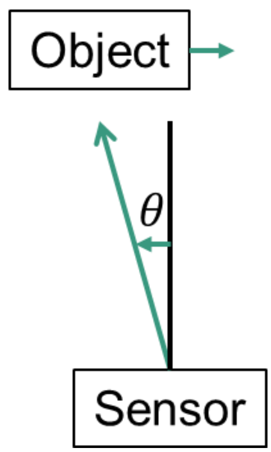
1) **Problem 2 (50 pts) - LQR:** Consider a tracking sensor which in steady state tracks an object. Let θ(t) = the angle (in radians) between the direction the sensor is pointing and the direction of the object being tracked. If the sensor loses the track (i.e., $\theta(t) \neq 0$) , in the absence of any feedback control to return the sensor to the object, the sensor will reorient itself to a given reference orientation. Assume this behavior may be modeled using:

$$\theta_{nc}(t+1) = 1.5\ \theta_{nc}(t).$$

where "nc" means "no control."

We want to build a Linear Quadradic Regulator (LQR) that will optimally return the system to tracking an object given it has lost track. Note: the control input provides the correction required to bring the sensor back into track (e.g., from a Kalman Filter).

a) (4) Draw a schematic diagram of the system.



b) (4) Write down the dynamic equation for the control system (θ(t)) that results when the control input (u(t)) is added.
   i) x(k+1) = 1.5 X(k) + u(k)
   ii) (Assume b = 1)

c) (4) Write down the Linear Quadratic Regulator (LQR) cost function that formulates the above problem where the cost of output error is equal to the cost of control, i.e., Q = R = 1.
   i) $J = X(N)^2 + \sum (X(k)^2 + u(k)^2)$
   ii) Sum is over k = 0 to N-1

d) (8) Write down the LQR equations to find the optimal control u(t).
   i) Riccati Difference (Value) Equation:
      (1) $P(k) = 1 + 1.5^2\ P(k+1) - 1.5^2\ P2\ (k+1) / (1 + P(k+1))$
         (a) Calculated backwards from P(k=N) = q
         (b) => $P(N-1) = 1 + 1.5^2 - 1.5^2 / 2$
         (c) … to P(1)

ii) Feedback Gain:

(1) L(k) = 1.5 P(k+1) / [1 + P(k+1)]

(2) => L(N-1) = 1.5 / 2

iii) Optimum Control Input:

(1) u(k) = - L(k) X(k)

e) (5) Write and submit a MATLAB program that will solve this this LQR problem where dt = 1 second that will:

i) Calculate P(t), L(t), Th(t) = θ_{nc}(t), and u(t)

ii) Plot Th(t) and u(t). Note that all graphs should be labeled.

f) (5) Run the program for Th(0) = θ(0) = 0.2 radians and t = 0-10 seconds, and provide the results below.

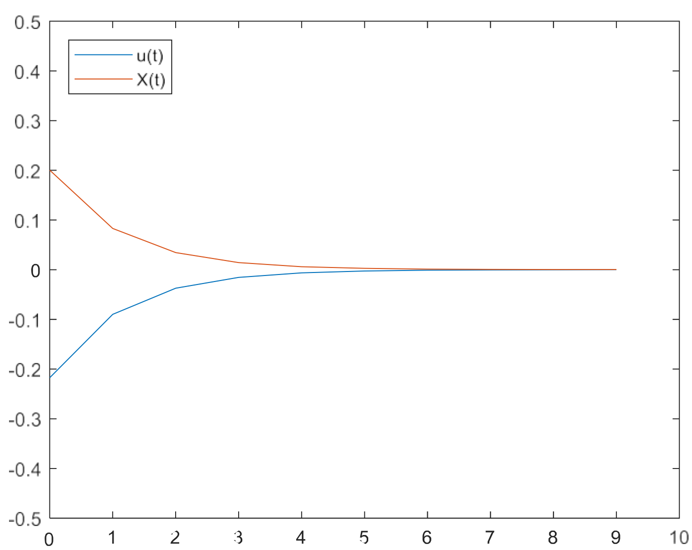
(1) Text Output:

u(N-1) = -1.5316e-04

X(N) =   1.5316e-04

>>

(2) Graphical Output:



g) (4) What do you conclude from these plots?

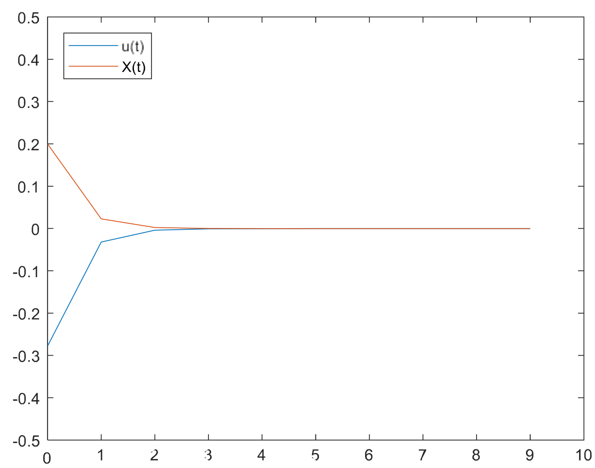
i) The system returns to steady state in ~6 seconds

h) (8) Run the program for each of the following test cases (excluding TC1) and provide the results below.

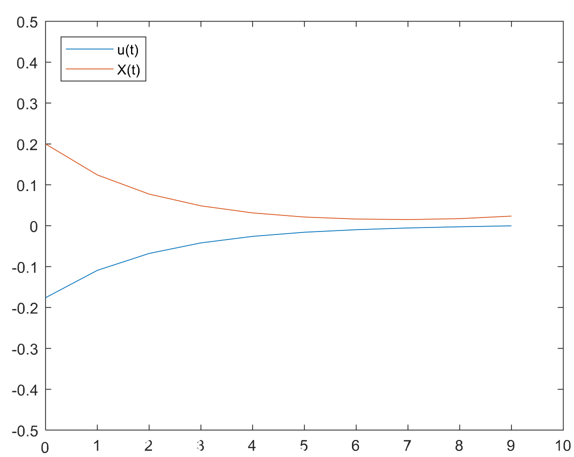
Test Case	Q	R
TC 1	1	1
TC 2	1	0.1
TC 3	1	10
TC 4	0.1	1
TC 5	10	1
TC 6	0.1	0.1
TC 7	10	10

i) TC1 is provided in f) above.

ii) TC2:



iii) TC3:



iv) TC4: Same as TC3

v) TC5: Same as TC2

vi) TC6 & TC 7: Same as TC1

i) (8) Explain the significance of each test case and behavior that results.

   i) TC2 (& TC5) => If the cost of a deviation from steady state exceeds the cost of control, the system returns more quickly to steady state (and larger control is applied).
   ii) TC3 (& TC4) => If the cost of a deviation from steady state is less than the cost of control, the system returns more slowly to steady state (and less control is applied).
   iii) It is the ratio of R to Q that matters (not the specific values).