing EM Algorithm on noisy data-sets.
Author: Sarvesh Thakur
Course: CMSC422 - Introduction to Machine Learning
```

1.0.1 Problem Statement

The purpose of this problem is to implement the Expectation-Maximization algorithm for the problem of grouping points into lines. That is, we assume that we are given a set of points, and we want to find two lines that explain them. In the expectation step, we find the line that minimizes the weighted sum of squares distance from points to lines. The variance is estimated using the distance between each line and the points. In the maximization step we assign (probabilistically) each point to each line based on its distance to the lines.

1. Line Fitting (20 points) Write a function that fits a line to data. Note that Weiss describes a method for doing this using weighted least squares, which essentially only looks at error in the y direction. This fits the examples below,in which noise is added to the y coordinate. If you are interested, you can also implement, for a small amount of extra credit, a total least squares method, that takes account of the Euclidean distance between each point and the line, and see what difference this makes. You will have to do a little research to see how this works. Test your function with the following set of points:

(i) x=0:0.05:1; y=2*x+1

(ii) x=0:0.05:1; y=2*x+1+0.1*randn(size(x))

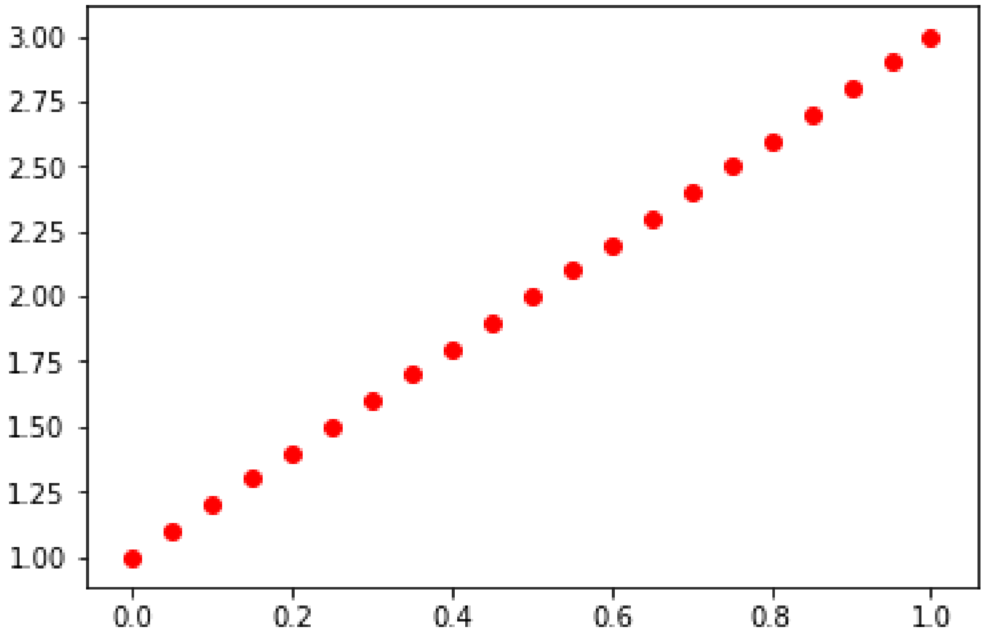
(iii) x=0:0.05:1; y=(abs(x-0.5) < 0.25).*(x+1)+(abs(x-0.5) >=0.25).*(-x);

In all cases plot the data and the best fitting lines.

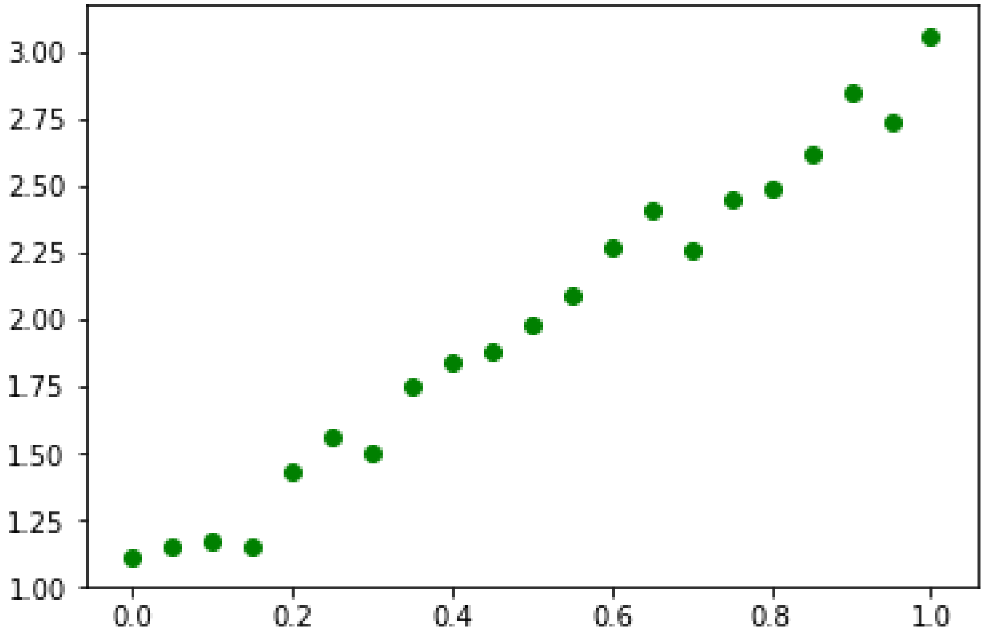
```
In [1]: %matplotlib inline
        import numpy as np
        import matplotlib.pyplot as plt
        import random
        import copy
        import math
```

1.0.2 0. Generate the Dataset

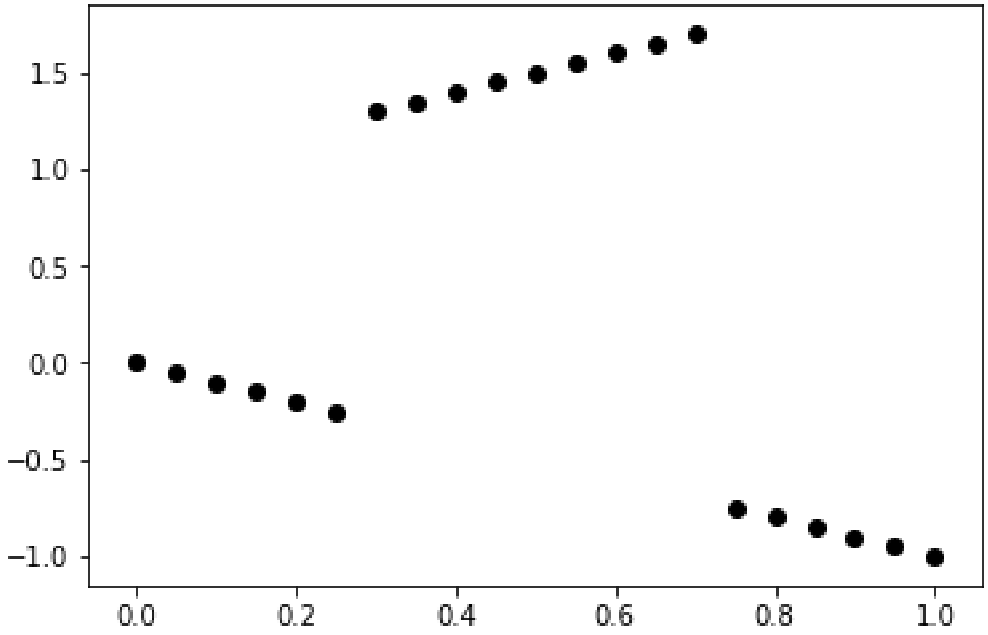
```
In [2]: # Creating Datasets
        x = np.round(np.arange(0, 1.05, 0.05), 3)

        # data set 1
        y1 = 2*x + 1
        dataset1 = list(zip(x, y1))

        # Visualize
        plt.scatter(*zip(*dataset1), color='red')
        plt.show()
```



```
In [618]: # data set 2
          y2 = np.round(2*x + 1 + 0.1*np.random.normal(0,1,len(x)), 3)
          dataset2 = list(zip(x, y2))

          # Visualize
          plt.scatter(*zip(*dataset2), color='green')
          plt.show()
```



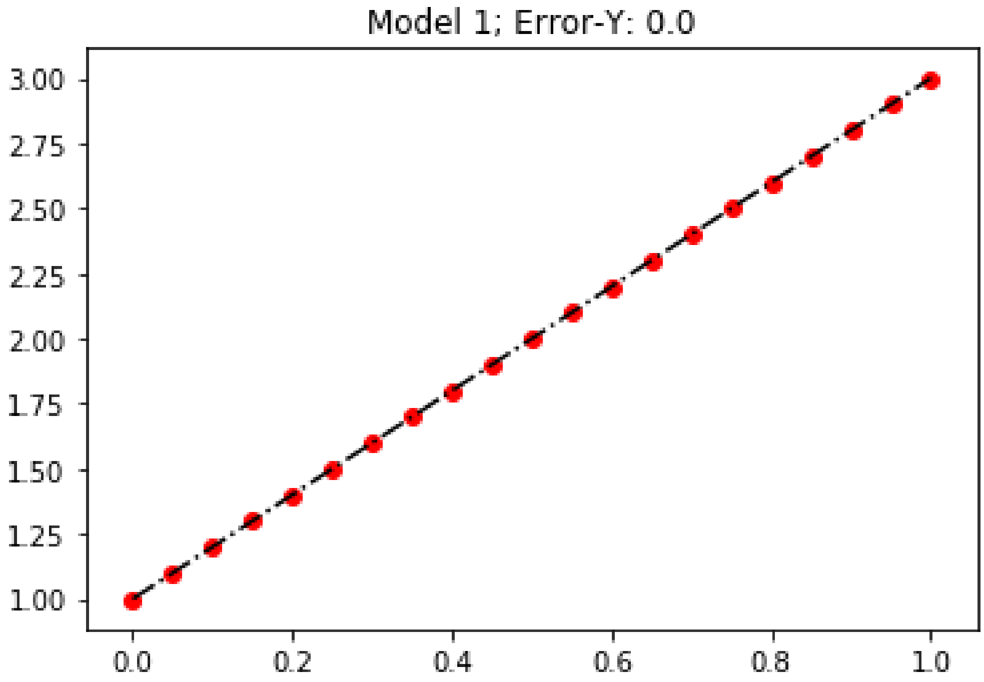
```
In [617]: # data set 3
          y3 = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x)
          dataset3 = list(zip(x, y3))

          # Visualize
          plt.scatter(*zip(*dataset3), color='black')
          plt.show()
```

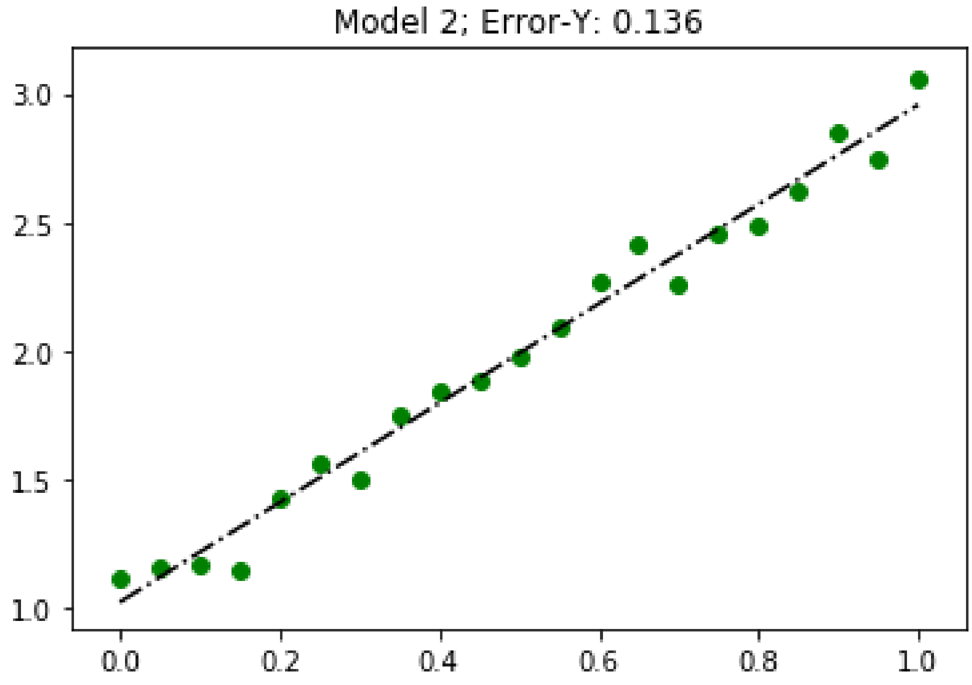


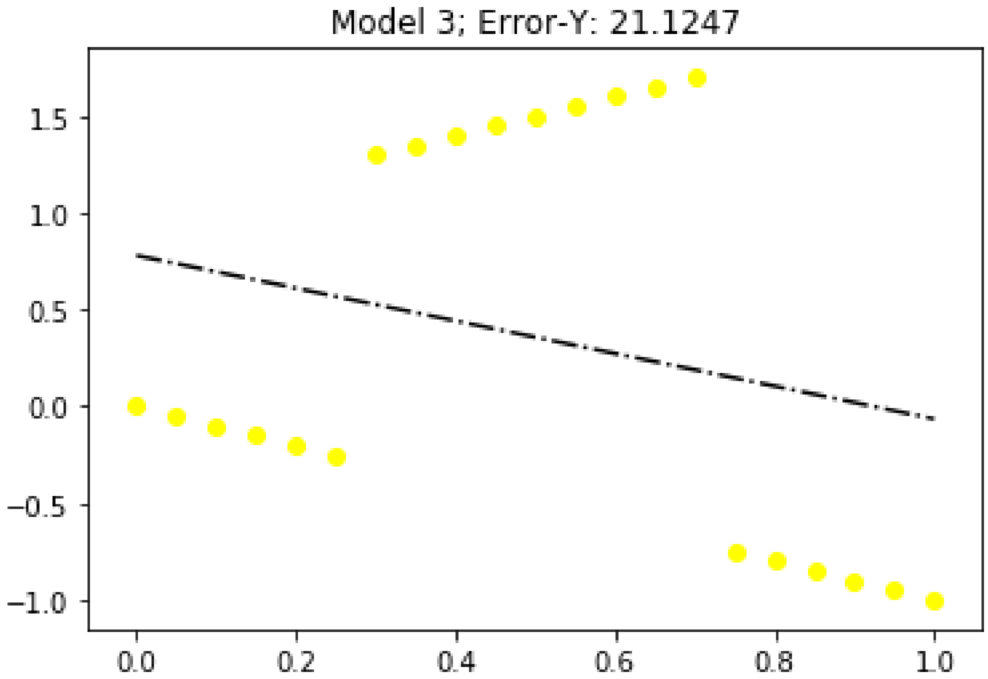
1.1 Part 1. Line Fitting (20 Points)

```
In [5]: def get_params(p1, p2):
            # p1 and p2(tuples) are points through which
            # we fit a line
            # @returns param: slope and offset for mx + by + c = 0
            x1, y1, x2, y2 = p1[0], p1[1], p2[0], p2[1]
            if x1 != x2: # if slope is not infinite
                m = (y2-y1)/(x2-x1)
                c = (y1*x2 - y2*x1)/(x2-x1)
                b = -1
            else:
                m = 1
                c = 0
                b = -x1
            params = [m, b, c]
            return params

        # get_params((0,1), (1,3))

        def error_y(params, dataset):
            # @return error for a fitted line on a dataset
            # dataset is a list of tuples, which each tuple being
            # a coordinate
            error = 0;
            m, b, c = params
            for point in dataset:
                xi, yi = point[0], point[1]
                error += math.pow((yi + (m*xi + c)/b),2)
            return error

        # error_y([2,-1,1], [(0,1), (1,3), (-1,-1)])

In [6]: def fitLineRANSAC(dataset, iterations = 100):
            # returns params for best fitted line for the dataset
            # dataset is a list of tuples, which each tuple being
            # a coordinate
            param_error = dict()
            for count in range(int(iterations)):
                # randomly select two points from the list
                p1, p2 = random.sample(dataset, 2)
                # find the line parameters for line passing through them
                params = get_params(p1, p2)
                # calculate error for the model
                error = error_y(params, dataset)
            # store the error for the model
            param_error[tuple(params)] = error
        best_model = min(param_error, key=param_error.get)
        least_error = param_error[best_model]
        return best_model, least_error

    def fitLineLeastSquares(dataset):
        N = len(dataset) # totalPoints
        dataset = np.array(dataset)
        X, Y = dataset[:,0], dataset[:,1]
        m = (N*(np.sum(X*Y)) - np.sum(X)*np.sum(Y))/(N*np.sum(X*X) - np.sum(X)*np.sum(X))
        c = (np.sum(Y) - m*np.sum(X))/N
        params = [m, -1, c]
        error = error_y(params, dataset)
        return params, error
```

```
In [723]: no_of_datapoints = len(dataset1)
          max_iterations = no_of_datapoints*(no_of_datapoints-1)/2;

          # Model 1
          modelOne, errorOne = fitLineLeastSquares(dataset1)
          plt.scatter(*zip(*dataset1), color='red')
          m, b, c = modelOne
          x1 = np.array([lis[0] for lis in dataset1])
          y1 = -(m*x1+c)/b
          plt.plot(x1, y1, linestyle='-.', color='black')
          plt.title('Model 1' + '; Error-Y: ' + str(np.round(errorOne, 4)))
          plt.show()

          # Model 2
          modelTwo, errorTwo = fitLineLeastSquares(dataset2)
          plt.scatter(*zip(*dataset2), color='green')
          m, b, c = modelTwo
          x2 = np.array([lis[0] for lis in dataset2])
          y2 = -(m*x2+c)/b
          plt.plot(x2, y2, linestyle='-.', color='black')
          plt.title('Model 2' + '; Error-Y: ' + str(np.round(errorTwo, 4)))
          plt.show()

          # Model 3
          modelThree, errorThree = fitLineLeastSquares(dataset3)
          plt.scatter(*zip(*dataset3), color='yellow')
          m, b, c = modelThree
          x3 = np.array([lis[0] for lis in dataset3])
          y3 = -(m*x3+c)/b
          plt.plot(x3, y3, linestyle='-.', color='black')
          plt.title('Model 3' + '; Error-Y: ' + str(np.round(errorThree, 4)))
          plt.show()
```


Model 1; Error-Y: 0.0
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.0
0.2
0.4
0.6
0.8
1.0


Model 2; Error-Y: 0.136
3.0
2.5
2.0
1.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0



1.2 Part 2: Expectation - Maximization Step

2. E-M (80 points) Write a function that estimates the parameters of two lines using E-M. It should get as input vectors x,y and return (a1,b1,c1), (a2,b2,c2) the parameters of the two lines as well as the weight vectors w_1 and w_2. (Set the free parameter in Eq. (2) and (3), sigma^2=0.1.) You must figure out how to initialize E-M appropriately, and how to set other parameters, if any. You should be able to manage things to that the algorithm converges to a reasonable answer.

```
In [626]: def generate_random_param(dataset):
              # Generates 2 random points from the sample space.
              # @dataset is a list of tuples of coordinates
              # @return params for line passing through that point.

              if isinstance(dataset, np.ndarray) is False:
                  dataset = np.array(dataset)

              x, y = dataset[:,0], dataset[:,1]
              min_x, max_x, min_y, max_y = min(x), max(x), min(y), max(y)

              # generate random points
              p1 = random.random()*(max_x - min_x)+min_x, random.random()*(max_y - min_y)+min_y
              p2 = random.random()*(max_x - min_x)+min_x, random.random()*(max_y - min_y)+min_y

              return get_params(p1, p2)# Get param
```

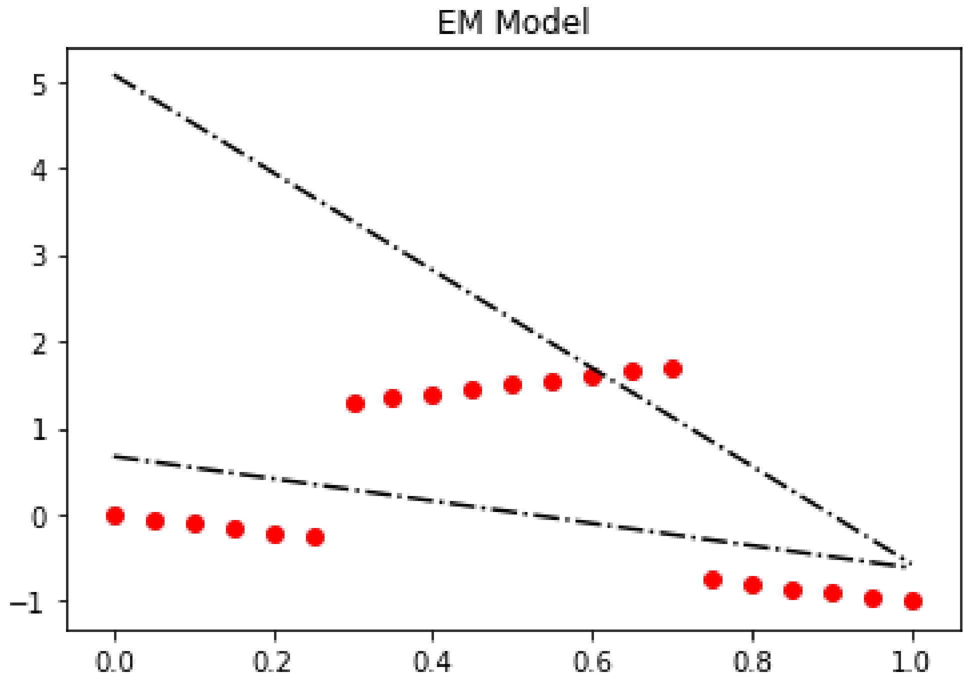
```
In [651]: def assignClass(params, dataset, sigma=0.01):
              # params = [[m1, b1, c1], [m2, b2, c2]]
              [[m1, b1, c1], [m2, b2, c2]] = params

              if isinstance(dataset, np.ndarray) is False:
                  datasetArr = np.array(copy.deepcopy(dataset))
              else:
                  datasetArr = copy.deepcopy(dataset)

              X, Y = datasetArr[:,0], datasetArr[:,1]

              # Calculate Residuals
              r1, r2 = m1*X + c1 - Y, m2*X + c2 - Y

              # Calculate Weights
              s1, s2 = np.exp(-np.power(r1/sigma, 2)), np.exp(-np.power(r2/sigma, 2))
              w1, w2 = s1/(s1+s2), s2/(s1+s2)

              # Not a Number considerations
              NAN1, NAN2 = np.isnan(w1), np.isnan(w2)
              w1[NAN1], w2[NAN2] = 0.5, 0.5
              weights = [w1, w2]

              # Assign Class
              pointClassArray = w1<w2
              pointClassArray = pointClassArray + 1
              pointClass = {tuple(datasetArr[i]): pointClassArray[i] for i in range(len(pointCla

              return weights, pointClass

          def didConverge(params_old, params_new):
              # params_old: [param1_old:[m1_o, -1, c1_o], param2_old:[m2_o, -1, c2_o]]
              # params_new: [param1_new, param2_new]
              # returns boolean if the algorithm converges
              param1_old, param2_old = params_old
              param1_new, param2_new = params_new

              m1_o, _, c1_o = param1_old
              m2_o, _, c2_o = param2_old

              m1_n, _, c1_n = param1_new
              m2_n, _, c2_n = param2_new

              if abs(m1_o-m1_n)<0.05 and abs(m2_o-m2_n)<0.05 and abs(c1_o-c1_n)<0.05 and abs(c2_
                  return True
              else:
                  return False
```

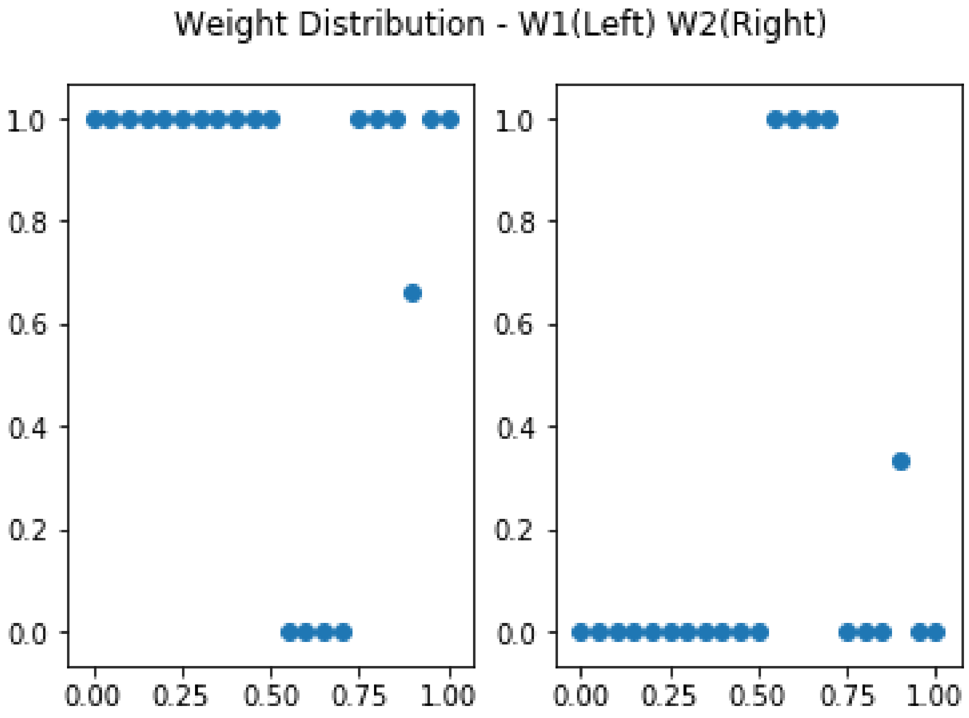
```
In [724]: def plotLines(params, dataset):
              plt.figure(figsize=(6,4))
              plt.scatter(*zip(*dataset), color='red')

              model1, model2 = params
              m1, b1, c1 = model1
              x3 = np.array([lis[0] for lis in dataset])
              y3_1 = -(m1*x3+c1)/b1
              plt.plot(x3, y3_1, linestyle='-.', color='black')

              m2, b2, c2 = model2
              y3_2 = -(m2*x3+c2)/b2
              plt.plot(x3, y3_2, linestyle='-.', color='black')

              plt.title('EM Model')
              plt.show()

          def plotWeights(weights, dataset):
              w1, w2 = weights
              x = np.array([lis[0] for lis in dataset])

              plt.figure(figsize=(6,4))

              plt.subplot(121)
              plt.scatter(x, w1)

              plt.subplot(122)
              plt.scatter(x, w2)

              plt.suptitle('Weight Distribution - W1(Left) W2(Right)')
              plt.show()
```

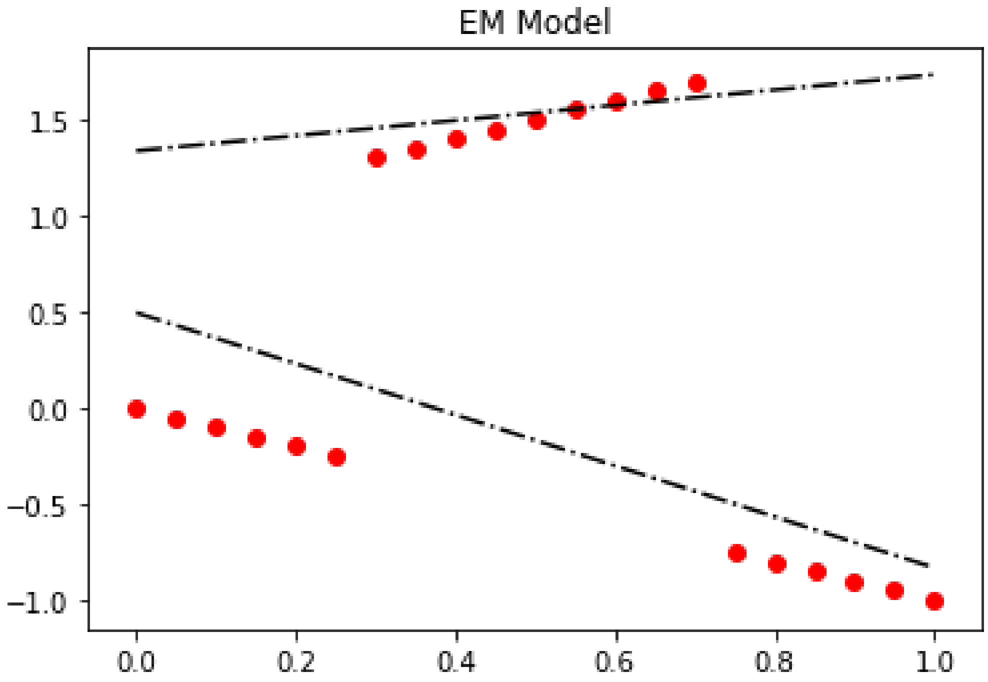
```
In [725]: def EM(dataset, sigma=0.1, iterations=100):

              ### Initialization ###
              # Random Param Values for 2 models
              params_old = [[0,-1, 0],[0,-1,0]]

              param1_new = generate_random_param(dataset)
              param2_new = generate_random_param(dataset)
              params_new = [param1_new, param2_new]

              ### Iterate Till Convergence ###
              count = 0
              while didConverge(params_old, params_new) == False and count < iterations:
                  count += 1
          #         print("Did Converge?", didConverge(params_old, params_new), "Iteration: ", c
        params_old = params_new

        # Expectation Step: Get the Weights for each line model
        sigma_val = copy.deepcopy(sigma)
        weights, pointsClass = assignClass(params_new, dataset, sigma_val)

        # Maximization Step
        w1, w2 = weights

        dataset = np.array(dataset)
        X, Y = dataset[:,0], dataset[:,1]

        # solving WLS for line 1 parameters
        A1 = np.matrix([[np.sum(w1*np.power(X,2)), np.sum(w1*X)], [np.sum(w1*X), np.su
        B1 = np.matrix([[np.sum(w1*X*Y)], [np.sum(w1*Y)]])
        A1_inv = np.linalg.inv(A1)
        param1 = np.matmul(A1_inv, B1)
        m1, c1 = np.asscalar(param1[0][0]), np.asscalar(param1[1][0])
        param1_new = [np.round(m1, 3), -1, np.round(c1, 3)] # Formatting
#         print("Line1: ",param1_new)

        # Solving WLS for Line 2 Parameters
        A2 = np.matrix([[np.sum(w2*np.power(X,2)), np.sum(w2*X)], [np.sum(w2*X), np.su
        B2 = np.matrix([[np.sum(w2*X*Y)], [np.sum(w2*Y)]])
        A2_inv = np.linalg.inv(A2)

        param2 = np.matmul(A2_inv, B2)
        m2, c2 = np.asscalar(param2[0][0]), np.asscalar(param2[1][0])
        param2_new = [np.round(m2, 3), -1, np.round(c2, 3)] # Formatting
#         print("Line2:",param2_new)


        params_new = [param1_new, param2_new]
#         print('Old: ', params_old)
#         print('New: ', params_new)

        # Plot Weights and Current Models
        plotLines(params_new, dataset)
        plotWeights(weights, dataset)

    weights, pointsClass = assignClass(params_new, dataset, sigma_val)
    print("EM Converged after {} iterations".format(count))
    return params_new, weights, pointsClass
```

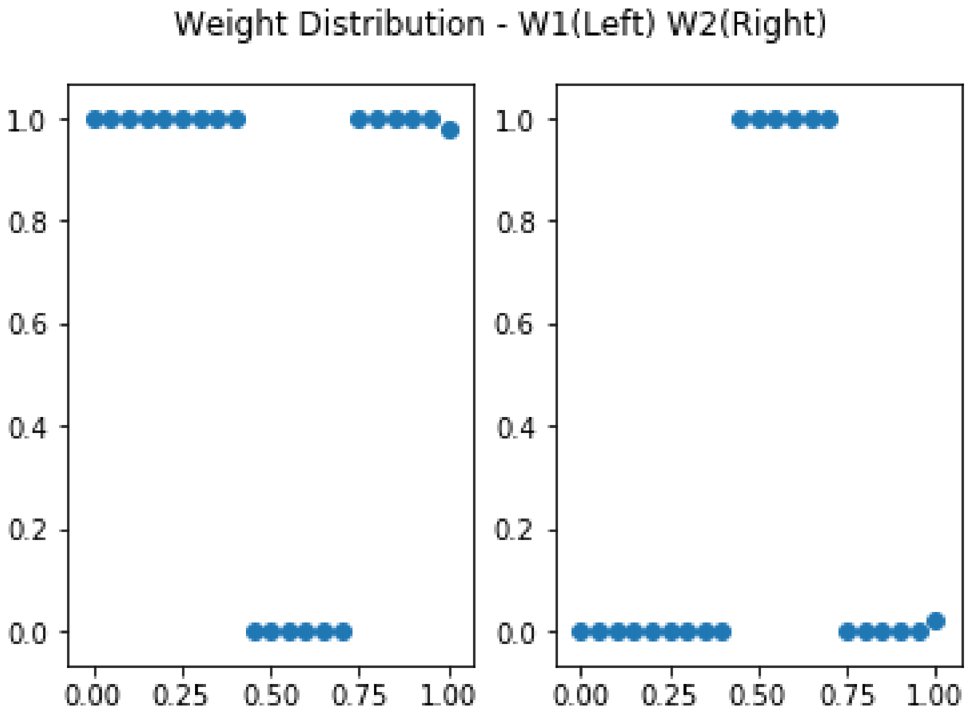
a. Test your function on the data in part (iii) of the previous question. Plot the data and the two fitted lines as estimated after each of the first five iterations. Also, show in separate plots the membership vectors after every iteration.

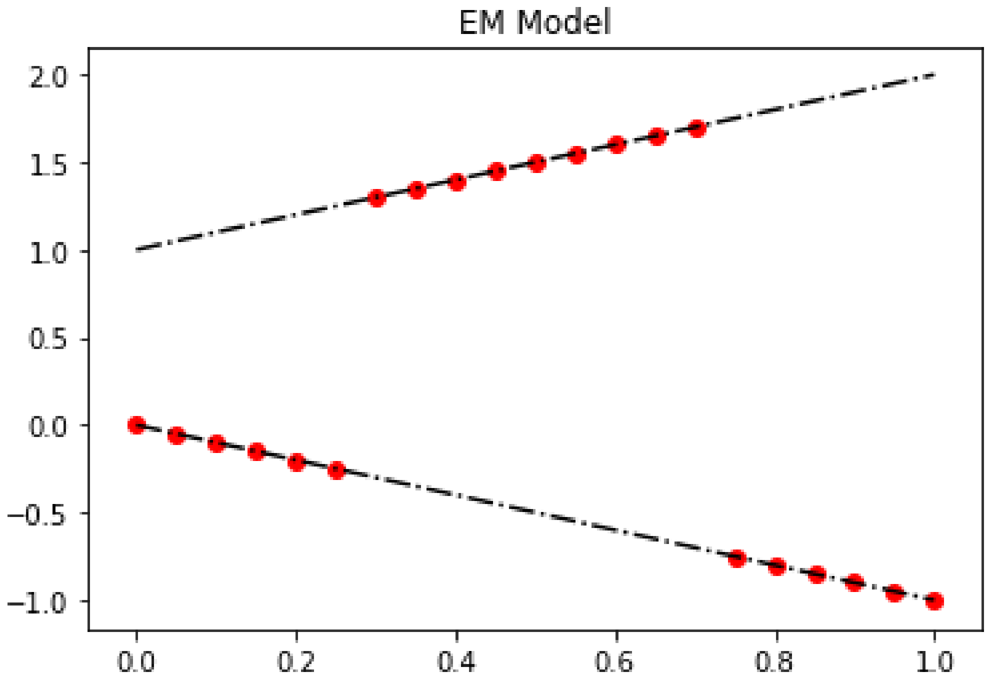
```
In [728]: models, weights, pointsClass = EM(dataset3, sigma=0.1, iterations=100)
```

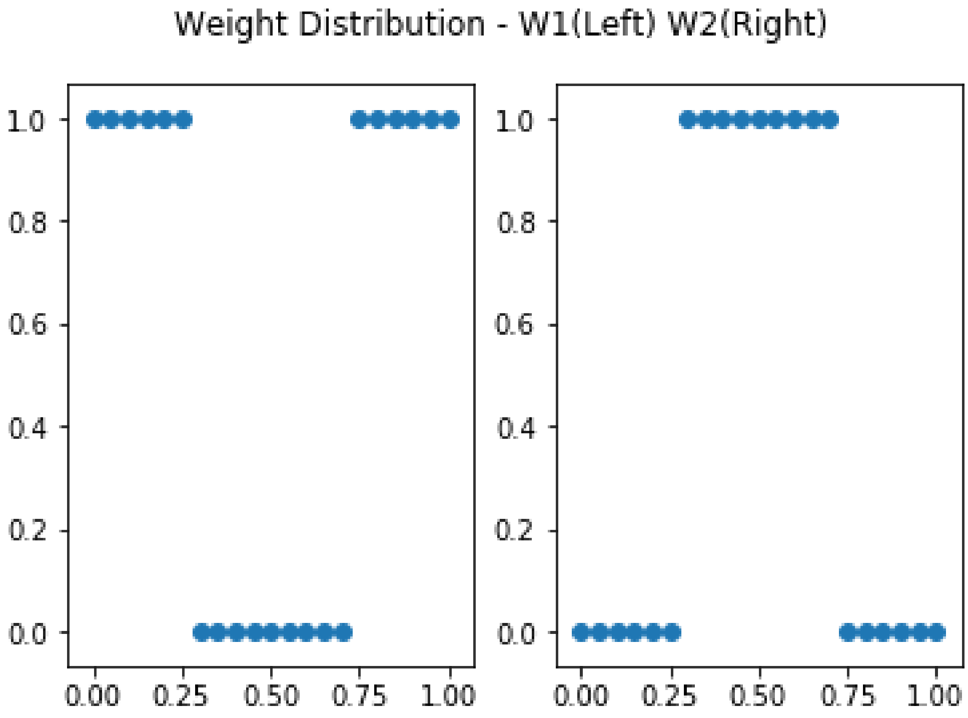

EM Model
5
4
3
2
1
0
−1
0.0
0.2
0.4
0.6
0.8
1.0

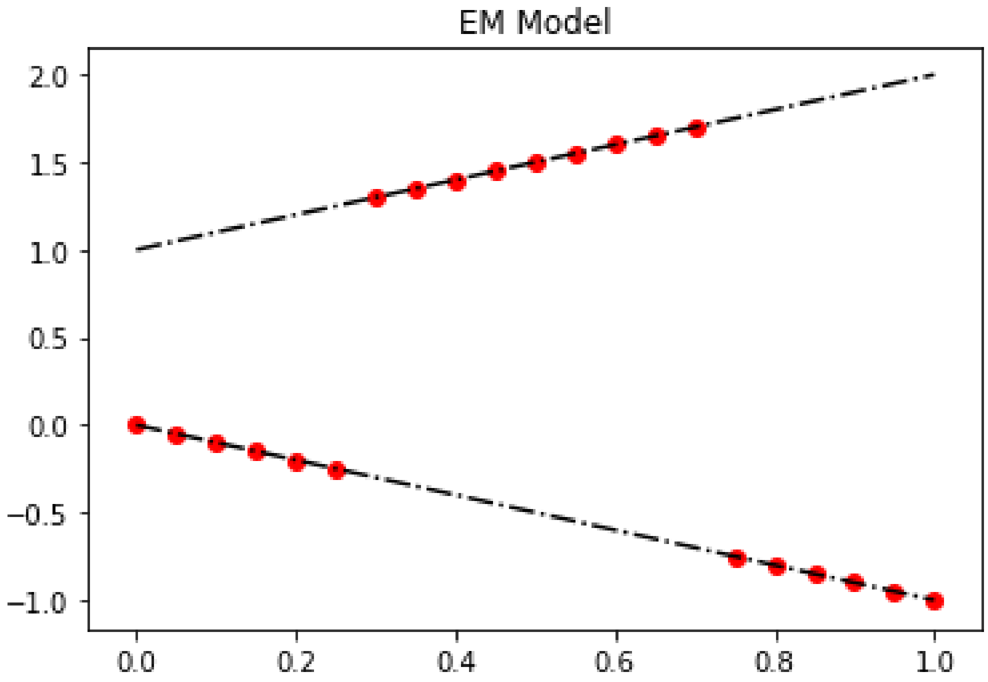

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

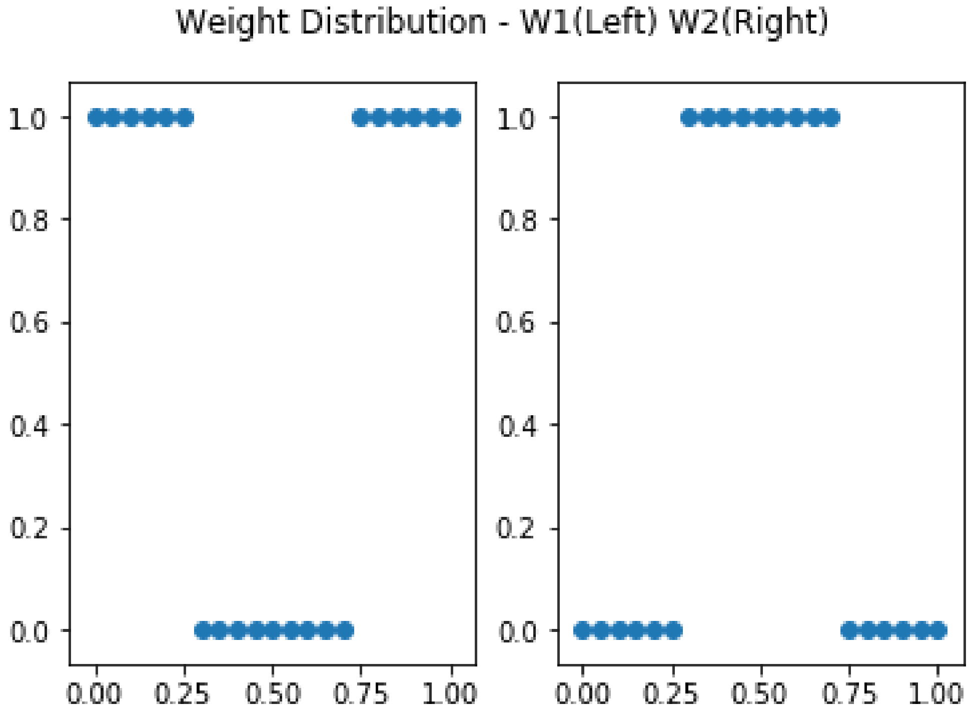

EM Model
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00



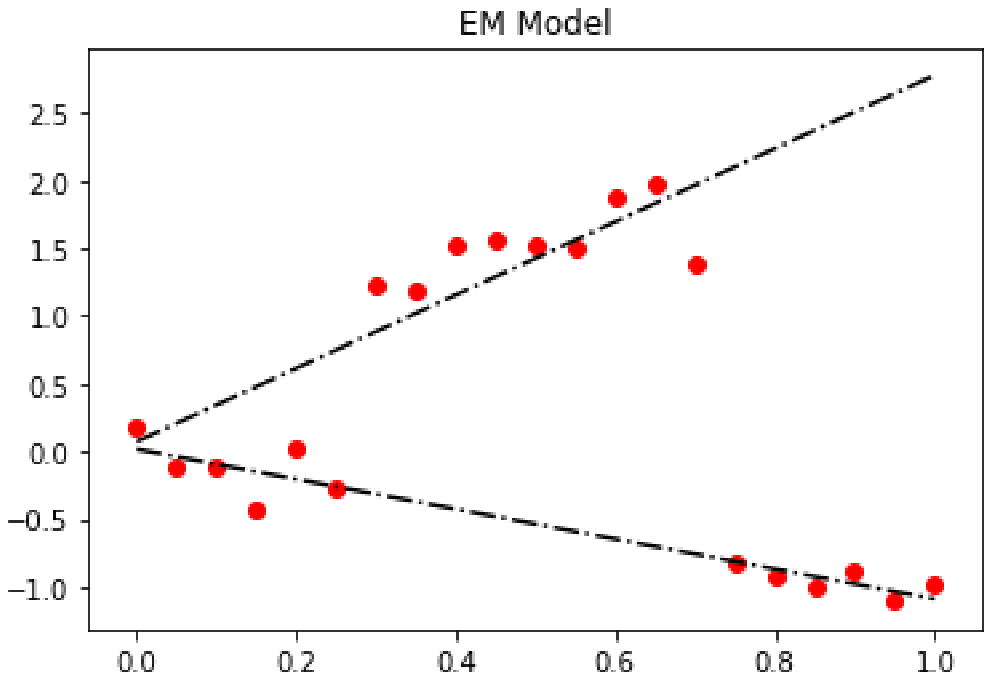


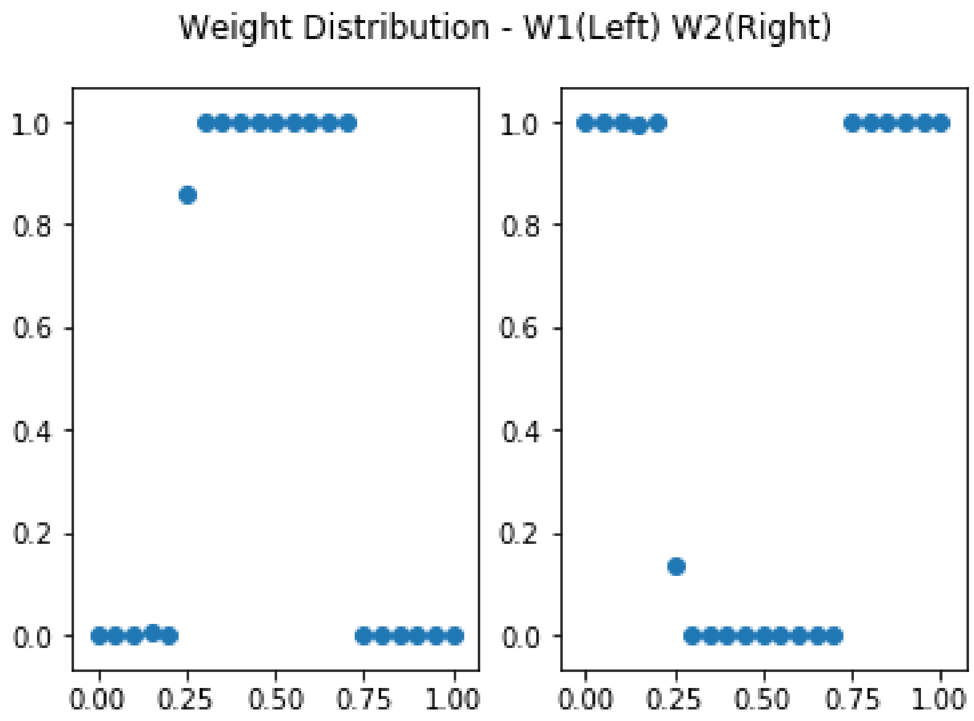
```
EM Converged after 4 iterations
```

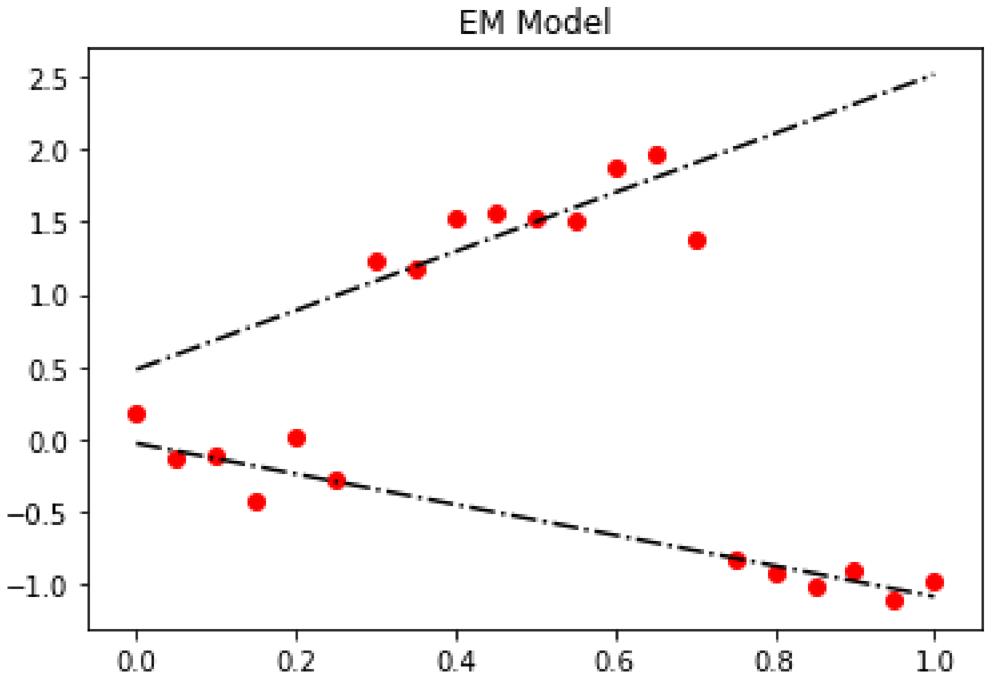
b. Experiment with adding Gaussian noise to the y coordinates. How much noise can you add before the algorithm breaks? Describe your experiment and illustrate with appropriate plot(s).

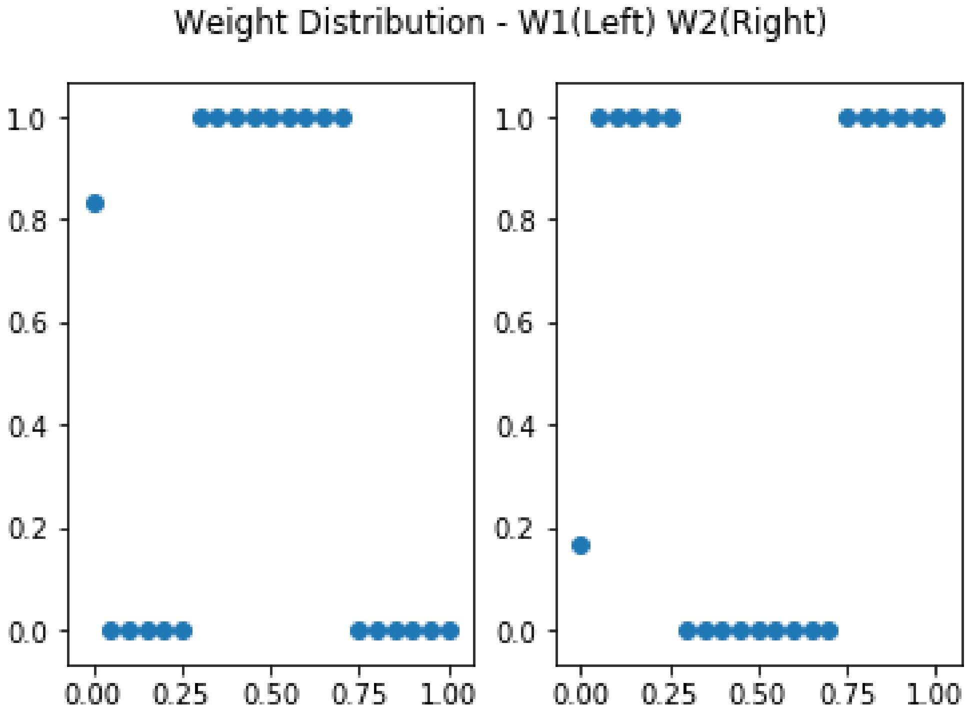
1.2.1 Experiment 1 NoiseLevel = 0.2

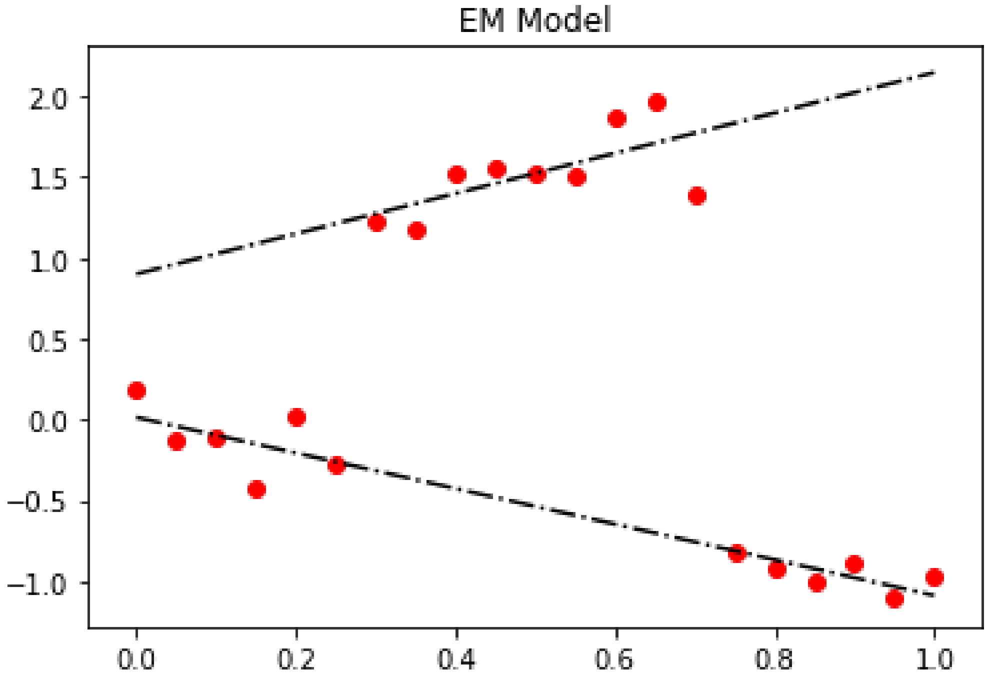
```
In [741]: # Experimenting with Noise

          # Experiment 1
          noiseLevel = 0.2

          x = np.round(np.arange(0, 1.05, 0.05), 3)
          y = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x) + noiseLevel*np.random
          NoisyDataset = list(zip(x, y))

          models, weights, pointsClass = EM(NoisyDataset, sigma=0.1, iterations=100)
```

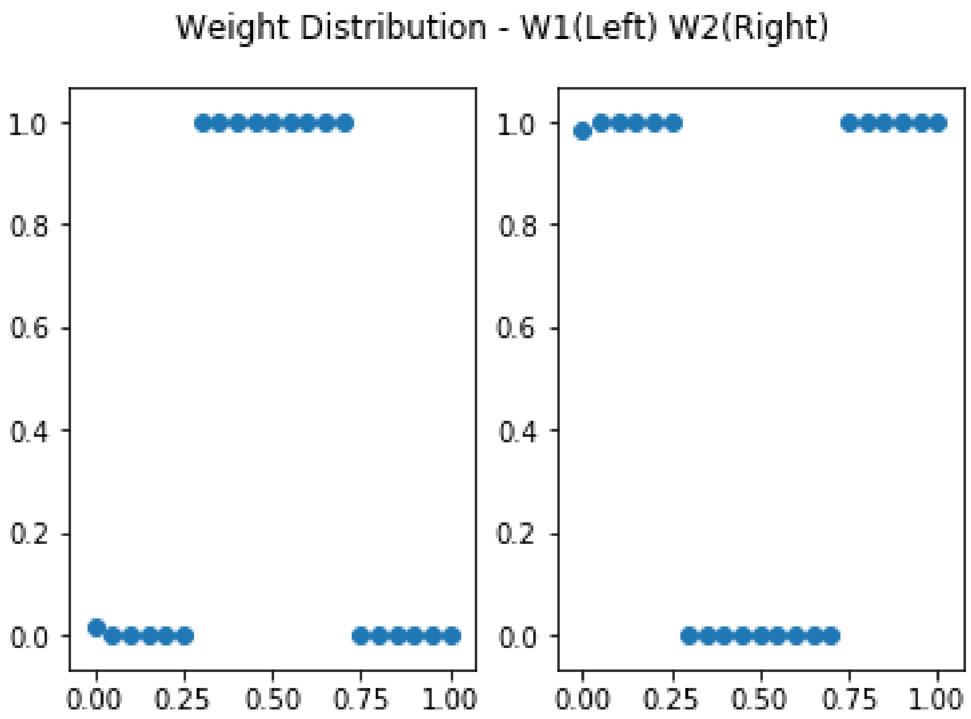


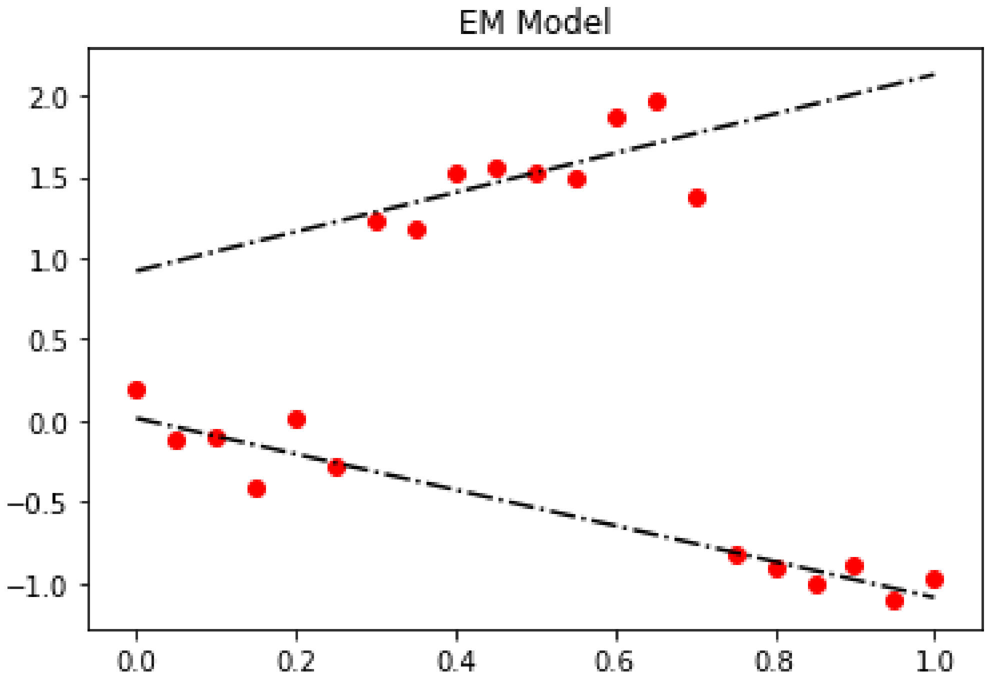

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

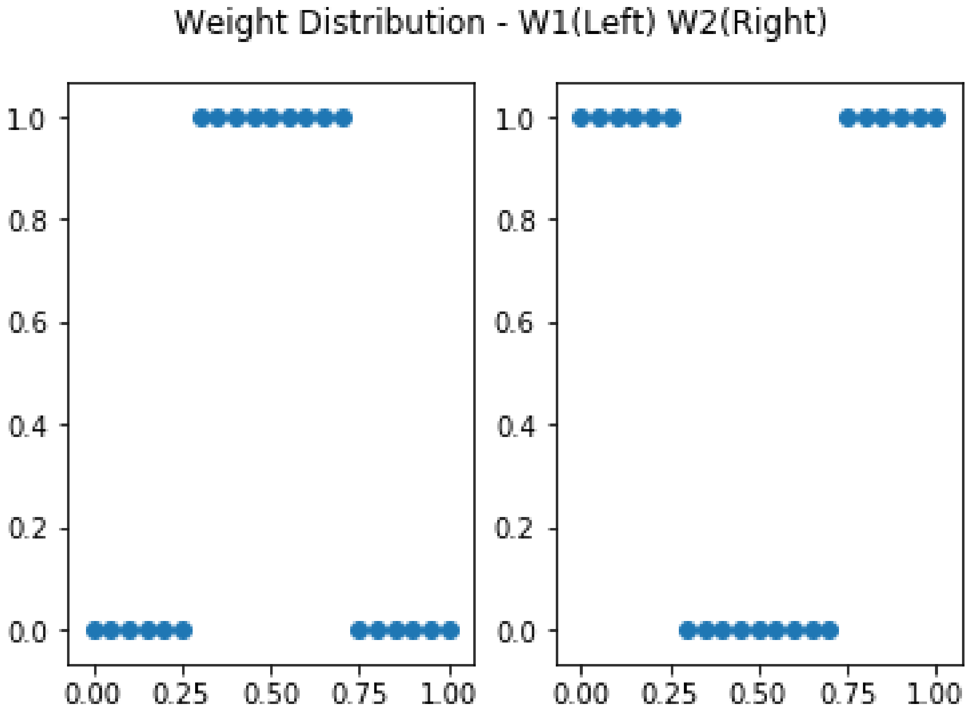

EM Model
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0

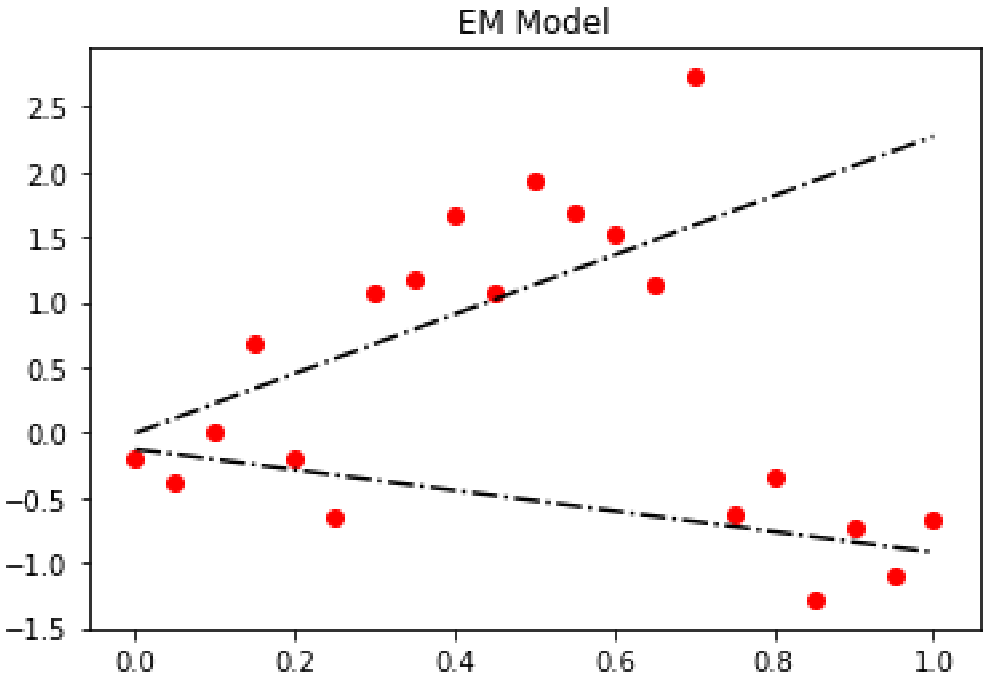


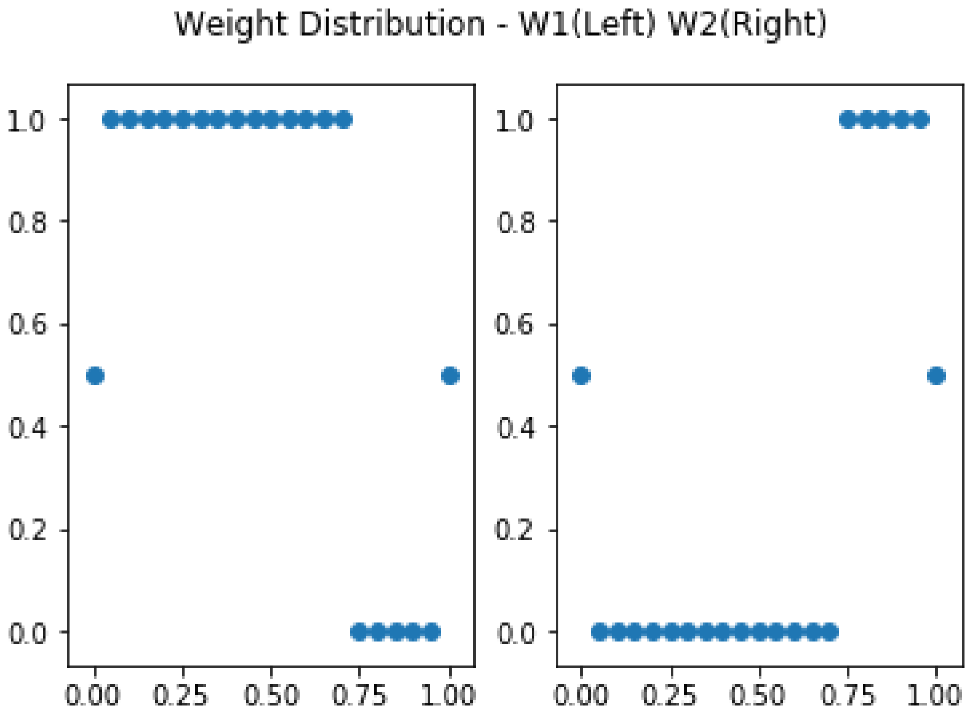
```
EM Converged after 4 iterations
```

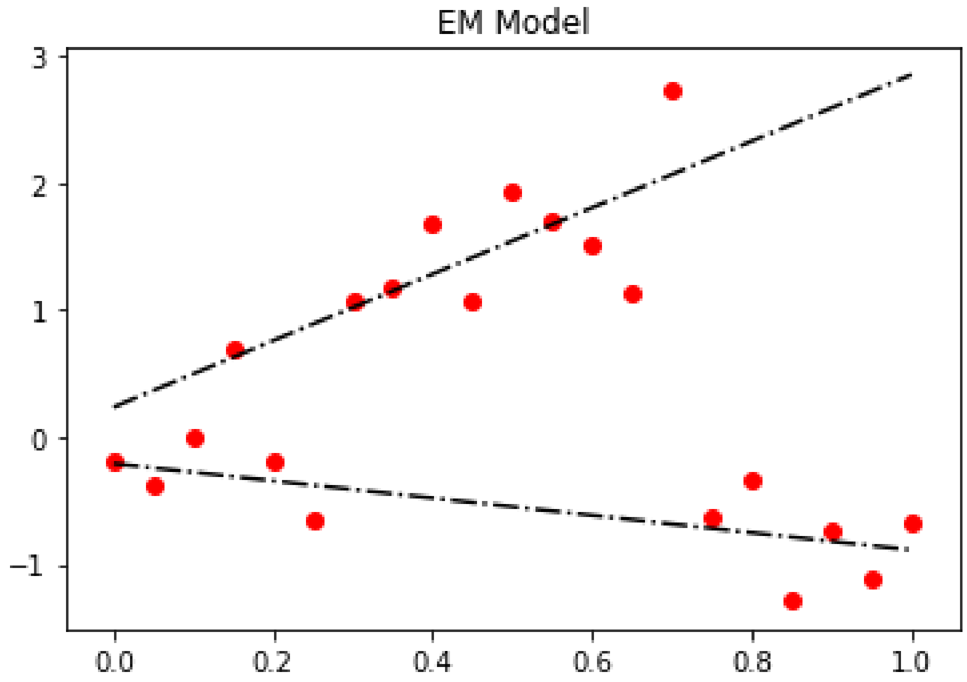
1.2.2 Experiment 2 NoiseLevel = 0.4

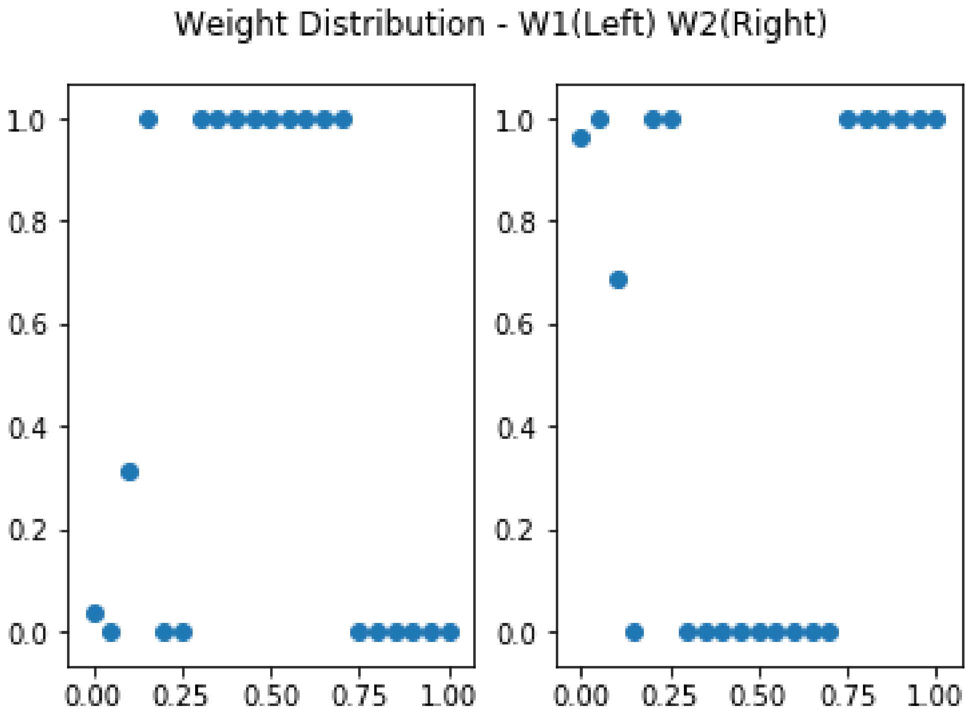
```
In [738]: # Experimenting with Noise

          # Experiment 2
          noiseLevel = 0.4

          x = np.round(np.arange(0, 1.05, 0.05), 3)
          y = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x) + noiseLevel*np.random
          NoisyDataset = list(zip(x, y))

          models, weights, pointsClass = EM(NoisyDataset, sigma=0.1, iterations=100)
```

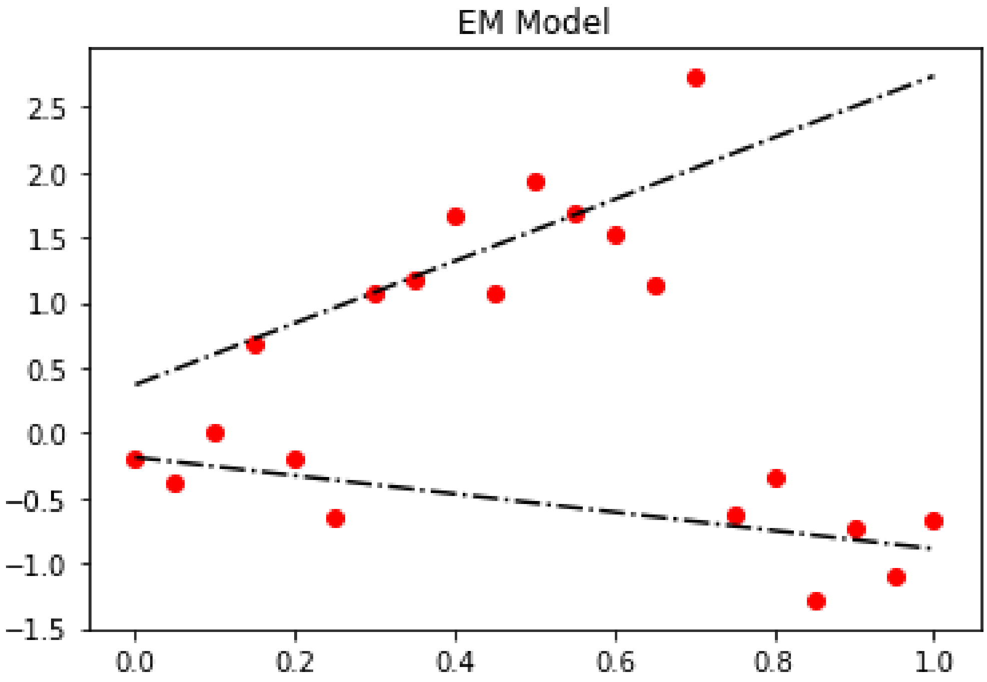
```
/home/detwieller/.local/lib/python3.5/site-packages/ipykernel_launcher.py:17: RuntimeWarning: in
```


EM Model
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.2
0.4
0.6
0.8
1.0

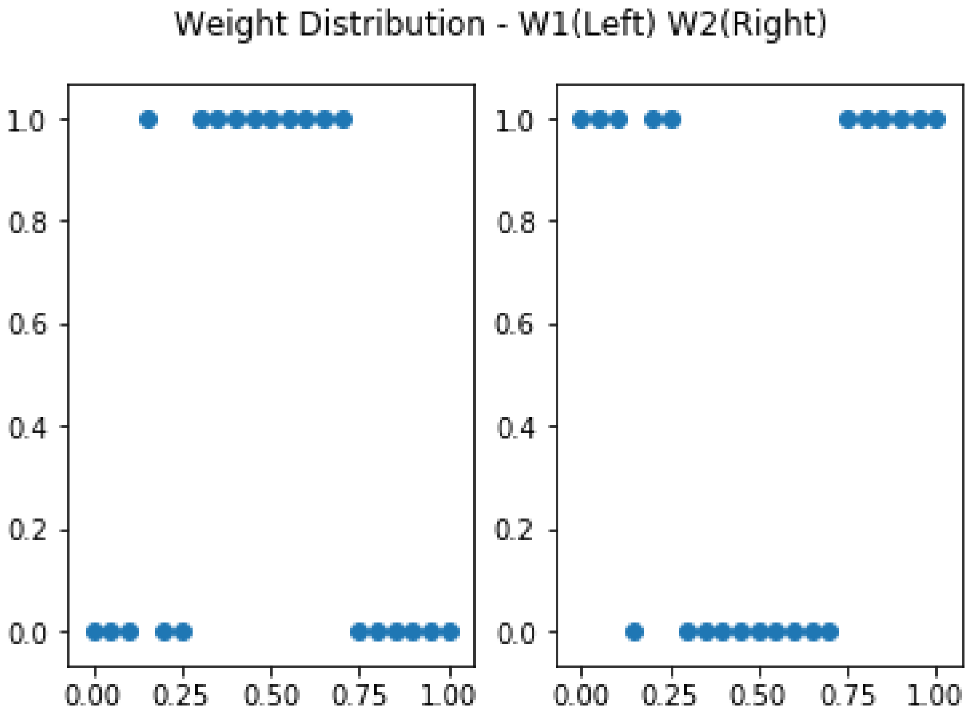

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
3
2
1
0
−1
0.0
0.2
0.4
0.6
0.8
1.0

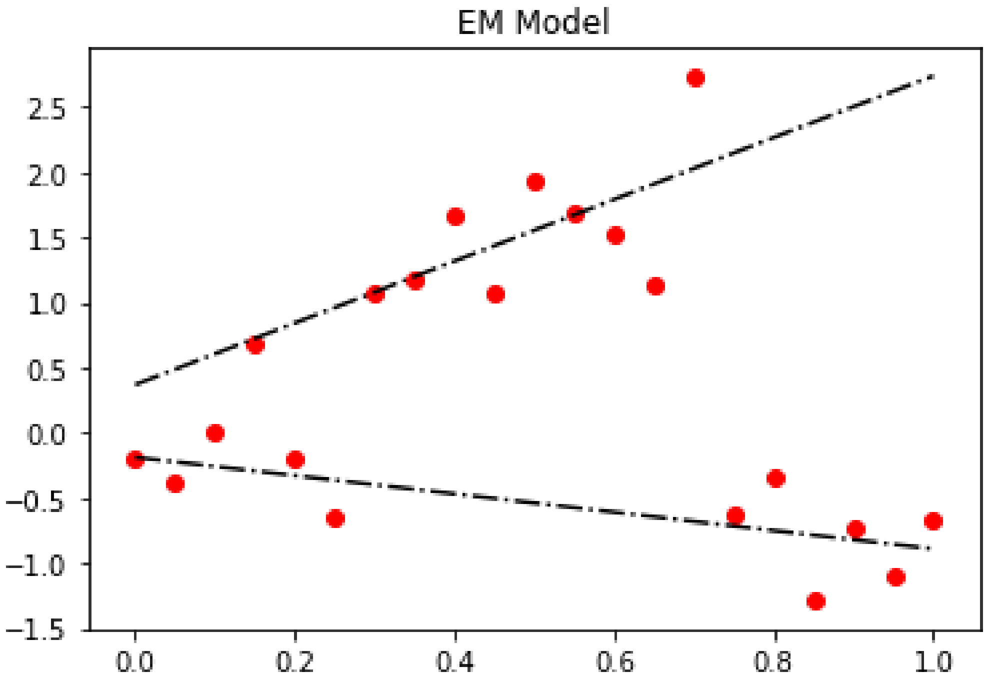

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

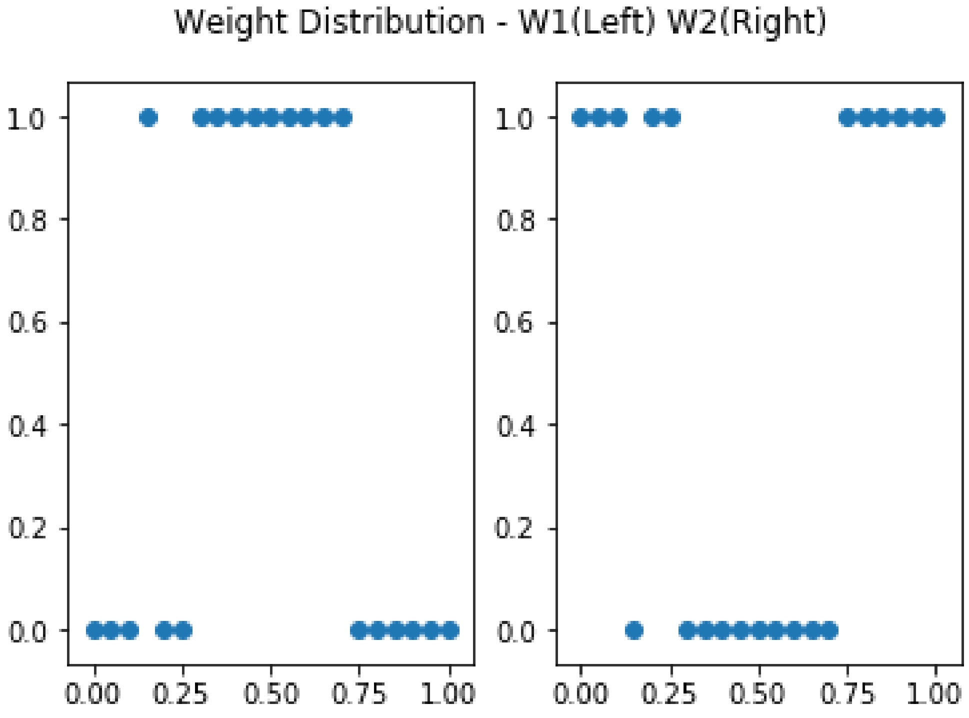


EM Model



Weight Distribution - W1(Left) W2(Right)

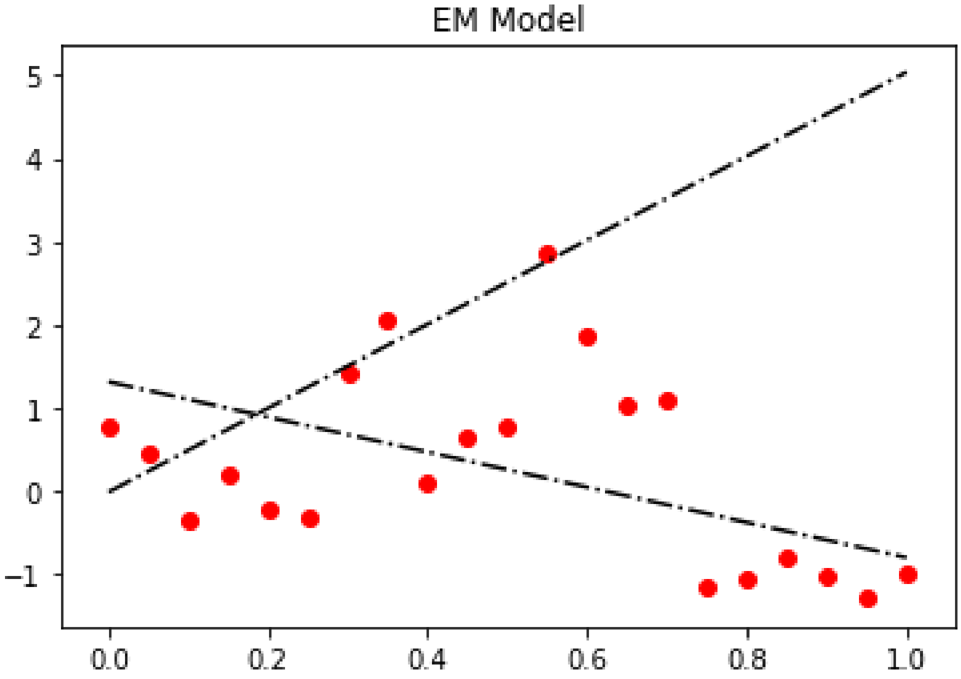


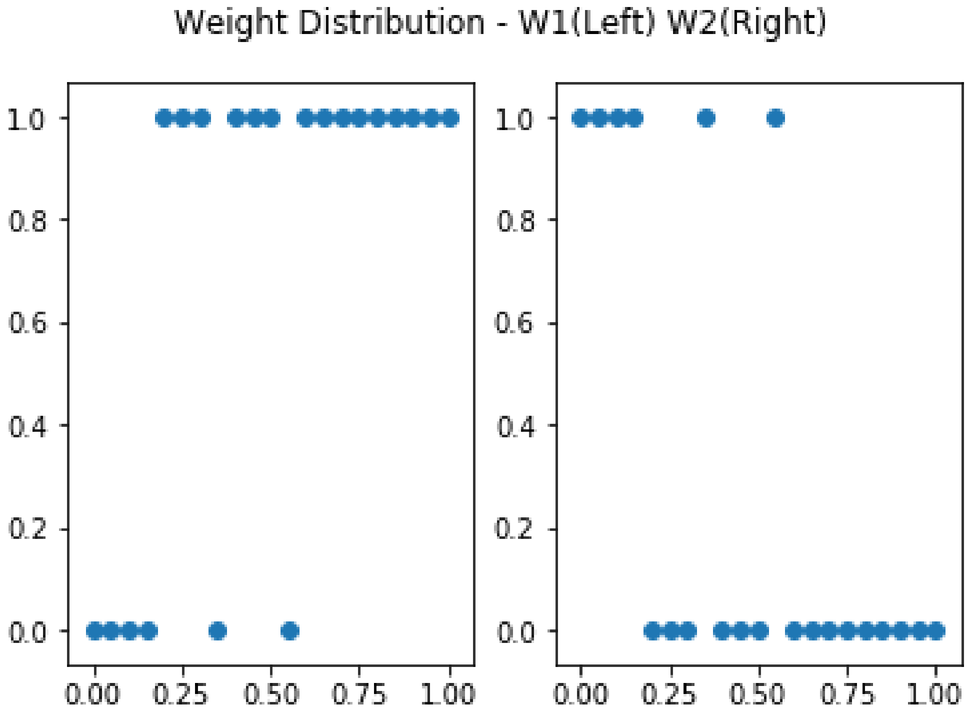


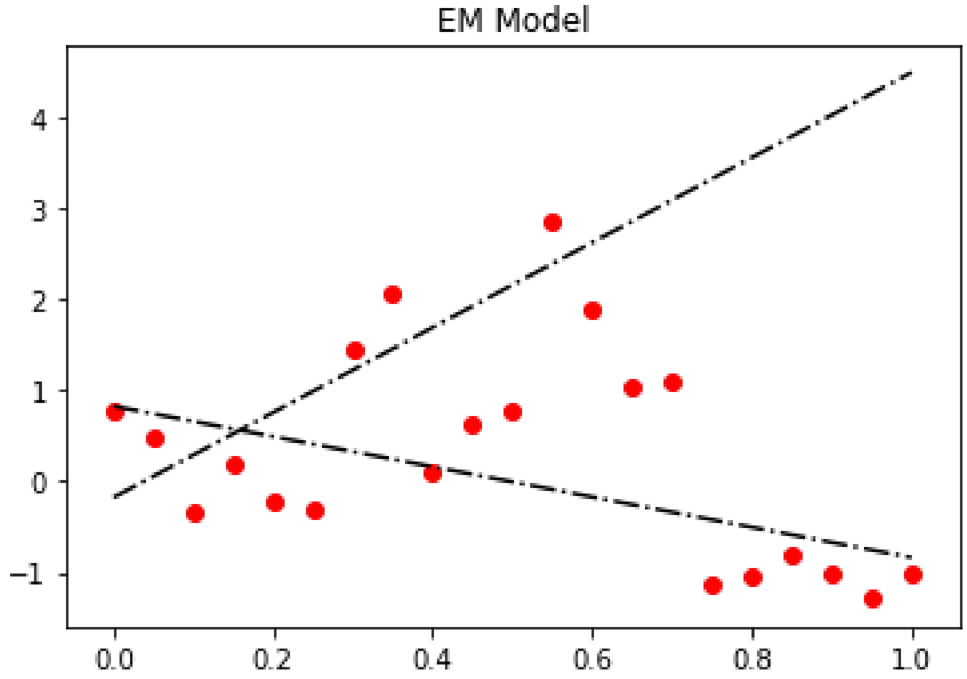
```
EM Converged after 4 iterations
```

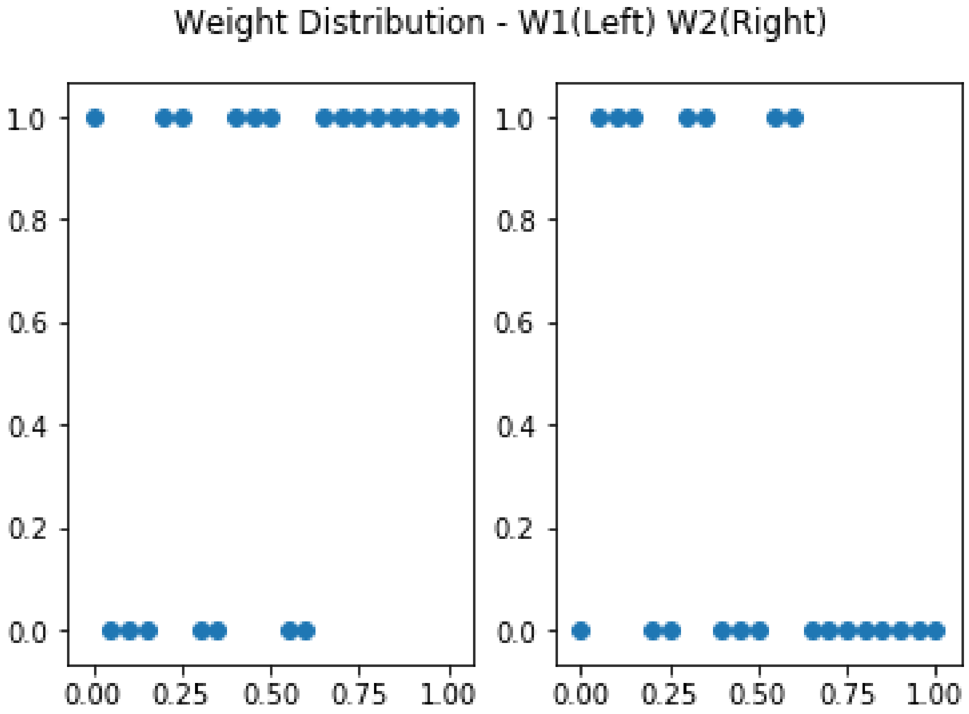
1.2.3 Experiment 3 NoiseLevel = 0.6

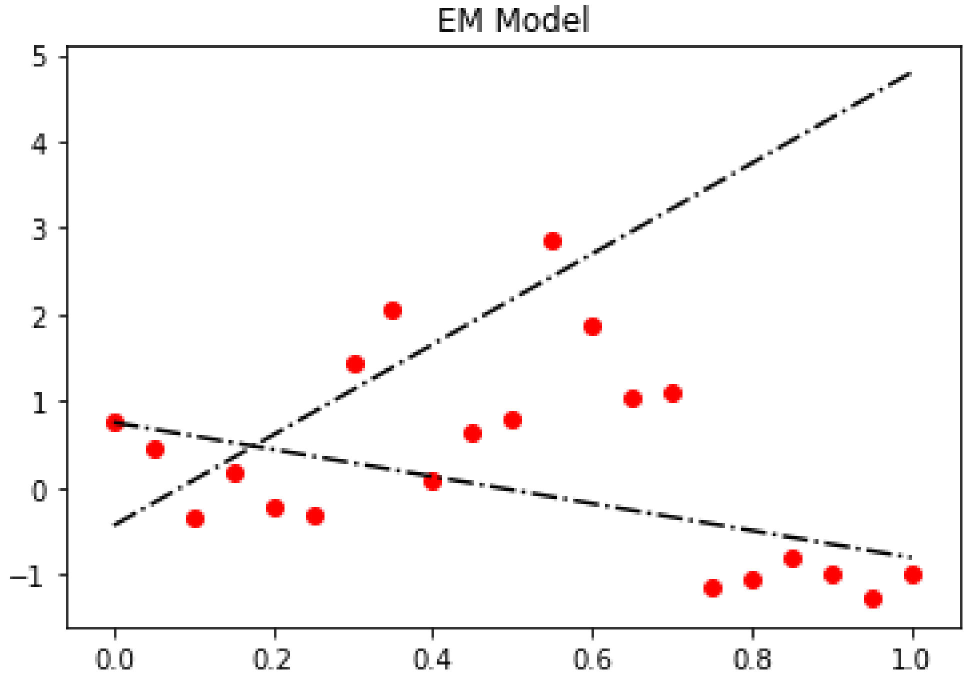
```
In [743]: # Experimenting with Noise

          # Experiment 3
          noiseLevel = 0.6

          x = np.round(np.arange(0, 1.05, 0.05), 3)
          y = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x) + noiseLevel*np.random
          NoisyDataset = list(zip(x, y))

          models, weights, pointsClass = EM(NoisyDataset, sigma=0.1, iterations=100)
```

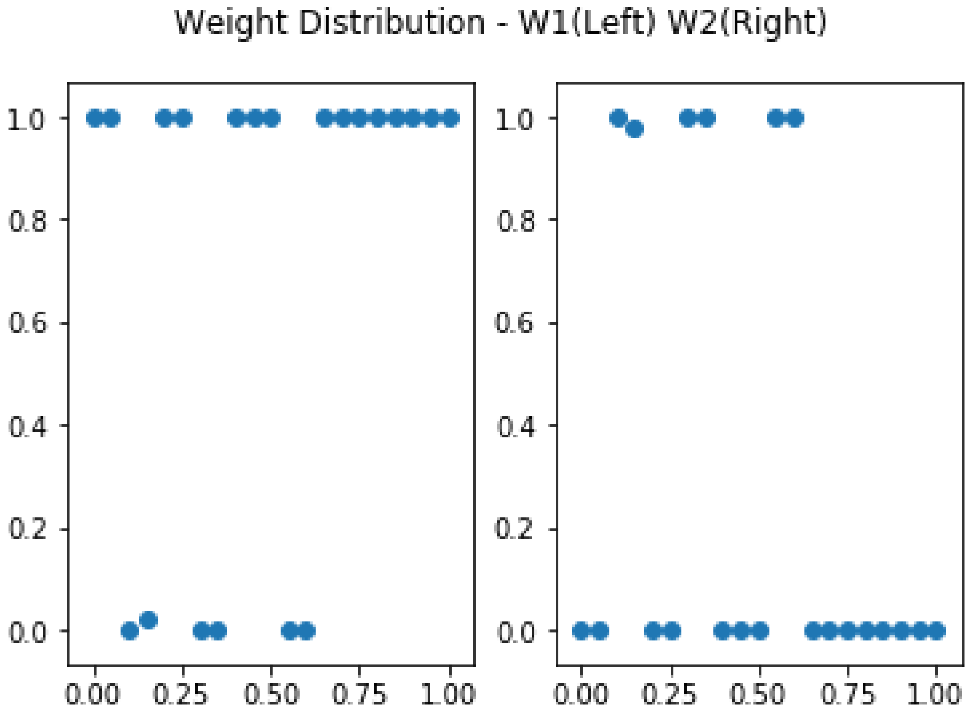


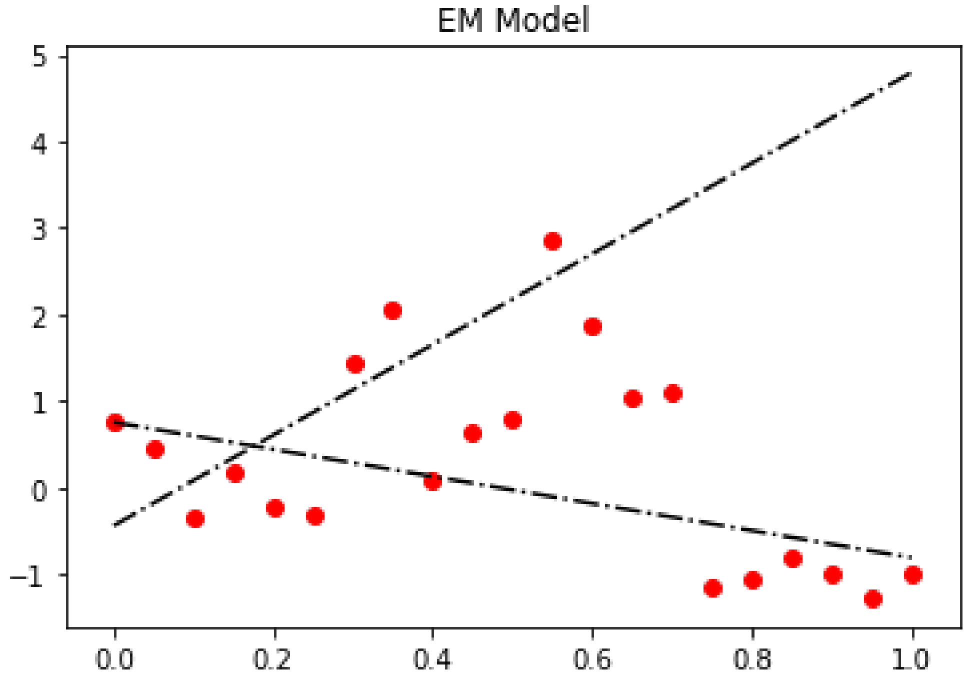

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

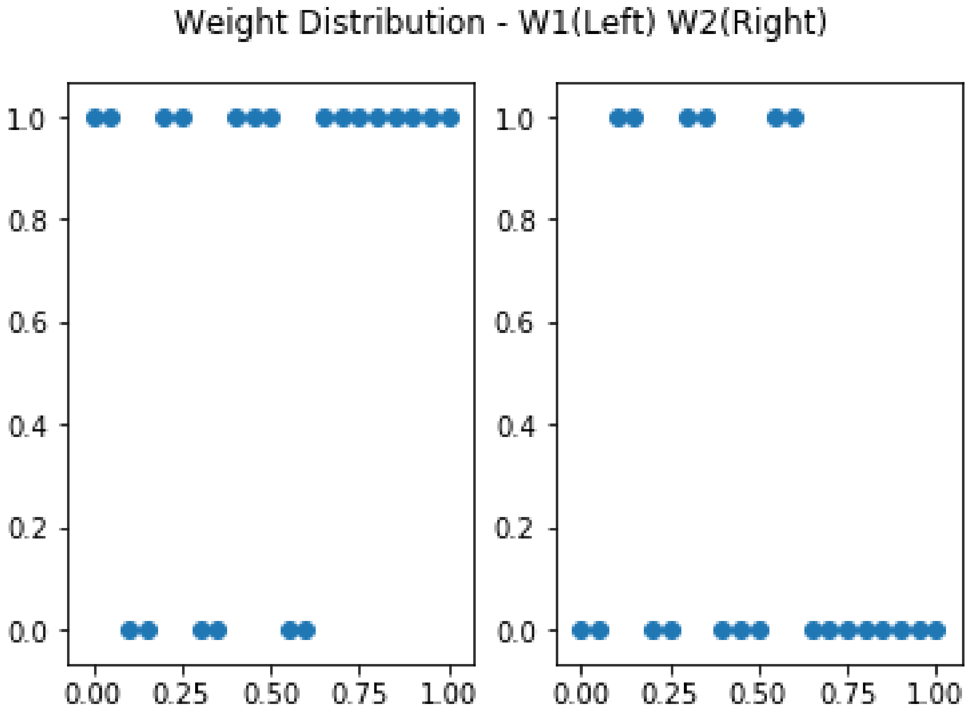

EM Model
4
3
2
1
0
−1
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
5
4
3
2
1
0
−1
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

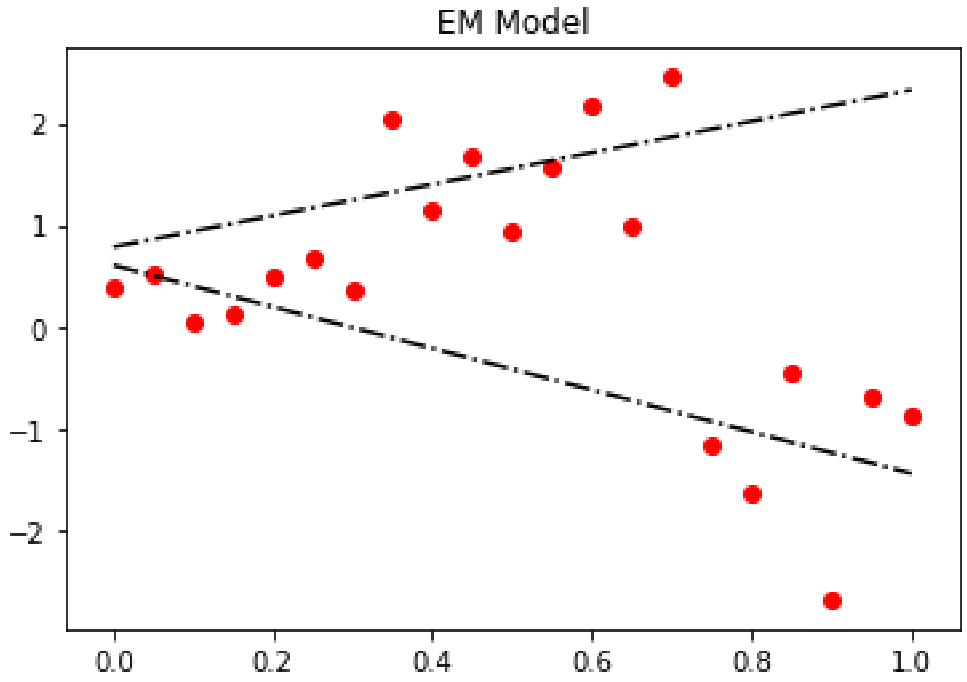

EM Model
5
4
3
2
1
0
−1
0.0
0.2
0.4
0.6
0.8
1.0

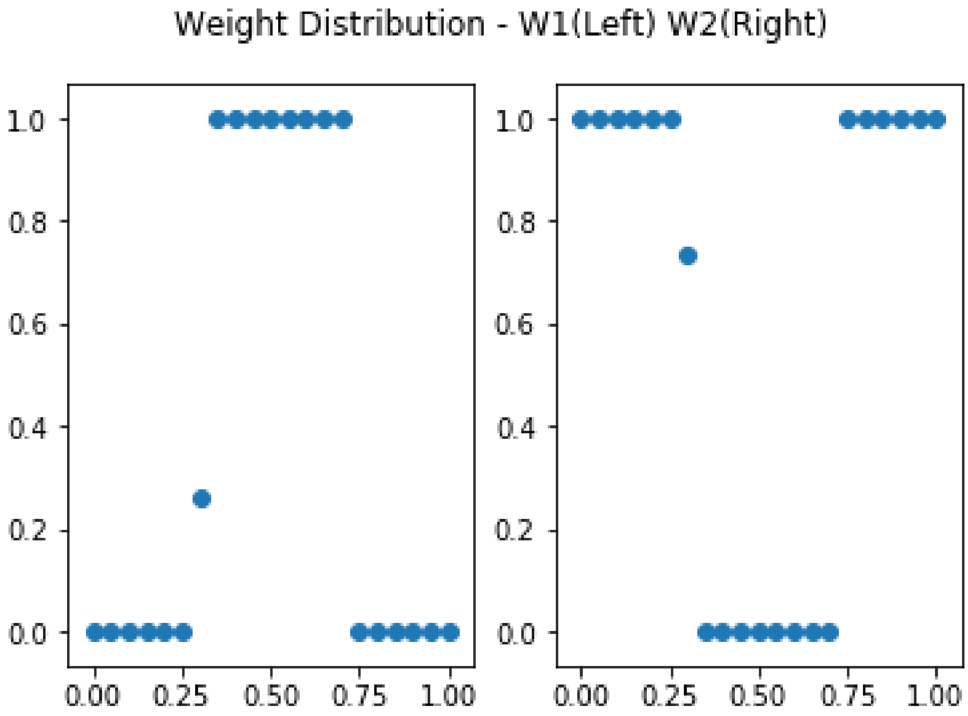


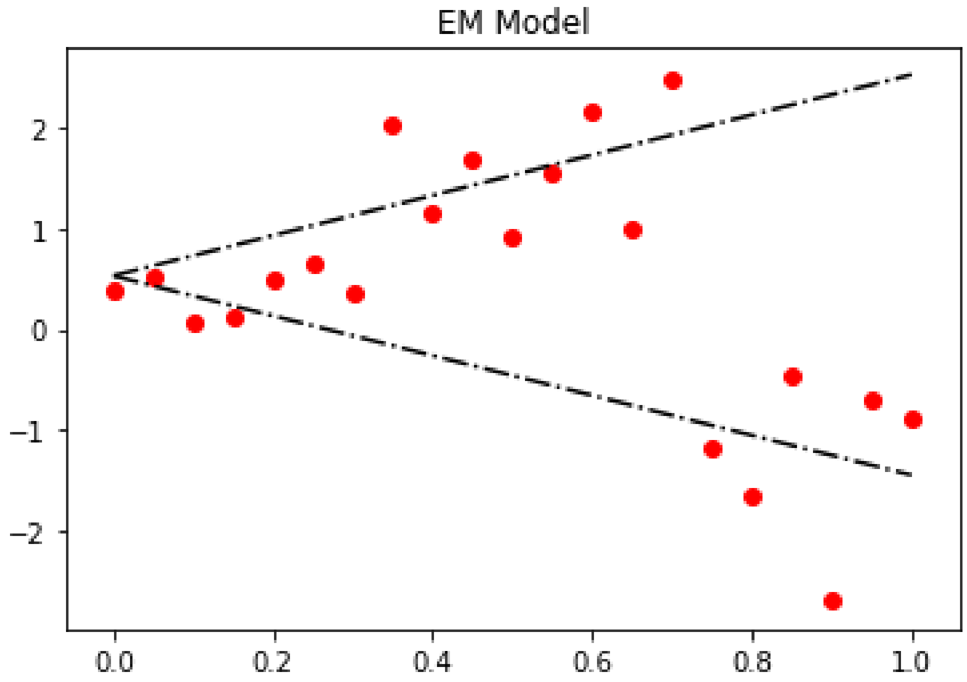
```
EM Converged after 4 iterations
```

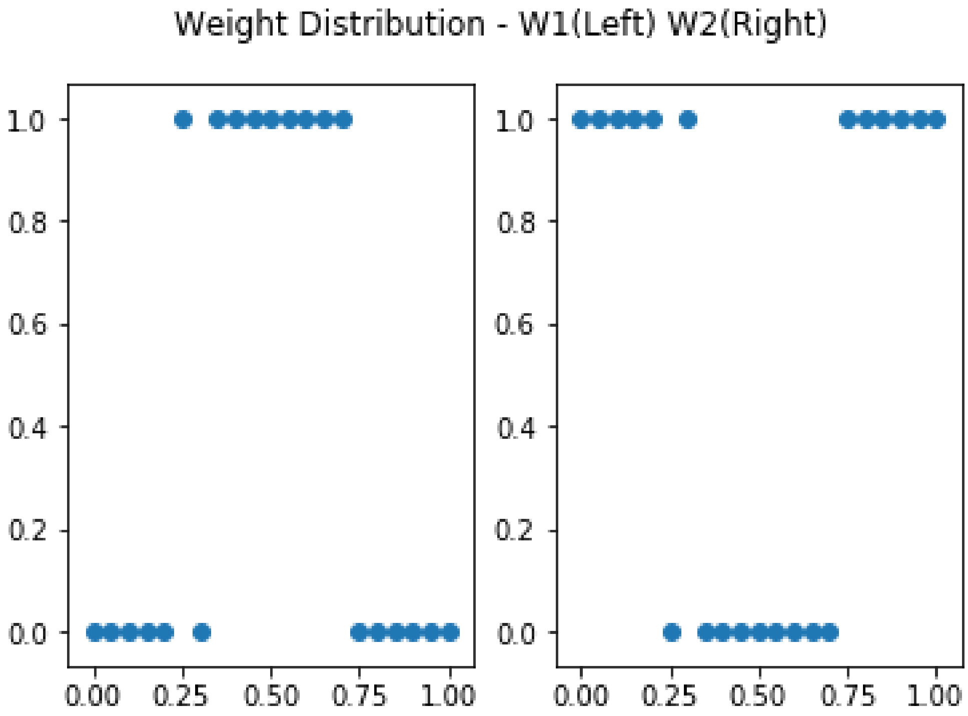
1.2.4 Experiment 4 NoiseLevel = 0.8

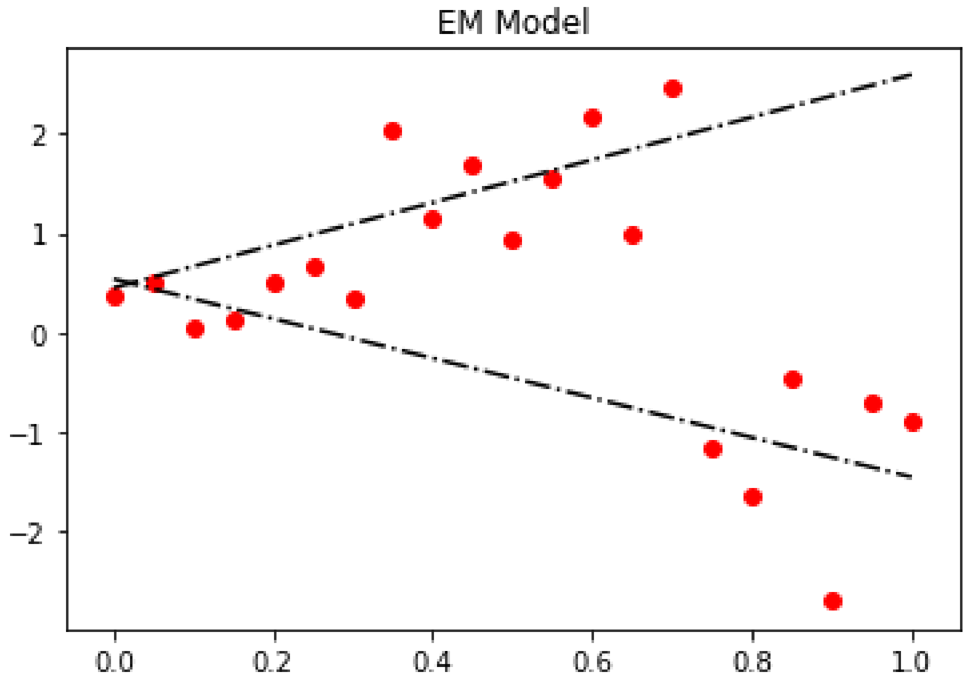
```
In [744]: # Experimenting with Noise

          # Experiment 4
          noiseLevel = 0.8

          x = np.round(np.arange(0, 1.05, 0.05), 3)
          y = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x) + noiseLevel*np.random
          NoisyDataset = list(zip(x, y))

          models, weights, pointsClass = EM(NoisyDataset, sigma=0.1, iterations=100)
```

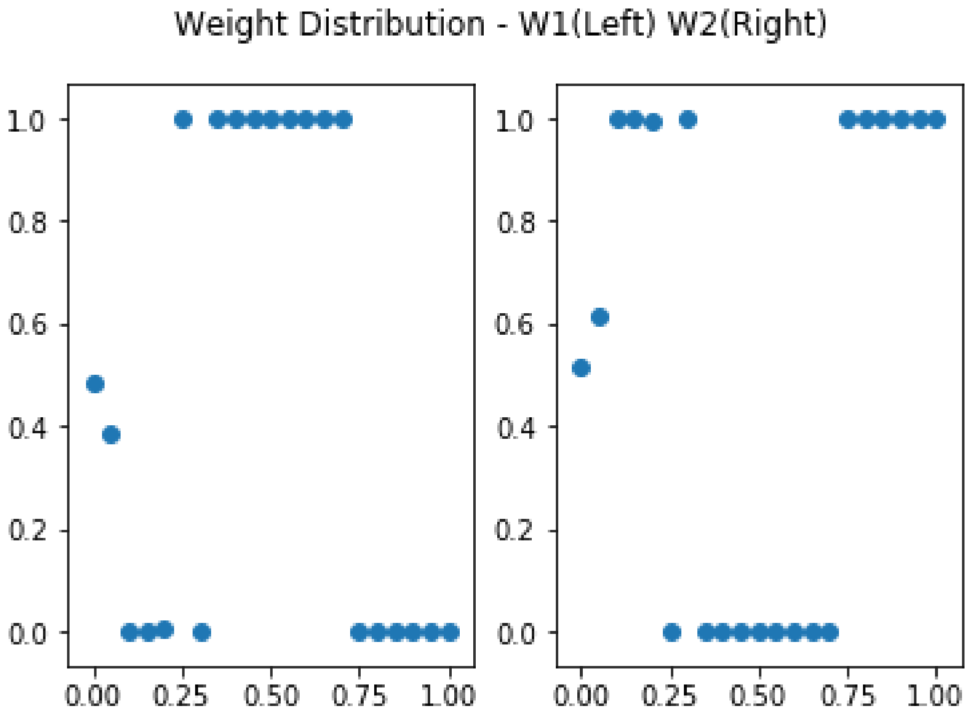

EM Model
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0

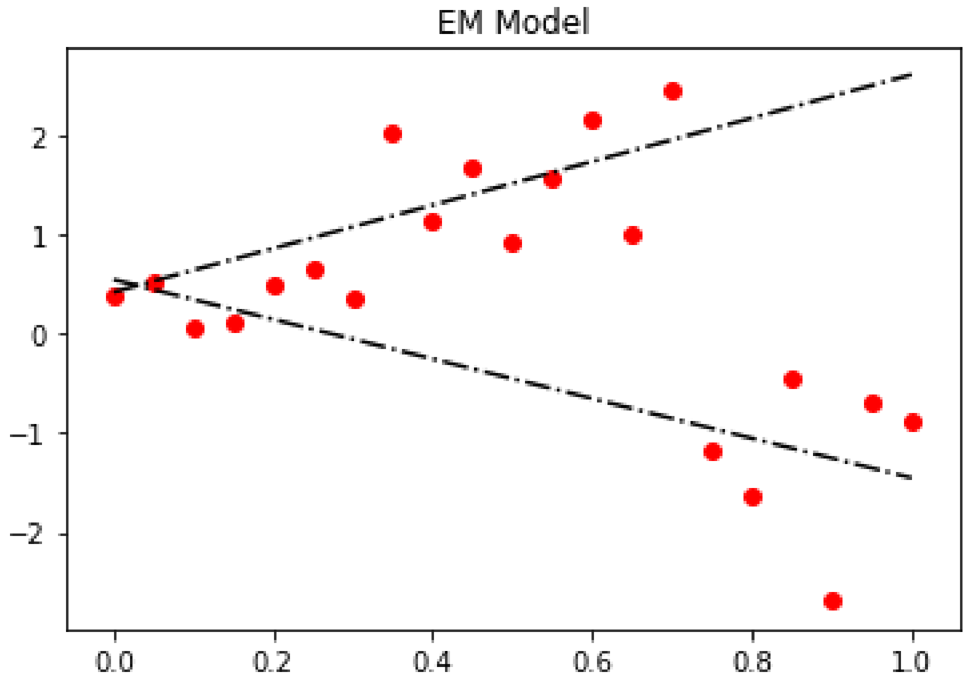

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

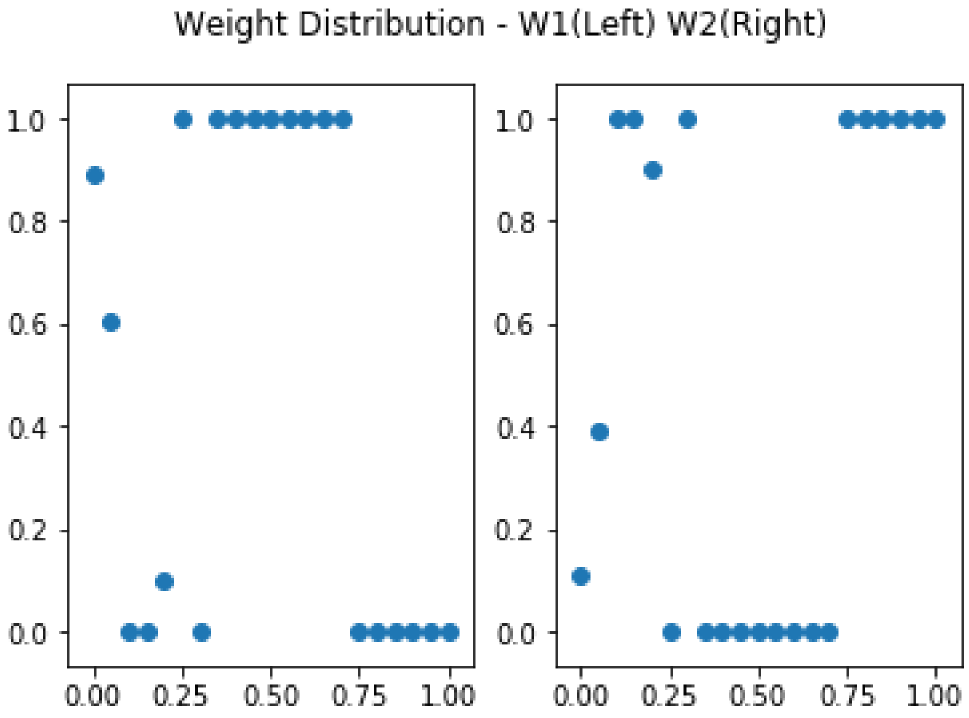

EM Model
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0

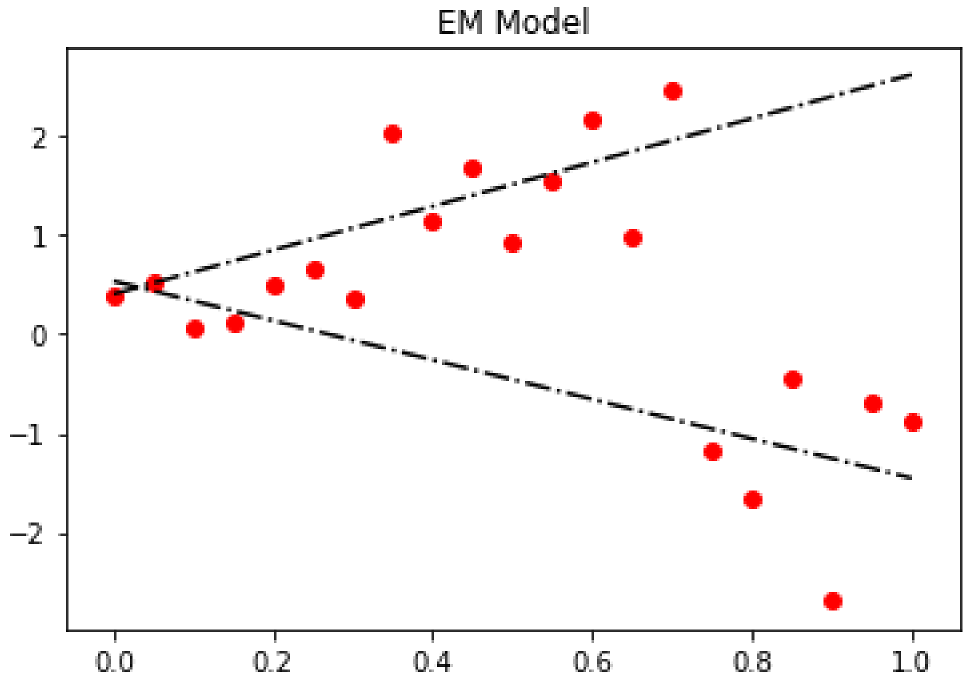

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

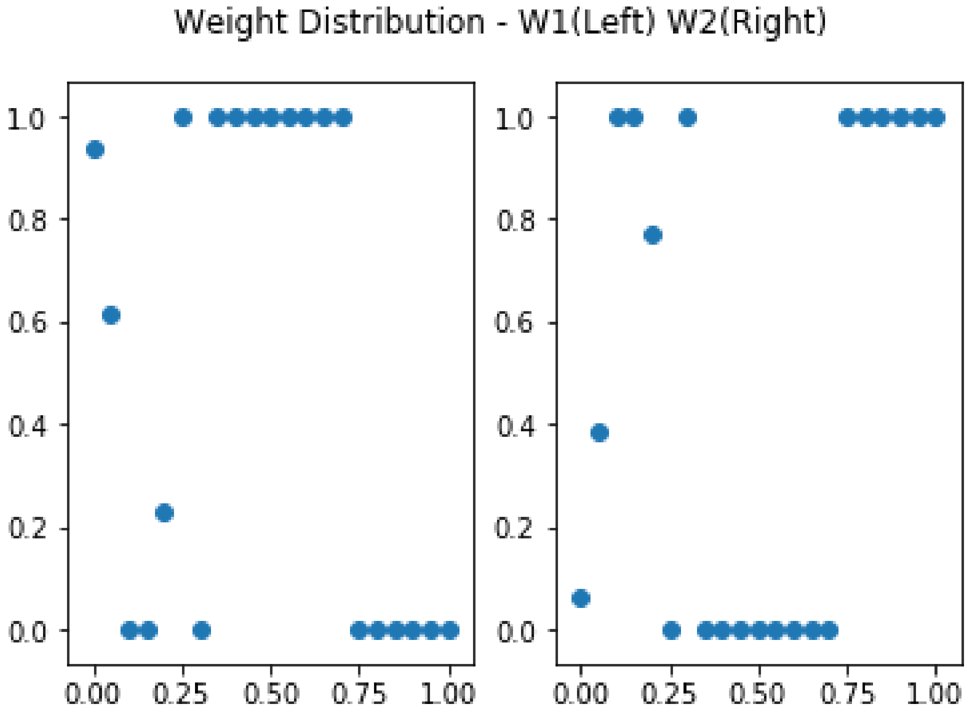

EM Model
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00



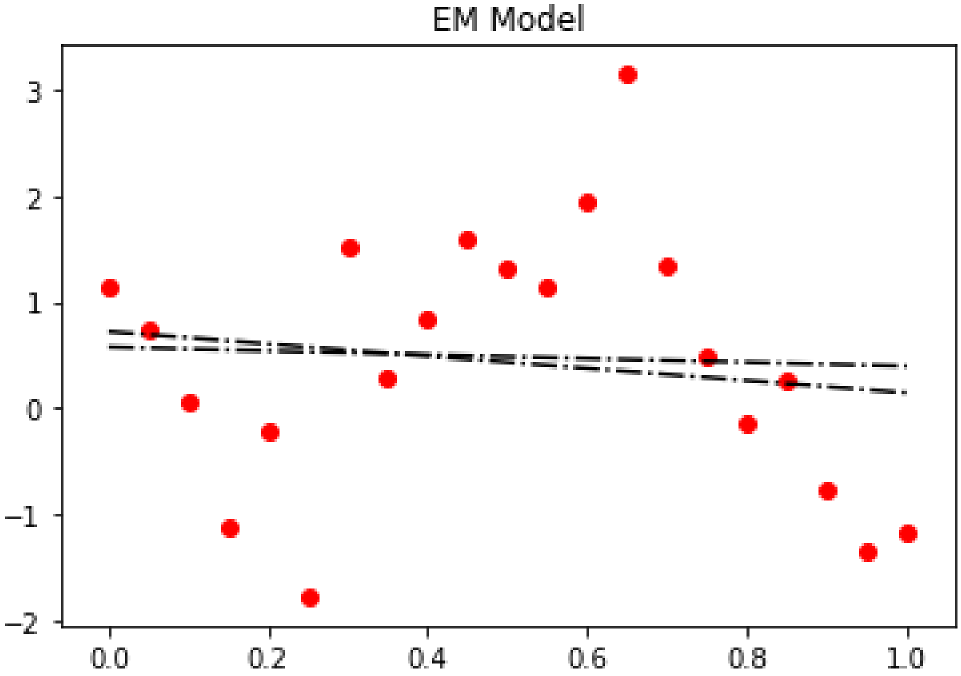


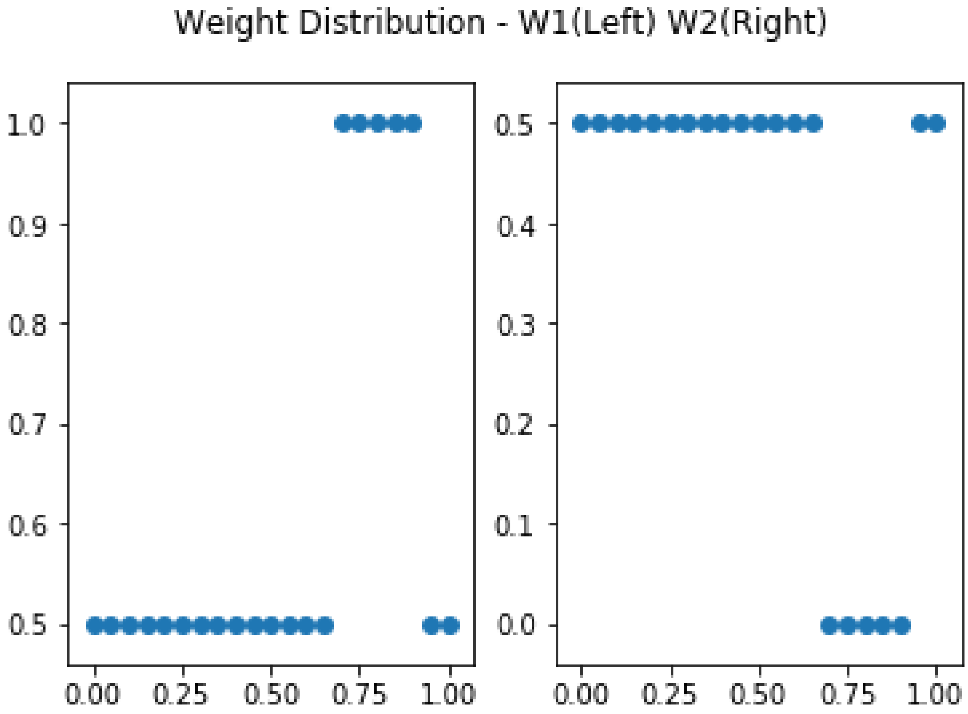
```
EM Converged after 5 iterations
```

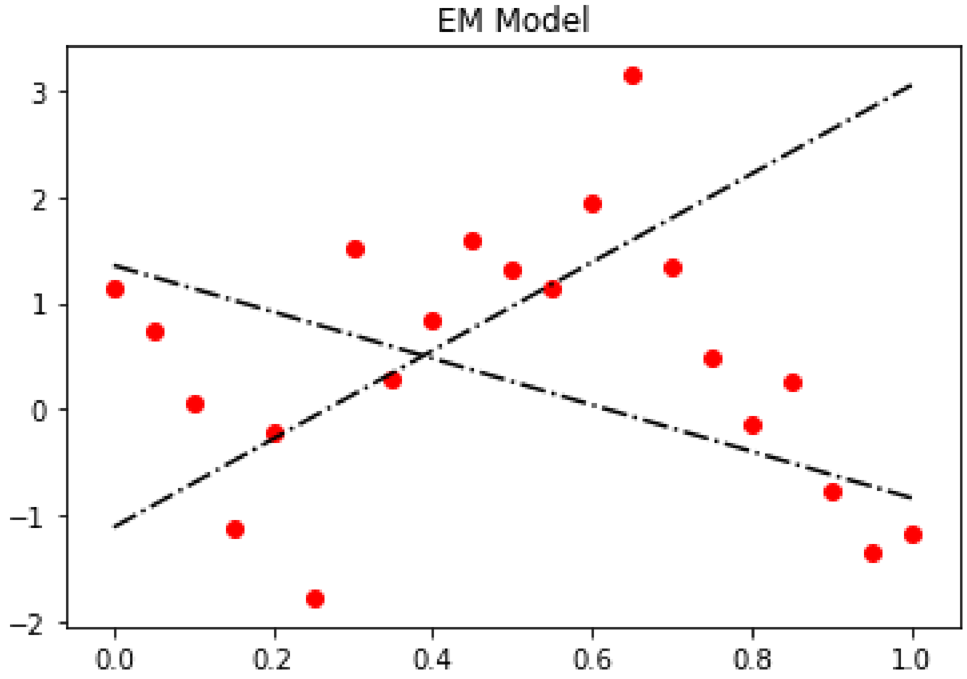
1.2.5 Experiment 5 NoiseLevel = 1.0

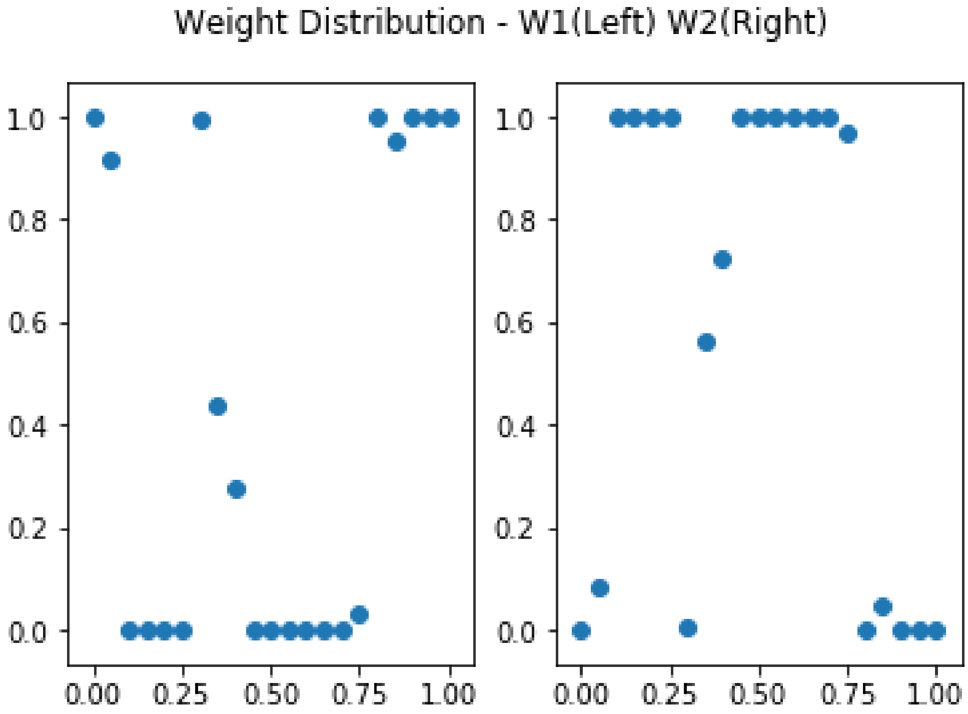
```
In [746]: # Experimenting with Noise

          # Experiment 5
          noiseLevel = 1.0

          x = np.round(np.arange(0, 1.05, 0.05), 3)
          y = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x) + noiseLevel*np.random
          NoisyDataset = list(zip(x, y))

          models, weights, pointsClass = EM(NoisyDataset, sigma=0.1, iterations=100)
          print(models)
```

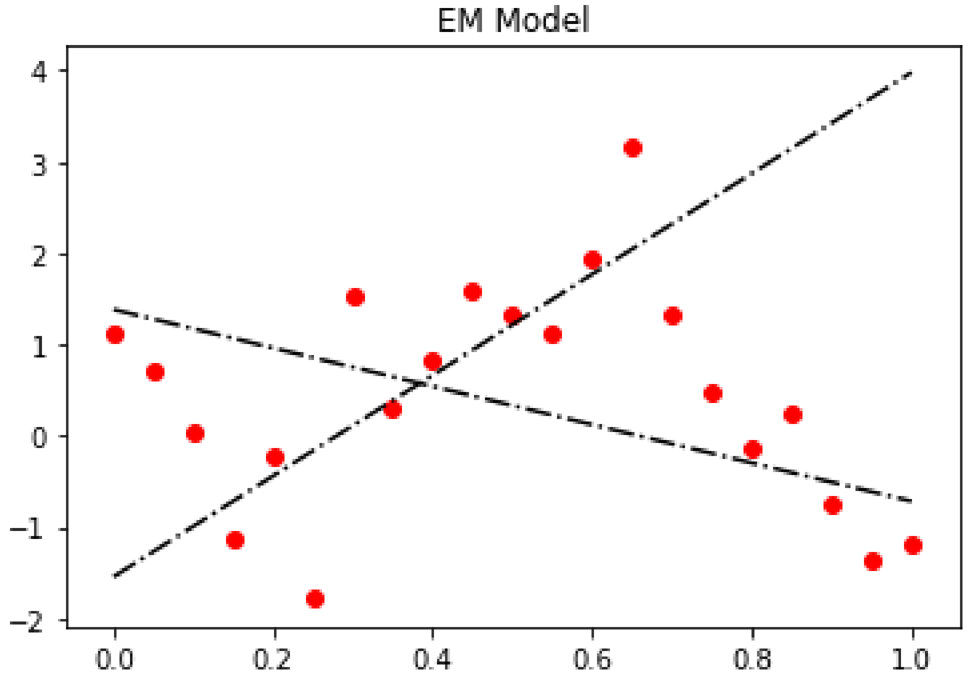
```
/home/detwieller/.local/lib/python3.5/site-packages/ipykernel_launcher.py:17: RuntimeWarning: in
```

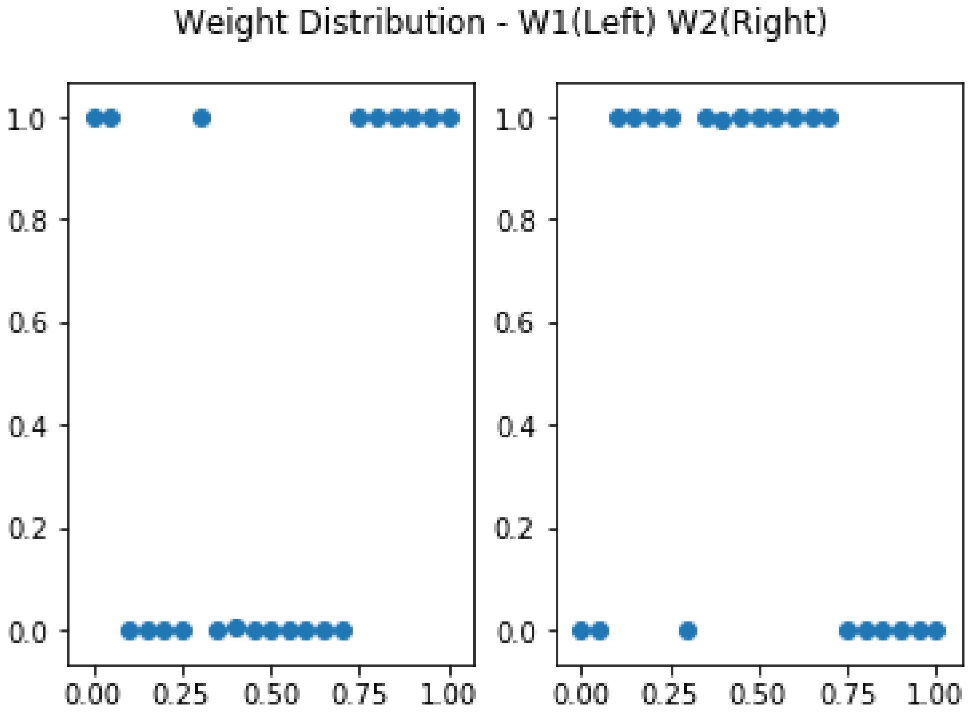


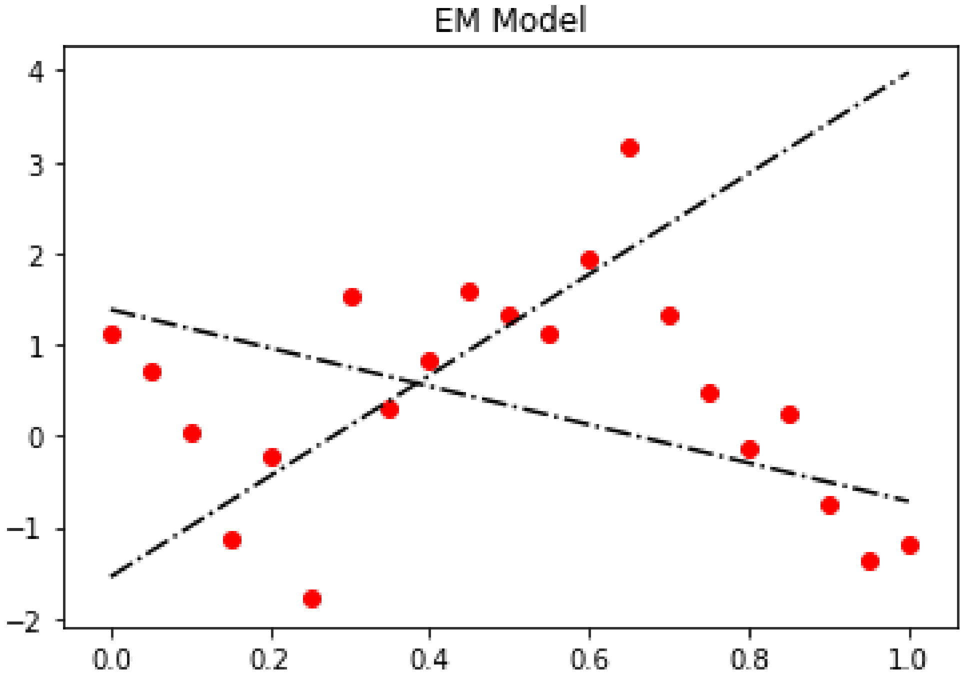

Weight Distribution - W1(Left) W2(Right)
1.0
0.9
0.8
0.7
0.6
0.5
0.00
0.25
0.50
0.75
1.00
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.25
0.50
0.75
1.00

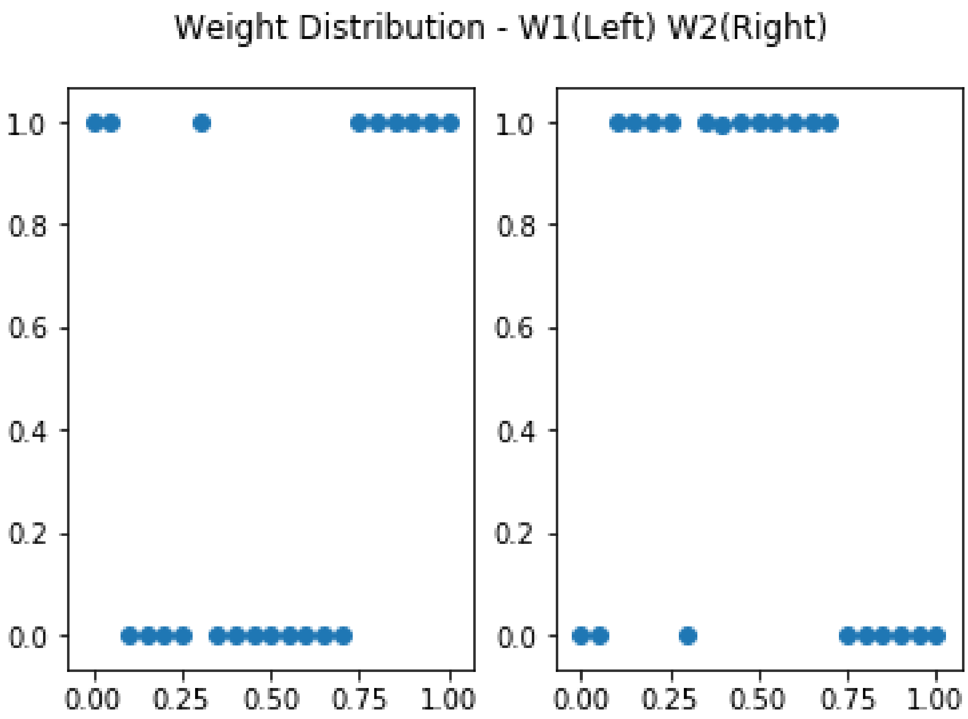

EM Model
3
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
4
3
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
4
3
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0

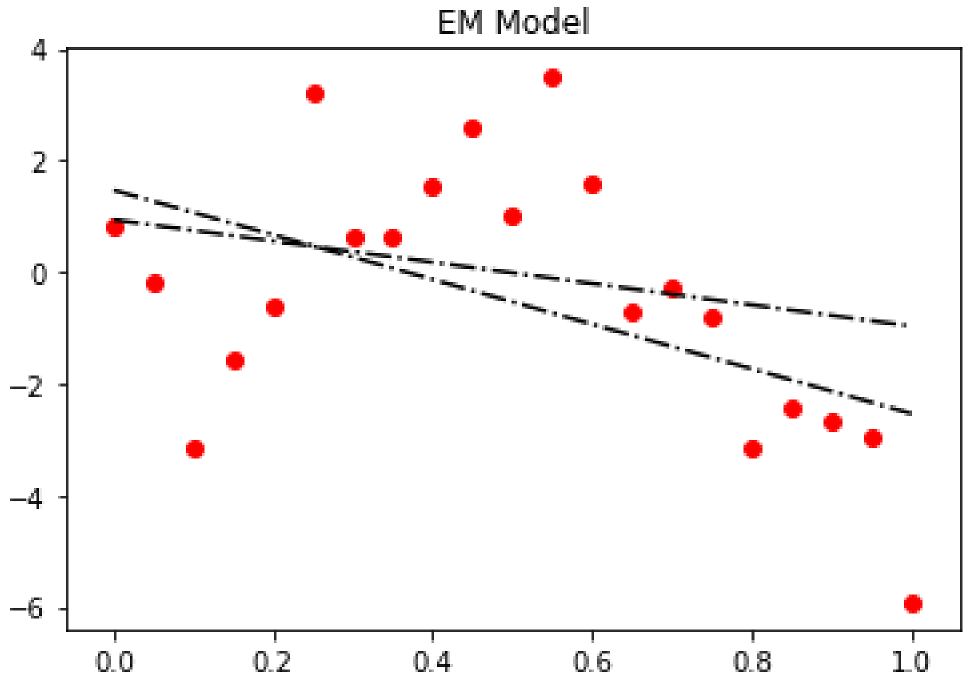


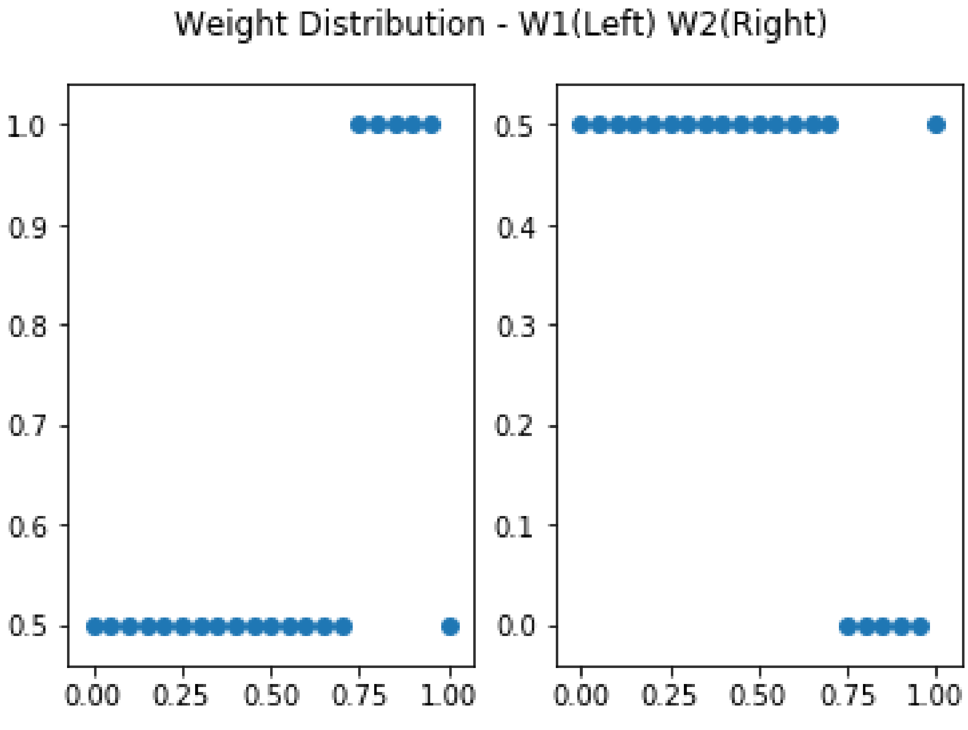
```
EM Converged after 4 iterations
[[-2.1, -1, 1.382], [5.523, -1, -1.545]]
```

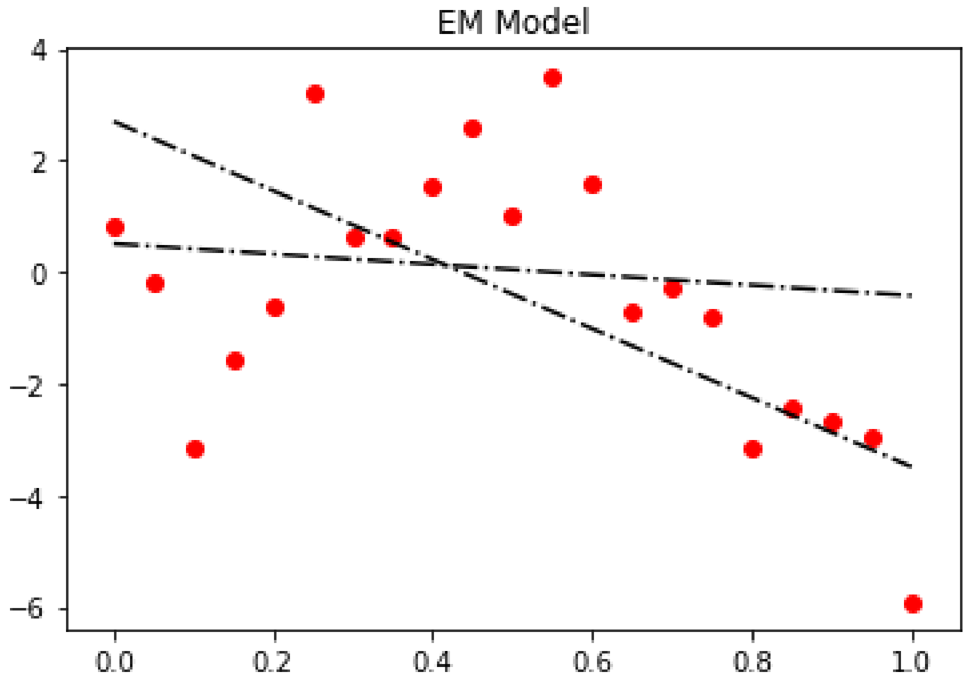
1.2.6 Experiment 6 NoiseLevel = 2

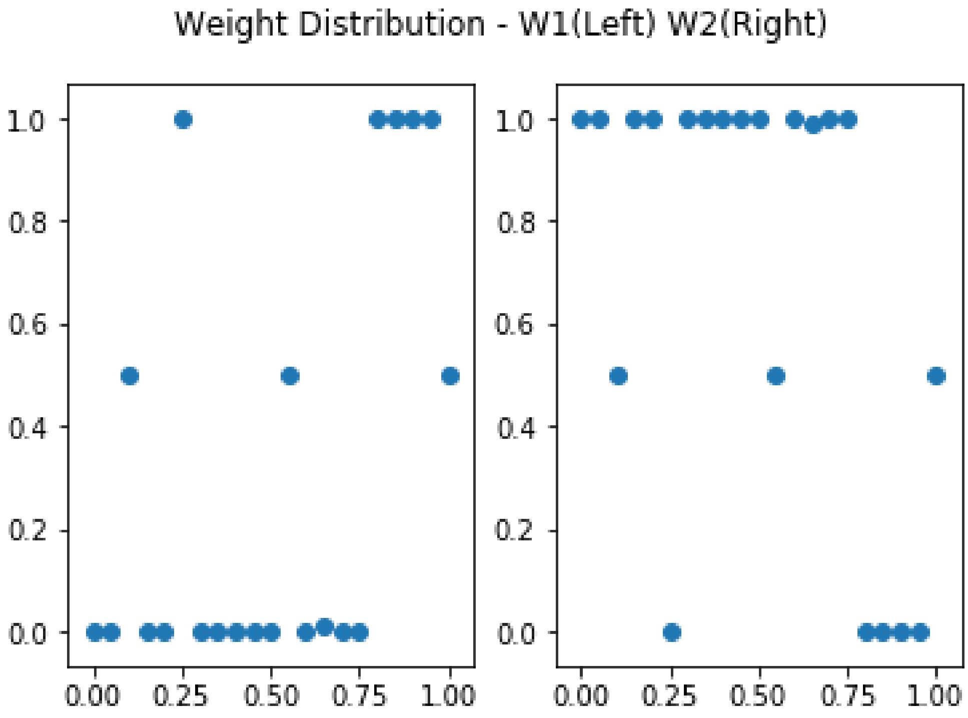
```
In [750]: # Experimenting with Noise

          # Experiment 6
          noiseLevel = 2

          x = np.round(np.arange(0, 1.05, 0.05), 3)
          y = (np.abs(x-0.5) < 0.25)*(x+1) + (np.abs(x-0.5) >= 0.25)*(-x) + noiseLevel*np.random
          NoisyDataset = list(zip(x, y))

          models, weights, pointsClass = EM(NoisyDataset, sigma=0.1, iterations=100)
          print(models)
```

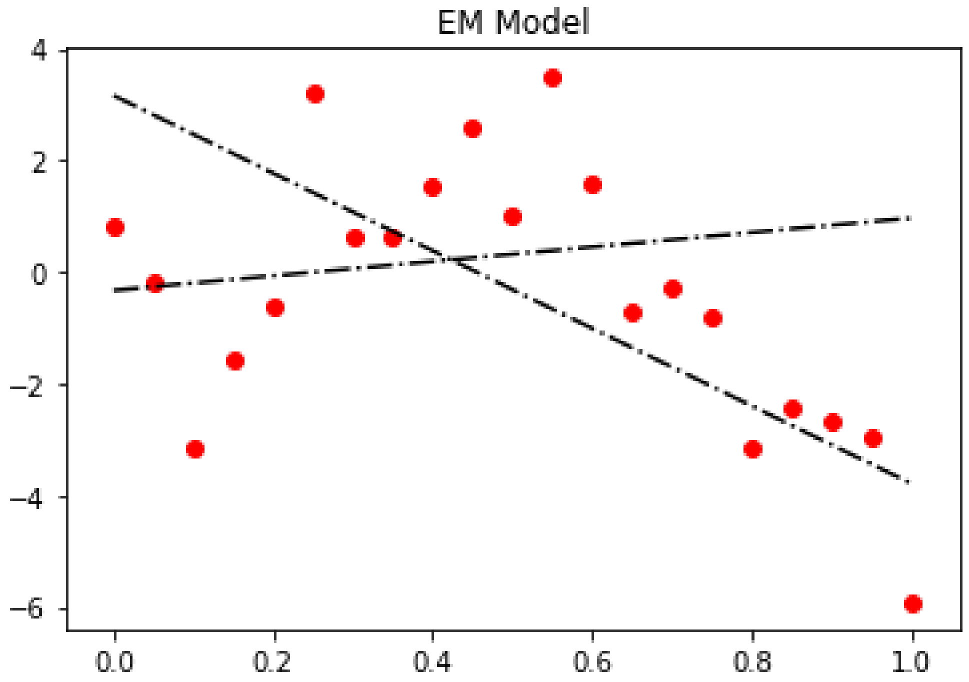
```
/home/detwieller/.local/lib/python3.5/site-packages/ipykernel_launcher.py:17: RuntimeWarning: in
```

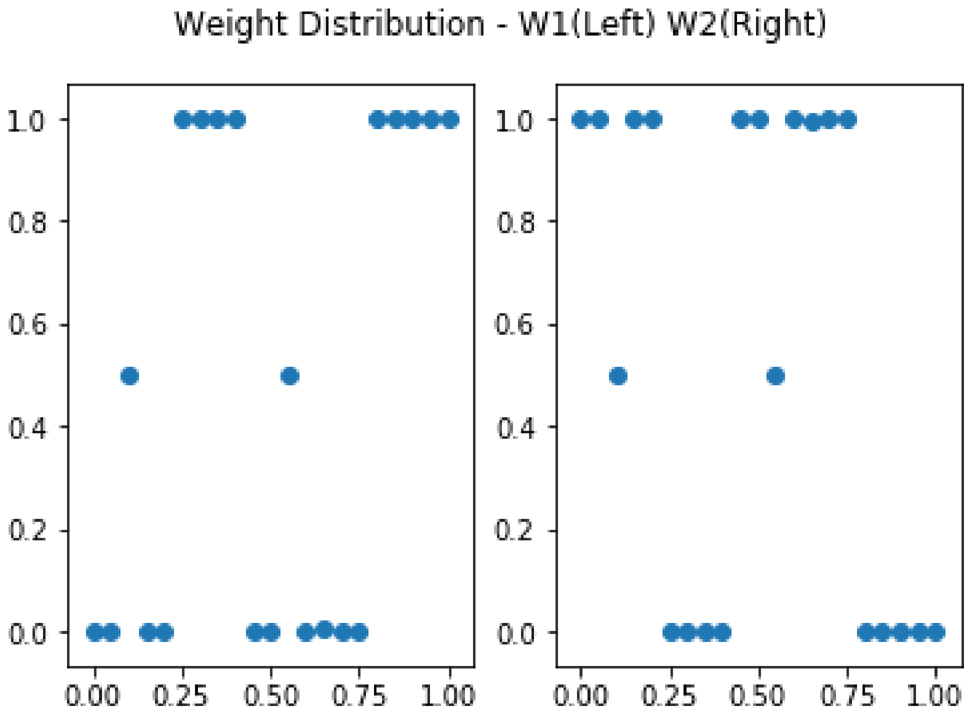

EM Model
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0

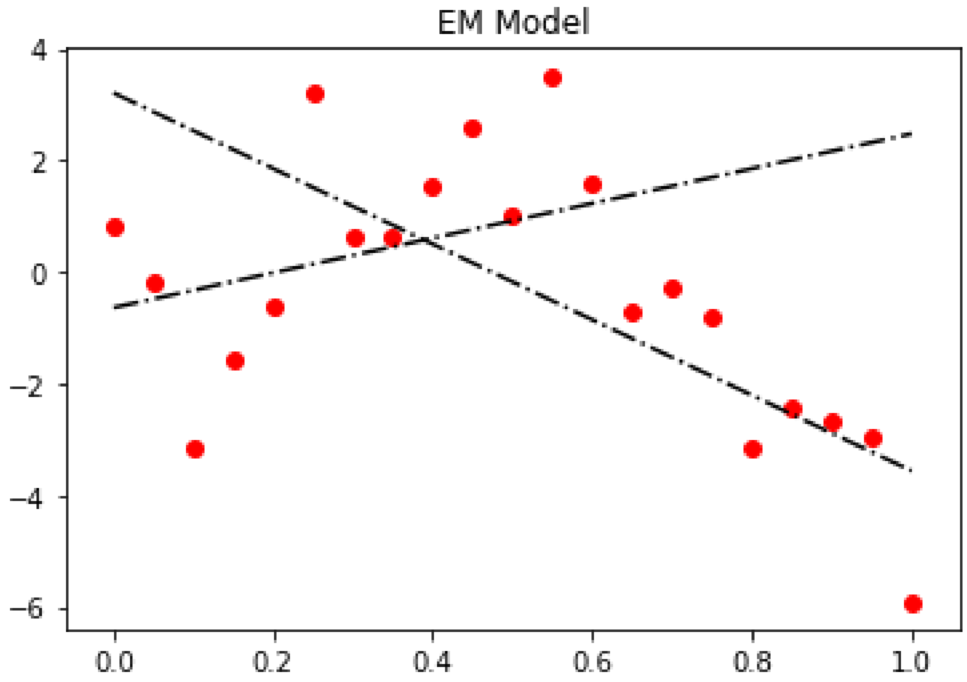

Weight Distribution - W1(Left) W2(Right)
1.0
0.9
0.8
0.7
0.6
0.5
0.00
0.25
0.50
0.75
1.00
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.25
0.50
0.75
1.00

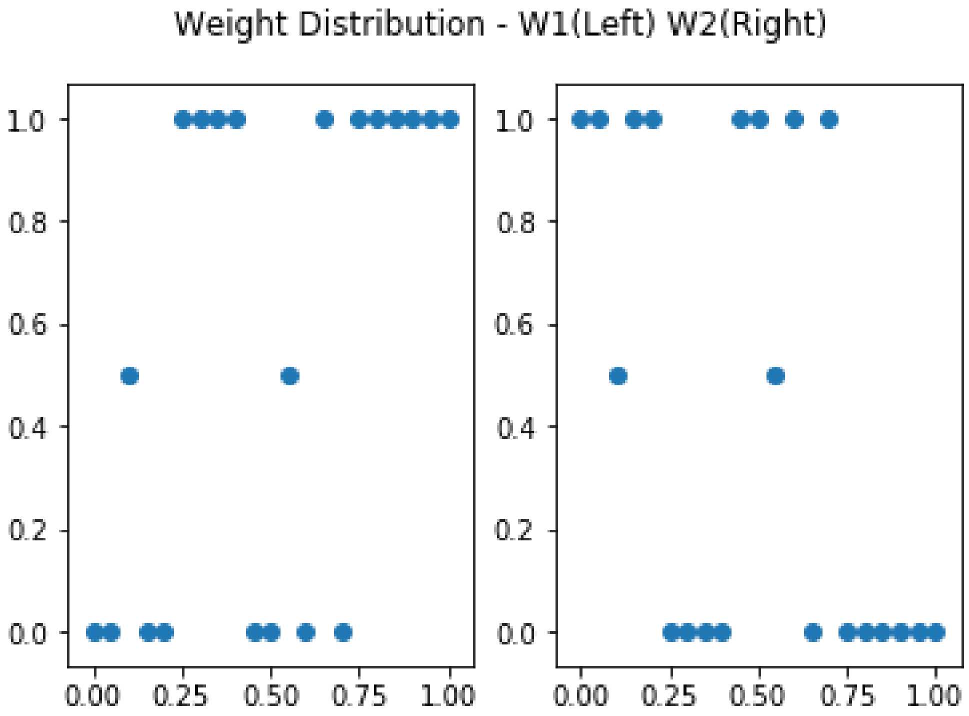

EM Model
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0

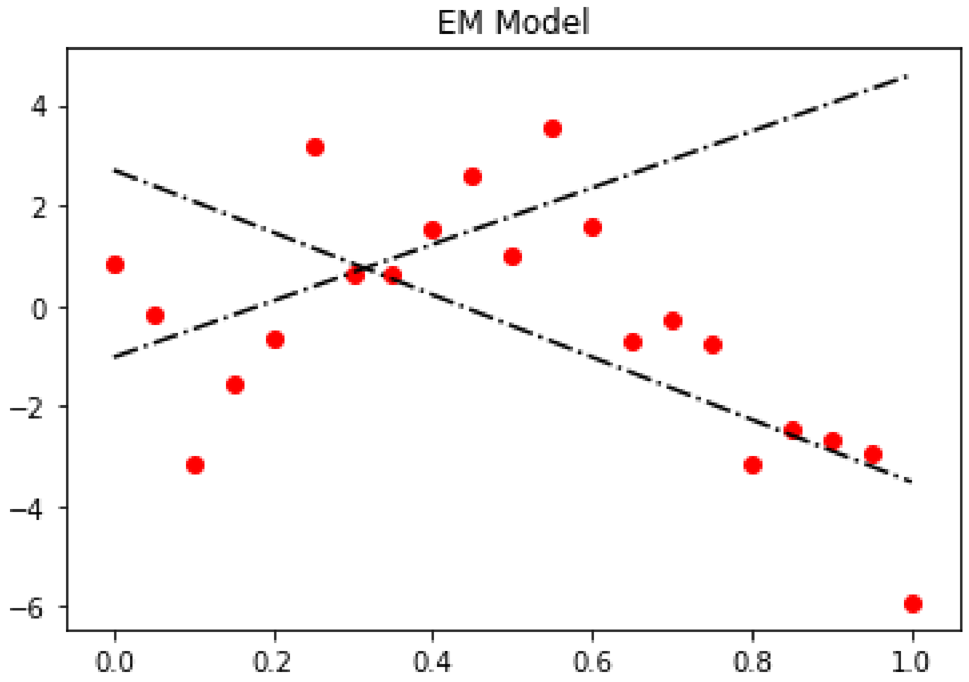

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

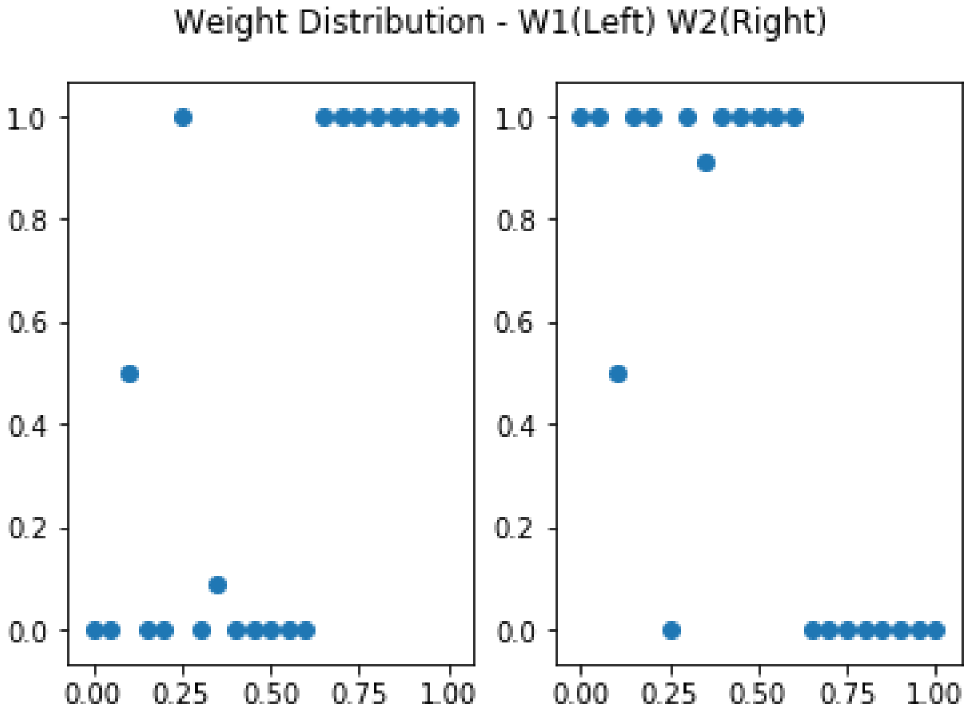

EM Model
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0

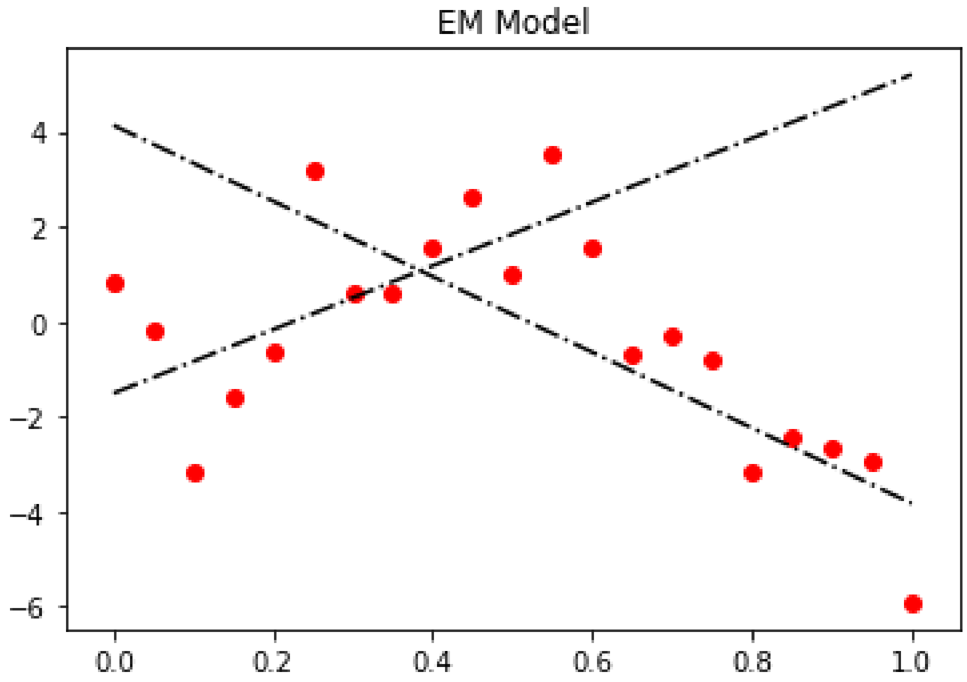

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0

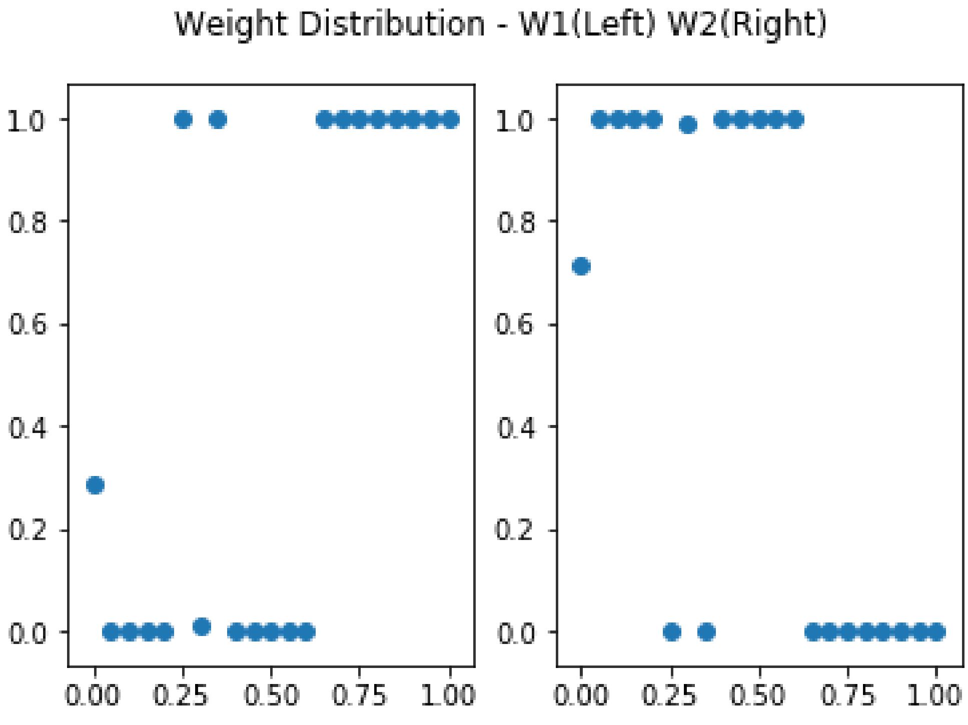

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00


EM Model
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0

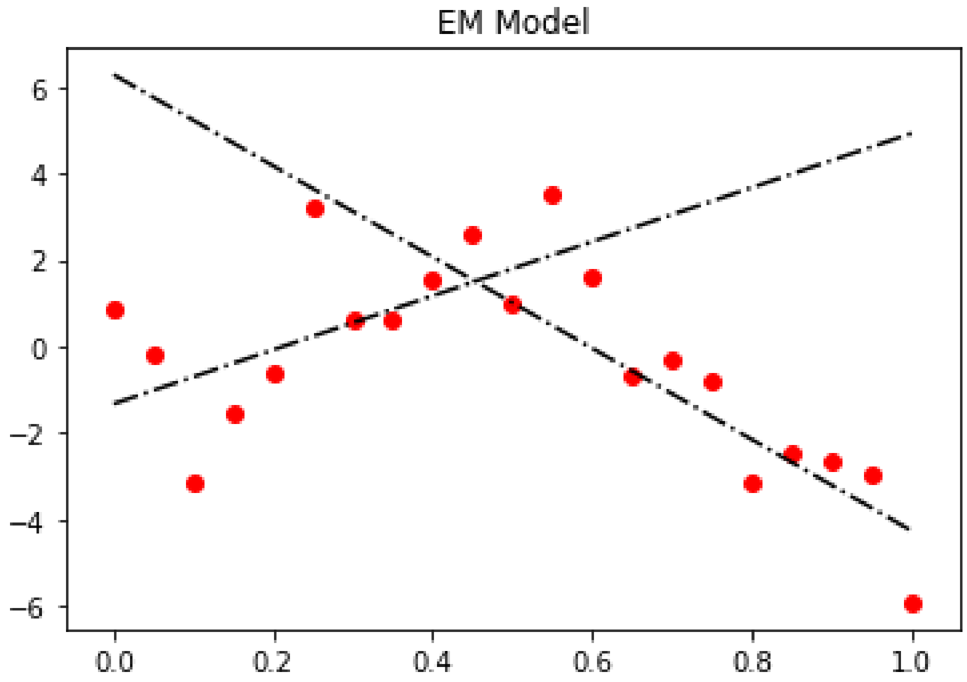

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

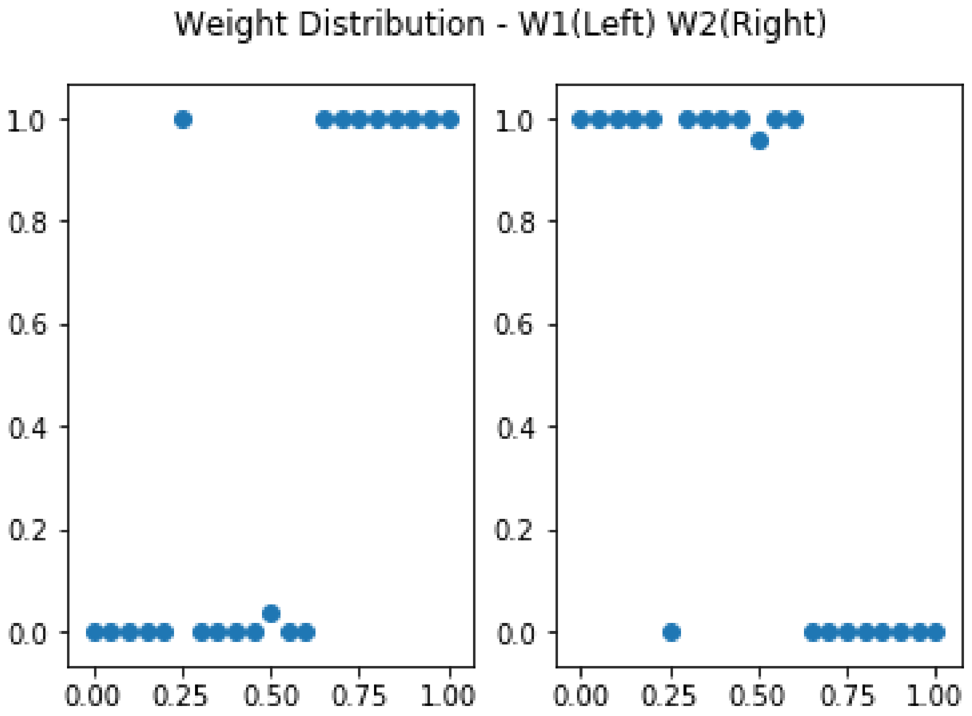


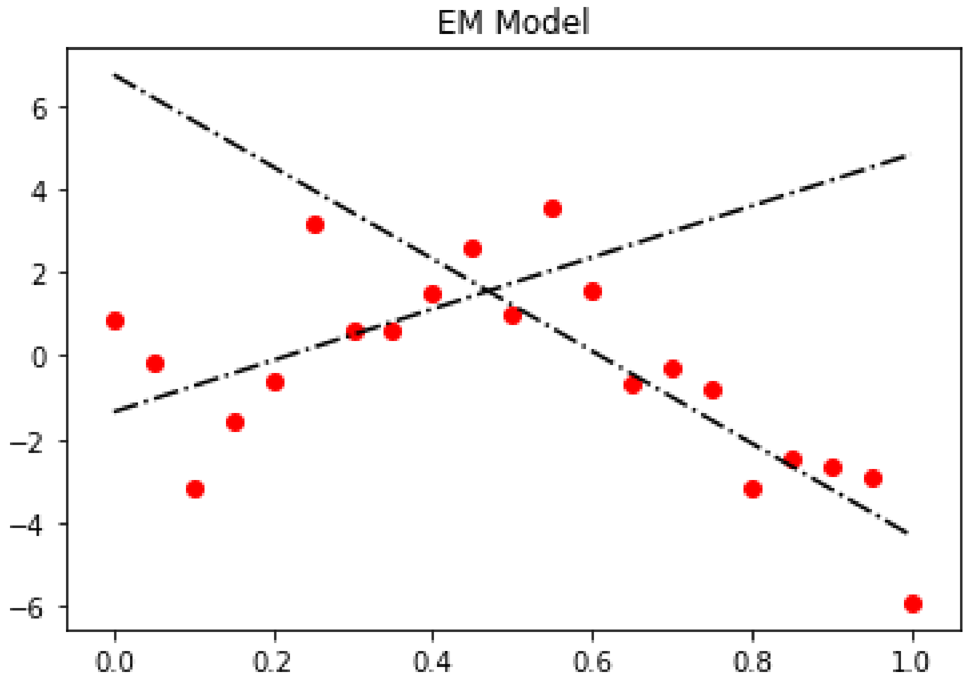
EM Model



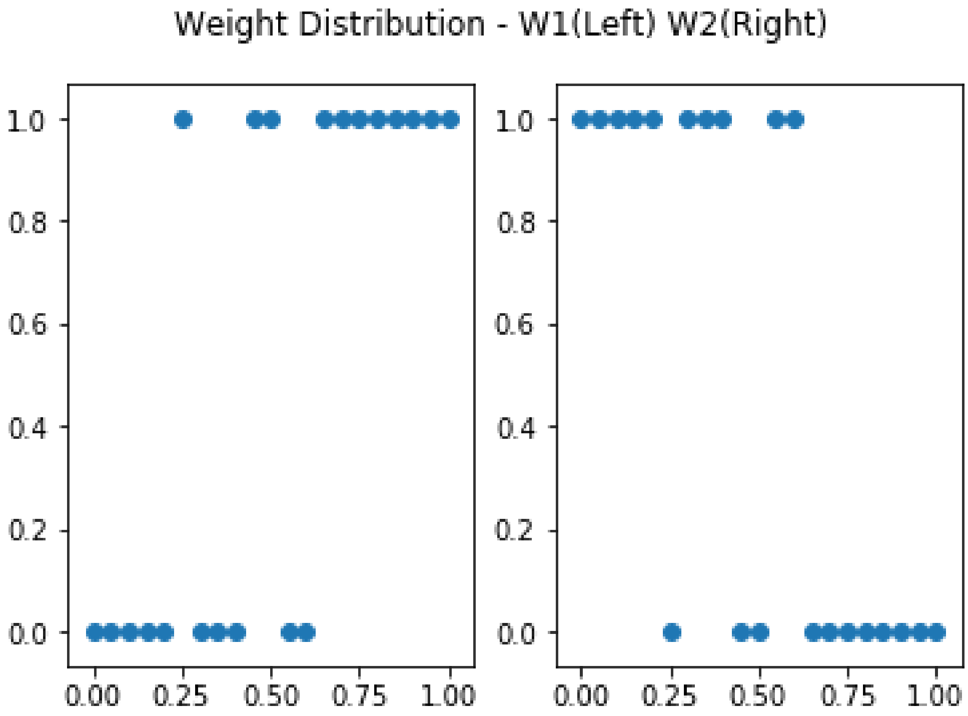
Weight Distribution - W1(Left) W2(Right)


EM Model
6
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0

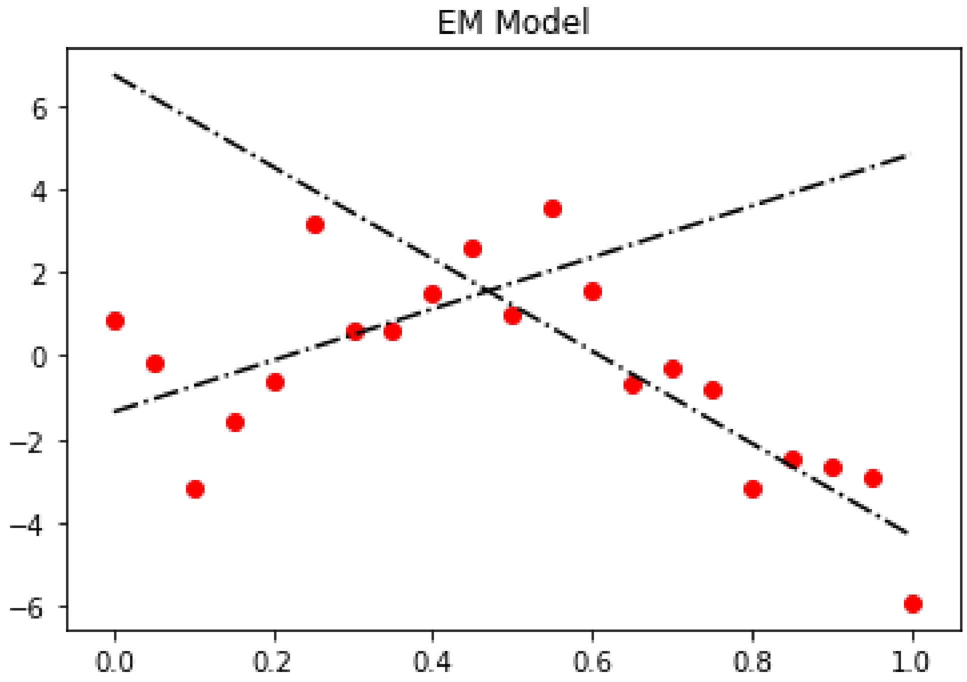

Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

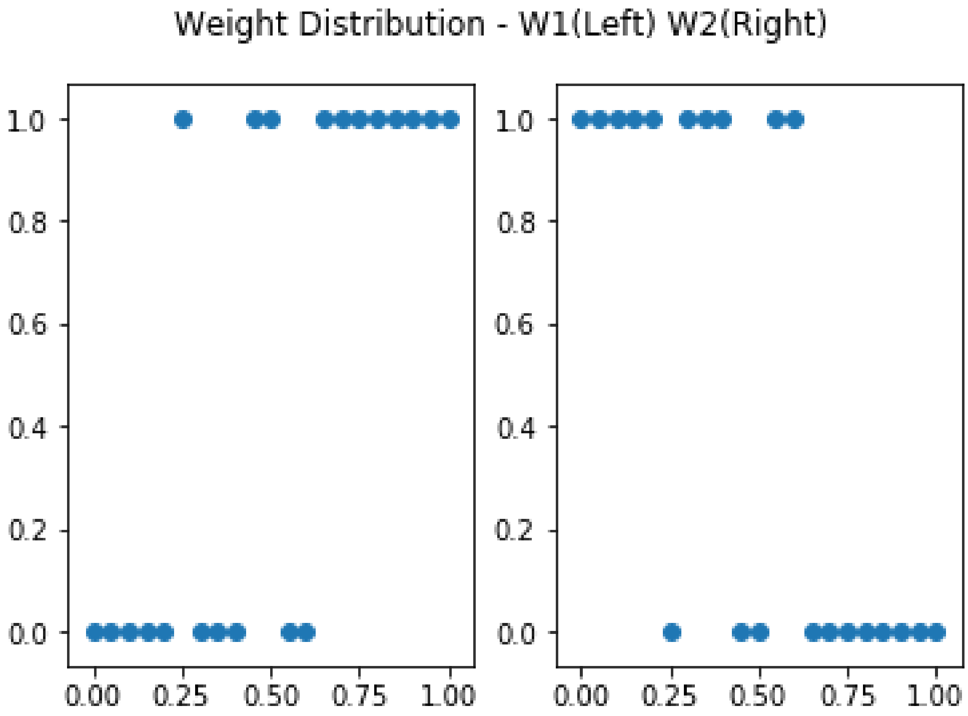


EM Model



Weight Distribution - W1(Left) W2(Right)


EM Model
6
4
2
0
−2
−4
−6
0.0
0.2
0.4
0.6
0.8
1.0


Weight Distribution - W1(Left) W2(Right)
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.25
0.50
0.75
1.00

```
EM Converged after 9 iterations
[[-11.052, -1, 6.739], [6.177, -1, -1.349]]
```

1.2.7 Result Part(b)

1) Algorithm seems to do a fairly well job even after adding very significant noise to the y-coordinates.

2) It does good job in producing similar sloped trend lines for noiseLevel of 0.5*np.random.normal(0,1,len(x)). It separates the points into two classes appropriately.(Till Experiment 2.

3) After Experiment 2, i.e. NoiseLevel>0.6 for NoiseLevel*np.random.normal(0,1,len(x)), the offset for one of the model changes considerably. That is EM starts estimating wrong model.(Till Experiment 4.)

4) After that, around NoiseLevel >= 1.0; EM breaks down. It gives models which are far from original.(Experiment 5)

5) In Experiment 6, with noise Level of 2.0, EM gives unacceptable results with Lines far far from original.

```
In [ ]:
```